Primordial Non-Gaussianity beyond the Bispectrum

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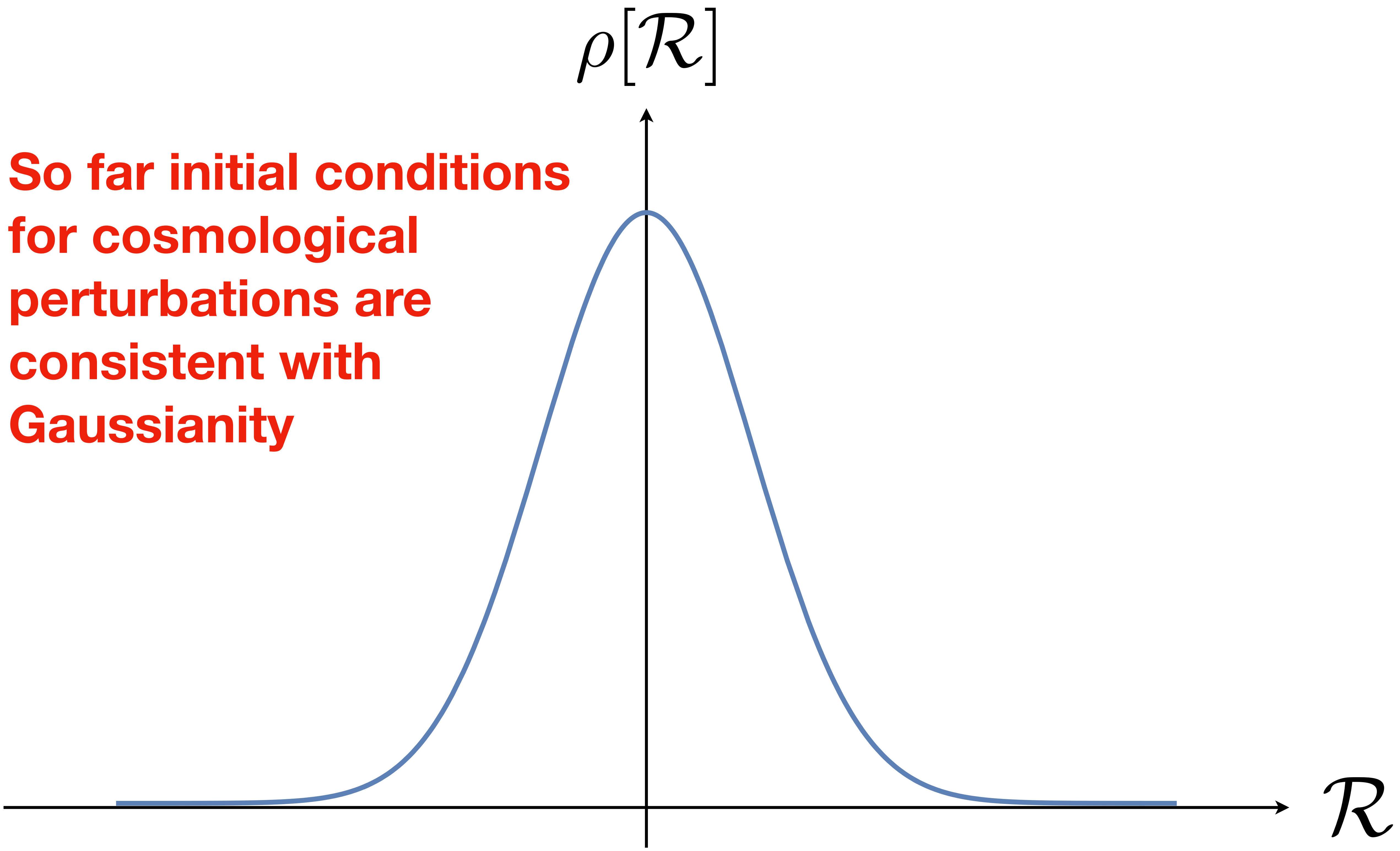
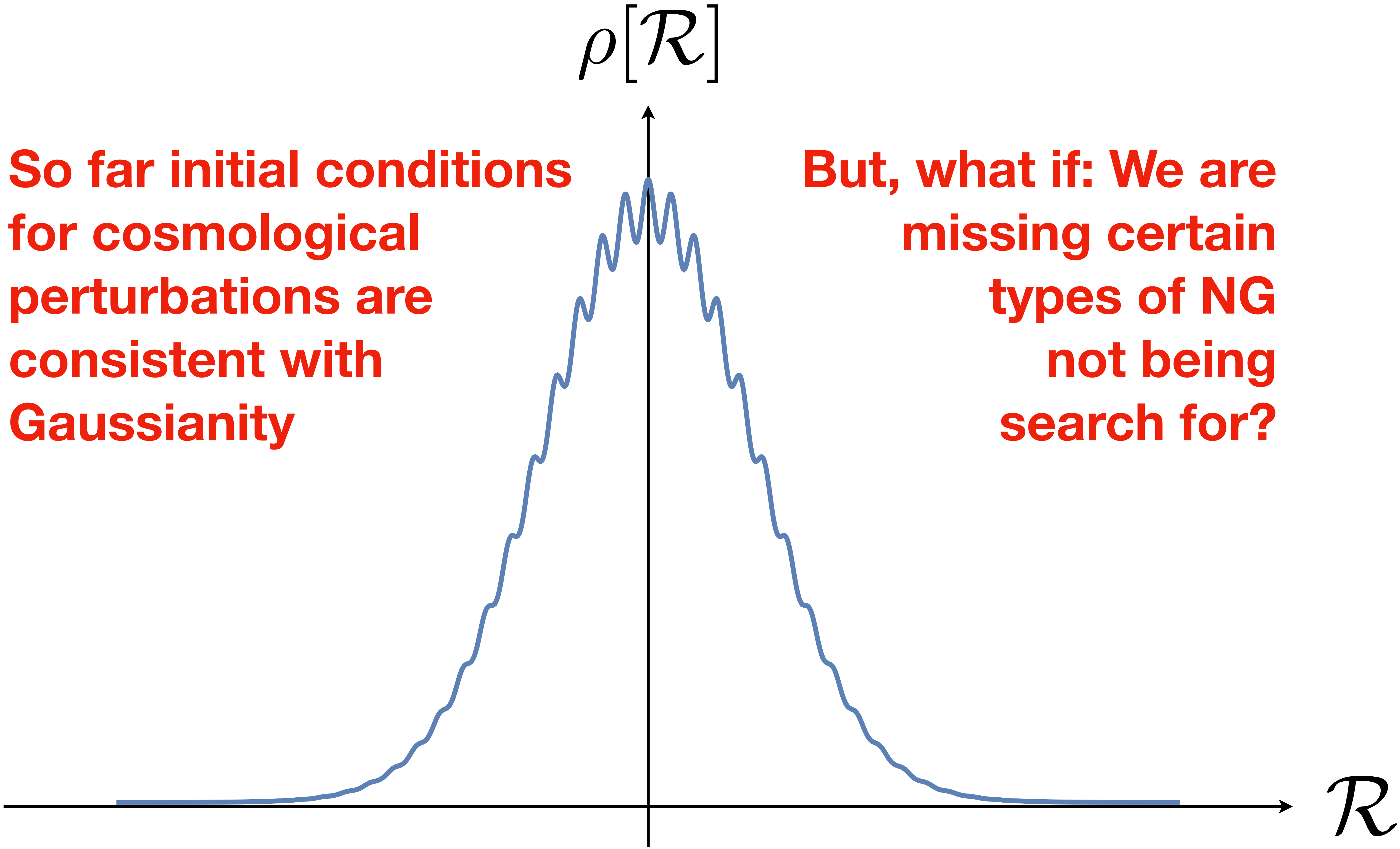
Preamble

So far initial conditions for cosmological perturbations are consistent with Gaussianity



Preamble



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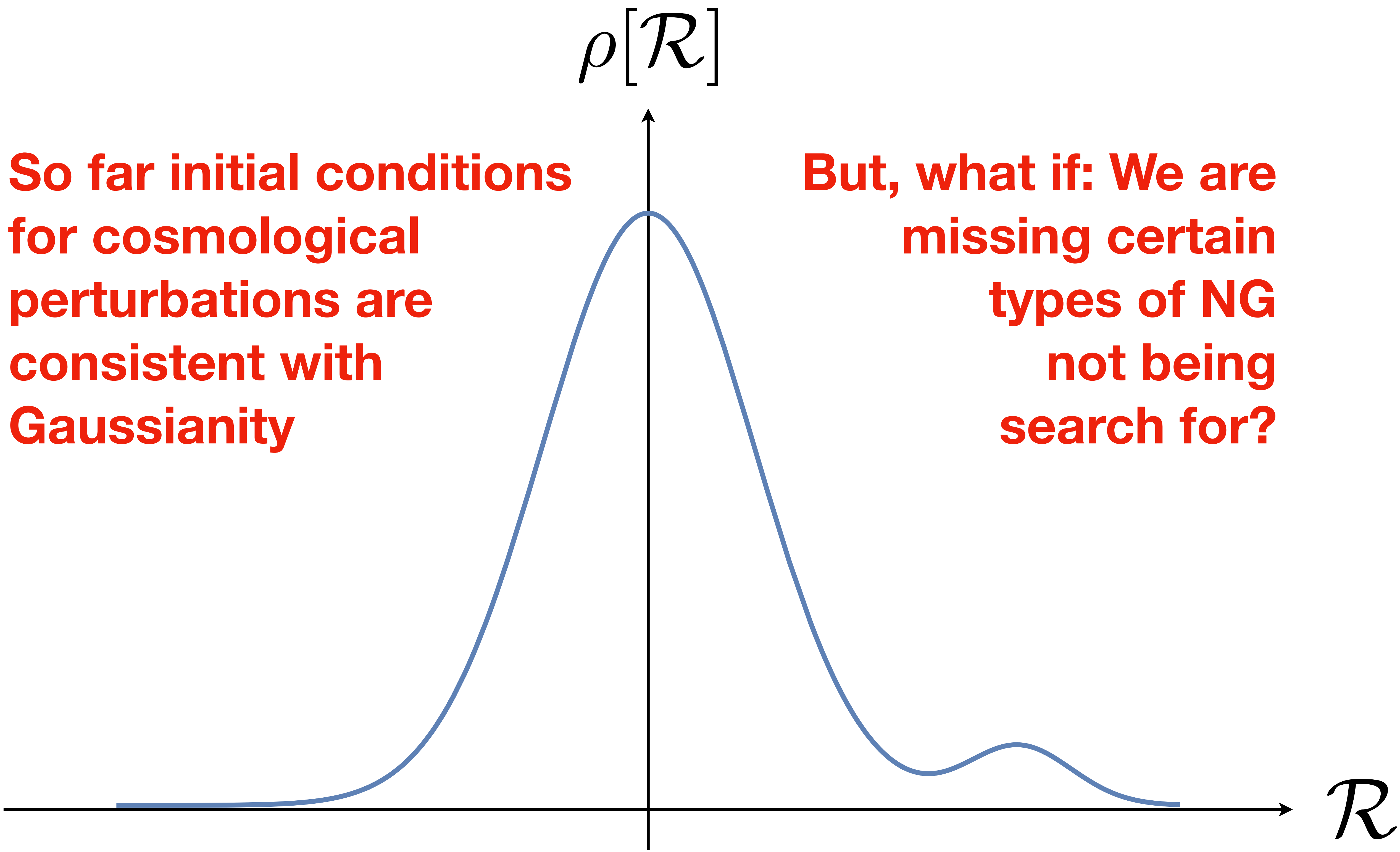

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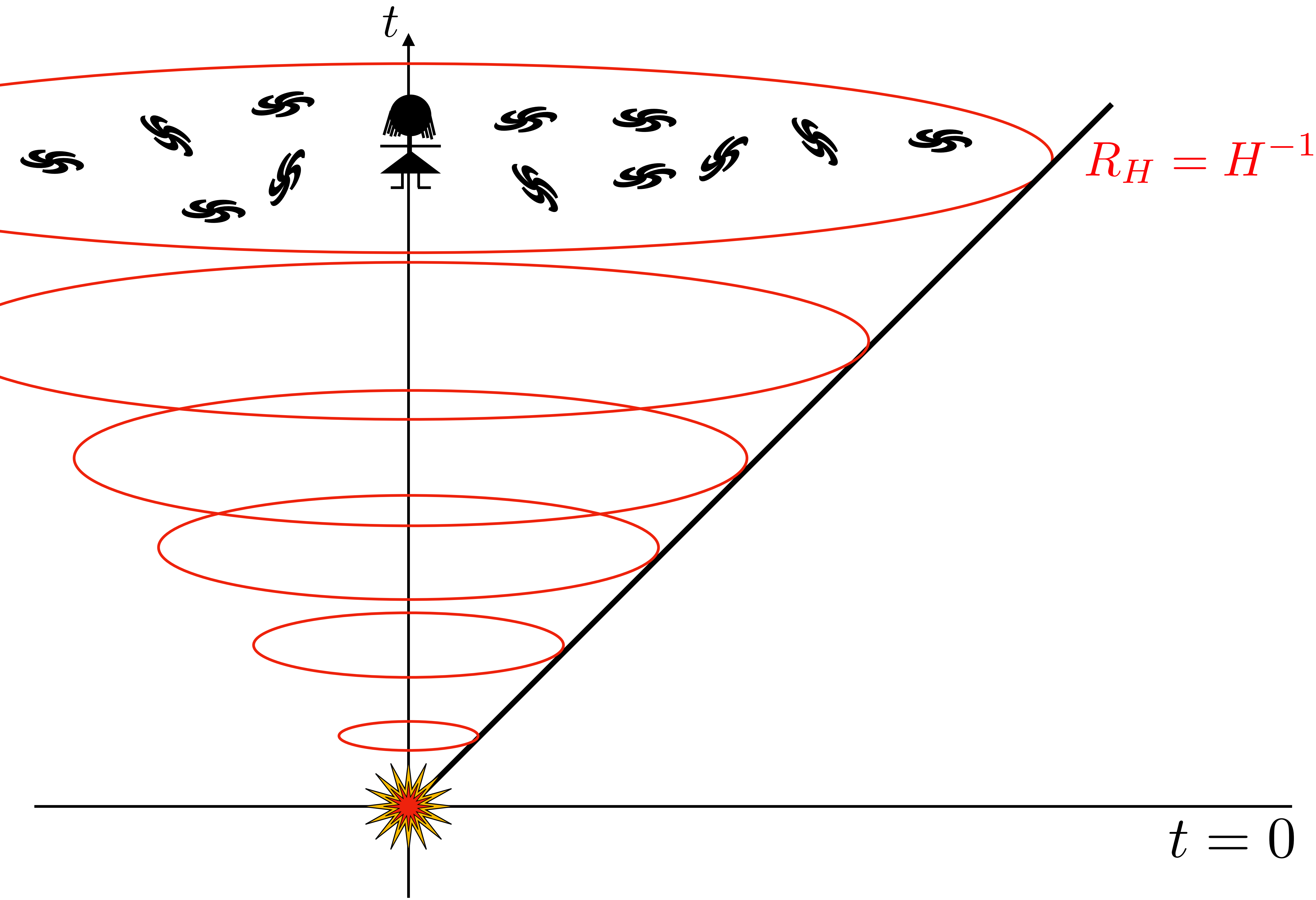
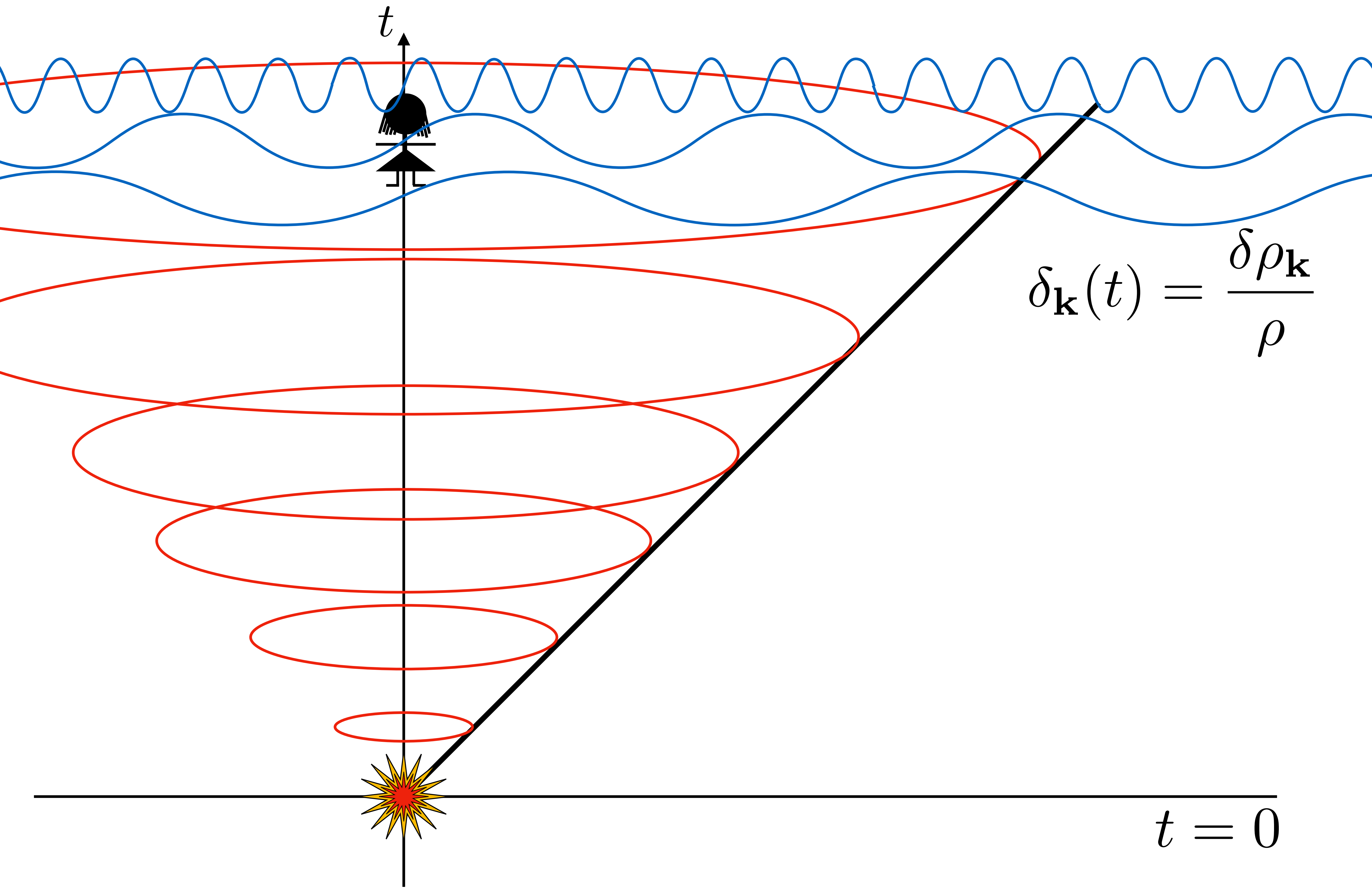
This may be relevant for the study of phenomena associated to large but rare fluctuations (PBH's)

Primordial fluctuations



Primordial fluctuations



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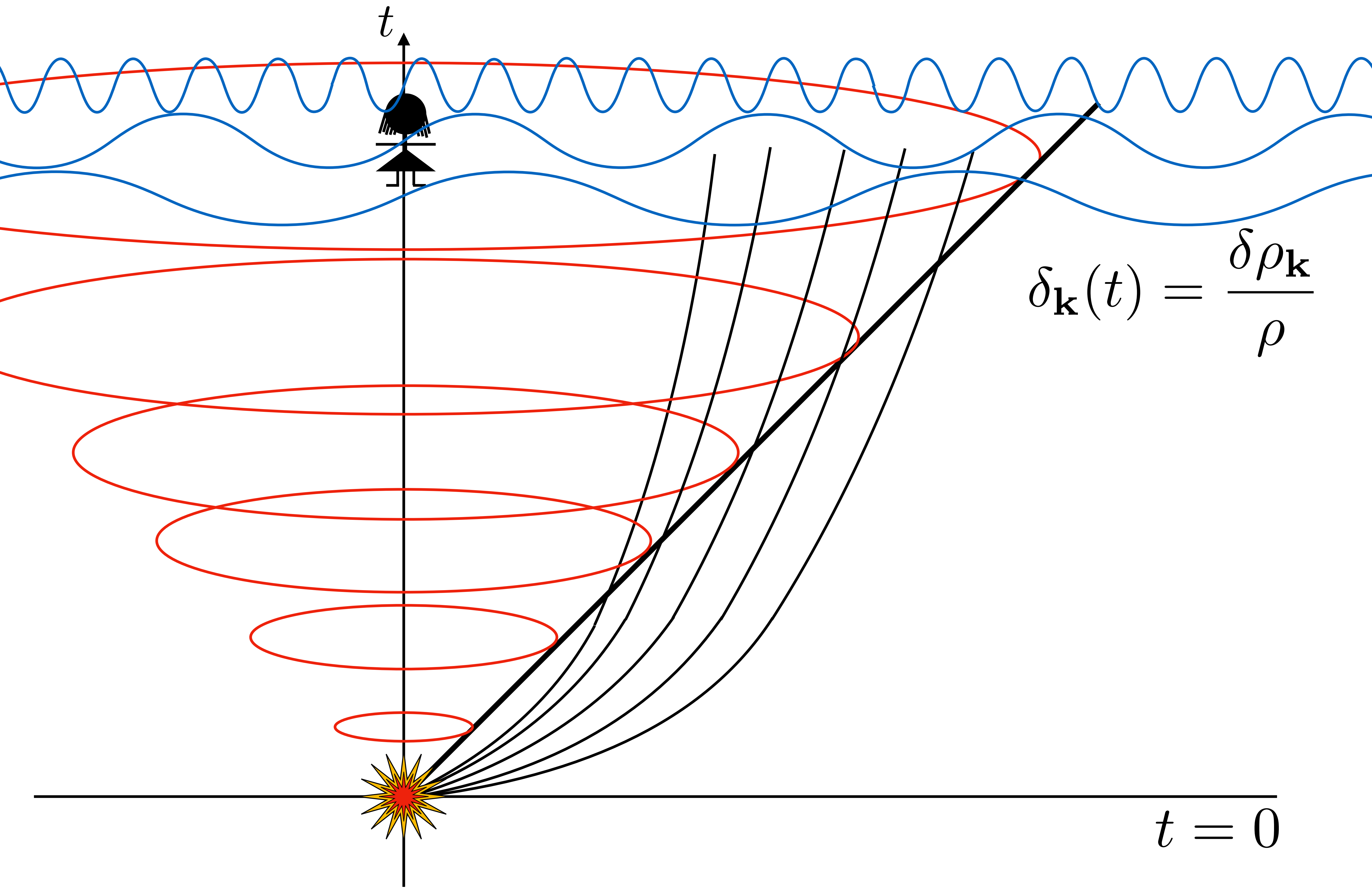
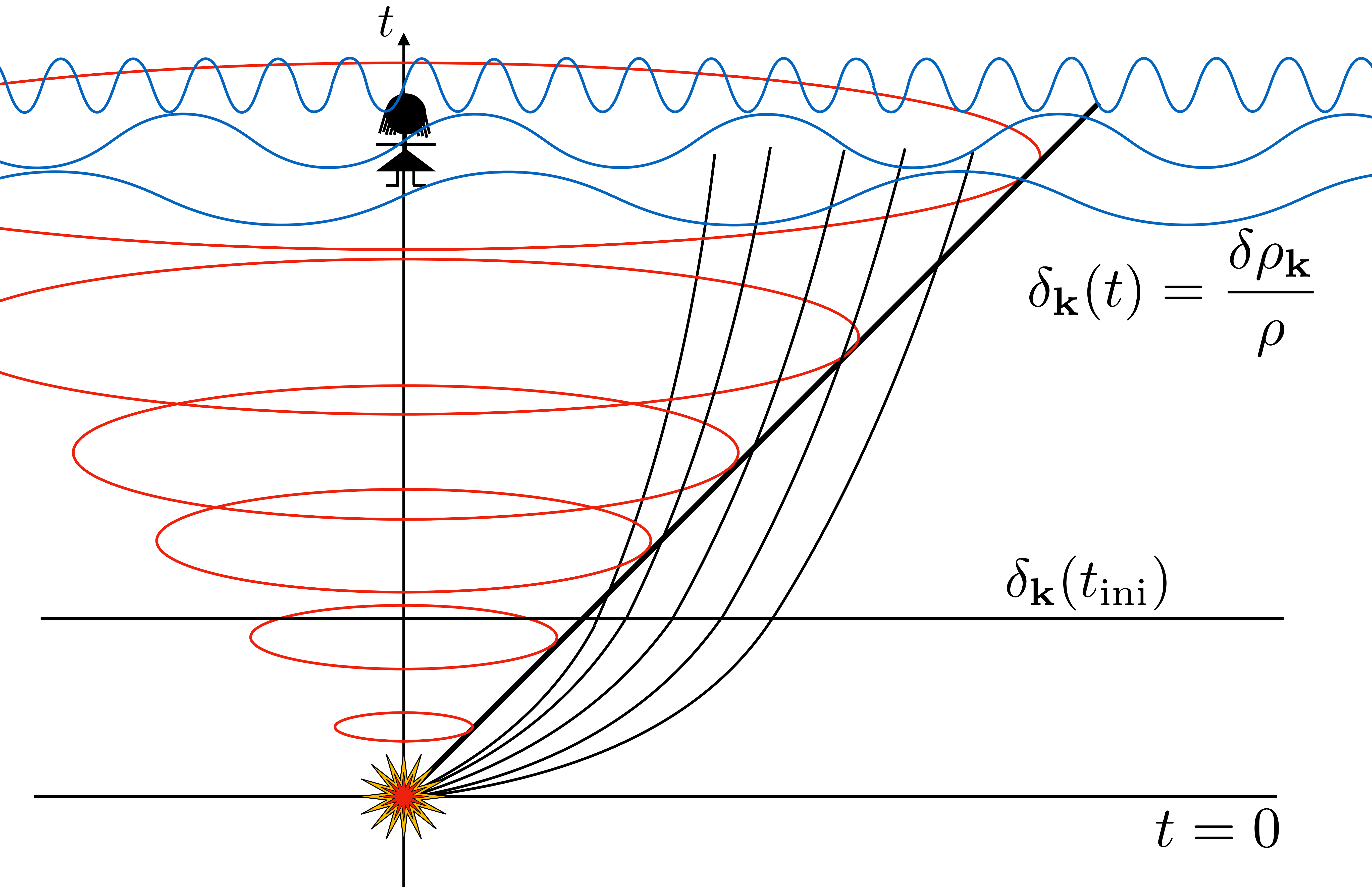


$$\delta_{\mathbf{k}}(t) = \frac{\delta\rho_{\mathbf{k}}}{\rho}$$

Primordial fluctuations



Primordial fluctuations

CMB and LSS tell us that the statistics of $\delta_{\mathbf{k}}(t_{\rm ini})$ is consistent with the following three characteristics:

* Adiabaticity
* Gaussianity
* Almost scale independency

* Adiabaticity

Every inhomogeneity is determined by a single fluctuation

$$\delta^{\gamma}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

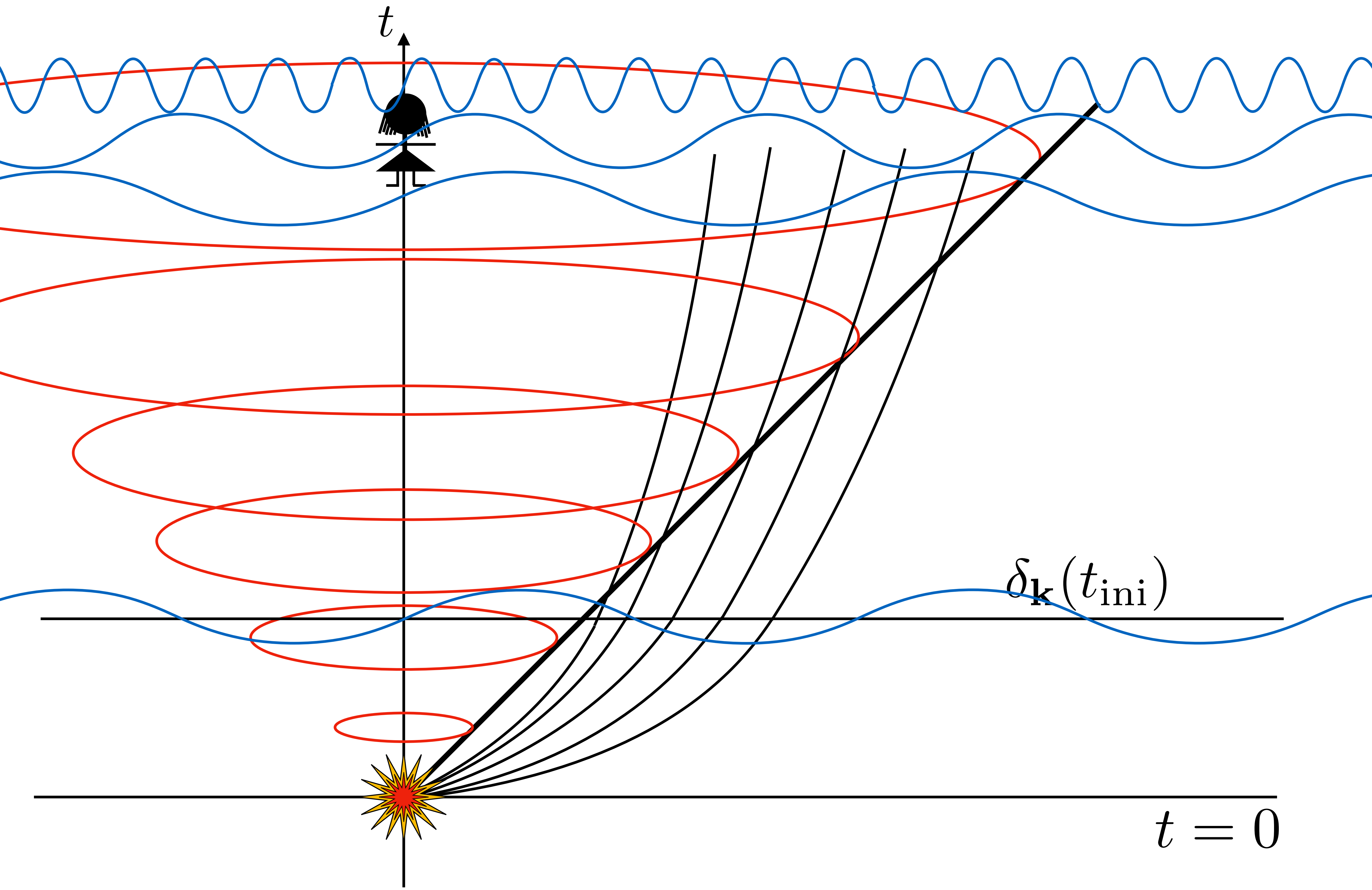
$$\delta^{\nu}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

$$\delta^{\rm Bar}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

$$\delta^{\rm DM}_{\mathbf{k}}(t_{\rm ini}) \propto \mathcal{R}_{\mathbf{k}}$$

$$ds^2 = -dt^2 + a^2(t)e^{2\mathcal{R}(t,\mathbf{x})}d\mathbf{x}^2$$

Primordial fluctuations



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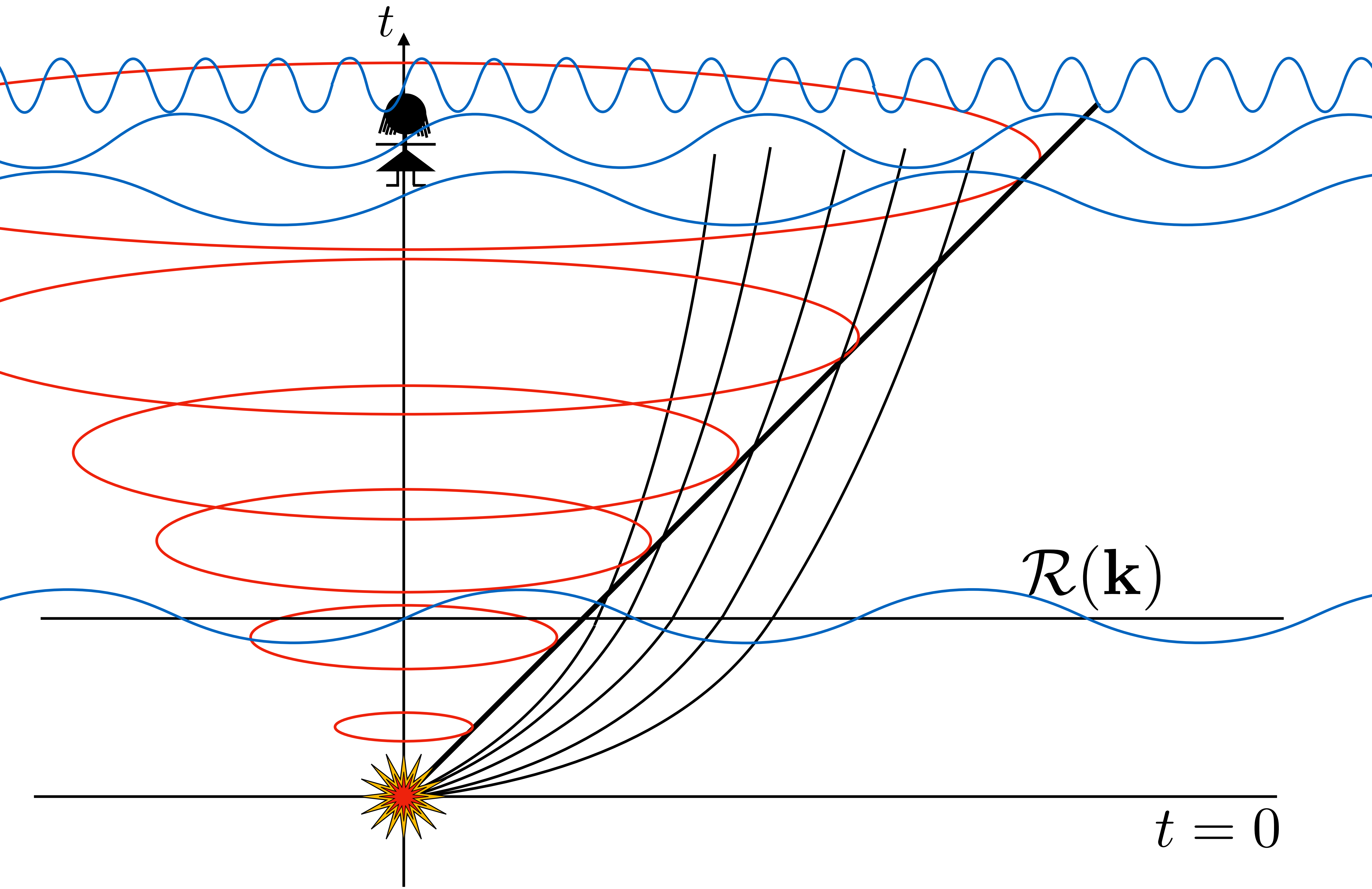
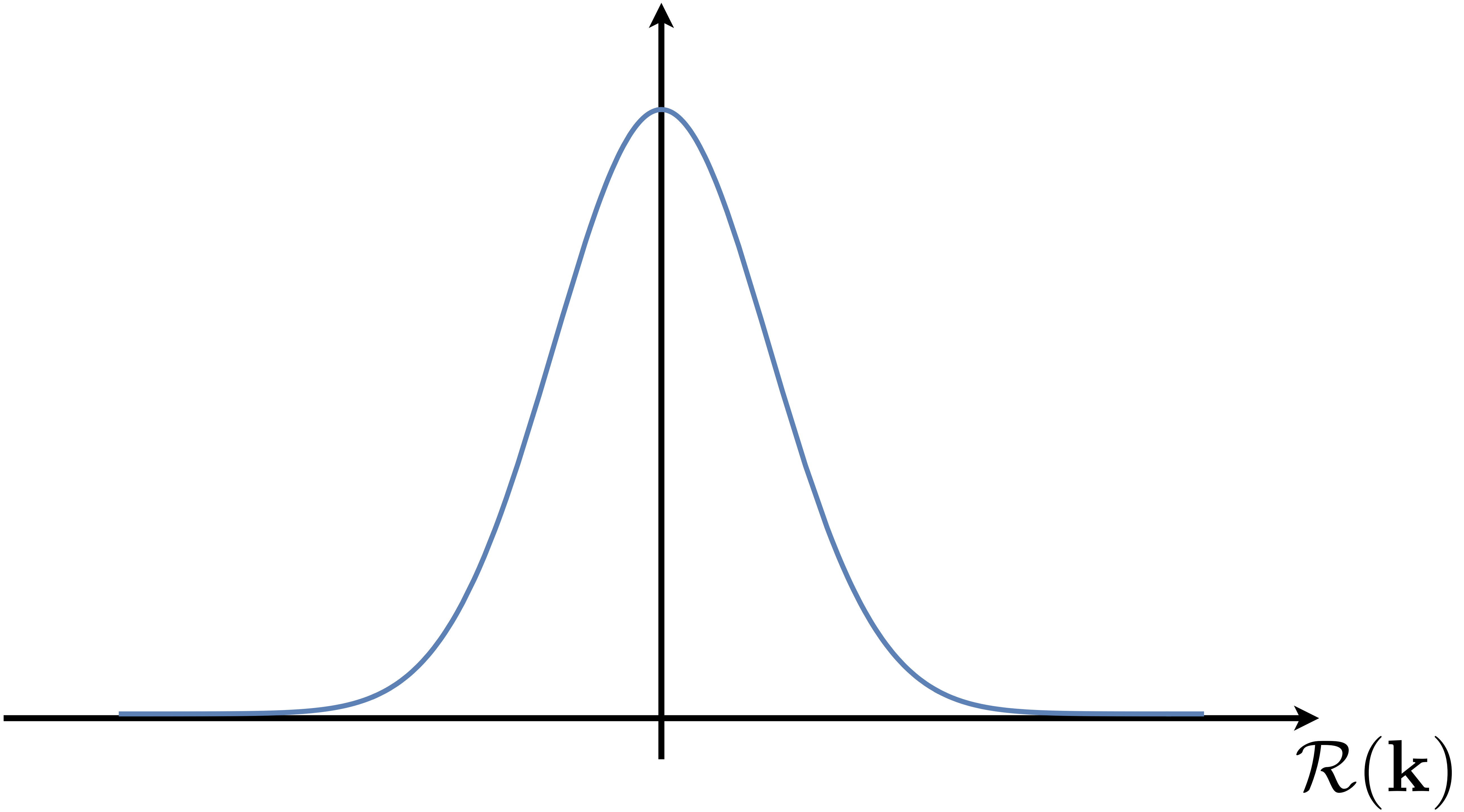


