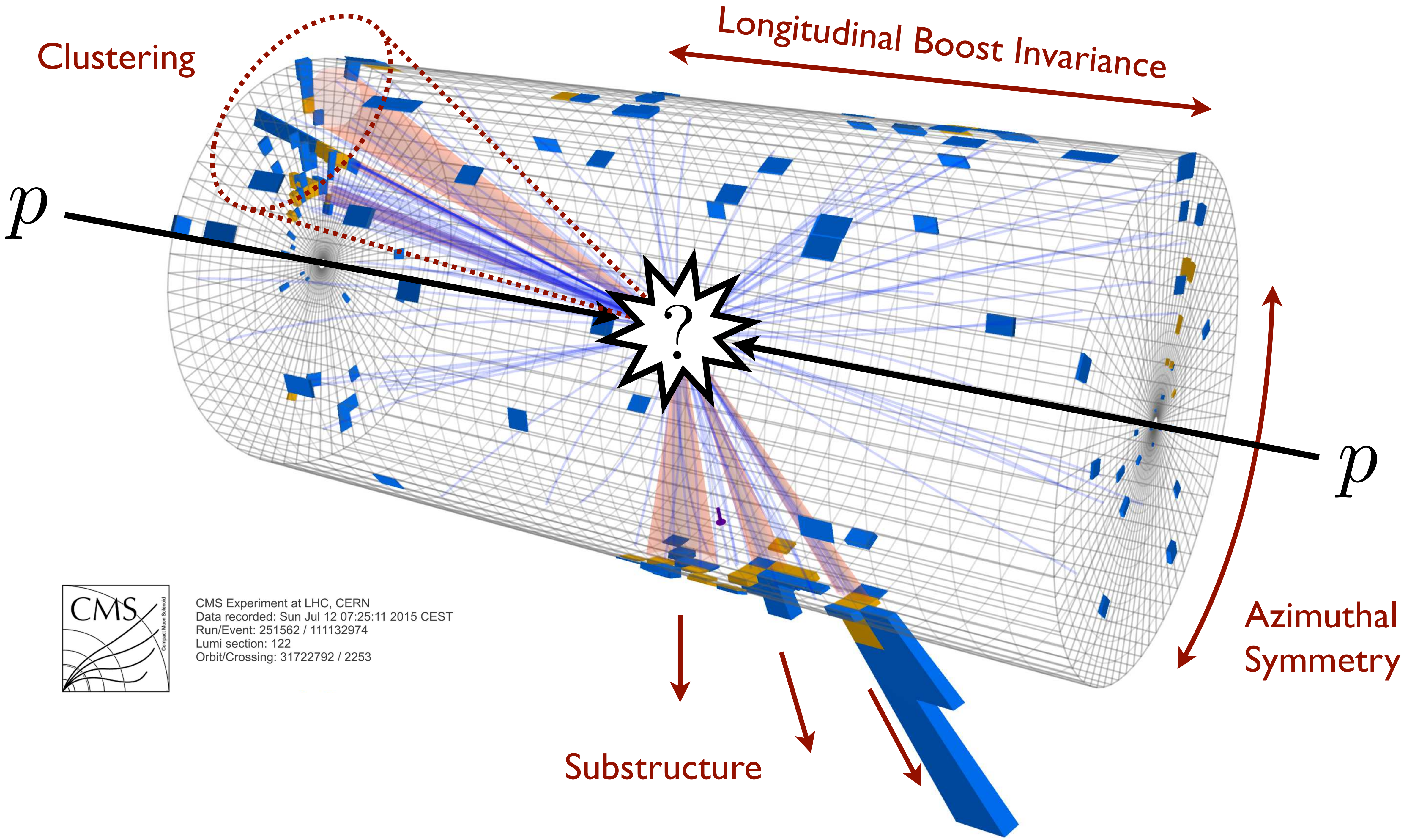
The Hidden Geometry of Particle Collisions

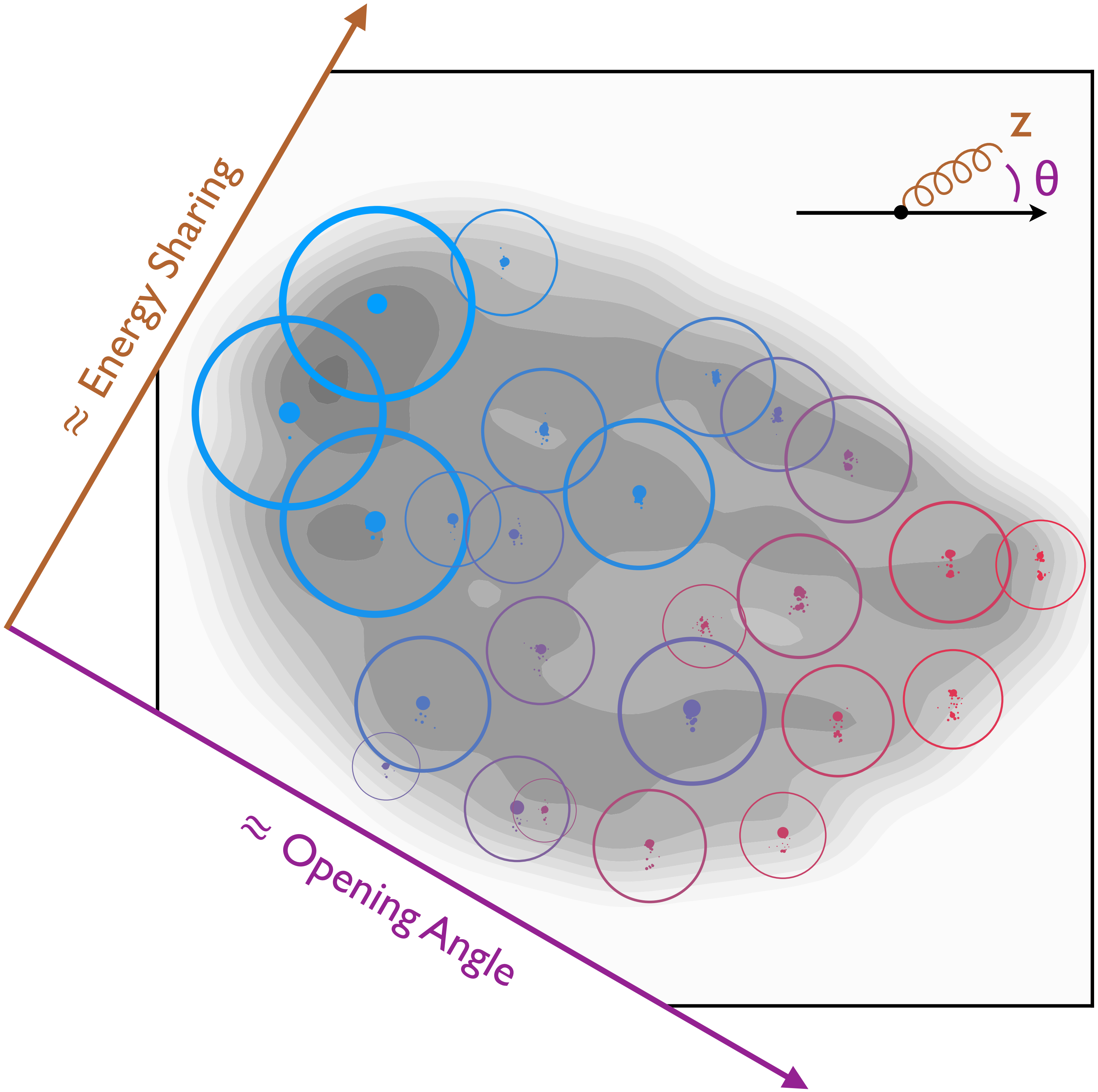
Jesse Thaler

CERN Theory Colloquium — May 20, 2020

The Manifest Geometry of One Collision



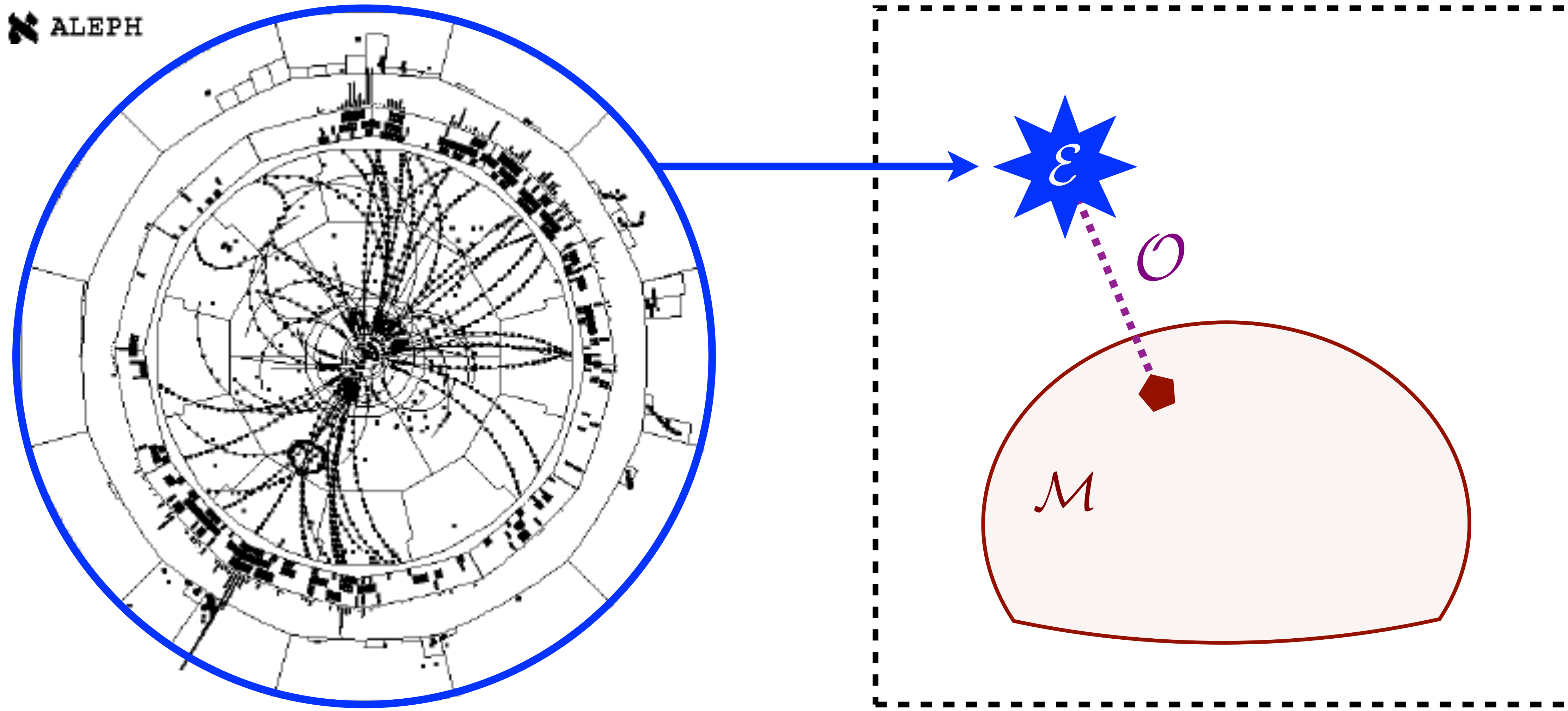
The Emergent Geometry of Many Collisions



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
based on Komiske, Metodiev, JDT, PRL 2019; using EnergyFlow and CMS Open Data]

The Hidden Geometry of Particle Collisions

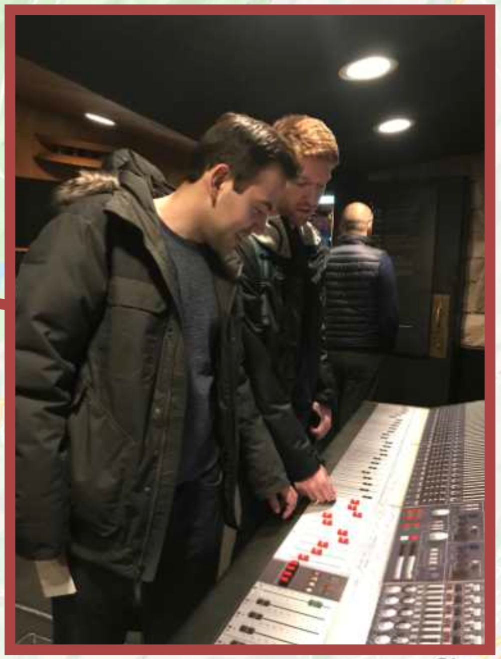
E.g. Classic QCD Event Shapes



One Electron-Positron Event *Distance to a Manifold in Event Space*

[Komiske, Metodiev, JDT, arXiv 2020]
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977]

An Idea Born at CERN

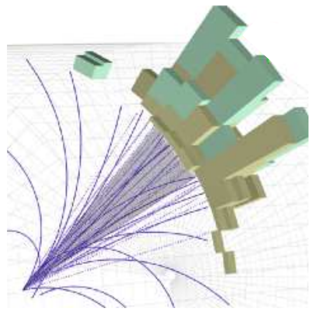


Eric Metodiev

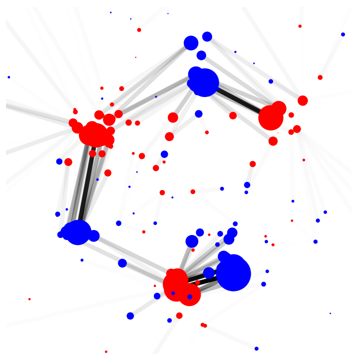
Patrick Komiske

[February 2019;
Simons Sabbatical Fellowship]

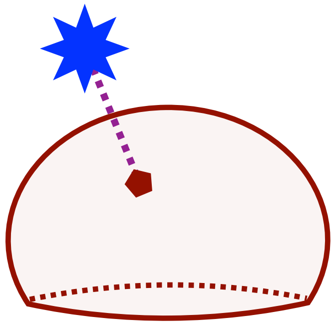
Outline



What is a Collider Event?



