# Theoretical model for the evaluation of the thermal stability and thermal stress of solid targets in a low emittance muon collider

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# Acknowledgments

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(INFN – LNF),

# Outline

- Introduction of the LEMMA project
- Numerical simulation of the deposited energy onto the target
- Temperature behaviour of the thermal parameters of Beryllium and Carbon
- Temperature field after a single bunch – temperature temporal evolution
- Target steady state temperature
- Thermal stress and quality factors to prevent target fractures
- Conclusions

# Introduction of the LEMMA project

**Low EMittance Muon Accelerator**

**INFN institutions involved: LNF, Roma1, Pd, Pi, Ts, Fe**
**Universities: Sapienza, Padova, Insubria**
**Contributions from: CERN, ESRF, LAL, SLAC**

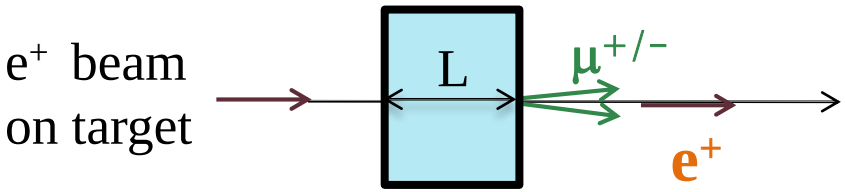
- A $\mu^+\mu^-$ collider offers an ideal technology to extend lepton high energy frontier in the multi-TeV range:
  - No synchrotron radiation (limit of $e^+e^-$ circular colliders)
  - No beamstrahlung (limit of $e^+e^-$ linear colliders)
  - but muon lifetime is 2.2 ms (at rest)

# Muon source

$e^+e^-$ annihilation - positron beam on target : very low emittance and no cooling needed, baseline for our proposal

$e^+$ on standard target (including crystals in channeling)
→ Need Positrons of ≈ 45 GeV

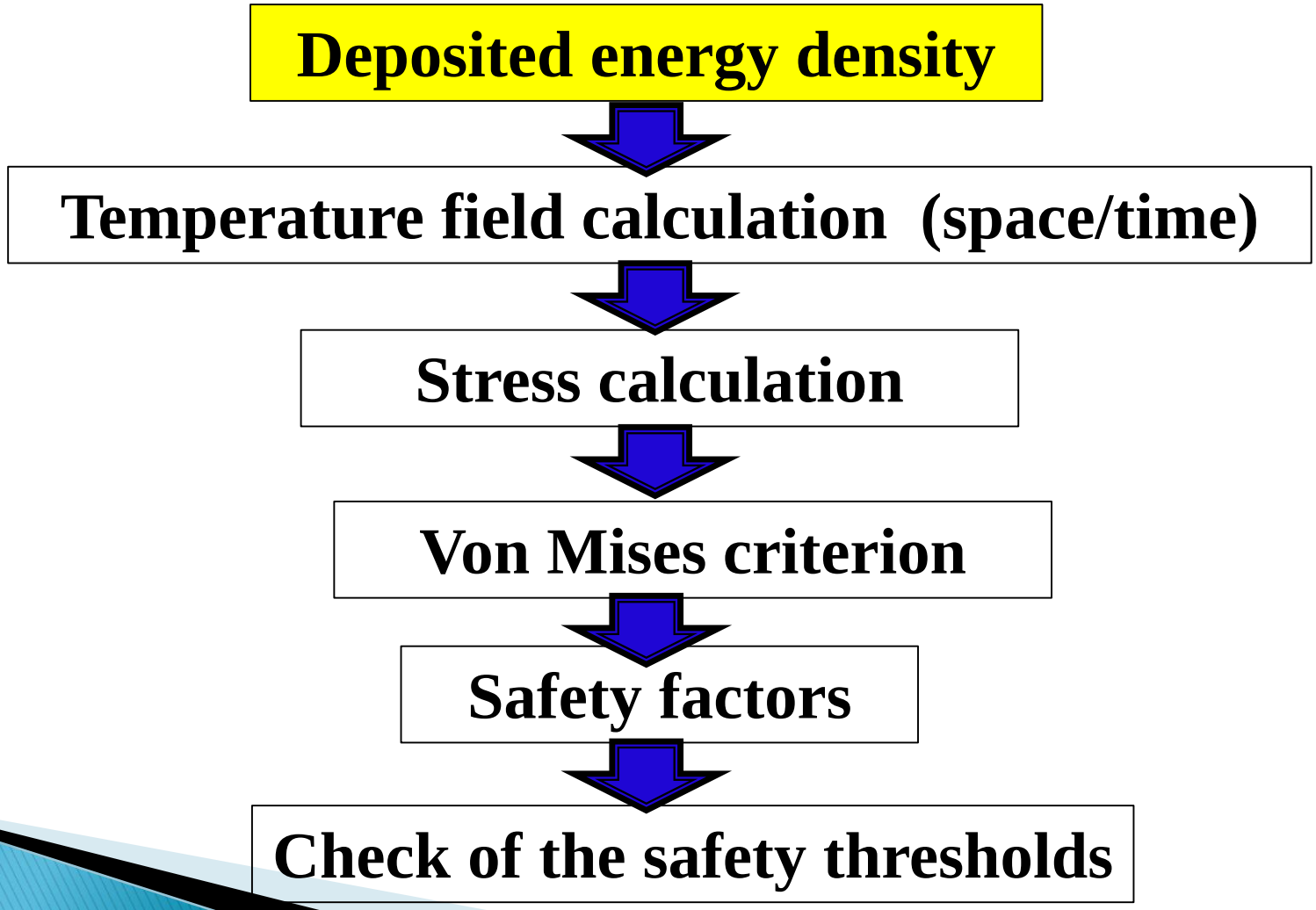
Ideally muons will *copy* the positron beam




(finite target length)

| Material | Density [g/cm$^3$] | Length [m] | Length [$X_0$] | eff [$10^{-6}\mu/e^+$] |
|---|---|---|---|---|
| Be | 1.85 | 0.106 | 0.3 | 1.3 |
| C | 2.27 | 0.057 | 0.3 | 1.0 |

