BRANS-DICKE ACCELERATED COSMOLOGIES WITH FERMIONIC SOURCES

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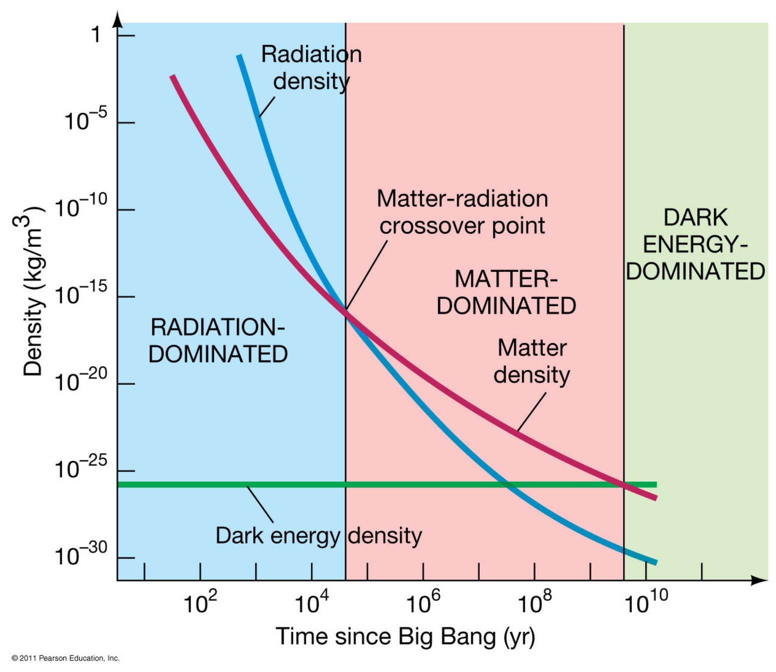
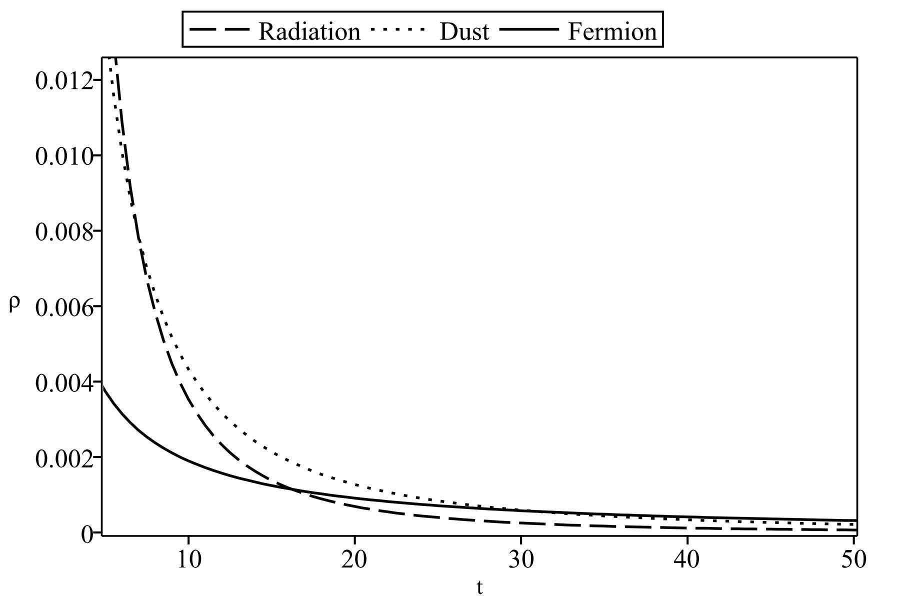
INTRODUCTION

The accelerated expansion of the universe, first observed using type 1a supernovae (1998), can be accounted for in several ways. In standard FLRW cosmological models, this may be done by considering exotic matter fields, such as perfect fluids with negative pressure. Although the simplest constituent of that kind, the cosmological constant $(p_\Lambda = -\Lambda)$, already provides adequate accordance with observational data, it is of theoretical interest to consider alternative dynamical possibilities. In this work, we consider a classical fermionic field in the framework of the simplest scalar-tensor theory of Gravitation, the Brans-Dicke theory (see Molinari *et al.*, *Phys. Rev. D* **105**, 043527 (2022) for more details).