Primordial fluctuations

* Gaussianity

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2}\int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}}$$



Primordial fluctuations

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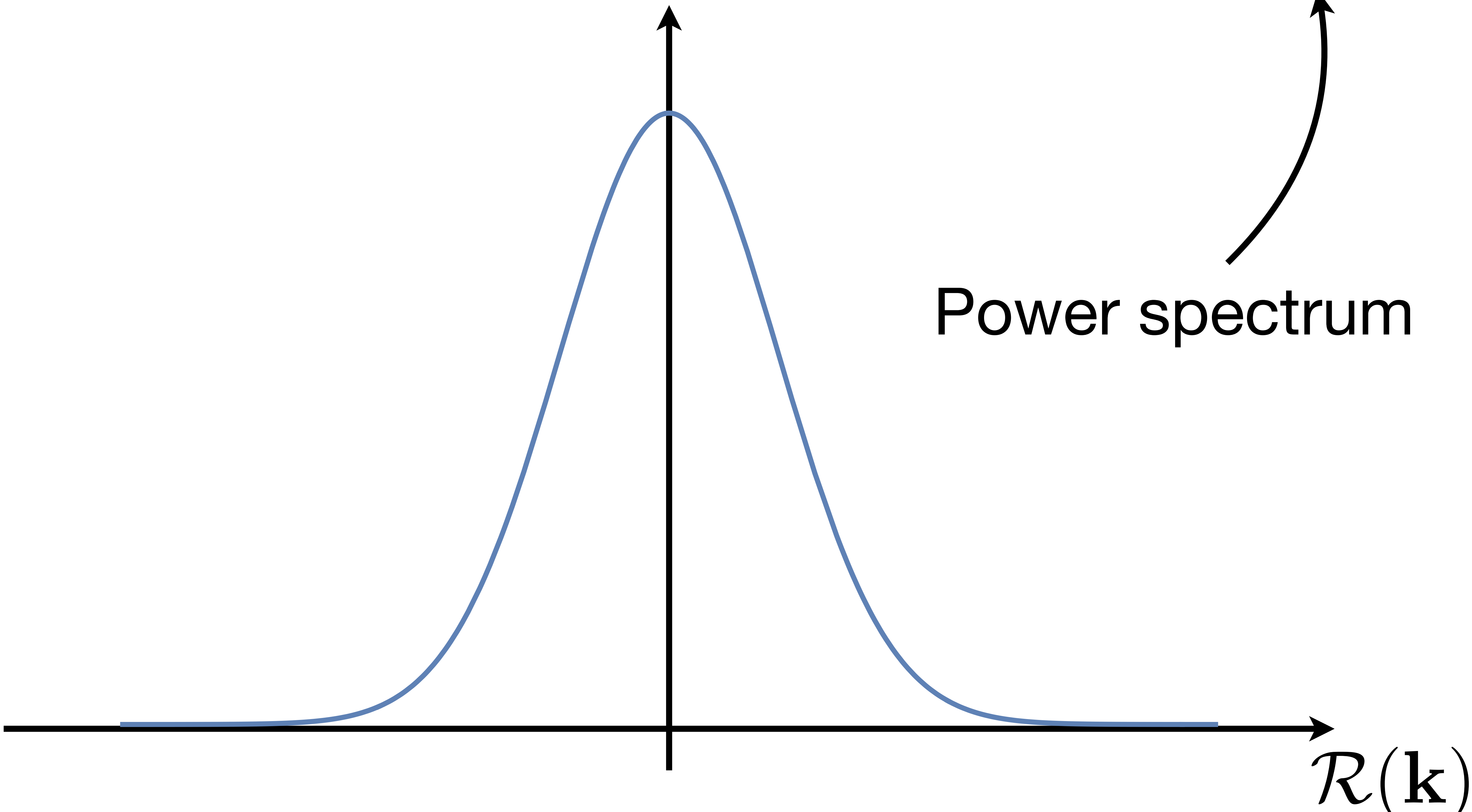
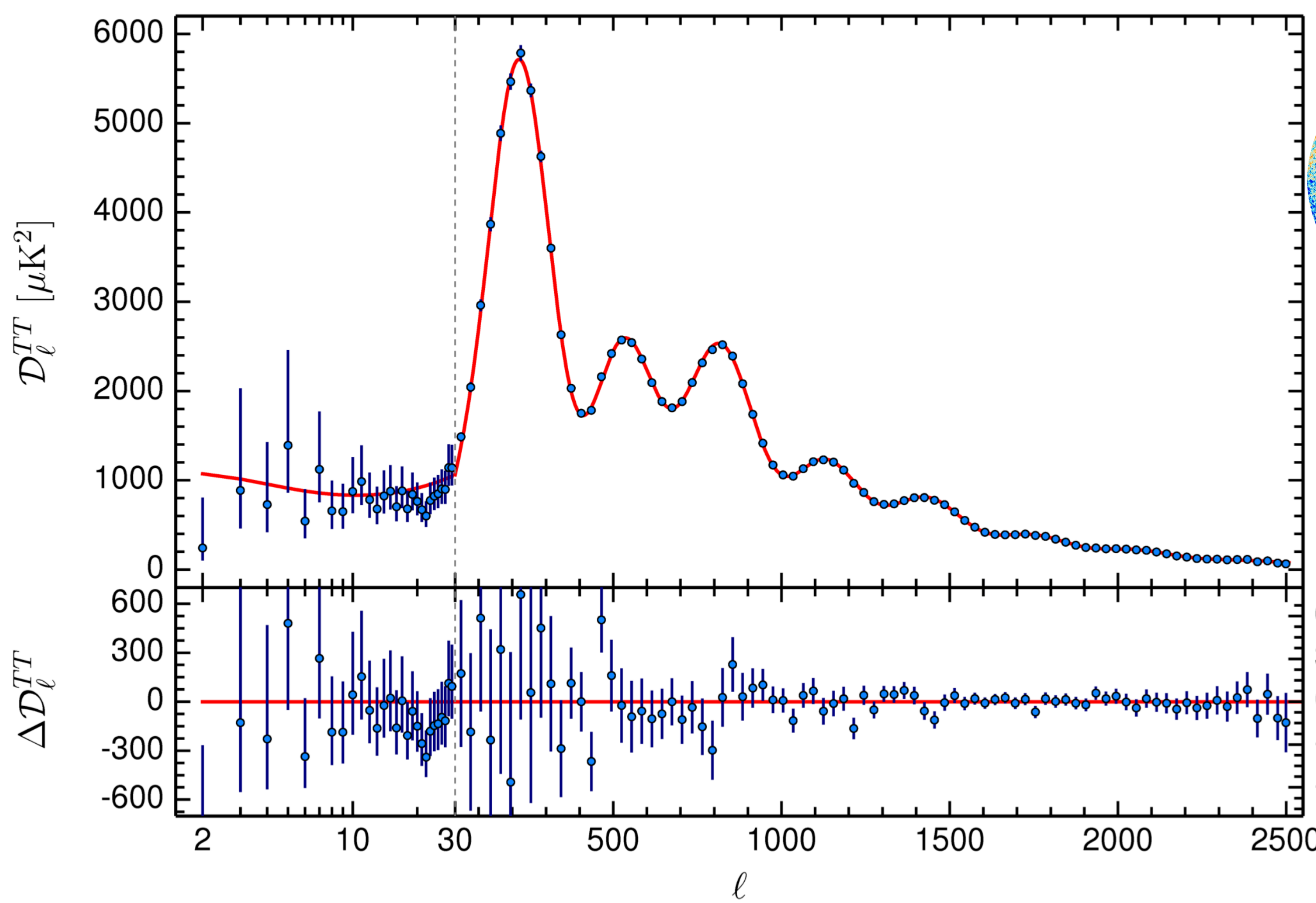
Power spectrum

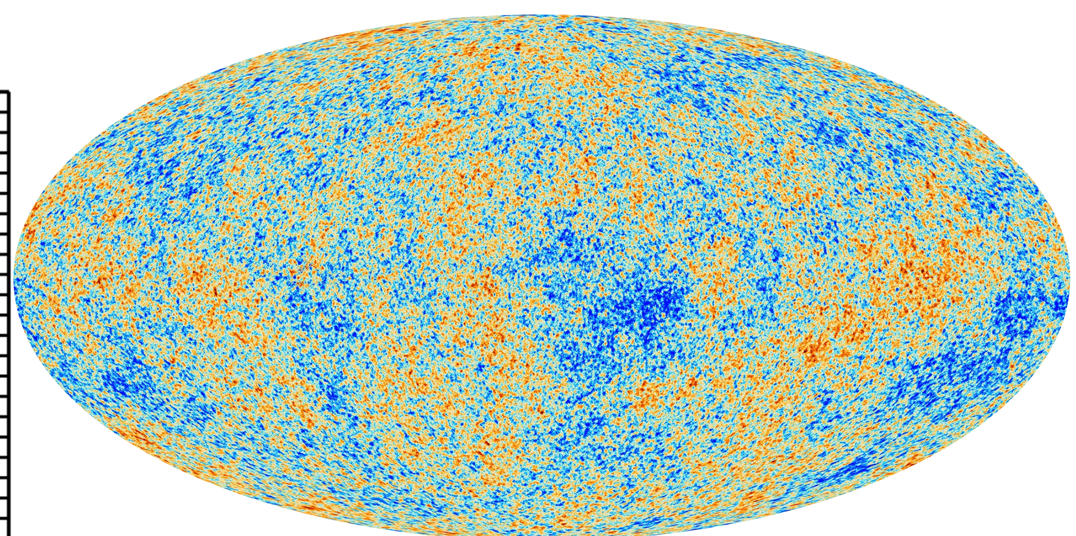


Primordial fluctuations

* Almost scale independent

$$P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3}\Delta_{\mathcal{R}}(k) \qquad \Delta_{\mathcal{R}}(k) = A\left(\frac{k}{k_*}\right)^{n_s-1}$$





$$n_s \simeq 0.96$$

Planck collaboration (2018)

Non-Gaussianity?

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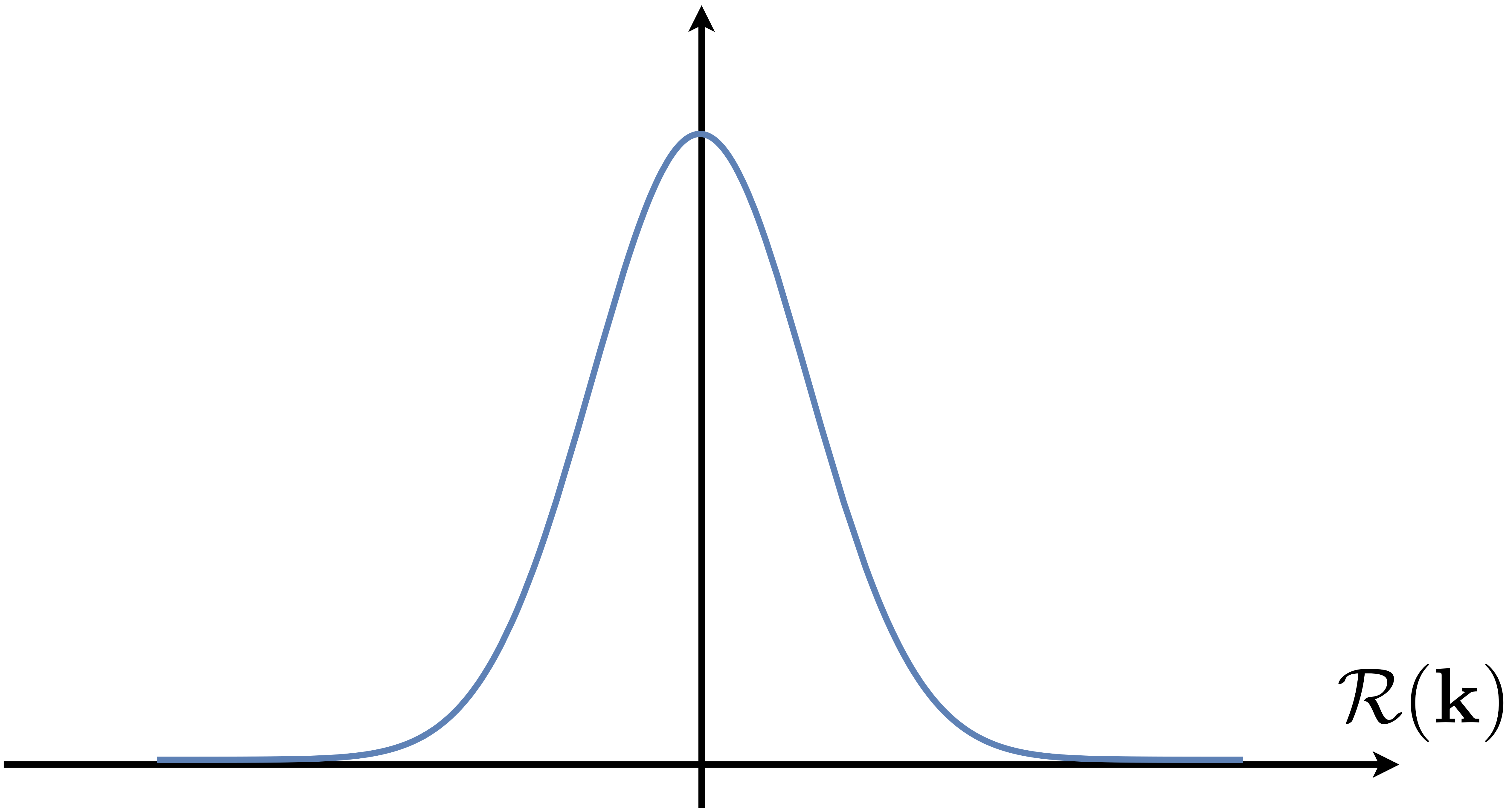
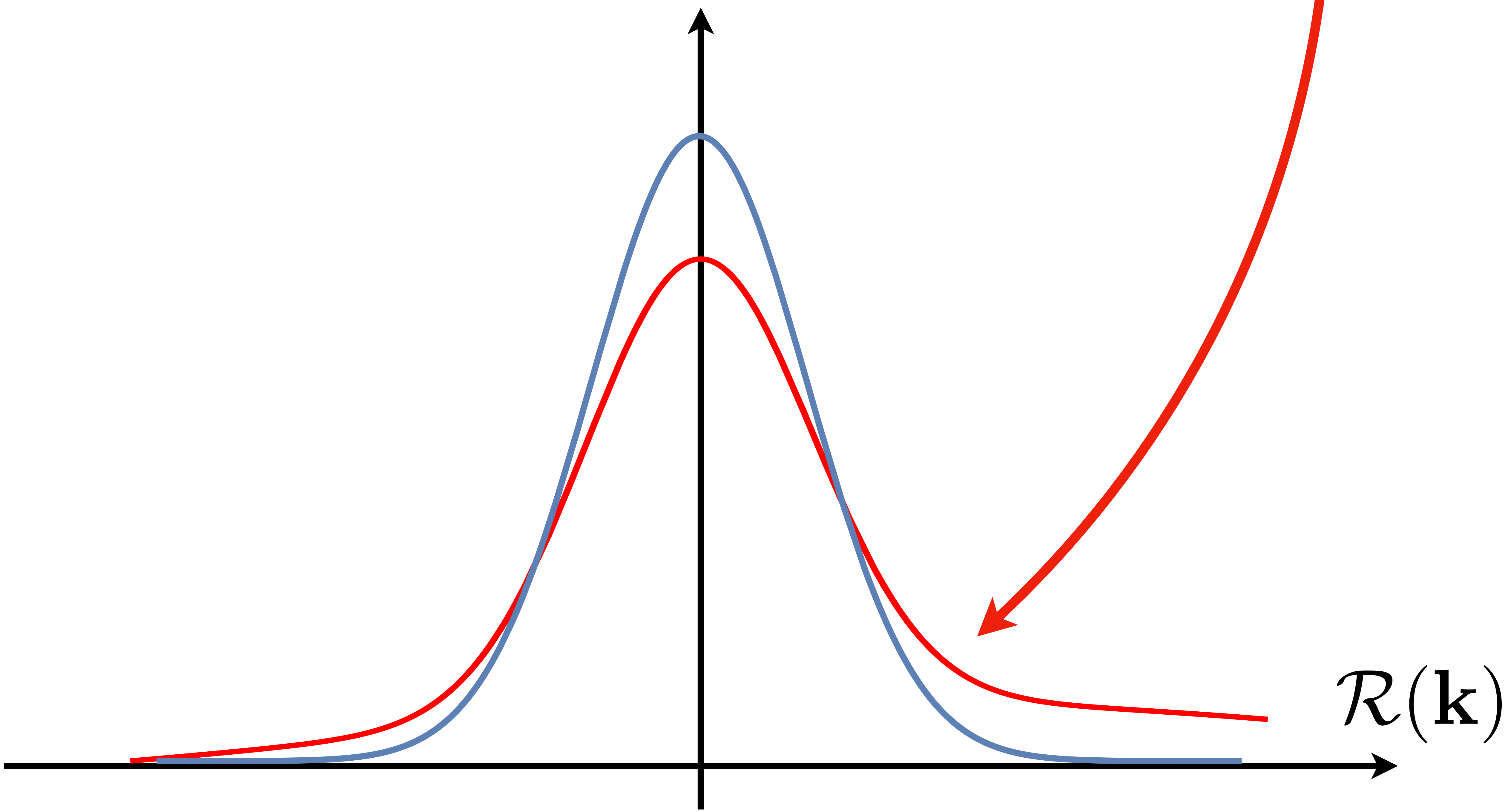


Non-Gaussianity?

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2}\int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \cdots$$

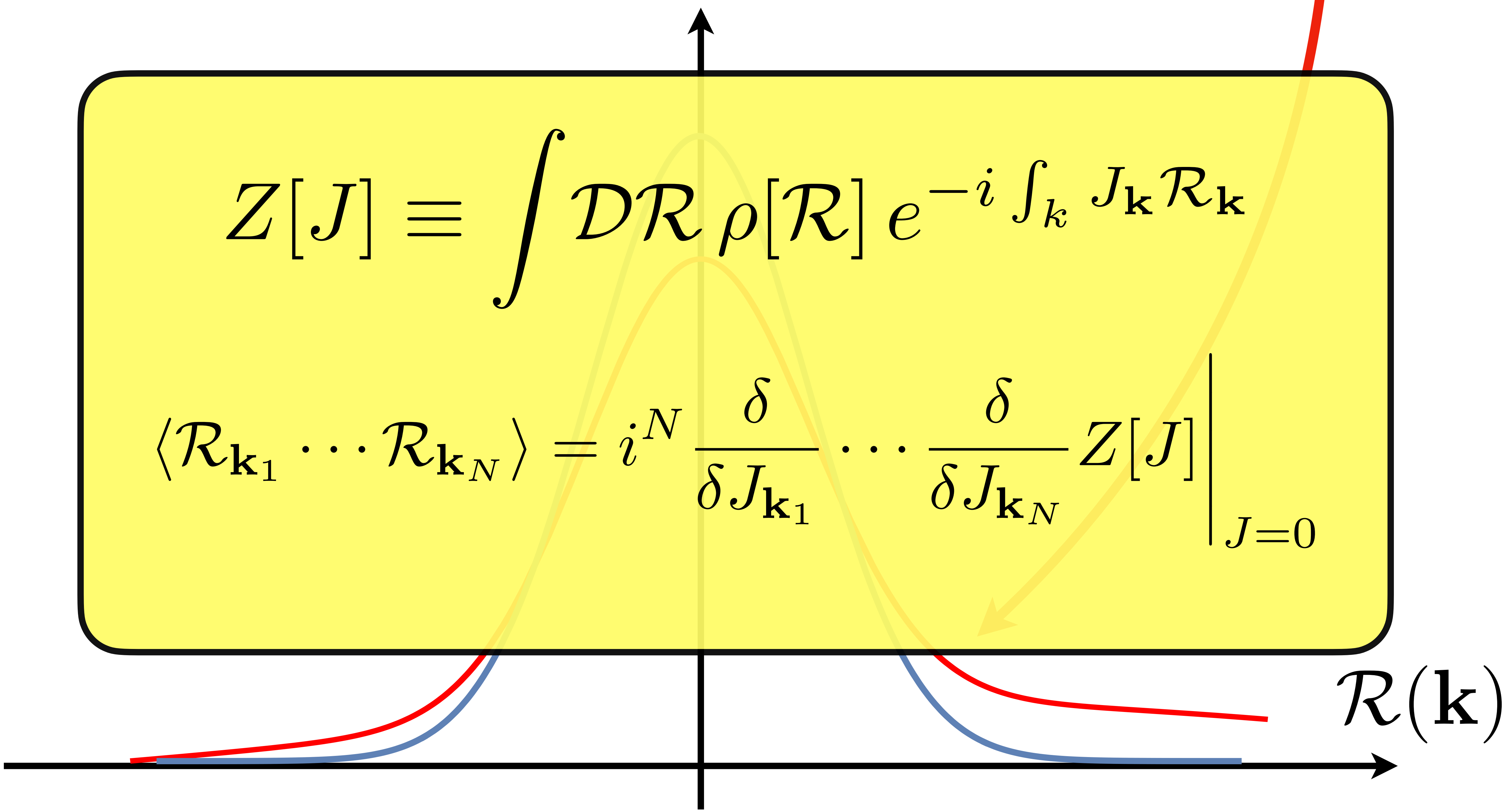


Non-Gaussianity?

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2}\int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \cdots$$

$$Z[J] \equiv \int \mathcal{D}\mathcal{R}\, \rho[\mathcal{R}]\, e^{-i\int_k J_{\mathbf{k}}\mathcal{R}_{\mathbf{k}}}$$

$$\langle \mathcal{R}_{\mathbf{k}_1} \cdots \mathcal{R}_{\mathbf{k}_N} \rangle = i^N \frac{\delta}{\delta J_{\mathbf{k}_1}} \cdots \frac{\delta}{\delta J_{\mathbf{k}_N}} Z[J] \Big|_{J=0}$$



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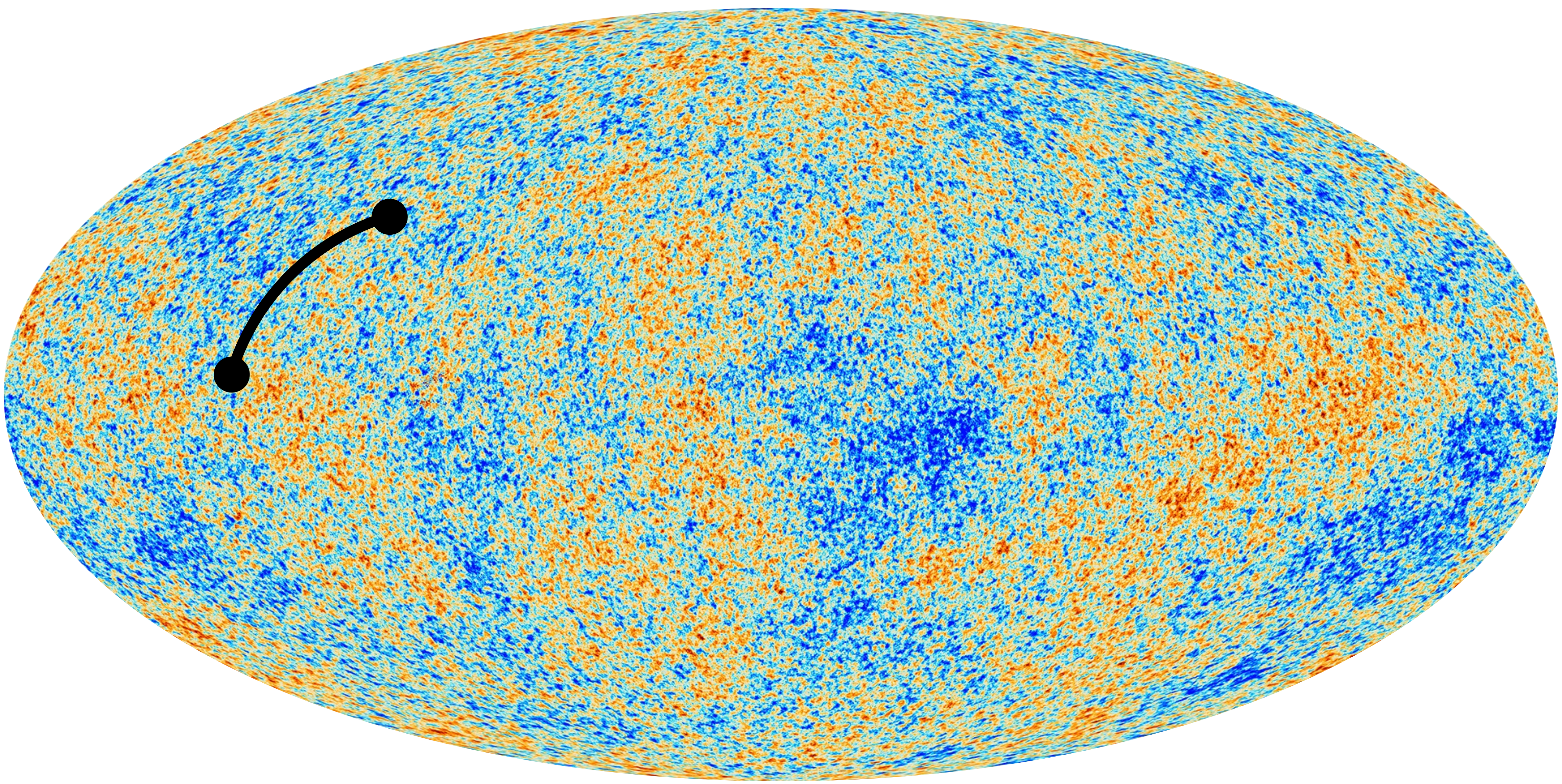

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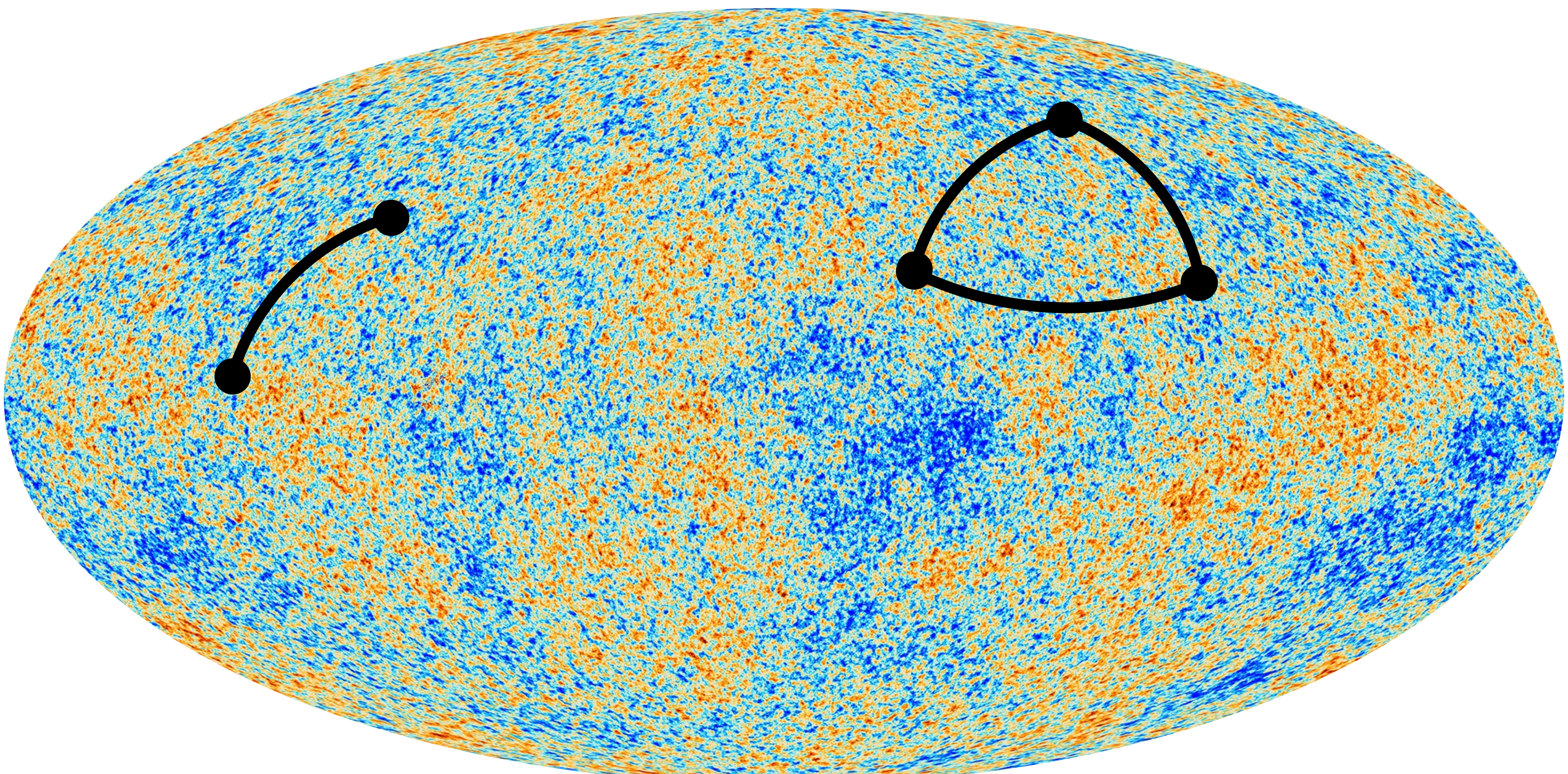

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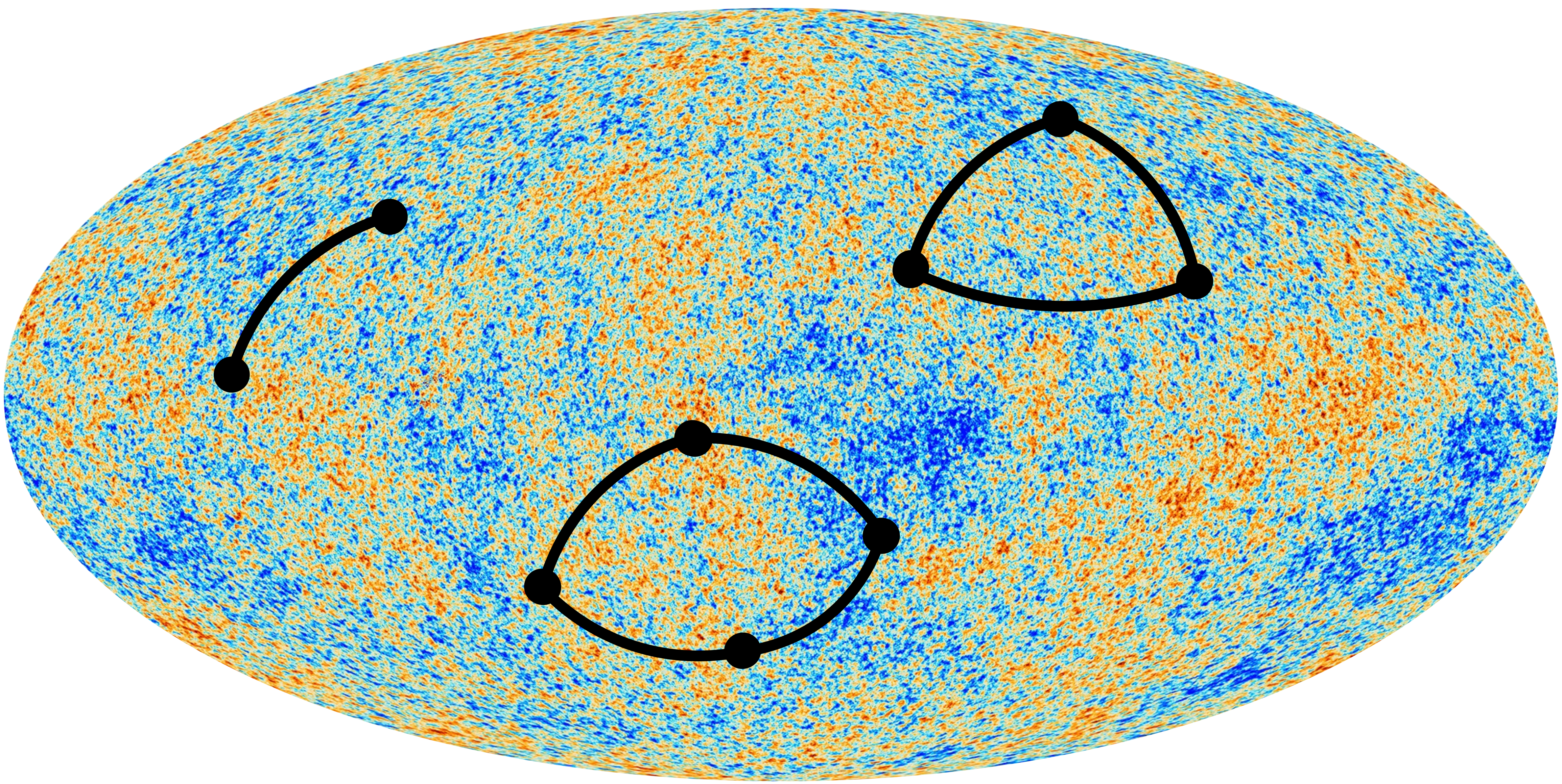

Non-Gaussianity?

