# Phase space coalescence model for Quarkonium production in heavy ion collision
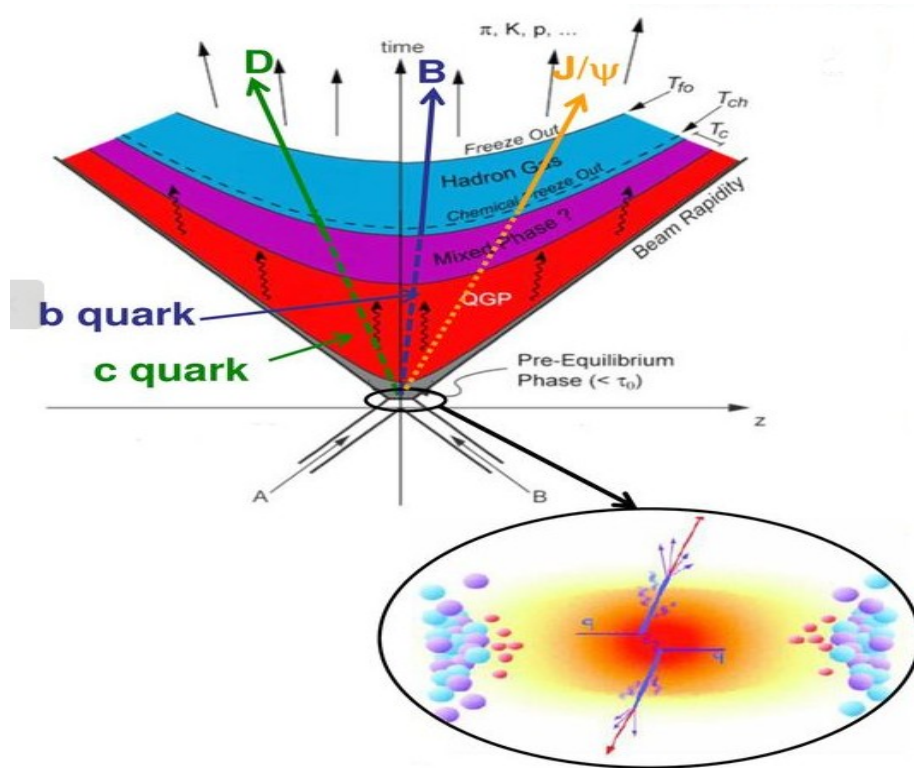
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in collaboration with
Pol Bernard Gossiaux and Joerg Aichelin

# Background



The QGP is studied through different probes. Among the most promising probes are the heavy Quarkonium (bound state of a quark and its respective antimatter quark). Due to their heavy mass ($m_c$ = 1.5 GeV and $m_b$=4.18 GeV) they are only created at the early stage of QGP in hard processes,they interact while traveling through the medium and leave without reaching thermal equilibrium.

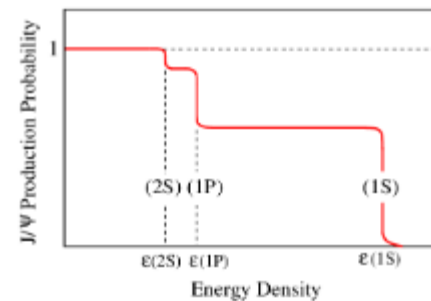Need for theoretical and phenomenological models to understand the experimental data
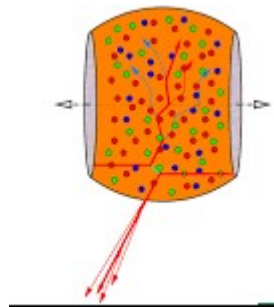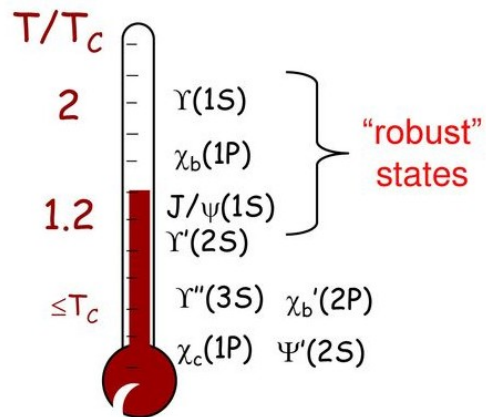
# Background

Quarkonium as Hard Probe

Thermometer

Transport properties

Medium density

# Model History

The idea of the formalism goes back to Remler's work   in which  a general formula connecting composite particle cross section with time-dependent density operators was presented. The formalism is able to deal with many particles (nucleons ⟶ deuterium )