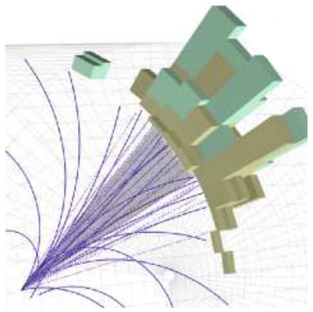
When are Events Similar?



What can be Geometrized?

Pause

Drop questions in the chat, and I'll try to answer them as I go



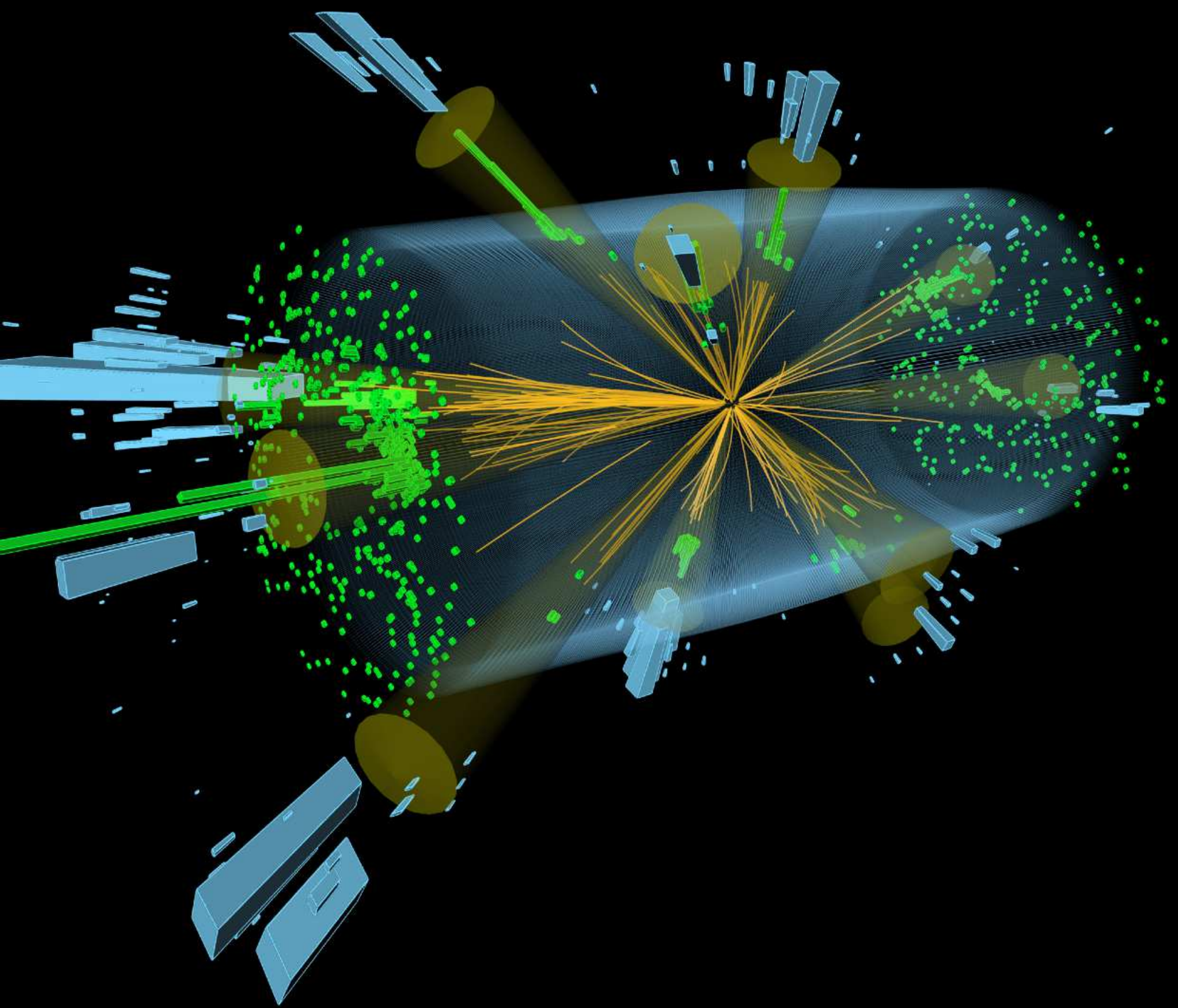
What is a Collider Event?

When are Events Similar?

What can be Geometrized?

Collider Event

Collection of points in (momentum) space



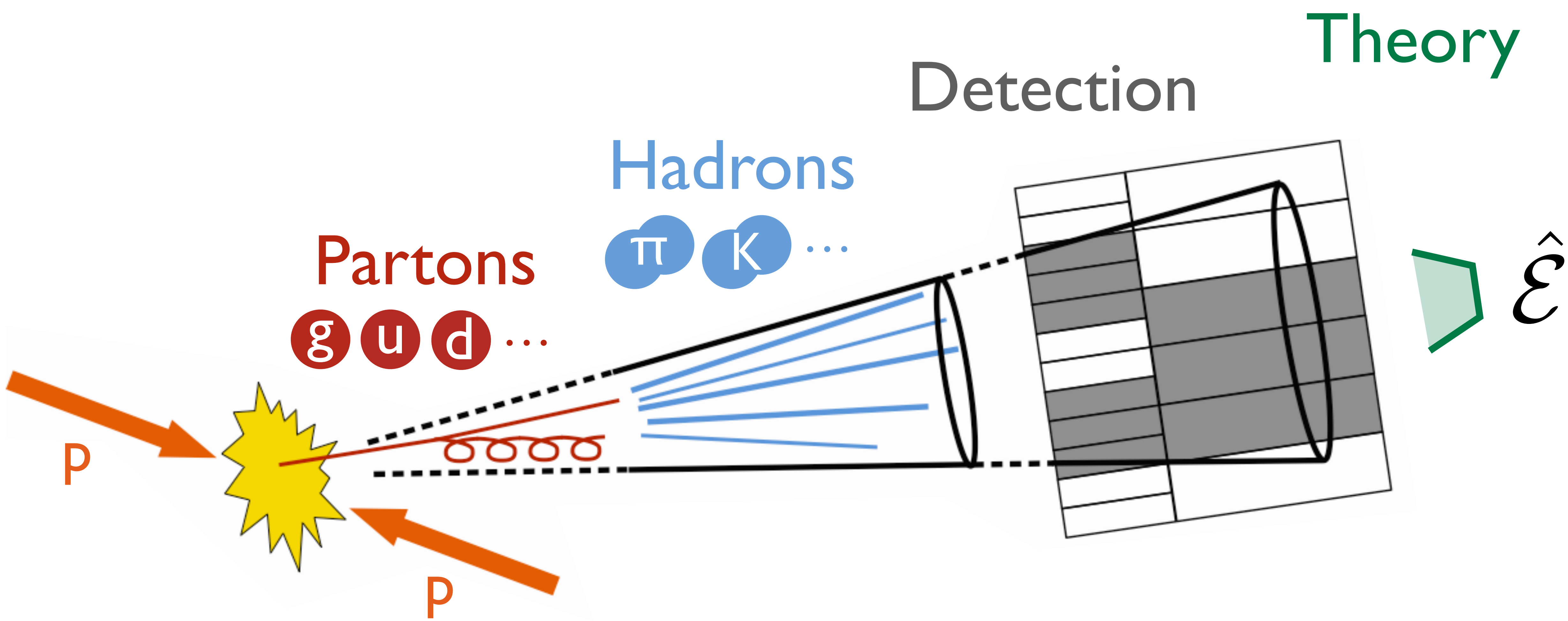
T	E	H	M			
	●			γ	photon	elementary
●	●			$e^{\pm}$	electron	elementary
●	•	•	●	$\mu^{\pm}$	muon	elementary
●	•	●		$\pi^{\pm}$	pion	composite
●	•	●		$K^{\pm}$	kaon	composite
	•	●		K_L^0	K-long	composite
●	•	●		$p/\overline{p}$	proton	composite
	•	●		$n/\overline{n}$	neutron	composite

Point Cloud

Collection of points in (position) space

[Popular Science, 2013]

Jet Formation Process



Stress-energy flow:

Robust to non-perturbative and detector effects
Well-defined for massless gauge theories

$$\boxed{\hat{\mathcal{E}} \simeq \lim_{t\to\infty} \hat{n}_i T^{0i}(t, vt\hat{n})}$$

[see e.g. Sveshnikov, Tkachov, PLB 1996; Hofman, Maldacena, JHEP 2008; Mateu, Stewart, JDT, PRD 2013; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, PRL 2014; Chen, Moult, Zhang, Zhu, arXiv 2020]

Jets as Weighted Point Clouds

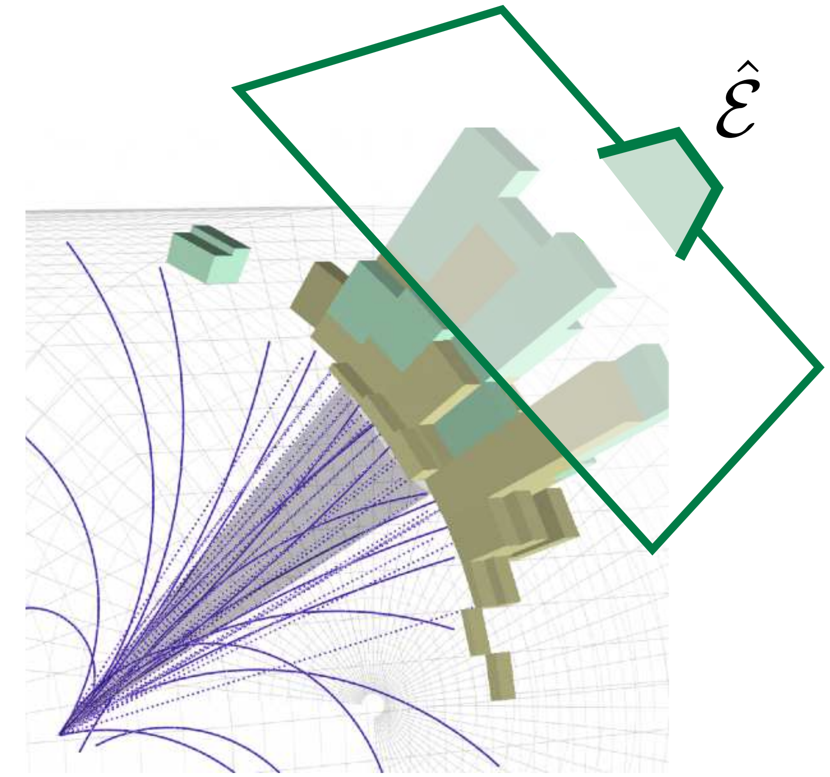
- Energy-Weighted Directions

$$\vec{p} = \{E, \hat{n}_x, \hat{n}_y, \hat{n}_z\}$$

Energy

Direction

(suppressing "unsafe" charge/flavor information)

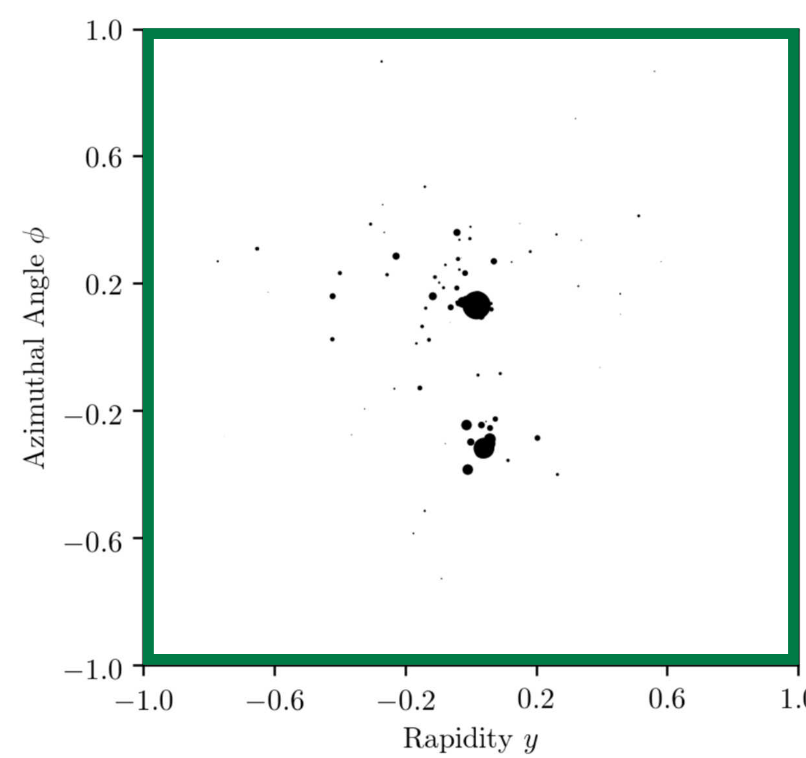


- Equivalently: Energy Density

$$\rho(\hat{n}) = \sum_{i \in \mathcal{J}} E_i \, \delta^{(2)}(\hat{n} - \hat{n}_i)$$

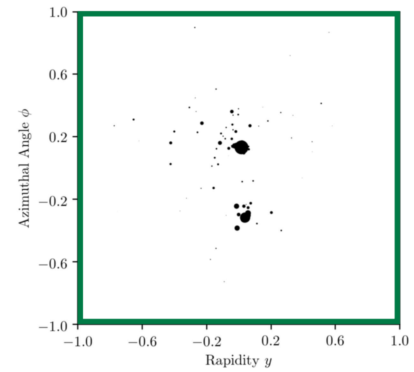
Energy

Direction



Pause

What is a Collider Event?

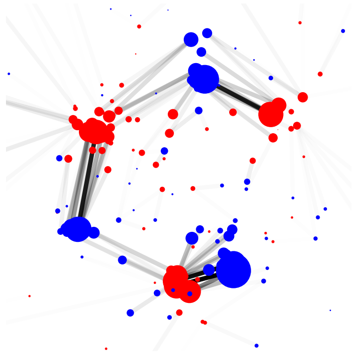


"Calo" Energy Density

$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$

(see backup for relevance to ML)

What is a Collider Event?

