Towards precision and accuracy in Galaxy Clusters simulations

Tiago Castro

In collaboration with: Alessandra Fumagalli, Raul Angulo, Sebastian Bocquet, Stefano Borgani, Klaus Dolag, Alex Saro, and others.

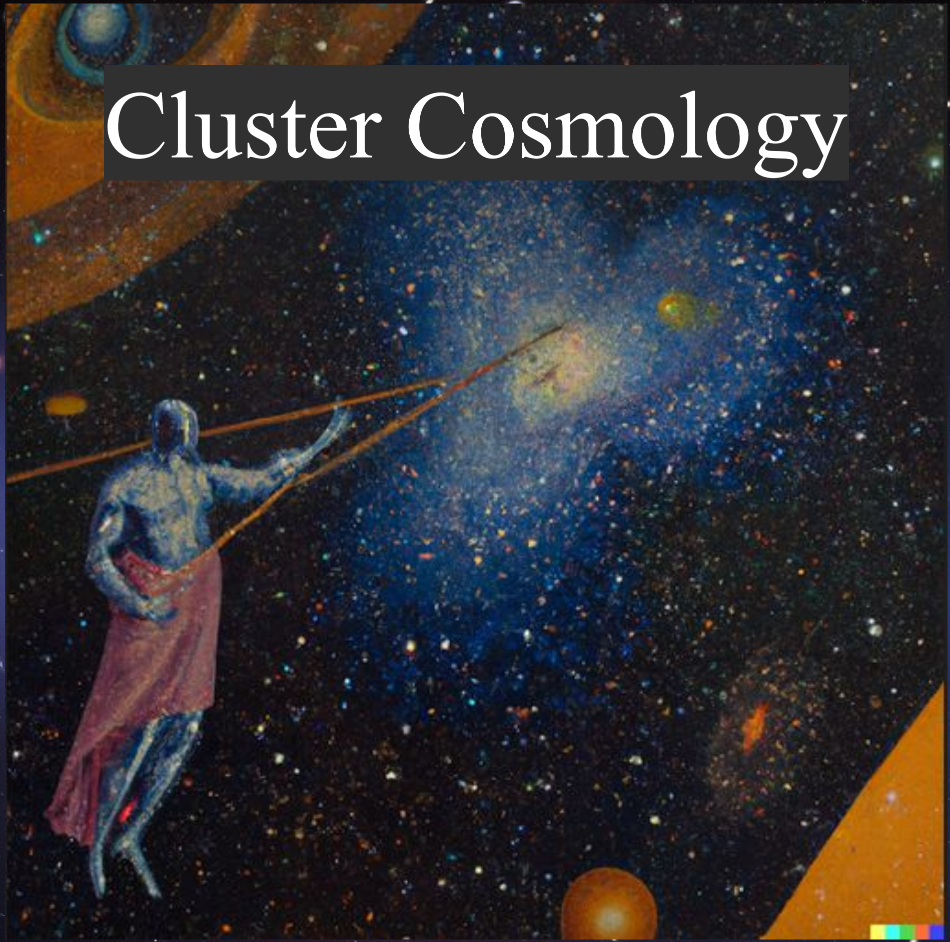
Summary

1. Cluster Cosmology:
    a. Cluster Counts;
    b. Cluster Clustering.
2. The HMF;
    a. Numerical/Theoretical systematics;
    b. Non-universality modelling;
    c. Effect of Baryons.
3. Theoretical requirements and challenges for future surveys;
4. Conclusion.

Euclid Collaboration: Castro+ 2022 (2208.02171)

Castro+ 2020 (2009.01775)

Cluster Cosmology



Cluster Counts

The number of clusters expected in a survey with sky coverage Ωsky within the i-th redshift bin Δzi and the j-th mass bin ΔMj is:

$$N_{i,j} = \frac{\Omega_{\mathrm{Sky}}}{8\pi} \int_{\Delta z_i} \int_{\Delta M_j} n(M, z) \times \mathrm{d}M \,,$$

where, n(M,z) is the differential halo mass function (HMF).

Cluster Clustering

The Cluster Clustering can be described (in linear scales) by its power-spectrum:

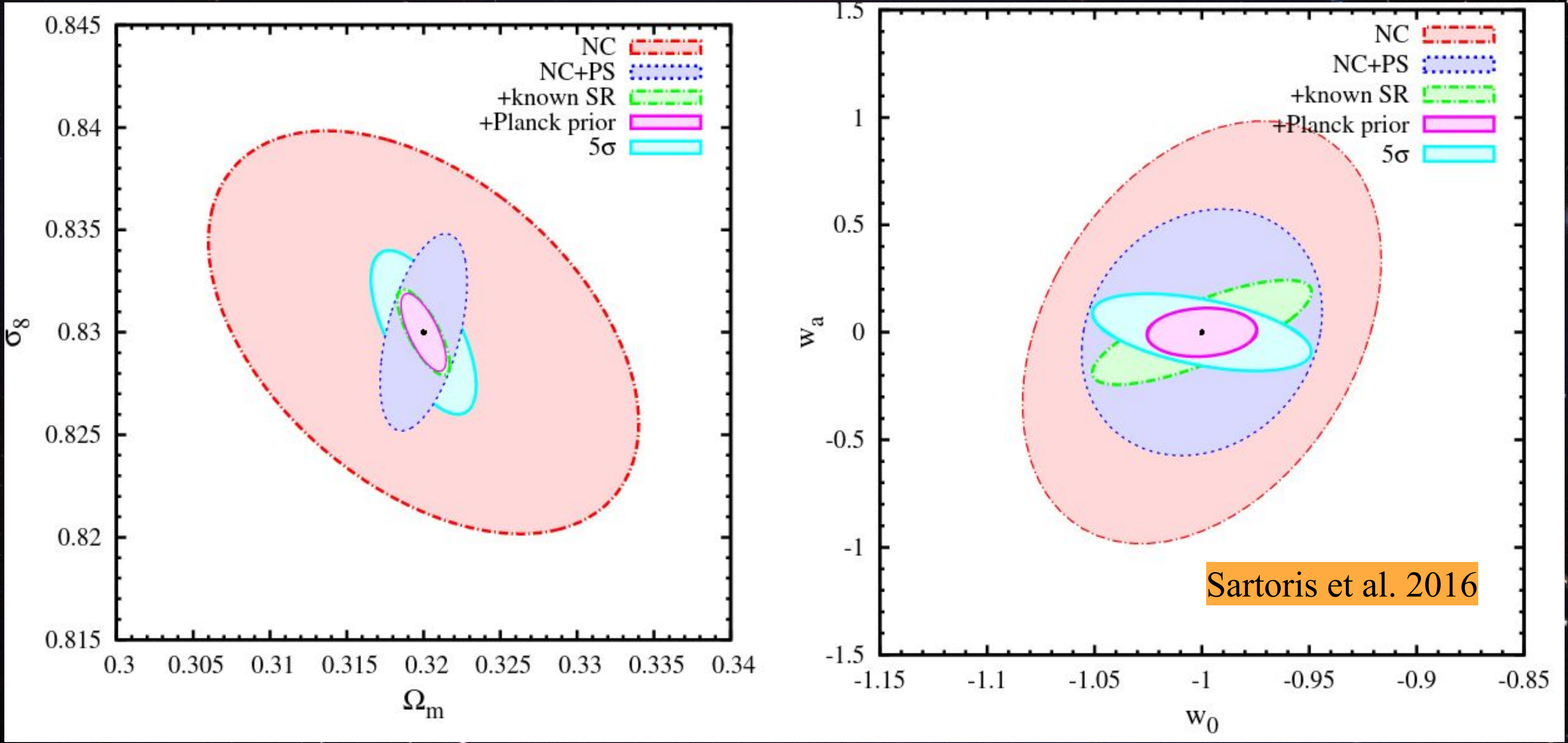
$$P_{\mathrm{Cl.}}(k, z) = b_{\mathrm{eff}}^2 \times P_{\mathrm{m}}(k, z)) \, ,$$