*E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)*

*Taesoo .S, J.Aichelin and E.Bratkovskaya , Physical Review C 96. 014907 (2017)*

The model  was also applied to Quarkonium production in pp  and  heavy ion collisions(only primordial).  For the case of pp collisions the model was able to reproduce the experimental data.
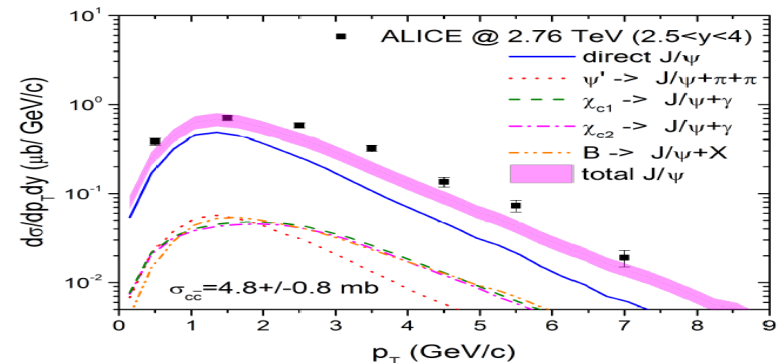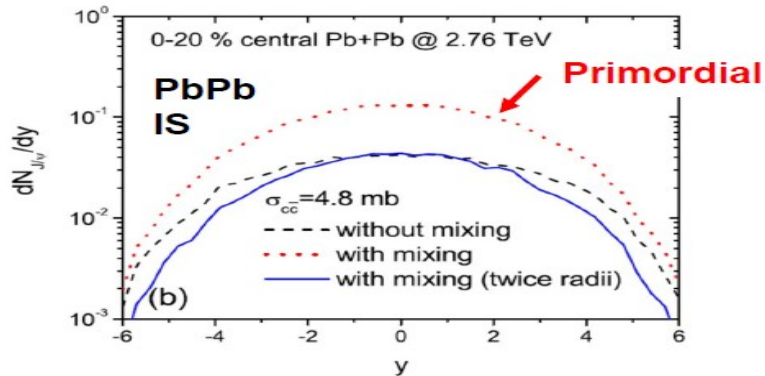
# Motivation

However in the same contribution was pointed out that for heavy ion collision a considerable enhancement of primordial (in the initial state) J/Psi was found when QGP effects are ignored.

*Taesoo .S, J.Aichelin and E.Bratkovskaya , Physical Review C 96. 014907 (2017)*

Apply the formalism to Quarkonium production in heavy ion collisions



- Interaction of heavy quarks with the bulk particles
- Expansion of the medium
- Formation of quarkonium states from HQ from difference vertex

# How does it work ?

Quarkonium interaction with QGP
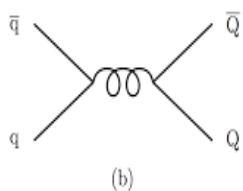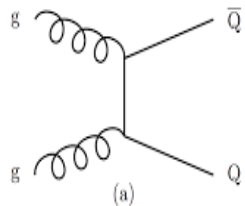


The probability of a Quarkonium state Ψ formation in the medium is given by

$$P^{\Psi}(t)=Tr[\rho_{QQ}^{\Psi}\rho_N(t)]$$
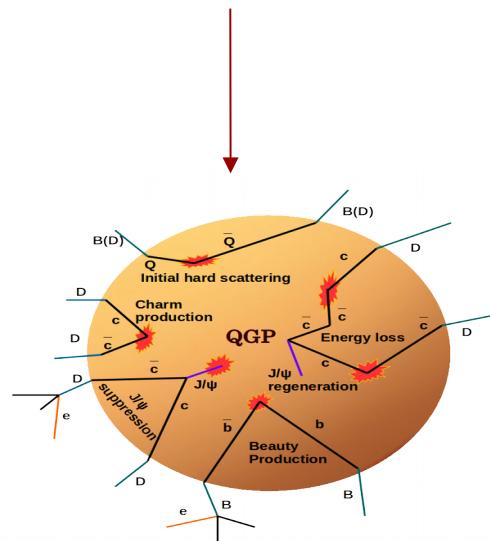
$$\rho_{QQ}^{\Psi}=\sum_i |\Psi_{Q\bar{Q}}^i\rangle\langle\Psi_{Q\bar{Q}}^i|$$

Two-body density matrix

$$\frac{\partial\rho_N(t)}{\partial t}=-i[H_N,\rho_N(t)]$$

Time evolution of the N-body system density matrix

# How does it work ?

The effective rate for Quarkonium state creation(dissociation) in the medium will be

$$\Gamma_{eff}^{\Psi}(t) = \frac{\partial P^{\Psi}(t)}{\partial t} = Tr\left[\frac{\partial(\rho_{QQ}^{\Psi}\rho_N(t))}{\partial t}\right]$$

in previous applications of Remler's model there was no influence of a medium (nuclear reactions and pp collision)

Since $\rho_{QQ}^{\Psi}$ is the density operator obtained from the ground state wave function, it is constant in time (vacuum basis).

If we also consider the influence of the medium, now the density operator of the ground state is a time dependent object (local basis)

*Original Remler's formalism(only one rate term)*

*"Updated" Remler's formalism (two rates contributions+QQbar SM interaction+relativistic Wigner)*

$$\Gamma_{eff}^{\Psi}(t) = \frac{\partial P^{\Psi}(t)}{\partial t} = Tr\left[\rho_{Q\bar{Q}}^{\Psi}\frac{\partial\rho_N(t)}{\partial t}\right]$$

$$\Gamma_{eff}^{\Psi}(t) = \frac{\partial P^{\Psi}(t)}{\partial t} = Tr\left[\rho_{Q\bar{Q}}^{\Psi}(t)\frac{\partial\rho_N(t)}{\partial t}\right] + Tr\left[\rho_N(t)\frac{\partial\rho_{QQ}^{\Psi}(t)}{\partial t}\right]$$

# How does it work ?

To work in the phase space we use the Wigner function

$$W^{\Psi^i} = \int d^3 y\, e^{ipy} \left\langle r - \frac{y}{2} \middle| \Psi^i \right\rangle \left\langle \Psi^i \middle| r + \frac{y}{2} \right\rangle$$

Double Gaussian approximation (Harmonic oscillator)

For the Wigner function of the full system   Semi-classical approach

$$W_N(t) = \prod_i \hbar^3 \delta(x_i - x_{i0}(t))\, \delta(p_i - p_{i0}(t))$$

$$W_{Q\bar{Q}}^{\Psi}(r_{rel}, p_{rel}) = C\, e^{\frac{-r_{rel}^2}{\sigma^2}}\, e^{\frac{-p_{rel}^2 \sigma^2}{\hbar^2}}$$