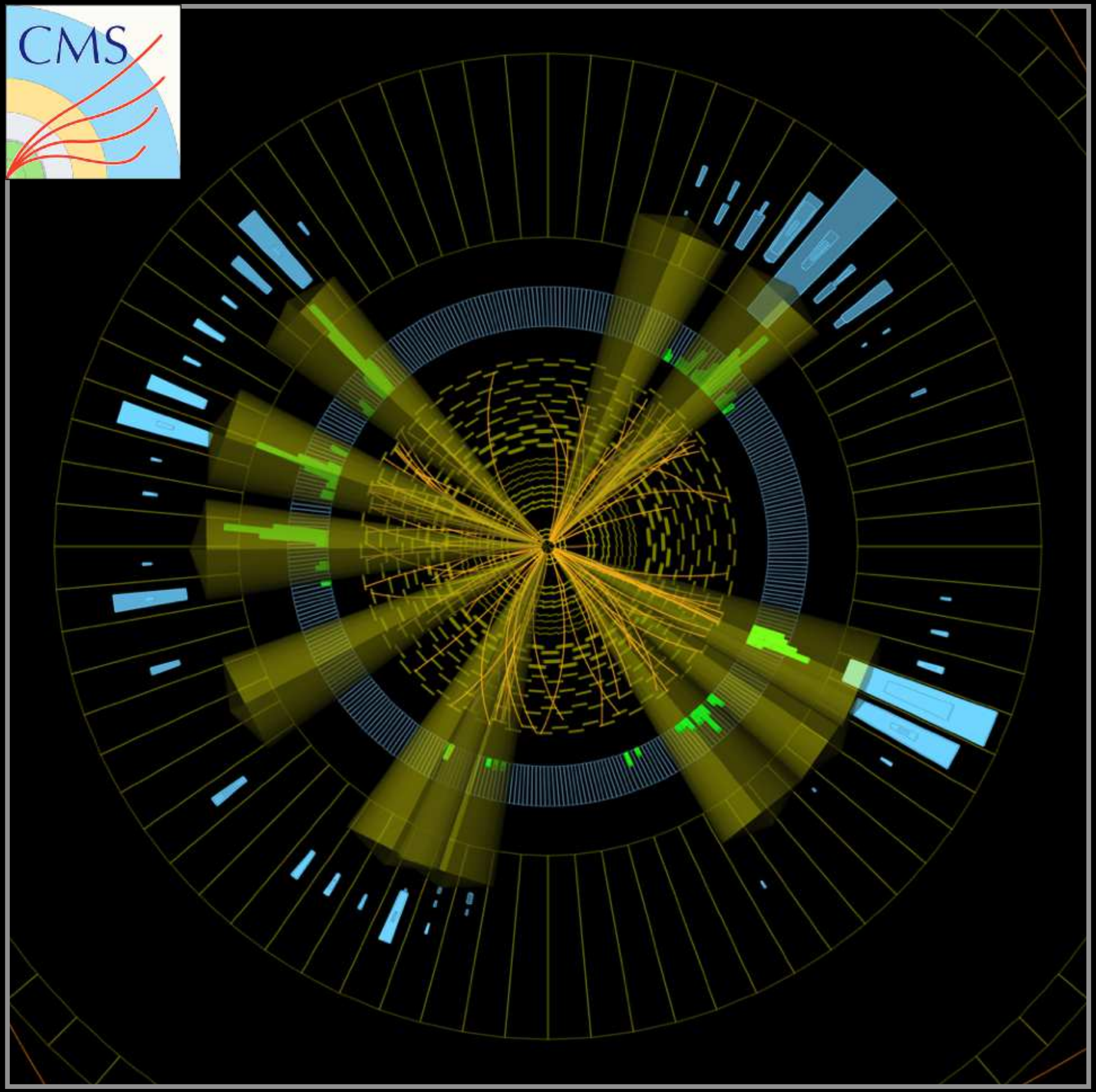


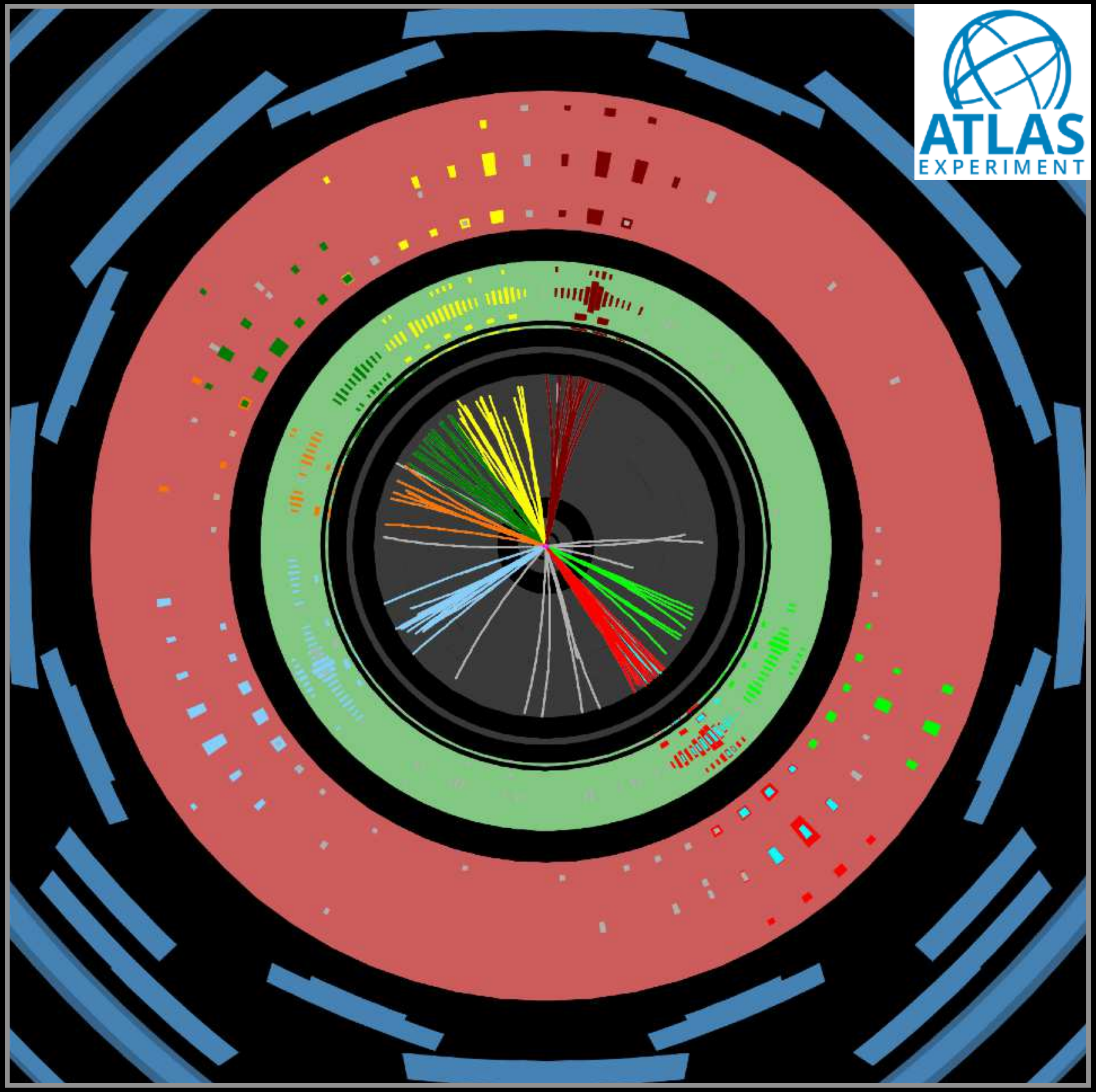
When are Events Similar?

What can be Geometrized?

Two Collider Events

Two collections of points in (momentum) space

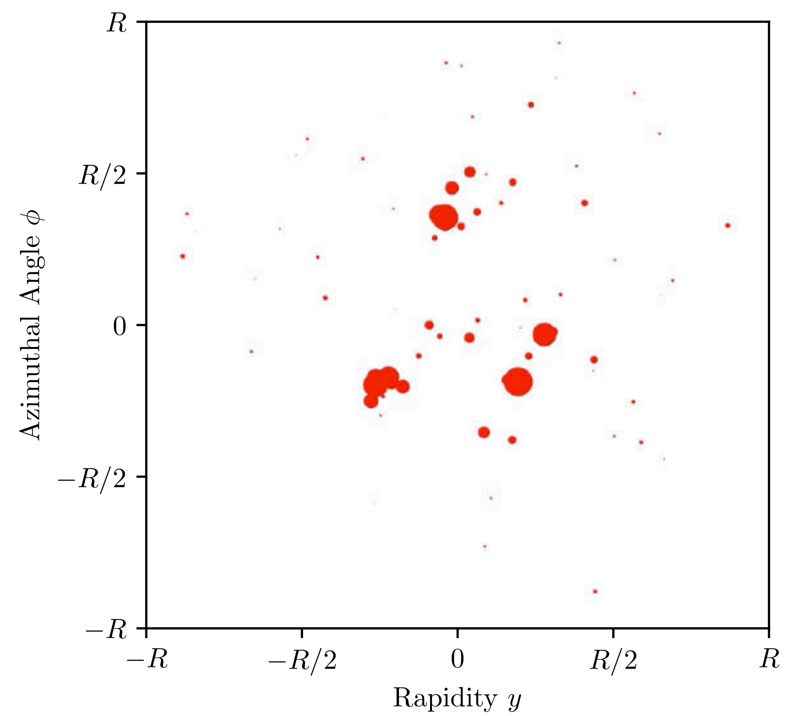




How "close" are these? (8.5 km?)

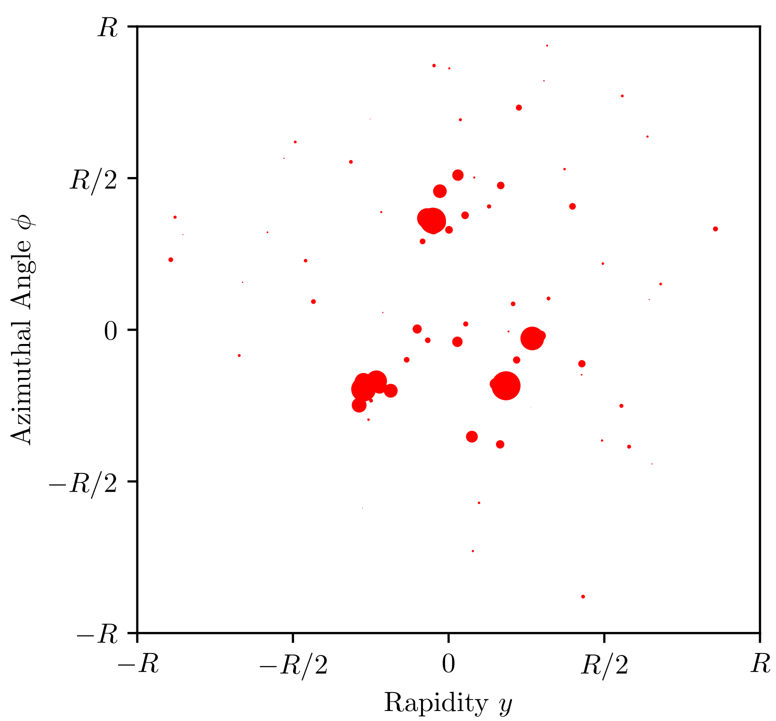
Similarity of Two Energy Flows?

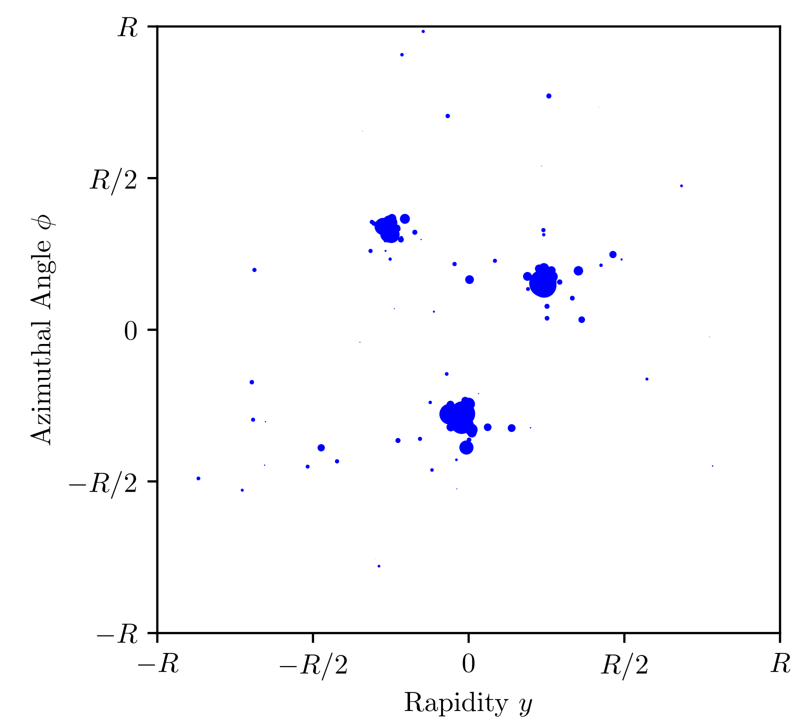
$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$



Optimal Transport:

Earth Mover's Distance
a.k.a. 1-Wasserstein metric





[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network]

The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, IEEE 1989;
Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000;
Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

Minimum "work" (stuff x distance) to make one distribution look like another distribution

Déblai

Remblai

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Vaseršteĭn, 1969; Wikipedia]

The Earth Mover's Distance

Optimal Transport: Minimum "work" (stuff x distance) to make one distribution look like another distribution