# Methodological approach



# Target size and discretization in the FLUKA code

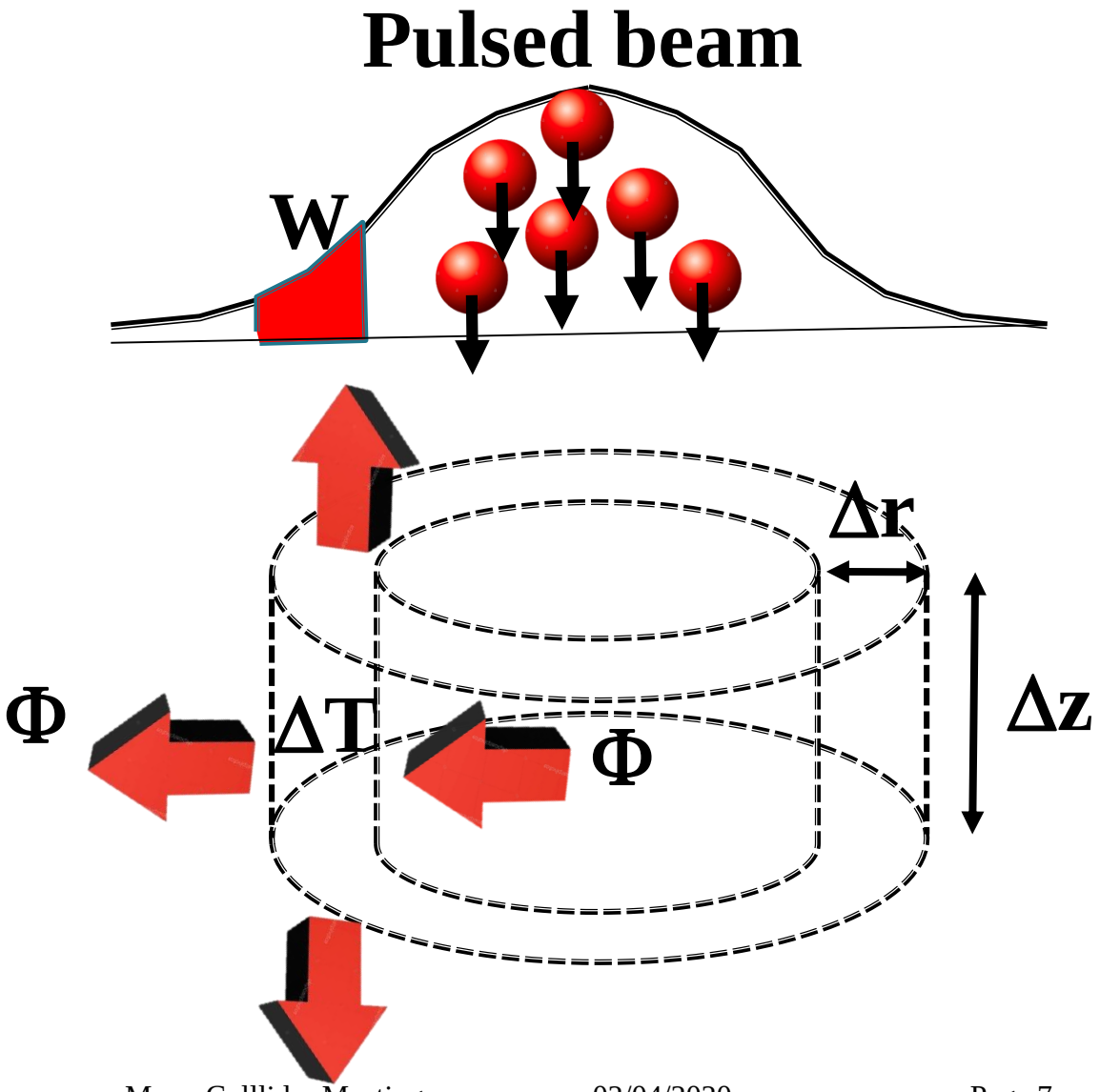
**Pulse duration** $\tau$ : 10 ps

**Spot size a** : three case studies
(10, 50, 140) μm

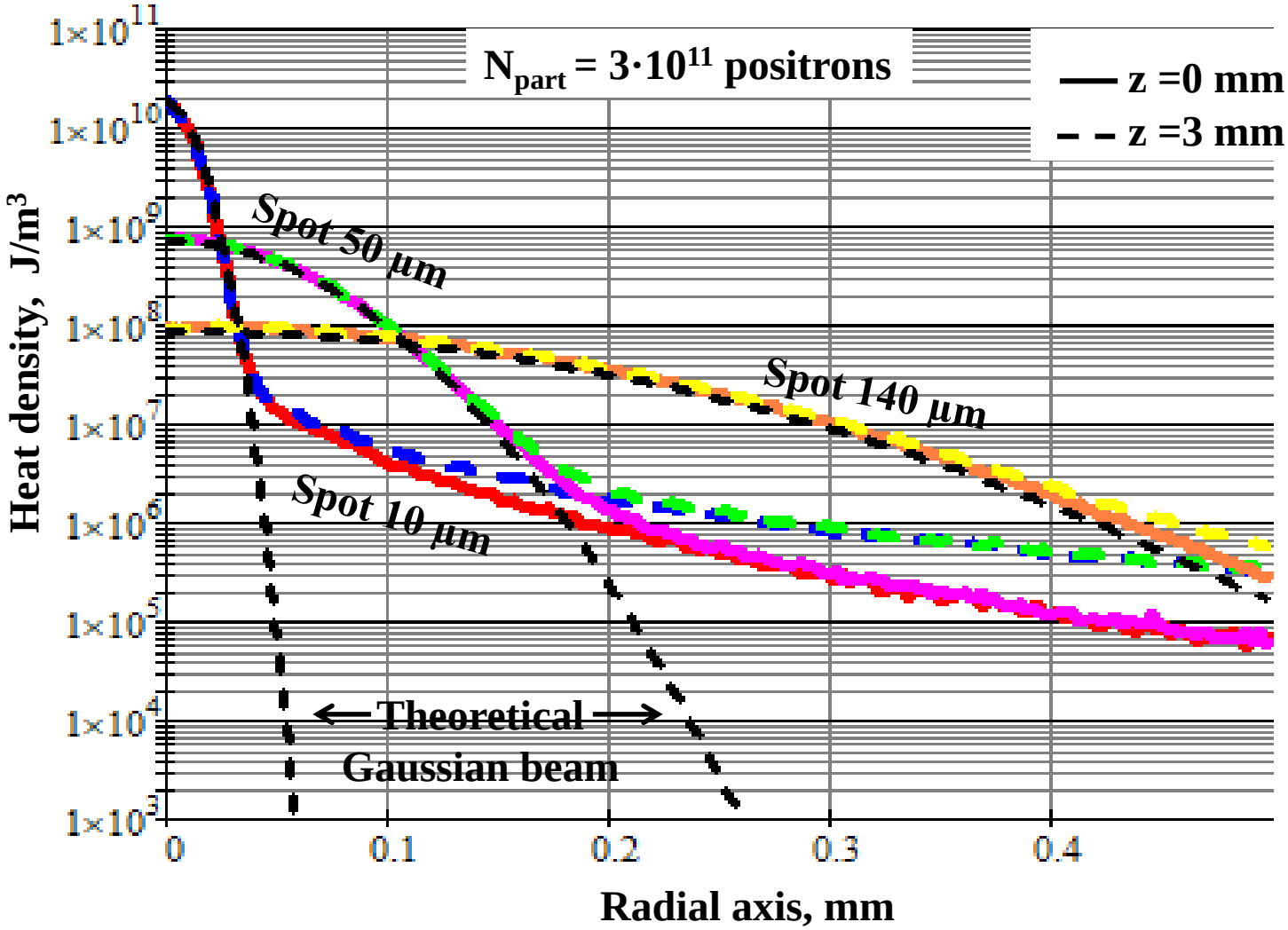
**Beryllium thickness**: 3 mm

**Beryllium radius** : 5 cm

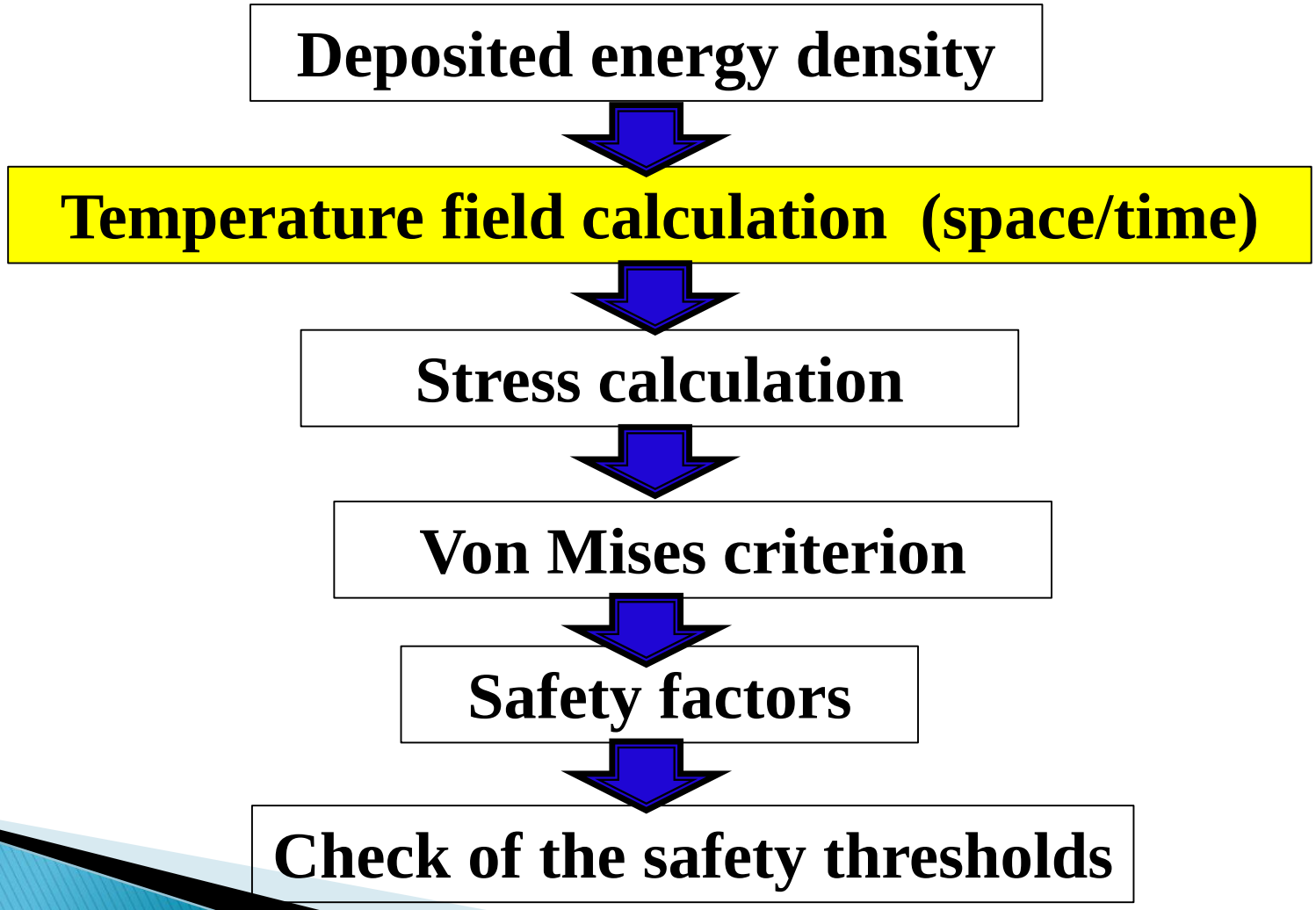
Radial simmetry in the distribution of deposited energy: use of cylindrical coordinates (r, φ=1) and discretization along z.



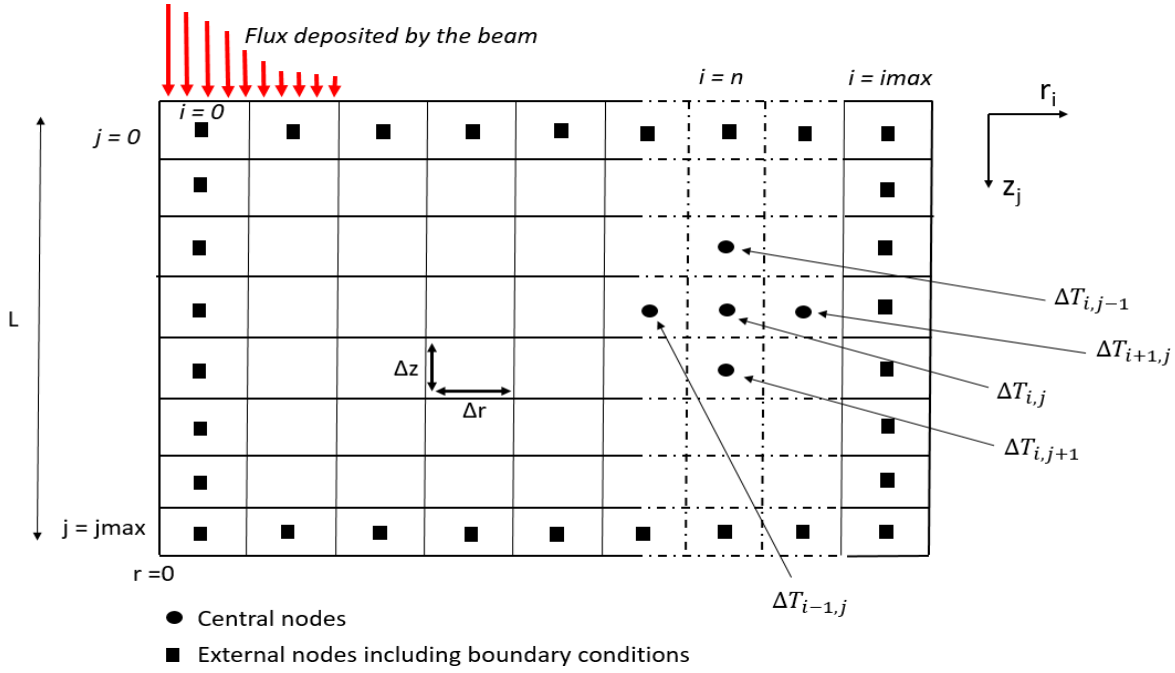
# Energy density deposited on the target



# Methodological approach



# Numerical model for temperature variation inside the material




**i** scan on r, **j** scan on z

**$\Delta T_{i,j}$'** temperature at time t', **$\Delta T_{i,j}$** at time t

**$W_{i,j}$** power deposited in element i,j

**$\Phi_{net}$** heat flow exchanged by the element i,j in the time unit

**$\Delta t$** time lapse

**V** element volume i,j

**$\rho$** density

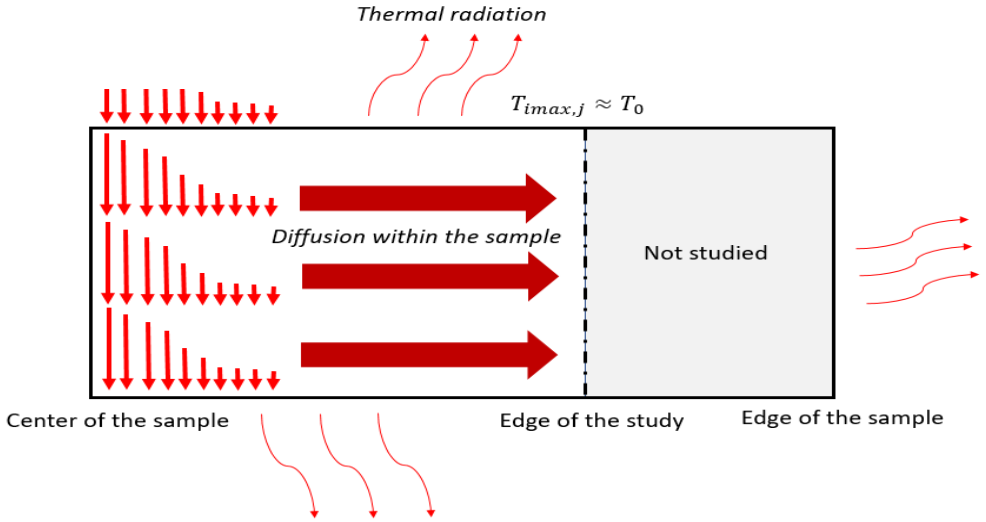
**$C_p$** specific heat

**D** thermal diffusivity

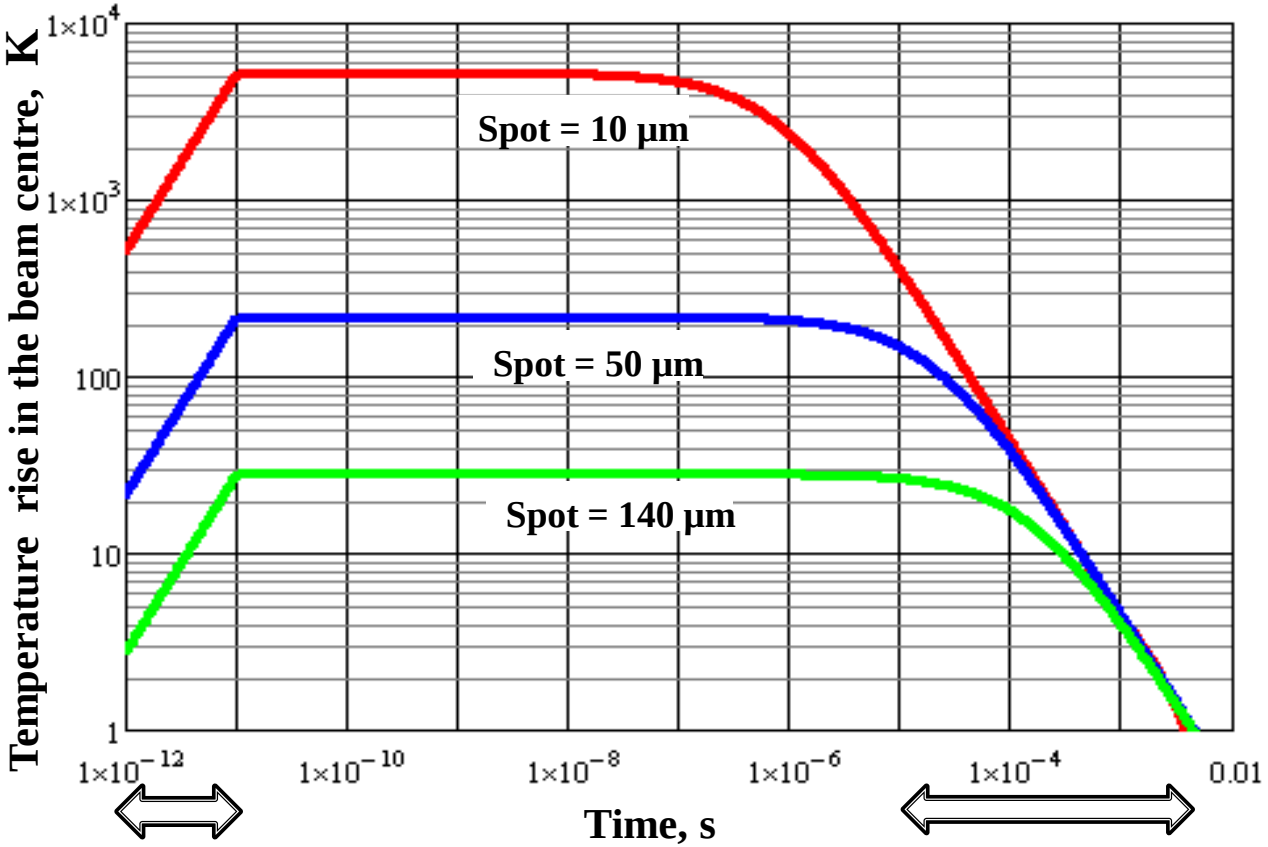
## Basic model equation

$$\Delta T'_{i,j} = \Delta T_{i,j} + \left(\frac{W_{i,j} + \Phi_{net}}{V_{i,j}}\right)\frac{\Delta t}{\rho c_p}$$