The Gaussian width $\sigma$

$$\frac{\int e^{-\frac{r^2}{\sigma^2}} r^2 d^3 r}{\int e^{-\frac{r^2}{\sigma^2}} d^3 r} \quad \longleftarrow \quad \langle r^2 \rangle = \frac{\langle \Psi_{Q\bar{Q}} | r^2 | \Psi_{Q\bar{Q}} \rangle}{\langle \Psi_{Q\bar{Q}} | \Psi_{Q\bar{Q}} \rangle} \quad \longleftarrow \quad [\frac{\hbar^2}{2\mu}\nabla^2 + V(r)]\Psi_{Q\bar{Q}}(r) = E_{Q\bar{Q}}\Psi_{Q\bar{Q}}$$

- The double Gaussian expression is a non relativistic Wigner function that can be applied in the center of mass, where the HQ have a low relative velocity(due to the large mass).

$$W_{y_\Psi, u_T}^{\Psi}(Y, u_T, r_{rel}, p_{rel}) = \delta(Y - y_\Psi)\,\delta(u_T - u_{T,\Psi})\, W_{Q\bar{Q},NR}^{\Psi}(r_{rel}^{cm}, p_{rel}^{cm}) \quad \longleftarrow$$

$$[r_{rel}^{cm}, p_{rel}^{cm}] \rightarrow f(r_{rel}^{lab}, p_{rel}^{lab})$$

- A fully relativistic version can be derive for a finite and well-defined center of mass velocity

# How does it work ?
# Collision rate

Substituting the expression for the Wigner functions of the density operator of Quarkonium state Ψ and the N-body system

$$\Gamma_{eff}^{\Psi}(t)=\frac{\partial P^{\Psi}(t)}{\partial t}=Tr[\rho_{Q\bar{Q}}^{\Psi}\frac{\partial \rho_N(t)}{\partial t}]$$

$$\Gamma_{coll,Q\bar{Q}}(t)=\sum_{i=1,2}\sum_{j\geq3}\delta(t-t_{ij}(\epsilon))\int\frac{d^3p_id^3x_i}{h^3}[W_{Q\bar{Q}}^{\Psi}(p_1,x_1;p_2,x_2)W_N(t+\epsilon)-W_{Q\bar{Q}}^{\Psi}(p_1,x_1;p_2,x_2)W_N(t-\epsilon)]$$

- The Quarkonium production in our model has a contribution from three process (see image)

- Collision rate do not discriminate between dissociation (destroys probability) like processes and recombination (creates) like processes
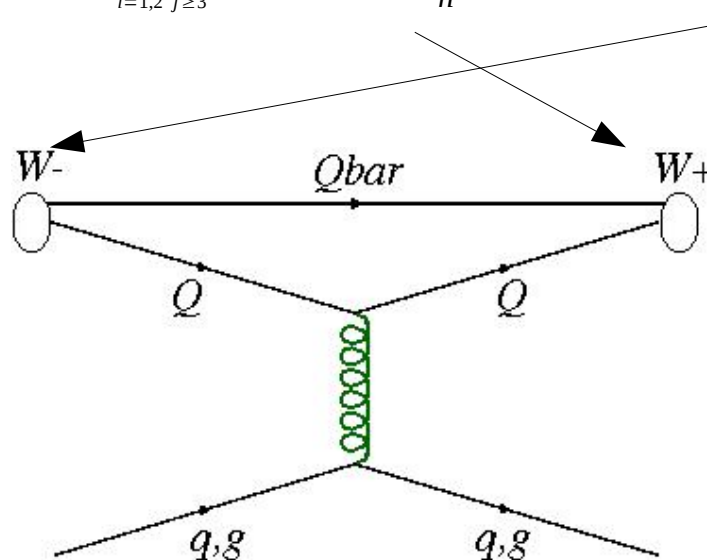


Interaction of HQ with the QGP were carried out by EPOSHQ

By integrating the equation of the effective rate we obtain a time dependent expression for the probability

$$P_{Q\bar{Q}}^{\Psi}(t) = P_{Q\bar{Q}}^{\Psi}(t_{init}^{Q}) + \int_{t_{init}^{\Psi}}^{t} \Gamma_{eff}(t') dt'$$

$$P_{QQ}^{\Psi}(t_{init}^{Q}) = \sum_{i=1}^{N_{init}^{Q}} W_{QQ}^{\Psi}(r_{rel}, p_{rel}, t_{init}^{Q})$$

Primordial(initial) probability

| state | $J/\psi(1S)$ | $\chi_c(1P)$ | $\psi'(2S)$ | $\Upsilon(1S)$ | $\chi_b(1P)$ | $\Upsilon(2S)$ | $\chi_b(2P)$ | $\Upsilon(3S)$ |
|-------|------|------|------|------|------|------|------|------|
| $T_d/T_c$ | 1.2 | 1.0 | 1.0 | > 3.0 | 1.2 | 1.2 | 1.0 | 1.0 |