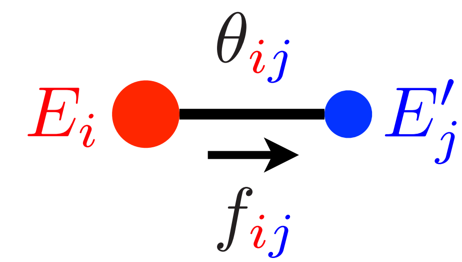
[Peleg, Werman, Rom, IEEE 1989;
Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000;
Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

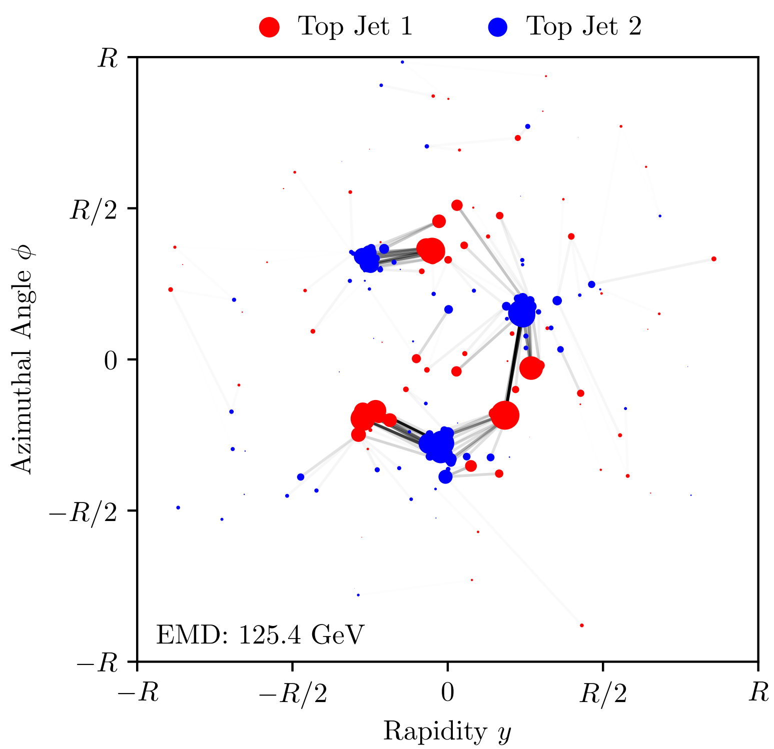
Additive Weight: ●
Ground Distance: ⊢—⊣

$\Rightarrow$ Distance Between Distributions

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Vaseršteĭn, 1969; Wikipedia]

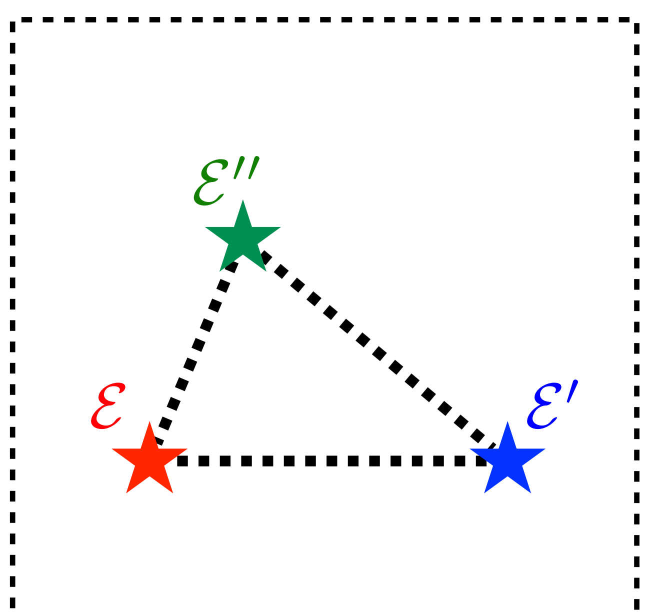
The Energy Mover's Distance





Optimal transport between energy flows…

$$\underset{\text{in GeV}}{\mathrm{EMD}(\mathcal{E},\mathcal{E}')} = \underbrace{\min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R}}_{\text{Cost to move energy}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost to create energy}}$$



…defines a metric on the space of events

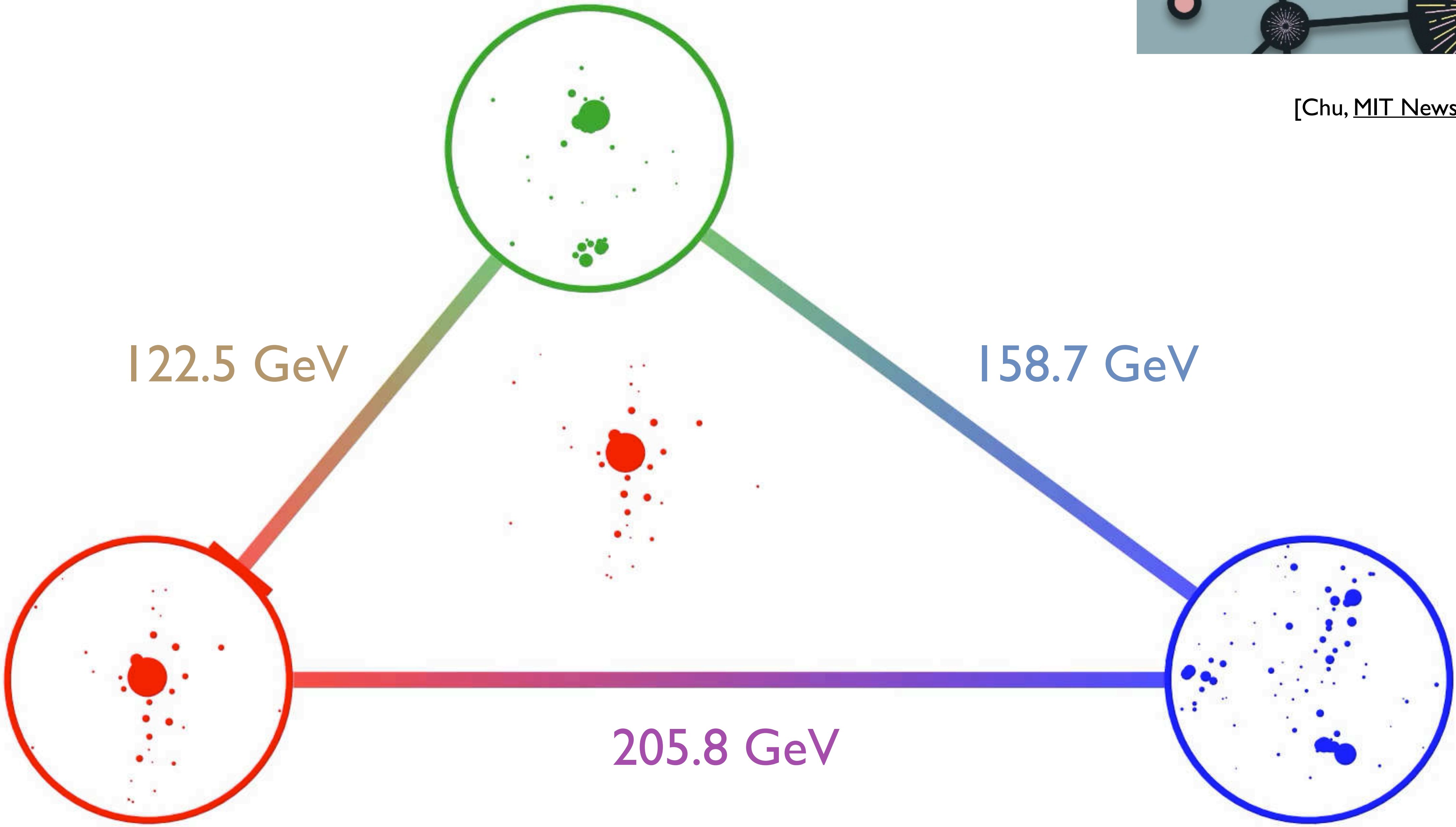
$$0 \leq \mathrm{EMD}(\mathcal{E},\mathcal{E}') \leq \mathrm{EMD}(\mathcal{E},\mathcal{E}'') + \mathrm{EMD}(\mathcal{E}',\mathcal{E}'')$$

(assuming R ≥ $\theta_{max}/2$, i.e. R ≥ jet radius for conical jets)

[Komiske, Metodiev, JDT, PRL 2019;
see also Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013;
see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, arXiv 2020]

Similarity of Three Energy Flows?

[Chu, MIT News July 2019]



[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network]

What can you do with a metric?

Dimensionality of Space of Jets

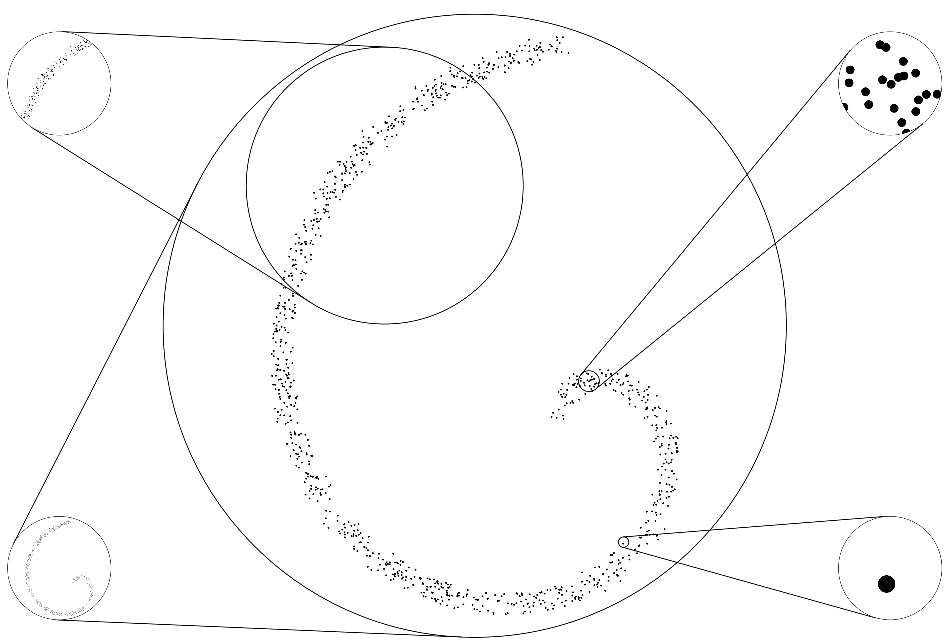
$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

$$\Rightarrow \quad \dim(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]

dim≃1

dim≃2



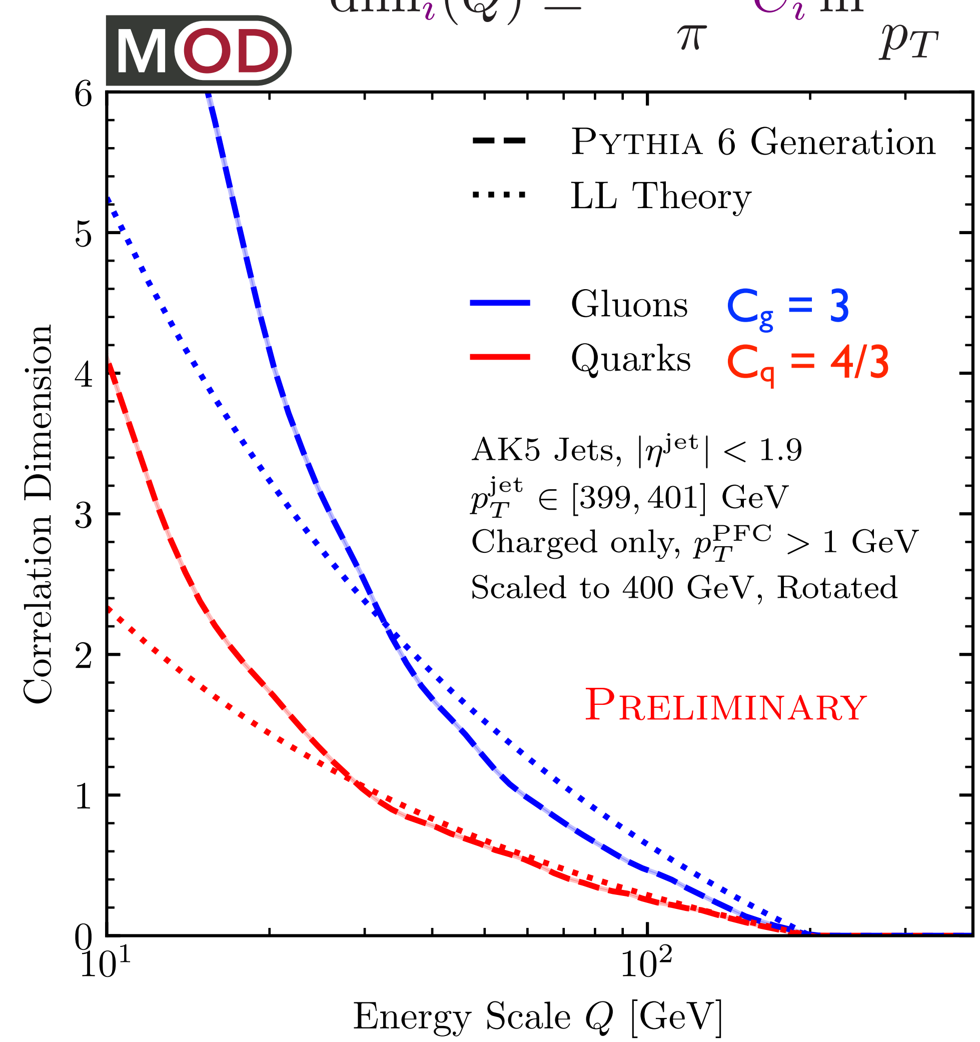
dim≃2

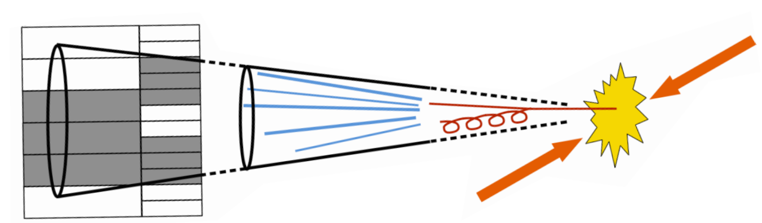
(eventually 0)

dim≃0

QCD Calculation

$$\dim_i(Q) \simeq -\frac{8\alpha_s}{\pi} C_i \ln \frac{Q}{p_T}$$





Dimensionality of Space of Jets

[http://opendata.cern.ch/]