Convergence condition
( Fourier number F**0** = Dt/L$^2$ ≤ ½): $\Delta t < \frac{min(\Delta r^2, \Delta z^2)}{4D_{max}}$



# Temperature simulations with constant thermal parameters: linear model – surface temperature rise



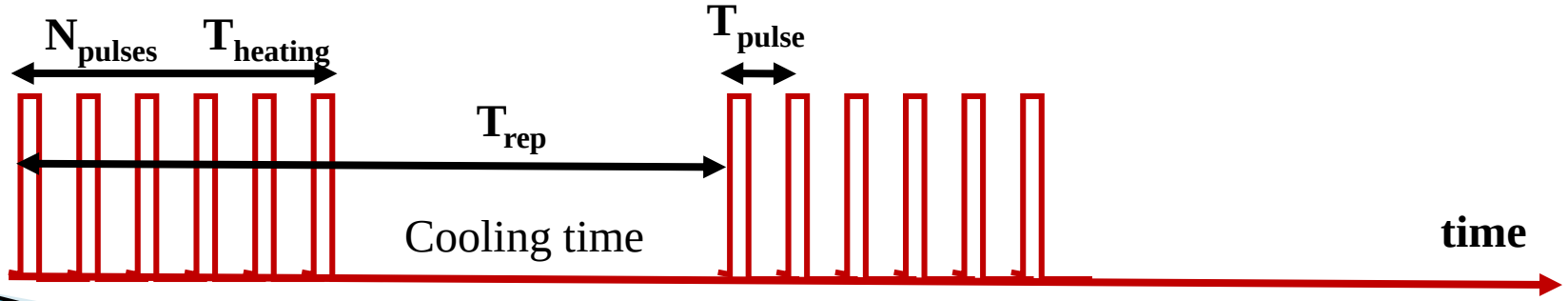

During the bunch

Heat diffusion process

Non-diffusive period

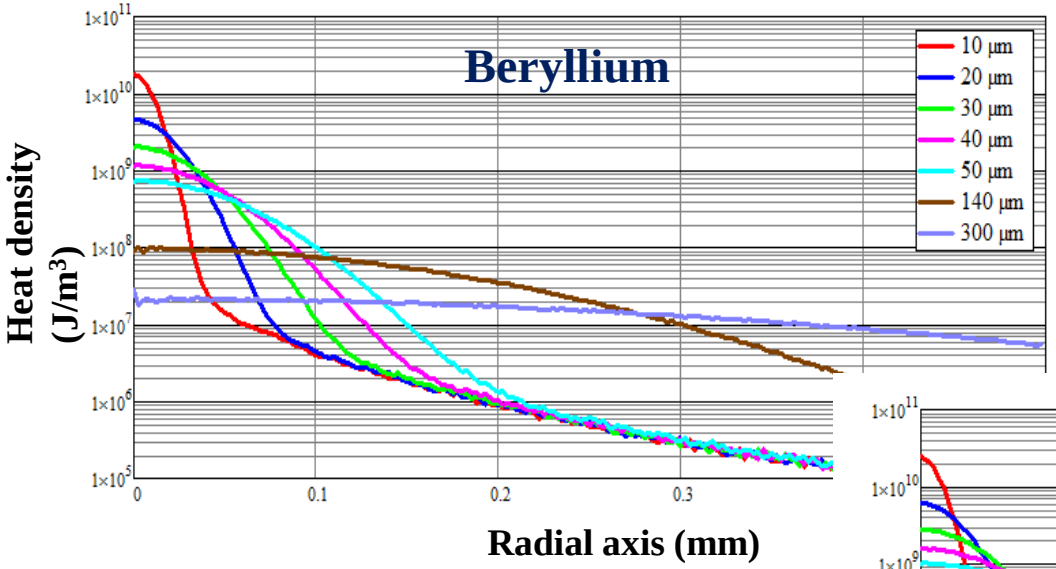
# Features of the benchmark positron beam

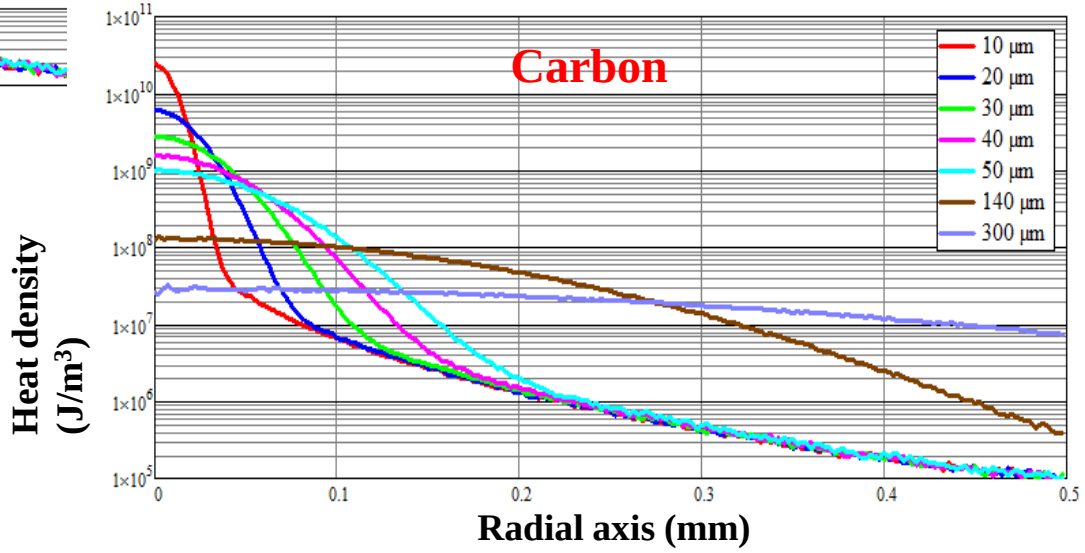
| Symbol | Description | Reference Value |
|---|---|---|
| $a$ | Gaussian beam spot size | 300 μm |
| $\tau$ | bunch duration | 10 ps |
| $N_{part}$ | positron number | $3 \cdot 10^{11}$ |
| $N_{pulses}$ | number of consecutive bunches | 100 |
| $T_{pulse}$ | time between two bunches | 400 ns |
| $T_{heating}$ | total time of $N_{pulses}$ | 40 μs |
| $T_{rep}$ | repetition time of the $N_{pulses}$ sequence | 0.1 s |



# Numerical simulation of the deposited energy onto the target

For this purpose Monte Carlo simulations have been performed with FLUKA both for Beryllium and Carbon (Low-Z materials). The figures show the heat deposited by a single bunch of $3 \cdot 10^{11}$ $e^+$ as a function of the radial distance from the center.



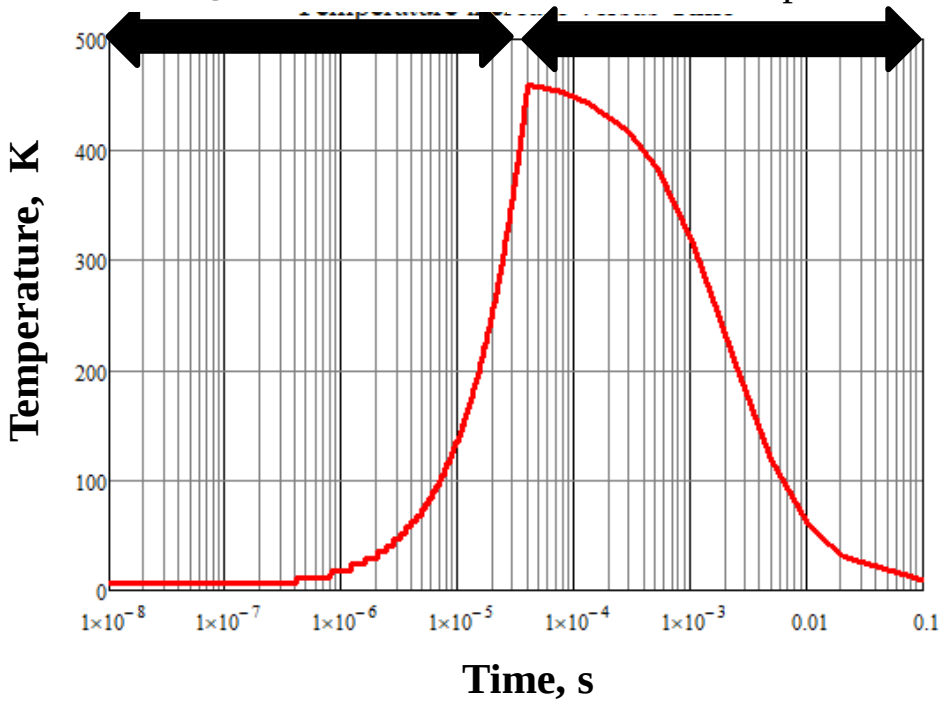



# Temperature temporal evolution in the beam spot center after a sequence of bunches

**After 100 bunches = 40 μs**

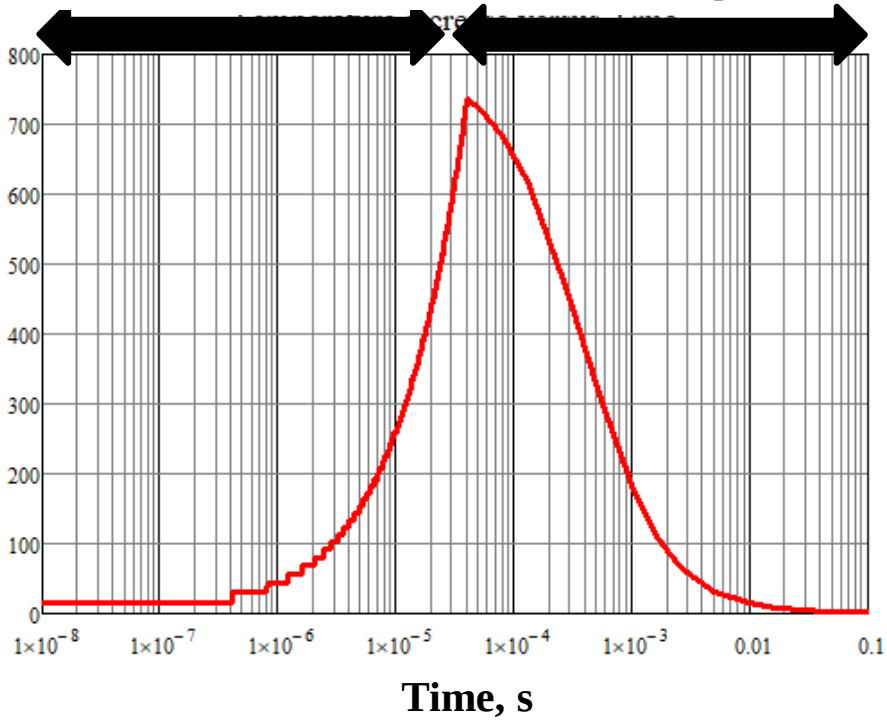
## Bervllium

During the 100 bunces Heat diffusion process




## Carbon

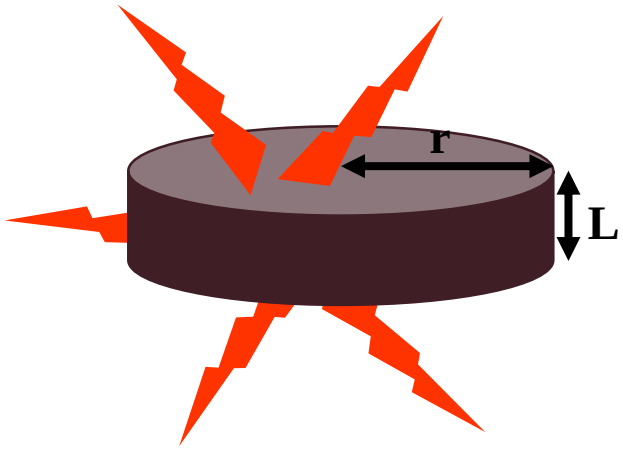
During the 100 bunces Heat diffusion process



