De-singularizing the extremal GMGHS black hole via higher derivatives corrections

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Phys.Lett.B 818 (2021) 136357 [2103.00884 [hep-th]]

☆Summary

- We construct a regular extension of the GMGHS extremal black hole in a model with $O(\alpha')$ corrections in the action. The de-singularization is supported by the $O(\alpha')$-terms.

- The regularized extremal GMGHS BHs are asymptotically flat, possess a regular (non-zero size) horizon of spherical topology, with an $AdS_2 \times S^2$ near horizon geometry.

- The near horizon solution is obtained analytically and some illustrative bulk solutions are constructed numerically.

[1] Introduction

☆The Gibbons~Maeda~Garfinkle~Horowitz~Strominger (GMGHS) black hole is an influential solution of the low energy heterotic string theory compactified to four spacetime dimensions.
(G.W. Gibbons, K.i. Maeda, Nucl. Phys. B 298 (1988) 741)
(D. Garfinkle, G.T. Horowitz, A. Strominger, Phys. Rev. D 43 (1991) 3140)

☆Property of the GMGHS solution

- Its extremal limit is singular.

- The area of the spatial sections of the horizon shrinks to zero.

- The Kretschmann scalar blows up at the horizon.

Nobody knows whether the possible stringy α'-corrections could de-singularize the extremal solution in the Einstein-Maxwell-dilaton action.

Cf)

- There is a perturbative extension of the extremal magnetic GMGHS BH.

(M. Natsuume, Phys. Rev. D 50 (1994) 3949, arXiv:hep-th/9406079)

- The corrected solution inherits all basic properties of the extremal GMGHS BH.

- The horizon area still vanishes.

$\Rightarrow$ The task of constructing the fully non-linear BH solutions of the $O(\alpha')$ corrected action has not yet been considered.

The main purpose of our work is to reply to the key question whether such corrections can de-singularize the extremal GMGHS solution.

[2] The model

- Starting with a general model for the O(α') corrections to the Einstein~Maxwell~dilaton action.

$$S_s = \int d^4x\sqrt{-\tilde{g}}\,\mathrm{e}^{-2\phi}\Big[\tilde{R} + 4\left(\tilde{\nabla}\phi\right)^2 - F^2 + \alpha'\Big\{a\left(\tilde{R}_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} - 4\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} + \tilde{R}^2\right)$$
$$+b\left(F^2\right)^2 + cF^2\left(\tilde{\nabla}\phi\right)^2 + h\tilde{R}^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\Big\}\Big]$$

Here, $F = dA$ is the U(1) field strength tensor, ϕ is a real scalar field and $a,\ b,\ c,\ h$ are constant coefficients.

☆Setup: static spherically symmetric solutions with a purely electric U(1) potential

$$ds^2 = -a(r)^2 dt^2 + c(r)^2 dr^2 + b(r)^2 d\Omega^2, \qquad \phi \equiv \phi(r), \qquad A = V(r)dt$$

- The field equations of the full model possess an exact solution describing a Robinson-Bertotti-type vacuum, with an $AdS_2 \times S^2$ metric, an electric field and a constant dilaton.

- There is no counterpart of this solution with a magnetic charge.

- Our choice here is to work with

$$a = \frac{1}{8}\,, \qquad b, c: \ \ \text{arbitrary}\,, \qquad h = 0$$

☆α′=0 limit: the GMGHS solution

(M. Natsuume, Phys. Rev. D 50 (1994) 3949, arXiv:hep-th/9406079)

$$ds^2 = -\left(1-\frac{r_+}{r}\right)dt^2 + \left(1-\frac{r_+}{r}\right)^{-1}dr^2 + r^2\left(1-\frac{r_-}{r}\right)d\Omega^2\,,$$

$$A = \frac{Q}{r}dt\,, \qquad \mathrm{e}^{2\phi} = \frac{1}{2}\left(1-\frac{r_-}{r}\right), \qquad M = \frac{r_+}{2}\,, \qquad Q = \left(\frac{r_- r_+}{2}\right)^{\frac{1}{2}}$$

The two free parameters $r_+,\ r_-$ (with $r_- < r_+$), corresponding to outer and inner horizon radius, respectively.