where:

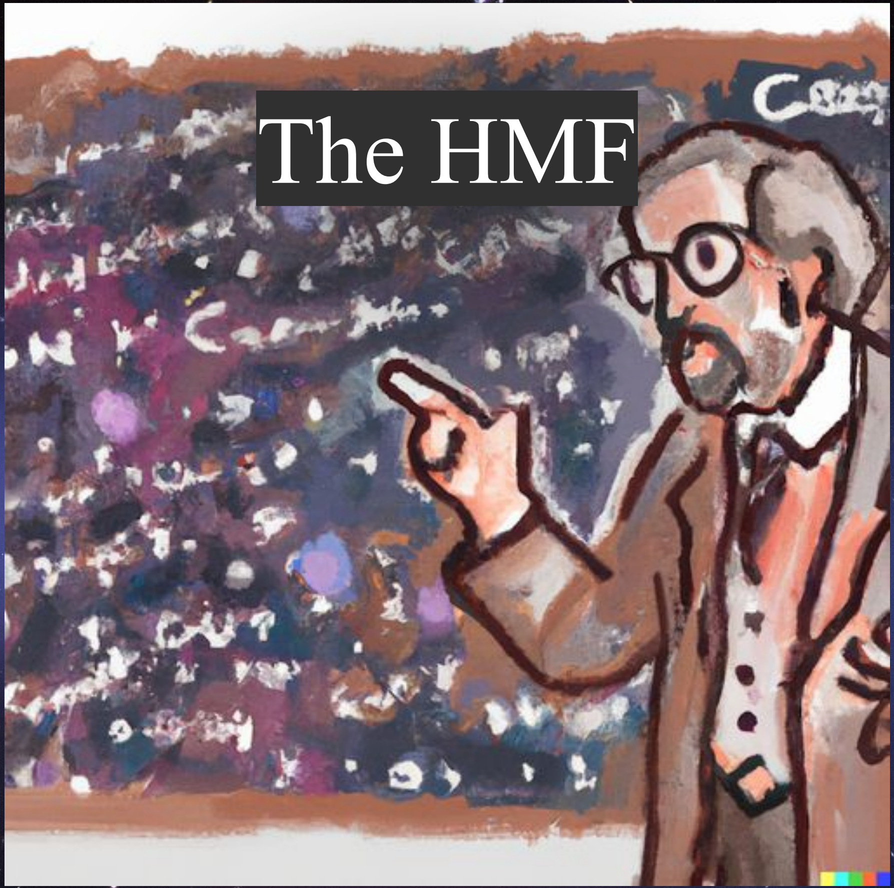
$$b_{\mathrm{eff}}(z) = \frac{1}{n(z)} \times \int_M n(M, z) \, b(M, z) \, \mathrm{d}M \, ,$$

measures the tracing bias of the cluster sample.

Cluster Cosmology (CC + CPS) capabilities:



The HMF



The HMF

The differential halo mass function gives the number density of halos of mass M at redshift z:

- Press–Schechter formalism:
    - Collapsed objects were in the past over densities above a threshold δc (Spherical Collapse prediction);
    - The abundance of Halos of mass M is proportional to the probability of fluctuations higher than δc on the Gaussian smoothed linear density field;
    - PS mass function offers a qualitative explanation to the observed halo abundances;
    - However, for a precise estimation, more complex mass functions (calibrated with N-body simulations) is required.

The HMF: calibration?

The formation of structures in the Universe is deep-seated in the non-linear regime. Assessing this regime is only possible with expensive cosmological simulations:

$$\frac{\mathrm{d}\, n(M, z)}{\mathrm{d}\, M}\,\mathrm{d}\, M = \frac{\rho_{\mathrm{m}}}{M}\, f(\nu)\,\mathrm{d}\,\nu\,,$$

$$f_{\mathrm{PS}}(\nu) = \sqrt{\frac{2}{\pi}}\nu \mathrm{e}^{\frac{-\nu^2}{2}}\,,$$

$$f_{\mathrm{ST}}(\nu) = A\sqrt{\frac{2a}{\pi}}\nu \mathrm{e}^{(-a\nu^2/2)}(1 + (a\nu^2)^{-p})\,.$$

$$f_{\mathrm{O-M}}(\nu) = f_1(\nu) \times f_2(n_{\mathrm{eff}}) \times f_3(a_{\mathrm{eff}})\,,$$

The HMF: emulators

- Emulators are a solution to circumvent the complex analytical modeling to predict the HMF (McClintock et al. 2019; Bocquet et al.2020);
- Although practical and straightforward, this strategy has limitations as emulators are not rigorously supported by a robust underlying model and are known to perform poorly outside the regime it has been built.
- Furthermore, building an emulator per se does not lead to a better understanding of nature, lacking a theoretical legacy. Still, it is essential to note that emulators had pushed the precision and accuracy of the HMF predictions to few percent, outperforming fitting functions that ranges in accuracy between 10−30 per cent.

The HMF: Numerical/Theoretical systematics

- Despite the choice for describing the HMF, cosmological simulations are vital for both approaches;
- Assessing the robustness of the simulations predictions is not an easy task:
    - For instance, tweaking code parameters and searching for convergence can result in inaccurate result.