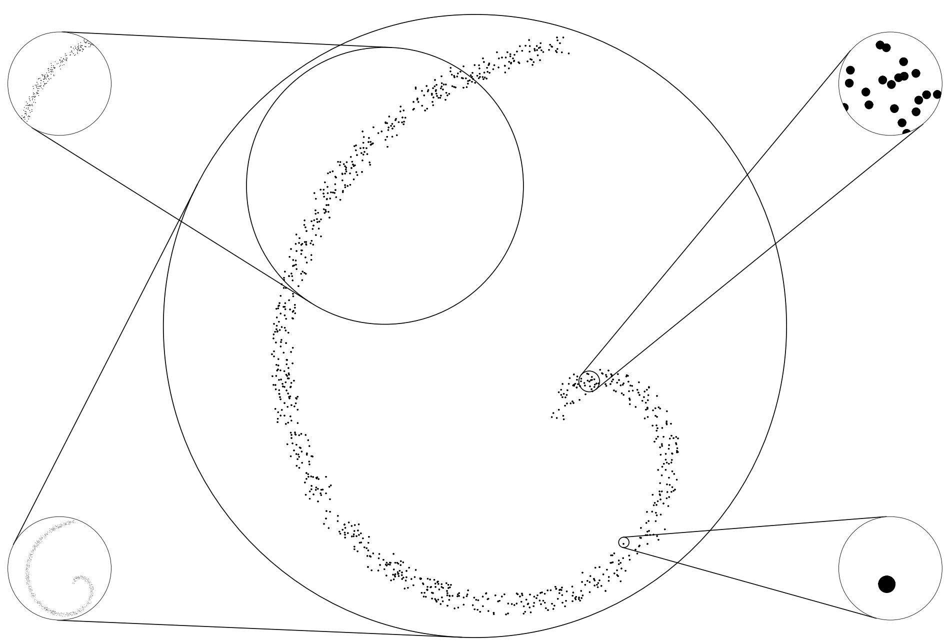
$$N_{\mathrm{neighbors}}(r) \sim r^{\mathrm{dim}}$$

$$\Rightarrow \quad \mathrm{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\mathrm{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]

dim≃1

dim≃2



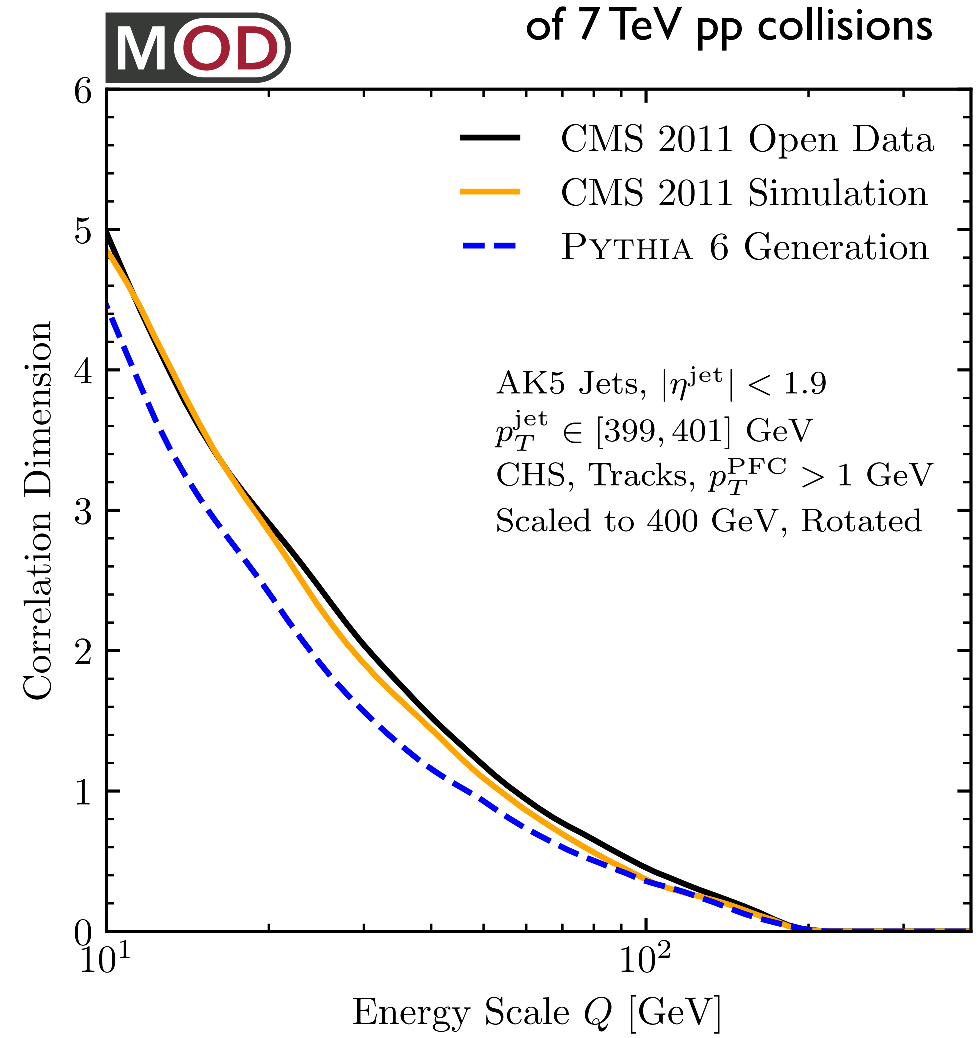
dim≃2

(eventually 0)

dim≃0

CMS Open Data

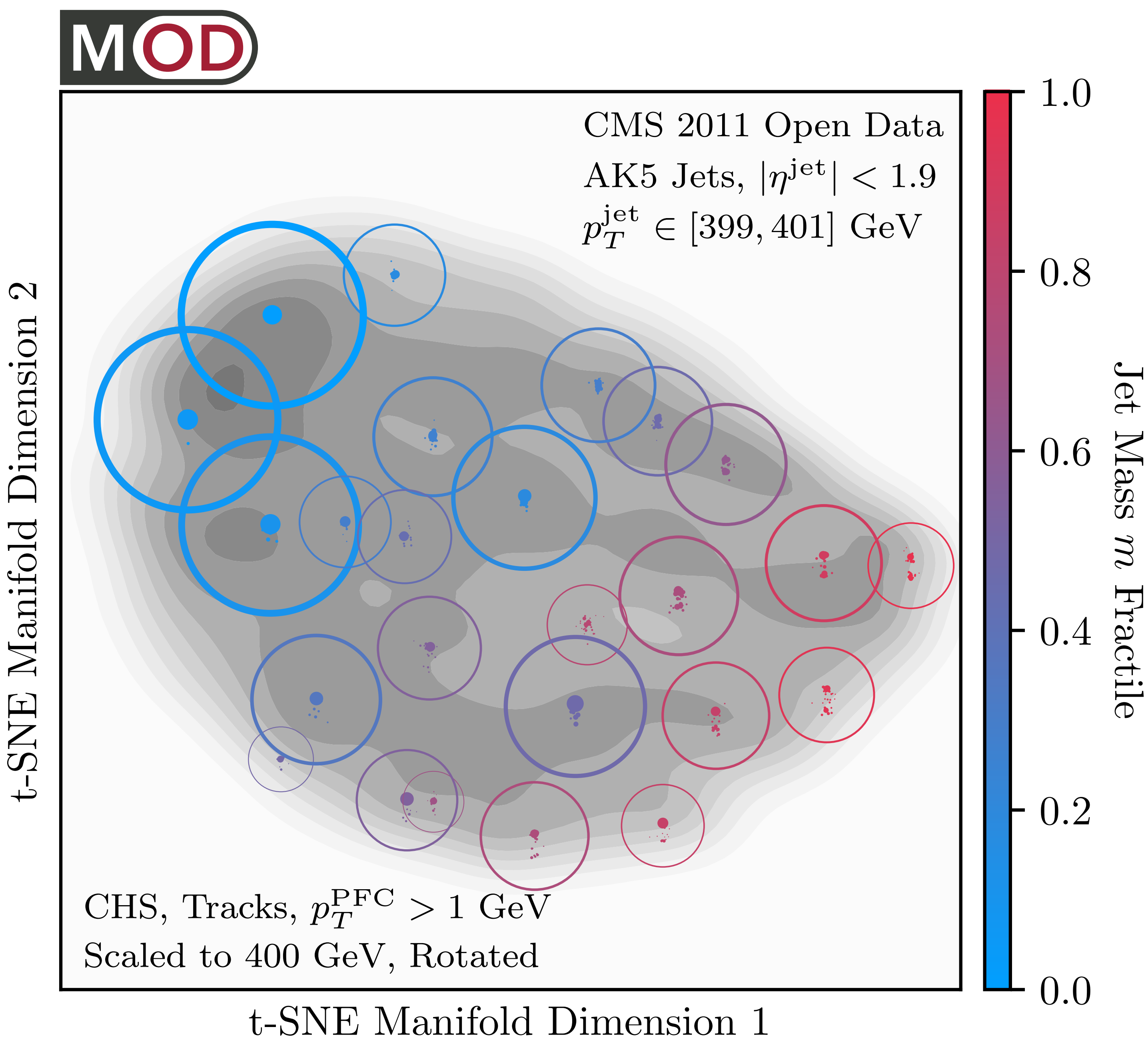
400 GeV jets in 2.3 fb⁻¹ of 7 TeV pp collisions



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]

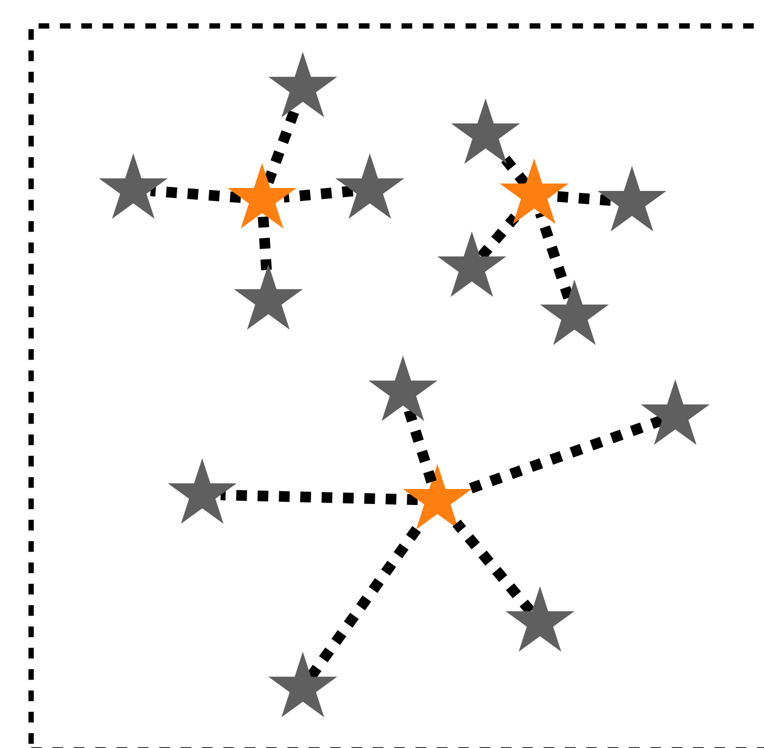
Most Representative Jets

[http://opendata.cern.ch/]