The bispectrum parametrizes the simplest deviation to NG

$$\rho[\mathcal{R}] \propto e^{-\frac{1}{2}\int_k \frac{|\mathcal{R}_{\mathbf{k}}|^2}{P(k)}} + \cdots$$

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$$+\cdots = \int_{k_1}\int_{k_2}\int_{k_3} B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_2)\mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3} + \cdots$$

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$$\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

Non-Gaussianity?

In the absence of a specific theory, a common parametrization for the bispectrum is the $f_{\rm NL}$ parameter

$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{\rm loc}\, \mathcal{R}_G^2(\mathbf{x})$$

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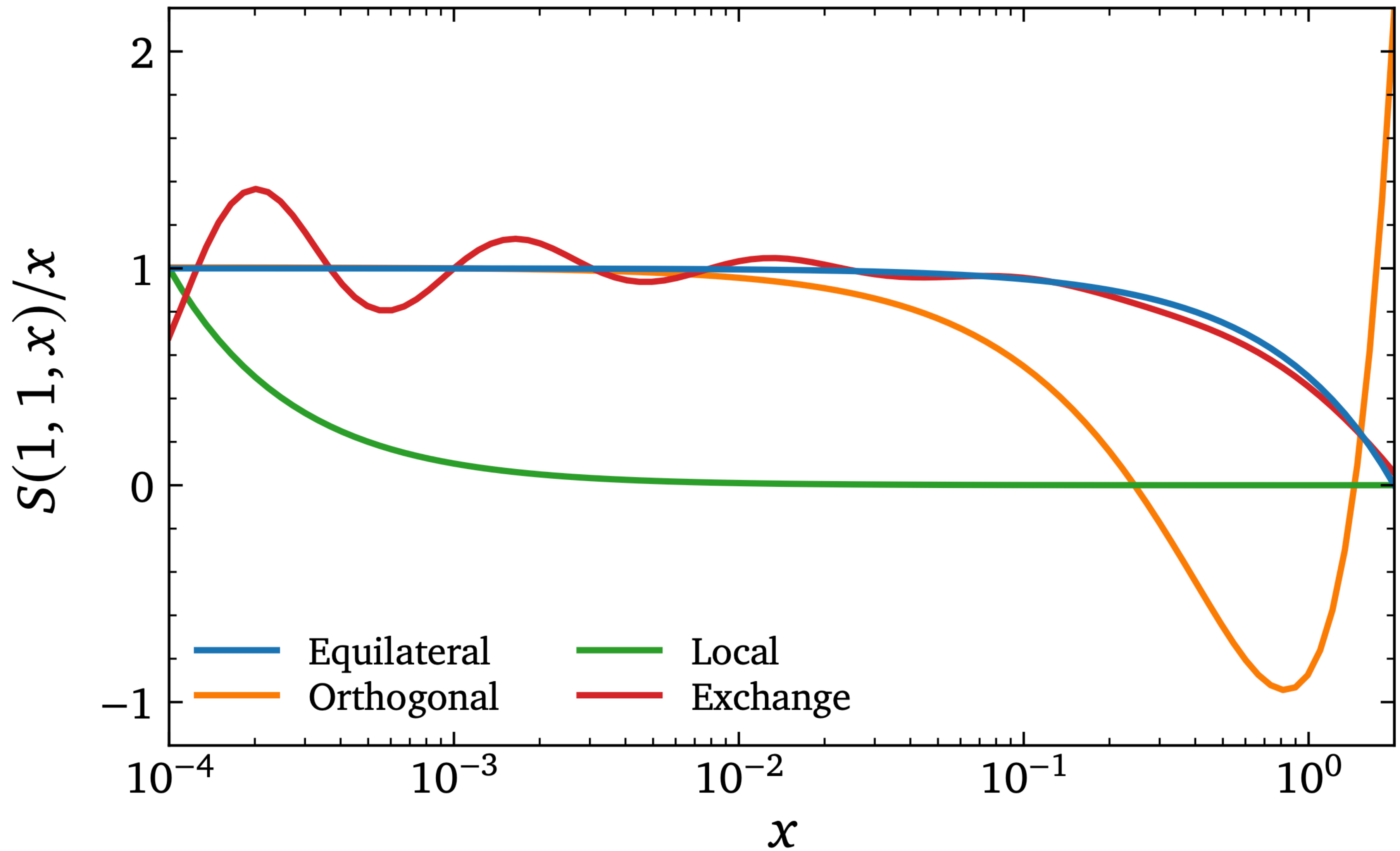
$$\boxed{B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{18}{5} A^2 \sum_{\rm type} f_{\rm NL}^{\rm type} S_{\rm type}(k_1, k_2, k_3)}$$

Non-Gaussianity?

In the … the
bispe…



Pimentel, Wallisch, Wu, Achúcarro et al. (2022)

$$B(\ldots \mathbf{k}_1))$$

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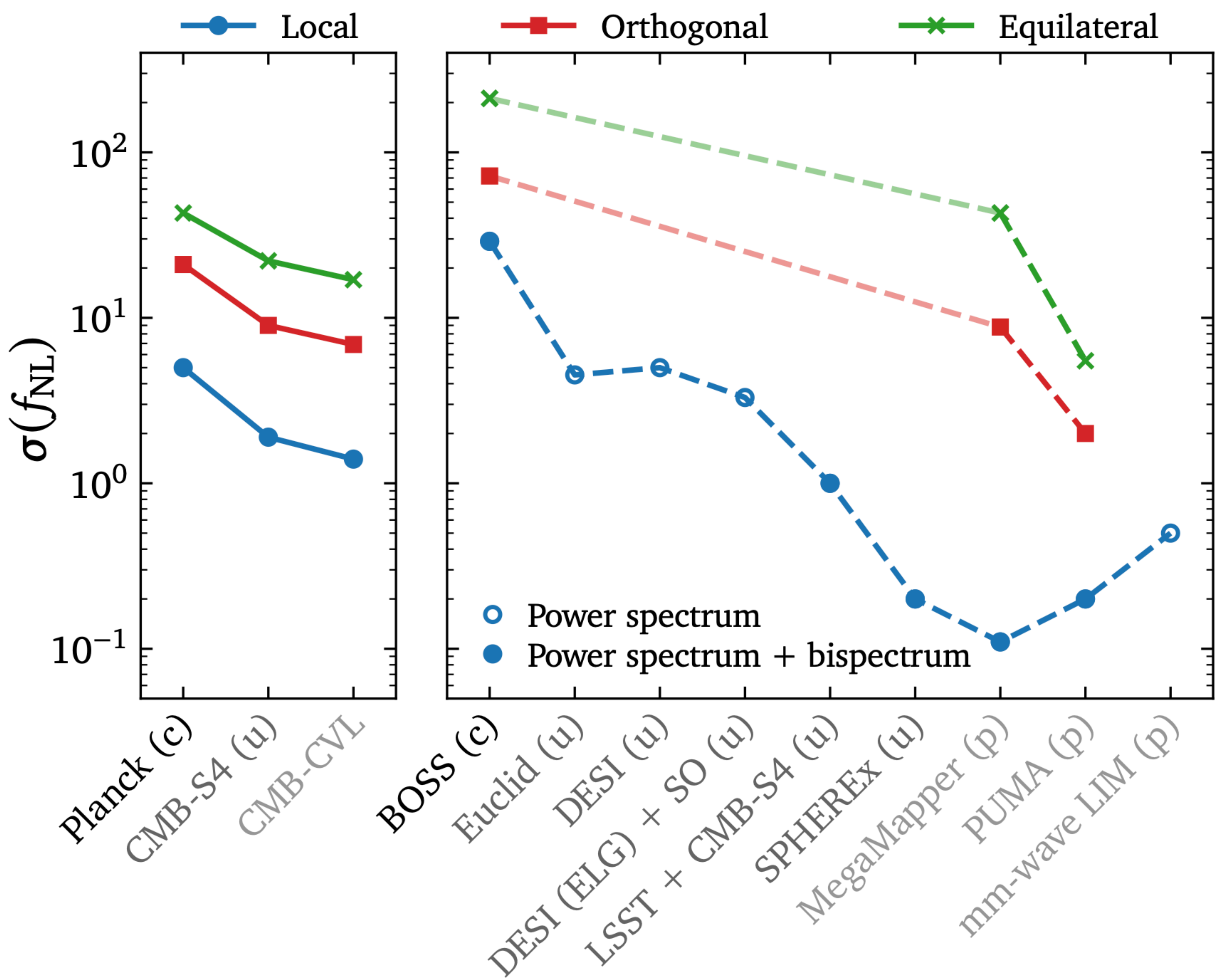

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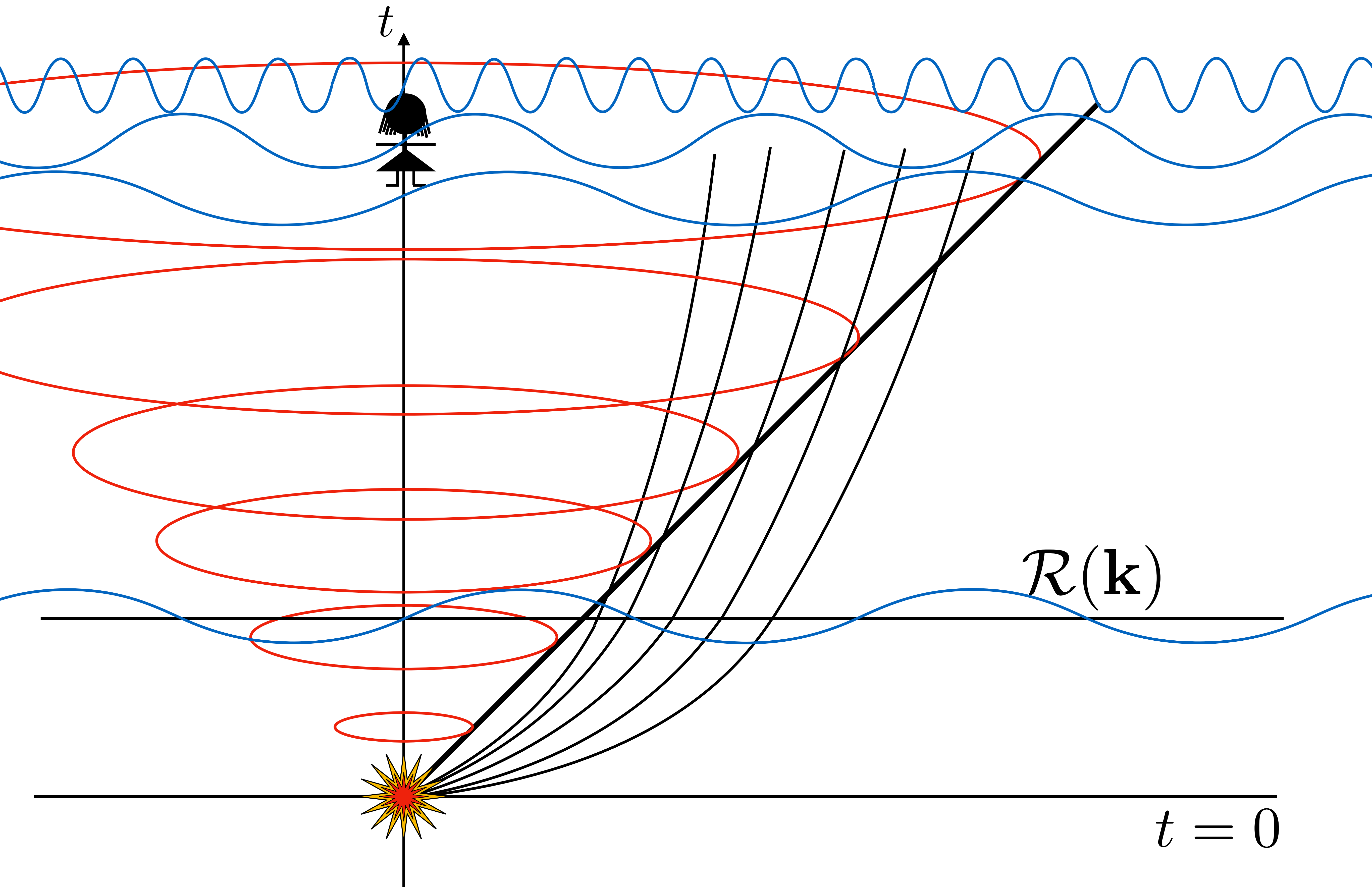
Inflation



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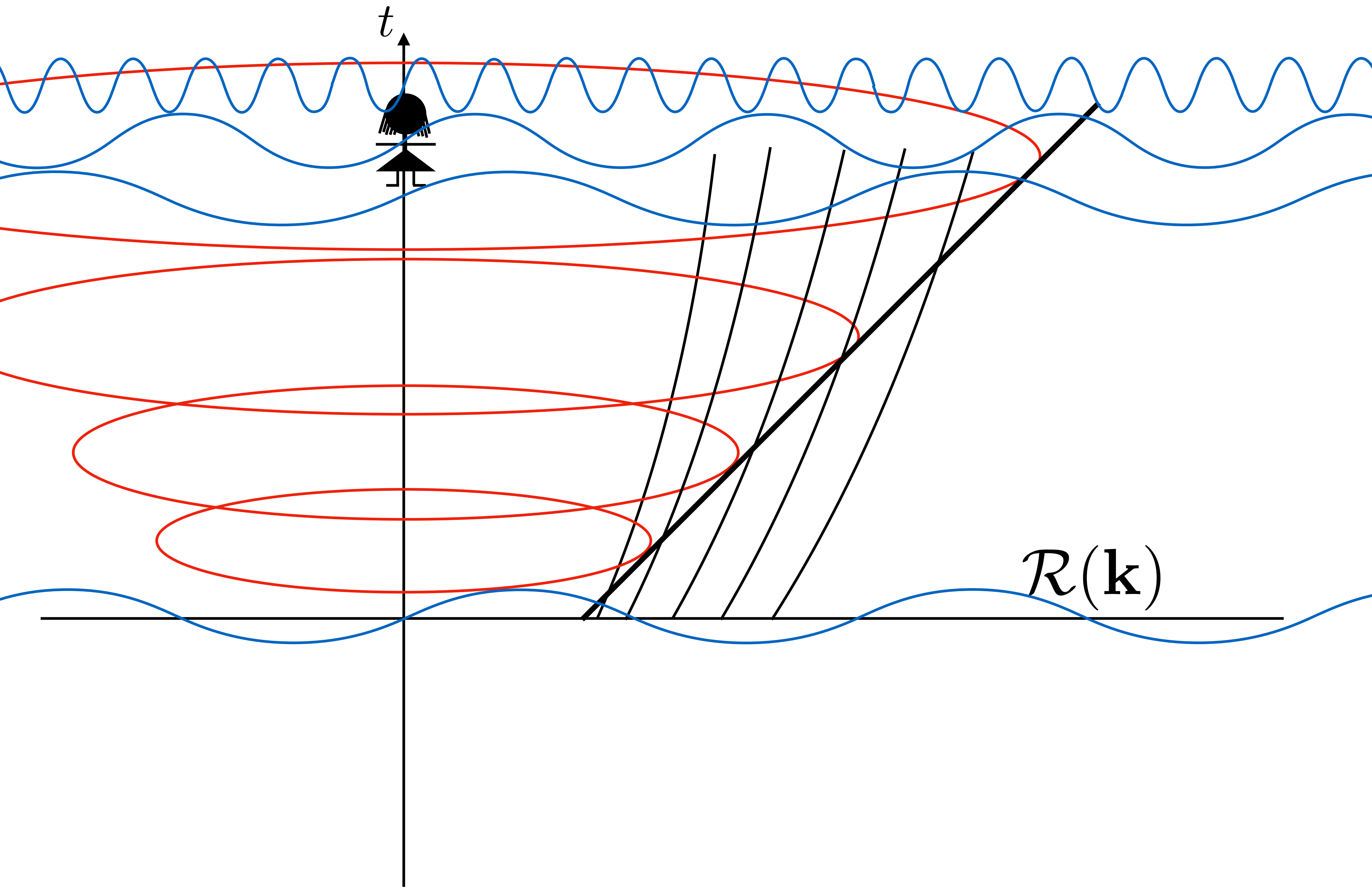

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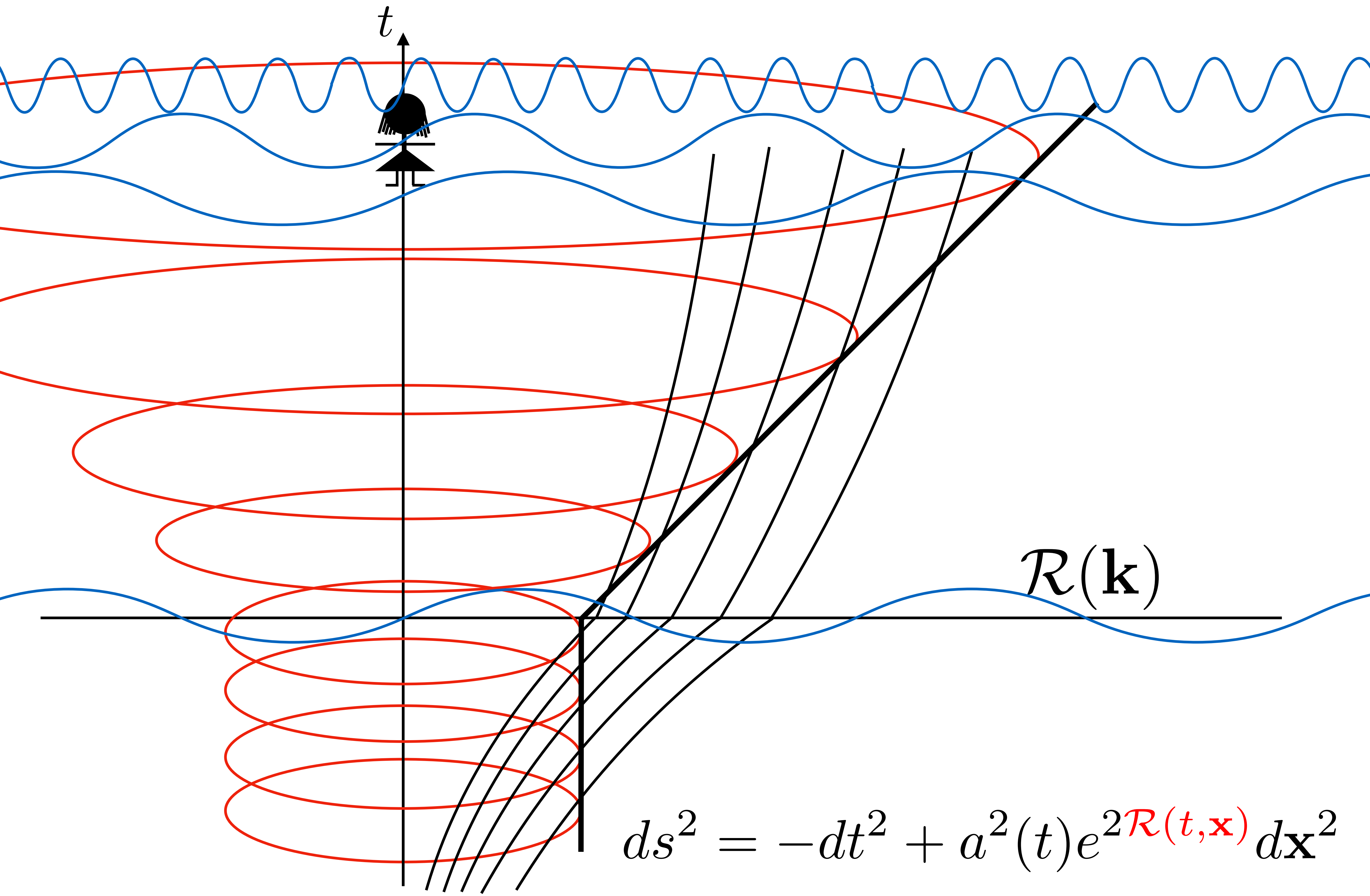
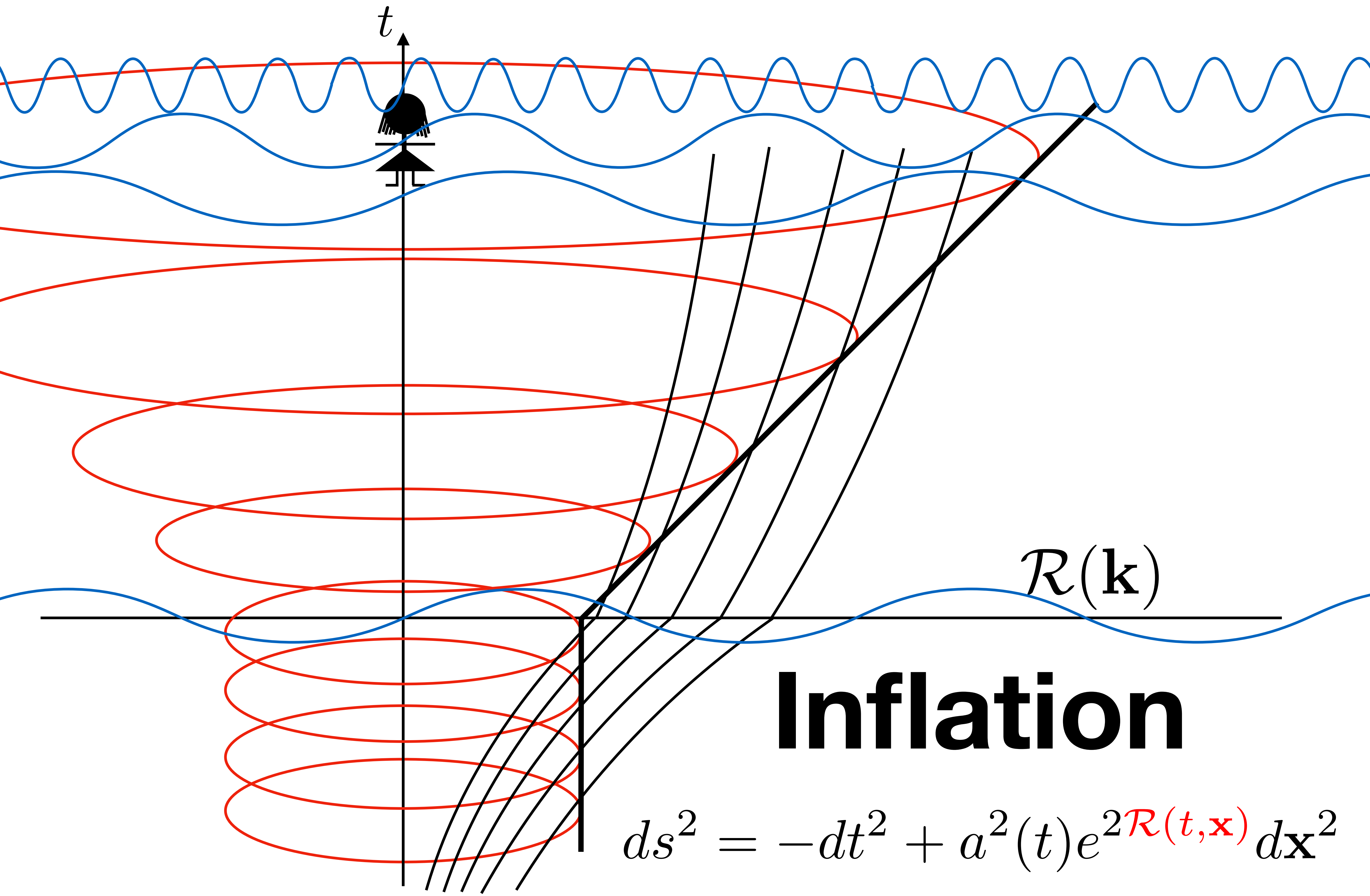


$$ds^2 = -dt^2 + a^2(t)e^{2\mathcal{R}(t,\mathbf{x})}d\mathbf{x}^2$$



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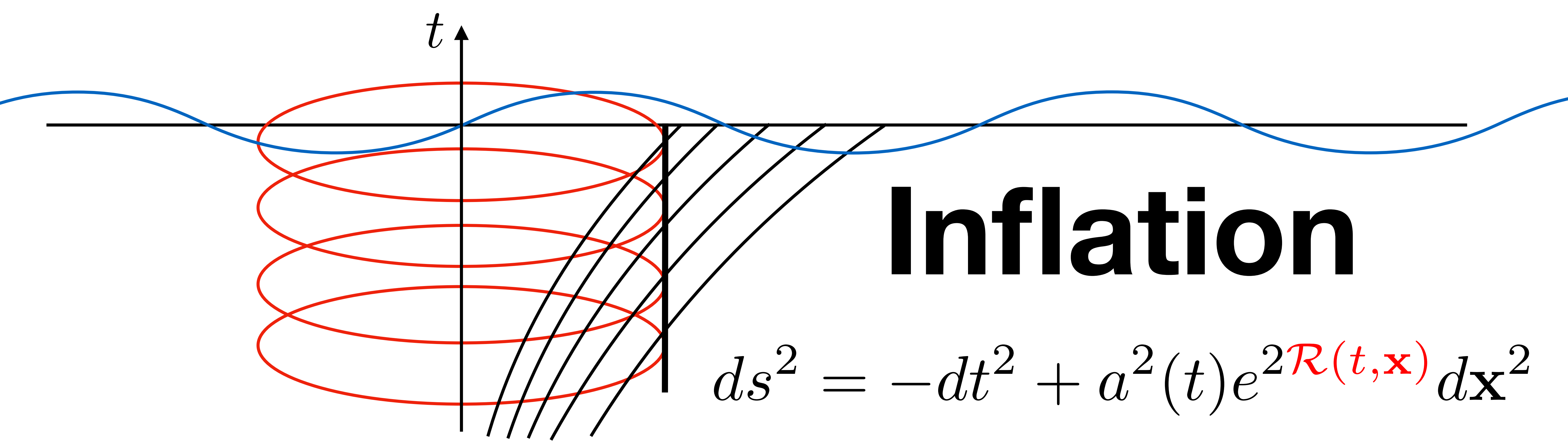

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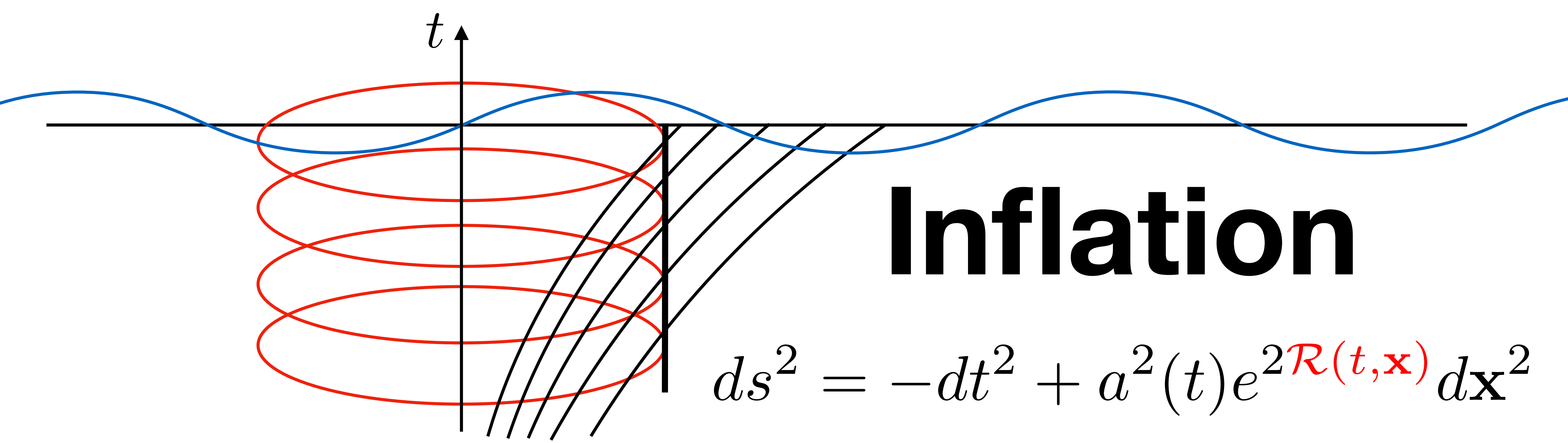
Inflation

Inflation is a concrete theory that allows us to come back to these three assumptions about initial conditions.



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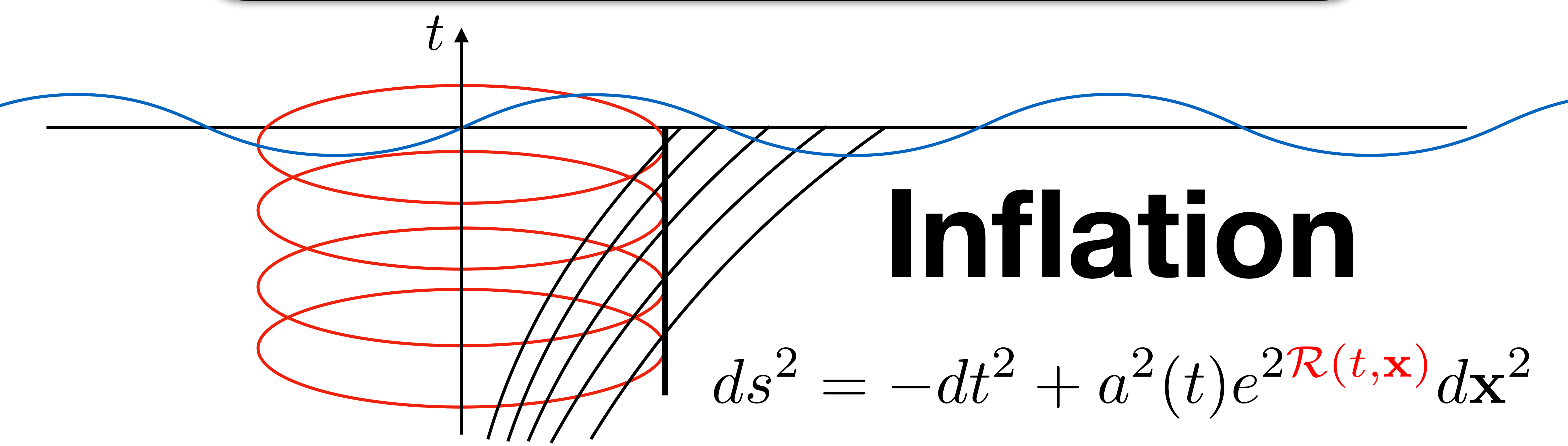

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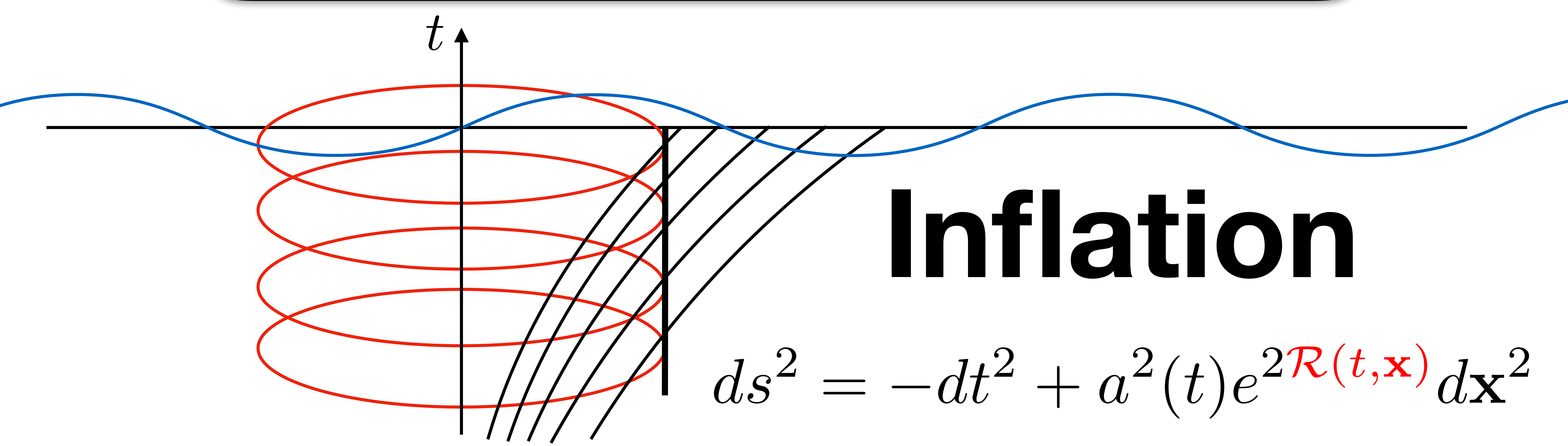
And so…

Are initial conditions truly adiabatic, Gaussian and nearly scale independent?