# Asymptotic temperature increase: Steady State Temperature

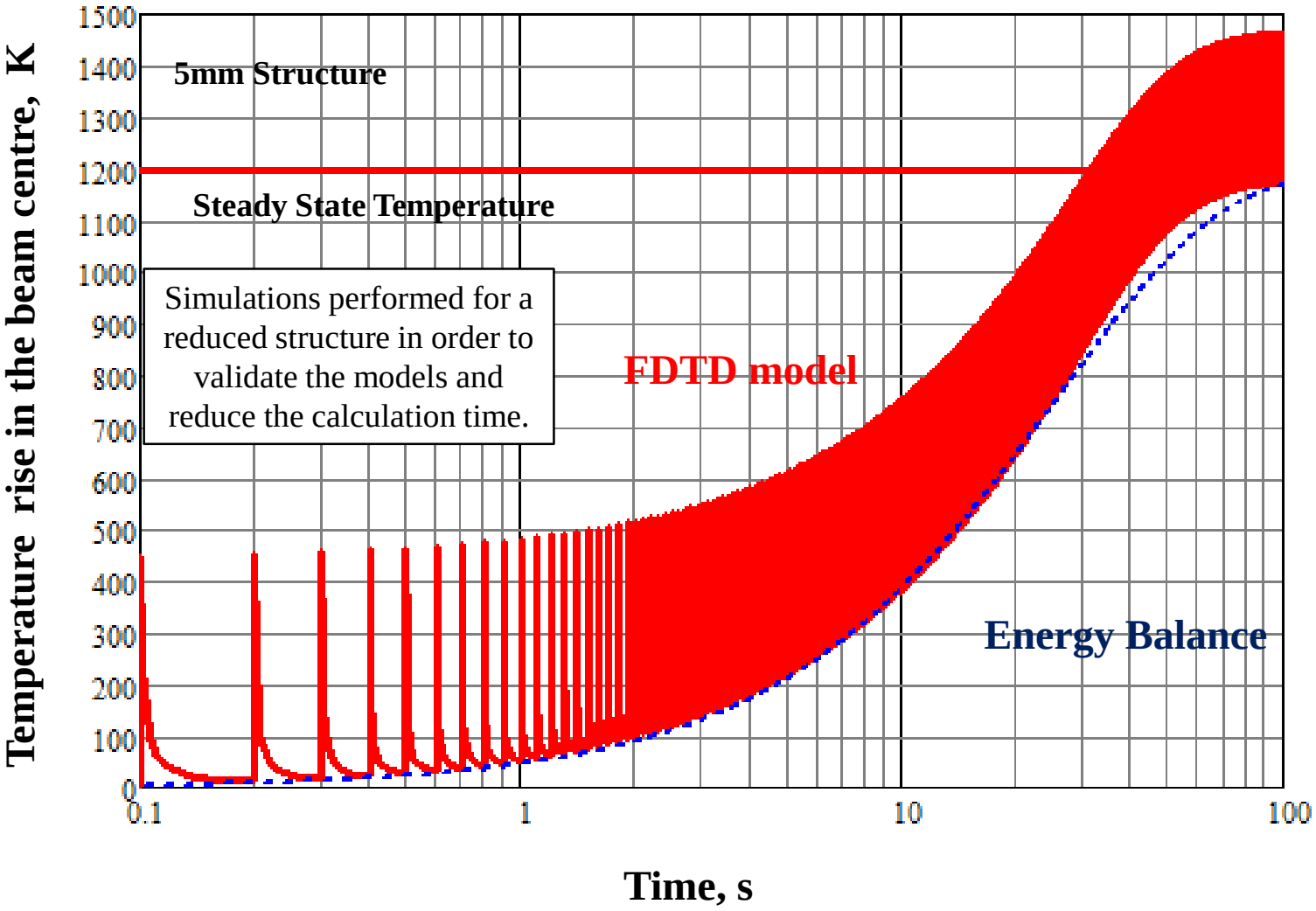
Obtained from the energy balance between the deposited energy and the dissipation by ***thermal radiation***

$$\Delta T = \sqrt[4]{T_{amb}^4 + \left(\frac{a^2 \cdot L}{r^2 + r \cdot L}\right)\frac{C_{\max,a} \cdot N_{part} \cdot N_{pulses}}{\varepsilon \cdot \sigma_B \cdot T_{rep}}} - T_{amb}$$

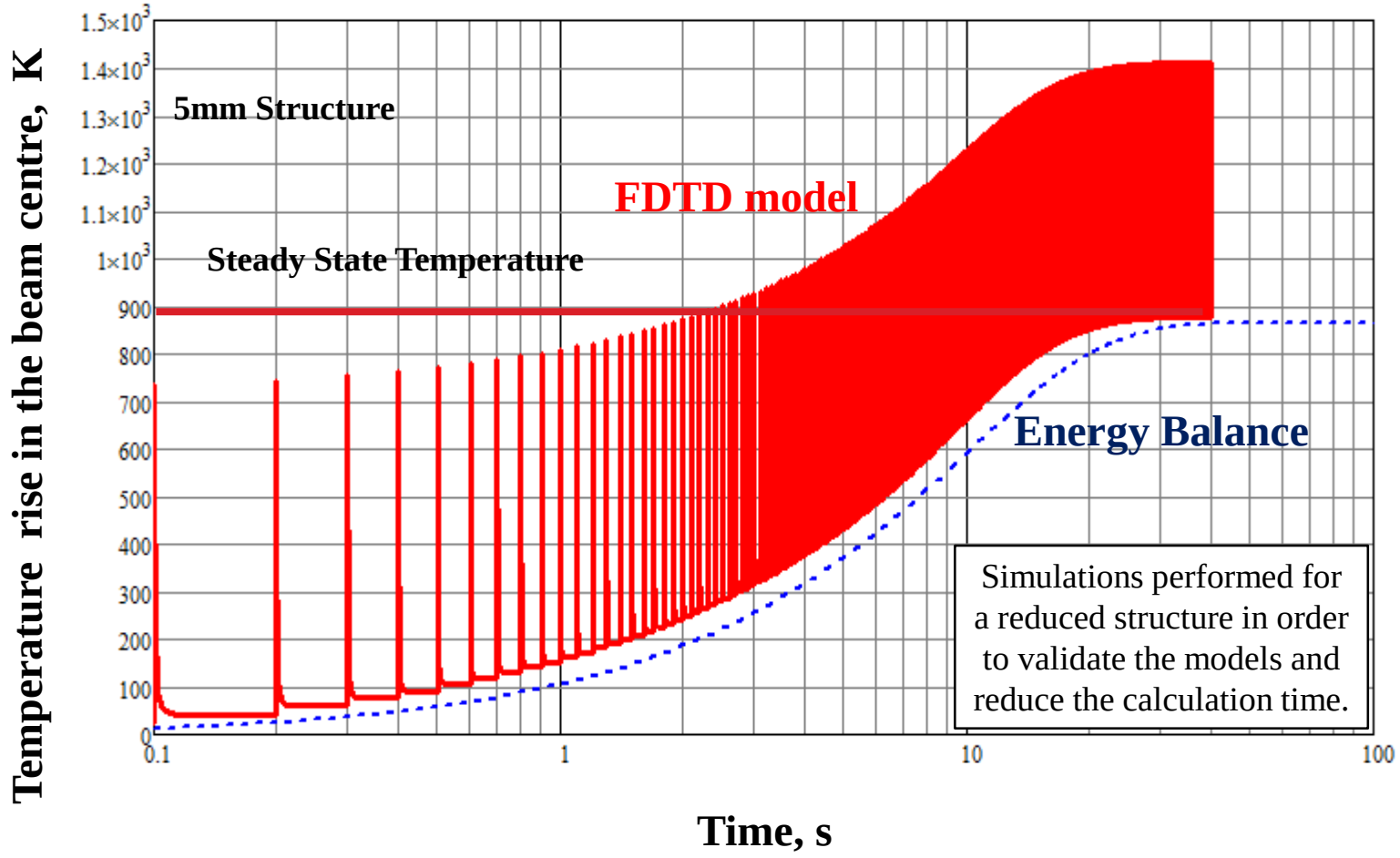



**ε** emissivity, $\boldsymbol{\sigma_B}$ Stefan-Boltzmann constant, $\mathbf{T_{rep}}$ pulse train repetition period, $\mathbf{C_{max,a}}$ deposited energy density peak by the Fluka data

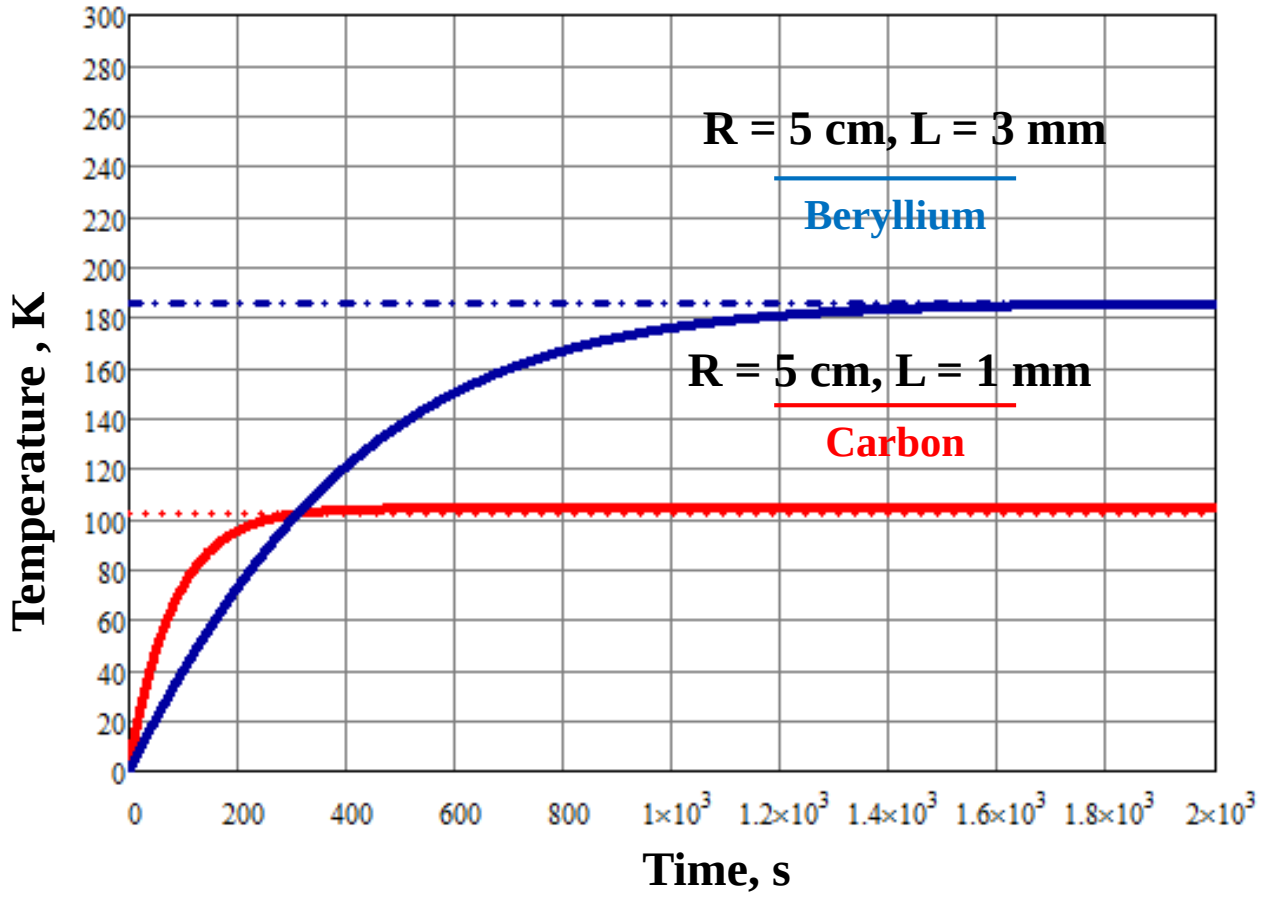
# Comparison between the numerical model and the model based on the energy balance for the Beryllium target