k-medoids

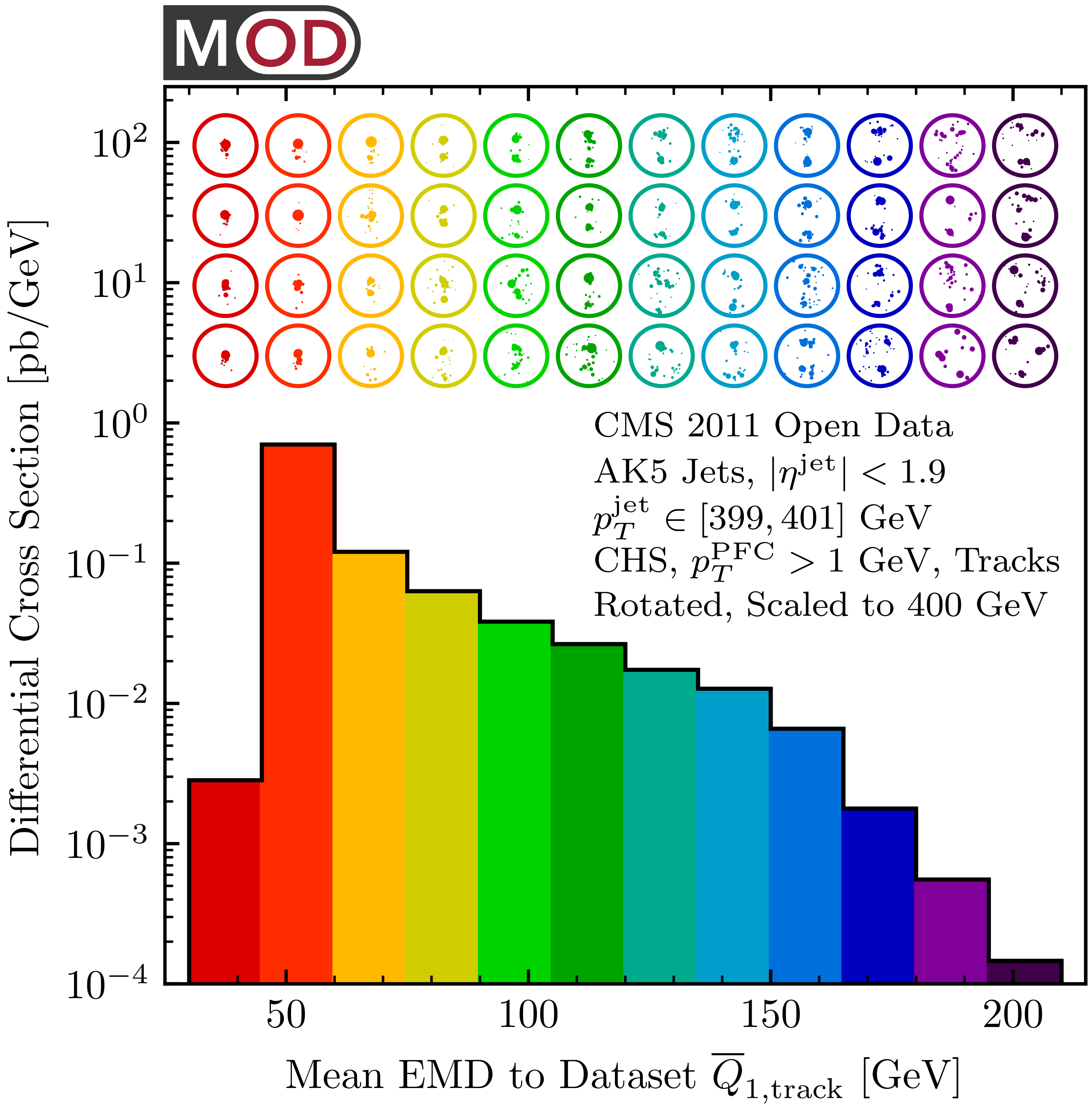
Arranged via t-SNE



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

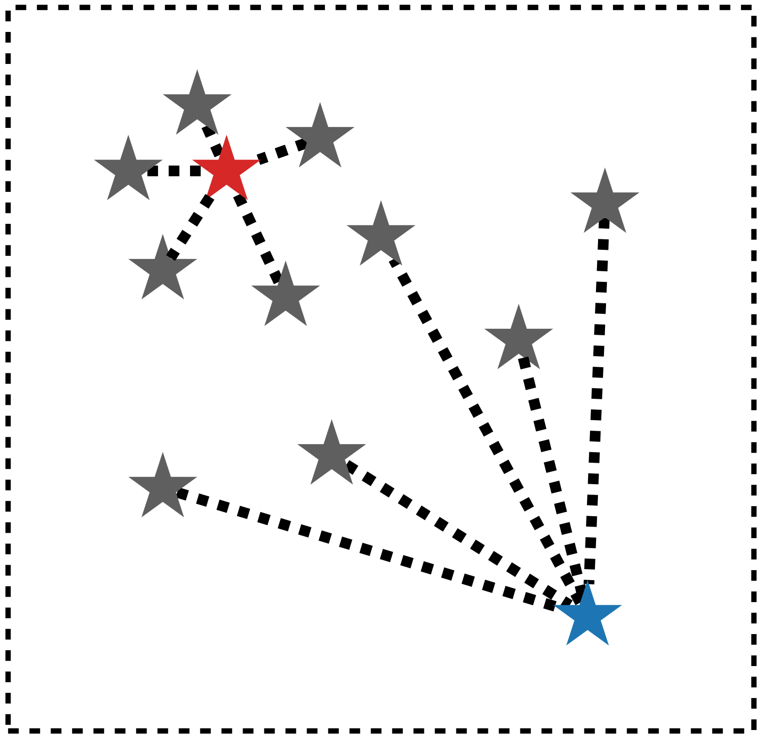
Least Representative Jets

[http://opendata.cern.ch/]



New Physics?

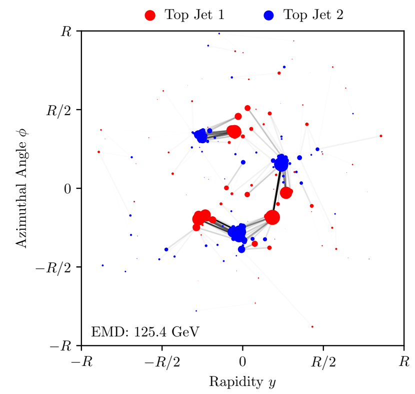
Or tails of QCD?



[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]

Pause

When are Events Similar?

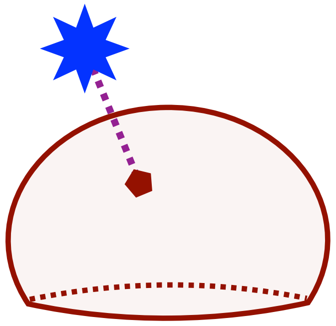


Small Energy Mover's Distance

$$\mathrm{EMD}(\mathcal{E},\mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_i E_i - \sum_j E'_j \right|$$

What is a Collider Event?

When are Events Similar?

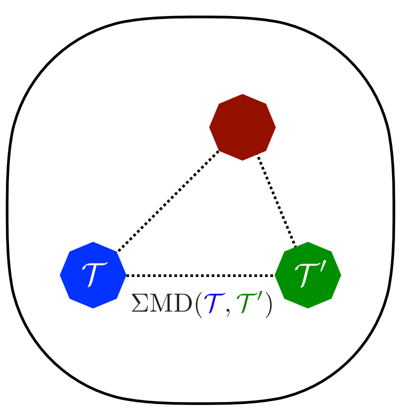


What can be Geometrized?

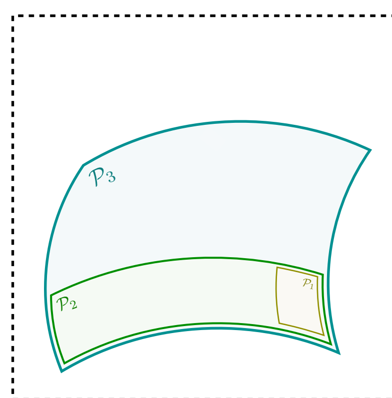
Master Formula for Collider Physics

$$\sigma_{\mathrm{obs}} \simeq \frac{1}{2E_{\mathrm{CM}}^2} \sum_{n=2}^{\infty} \int \mathrm{d}\Phi_n \, |\mathcal{M}_{AB\to 12\ldots n}|^2 \, f_{\mathrm{obs}}(\Phi_n)$$

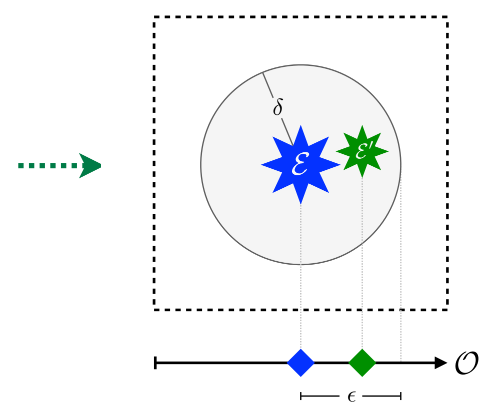
cross section



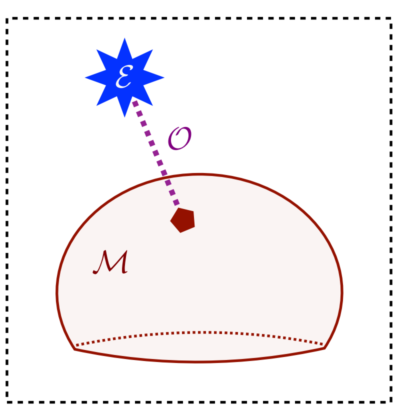
phase space



amplitude



observable

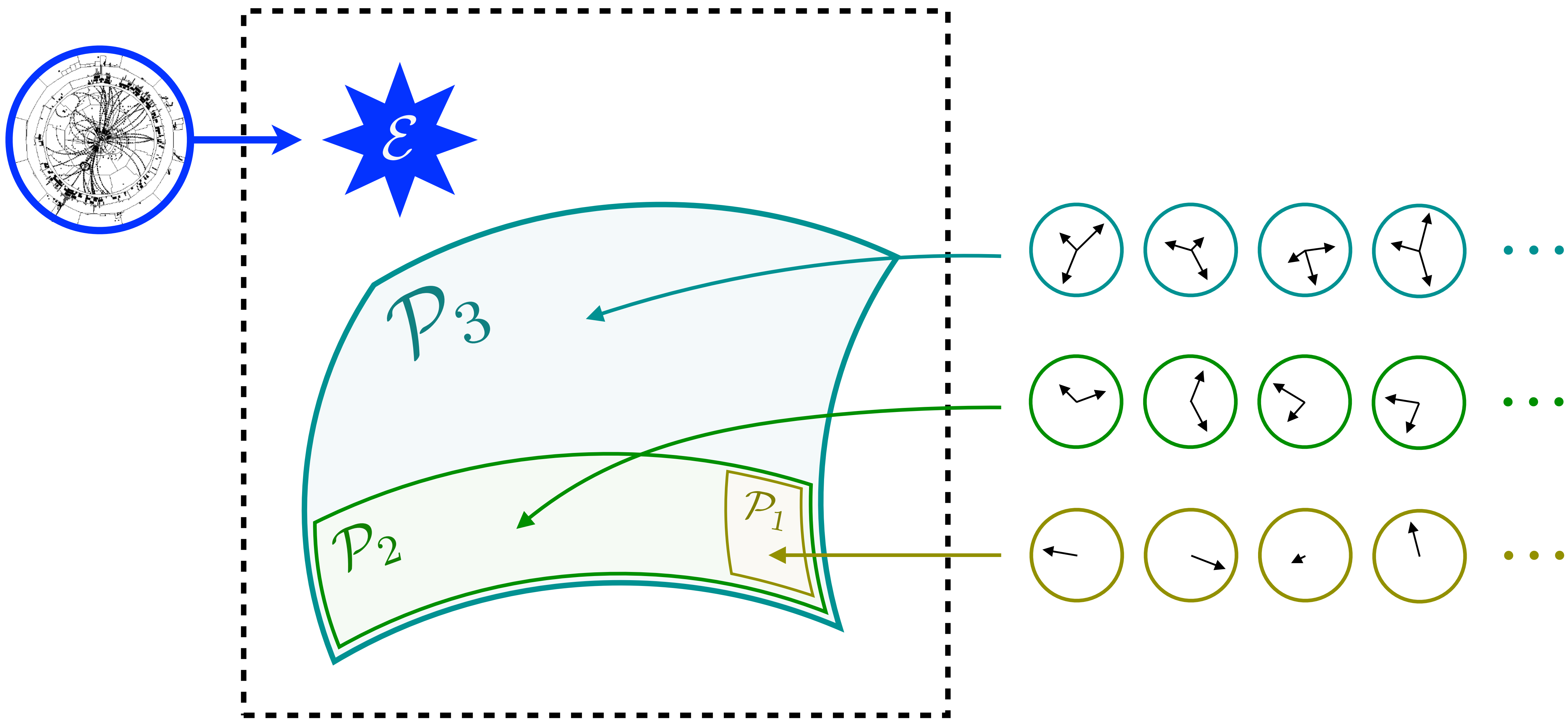


[Komiske, Metodiev, JDT, arXiv 2020]

Introducing N-particle Manifolds

$$\sum_{n=2}^{\infty} \int \mathrm{d}\Phi_n$$

$\mathcal{P}_N$ = set of all N-particle configurations

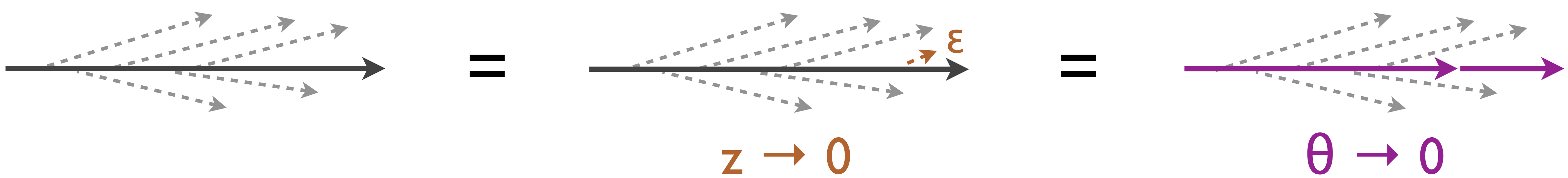


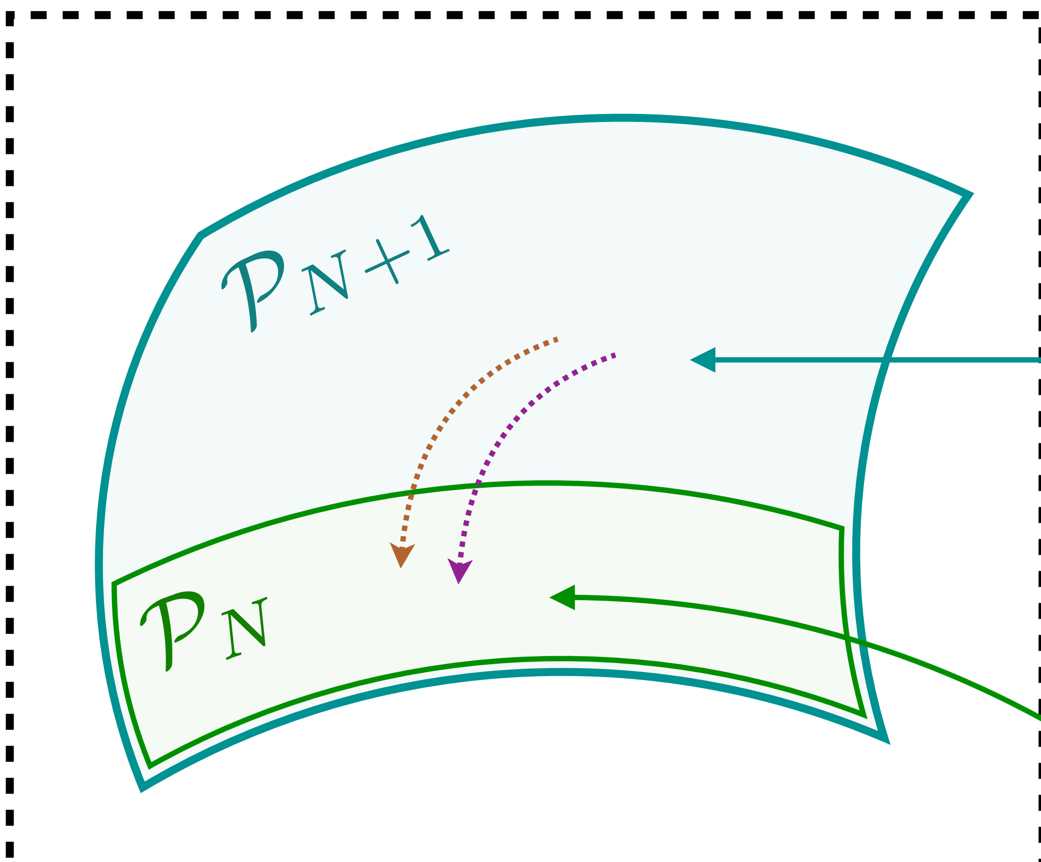
$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \ldots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by soft/collinear limits

When are Two Events the Same?

$$\mathcal{E}(\hat{n}) = \sum_i E_i\,\delta(\hat{n} - \hat{n}_i)$$

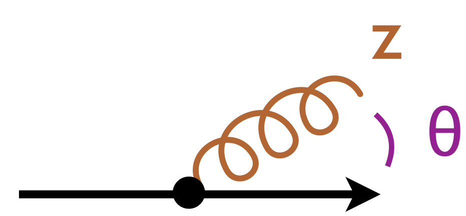
Energy Flow unchanged by infinitesimal soft/collinear emissions





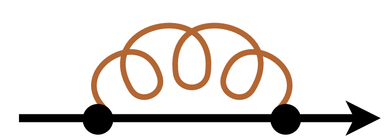
Infrared divergences "live" together!