- Due to the dissociation temperature (melt down any bound state) and the anisotropy of the temperature in QGP, the initial probability dependent on time.
- Each time that our code find a HQ quark below the its dissociation temperature (active quark) for the first time, its contributions will be accumulated into the initial probability.
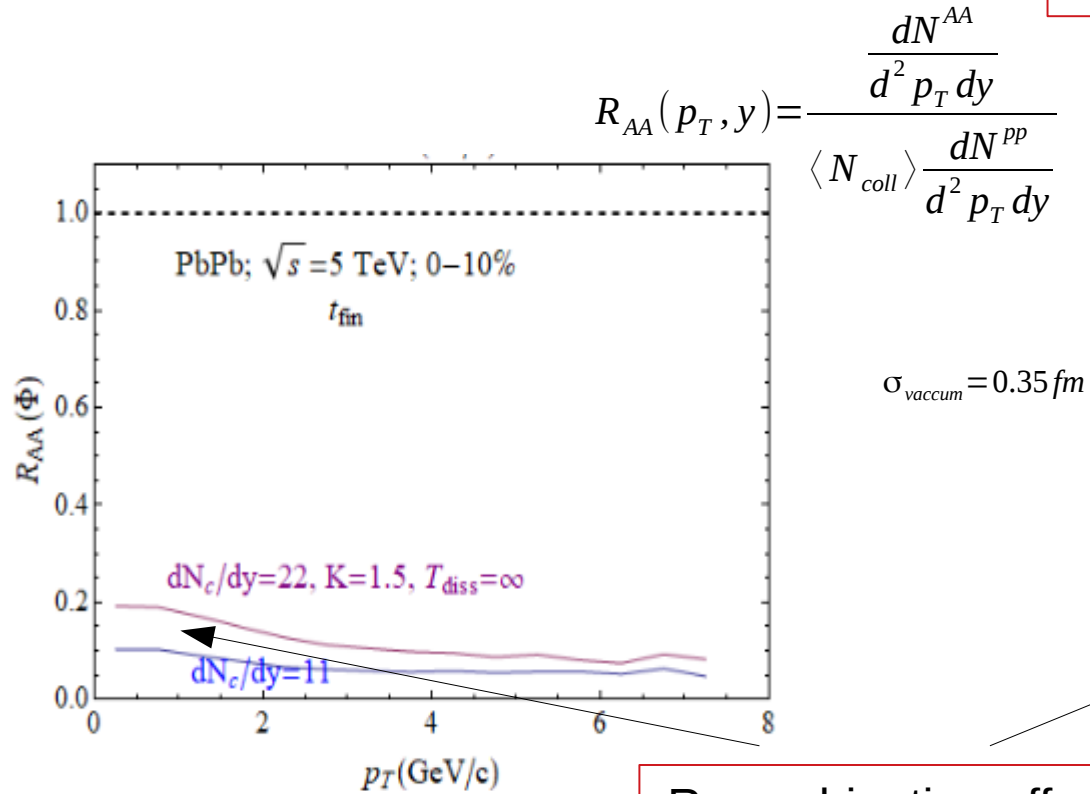- After that, further contributions will be set in the rates terms.

The probability equation only refers to a single HQ pair , but this analysis can be extended to all possible combinations (pairs) active at a given time.

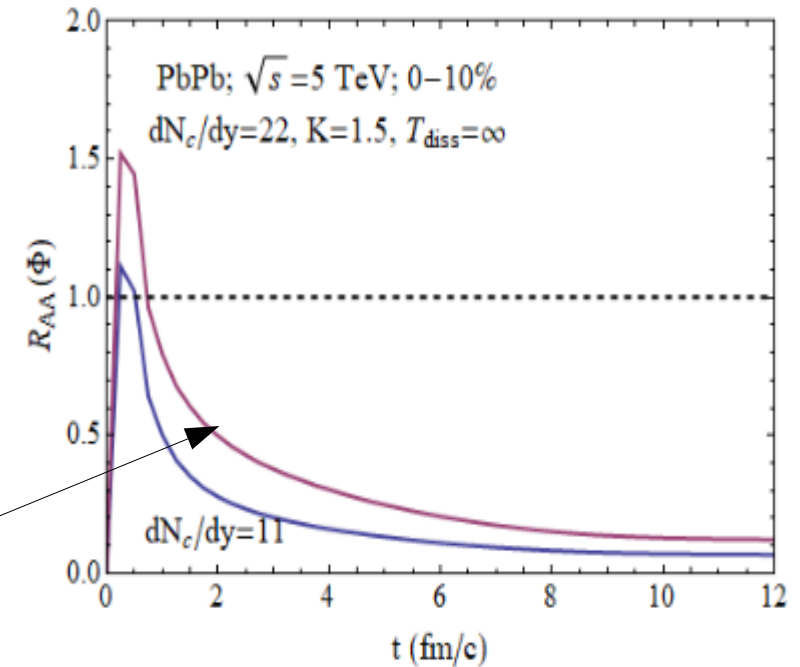$$P^{\Psi, tot}(t) = \sum_{j=1}^{N_Q^{active} \times N_Q^{active}} P_{Q\bar{Q}}^{\Psi}(t)$$

# Vacuum basis results (Original Remler's Formalism)

Effect of primordial production (uncorrelate ccbar pairs )

$$R_{AA}(p_T, y) = \frac{\dfrac{dN^{AA}}{d^2 p_T \, dy}}{\langle N_{coll} \rangle \dfrac{dN^{pp}}{d^2 p_T \, dy}}$$