# Comparison between the numerical model and the model based on the energy balance for the Carbon target



# Energy Balance Model and Steady State Temperature



# Steady State Temperature

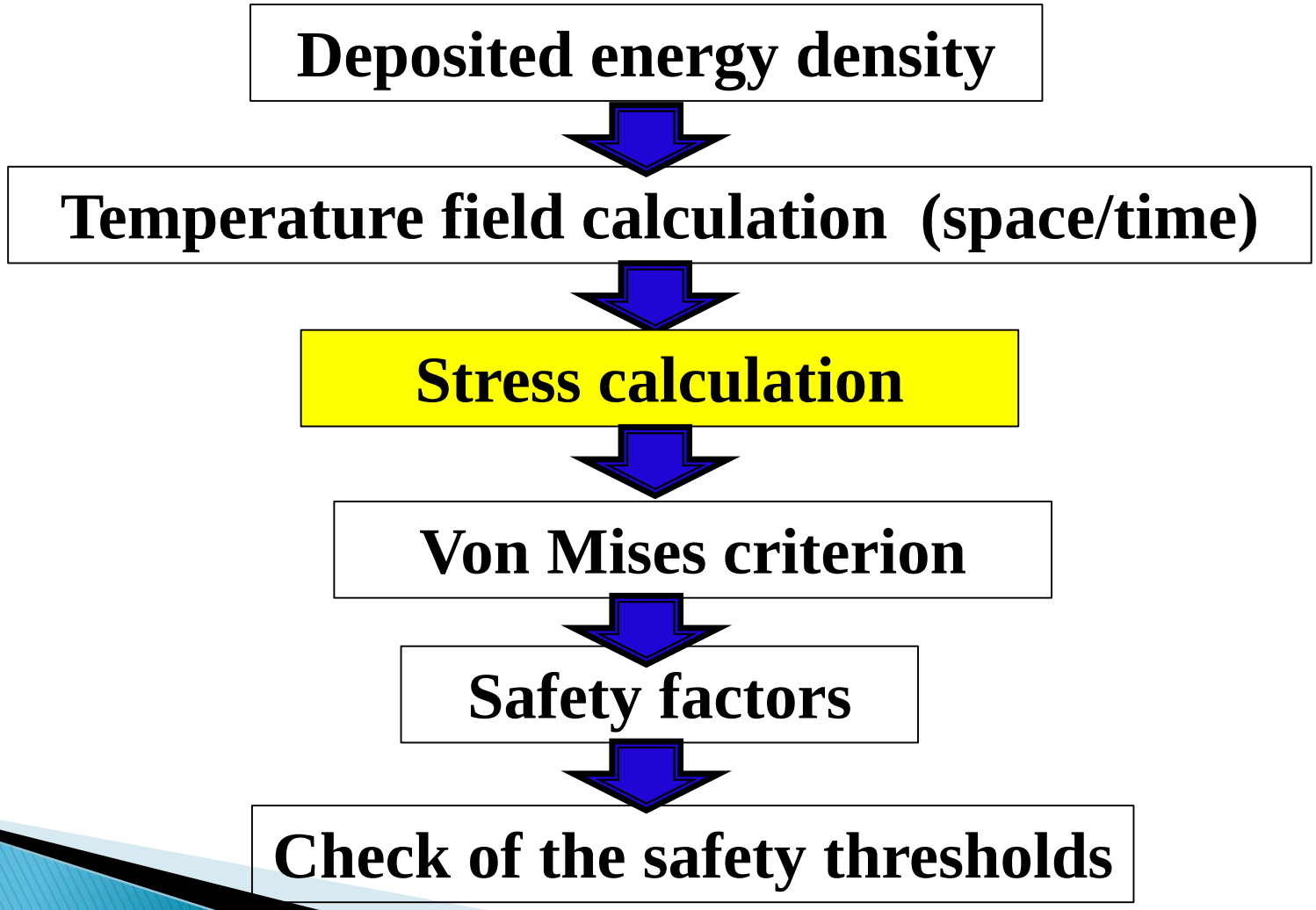
**Beryllium target**
radius r = 5 cm, thickness L = 3 mm;
Beam spot size: a = 300 μm;
Number of positrons: N = $3 \cdot 10^{11}$
Cooling time: $T_{Rep}$ = 0.1 s.

Steady state temperature increase: $\mathbf{\Delta T_{SS}}$ = **185.5 K**
Melting point: 1551 K

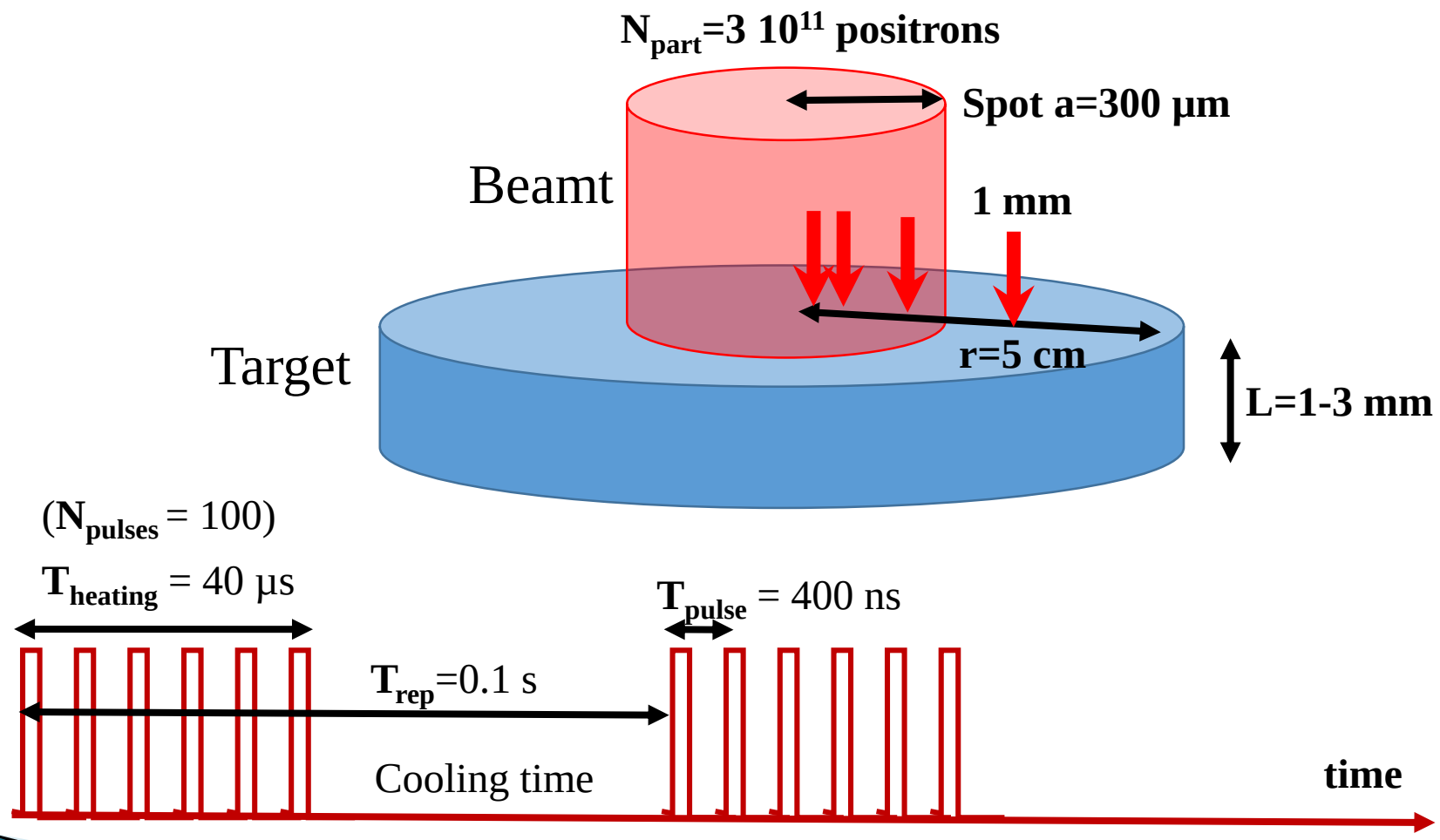
**Carbon target**
radius r = 5 cm, thickness L = 1 mm;
Beam spot size: a = 300 μm;
Number of positrons: N = $3 \cdot 10^{11}$
Cooling time: $T_{Rep}$ = 0.1 s.

Steady state temperature increase: $\mathbf{\Delta T_{SS}}$ = **102.5 K**
Melting point: 3923 K

# Methodological approach



# Stress and Safety factors



# Stress Relations

## Radial Stress

$$\sigma_{rr} = \frac{E(r)}{1-\nu}\left[\frac{1}{R^2}\int_0^R \alpha T(r,t) r dr - \frac{1}{r^2}\int_0^r \alpha T(r,t) r dr\right]$$

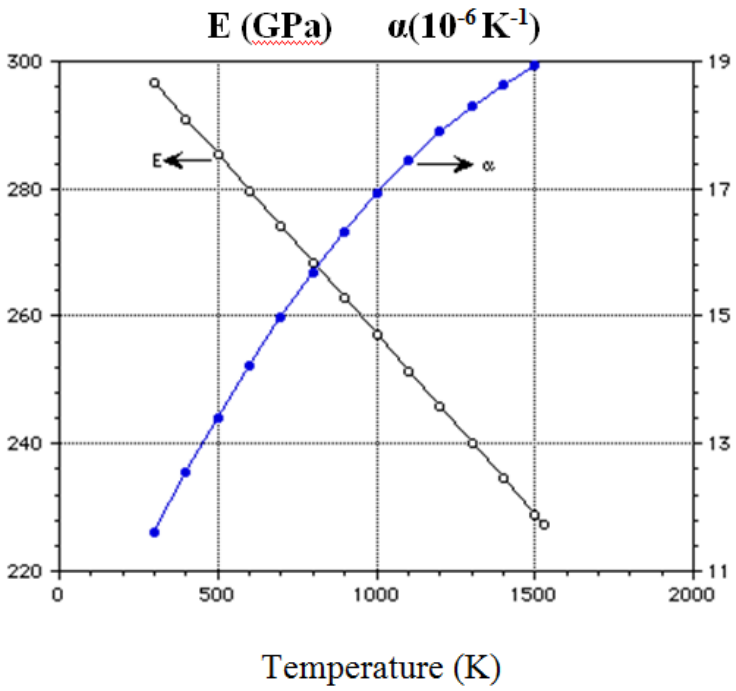
## Hoop Stress

$$\sigma_{\theta\theta} = \frac{E(r)}{1-\nu}\left[\frac{1}{R^2}\int_0^R \alpha T(r,t) r dr + \frac{1}{r^2}\int_0^r \alpha T(r,t) r dr - \alpha T(r,t)\right]$$

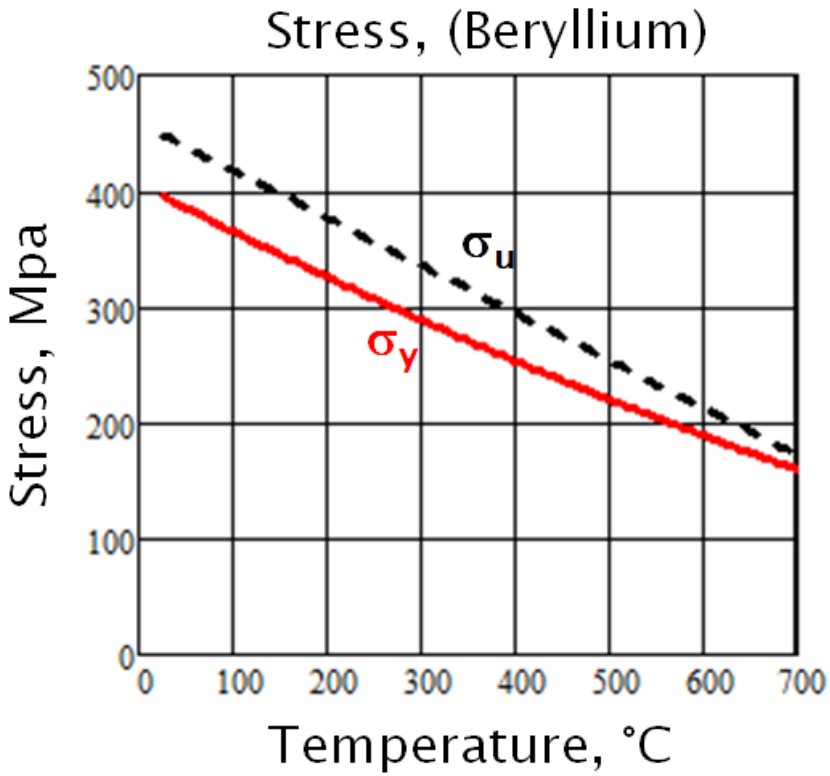
## Axial Stress

$$\sigma_{zz} = \frac{E(r)}{1-\nu}\left[\frac{2}{R^2}\int_0^R \alpha T(r,t) r dr - \alpha T(r,t)\right]$$