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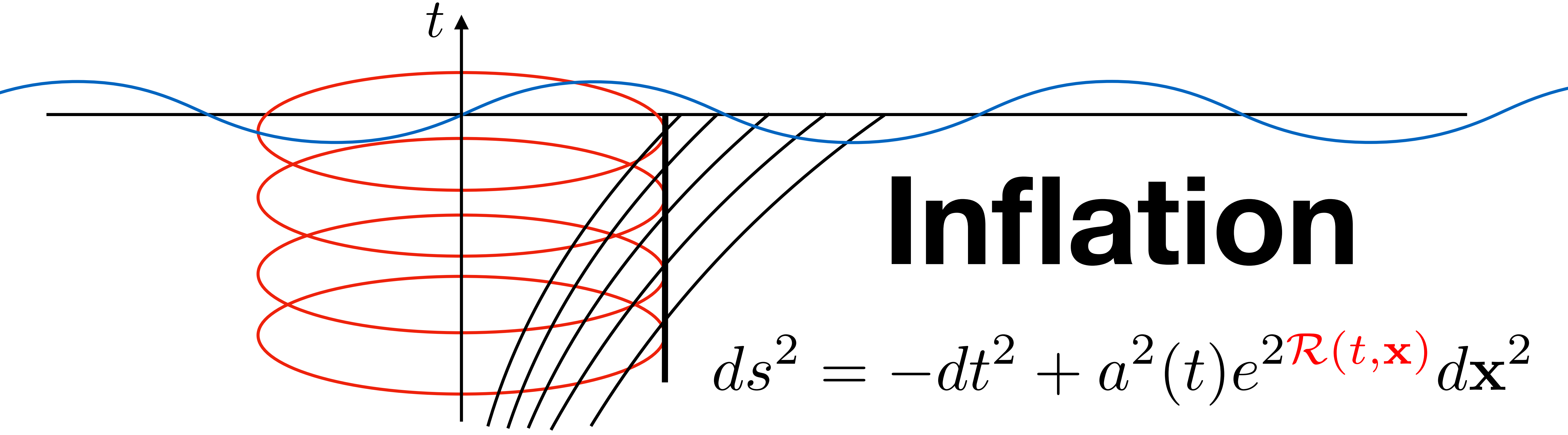

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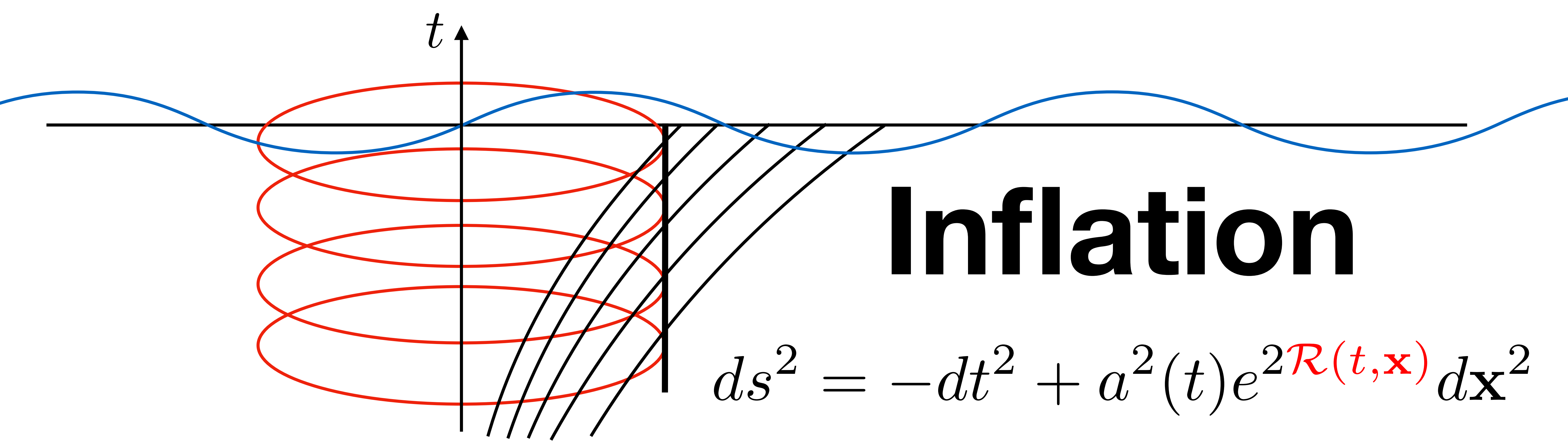
Inflation

$$S = \int d^4x\, a^3 \epsilon \left[\dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 + \mathcal{O}(\mathcal{R}^3) + \mathcal{O}(\mathcal{R}^4) + \cdots \right]$$



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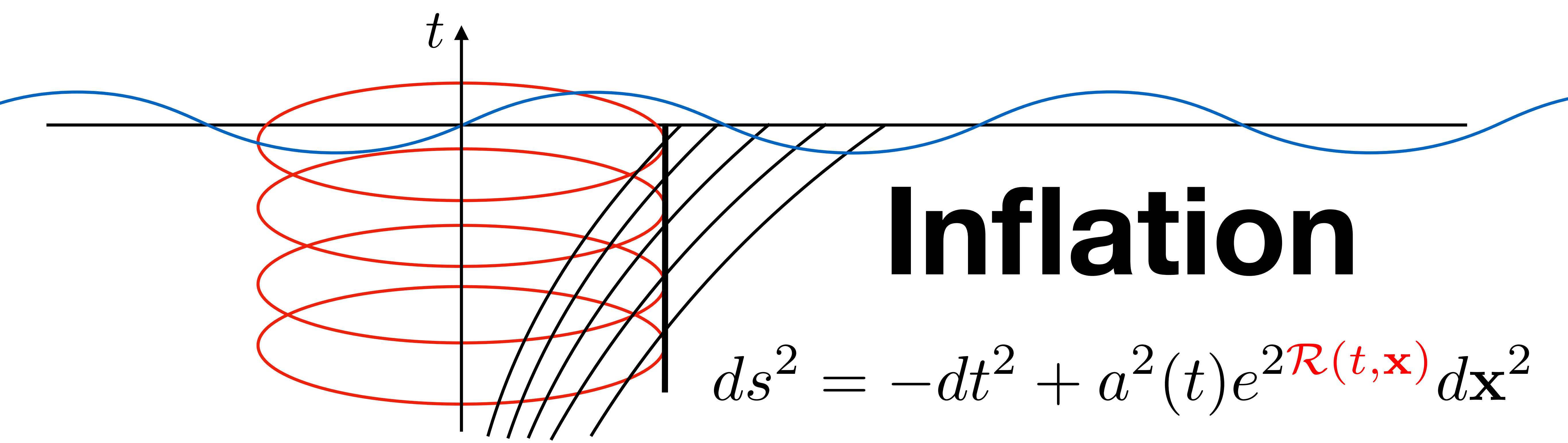
$$H(t) = H^{(2)}(t) + H^{(3)}(t) + \cdots$$



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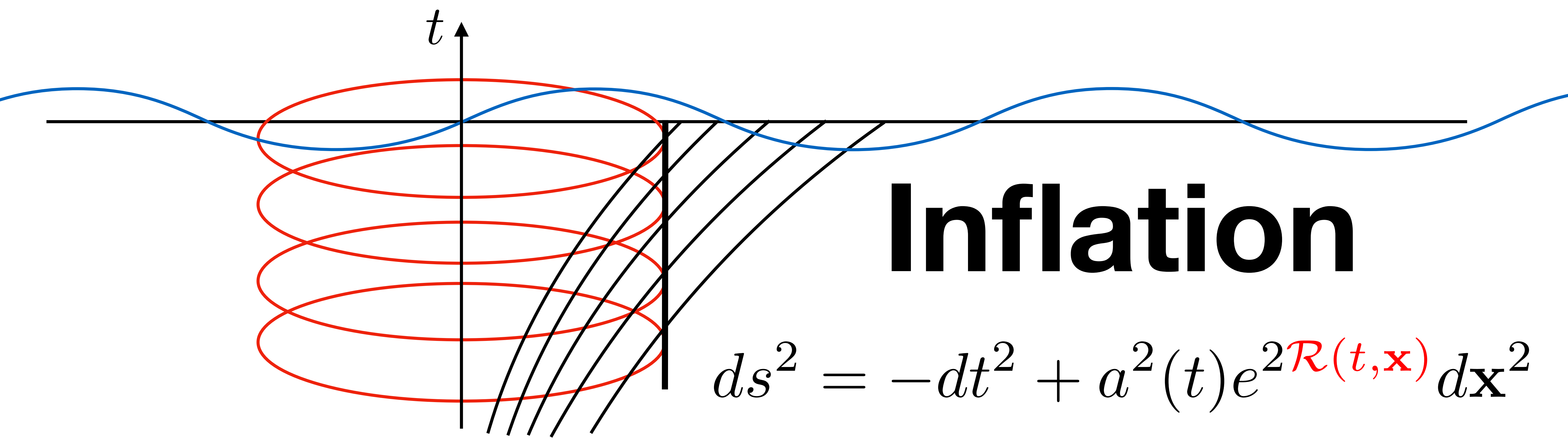
$$U(t) = \mathcal{T} \exp\left\{ -i \int_{-\infty}^{t} dt'\; H_I(t') \right\}$$

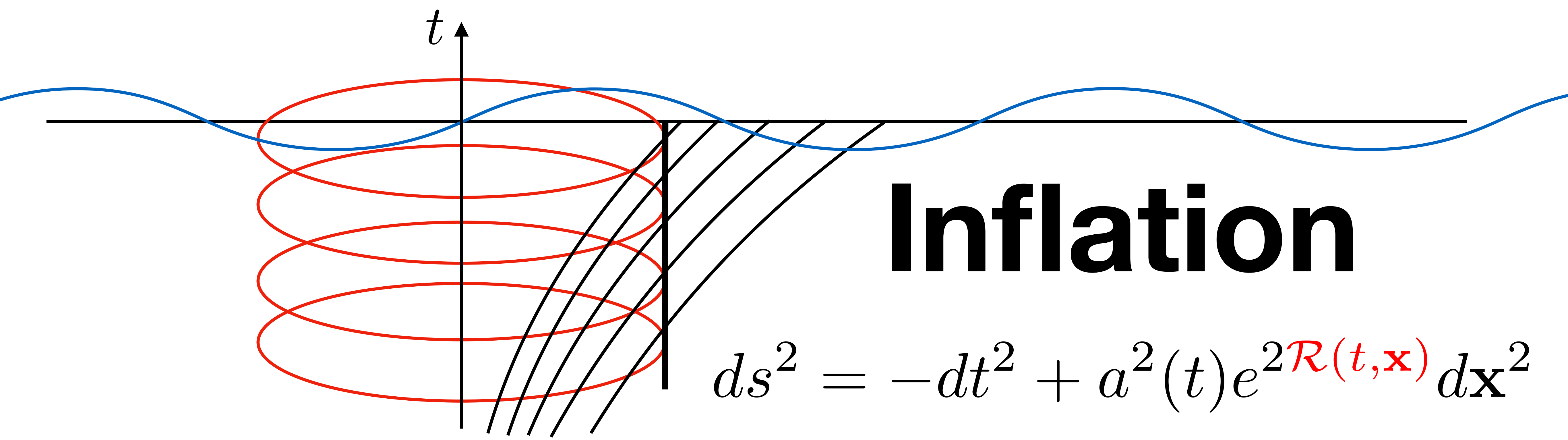


Inflation

$$ds^2 = -dt^2 + a^2(t) e^{2\mathcal{R}(t,\mathbf{x})} d\mathbf{x}^2$$

Inflation

$$\langle \mathcal{R}_{\mathbf{k}_1}(t) \cdots \mathcal{R}_{\mathbf{k}_N}(t) \rangle = \langle 0 | U^\dagger(t) \mathcal{R}^I_{\mathbf{k}_1}(t) \cdots \mathcal{R}^I_{\mathbf{k}_N}(t) U(t) | 0 \rangle$$



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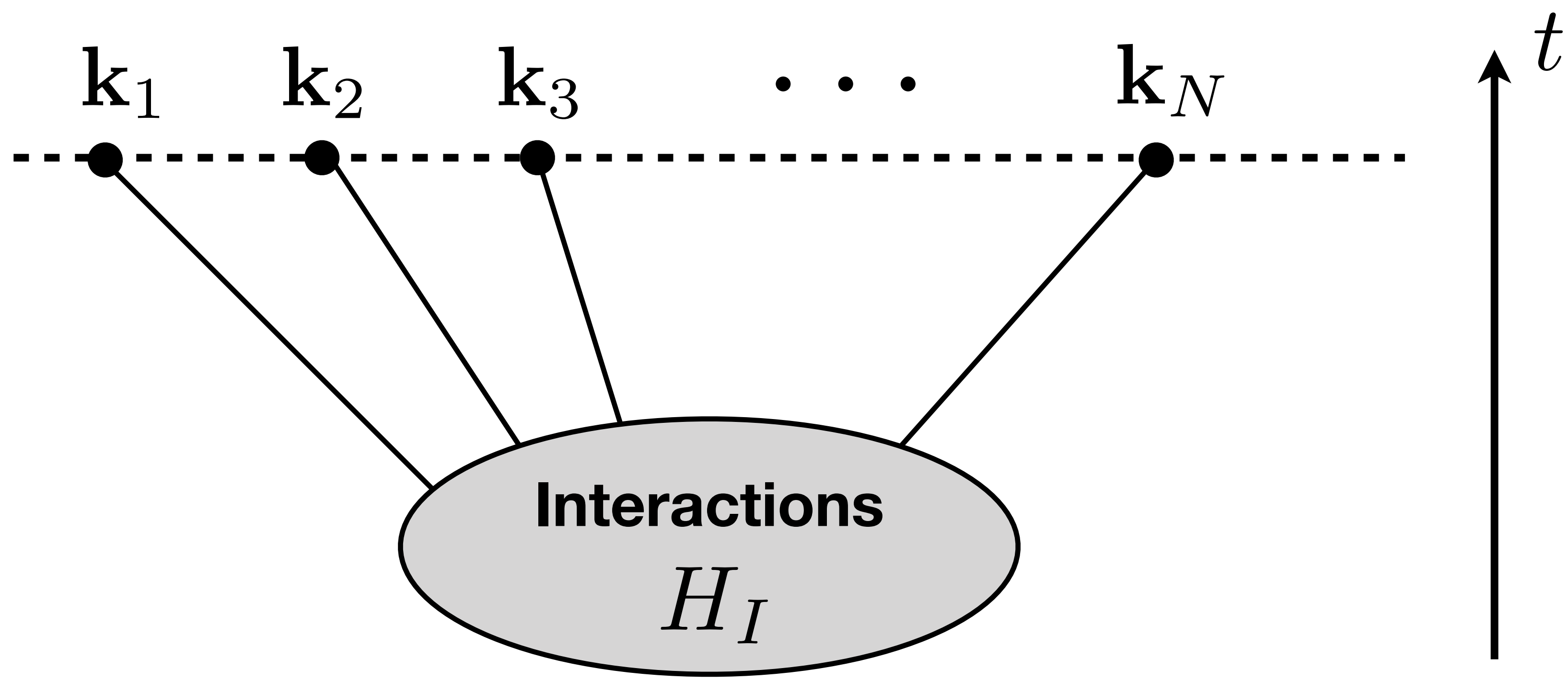

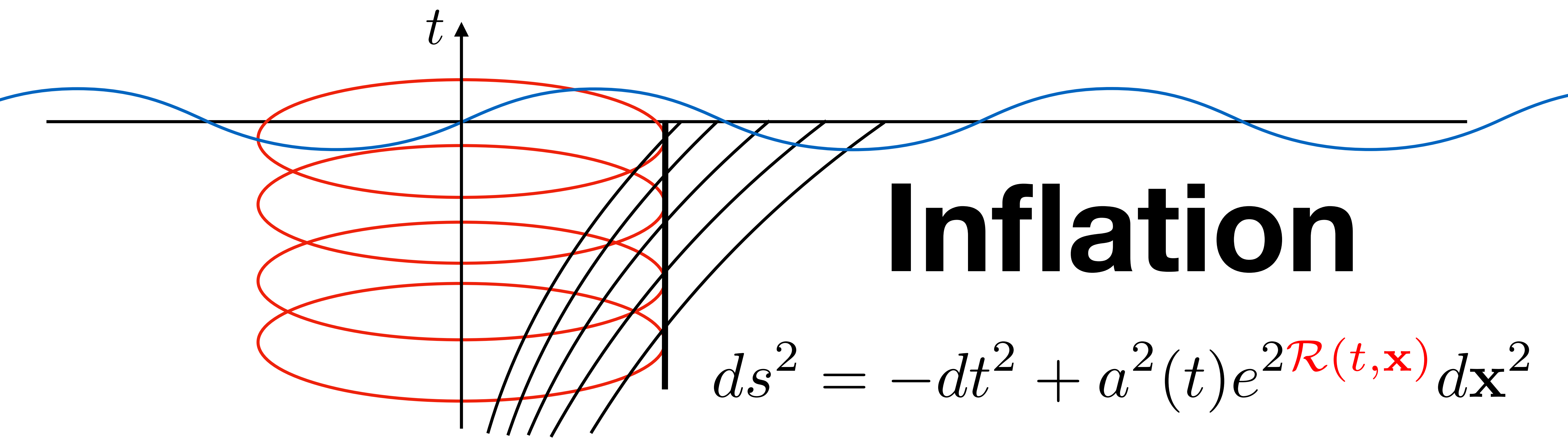

Inflation

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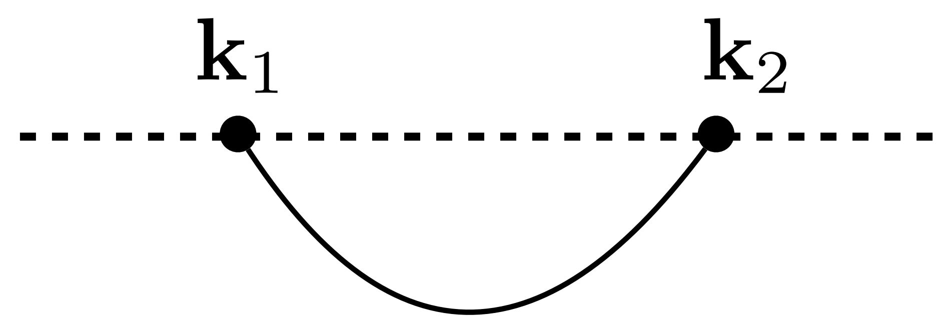
Power spectrum

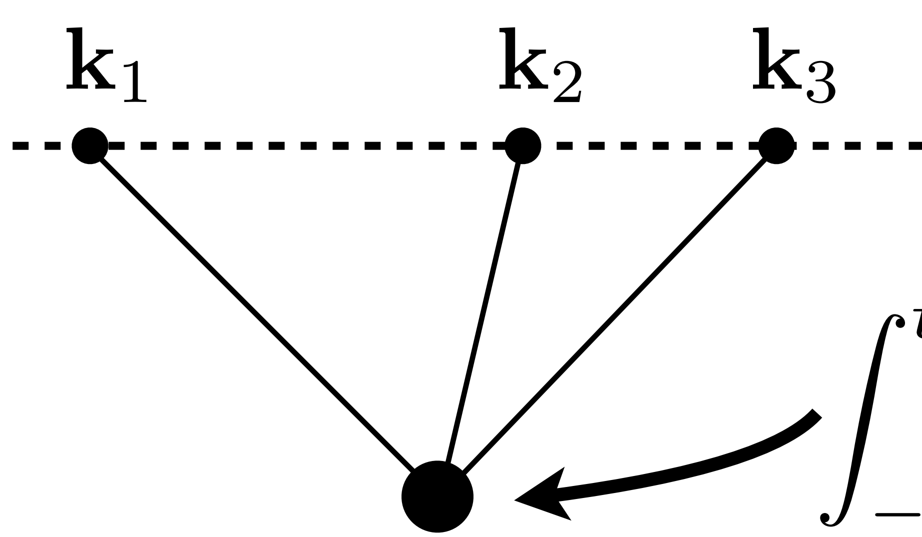
$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \rangle =$$



$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k)$$

Bi-spectrum

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle =$$



$$\int_{-\infty}^{t} dt' \; H_I^{(3)}(t') = \mathcal{O}(\mathcal{R}_I^3)$$

$$= (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

Inflation

Single-field slow-roll inflation predicts small amounts of NG:

$$f_{\rm NL}^{\rm type} \simeq \mathcal{O}(\epsilon, \eta)$$

Maldacena (2002)

$$f_{\rm NL}^{\rm loc} = 0$$

Tanaka & Urakawa (2011)
Pajer, Schmidt & Zaldarriaga (2013)

More general types of single-field inflation can enhance NG:

$$\mathcal{L} = \epsilon \left(c_s^2 \dot{\mathcal{R}}^2 - \frac{1}{a^2} (\nabla \mathcal{R})^2 \right) + \left(\frac{1}{c_s^2} - 1 \right) \times \mathcal{O}(\mathcal{R}^3) + \cdots$$

$$f_{\rm NL}^{\rm equil} \simeq \mathcal{O}\left(\frac{1}{c_s^2} - 1 \right)$$

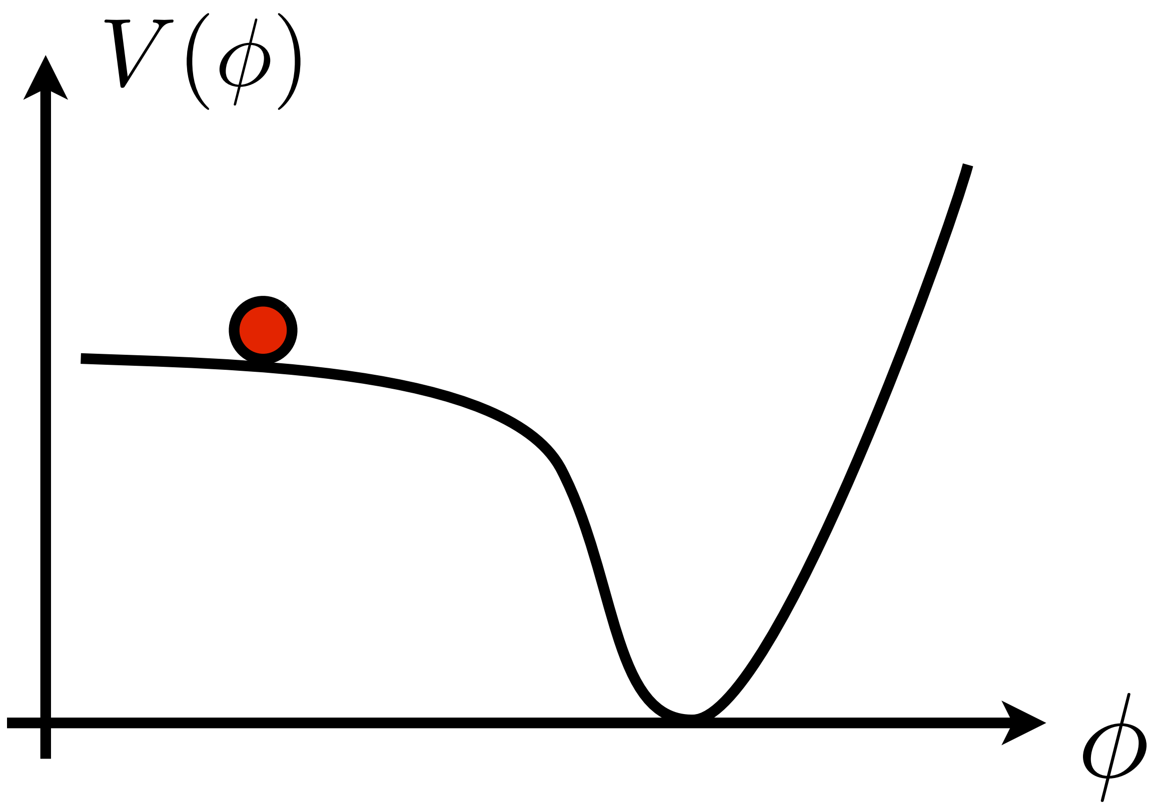
Chen, Huang, Kachru & Shiu (2007)

$$f_{\rm NL}^{\rm loc} = 0$$

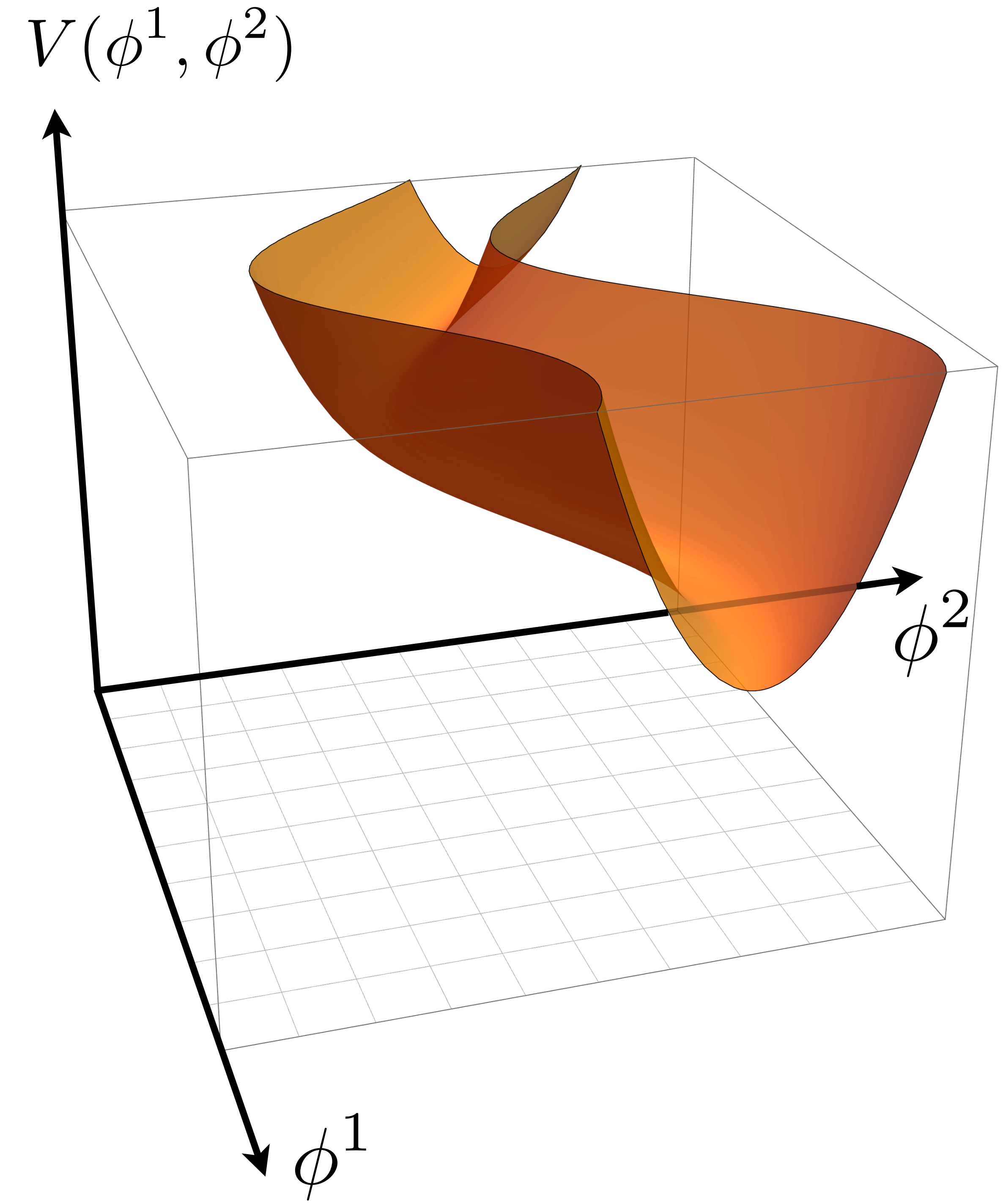
Creminelli & Zaldarriaga (2004)

Multi-field inflation

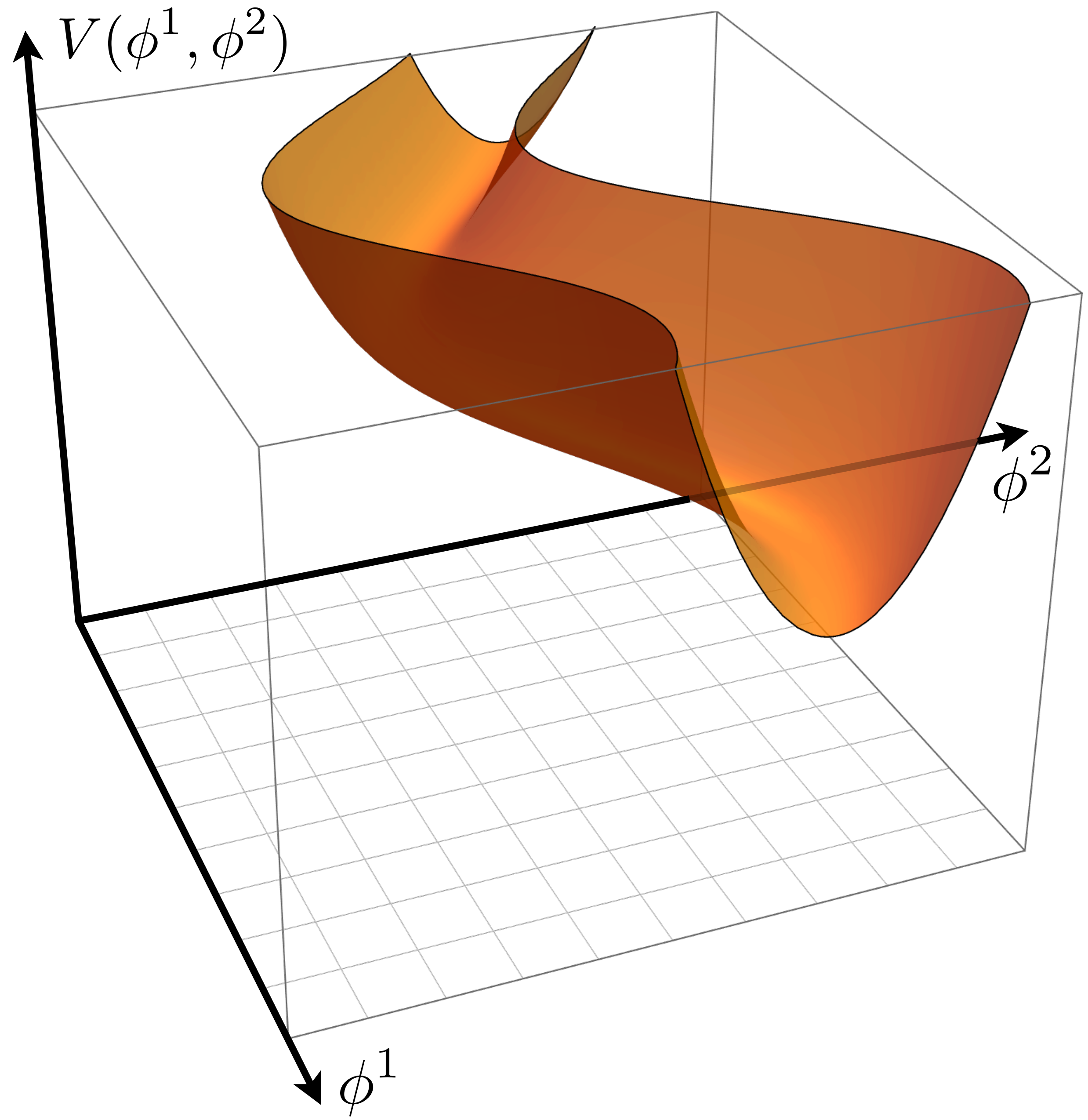
The primordial universe might be a bit more contrived:



Vs



Multi-field inflation



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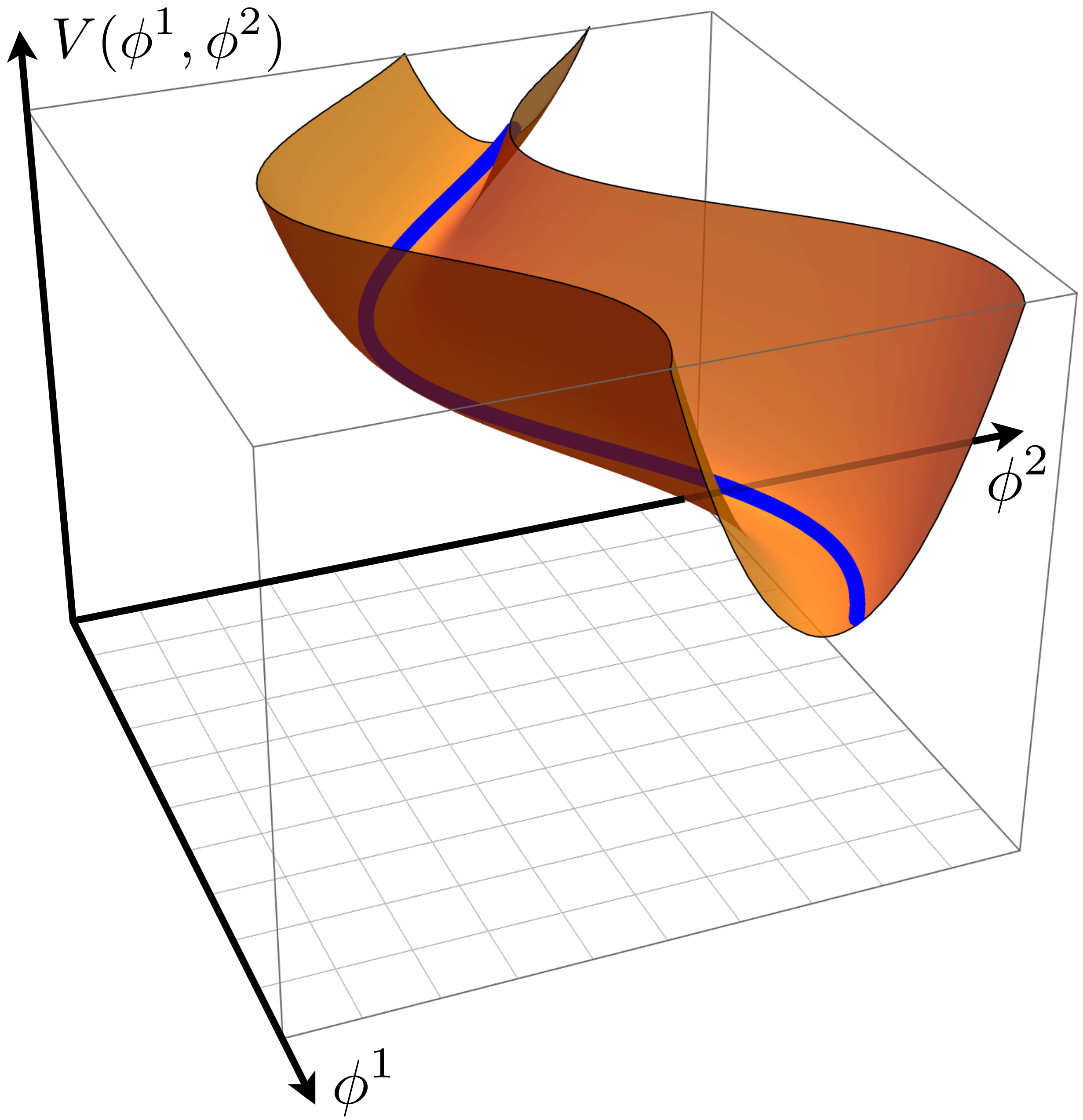