Low accuracy and low precision

Low accuracy and high precision

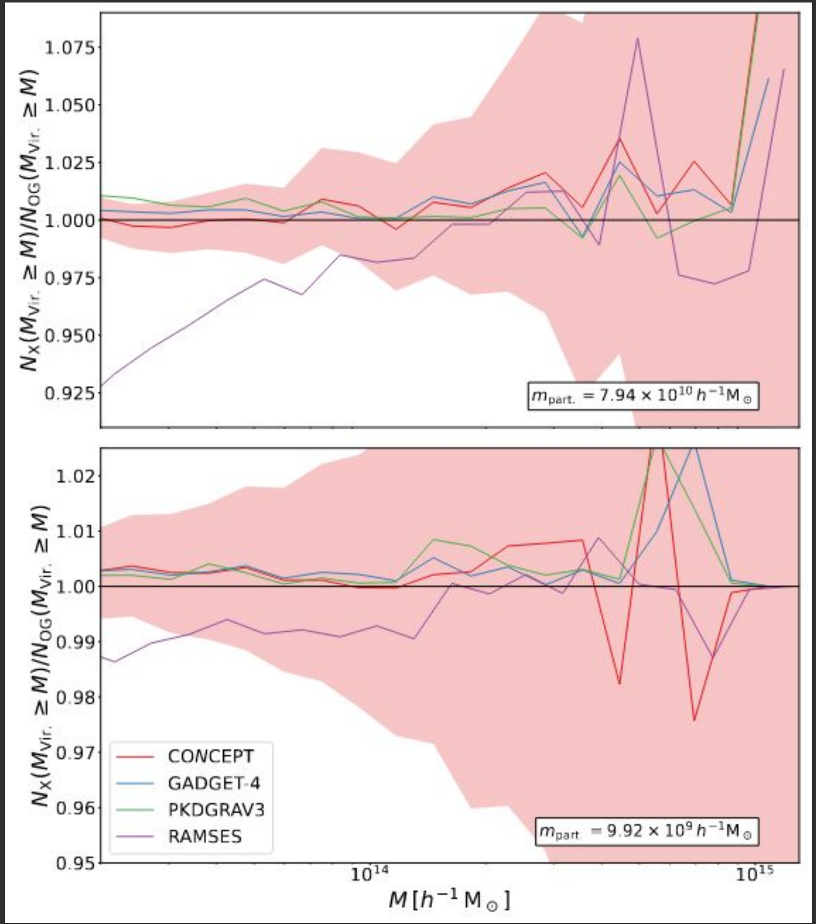
The HMF: Numerical/Theoretical systematics

- Designing an accurate and precise set of simulations for Cluster Cosmology:

Set	$L_{\rm box}$ (h^{-1} Mpc)	$N_{\rm p}$	Background	$P_{\rm lin.}(k)$	Initial Conditions			Grav. Solver
					Code	LPT Order	z	
TEASE	500	256^3 512^3 1024^3 4×160^3 4×320^3 4×640^3 4×1280^3	$C0$	ΛCDM	MUSIC	Zel.	99	Tree-PM, FMM-PM, FMM, P^3M, AMR†
					monofonIC	3LPT	24	Tree-PM, FMM-PM, FMM, P^3M
AETIOLOGY	1000	1024^3	EdS	Power-law ΛCDM ($C0$)	GADGET-4	2LPT	99	FMM-PM
			$C0$	Power-law ΛCDM				
PICCOLO	2000	4×1280^3	$C0-C8$	ΛCDM	monofonIC	3LPT	24	Tree-PM

The HMF: Numerical/Theoretical systematics

- Assessing the results robustness through code comparison:



The HMF: Numerical/Theoretical systematics

- Sensitivity of the HMF on initial conditions:
  - Due to the break of the commutative property due to limited precision, the HMF is sensitive to small perturbations on the initial conditions.

```
import numpy as np

# Random sample size
NMAX = 10000000
# Creating an array with random variables
# following a normal distribution
arr = np.random.randn(NMAX)
# Permutantion indexes
idxs = np.random.permutation(NMAX)
# Comparing the mean of the original array
# and the permuted one
print(arr.mean()/arr[idxs].mean())
```

```
(base) [tcastro@tiago-pc ~] $ python sum.py
1.0000000000000009
(base) [tcastro@tiago-pc ~] $ python sum.py
0.9999999999999982
(base) [tcastro@tiago-pc ~] $ python sum.py
0.9999999999999922
(base) [tcastro@tiago-pc ~] $ python sum.py
1.0000000000000002
(base) [tcastro@tiago-pc ~] $ python sum.py
0.9999999999999896
(base) [tcastro@tiago-pc ~] $ python sum.py
0.999999999999999
```

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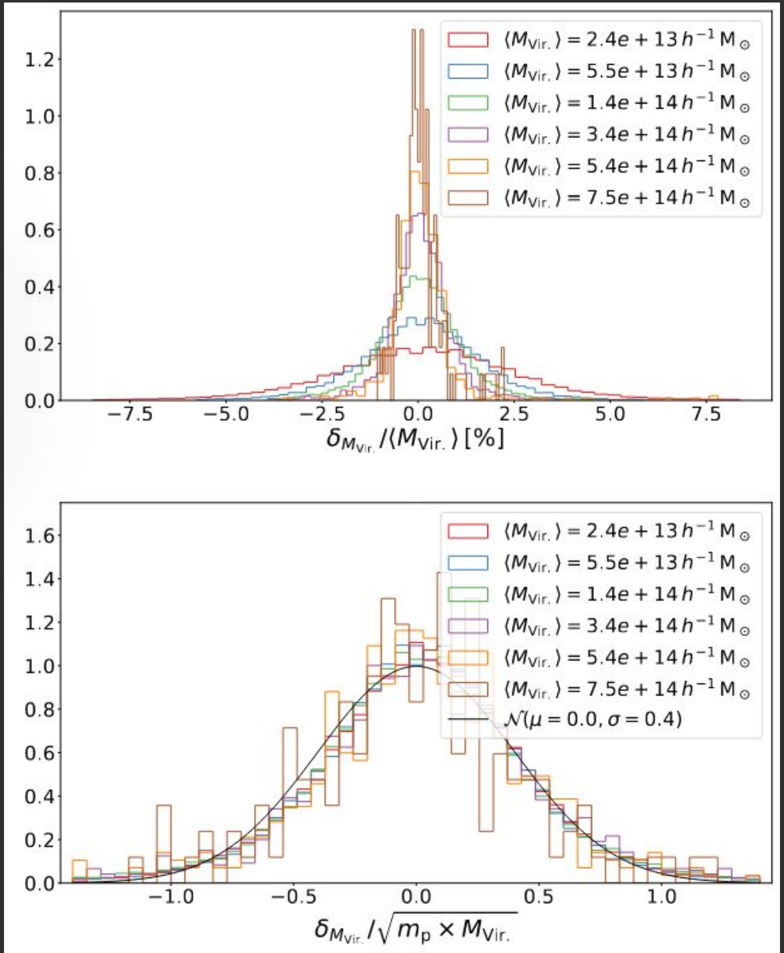
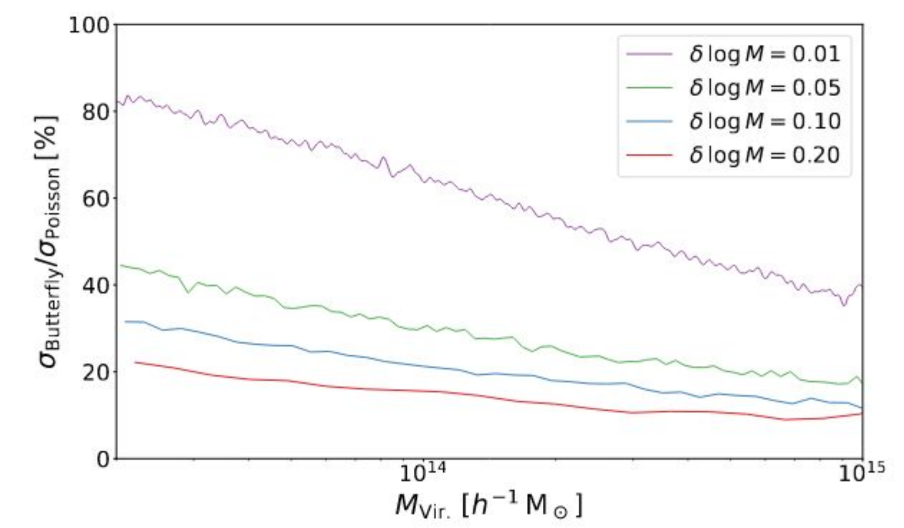