$(n = 0.3)$

THEORETICAL MODEL

A fermionic field is introduced in a general relativistic setting by means of the tetrad formalism. We thus consider the tetrad fields $e^a{}_\mu(x)$ [1], which allow to define fermionic variables in a locally Minkowskian frame. A fermionic covariant derivative is introduced via the Fock-Ivanenko coefficients $\Omega_\mu = \frac{1}{2}\sigma^{ab}\omega_{ab\mu}$, where $\sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$ and $\omega^{ab}{}_\mu = e^{a\nu}\nabla_\mu e^b{}_\nu$. We shall consider an unperturbed FLRW universe filled with two non-interacting fluids and a fermionic field, with the action $S = \int d^4x\sqrt{-g}(\mathscr{L}_{BD} + \mathscr{L}_D + \mathscr{L}_M)$, where $\mathscr{L}_{BD} = -(1/2)\left[\phi R - (\omega/\phi)\phi_{,\alpha}\phi^{,\alpha}\right]$ and $(\Gamma^\mu \equiv e_a{}^\mu\gamma^a)$

$$\mathscr{L}_D = \frac{i}{2}\left[\overline{\psi}\Gamma^\alpha D_\alpha\psi - (D_\beta\overline{\psi})\Gamma^\beta\psi\right] - m\overline{\psi}\psi - V \quad ; \quad V = V_0\left[\beta_1(\overline{\psi}\psi)^2 + \beta_2(i\overline{\psi}\gamma^5\psi)^2\right]^n .$$

Apart from the usual Einstein-Brans-Dicke field equations and conservation equations for the perfect fluids, one also has the generalized Dirac equation $(i\Gamma^\mu D_\mu - m)\psi - \partial_{\overline{\psi}}V = 0$. It can be shown that, in the homogeneous and isotropic case, the fermion's energy-momentum tensor takes the form a perfect fluid with

$$\rho_\psi = m\overline{\psi}\psi + V \quad ; \quad p_\psi = (2n-1)V .$$

DYNAMICAL EQUATIONS AND RESULTS

Specialization of the general equations of motion (obtained by varying S with respect to its dynamical fields) to the FLRW case yields the system of equations

$$\diamond \quad \frac{\ddot{a}}{a}\phi + \left(\frac{\dot{a}}{a}\right)^2\phi + \ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} + \frac{\omega}{2\phi}\dot{\phi}^2 = -w_1\rho_1 - w_2\rho_2 - (2n-1)V ,$$

$$\diamond \quad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{1}{3+2\omega}\left\{(1-3w_1)\rho_1 + (1-3w_2)\rho_2 + m\overline{\psi}\psi + (4-6n)V\right\} ,$$

$$\diamond \quad \dot{\psi} + \frac{3\dot{a}}{2a}\psi + \mathscr{V}_0\gamma^0\psi + \mathscr{V}_5\gamma^0\gamma^5\psi = 0 \quad \text{(4 equations)}$$

$$\mathscr{V}_0 \equiv im + 2i\beta_1 n(\overline{\psi}\psi)V_0\tilde{V}^{n-1} \quad ; \quad \mathscr{V}_5 \equiv -2i\beta_2 n(\overline{\psi}\gamma^5\psi)V_0\tilde{V}^{n-1} ,$$

with $\tilde{V} \equiv \beta_1(\overline{\psi}\psi)^2 + \beta_2(i\overline{\psi}\gamma^5\psi)^2$; and the conservation equations $\dot{\rho}_i + 3\frac{\dot{a}}{a}(1+w_i)\rho_i = 0$.