Real:



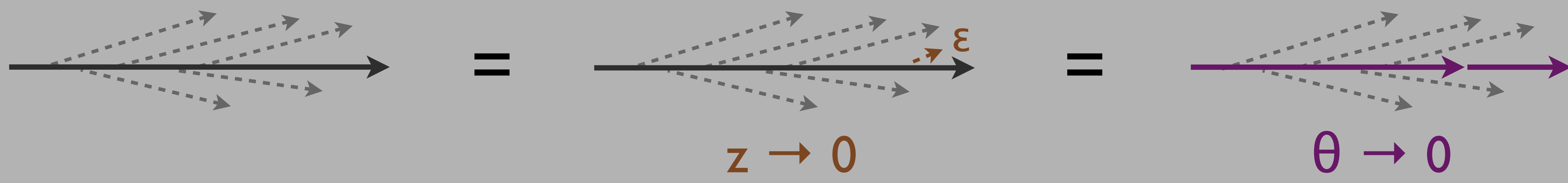
$$\mathrm{d}P_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{\mathrm{d}z}{z}\frac{\mathrm{d}\theta}{\theta}$$

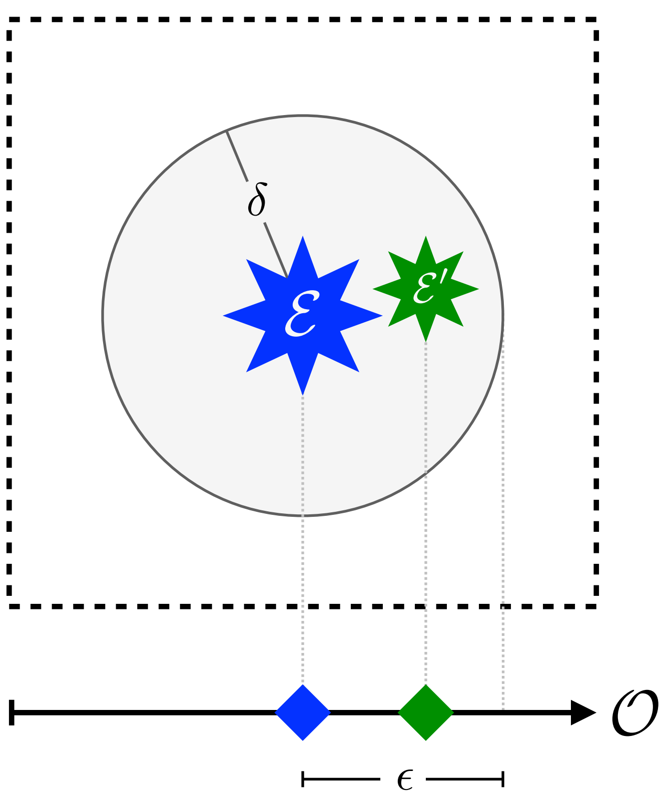
Virtual:



When are Two Events the Same?

Energy Flow unchanged by infinitesimal soft/collinear emissions





Infrared & Collinear Safety

≈ calculable in perturbative quantum field theory

*is**

Continuity in EMD Space

[Komiske, Metodiev, JDT, arXiv 2020]
[Sterman, Weinberg, PRL 1977; Sterman, PRD 1979]
[see also Banfi, Salam, Zanderighi, JHEP 2005]

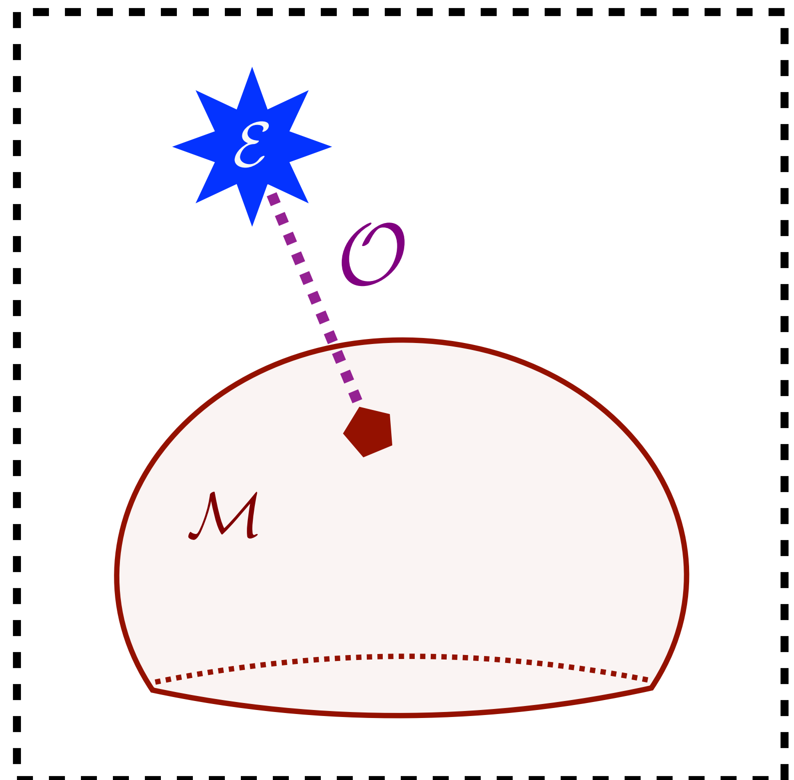
EMD seems to define the "natural" geometry for massless gauge theories

Open question: Can you define $\left|\mathcal{M}_{AB\to 12\ldots n}\right|^2$ directly in this space?

Manifolds for Observables

$f_{\text{obs}}(\Phi_n)$

One Event



Set of Events

Distance of Closest Approach ⇒ Observable

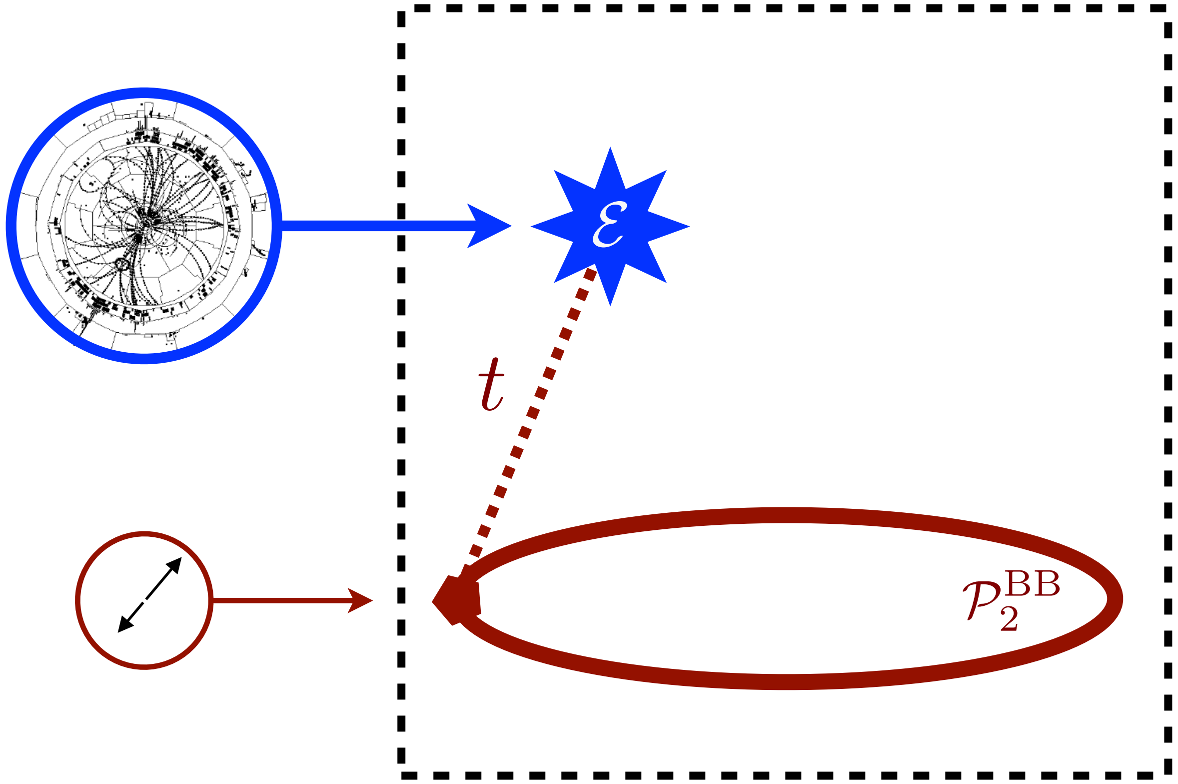
$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}(\mathcal{E}, \mathcal{E}')$$

[Komiske, Metodiev, JDT, arXiv 2020]

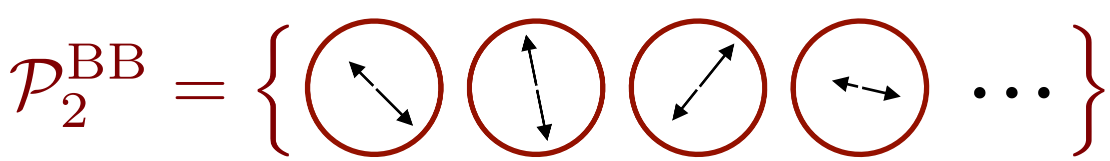
E.g. Thrust

How dijet-like is an event?

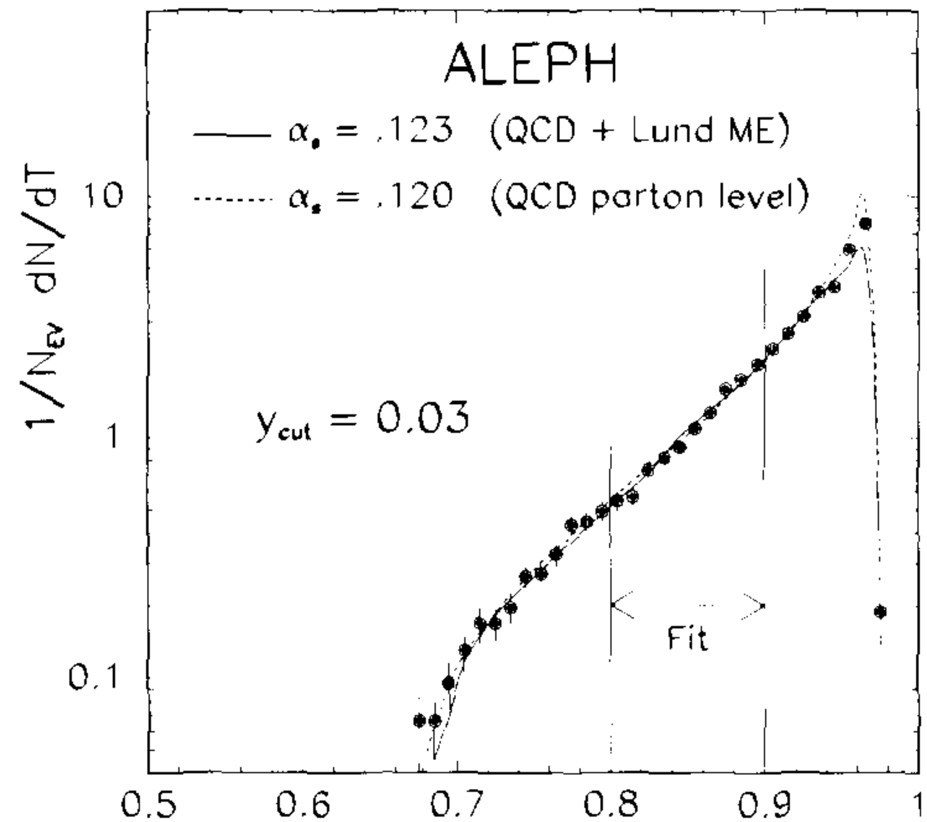
$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\mathrm{BB}}} \mathrm{EMD}_2(\mathcal{E}, \mathcal{E}')$$