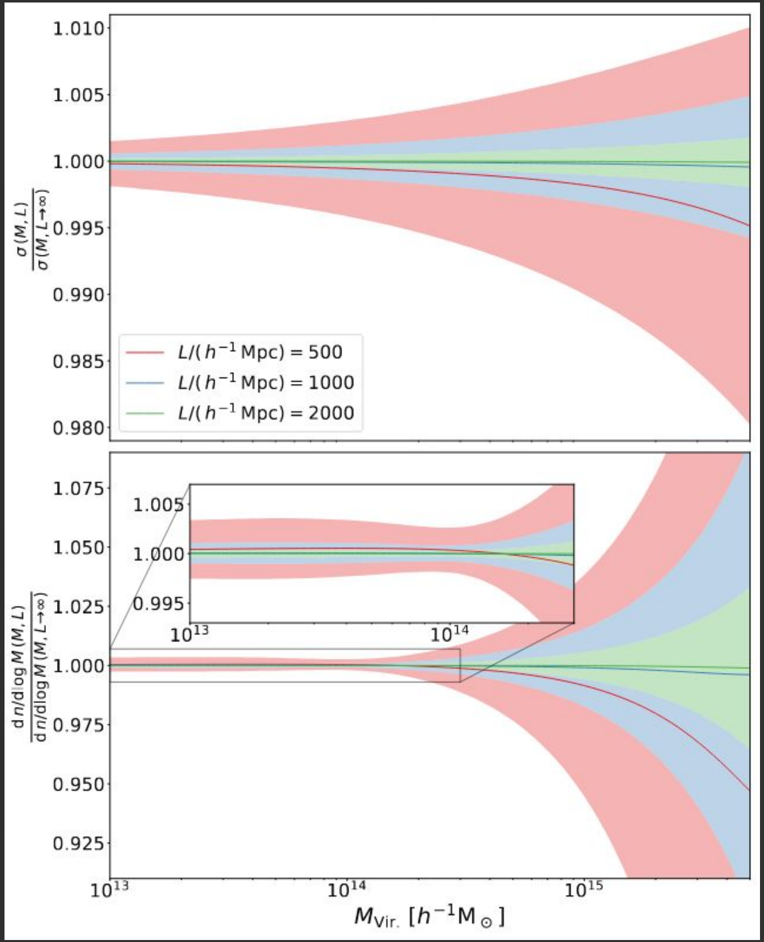
The HMF: Numerical/Theoretical systematics

- Sensitivity of the HMF on initial conditions:
  - Due to the break of the commutative property due to limited precision, the HMF is sensitive to small perturbations on the initial conditions;
  - This introduces a further scatter on the binned statistics depending on the bin width.



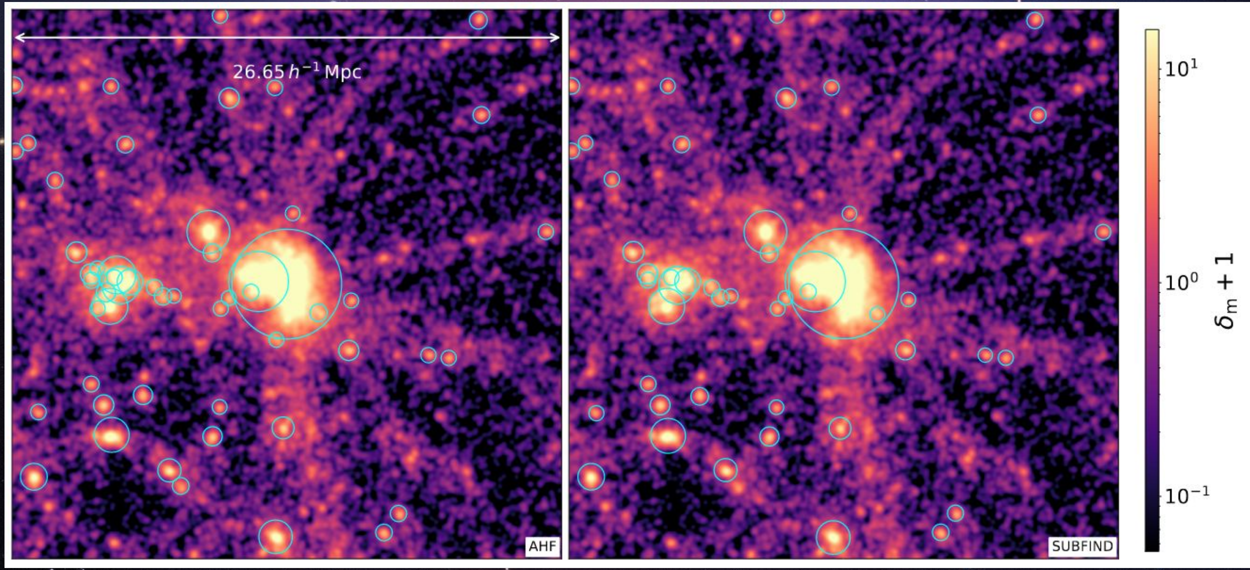
The HMF: Numerical/Theoretical systematics

- Impact of the simulated volume:
  - The simulated volume introduces further scatter to the HMF due to the lack of super-sample modes and the sample variance of the independent modes.