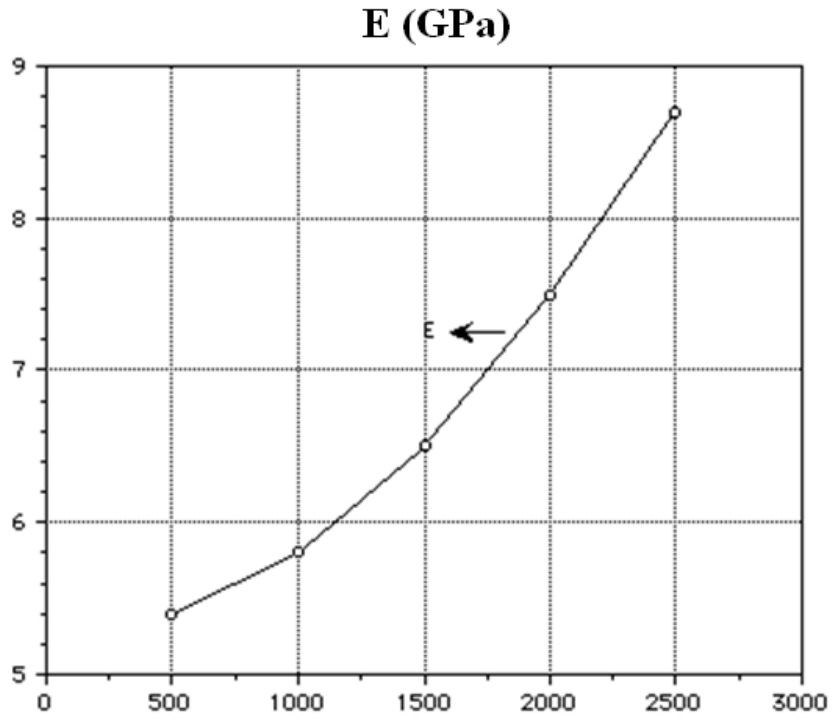
# Elastic properties: Beryllium




Elastic modulus and coefficient of thermal expansion for berylliu



# Elastic properties: Carbon




Elastic modulus of pyrolytic graphite

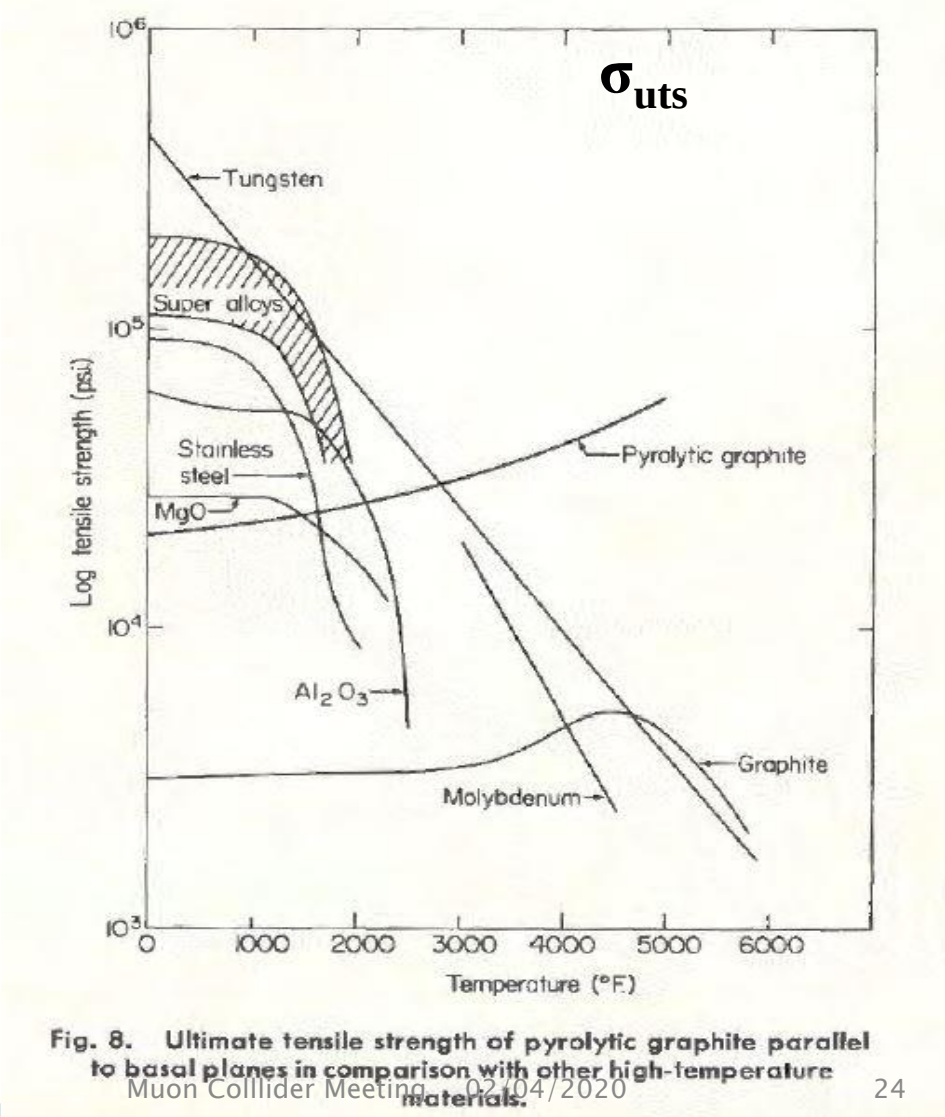
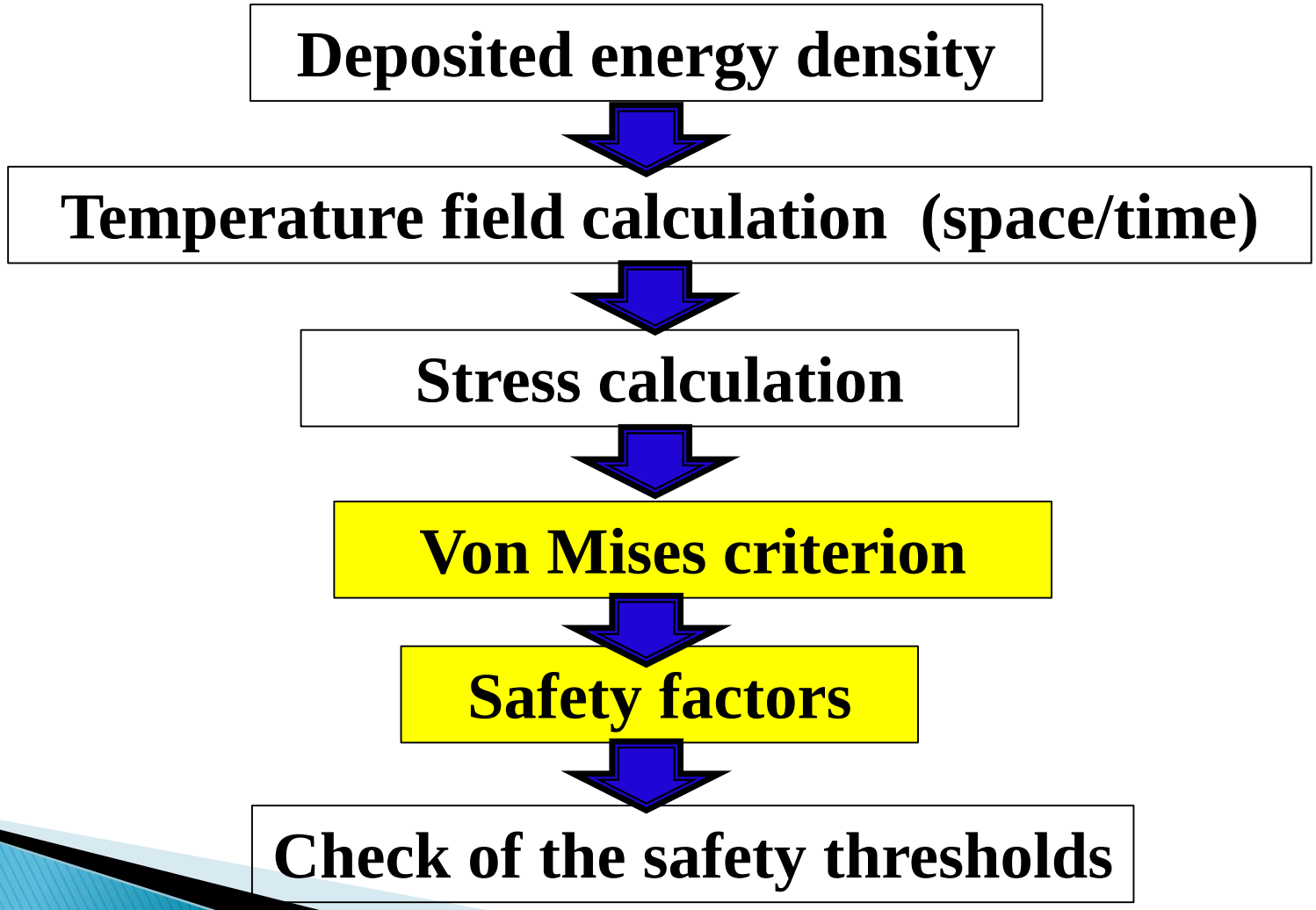
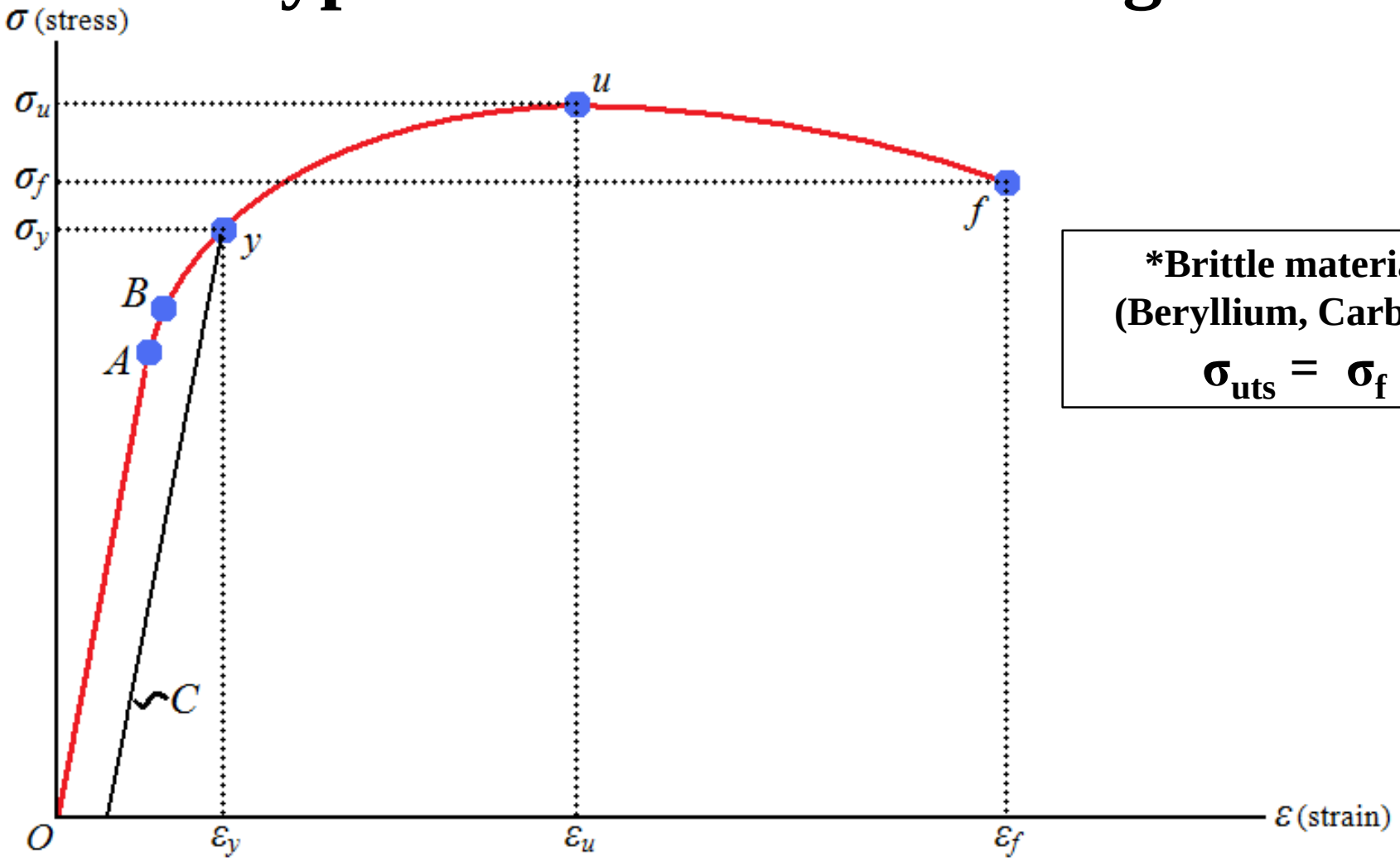



Fig. 8. Ultimate tensile strength of pyrolytic graphite parallel to basal planes in comparison with other high-temperature materials.