☆Horizon area A_H and Hawking temperature T_H :

$$A_H = 4\pi r_+^2\left(1-\frac{r_-}{r_+}\right), \qquad T_H = \frac{1}{4\pi r_+}$$

- In the extremal limit, $A_H \to 0$ while T_H approaches a constant. (It gives singular solution)

$$A_H = 4\pi r_+^2 \left(1 - \frac{r_-}{r_+}\right), \qquad T_H = \frac{1}{4\pi r_+}$$

- Magnetic version of the GMGHS solution

$$ds^2 = -\left(1 - \frac{r_+}{r}\right) dt^2 + \left(1 - \frac{r_+}{r}\right) dr^2 + r^2 \left(1 - \frac{r_-}{r}\right) d\Omega^2$$

$$A = Q \cos\theta d\varphi, \qquad \mathrm{e}^{-2\phi} = \frac{1}{2}\left(1 - \frac{r_-}{r}\right)$$

- Magnetic solution is also singular in the extremal limit.

[3] Electrically charged solutions

◆ Taking into account α' corrections gives the presence of a Robinson-Bertotti-type exact solution

$\Rightarrow$ the possible existence of non-perturbative generalizations of the extremal GMGHS BHs with a nonzero horizon size.

- The existence of such near-horizon geometry is closely connected with the attractor mechanism.
  (A. Sen, J. High Energy Phys. 05 (2005) 059, arXiv:hep-th/0411255 [hep-th])
  (A. Sen, J. High Energy Phys. 0509 (2005) 038, arXiv:hep -th/0506177)

☆ Ansatz:

$$ds^2 = v_0^2 \left(\frac{dr^2}{r^2} - r^2 dt^2 \right) + v_1^2 d\Omega_{(2)}^2, \qquad \phi(r) = \phi_0, \qquad V = qr$$

- Near horizon field configuration for the ansatz is consistent with the SO(2, 1)×SO(3) symmetry of $AdS_2 \times S^2$

(A. Sen, J. High Energy Phys. 0509 (2005) 038, arXiv:hep-th/0506177)

- We choose to determine the unknown parameters by extremizing an entropy function.

(A. Sen, Gen. Relativ. Gravit. 40 (2008) 2249, arXiv:0708.1270 [hep-th])

The entropy function is defined by

$$F(v_0, v_1, q, Q, \phi_0) = 2\pi \left(qQ - f(v_0, v_1, q, \phi_0) \right) \quad Q: \text{ electric charge of the solutions}$$

The black hole entropy $S_{\rm BH}$ is given by the value of the function F at the extremum with v_0 , v_1 , ϕ_0 :

$$S_{\rm BH}(q\,,\,Q) = F\,(v_0\,,\,v_1\,,\,q\,,\,Q\,,\,\phi_0)$$

The Lagrangian density $f(v_0, v_1, q, \phi_0)$ can be evaluated as

$$\begin{aligned} f(v_0, v_1, q, \phi_0) &= \frac{1}{4\pi} \int d\theta d\varphi \sqrt{-g} \mathcal{L} \\ &= \frac{v_0^2 - v_1^2}{2} + e^{-2\phi_0} q^2 \frac{v_1^2}{2v_0^2} - \frac{1}{4} \alpha' e^{-2\phi_0} \left(1 - 4b \frac{e^{-4\phi_0} q^4 v_1^2}{v_0^6} \right) \end{aligned}$$

☆The scalar and the metric field equations in the near horizon geometry correspond to extremizing F :

$$\frac{\partial F}{\partial v_0} = 0\,, \quad \frac{\partial F}{\partial v_1} = 0\,, \quad \frac{\partial F}{\partial \phi_0} = 0\,, \quad \frac{\partial F}{\partial q} = 0$$

- Solution :

$$v_0 = \frac{\sqrt{\alpha'}e^{-\phi_0}}{\sqrt{2}}\,, \qquad v_1 = \sqrt{\alpha'}e^{-\phi_0}\frac{\sqrt{2b}}{\sqrt{1+12b-\sqrt{1+16b}}}\,,$$

$$q = \sqrt{\alpha'}\frac{\sqrt{\sqrt{1+16b}-1}}{4\sqrt{b}}\,, \qquad Q = \sqrt{\alpha'}e^{-2\phi_0}\frac{\sqrt{b(1+16b)(\sqrt{1+16b}-1)}}{1+12b-\sqrt{1+16b}}$$