The HMF: Numerical/Theoretical systematics

- Impact of the halo definition:
  - Centering;
  - Boundedness condition;
  - **Hierarchy conditions.**



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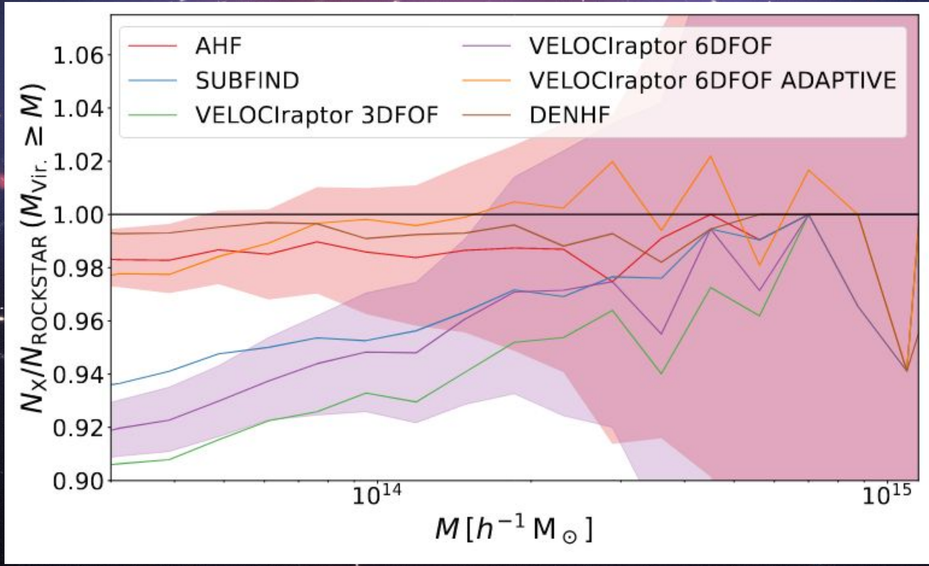

The HMF: Non-universality modelling

- We have adopted a bottom-up approach to develop our HMF model:
  - Selecting the fitting-function to be calibrated using scale-free simulations;
  - Modelling the evolution of the parameters as a function of the matter power spectrum shape;
  - Using simulations with composed initial conditions to discriminate between the impact of the background evolution and the matter power spectrum shape.

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$$f_{\mathrm{ST}}(\nu) = A\sqrt{\frac{2a}{\pi}}\nu \mathrm{e}^{(-a\nu^2/2)}\left(1 + (a\nu^2)^{-p}\right).$$

$$f(\nu) = f_{\mathrm{ST}}(\nu)(\nu\sqrt{a})^{q-1}.$$

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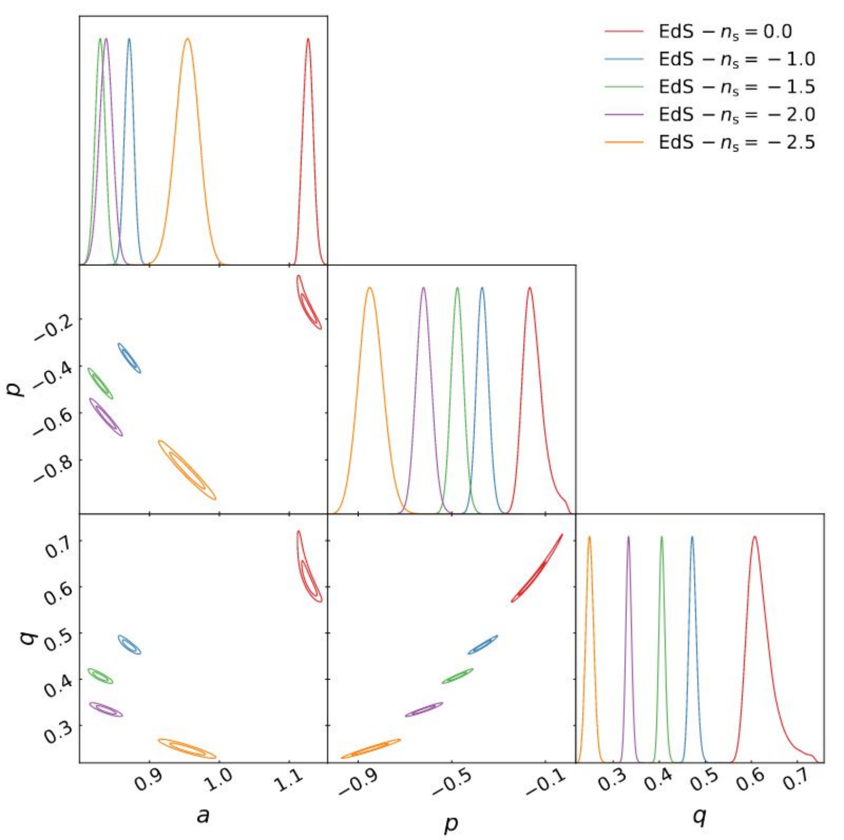

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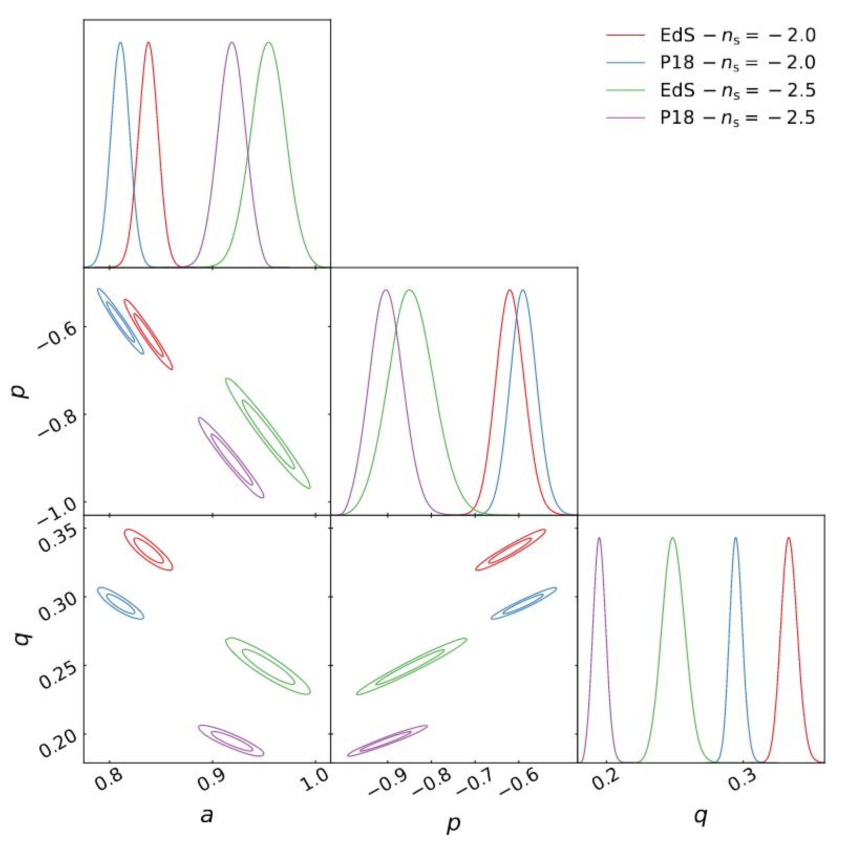

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$$q = q_R \times \Omega_{\rm m}(z)^{q_z}$$

$$q_R = q_1 + q_2 \times \left(\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln R} + 0.5\right)$$