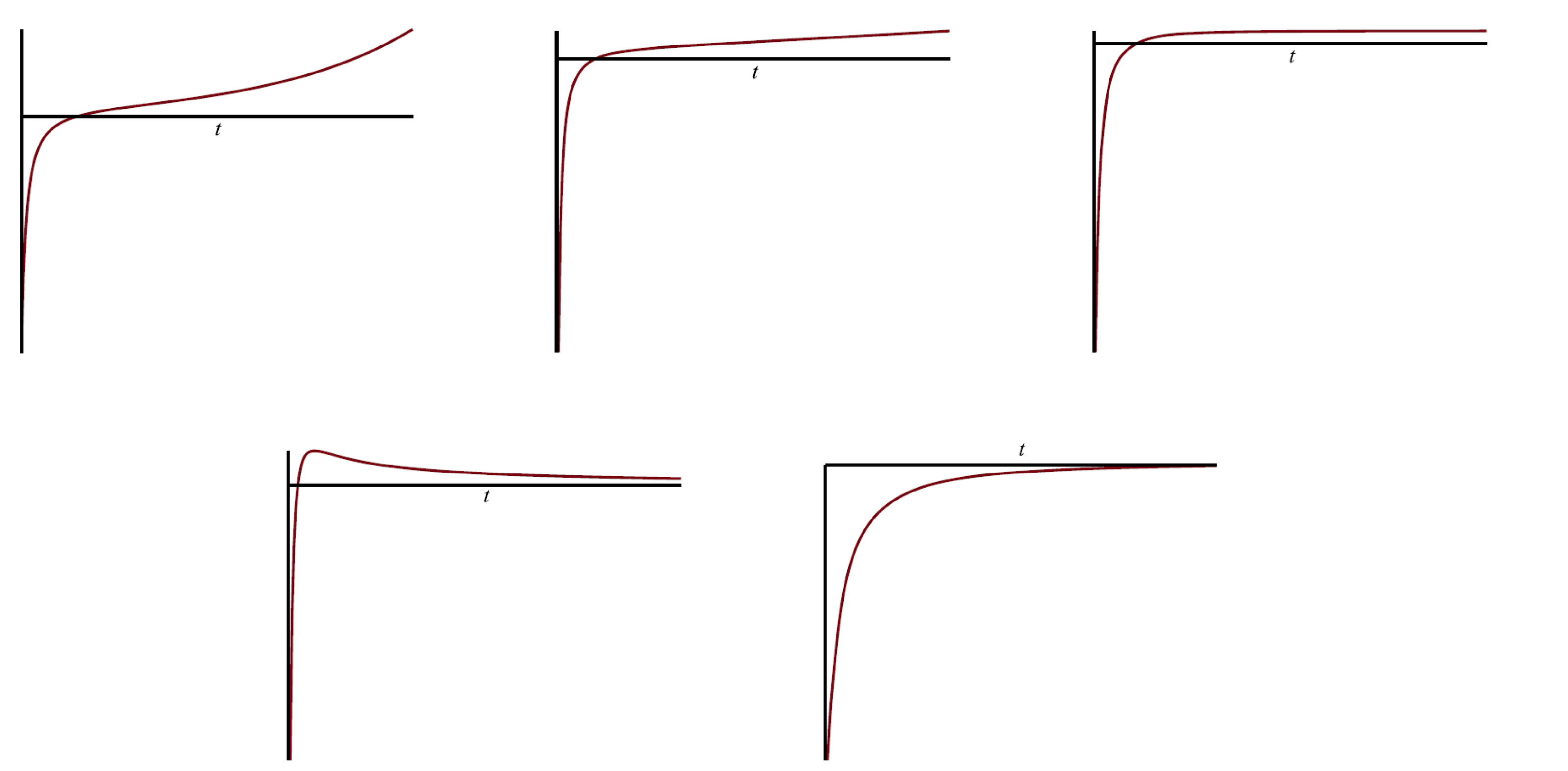


Fig. 3: Qualitative aspect of the acceleration $\ddot{a}$ for different values of n. From left to right: $n = \{0,\ 0.1,\ 1/6,\ 0.3,\ 0.5\}$.

Given the nonlinearity of the field equations, and since we are interested only in the qualitative evolution of this model, we follow [2, 3] and look for numerical solutions. By choosing suitable values for the initial conditions of field configuration and velocities, we investigated the evolution of the system specifying $w_1 = 0, w_2 = 1/3$ for dust and radiation fields, respectively. We considered $n = 0.3$ and $\omega = 5 \times 10^4$, which recovers Einstein gravity, although smaller values yielded similar results.

For the fermionic exponent $n < 0.5$ we have a negative pressure. Below a given threshold smaller than 0.5 (in this case ≈ 0.34), the fermion is able to promote transitions to an accelerated regime, after its energy density becomes dominant (with a radiation → dust → fermion transition).