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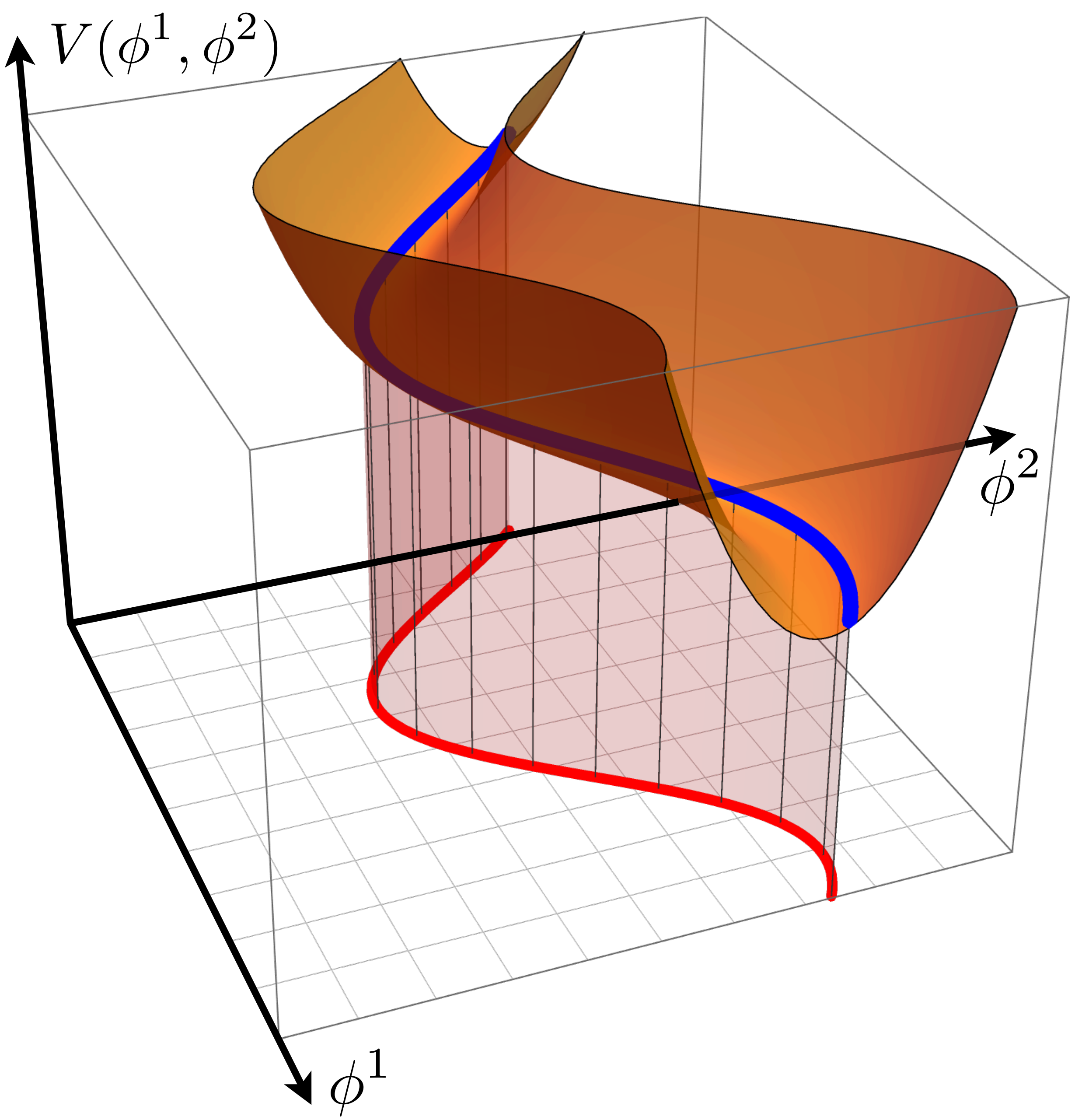
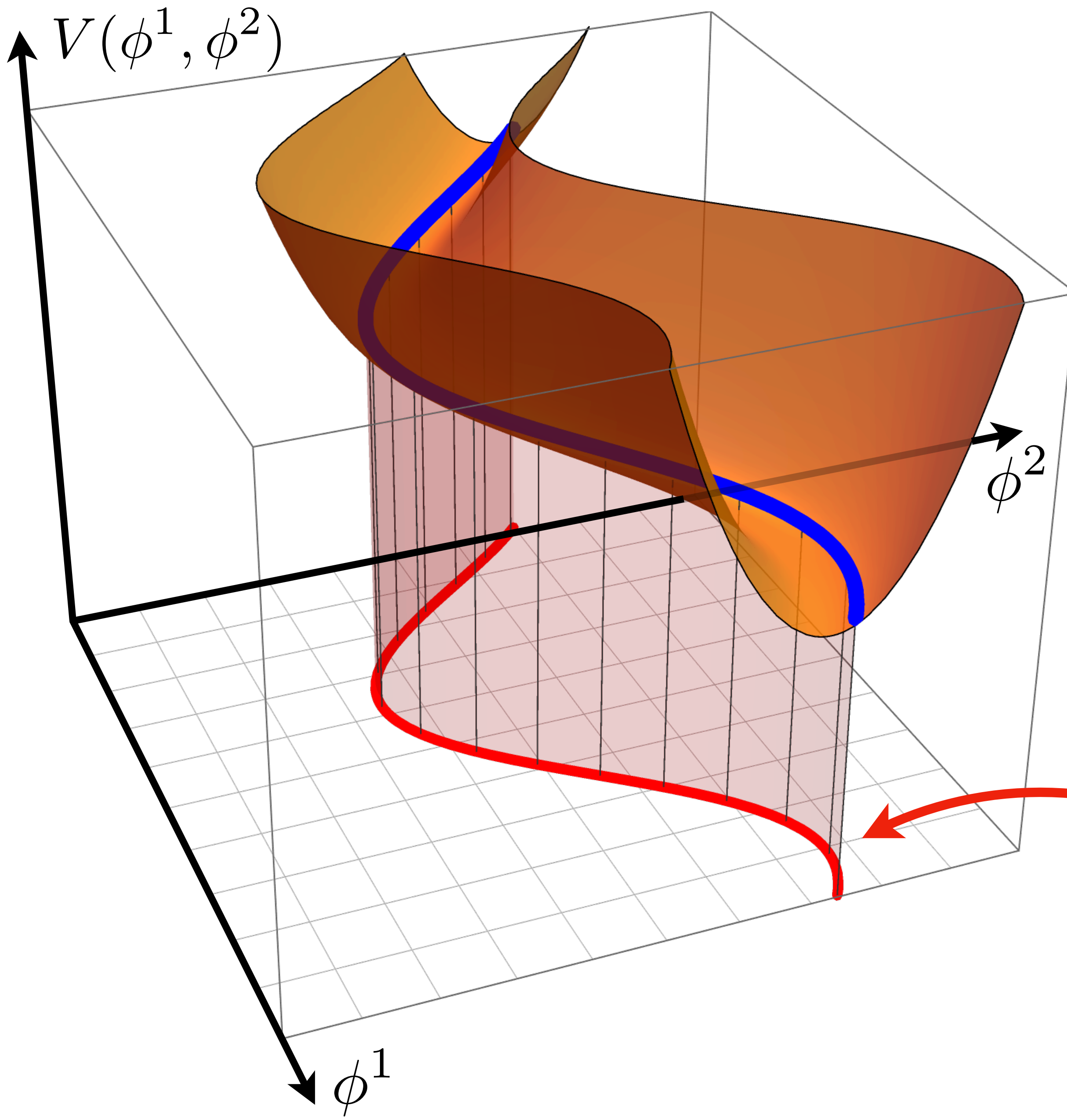


Multi-field inflation

$$\mathcal{L} = \epsilon\left(\dot{\mathcal{R}} - \alpha\,\psi\right)^2 - \frac{\epsilon}{a^2}(\nabla\mathcal{R})^2 + \frac{1}{2}\dot{\psi}^2 - \frac{1}{a^2}(\nabla\psi)^2$$
$$+\,\epsilon\left(\dot{\mathcal{R}} - \alpha\,\psi\right)^3 - V(\psi) + \cdots$$



$\alpha \neq 0$

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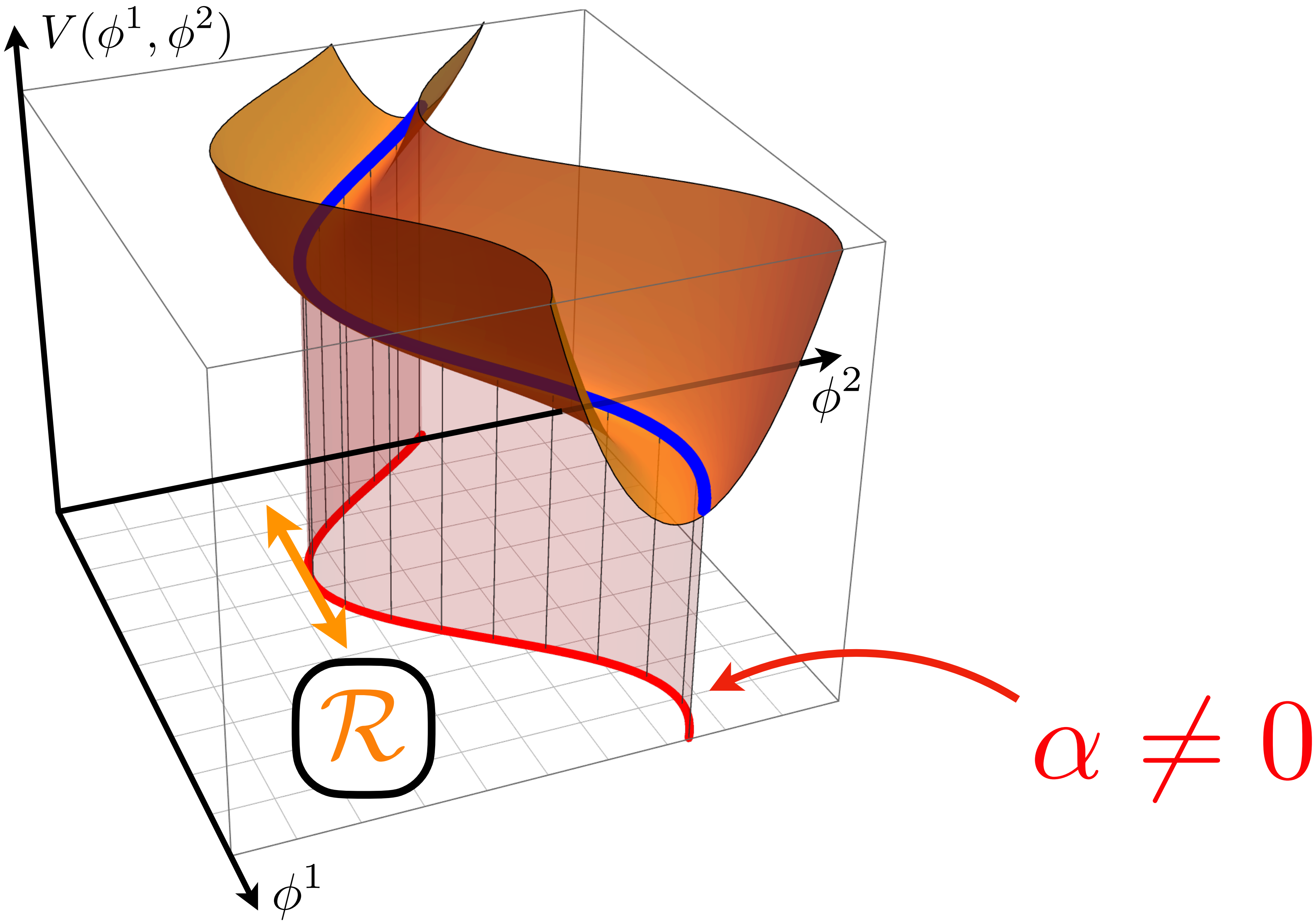

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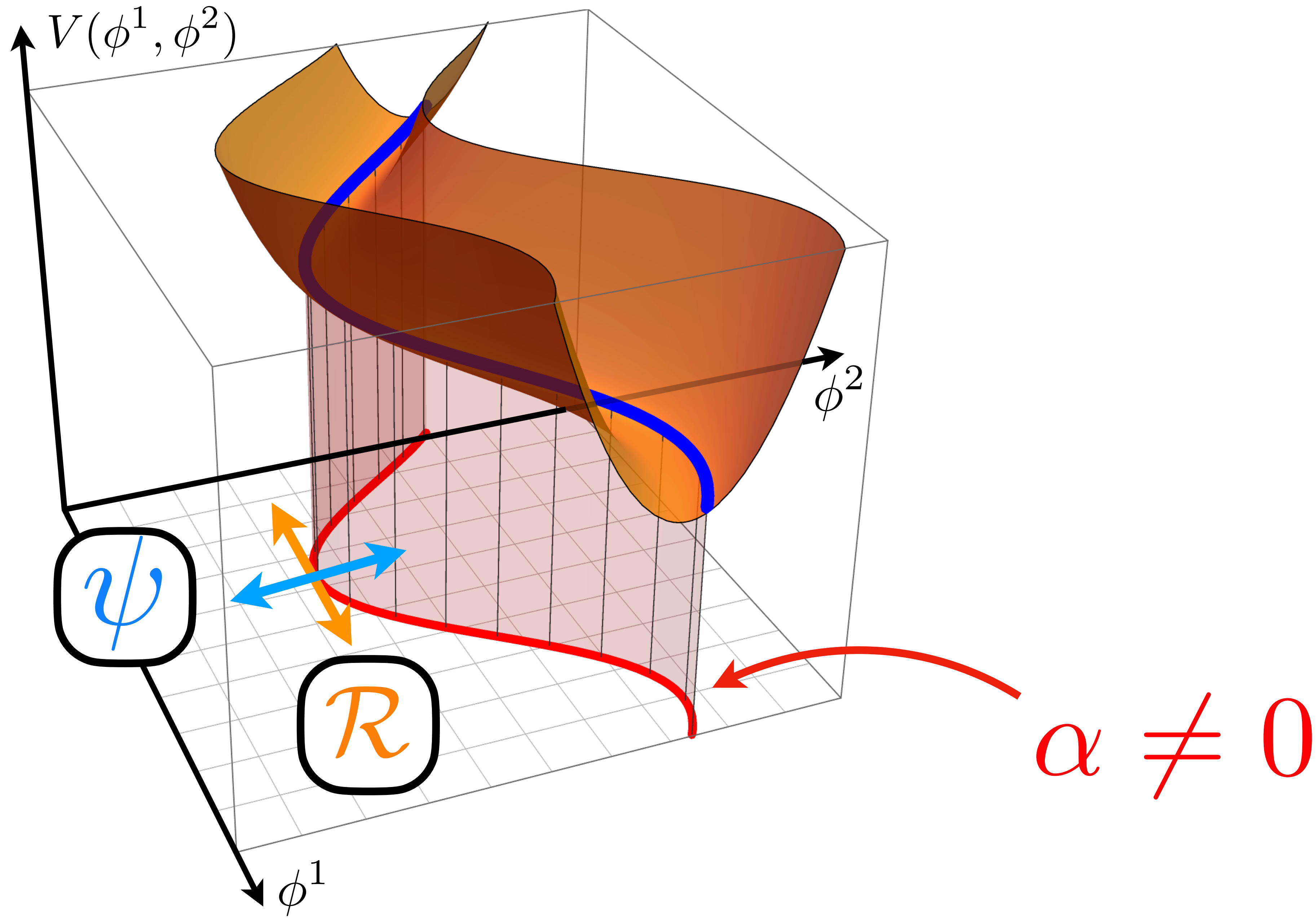

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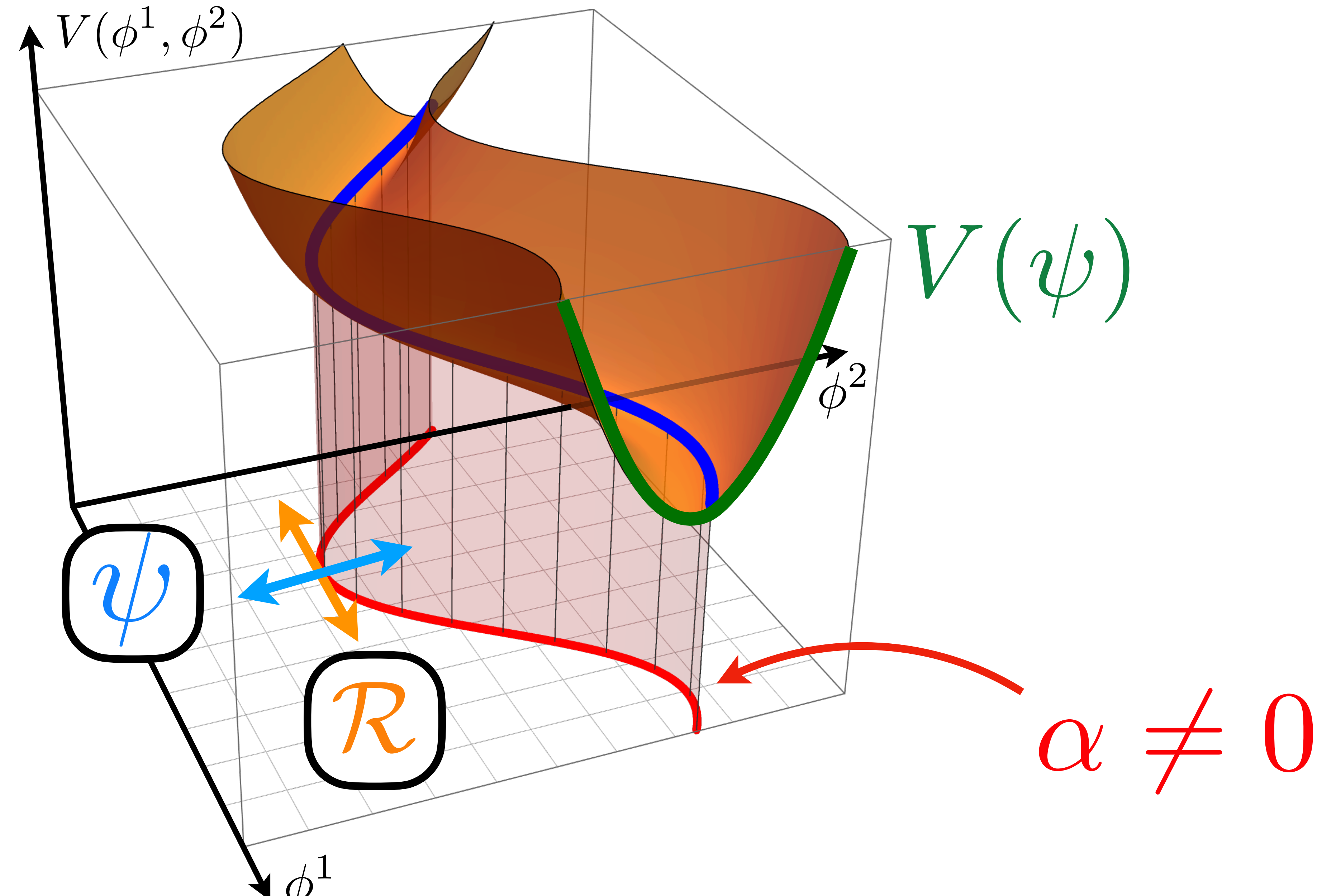

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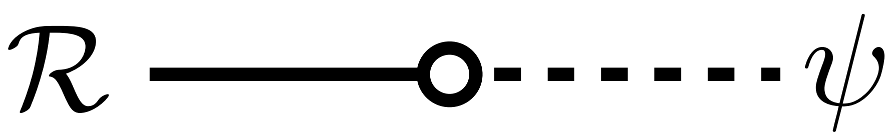

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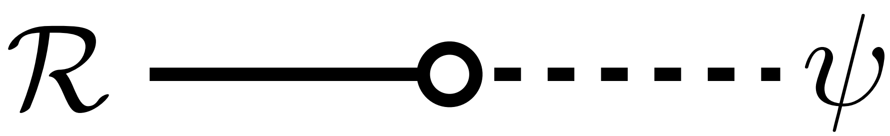

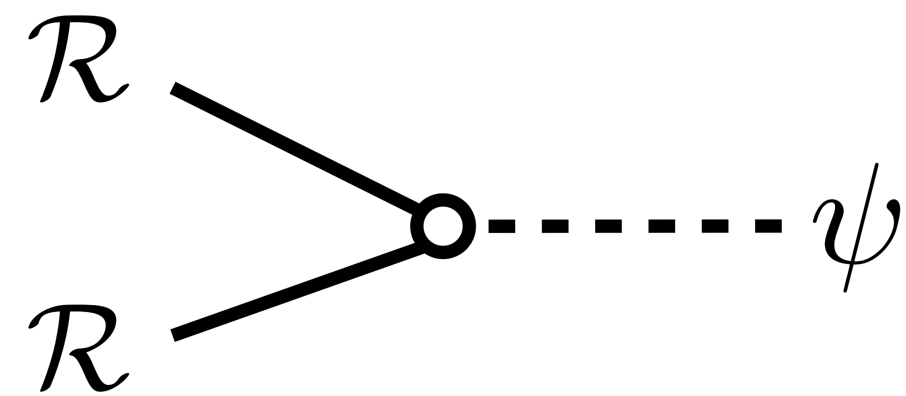

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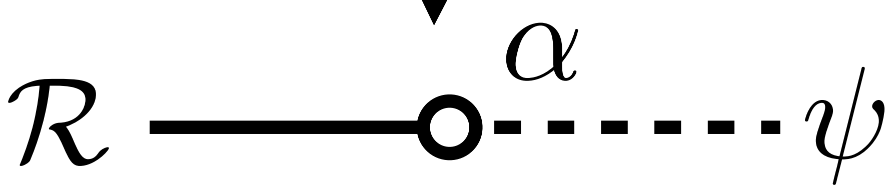
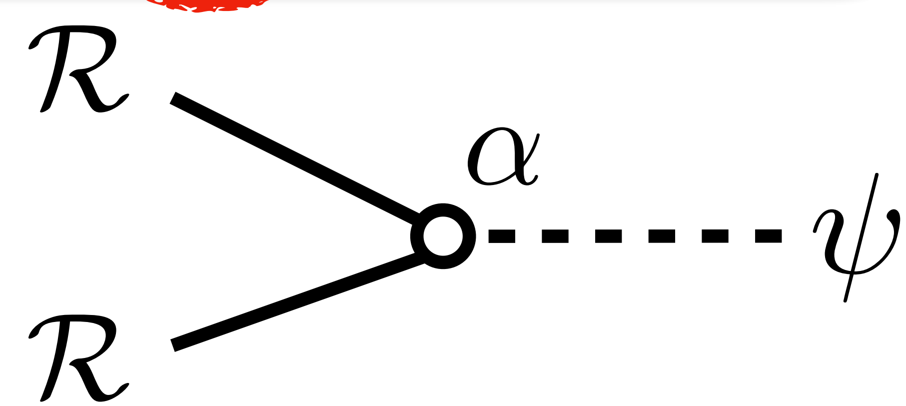
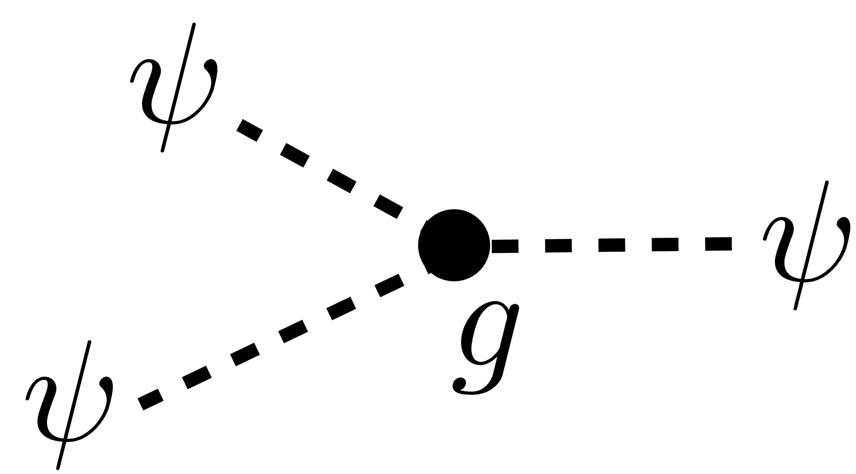




$$V(\psi) = \frac{1}{2}\mu^2\psi^2 + \frac{1}{3}g\,\psi^3 + \cdots$$

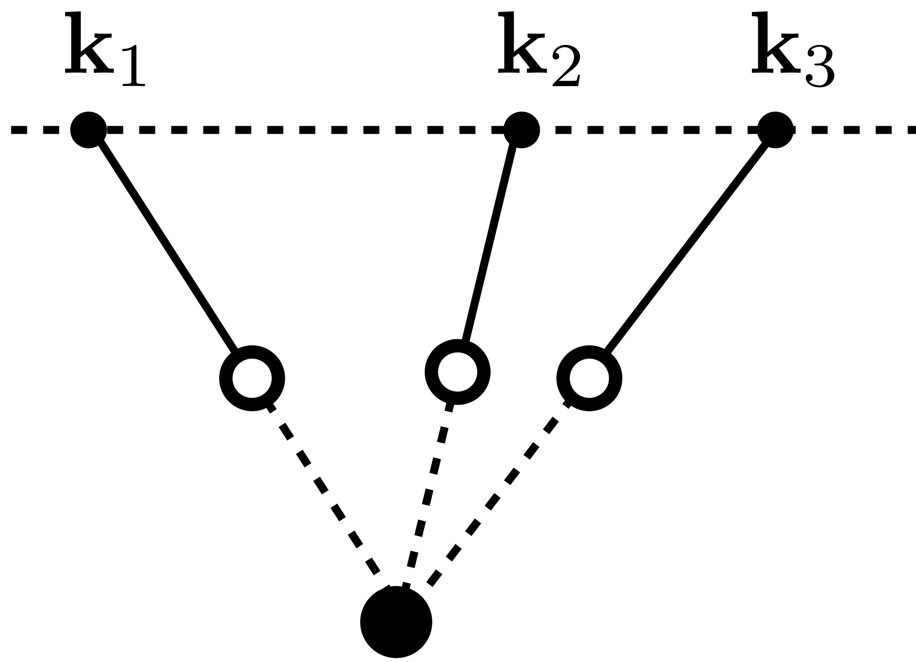


Multi-field inflation

Three-point statistics:

$\mu \neq 0$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle =$$



(Quasi-single field)

Chen & Wang (2012)
see also Assassi et al. (2013)

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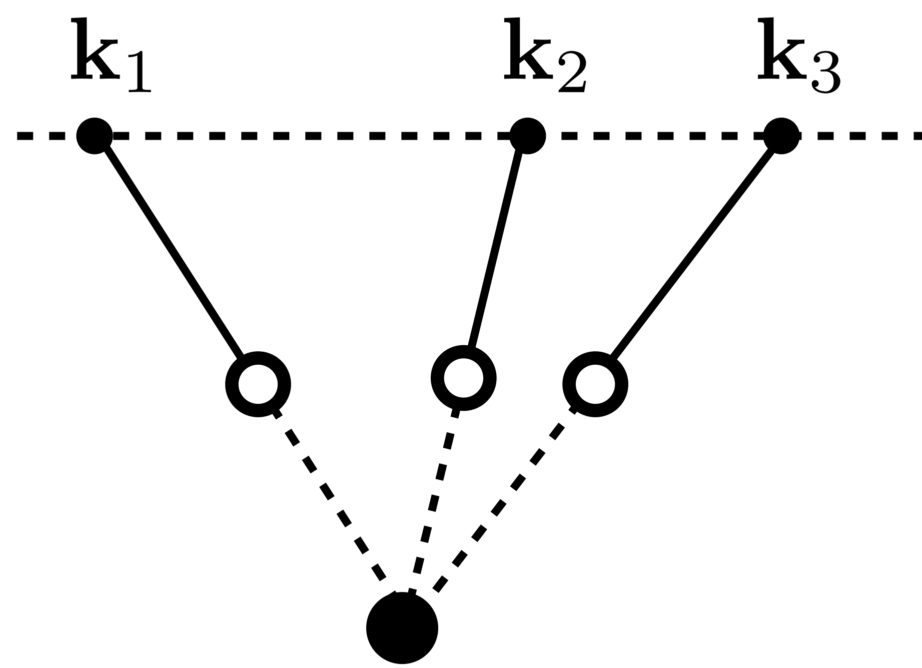

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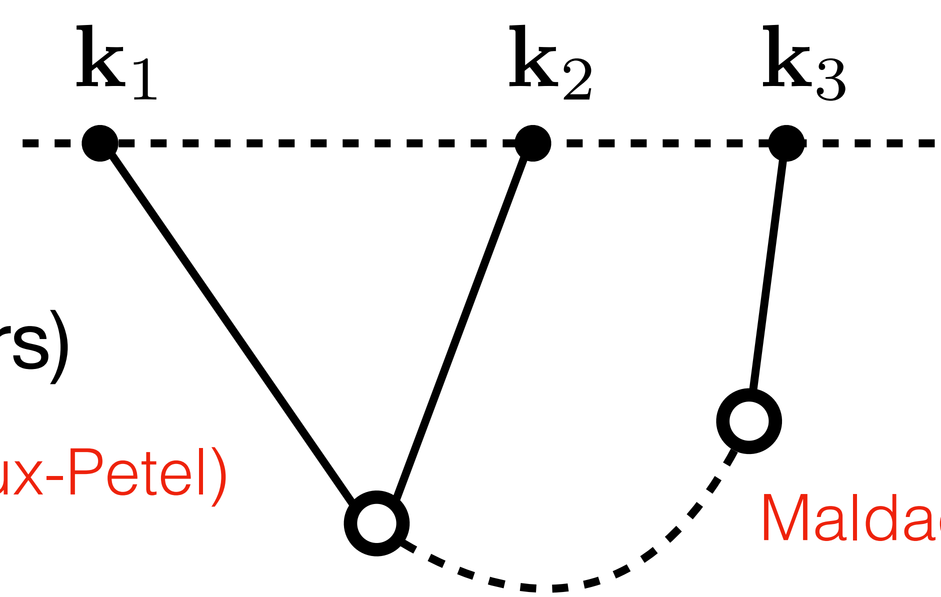


(Cosmological colliders)

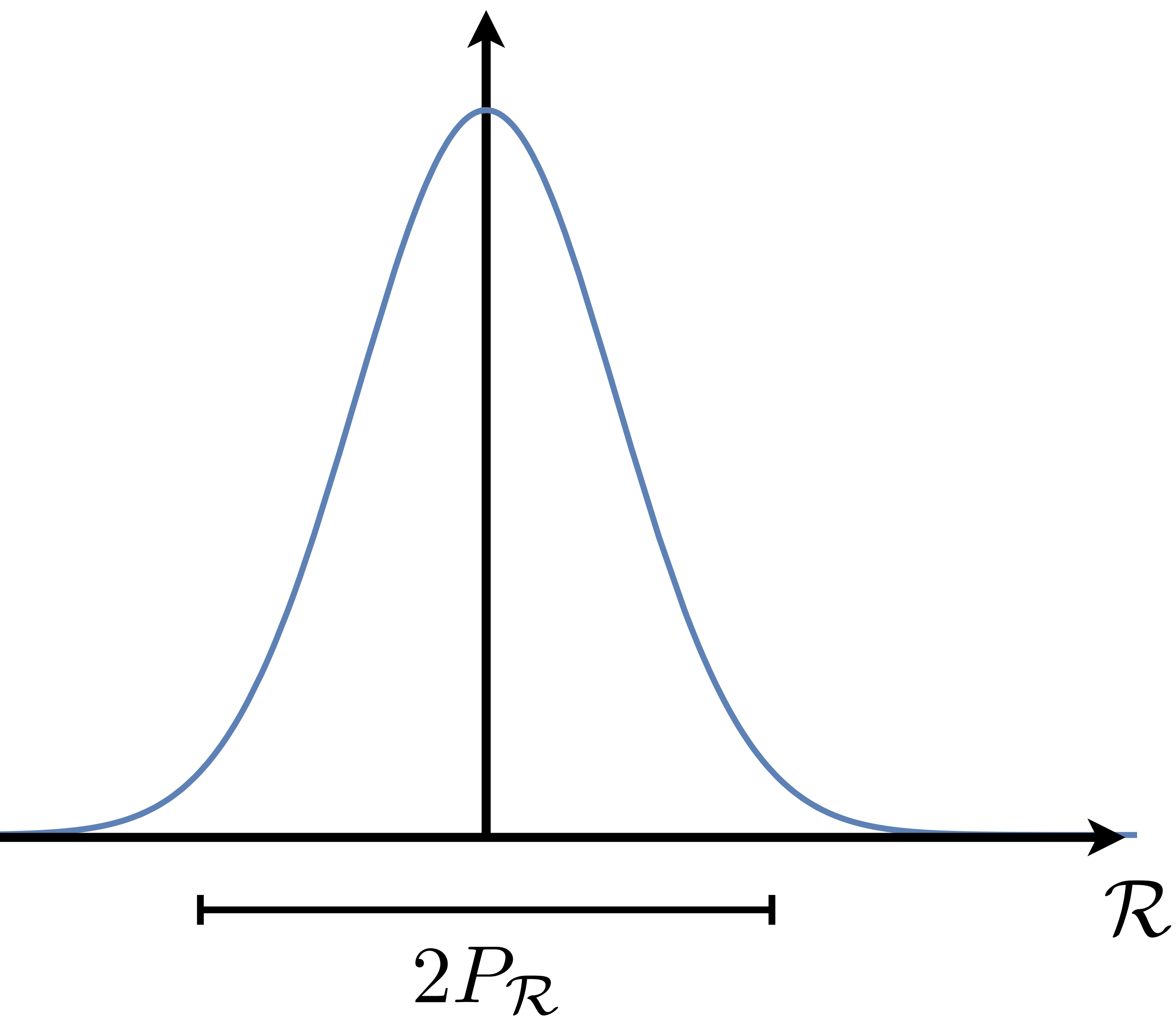
(See talk by Sebastien Renaux-Petel)

Maldacena & Arkani-Hamed (2016)
Chen & Wang (2016)
Lee, Baumann & Pimentel (2016)

Beyond the bispectrum

Question: Do we have any **theoretical** justification for searching non-Gaussianity beyond the three-point function?

$$\rho[\mathcal{R}] \sim \exp\left\{-\frac{\mathcal{R}^2}{2P_{\mathcal{R}}}\left(1+f_{\rm NL}\mathcal{R}+g_{\rm NL}\mathcal{R}^2+\cdots\right)\right\}$$



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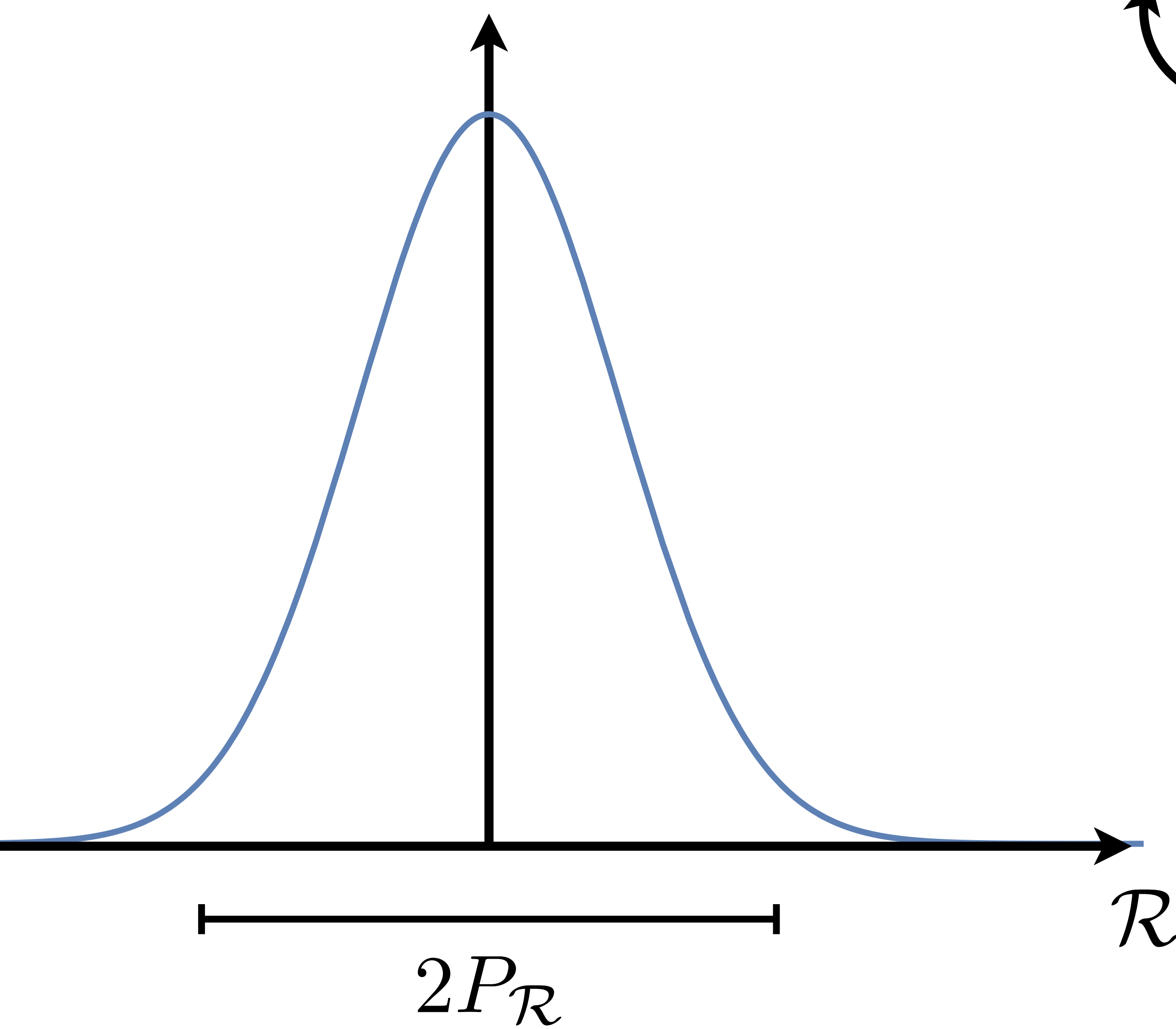