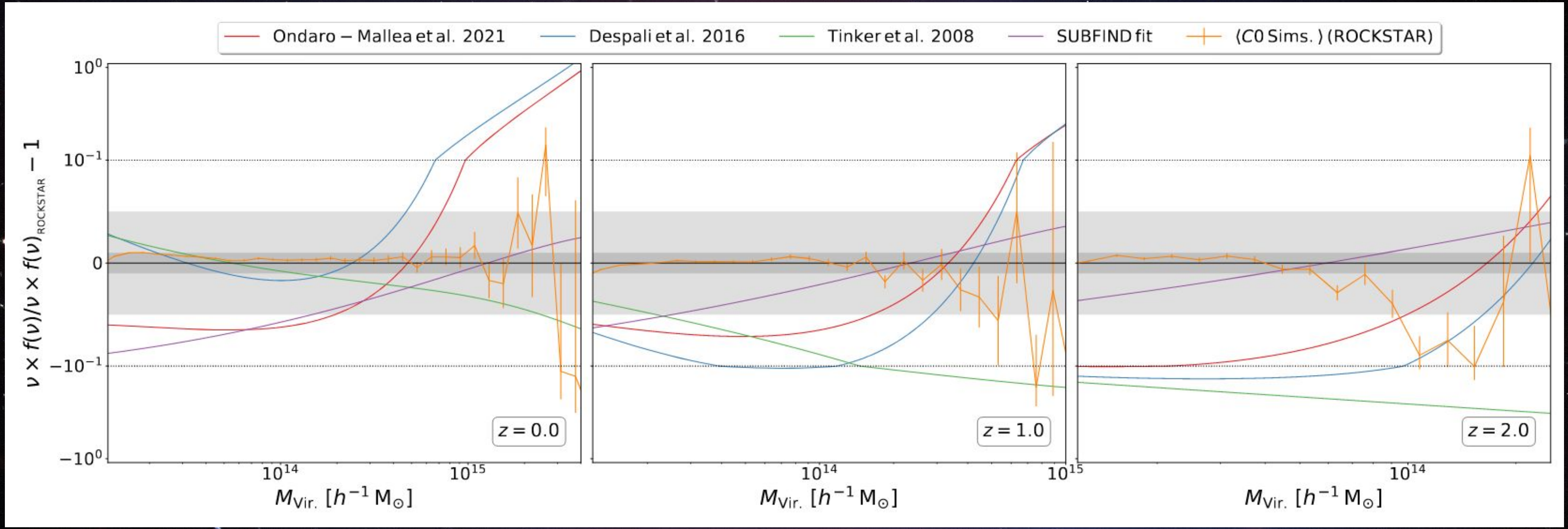
$$p = p_1 + p_2 \times \left(\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln R} + 0.5\right)$$

$$a = a_R \times \Omega_{\rm m}(z)^{a_z}$$

$$a_R = a_1 + a_2 \times \left(\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln R} + 0.6125\right)^2 .$$

The HMF: Non-universality modelling

- Calibration accuracy:



The HMF: Non-universality modelling

- Calibration accuracy:
  - Similar results were obtained by comparing our calibration directly with the simulations used and made available by Ondaro-Mallea et al. (2021), reassuring the robustness of our calibration.
  - 1% agreement with the Uchuu results at z=0 despite the factor of >1.000 in the computational cost of the simulations.

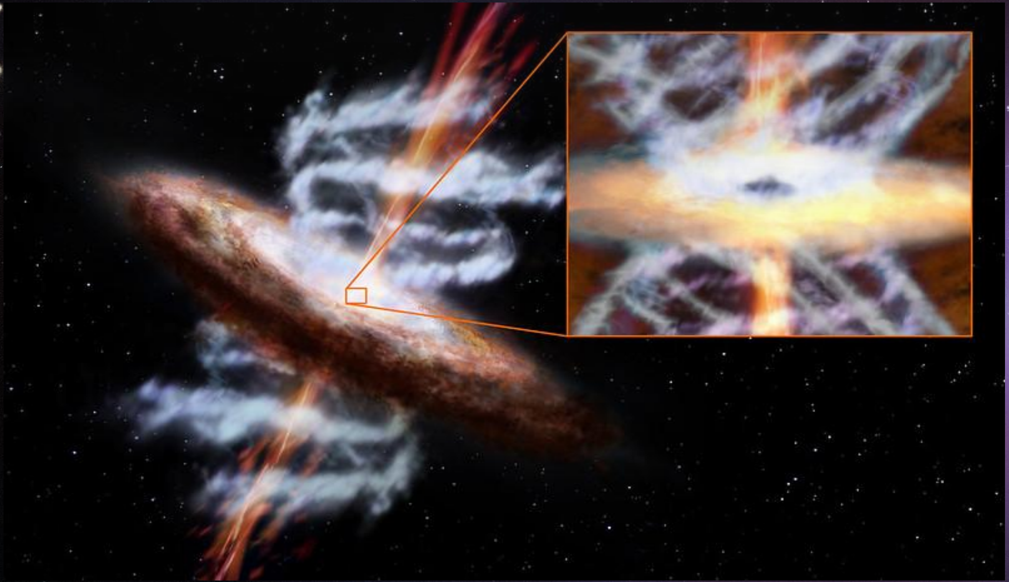
The HMF: Effect of Baryons

MAGNETICUM

- Magneticum takes into account:
  - Cooling, star formation, winds (Springel & Hernquist 2003);
  - Metals, stellar population and chemical enrichment;
  - SN-Ia, SN-II, AGB (Tornatore et al. 2003/2006);
  - BH and AGN feedback (Springel & Di Matteo 2006, Fabjan et al. 2010);
  - Thermal Conduction (1/20th Spitzer) (Dolag et al. 2004);
  - Low viscosity scheme to track turbulence (Dolag et al. 2005);
  - Higher order SPH Kernels (Dehnen & Aly 2012);
  - Magnetic Fields (passive) (Dolag & Stasyszyn 2009).

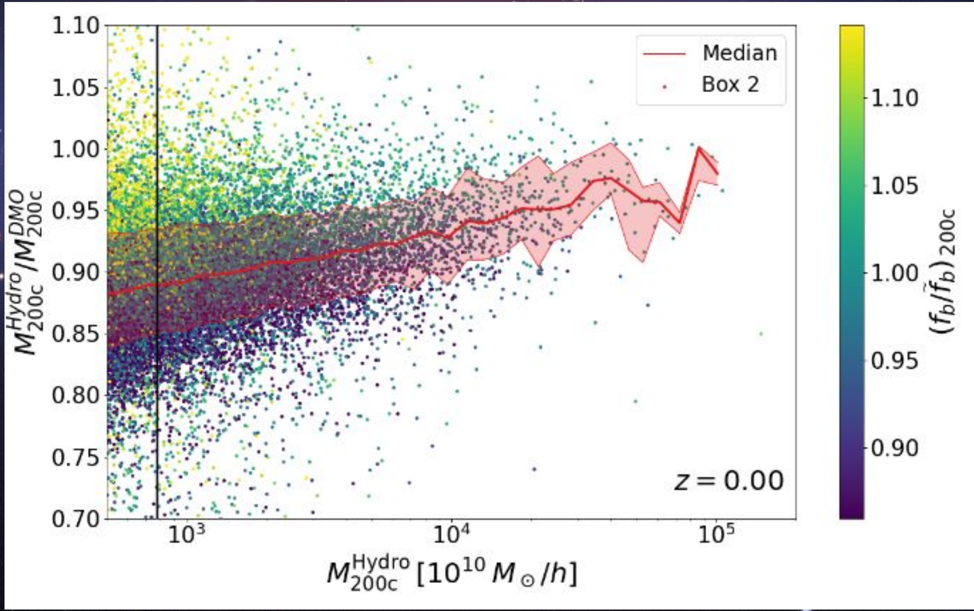
The HMF: Effect of Baryons

- A more consistent picture of AGN feedback:



The HMF: Effect of Baryons

- Baryons affect the LSS:
  - **The net effect is that matched halos has systematically lower masses on hydro than on the DMO;**
  - Cluster abundance is suppressed by 5-15%;



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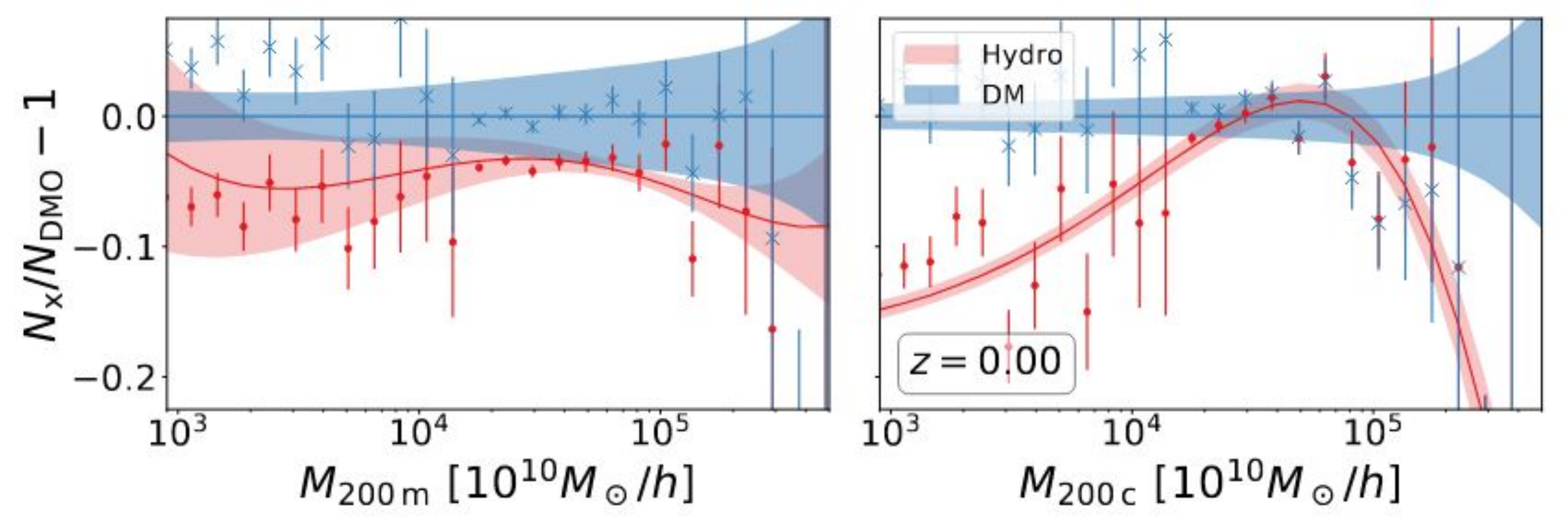
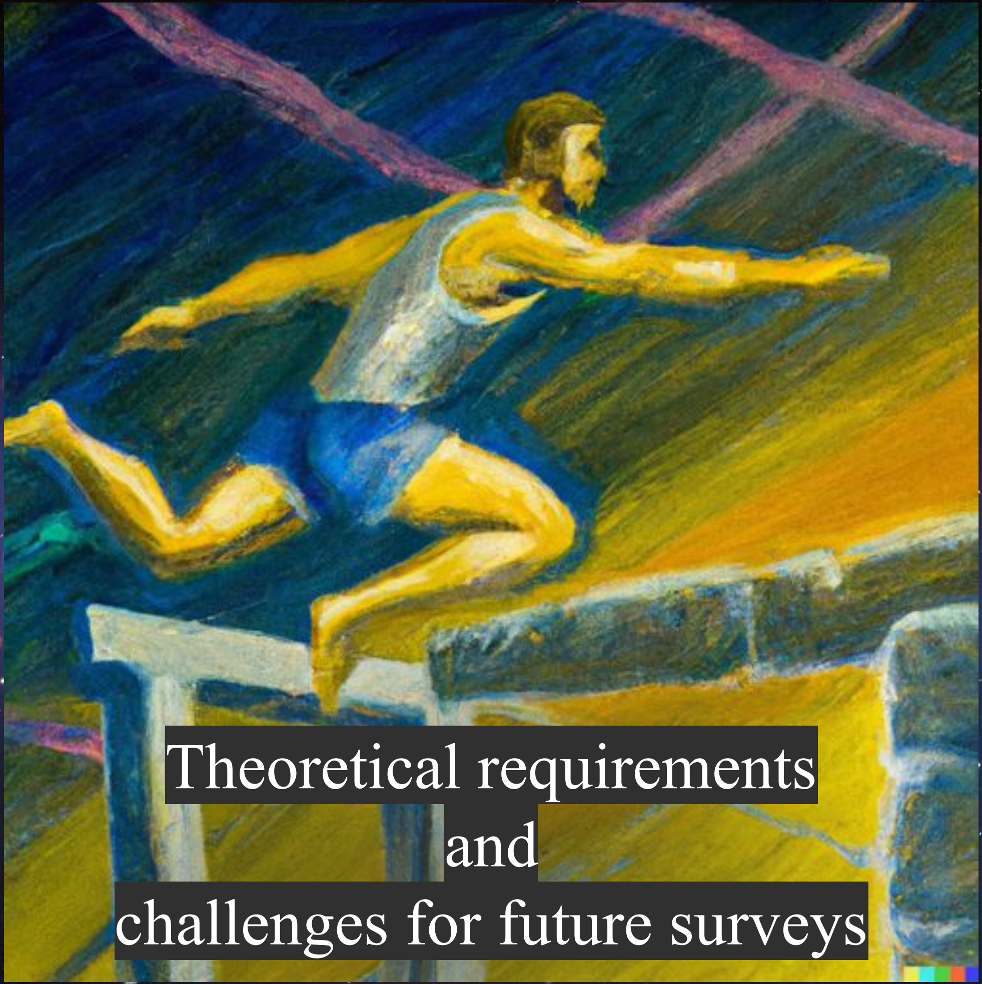