$$\sigma_{vaccum} = 0.35 \, fm$$



PbPb; $\sqrt{s}$ =5 TeV; 0−10%

$t_{fin}$

$dN_c/dy$=22, K=1.5, $T_{diss}$=∞

$dN_c/dy$=11



PbPb; $\sqrt{s}$ =5 TeV; 0−10%

$dN_c/dy$=22, K=1.5, $T_{diss}$=∞

$dN_c/dy$=11

Recombination effects

# Preliminary Results J/Psi

*PhD Thesis : Raúl Tonaituh Jiménez*
*ALICE Collaboration*

Mid-rapidity and $\sqrt{s} = 2.76\,TeV$



PbPb; $\sqrt{s}$ =5 TeV; 0−10%    $t_{fin}$

$dN_c/dy$=22, K=1.5, $T_{diss}$=0.3

$\sigma$=0.5 fin

$\sigma$=0.35 fin



This work, Pb–Pb $\sqrt{s_{NN}}$ = 5.02 TeV

Inclusive J/ψ → e⁺e⁻, |y| < 0.9

Centrality 0-20%
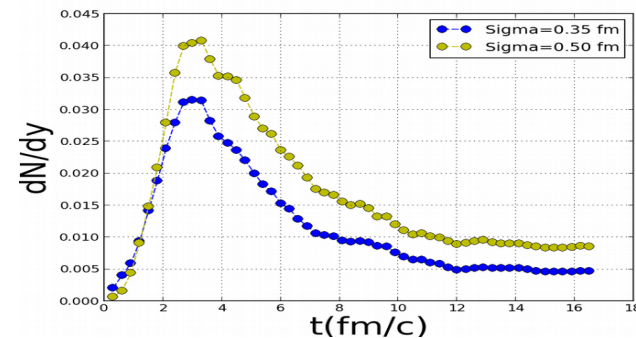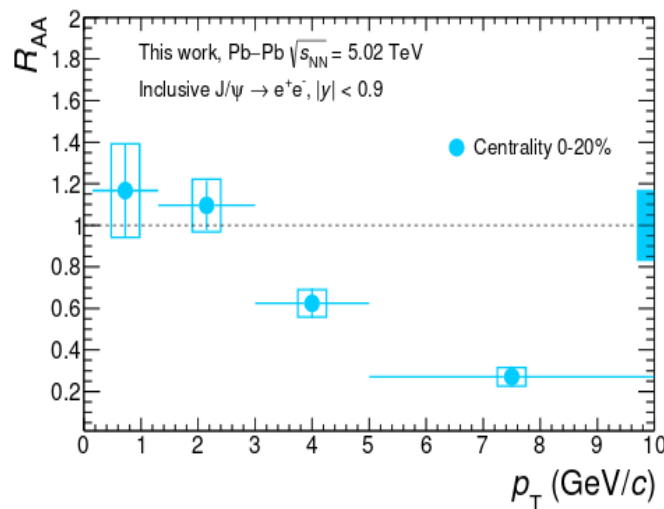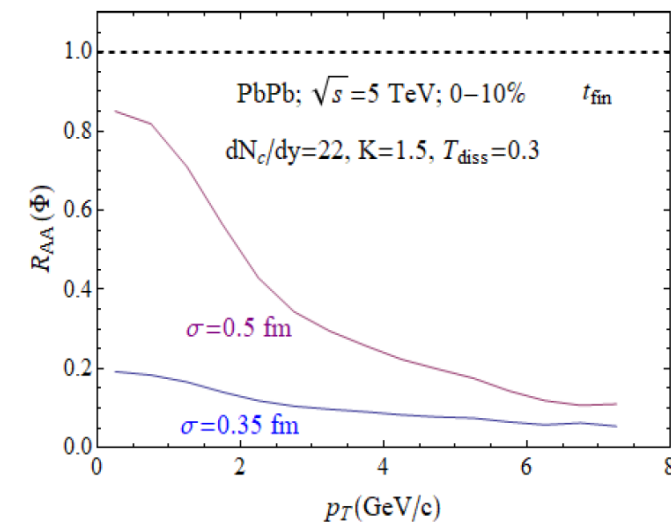


*PhD Thesis : Nirupam Dutta*
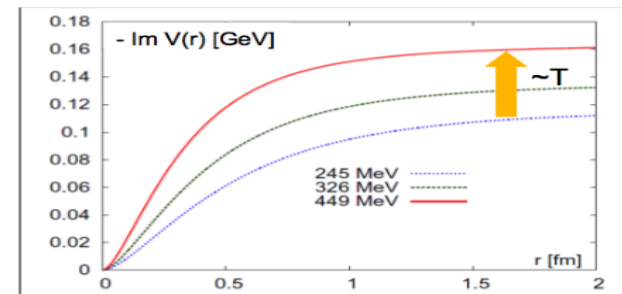
Need for temperature dependent Wigner function



- Im V(r) [GeV]

~T

245 MeV
326 MeV
449 MeV

# Preliminary Results J/Psi

Elliptic flow (uncorrelated ccbar pairs)
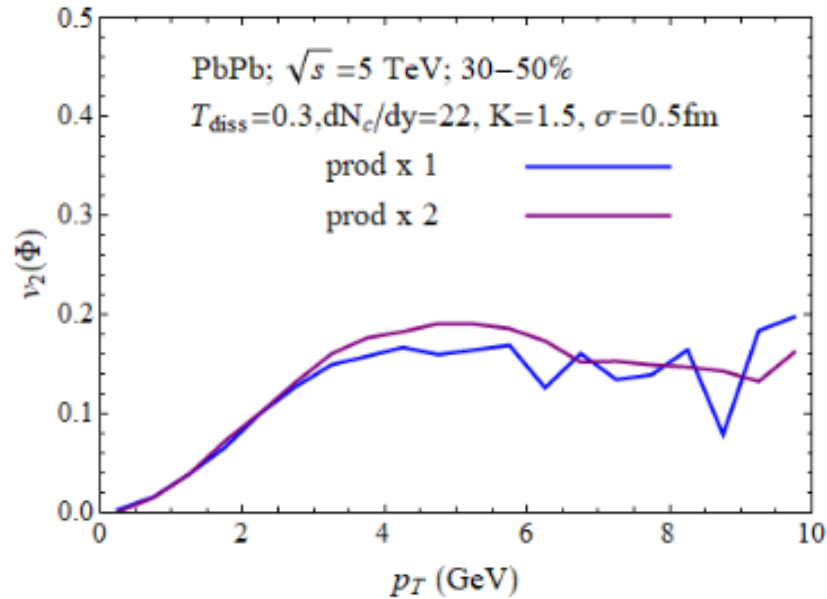


