Optimal Transverse Bunch Feedback to Minimize Emittance Growth

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Motivation

Theory

Numerical Results

Summary & Outlook
Noise

- Theory predicts that the LHC beam screen filters noise beyond $f_{\text{cutoff}} < 150$ Hz. [M. Morrone et al.]
- Measurements show 50 Hz lines up to $f_{\text{max}} \sim 10$ kHz. [S. Kostoglou et al.]
- The noise will only excite multi-bunch modes of frequency $f_c < f_{\text{max}}$.
- **Idea**: Investigate how different feedback filters damp the coherent multi-bunch motion.
Theory
Transverse feedback

- Feedback limits emittance growth rate and maintains stability.
- **Ideal** feedback acts on bunch position $x_b$ (excl. phase advance)
  \[ x_b \rightarrow x_b - gx_b = x_b(1 - g) \]
- **Real** feedback acts on measured position $x_{bm} = x_b - \delta_{bm}$
  \[ x_b \rightarrow x_b - gx_{bm} = x_b(1 - g) + g\delta_{bm} \]
- Do not need full bandwidth (FBW) feedback to damp low-frequent beam modes.

- Can reduce the term $g\delta_{bm}$ without reducing feedback efficiency!
Low bandwidth (LBW) feedback

- **Low bandwidth** real feedback depends on neighbors

\[ x_b \rightarrow x_b - g \sum_{b'} w_{bb'} x_{b'm} \]

- Efficient gain damping a mode of frequency \( f_c = \omega_c / 2\pi \)

\[ g_c = g \sum_{b'} w_{bb'} \cos[\omega_c (t_{b'} - t_b)] \]

- Effect depends on the coherent mode frequency relative to the filter’s cutoff frequency
  - \( f_c \ll f_{\text{cutoff}} \rightarrow g_c > g \)
  - \( f_c \gg f_{\text{cutoff}} \rightarrow g_c \ll g \)
Emittance growth rate

- The emittance growth rate without feedback is equal to
  \[
  \frac{\dot{\epsilon}_{\text{tot}}}{\epsilon_0} = \frac{\sigma_{\text{Noise}}^2}{2}
  \]

- The feedback reduces the growth rate for FBW feedback
  \[
  \frac{\dot{\epsilon}_{\text{tot}}}{\epsilon_0} = \frac{\sigma_{\text{Noise}}^2 + g^2\sigma_{\text{BPM}}^2}{2} \cdot \left\langle \frac{(1 - \frac{g}{2})^2 4\pi^2 \Delta Q^2}{\left(\frac{g}{2}\right)^2 + (1 - \frac{g}{2}) 4\pi^2 \Delta Q^2} \right\rangle
  \]

- The LBW feedback can further reduce the growth rate caused by the machine noise if \( g_c > g \)
  \[
  \frac{\dot{\epsilon}_{\text{Noise}}}{\epsilon_0} = \frac{\sigma_{\text{Noise}}^2}{2} \cdot \left\langle \frac{(1 - \frac{g_c}{2})^2 4\pi^2 \Delta Q^2}{\left(\frac{g_c}{2}\right)^2 + (1 - \frac{g_c}{2}) 4\pi^2 \Delta Q^2} \right\rangle
  \]

- The BPM noise induced growth rate is challenging to tackle analytically. Wait for the numerical results!
Numerical Results
Numerical setup

- 64 bunches.
- $10^6$ particles per bunch.
- $10^5$ turns.
- Bunch separation $\Delta t = 25$ ns.
- Single bunch feedback gain $g = 0.01 = 2/\tau$, unless stated otherwise.
- Linear detuning $Q_x = Q_{x0} + 10^{-3} J_x - 7 \times 10^{-4} J_y$ (normalized actions).
- Different noise sources of rms amplitude $\sigma_{\text{HF,LF,BPM}}$:
  - High frequent noise, uncorrelated between bunches.
  - Low frequent flat noise spectrum up to $f_{\text{max}} = f_{\text{rev}} = 11.245 \text{ kHz}$.
  - Uncorrelated pickup noise $\rightarrow$ LBW feedback correlates the kicks.
High-frequency (HF) machine noise

- No correlation between the kicks on neighboring bunches.
- Lead to uncorrelated multi-bunch motion.
- LBW feedback will not help, $g_c < g$.
- HF noise is qualitatively different from the LHC noise.
Low-frequency (LF) machine noise

- Correlation between the kicks on neighboring bunches.
- Lead to correlated multi-bunch motion of $f_c \leq f_{\text{max}}$.
- LBW feedback will help if $f_{\text{max}} = f_{\text{rev}} \ll f_{\text{cutoff}}$.
- This noise is qualitatively similar to the LHC noise.
Beam position monitor (BPM) noise

- Position measurement error is assumed uncorrelated.
- BPM noise is inflicted on the beam by the feedback. → BPM noise has the same spectrum as the feedback.
- LBW feedback helps marginally for symmetric feedback only.
Gain and bandwidth scan

- For each $g$, $f_{\text{cutoff}}$, present average growth rate over 64 bunches, using a symmetric LBW feedback.

- White area signifies that the beam went unstable due to $g_c > 1.02$. 
LF noise and BPM noise

- Different noise sources are uncorrelated → Assume that the emittance growth rate is the sum of the individual growth rates.

- Relative difference has standard deviation of 3.2%.
- Expect a nonzero difference between equal simulations due to noise.
Optimal transverse feedback

- $\sigma_{\text{BPM}} = 50\sigma_{\text{LF}}$, similar to LHC. [X. Buffat et al.]
- $\sigma_{\text{BPM}} = 25\sigma_{\text{LF}}$, if one could half the pickup error.

- BPM noise moves the optimum towards smaller $f_{\text{cutoff}}$ and $g$.
- Reducing the pickup error is important if full bandwidth is needed.
Summary & Outlook

- Using a **LBW feedback can reduce the emittance growth**
  - caused by LF machine noise, as in the LHC, and
  - caused by BPM noise IFF the feedback is symmetric.

- The emittance growth rate from different noise sources can to first order be summed, even if the feedback couples the noise.

- **Reducing the pickup error** will reduce the emittance growth rate, and **allows more optimal scenarios with full bandwidth**.

- **Outlook**:
  - Implement the response function of the ADT in the LHC.
  - Test more LHC relevant beam constellations (8b4e etc.).
  - Include beam-beam and wakefields to see if the LBW feedback still manages to stabilize multi-bunch instabilities.
  - Test the model’s prediction in MDs and, if successful, implement an optimal setup for physics in run 3.
Thank you for your attention!
Backup: LF noise and BPM noise

- Include both LF and BPM noise with $f_{\text{cutoff}} = 1 \text{ MHz}$.
  - Max amplitude of either noise type is equal to those studied on previous slides.
- Different noise sources are uncorrelated $\rightarrow$ Emittance growth rate is the sum of the individual growth rates.