Theoretical requirements
and
challenges for future surveys

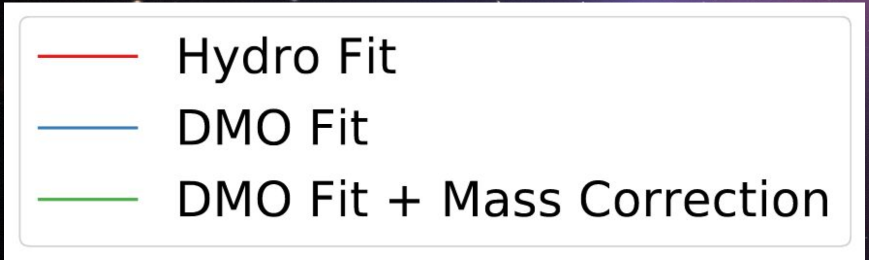
Theoretical requirements and challenges for future surveys

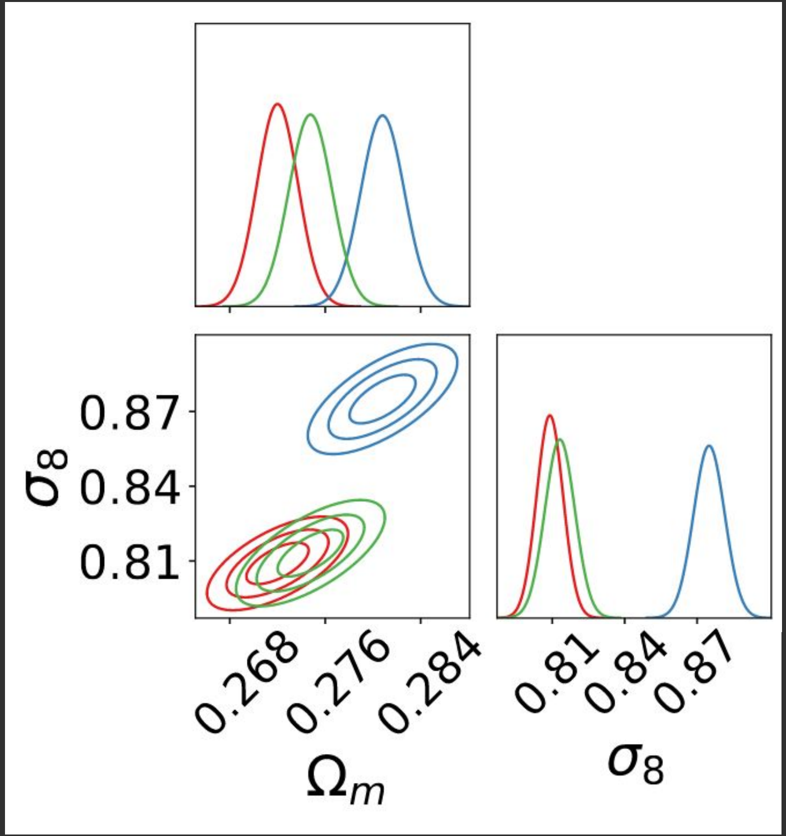
- Impact of the halo definition (assuming different halo-finders):

Summary statistics	richness-mass relation priors	Analysis	Synthetic catalog	Value
IOI	1 %	VELOCIraptor (Fixed)	ROCKSTAR	1.66 ± 0.01
	3 %			0.77 ± 0.01
	5 %			0.65 ± 0.01
	1 %	SUBFIND (Fixed)	ROCKSTAR	1.70 ± 0.02
	3 %			0.84 ± 0.01
	5 %			0.61 ± 0.01
	1 %	AHF (Fixed)	ROCKSTAR	0.90 ± 0.02
	3 %			0.61 ± 0.01
	5 %			0.47 ± 0.00
$\frac{\Delta FOM}{FOM}$	1 %	ROCKSTAR (Marginalized)	ROCKSTAR	0.04 ± 0.05
	3 %			0.06 ± 0.04
	5 %			-0.01 ± 0.02
	1 %	VELOCIraptor (Marginalized)	VELOCIraptor	-0.09 ± 0.05
	3 %			0.00 ± 0.03
	5 %			-0.02 ± 0.03

Theoretical requirements and challenges for future surveys

- Neglecting Baryons:





Conclusions

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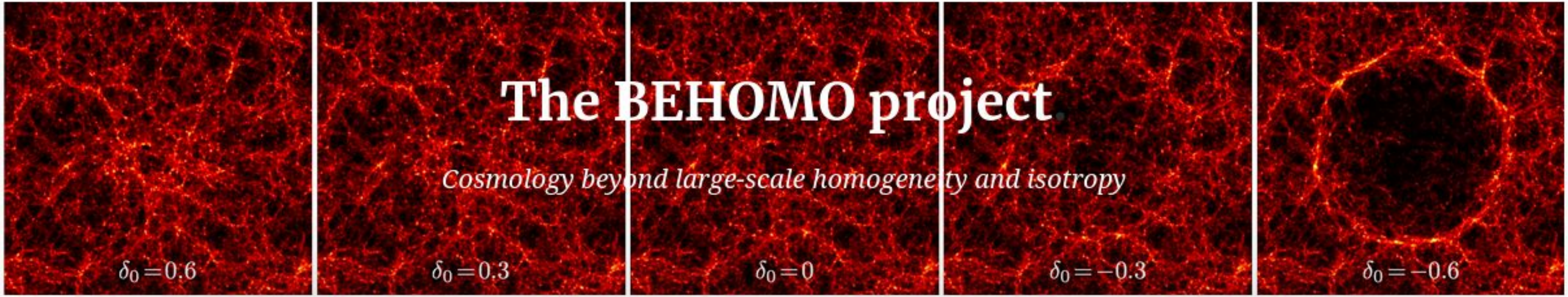
1. We presented a precise and accurate model for the virial HMF in LCDM:
    a. 1% agreement for the range of masses relevant for CC;
    b. Minimal impact on the Cluster Counts FOM.
2. The impact of the halo-finder choice might bias the cosmological inference:
    a. The impact is smaller than previously discussed in Salvatti et al. 2021 and Artis et al. 2021;
    b. Still, it raise awareness that the halo definition is an important systematic and should be better understood.
3. The impact of baryons remains as the main theoretical systematic effect:
    a. Assessing the robustness of hydro-simulations is a completely non-trivial task;
    b. The cost of hydro-simulations is impeditive for many of the approaches.



1. The BEHOMO project studies the evolution of the large-scale structure on an inhomogeneous background.
2. The basic idea is to add non-standard large-scale perturbations on top of the FLRW metric and understand how cosmological structures evolve and how the corresponding observables are modified in an inhomogeneous universe.
3. This will allow us to constrain, with present and future data, deviations from large-scale homogeneity and isotropy and place the standard model of cosmology on more solid grounds.
4. https://valerio-marra.github.io/BEHOMO-project/, for more details.

Thanks / Grazie / Obrigado!