# Methodological approach



# Typical Stress-Strain Diagram




***Brittle materials (Beryllium, Carbon):**

$\sigma_{uts} = \sigma_f$

# Von Mises Equivalent Stress and Safety Factor

**Von Mises Equivalent Stress**

$$\sigma_e = \sqrt{\frac{(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{rr} - \sigma_{zz})^2 + (\sigma_{\theta\theta} - \sigma_{zz})^2}{2}}$$

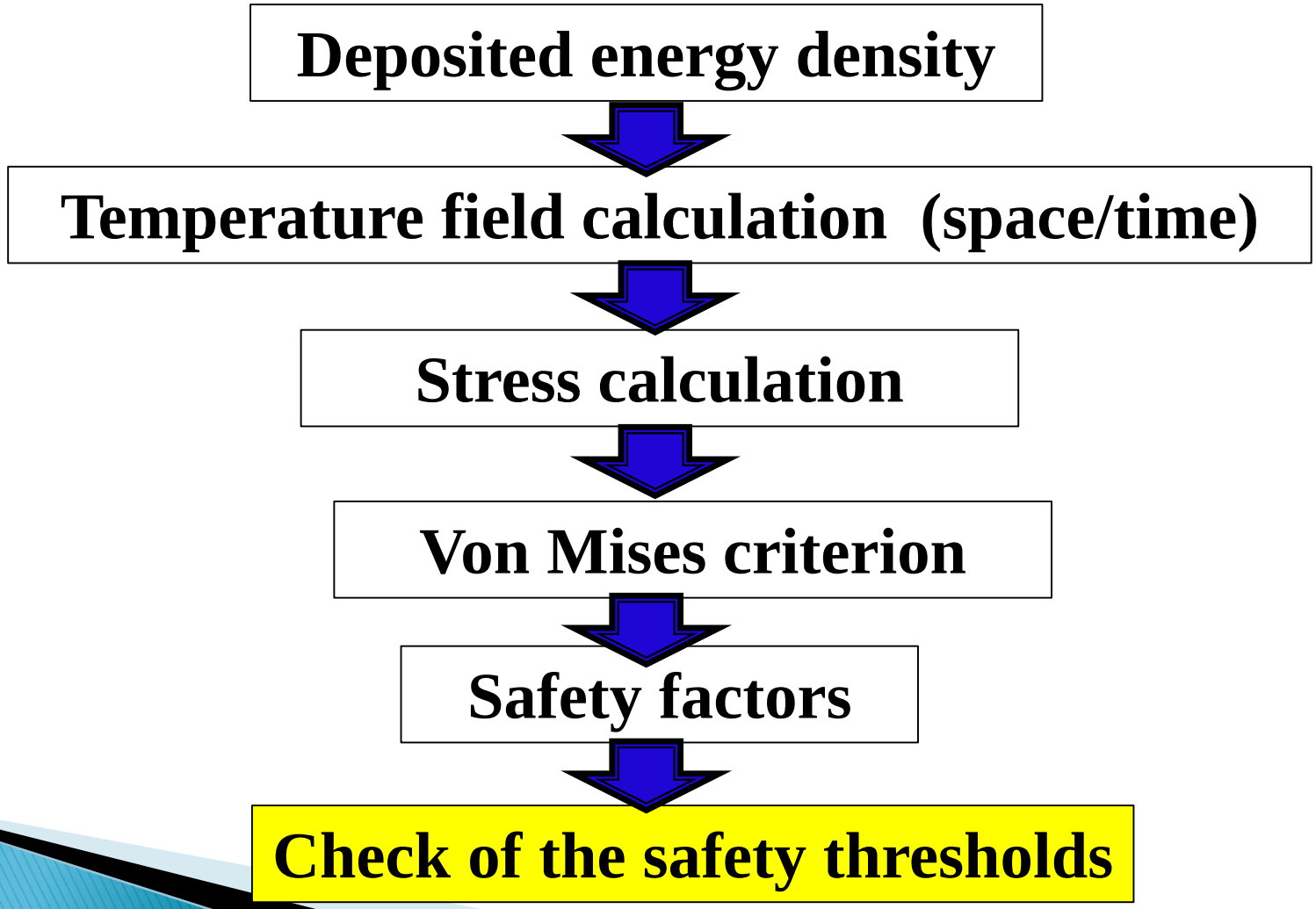
$$\frac{\sigma_y}{\sigma_e} > 1$$

**Safety factor and threshold for Beryllium target**

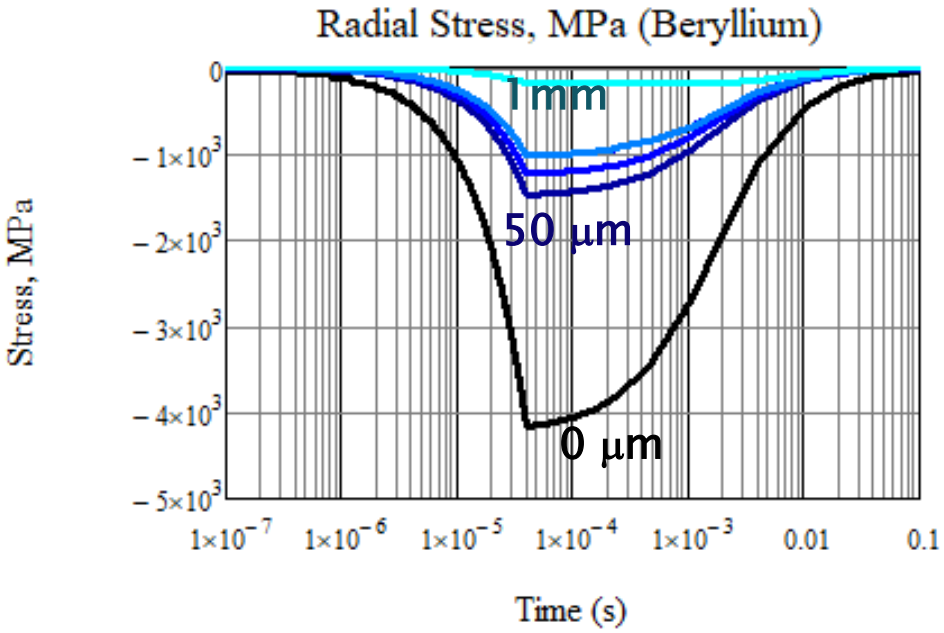
$$\frac{\sigma_{uts}}{\sigma_e} > 2$$

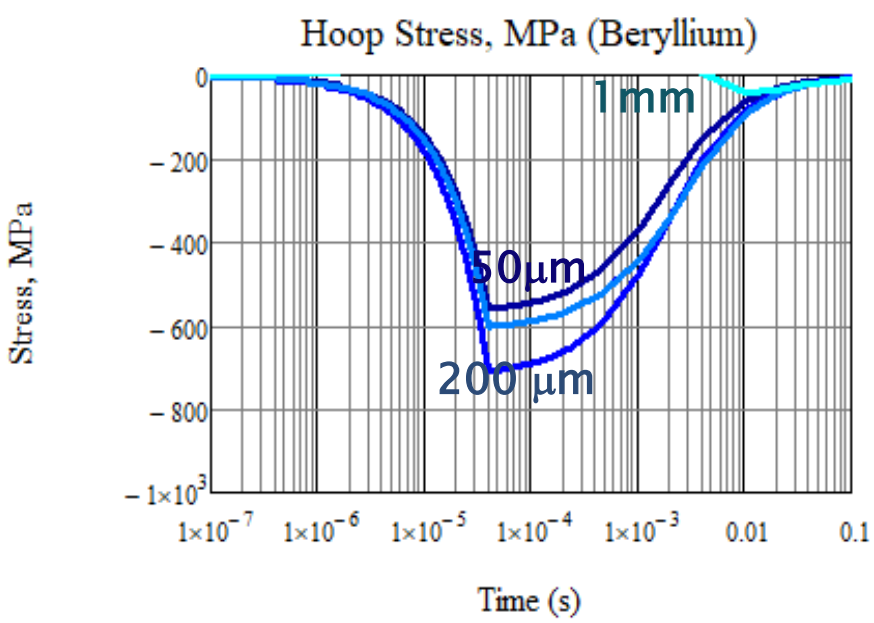
**Safety factor and threshold for Carbon target**