v$_2$ that extend at rather large p$_T$
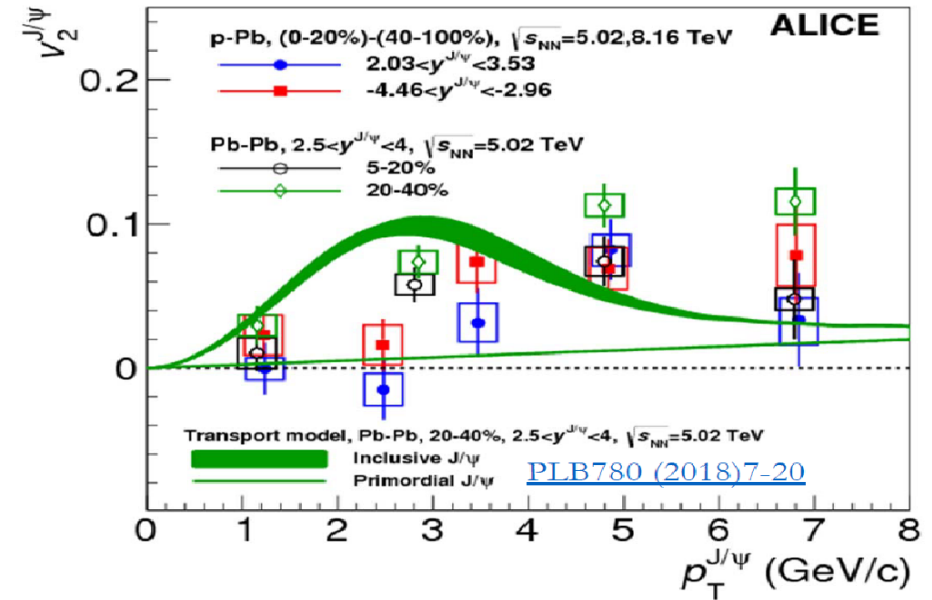
# "Updating" the Remler's formalism: local basis and QQbar semi-classical interaction

# Local rate

Considering now that $\rho_{QQ}^{\Psi}(t)$ depend on time through the temperature. Assuming that the temperature dependence of our two body Wigner function is located in the Gaussian width $\sigma(T)$ through its relation with the mean square radius

$$\Gamma_{loc}(t) = Tr\left[\rho_N(t)\frac{\partial \rho_{QQ}^{\Psi}(t)}{\partial t}\right]$$

$$\sigma^2(T) = \frac{2}{3}\langle r^2(T)\rangle$$

*Roland Katz Ph.D thesis*

Applying the trace operator

$$\Gamma_{loc}(t) = \int\int d^3r d^3r' \frac{\partial \rho_{QQ}^{\Psi}(t)}{\partial t}(r,r',T(t))\rho_{N=2}^{Q\bar{Q}}(r,r',t)$$
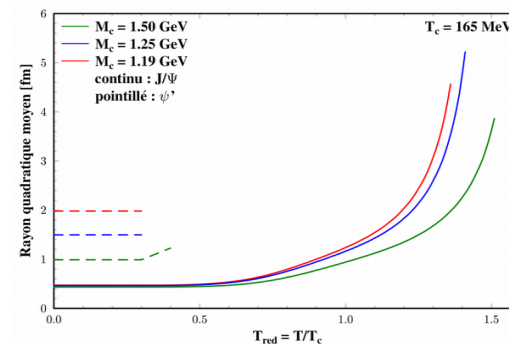
- The local rate comes from the time variation of the ground state density

- We assume that the temperature modification affect the mean square radius but not the form of the two body Wigner function

Moving to the phase space by substituting the Wigner function

$$\Gamma_{loc}(t) = 16\frac{\partial \sigma(T(t))}{\partial t}\left(\frac{r_{rel}^2}{\sigma^3} - \frac{\sigma p_{rel}^2}{\hbar^2}\right)e^{\frac{-r_{rel}^2}{\sigma^2}}e^{\frac{-p_{rel}^2\sigma^2}{\hbar^2}}$$

# Semi-classical QQbar relativistic interaction
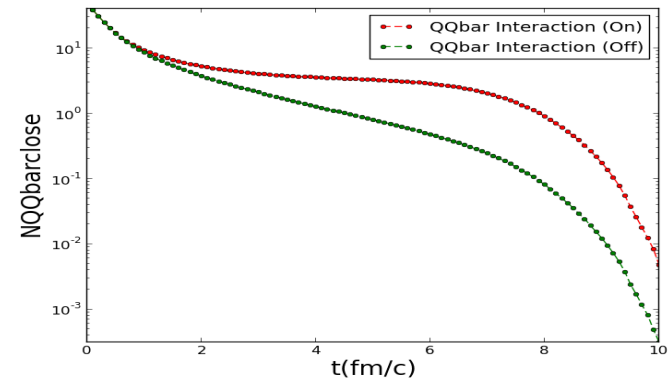
- In the original Remler's model, the interaction between the pair members was not take it into account.
- But due to the influence of the medium expansion an interaction potential is need to keep HQ pairs close to each other.
- This relativistic interaction is based on an instantaneous potential, meaning that the relative momentum of the HQ pair should be small.