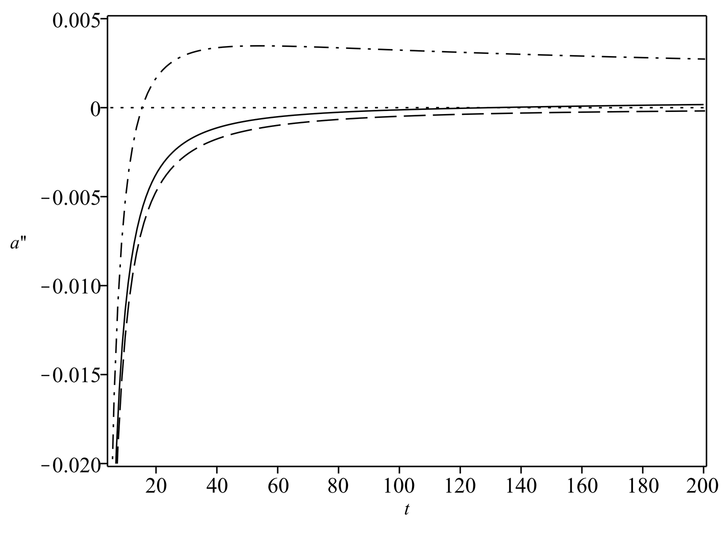


Fig. 4: Straight and dashed line: $n = 0.3$ and $n = 0.6$, respectively; dash-dotted line: $n = 0.2$

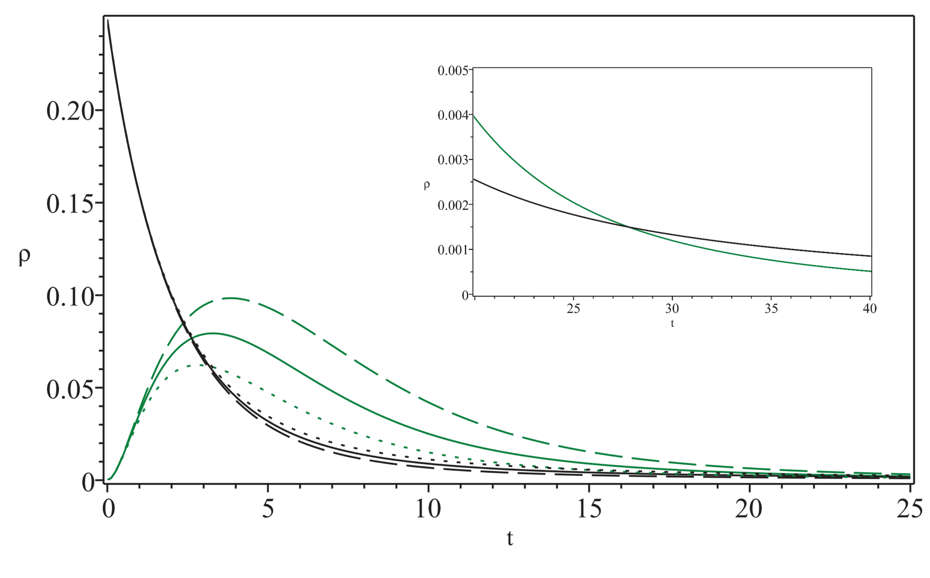


Fig. 5: Straight lines: $\tau = 1$; dash-dotted lines: $\tau = 1.2$; dashed lines: $\tau = 0.8$.

EARLY UNIVERSE ENERGY TRANSFER

We consider dissipative processes in an early stage of the universe where the fermion prevails in order to model matter creation (radiation, in this case). Using the (second order) dynamical pressure formalism [4, 5], we introduce the non-equilibrium pressure $\varpi(t)$, which contributes as a standard pressure in the field equations and satisfies