✩Bulk extremal BH

$$ds^2 = -\mathrm{e}^{-2\delta(r)} N(r) dt^2 + \frac{dr^2}{N(r)} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

If we assume the existence of a horizon located at $r = r_H > 0$, one finds the following approximate solution:

$$N(r) = N_2 (r - r_H)^2 + \dots \ , \qquad \delta(r) = \delta_0 + \delta_1 (r - r_H) + \dots \ ,$$

$$\phi(r) = \phi_0 + \phi_1 (r - r_H) + \dots \ , \qquad V(r) = v_1 (r - r_H) + \dots \ ,$$

with

$$N_2 = \frac{2}{\alpha'} \ , \qquad r_H = \frac{\mathrm{e}^{-\phi_0} \sqrt{\alpha'} \sqrt{2b}}{\sqrt{1 + 12b - \sqrt{1 + 16b}}} > 0 \ , \qquad v_1 = \frac{\mathrm{e}^{-\delta_0} \sqrt{\sqrt{1 + 16b} - 1}}{2\sqrt{\alpha'}\sqrt{b}}$$

☆For large r , the asymptotic form of the solution becomes

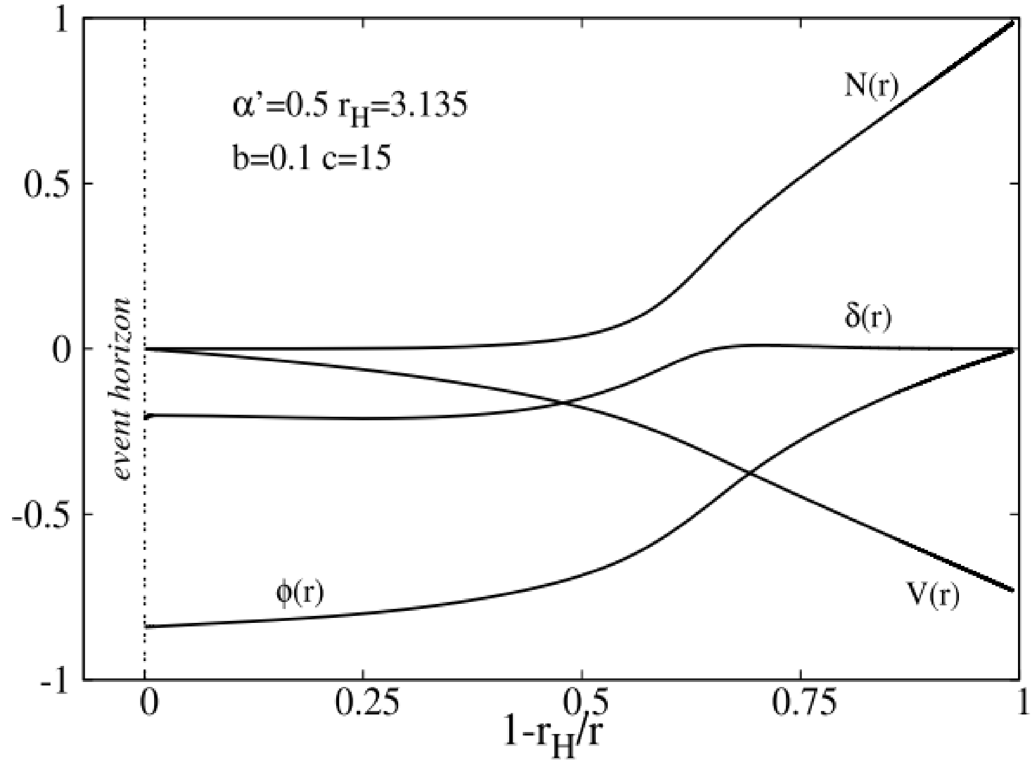
$$N(r) = 1 - \frac{Q^2 + Q_s^2}{r} + \dots \ , \qquad \phi(r) = \frac{Q_s}{r} + \dots \ ,$$