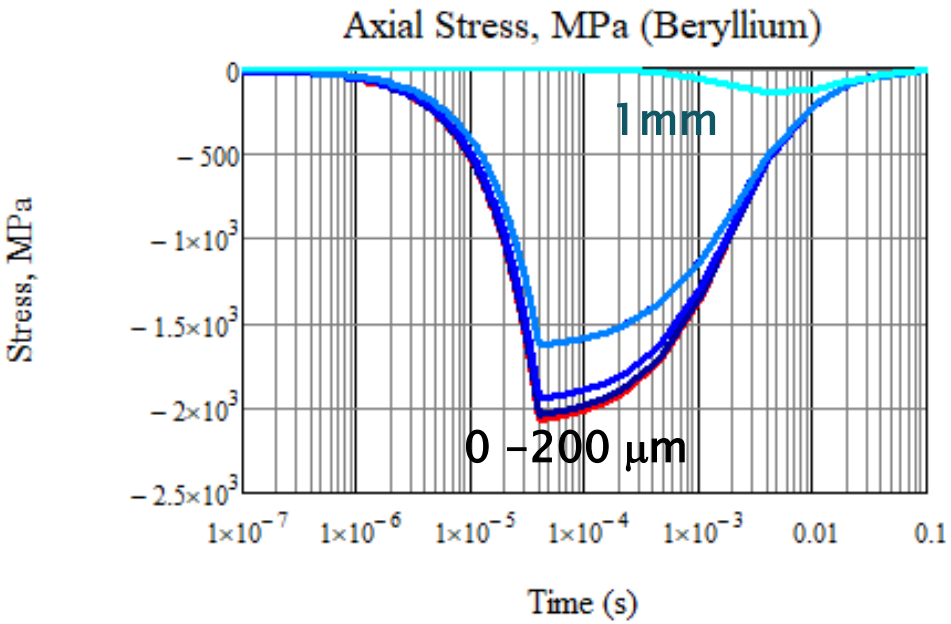
# Methodological approach

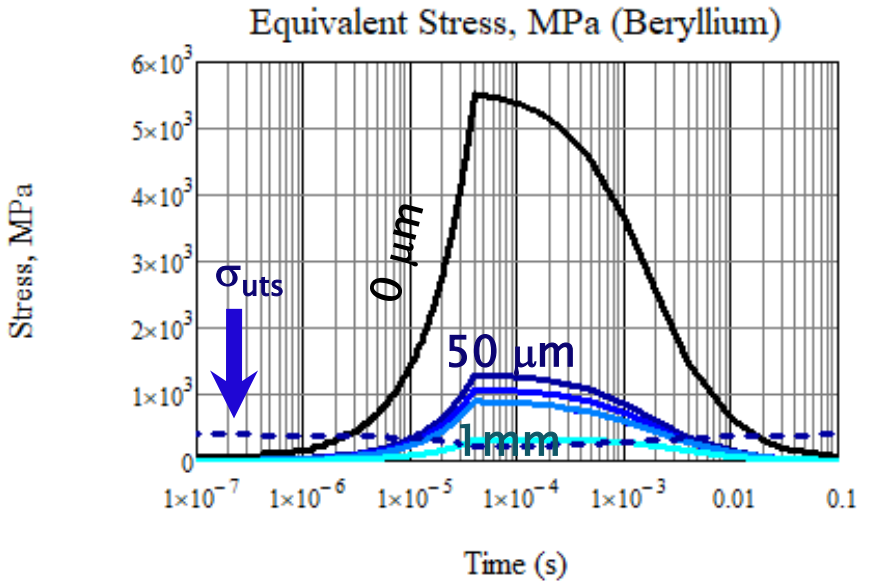


# Stress and Safety factors – Beryllium – Spot 300 um

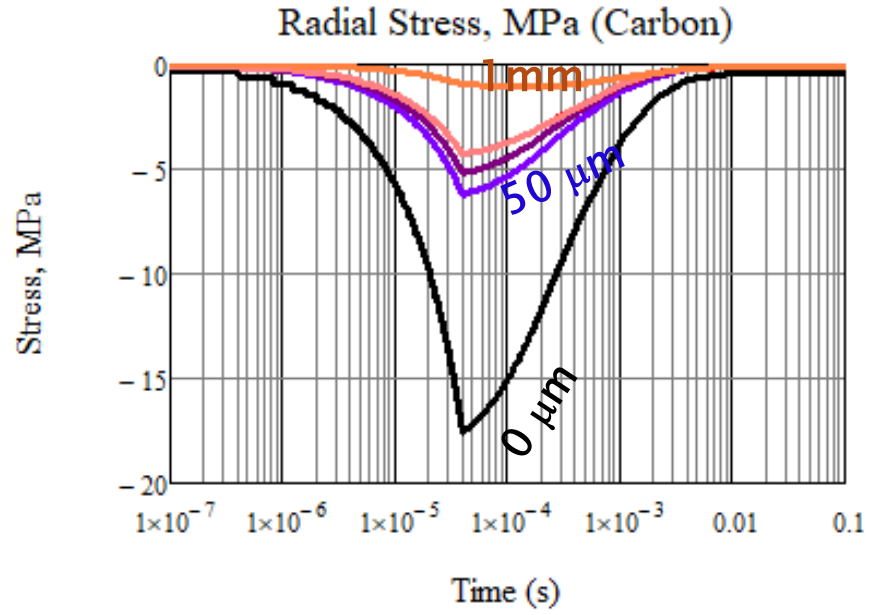


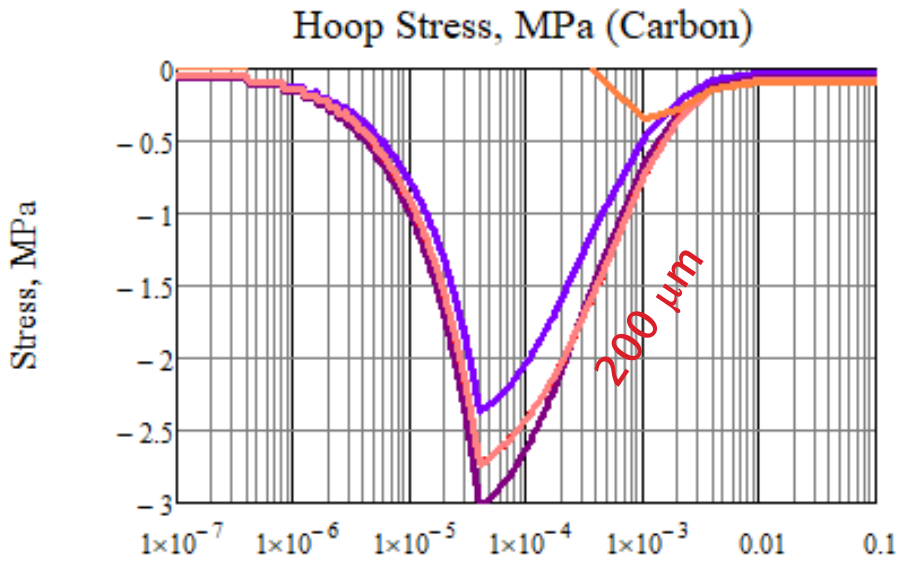



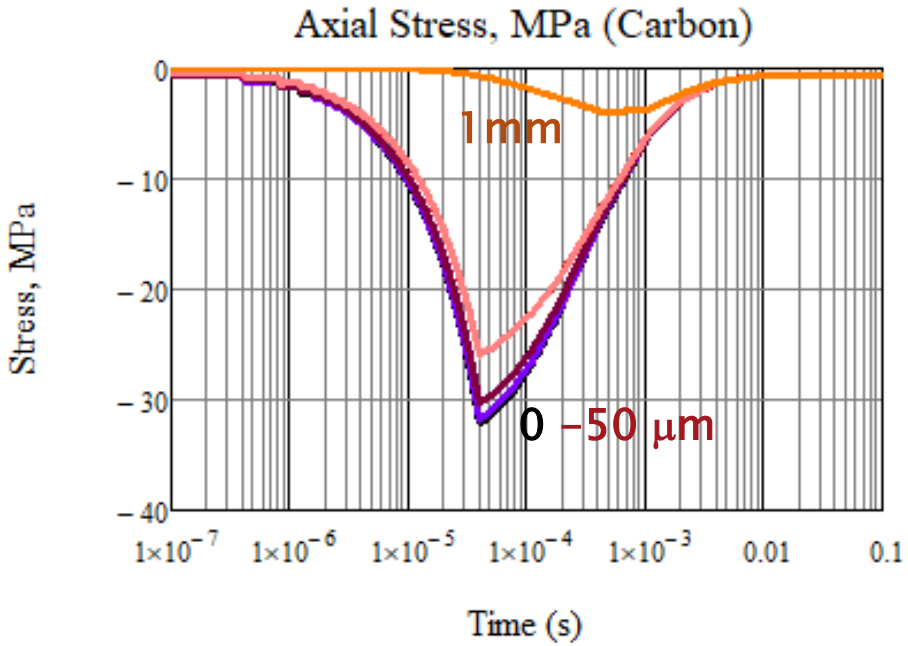



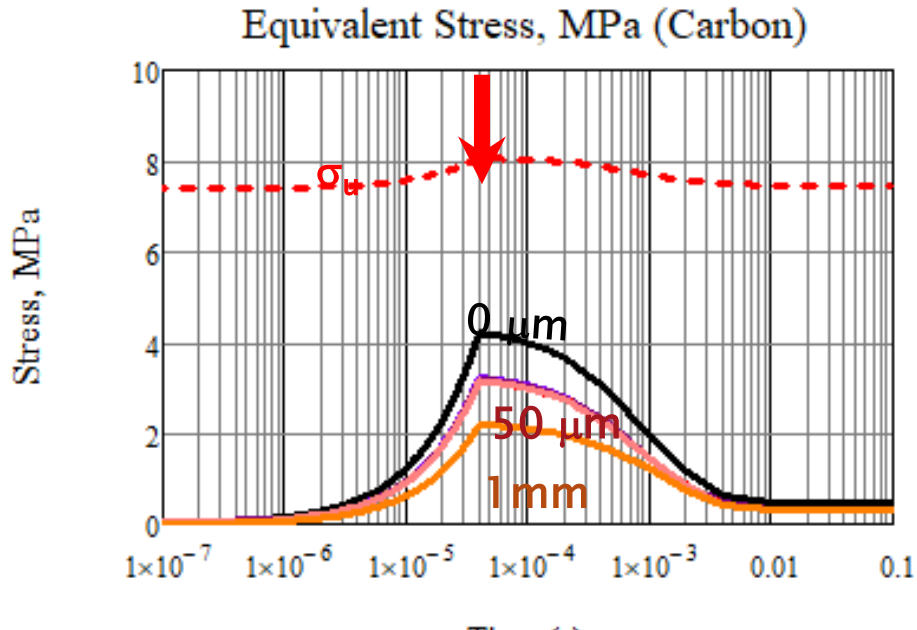



# Stress and Safety factors – Carbon – Spot 300 um



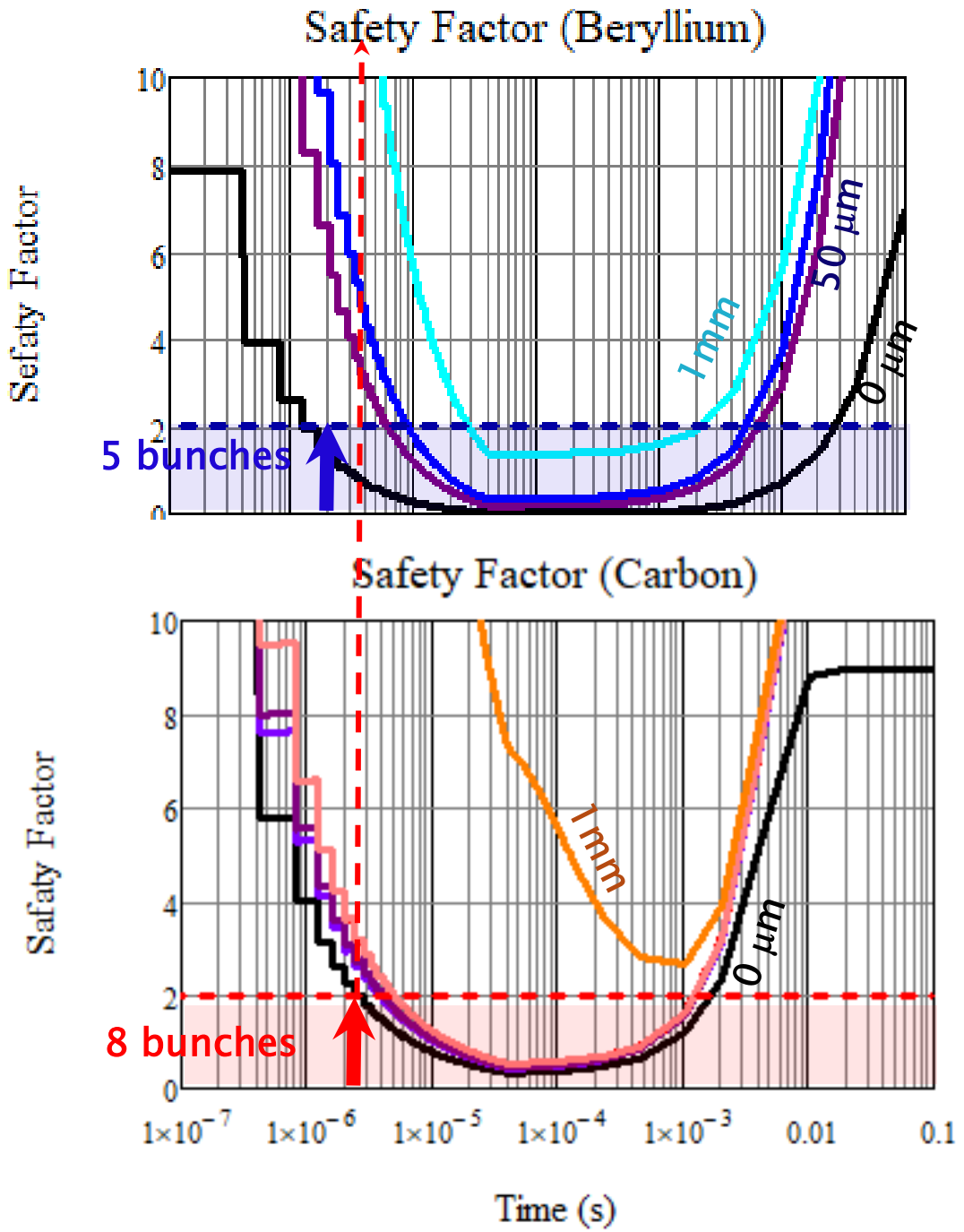



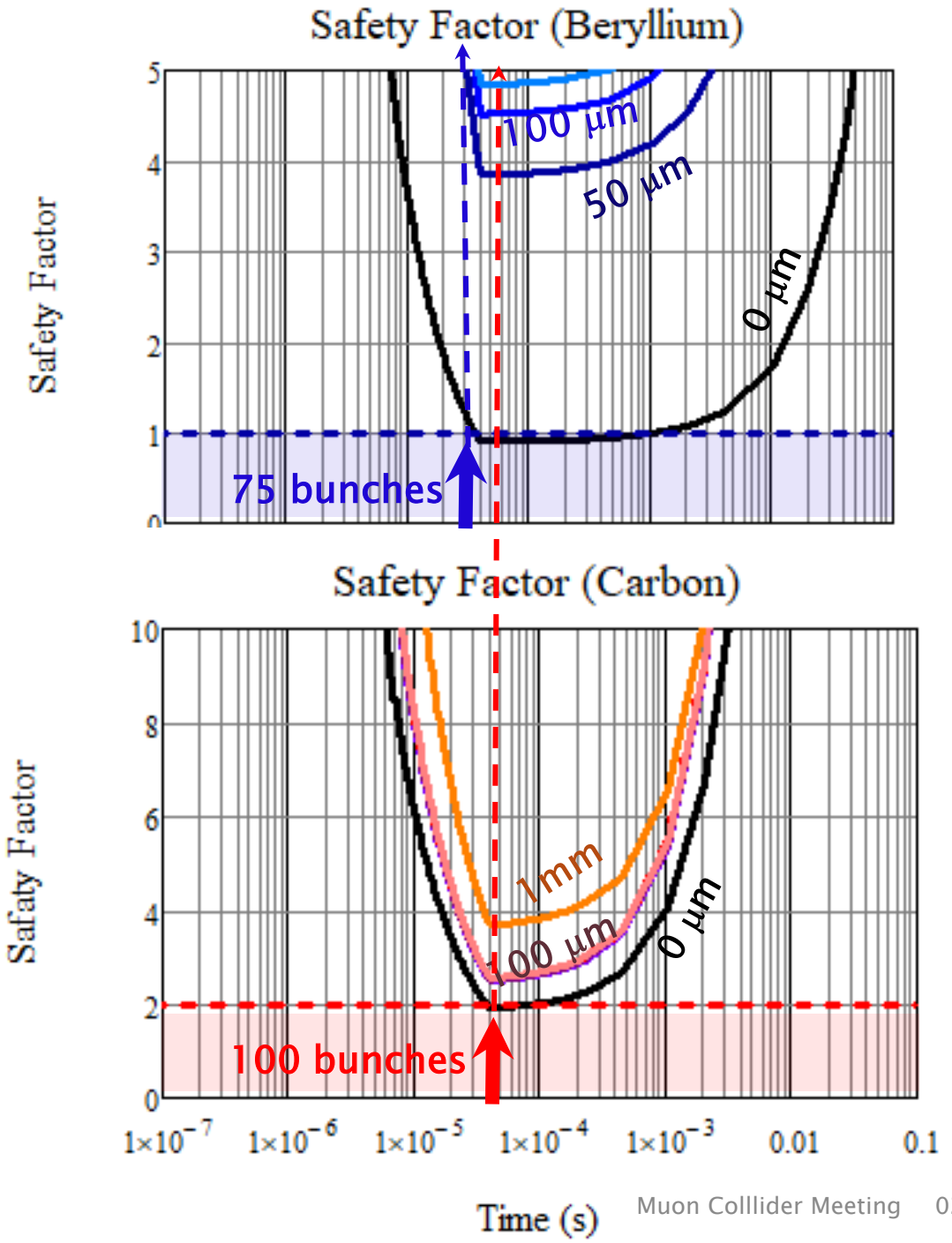
Spot 300 μm

$$\frac{\sigma_{uts}}{\sigma_e} > 2$$



# Spot 1000 μm

$$\frac{\sigma_{uts}}{\sigma_e} > 2$$



# Conclusions

- We used the numerical model (FDTD) in order to evaluate the spatial and temporal gradients of temperature due to a single bunch or to sequence of bunches that can cause thermomechanical stresses and therefore damage or fractures of the target;

- We used the model based on the energy balance to obtain the steady state temperature and evaluate its sustainability both for a static configuration of the target.

- Future perspectives: characterization of the thermo elastic properties of solid targets. Real experiments by monitoring the temperature with an infrared camera

# Thanks for your attention