In single field inflation: $g \sim \epsilon$

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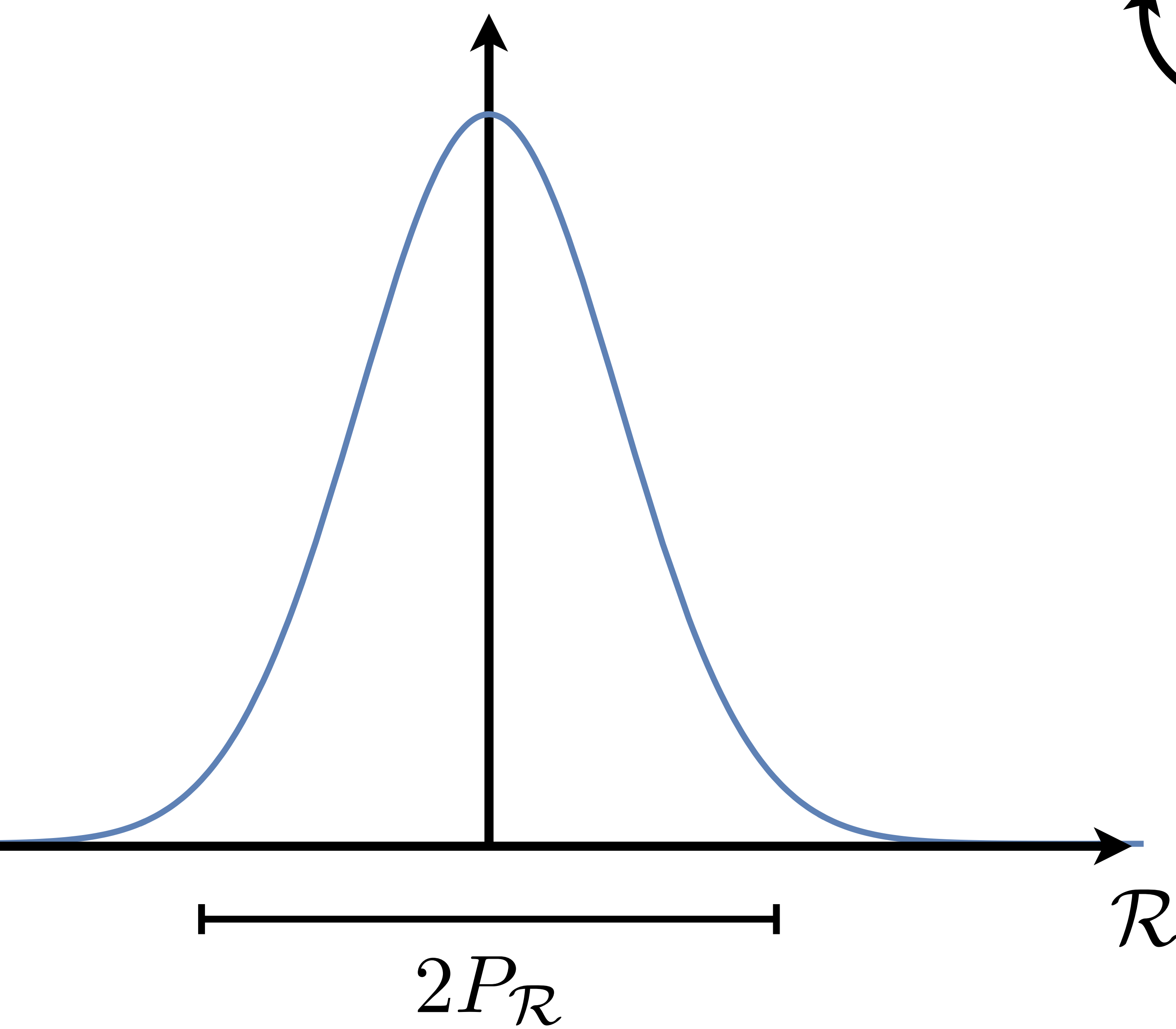

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$$\mathcal{R} \sim \frac{1}{g}$$

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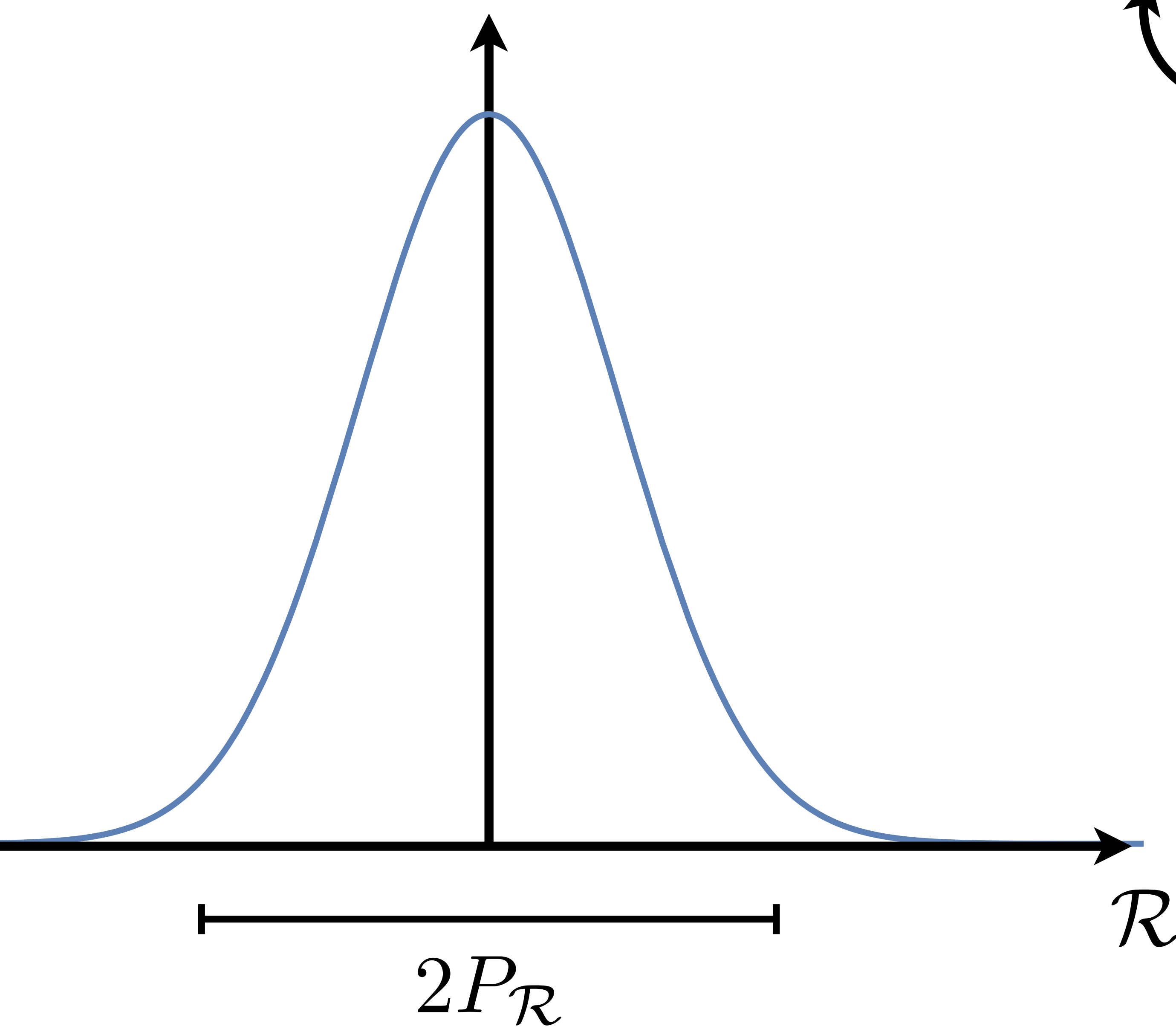


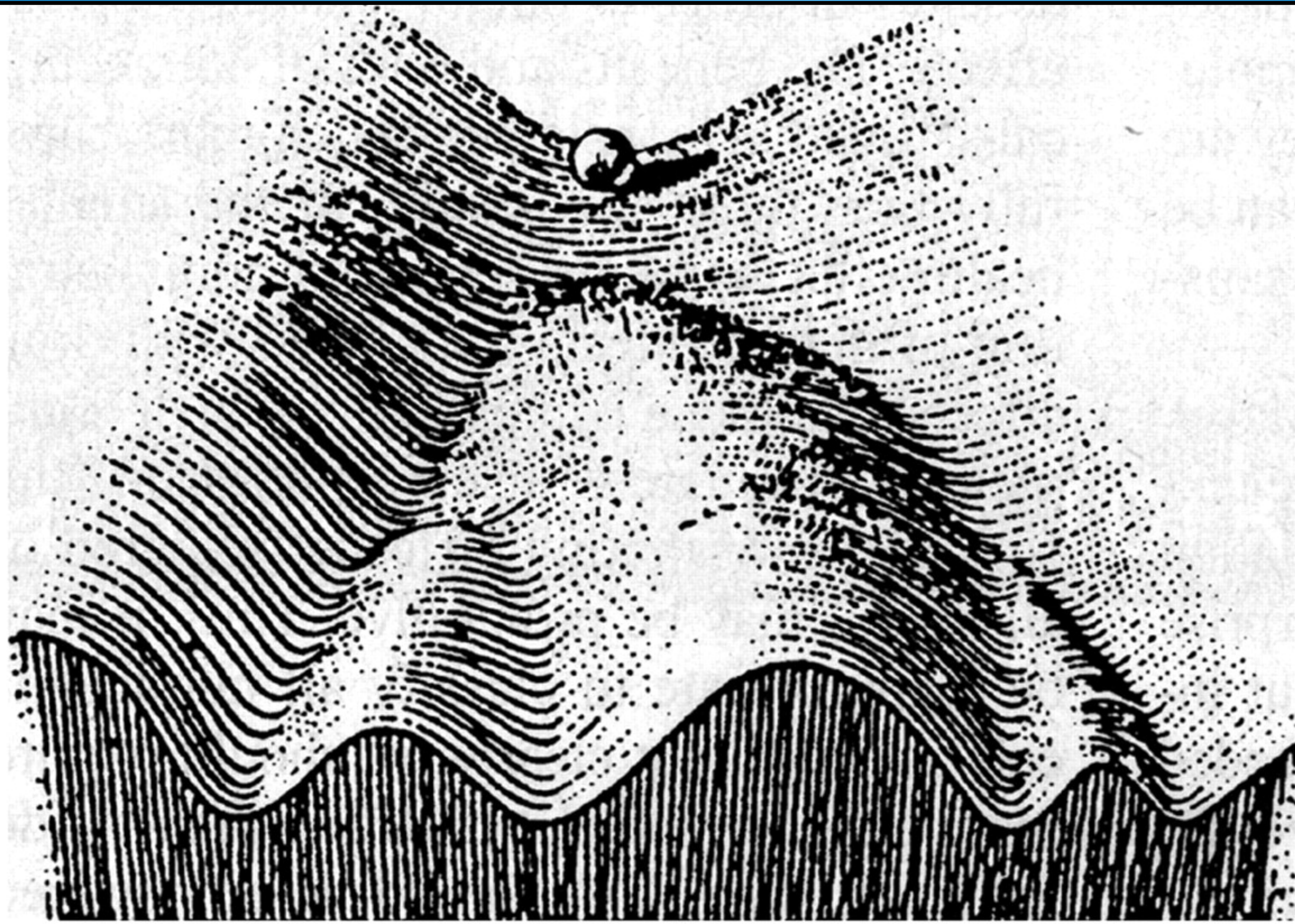
In single field inflation: $g \sim \epsilon$

$$\mathcal{R} \sim \frac{1}{g}$$

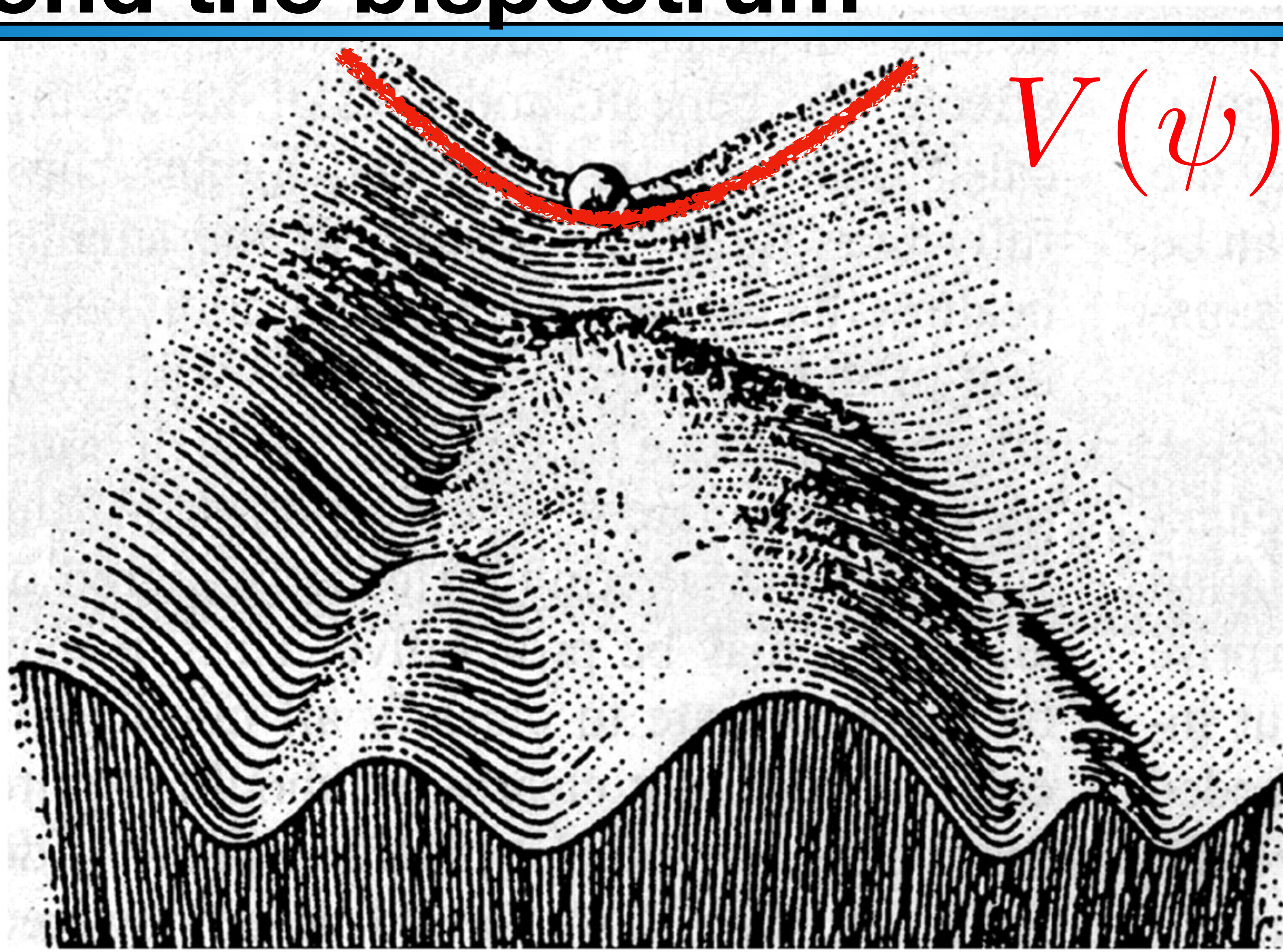
But if

$$g \gtrsim 1$$

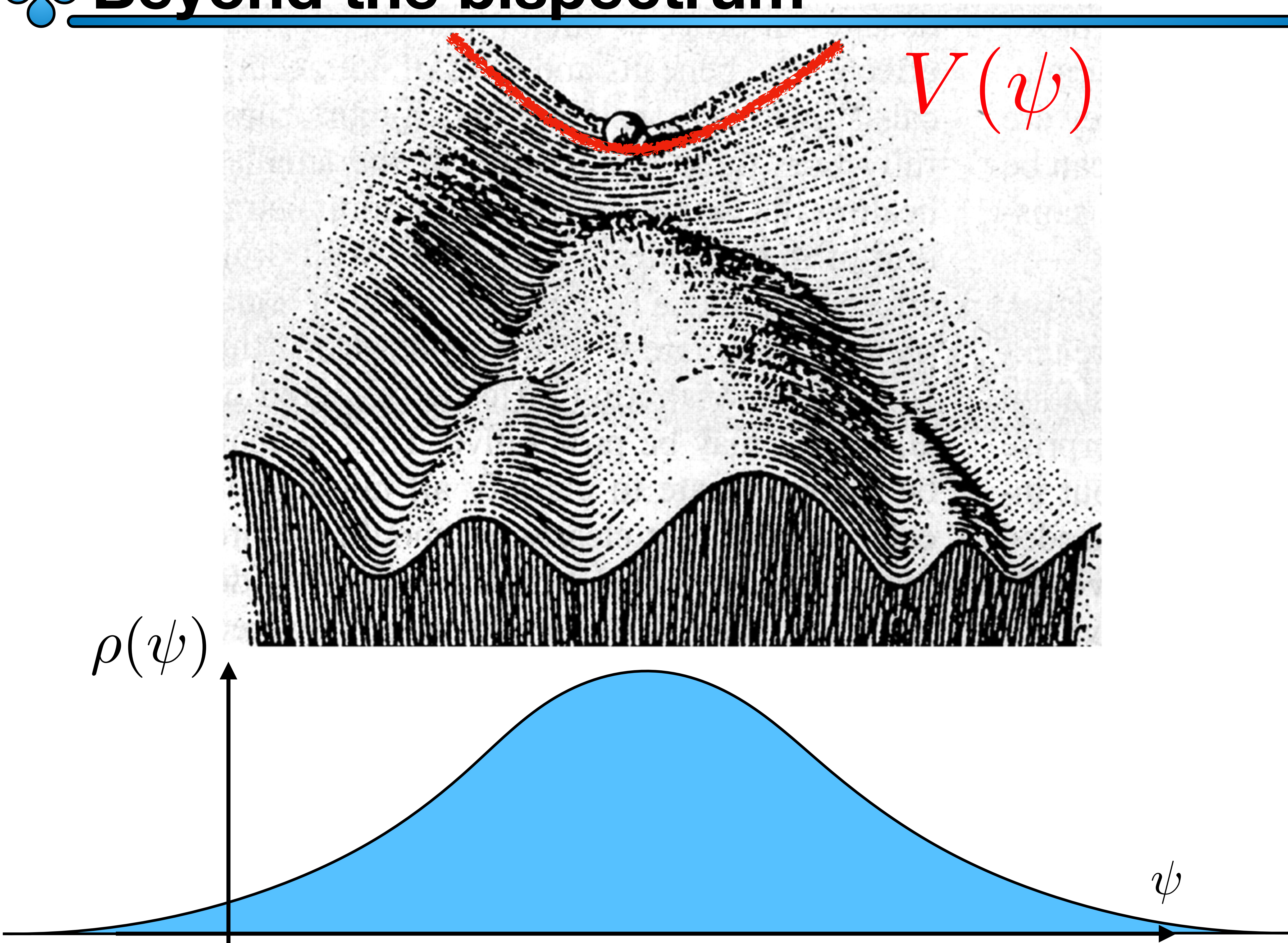
Beyond the bispectrum



Beyond the bispectrum



Beyond the bispectrum



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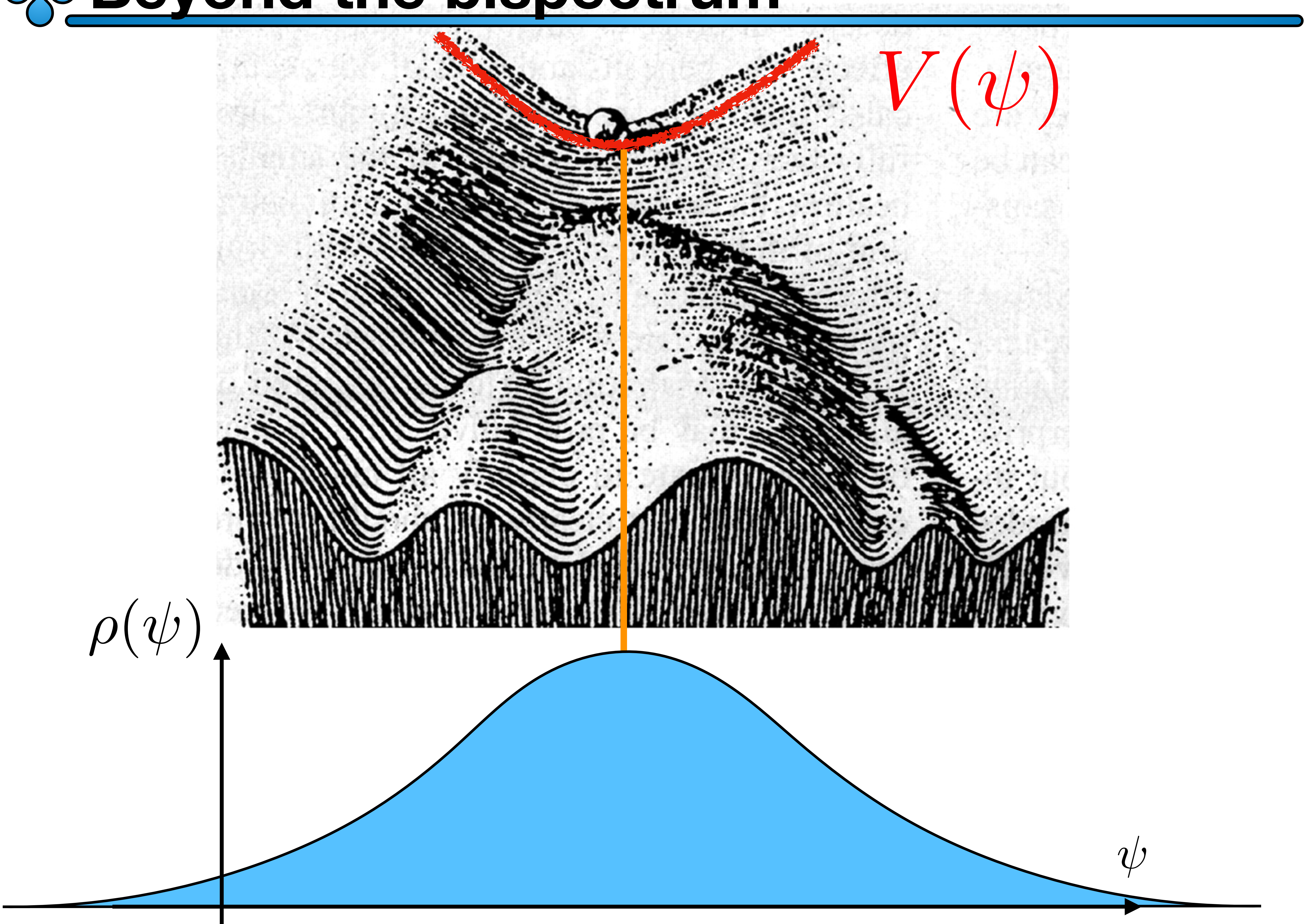

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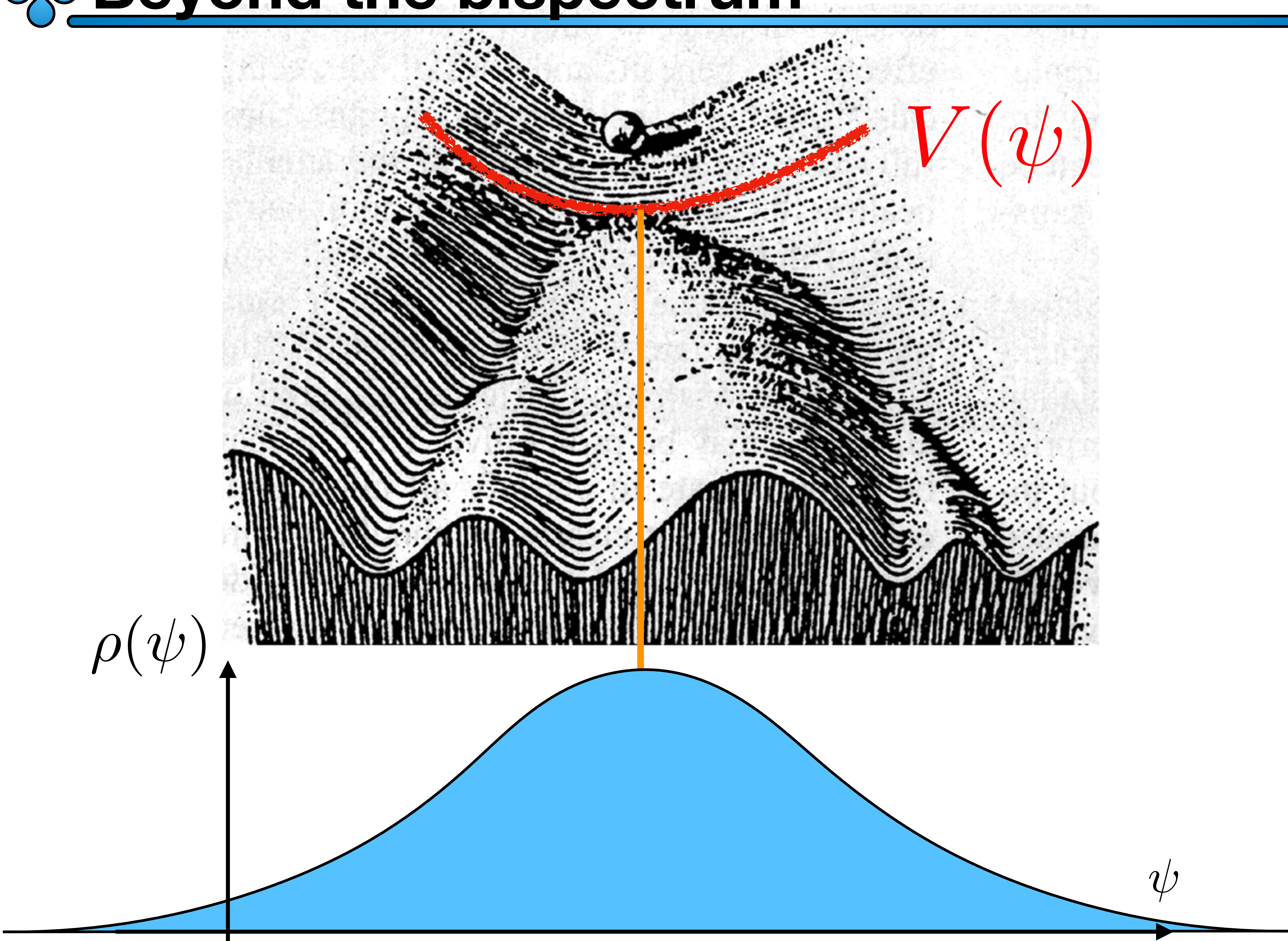
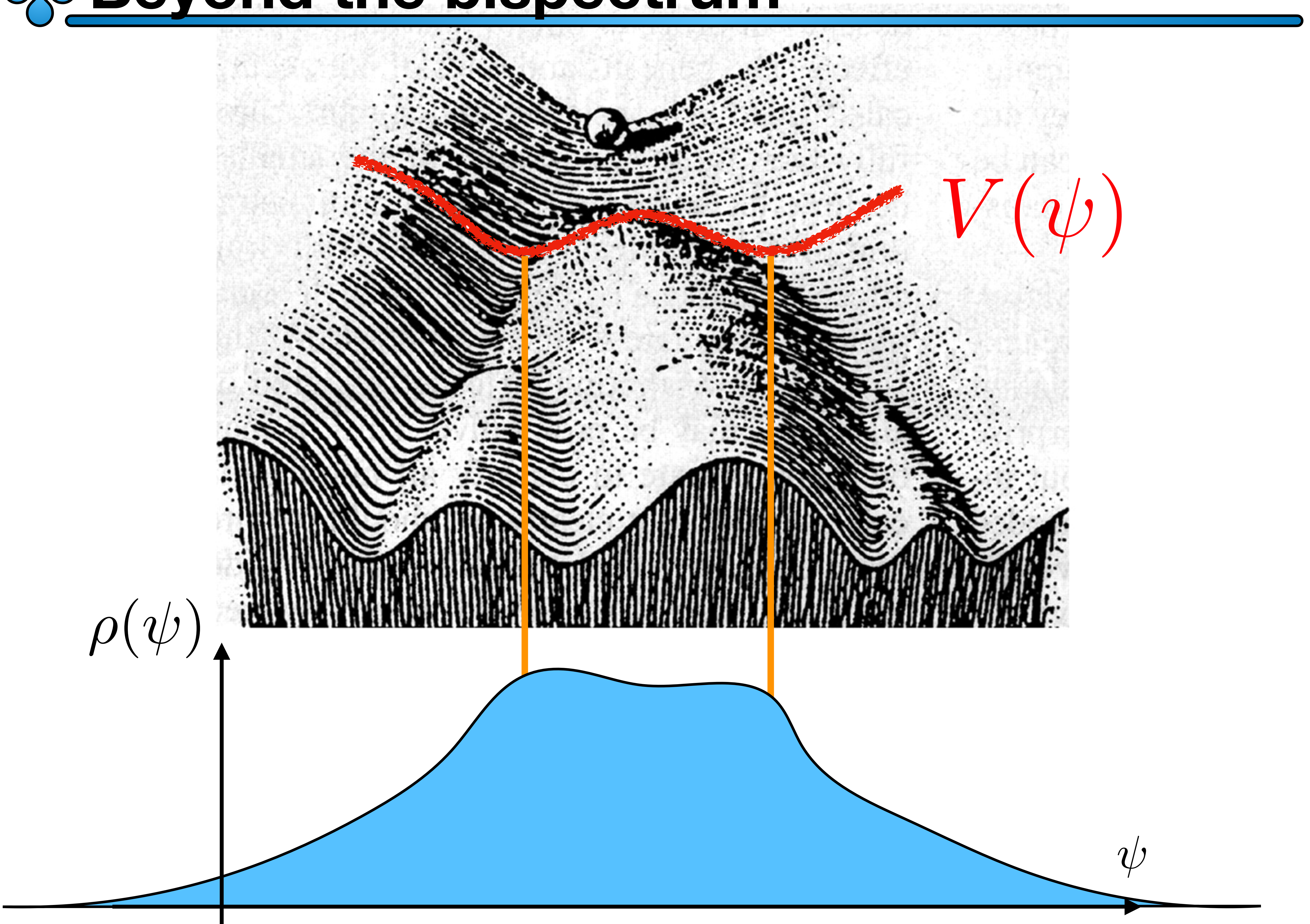


Beyond the bispectrum



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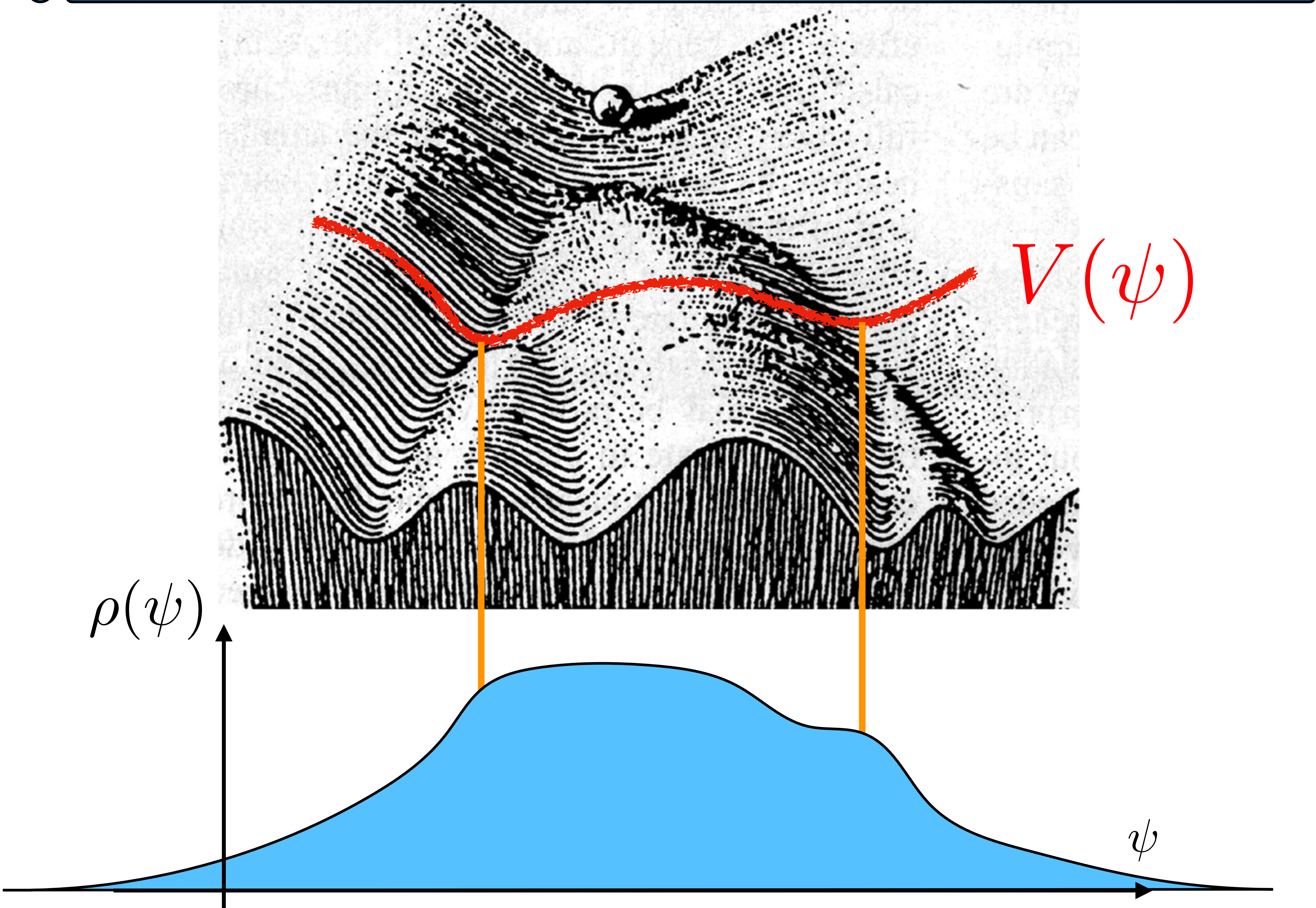