The simplest relativistic semi classical modification to movement of a QQbar pairs is to define a Lagrangian which contains the relativistic energy and the gluon fields in a Coulomb approximation

$$L = -\gamma mc^2 + \frac{C}{r} \longrightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$
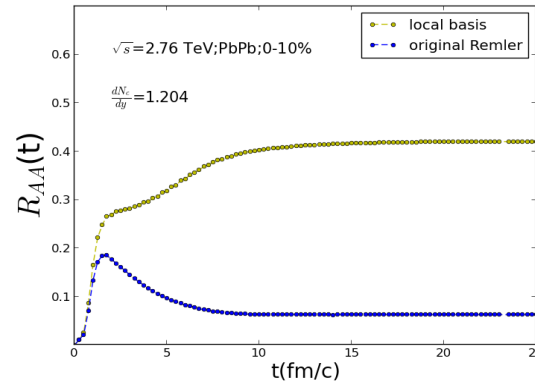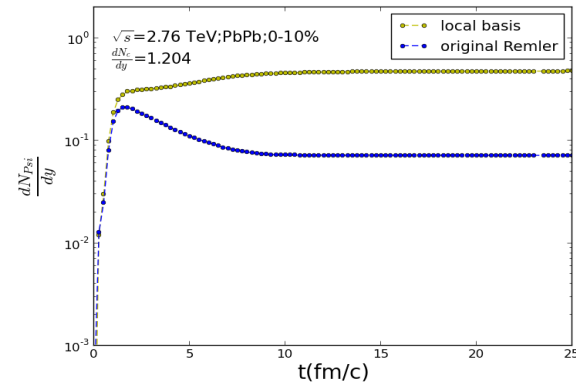
$$\gamma mr\,\dot{\theta} = \frac{l^2}{\gamma\,mr^3} \qquad l^2\frac{d^2}{d\theta^2}\frac{1}{r} - \gamma\frac{C}{m} + \frac{l^2}{r} = 0$$

# Preliminary Results J/Psi (local vs Orignal Remler's model)

Comparison between the original Remler's model and local basis prediction for the full probability and nuclear modification factor



The $p_T$ spectra for the $J/\Psi$ production, $R_{AA}$ and $v_2$ is a work in progress !!!

Important contribution coming from the binding potential (keeping close the HQ pairs for a bit more longer) and the Wigner functions which dependent on the temperature (local basis) are observed in the results. The combined effects of the interaction potential and mean square radius are able to repopulate the quarkonium states contrarily to the vacuum basis.

# Conclusions

We have presented a model based on the density operator that allows us to obtain the time evolution of the formation probability of Quarkonium through an effective rate in the QGP.
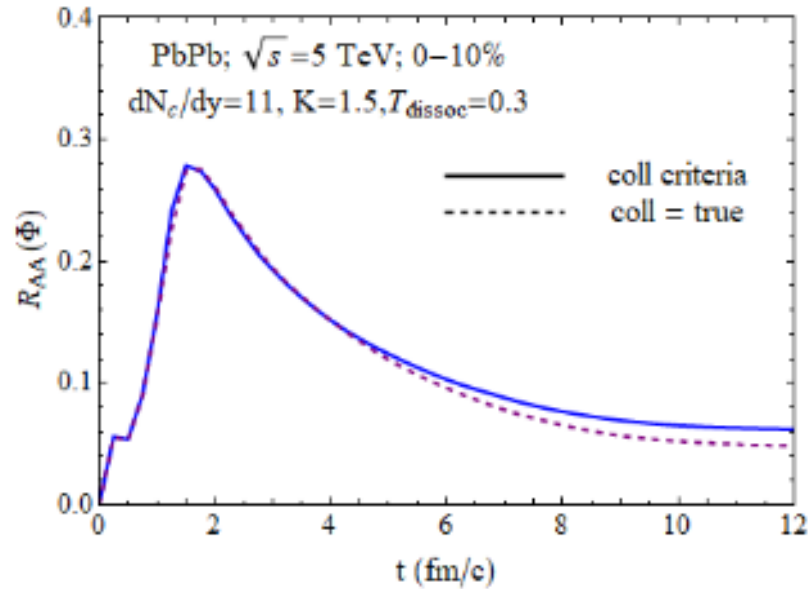
According with the results, the implementation of a temperature (time) dependent Wigner function and relativistic HQ pair interaction modifies the massive suppression that we obtained while using the vacuum basis.

Although the relativistic HQ pair interaction has been implemented , this formalism is base in a instantaneous potential, which is able to deal with a relativistic pairs as long as their relative momentum is not large (non relativistic behavior in the center of mass)
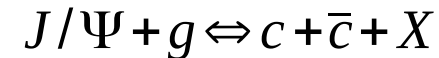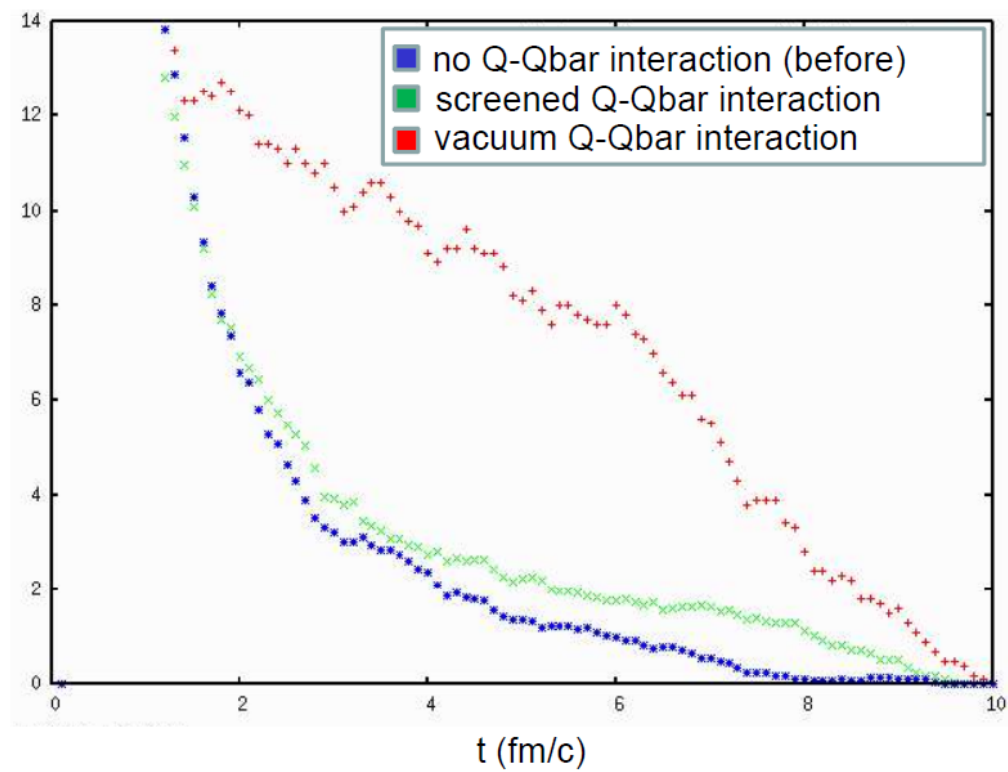
# Thanks!!!