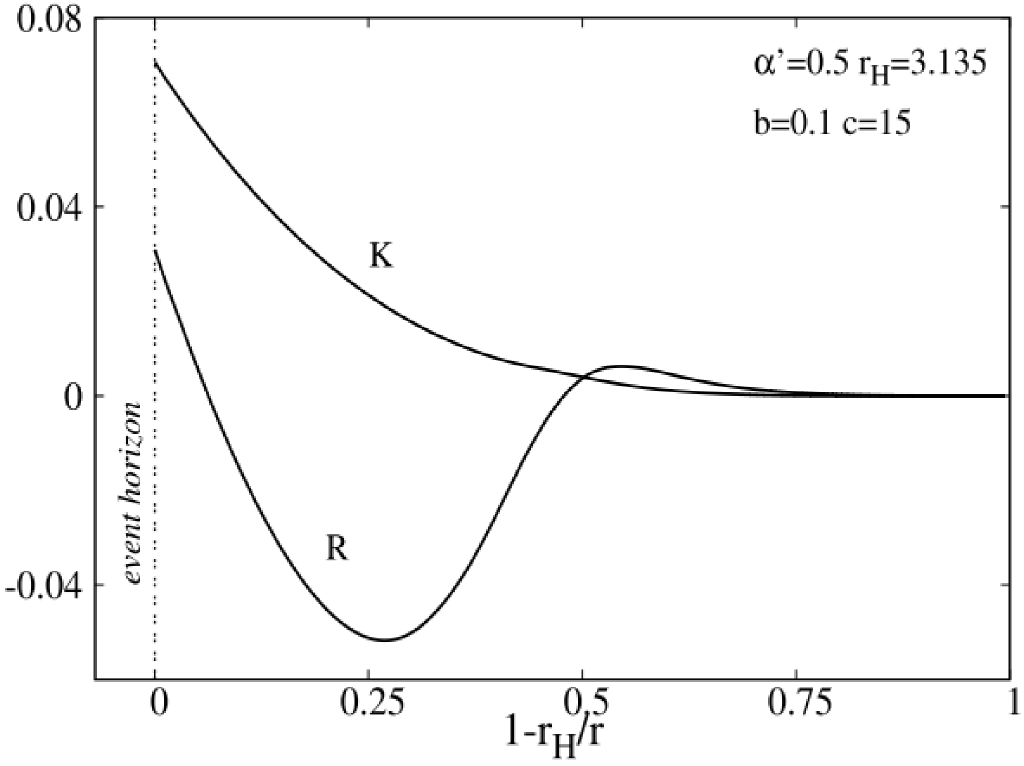
$$V(r) = V_0 + \frac{Q}{r} + \dots , \qquad \delta(r) = \frac{Q_s^2}{2r^2} + \dots \ .$$

Q_s : scalar charge

- These extremal BHs have finite global charges $M, \ Q$ as well as a finite scalar "charge" Q_s while their Hawking temperature vanishes.