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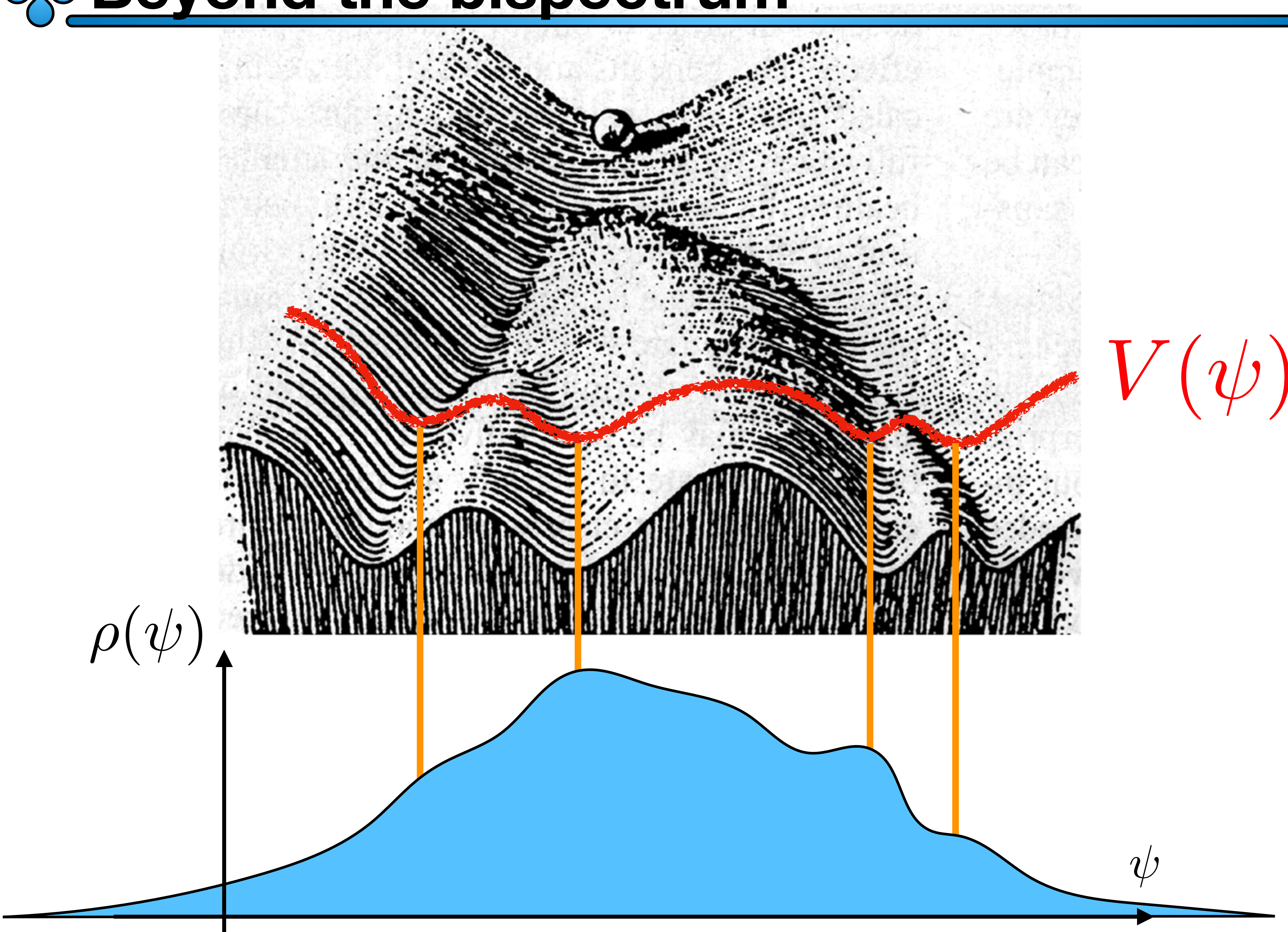

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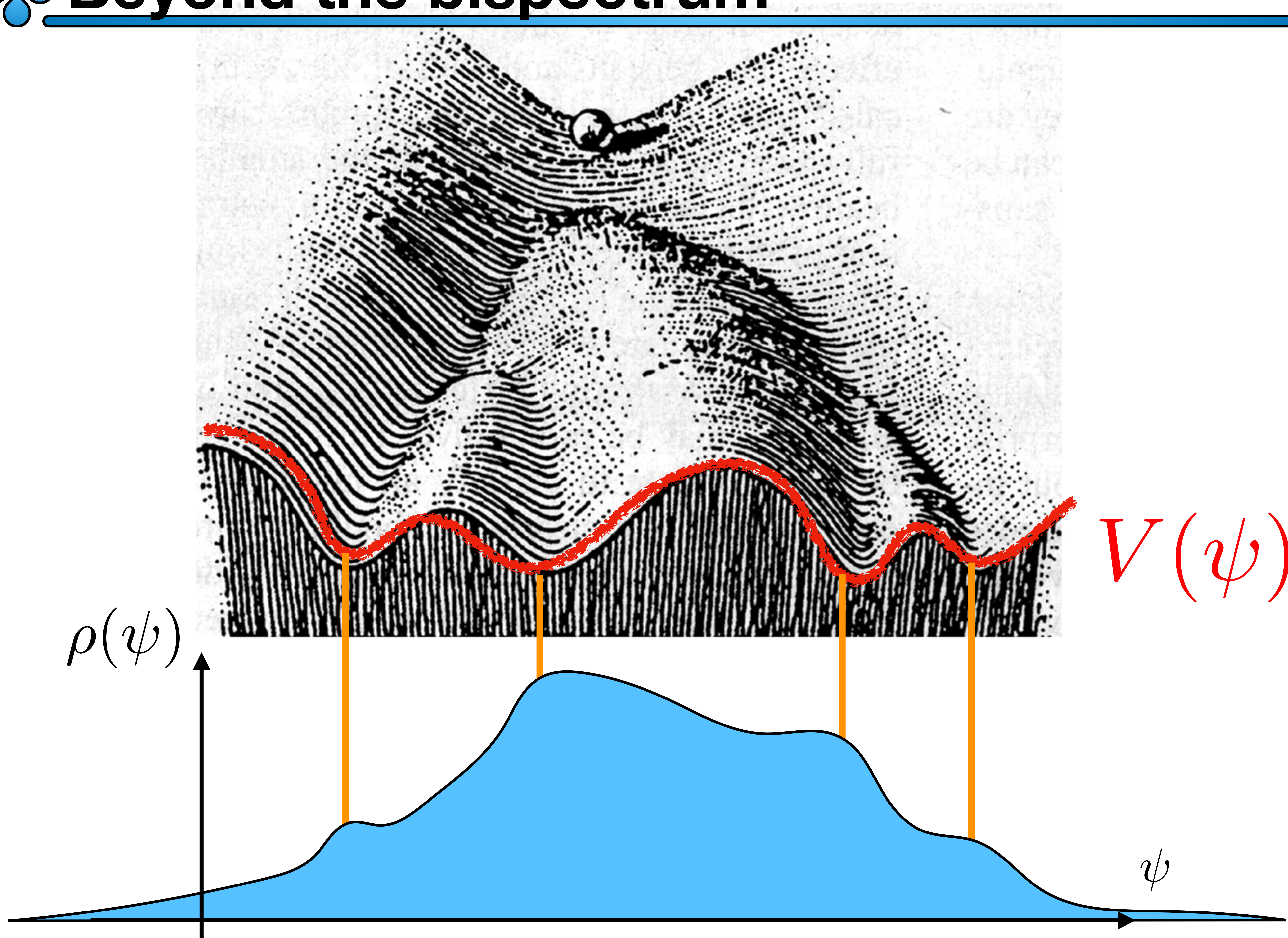
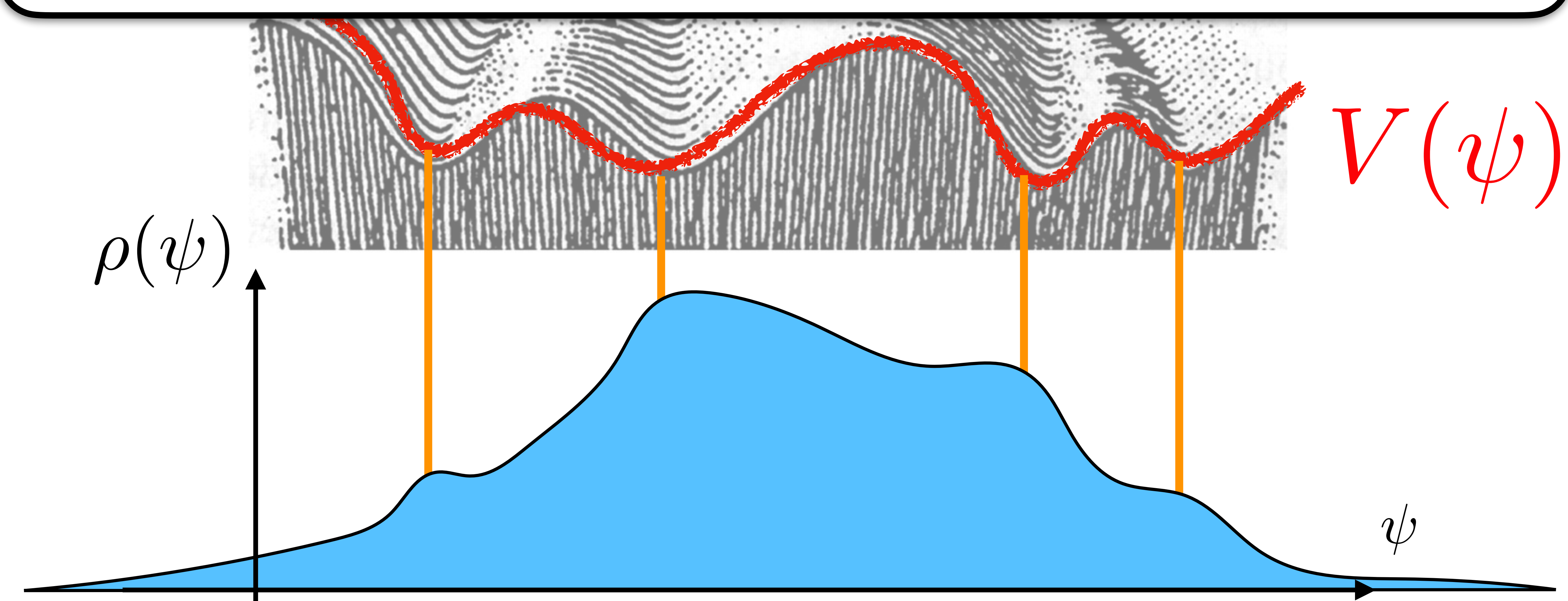


Beyond the bispectrum

$$\rho(\psi) \propto e^{-\frac{1}{H^4}\left(\alpha V'' + \beta \psi V'\right)}$$

GAP & Riquelme (2017)



Beyond the bispectrum

$$\mathcal{L} = \epsilon \left(\dot{\mathcal{R}} - \alpha\, \psi \right)^2 - \frac{\epsilon}{a^2} (\nabla \mathcal{R})^2 + \frac{1}{2} \dot{\psi}^2 - \frac{1}{a^2} (\nabla \psi)^2 - V(\psi) + \cdots$$

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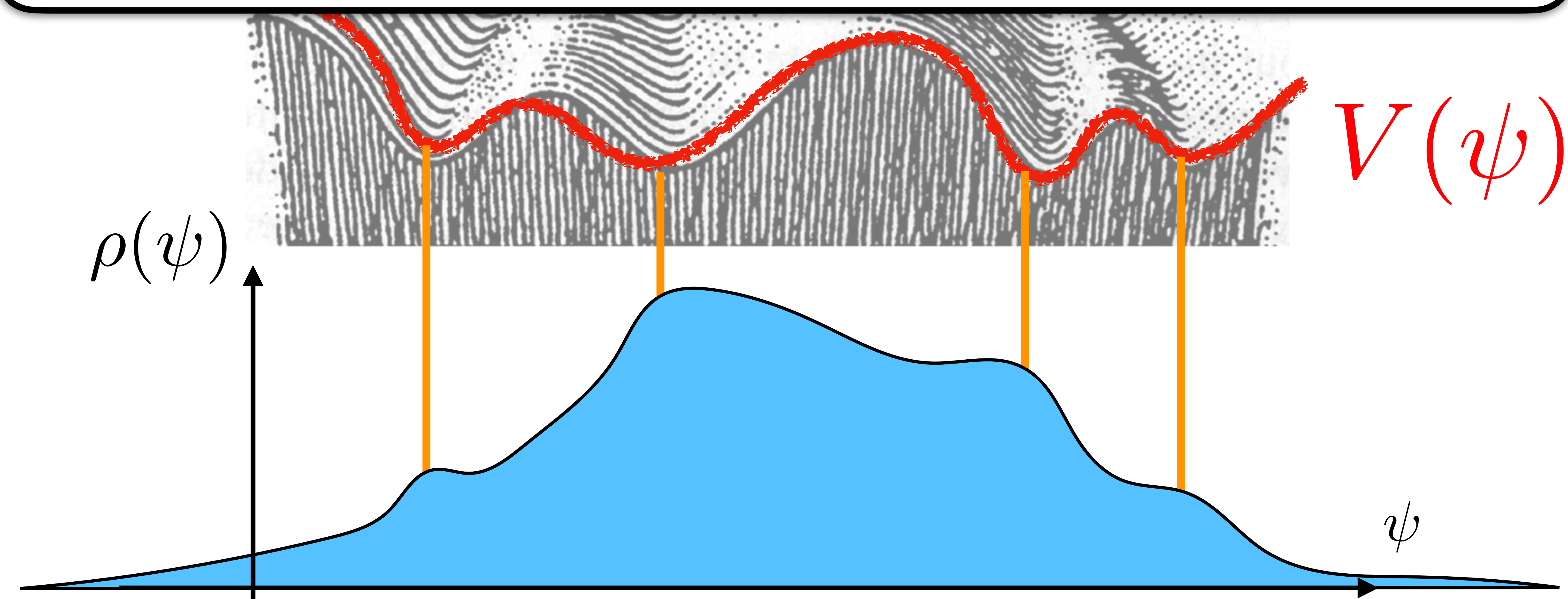
Chen, **GAP**, Scheihing & Sypsas (2019)



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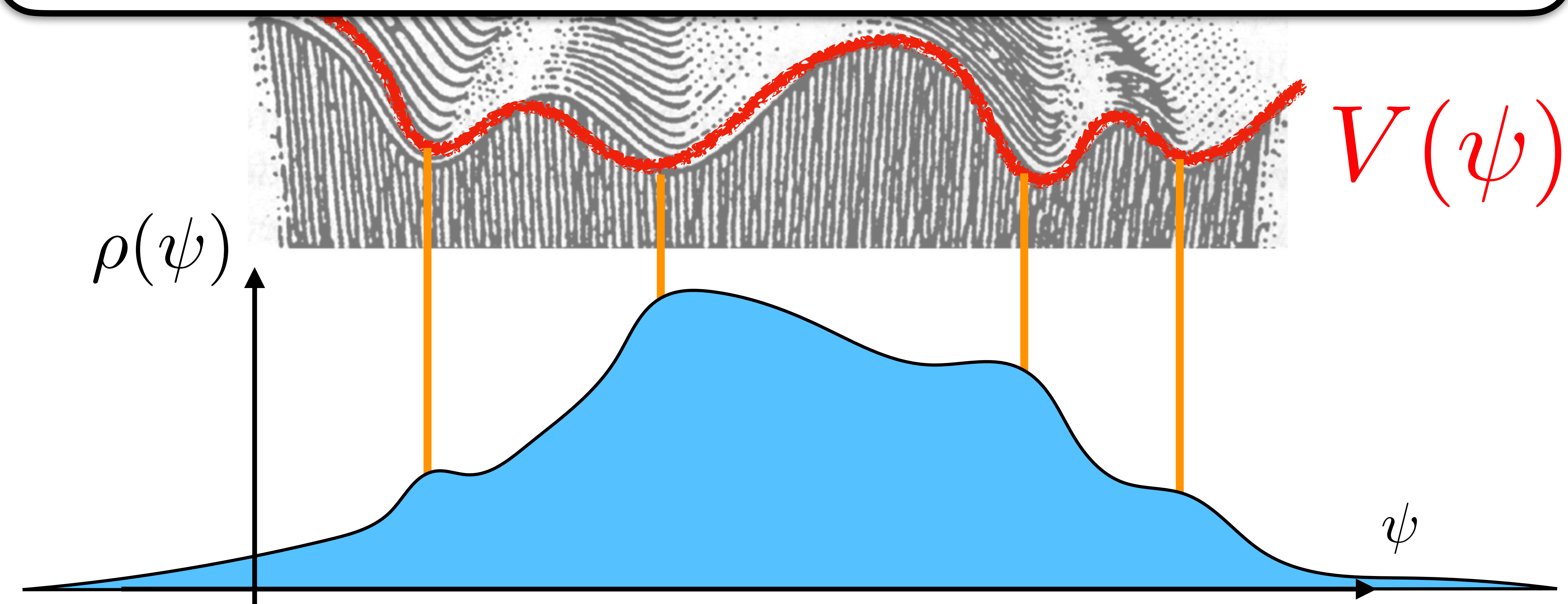

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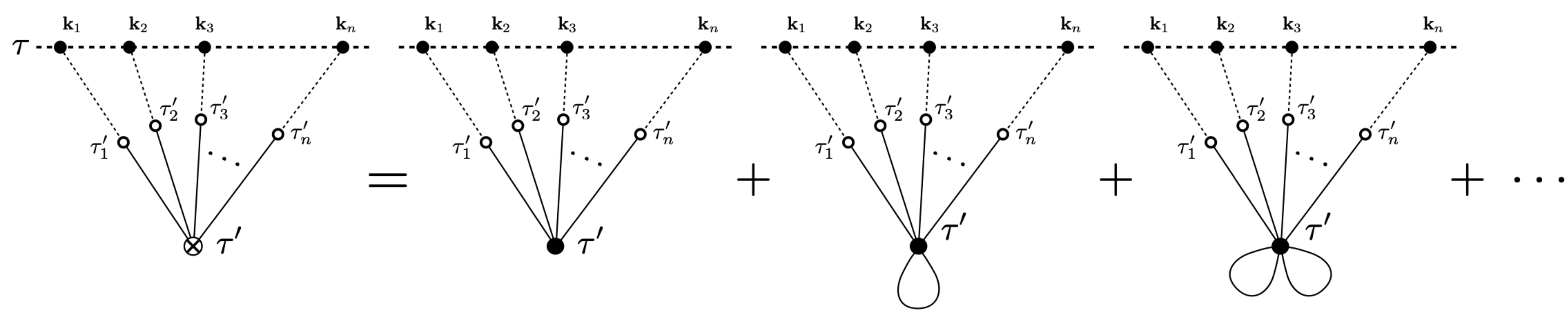

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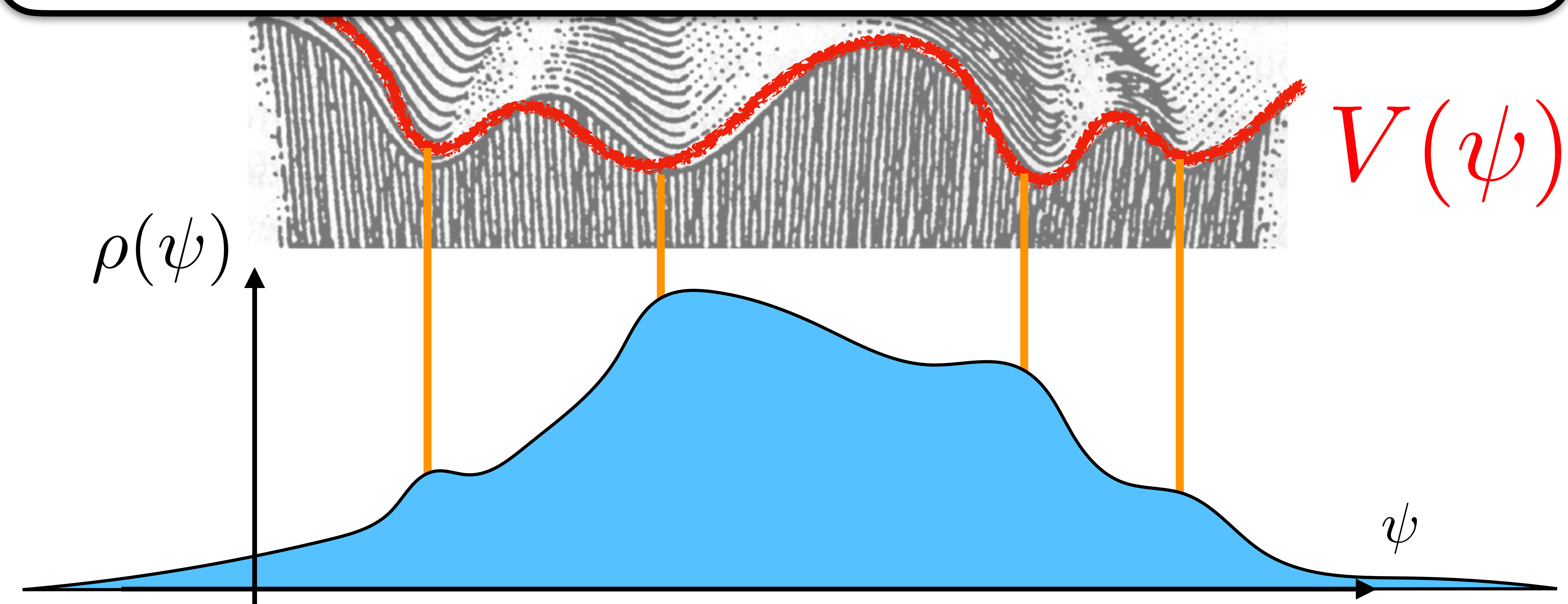
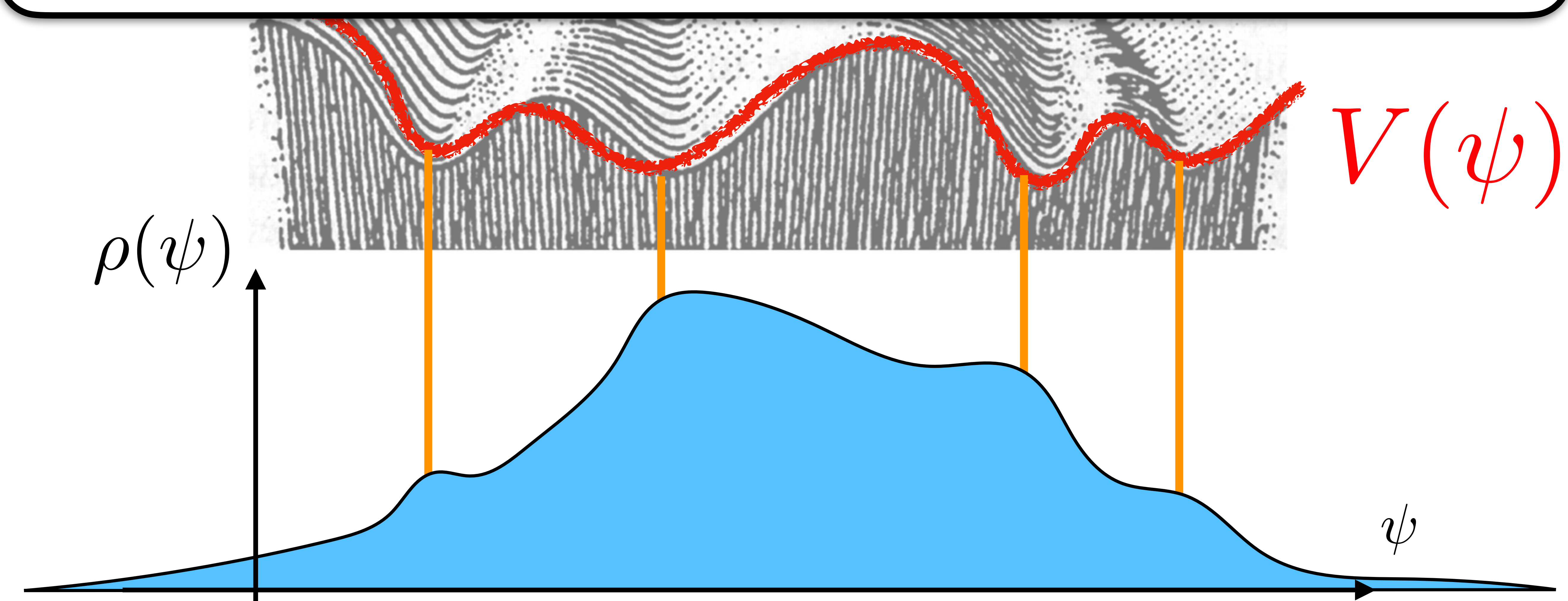


Beyond the bispectrum

$$\rho(\mathcal{R}) = \frac{1}{\sqrt{2\pi}\sigma_{\mathcal{R}}} e^{-\frac{\mathcal{R}^2}{2\sigma_{\mathcal{R}}^2}} \left[1 + \Delta(\mathcal{R})\right]$$

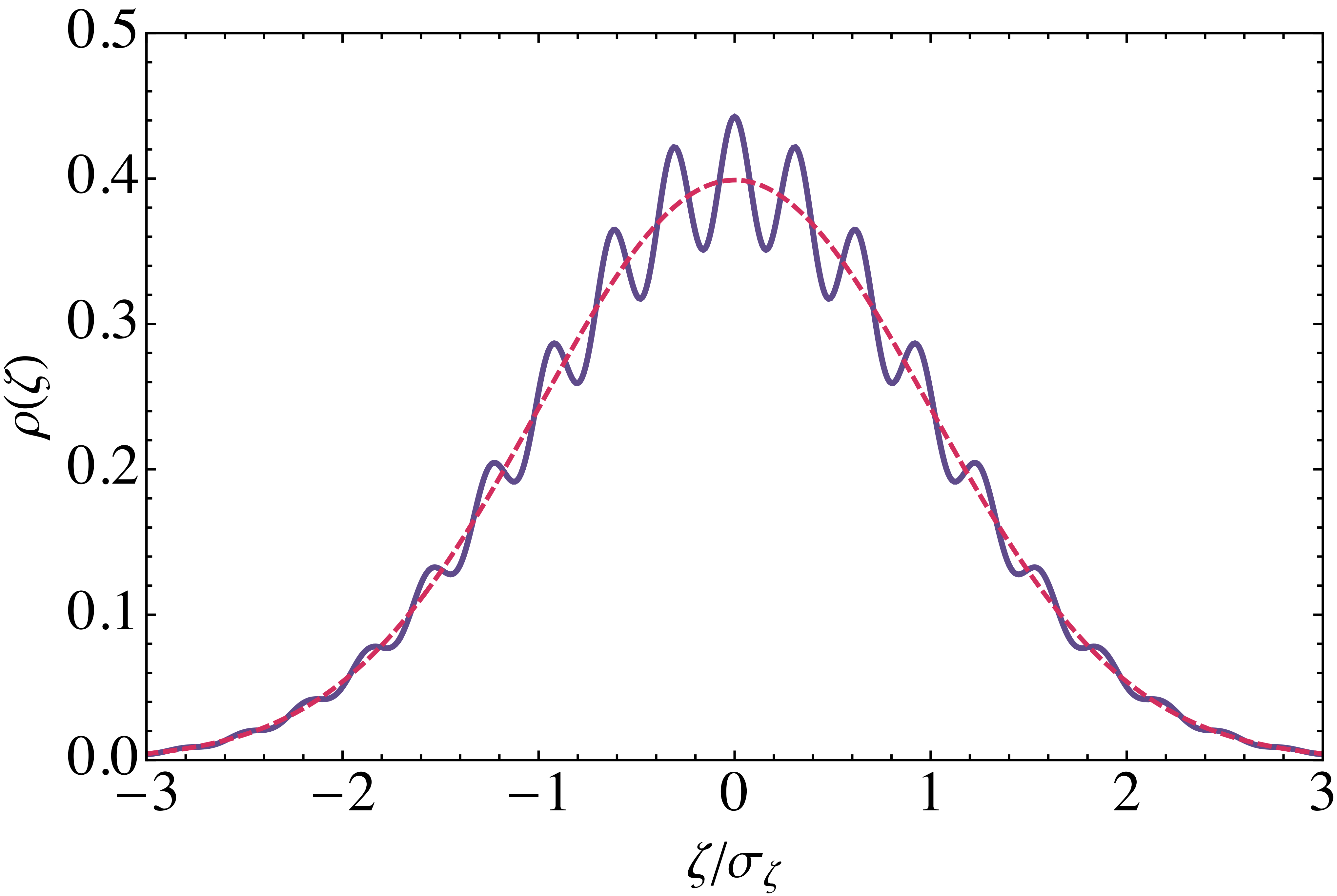
$$\Delta(\mathcal{R}) \propto \int_0^\infty \frac{dx}{x} \mathcal{K}(x) \int_{-\infty}^{\infty} d\bar{\mathcal{R}}\, \frac{\exp\left[-\frac{\left(\bar{\mathcal{R}} - \mathcal{R}(x)\right)^2}{2\sigma_{\mathcal{R}}^2(x)}\right]}{\sqrt{2\pi}\sigma_{\mathcal{R}}(x)} \times \left(\sigma_{\mathcal{R}}^2 \frac{\partial}{\partial \bar{\mathcal{R}}} - \bar{\mathcal{R}}\right) V(\psi_{\bar{\mathcal{R}}})$$

Chen, **GAP**, Scheihing & Sypsas (2019)



Beyond the bispectrum

$$V(\psi) \propto [1 - \cos(\psi/f)]$$



Chen, **GAP**, Riquelme, Scheihing & Sypsas (2019)

Beyond the bispectrum

The previous idea can be examined non-perturbatively. On long wavelengths a spectator field satisfies the Fokker-Planck equation:

$$\frac{\partial \rho}{\partial t} = \frac{1}{3H} \frac{\partial}{\partial \psi} \Big(V'(\psi) \rho \Big) + \frac{H^3}{8\pi^2} \frac{\partial^2 \rho}{\partial \psi^2}$$

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Starobinsky & Yokoyama (1994)

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Starobinsky & Yokoyama (1994)

$$\boxed{\rho(\mathcal{R}) \sim e^{-\frac{\mathcal{R}}{\kappa}}}$$

Panagopolous & Silverstein (2020)

Beyond the bispectrum

In the case of multi-fields:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial \mathcal{R}}\left(\dot{\mathcal{R}}\rho\right) + \frac{\partial}{\partial \dot{\mathcal{R}}}\left[\left(3H\mathcal{R} + \alpha^2\psi^2 + \alpha\dot{\mathcal{R}}\psi\right)\rho\right] + \cdots$$

Achúcarro, Céspedes, Davis, **GAP** (2022)

See also: Pinol, Renaux-Petel &Tada (2021)

Beyond the bispectrum

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$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial \mathcal{R}}\left(\dot{\mathcal{R}}\rho\right) + \frac{\partial}{\partial \dot{\mathcal{R}}}\left[\left(3H\mathcal{R} + \alpha^2\psi^2 + \alpha\dot{\mathcal{R}}\psi\right)\rho\right] + \cdots$$

$$\rho[\mathcal{R}, \psi] \sim \exp\left[-\frac{\psi^2}{2P_\psi^2} - \frac{1}{2P_\psi^2}\left(\mathcal{R} - \kappa\frac{\psi^2}{2P_\psi^2}\right)^2 + \cdots\right]$$

Achúcarro, Céspedes, Davis, **GAP** (2022)

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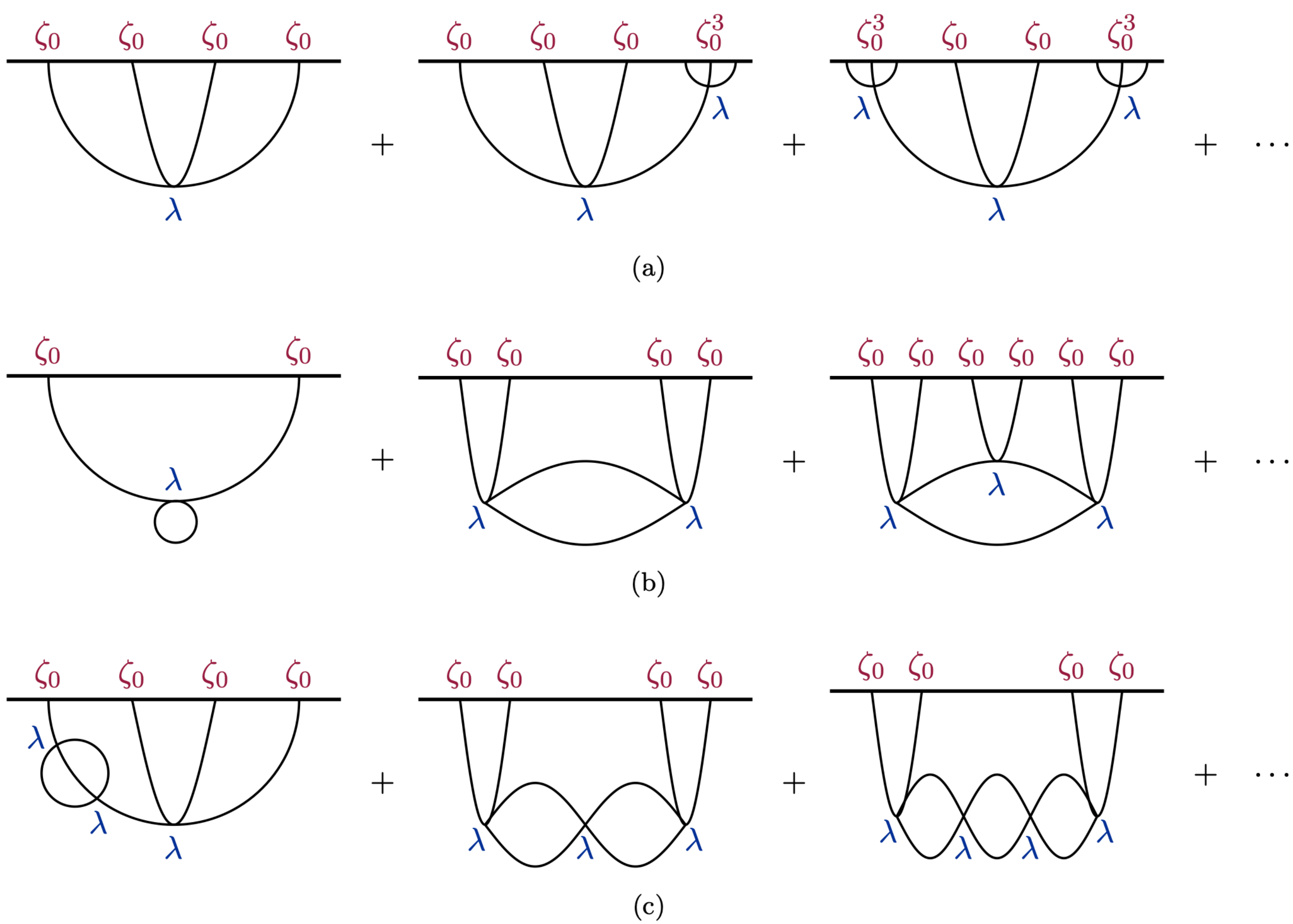
Achúcarro, Céspedes, Davis, **GAP** (2022)

See also: Pinol, Renaux-Petel &Tada (2021)

Beyond the bispectrum

NG tails also possible in single field inflation:

$$\mathcal{L} = \epsilon\left(\dot{\mathcal{R}}^2 - (\nabla\mathcal{R})^2 + \frac{\lambda}{4!H^2}\dot{\mathcal{R}}^4\right)$$



Celoria, Creminelli, Tambalo & Yingcharoenrat (2021)