# Preliminary Results J/Psi

PbPb; $\sqrt{s}$ =5 TeV; 0−10%
$dN_c/dy$=11, K=1.5, $T_{dissoc}$=0.3

coll criteria
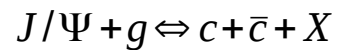coll = true

$R_{AA}(\Phi)$

t (fm/c)

- The effect of collision increase with time together with the thermalization degree

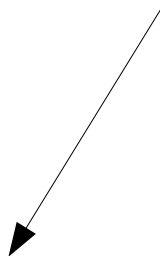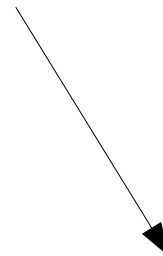- As the fireball expand in time most of the collision leads to a dissociation process

$$J/\Psi + g \Leftrightarrow c + \bar{c} + X$$

| | no Q-Qbar interaction (before) |
| | screened Q-Qbar interaction |
| | vacuum Q-Qbar interaction |

t (fm/c)

The detail balance law
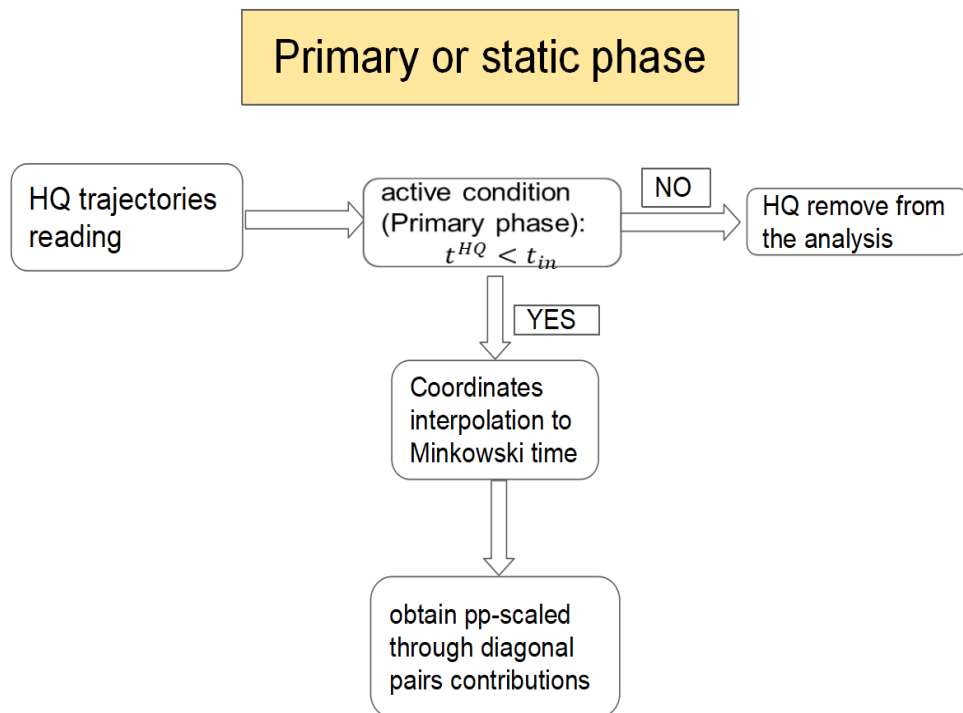
$$J/\Psi + g \Leftrightarrow c + \bar{c} + X$$

Rates Equation

$$\frac{dN_\Psi}{d\tau} = \Gamma_{recomb} N_c N_{\bar{c}} [V_{FB}(\tau)]^{-1} - \Gamma_{diss} N_\Psi$$

$$\frac{dN_\Psi}{d\tau} = -N_\Psi L\tau + G(\Psi) = \frac{-1}{\tau_\Psi}[N_\Psi(\tau) - N_\Psi^{eq}(\tau)]$$

# Analysis implementation

# Analysis implementation



Secondary or dynamical phase

coordinates interpolation to Minkowski time for each time step

active condition (secondary phase): $t^{HQ} \in [t_{in}, t^{HQ}_{lim}]$

NO → HQ remove from the analysis

YES

check if the $T_{HQ} < T_{diss}$

1st time

NO → check if collision happens

YES → Will contribute to initial state

YES → Will contribute to the rate