$$\tau\dot{\varpi} + \varpi = -3H\rho_T .$$

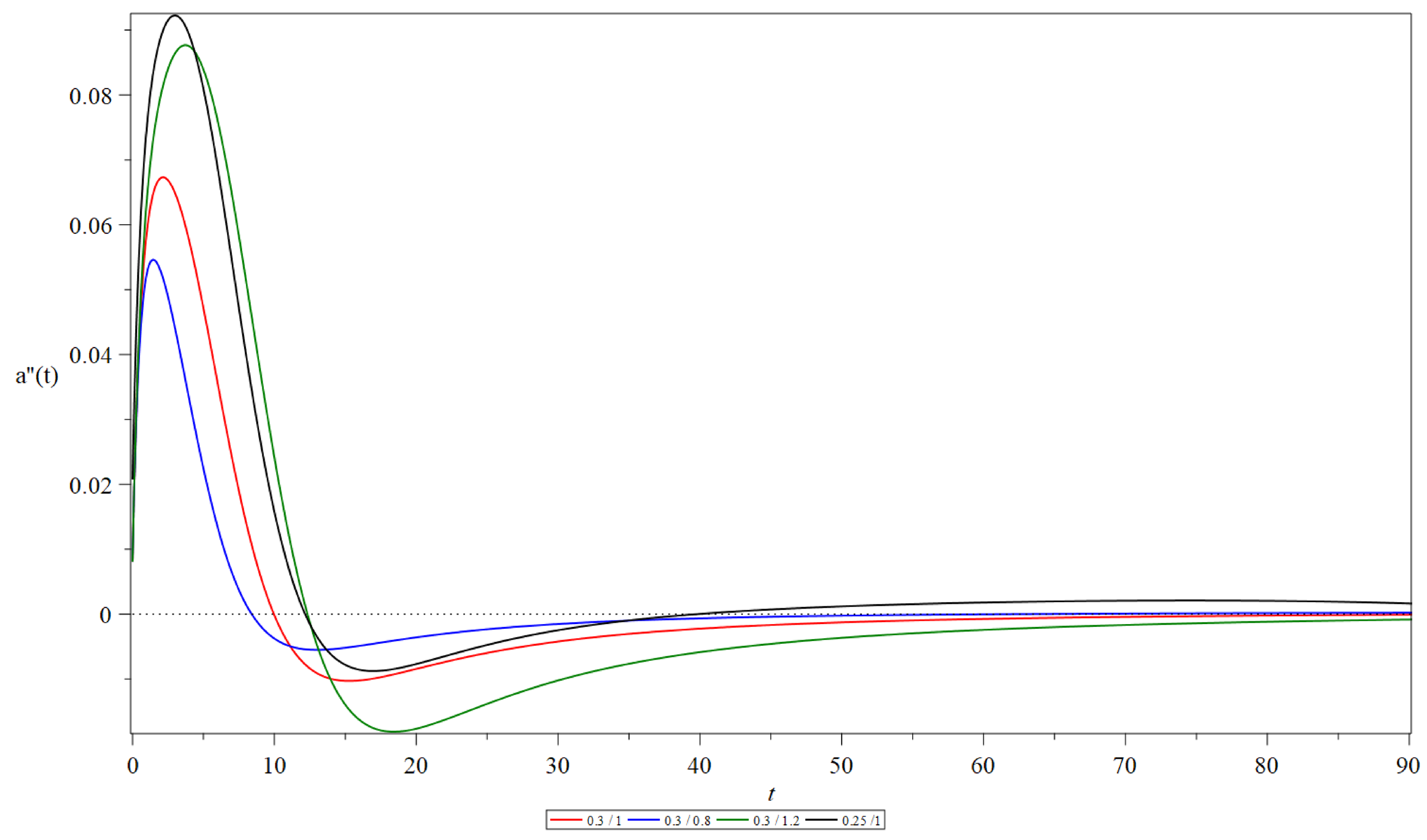


Fig. 6: Acceleration $\ddot{a}$ against time for different values of n and α (first and second numbers in caption, respectively).

It is seen that this approach gives a three-era cosmology, where the fermionic field can answer for an early and later accelerated period, with an intermediary decelerated matter domination. A similar result was obtained by considering two matter fields, each with an associated $\varpi_i(t)$.

CONCLUSIONS

- Considering a self-interacting fermionic field, we have obtained varied scenarios, some of which identify it as a dark energy-like constituent
- The Brans-Dicke field, for $\omega = \mathcal{O}(10^{>0})$, does not affect the general qualitative behavior
- Using the dynamical pressure approach, it was shown that the fermion can also model matter production in an early accelerated regime, followed by a finite period of matter domination.

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