☆ Profile of a typical BH solution

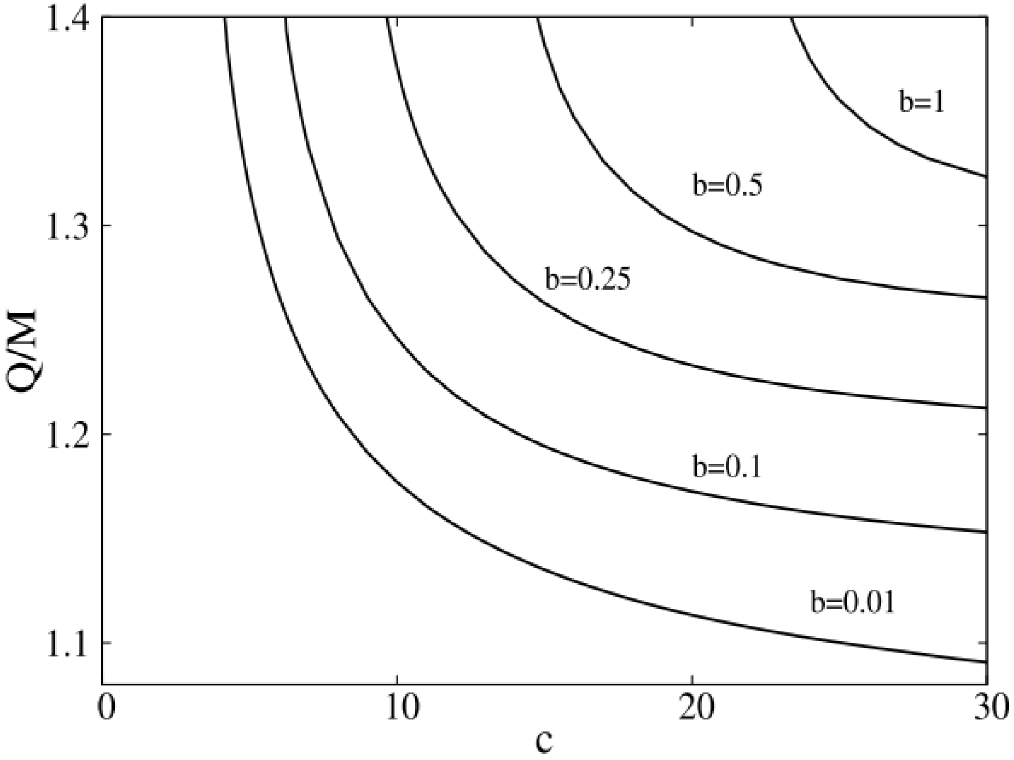




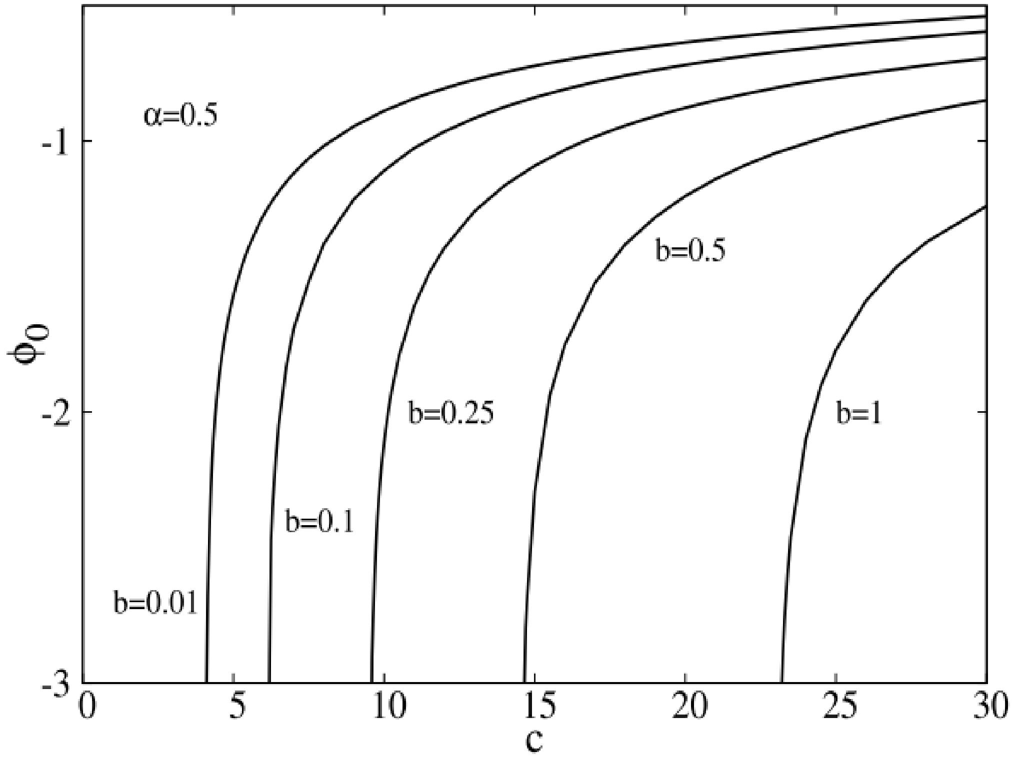
Profile functions of the extremal BH

Ricci (R) and Kretschmann (K) scalars

Charge to mass ratio

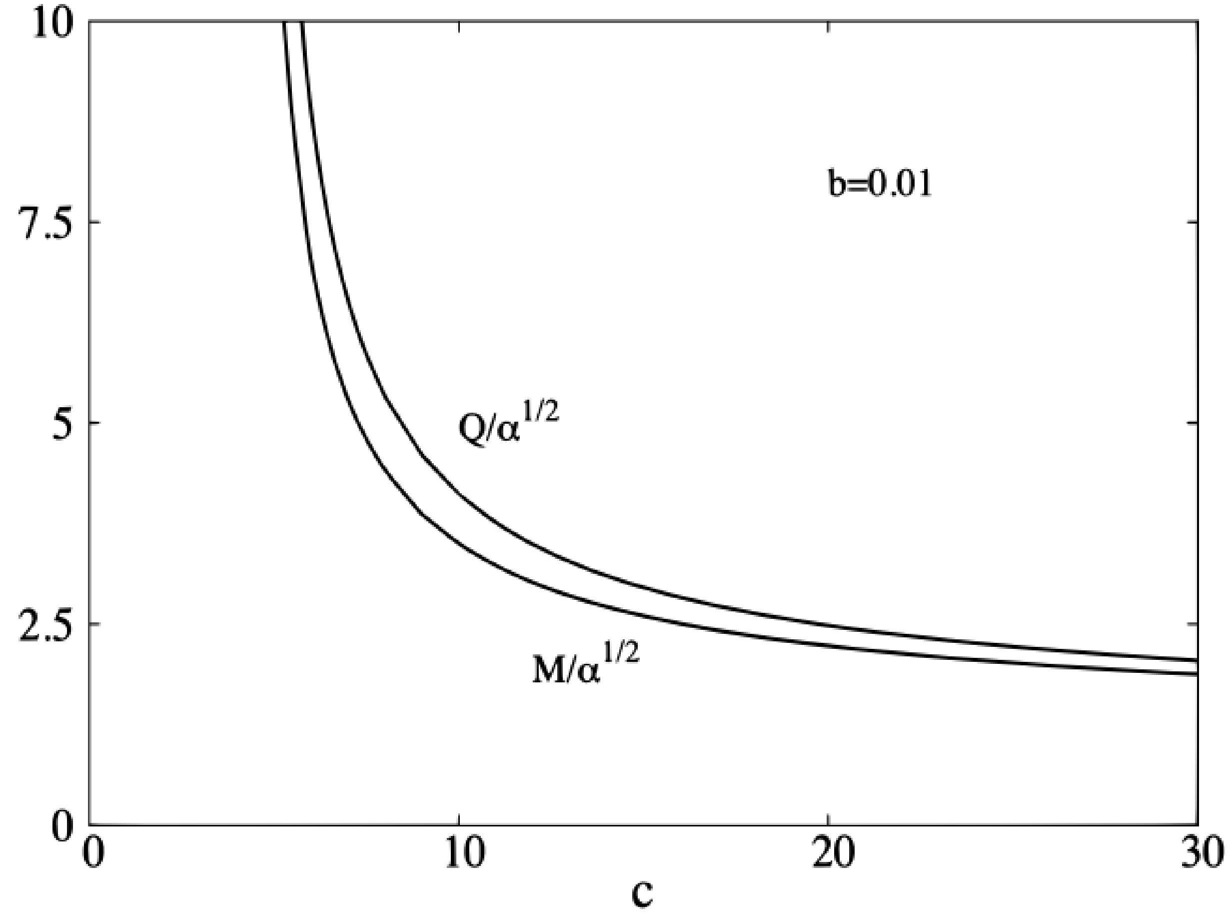


Value of dilaton



- The most interesting feature of the solutions found so far is that the charge-to-mass ratio Q/M is always greater than one.

- The ratio Q/M decreases when c increases for fixed b .



- As the parameter c grows, the BH mass M decreases for fixed value of b . This feature seems to be at tension with the rationale of the weak gravity conjecture.

[4] Discussion and remarks

- In this work, we have focused on static BHs but there are also studies with rotating BHs with first order correction in α'.

- We have confirmed that α' corrections can de-singularize the extremal GMGHS solution, an influential stringy BH whose extremal limit is long known to be singular.

- The charge-to-mass ratio Q/M decreases when the BH mass M decreases for fixed value of a parameter.

- Although we get Q/M>1, Q/M decreases as the mass decreases. Hence, our results do not assure that an extremal BH is always able to decay to smaller extremal BHs of marginally higher Q/M.

(C. Cheung, J. Liu and G. N. Remmen, JHEP 1810 (2018) 004)
(G. J. Loges, T. Noumi and G. Shiu, Phys. Rev. D 102 (2020) no.4, 046010)
(C. Herdeiro, E. Radu, K. Uzawa, in preparation)