Beyond the bispectrum

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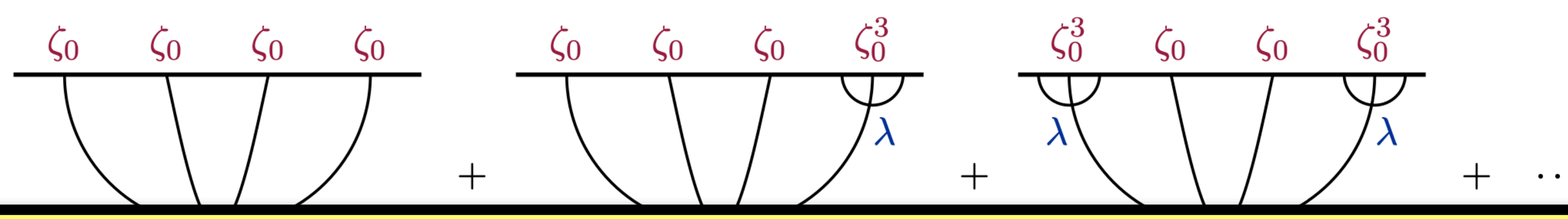
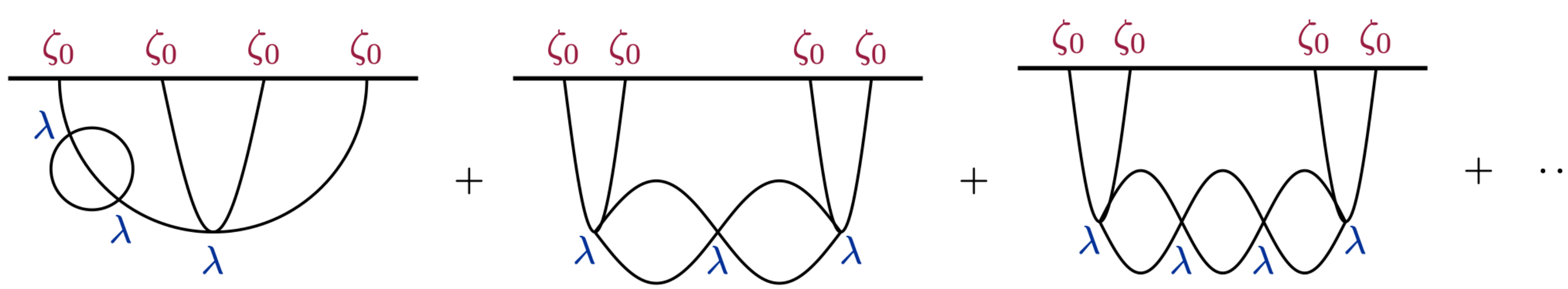


$$\rho(\mathcal{R}) \sim \exp\left[-\frac{\mathcal{R}^{3/2}}{\lambda^{1/4}}\right]$$

(b)

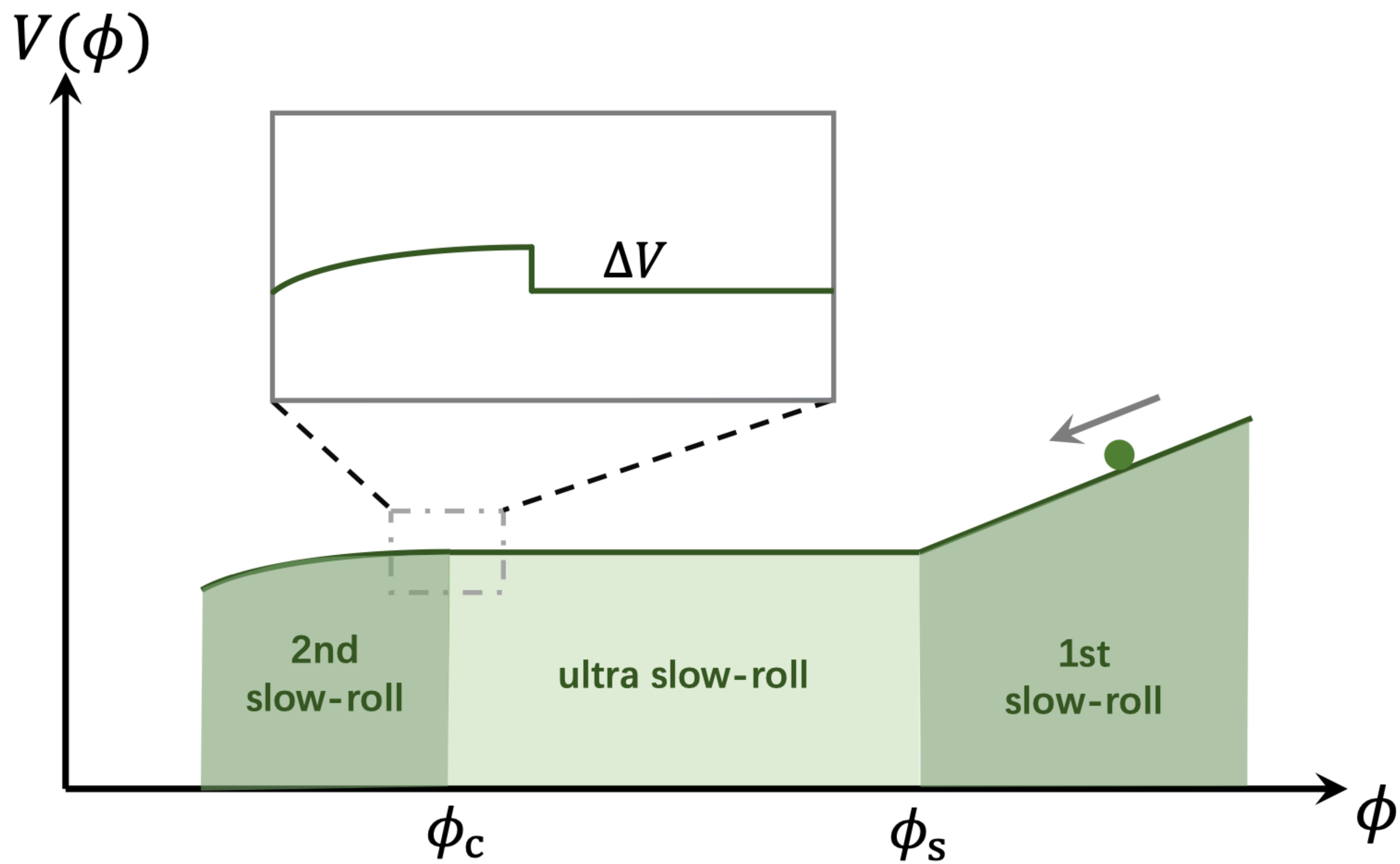


(c)

Celoria, Creminelli, Tambalo & Yingcharoenrat (2021)

Beyond the bispectrum

Other recent results on highly non-Gaussian tales:



Cai, Ma, Sasaki, Wang & Zhou (2022)

Beyond the bispectrum

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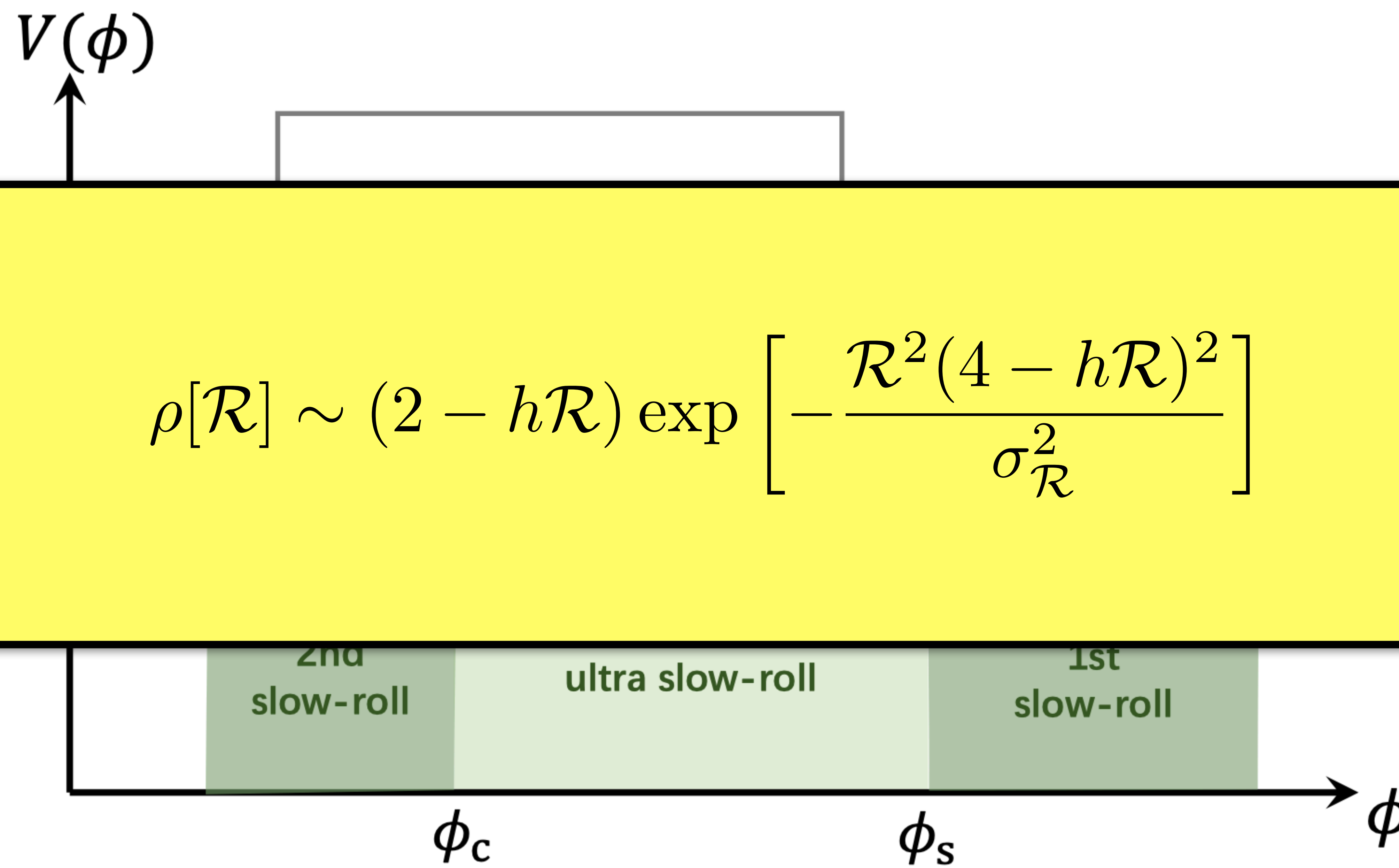
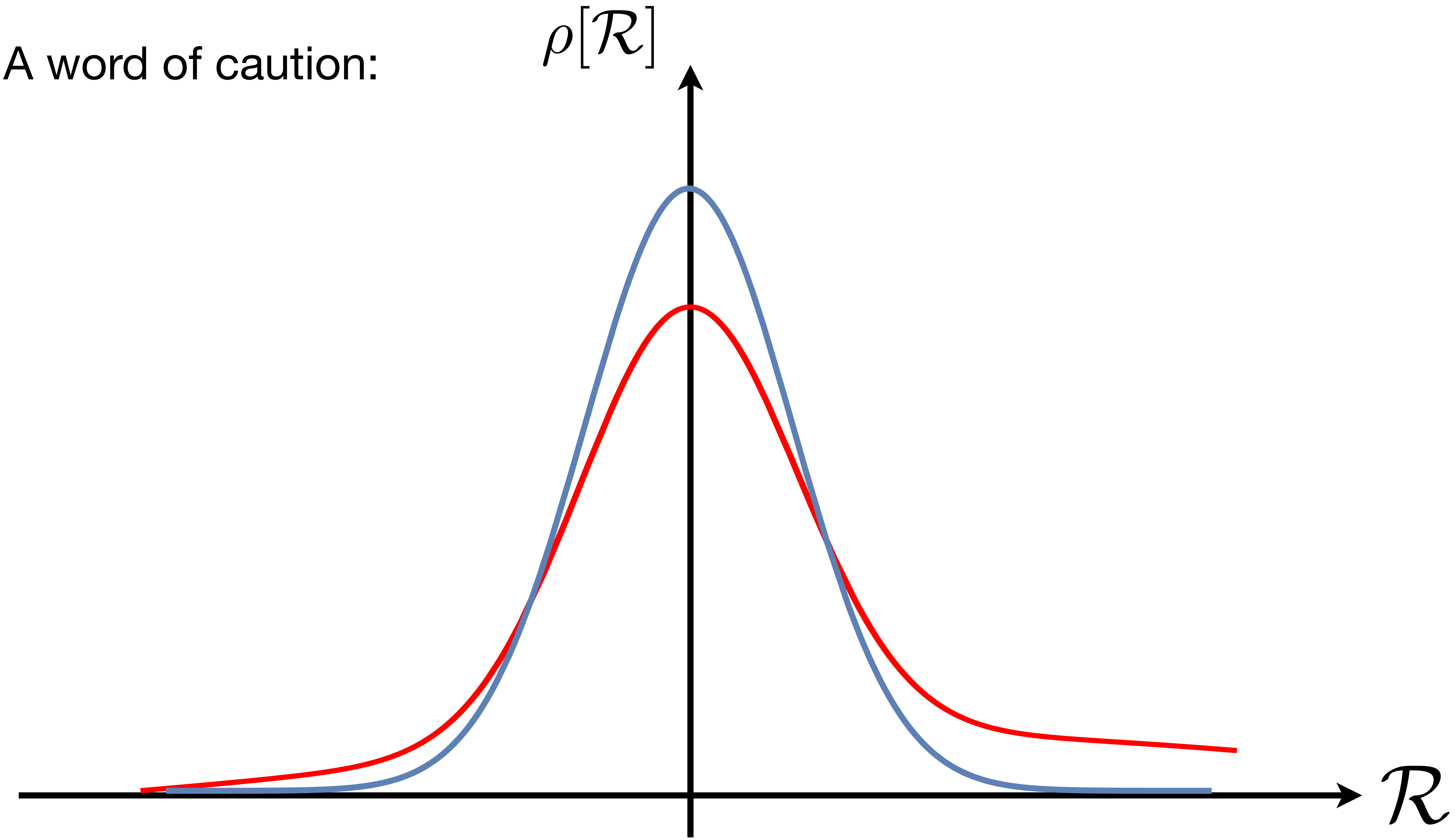


$$\rho[\mathcal{R}] \sim (2 - h\mathcal{R}) \exp\left[-\frac{\mathcal{R}^2(4 - h\mathcal{R})^2}{\sigma_{\mathcal{R}}^2}\right]$$

Cai, Ma, Sasaki, Wang & Zhou (2022)

Beyond the bispectrum

A word of caution:



De Luca & Riotto (2022)

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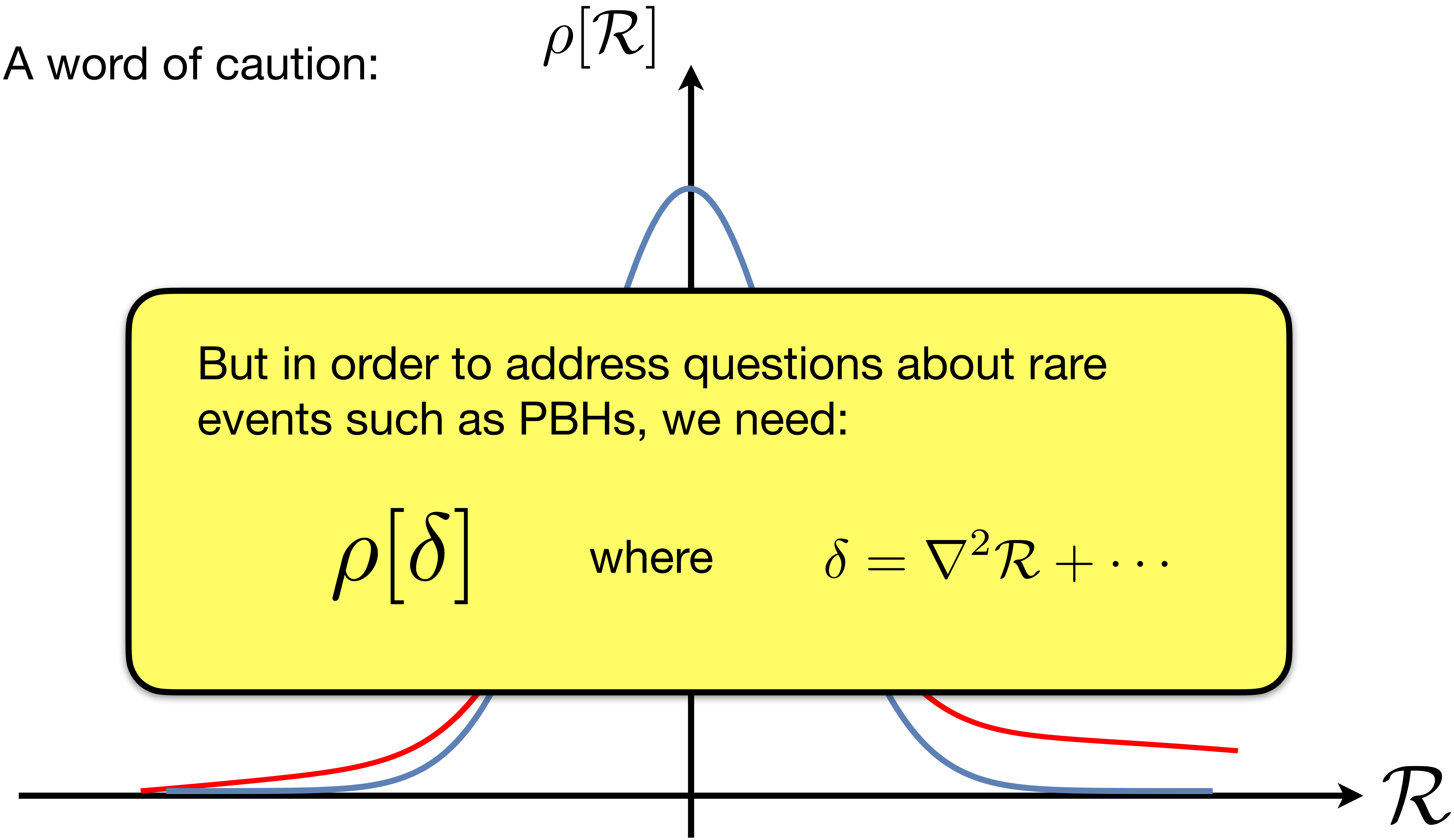


But in order to address questions about rare events such as PBHs, we need:

$$\rho[\delta] \quad \text{where} \quad \delta = \nabla^2\mathcal{R} + \cdots$$

De Luca & Riotto (2022)

Conclusions

- The primordial statistics may deviate significantly from Gaussianity in a way not parametrized by the bispectrum

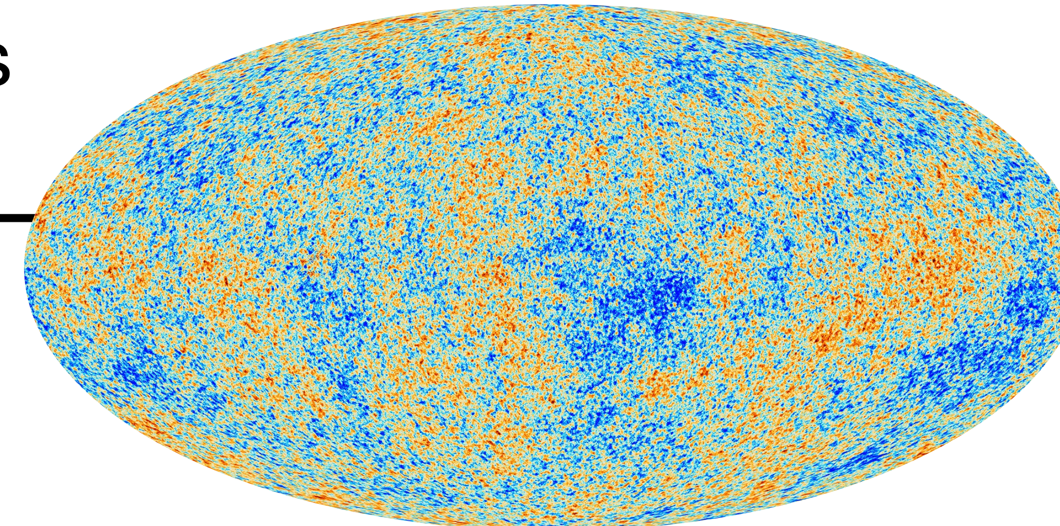
- These effects could escape conventional data analysis

- New (non-perturbative) techniques are necessary to uncover this type of NG

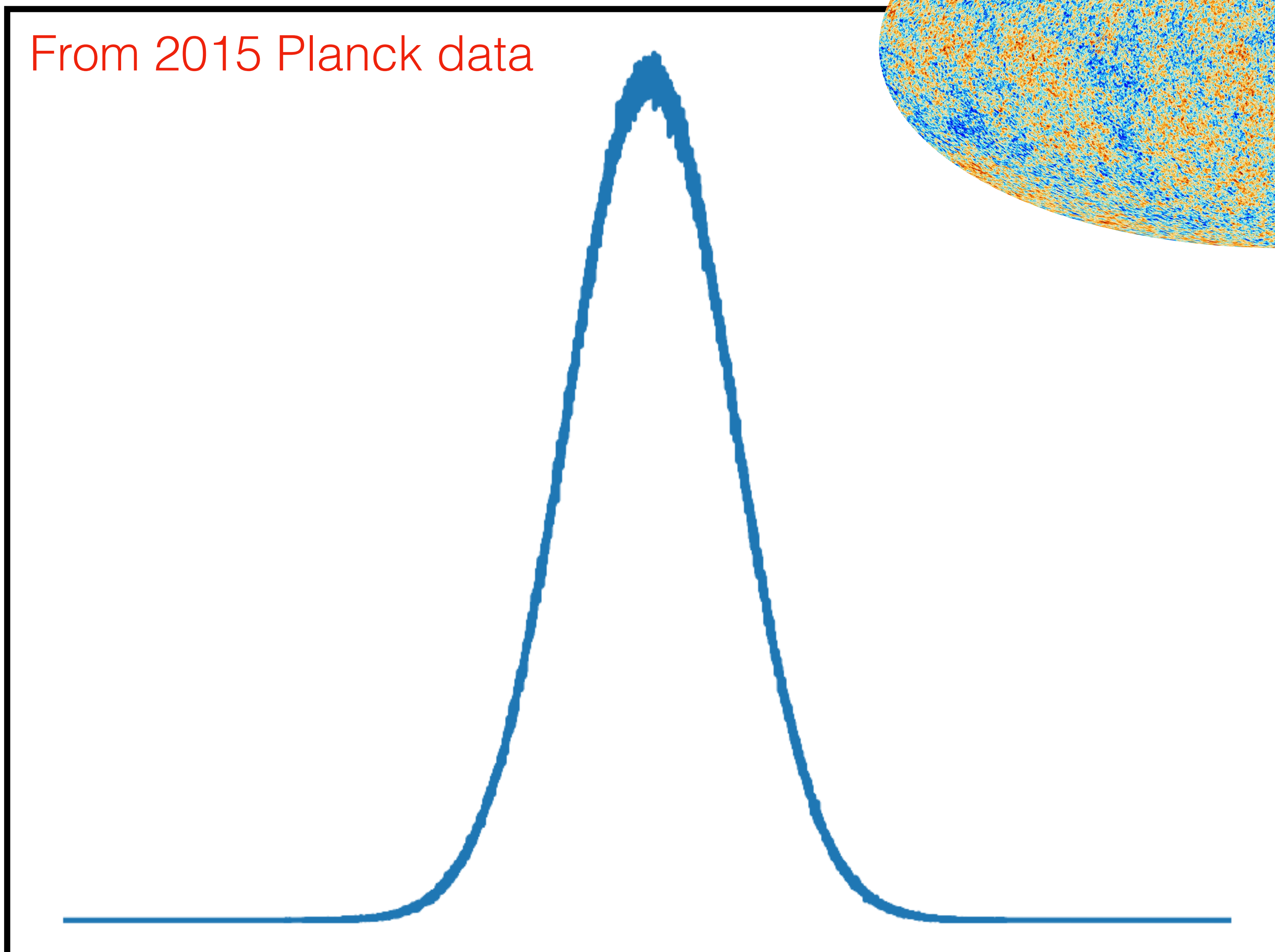
Thanks!

Reconstructing the landscape

The reconstructed PDF from CMB data is



$\rho(\Theta)$



$$\Theta = \frac{\Delta T}{T}$$

See also:
Buchert, France, Steiner (2017)

Reconstructing the landscape

$$a_n \equiv \int d\Theta \, \rho(\Theta) He_n(\Theta/\sigma_\Theta)$$

Chen, GAP, Scheihing & Sypsas (2018)

Reconstructing the landscape

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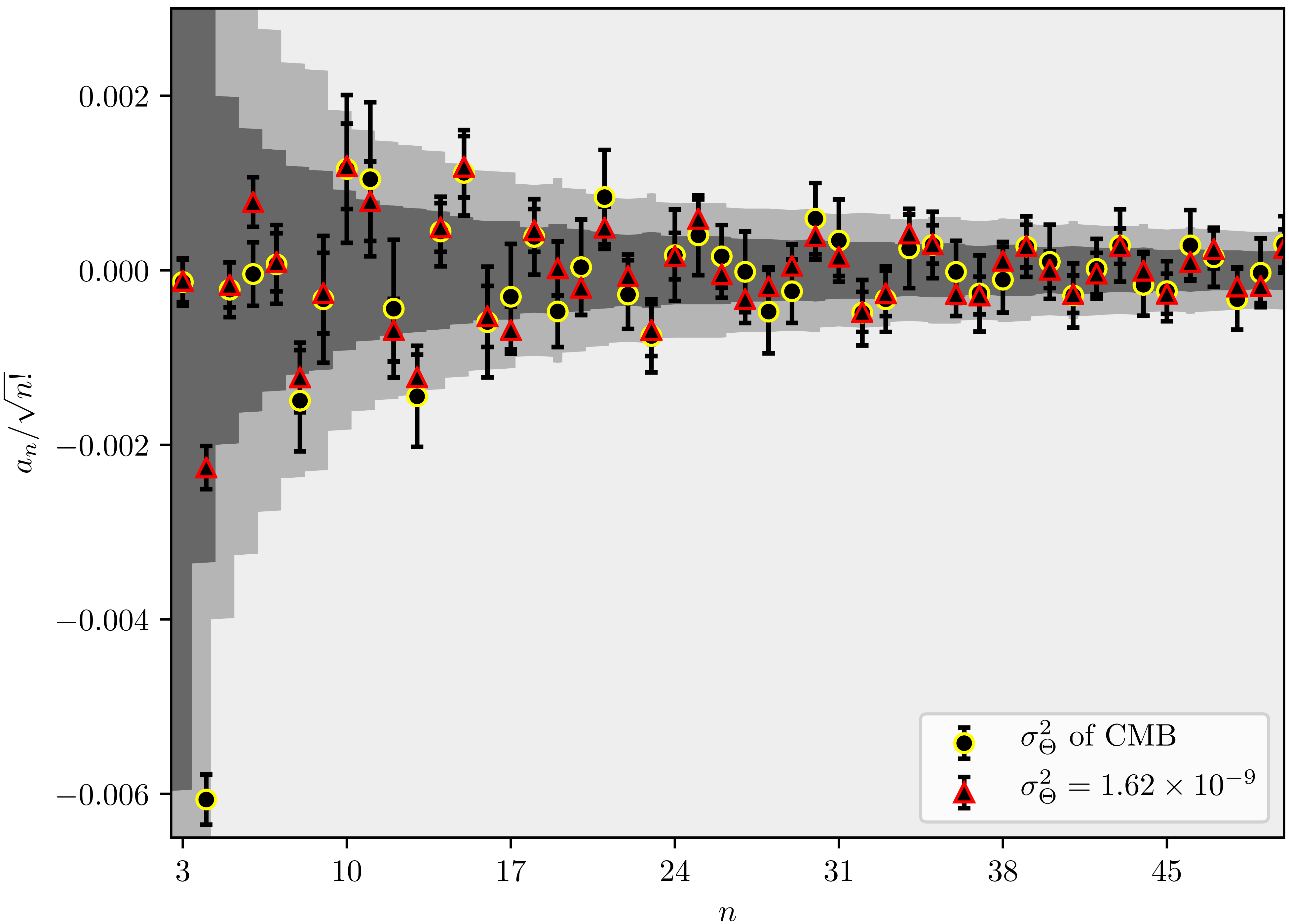

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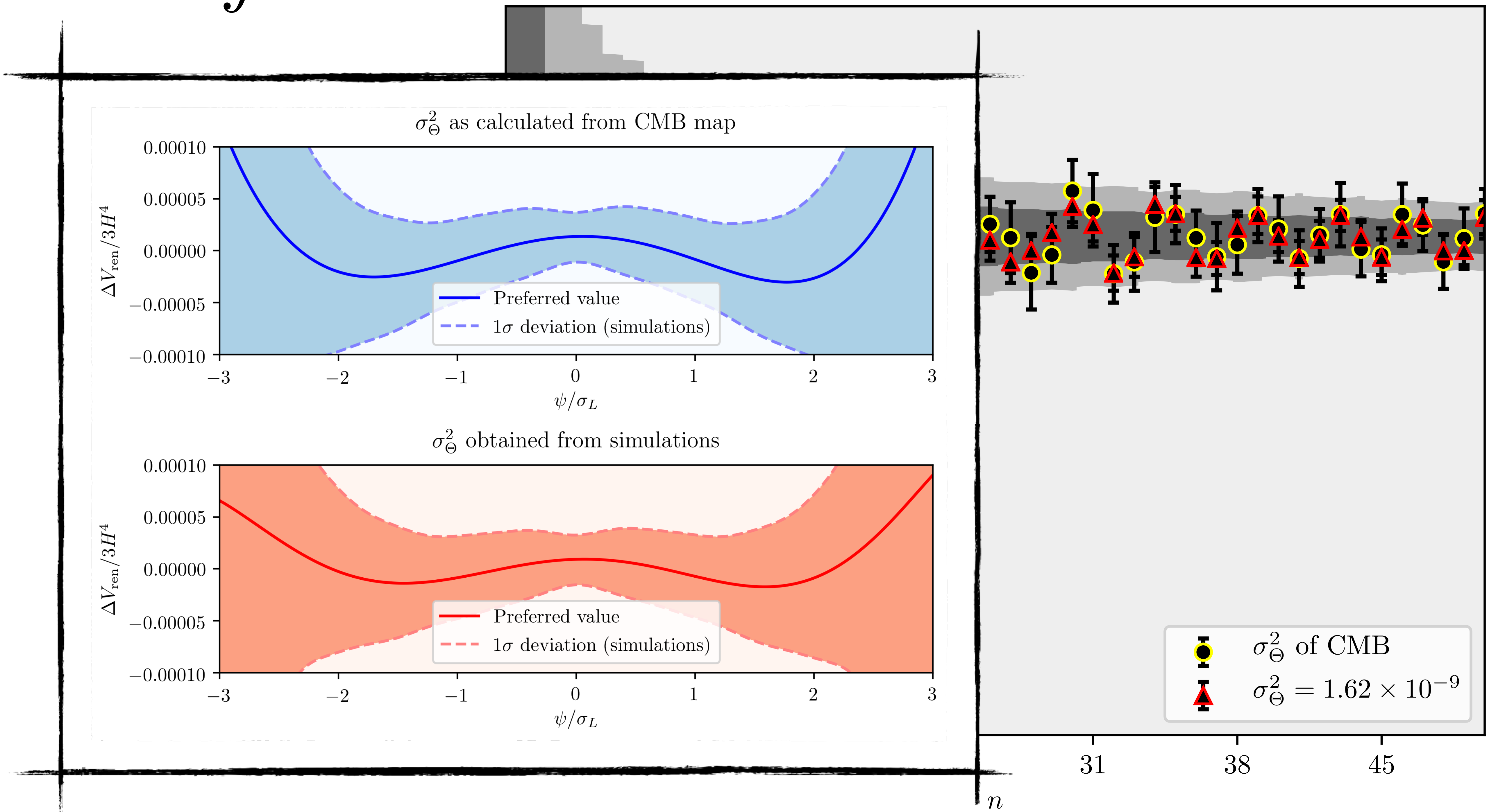

Chen, GAP, Scheihing & Sypsas (2018)

Other statistical tools

A generalized local ansatz

$$\zeta = \zeta_G + \mathcal{F}_{\rm NG}(\zeta_G)$$

GAP, Scheihing & Sypsas (2018)

Other statistical tools

A generalized local ansatz

$$\zeta = \zeta_G + \mathcal{F}_{\mathrm{NG}}(\zeta_G)$$

From it, one can derive a functional probability distribution

$$\mathrm{P}_{\mathrm{NG}}[\zeta] = \mathrm{P}_{\mathrm{G}}[\zeta] \times \exp\left[- \int_{\boldsymbol{y}} \int_{\boldsymbol{k}} e^{-i\boldsymbol{k}\cdot\boldsymbol{y}} \left(\frac{\delta}{\delta\zeta(-\boldsymbol{k})} - \frac{\zeta(\boldsymbol{k})}{P_\zeta(k)} \right) \mathcal{F}_{\mathrm{NG}}[\zeta](\boldsymbol{y}) \right]$$

GAP, Scheihing & Sypsas (2018)

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From where one recovers

$$\rho(\bar\zeta) = \int D\zeta \, \mathrm{P}_{\rm NG}[\zeta] \, \delta(\zeta(\boldsymbol{x}) - \bar\zeta)$$

GAP, Scheihing & Sypsas (2018)

Other statistical tools

The functional allows for other means of analysing data:

$$\rho(\zeta_1, \zeta_2, |\mathbf{x}_1 - \mathbf{x}_2|) = \int D\zeta \, \mathrm{P}_{\mathrm{NG}}[\zeta] \, \delta(\zeta(\boldsymbol{x}_1) - \zeta_1)\delta(\zeta(\boldsymbol{x}_2) - \zeta_2)$$

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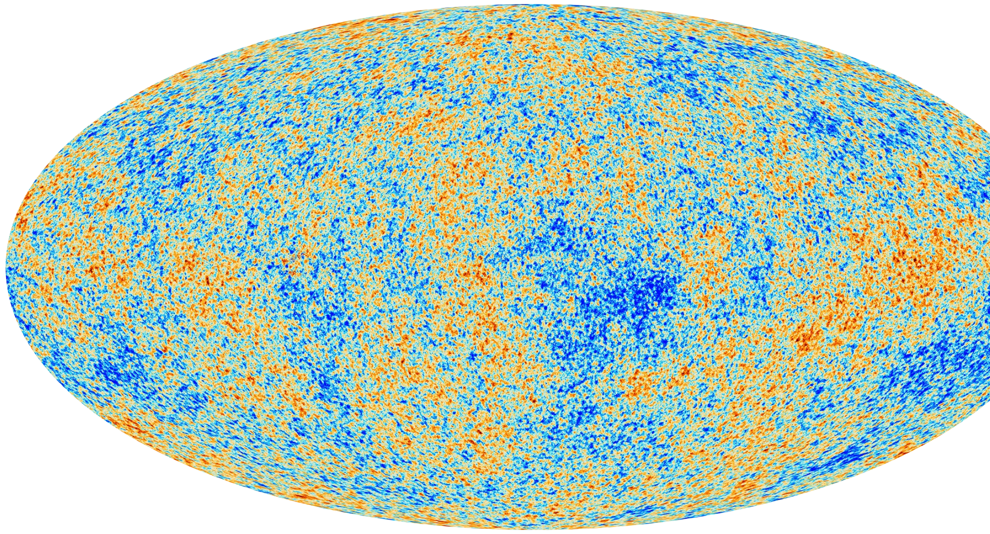

This leads to two point PDF’s:

$$\rho(\Theta_1, \Theta_2, \hat{n}_1 \cdot \hat{n}_2) = \rho_G \left[1 + \mathcal{O}(\mathcal{F}_{NG})\right]$$

Other statistical tools

- Dark matter halos

$$\mu^{>}(M,z) \equiv \int_{\nu_c(z)}^{\infty} d\nu P(\nu)$$

$$\frac{dn}{dm} \propto \left.\frac{dn}{dm}\right|_G (1 + \mathcal{O}(\mathcal{F}_{\mathrm{NG}}))$$

LoVerde, Miller, Shandera & Verde (2008)

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- Halo bias

$$\Delta b(k) = \frac{2\delta_c(b_{\rm G} - 1)}{\alpha(k)} f_{\rm NL}$$

Dalal, Doré, Huterer & Shirokov (2008)

$$\Delta b(k) = \frac{2\delta_c(b_{\rm G} - 1)}{\alpha(k)} \mathcal{O}(\mathcal{F}_{\rm NG})$$

GAP, Scheihing & Sypsas (2018)