All Back-to-Back Two Particle Configurations



(using β=2 EMD variant)



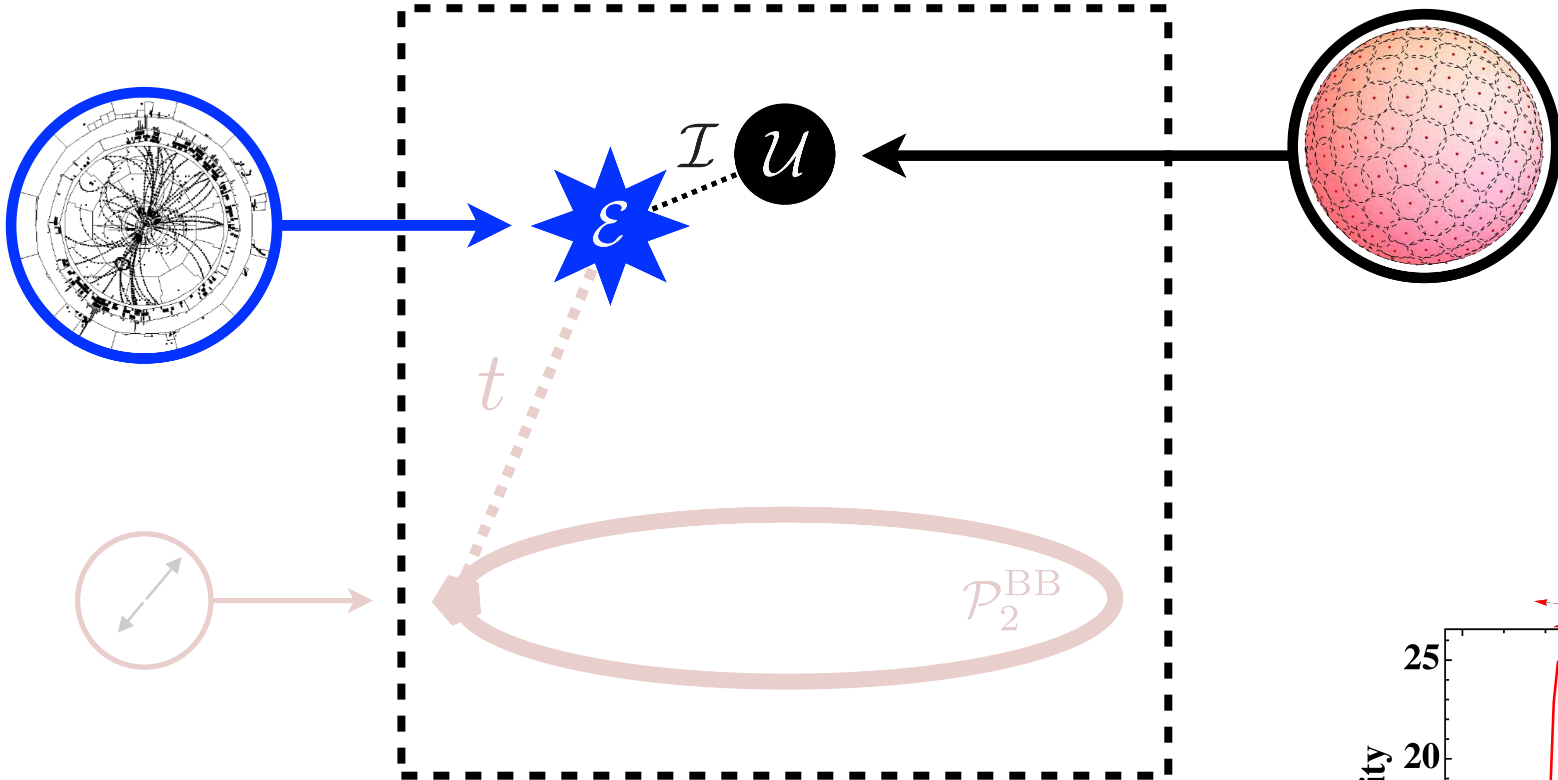
$$1 - \frac{t}{2E_{\mathrm{CM}}}$$

cf. $T(\mathcal{E}) = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_j |\vec{p}_j|}$

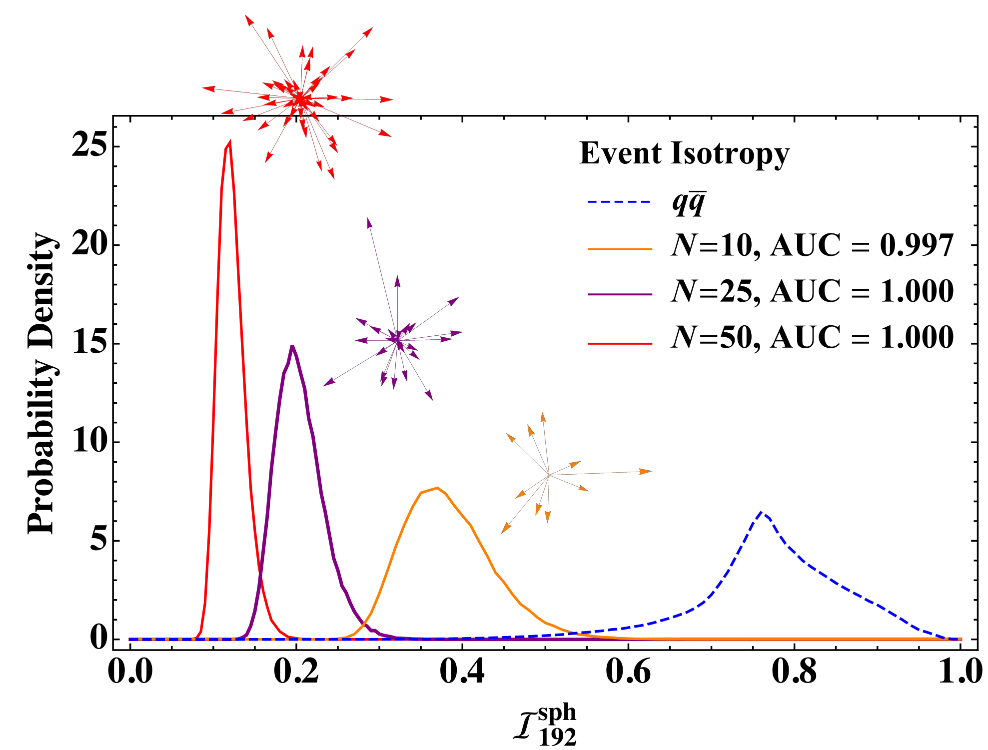
[Komiske, Metodiev, JDT, arXiv 2020]
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

New! Event Isotropy

How isotropic is an event?



$$\mathcal{I}(\mathcal{E}) = \mathrm{EMD}(\mathcal{E}, \mathcal{U})$$

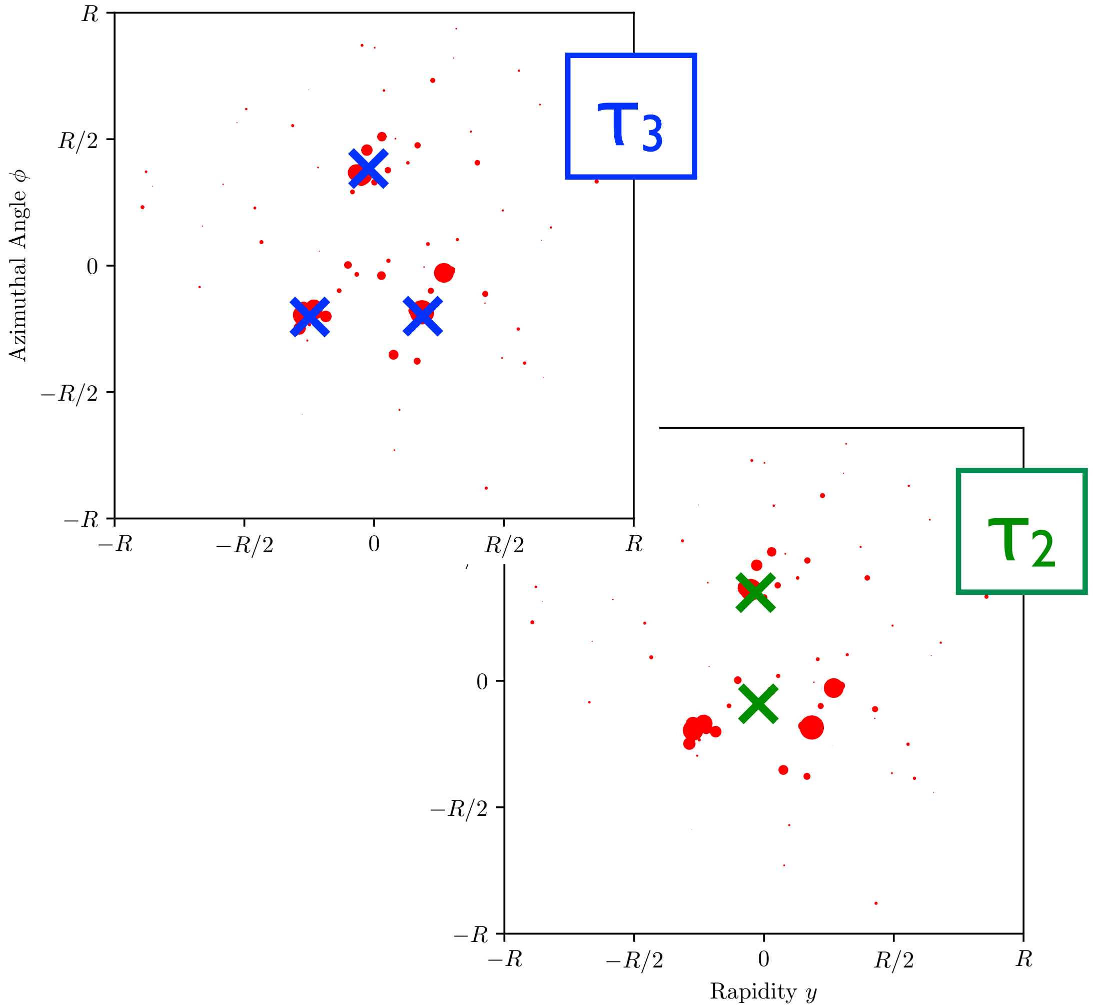


[Cesarotti, JDT, arXiv 2020]

N-subjettiness

Ubiquitous jet substructure observable used for almost a decade…

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_i E_i \min\{\theta_{1,i}, \theta_{2,i}, \ldots, \theta_{N,i}\}$$

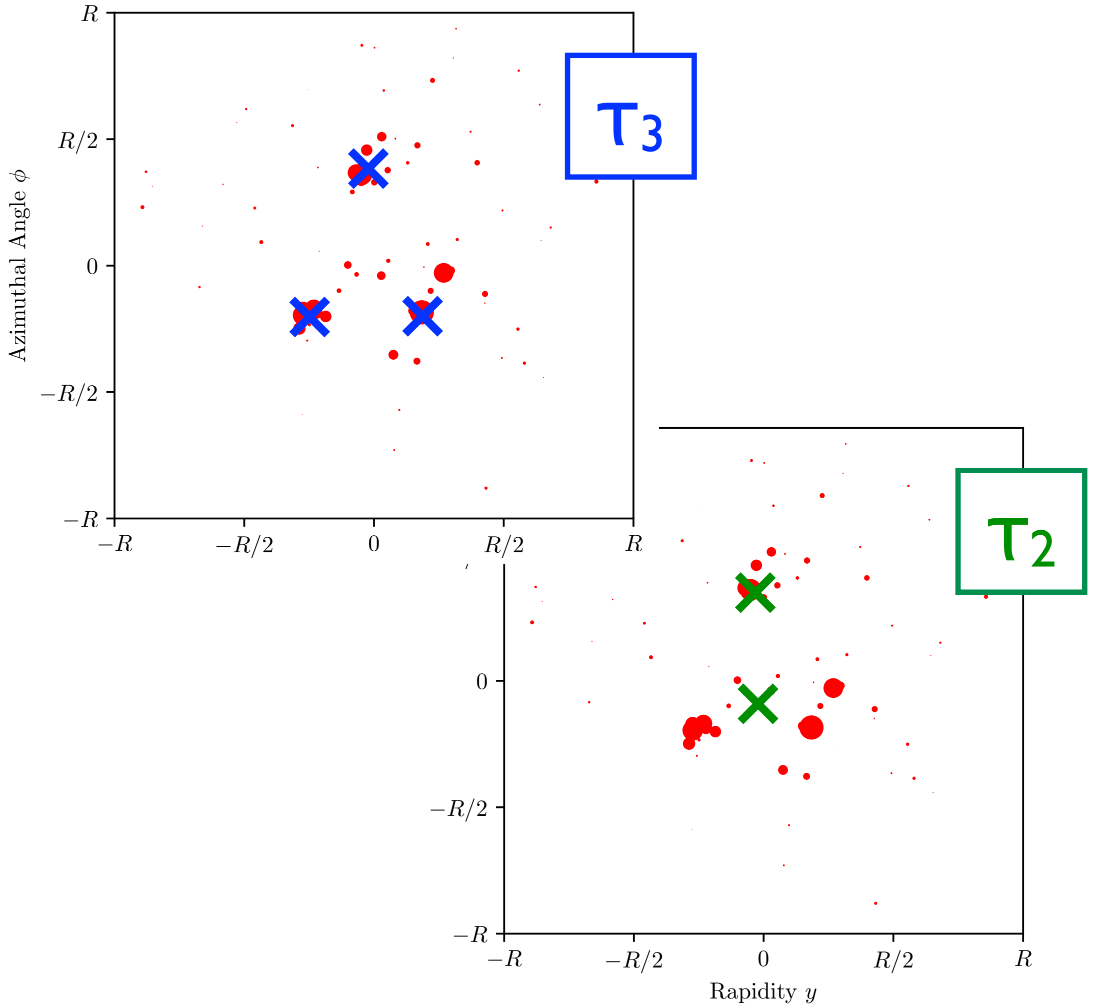


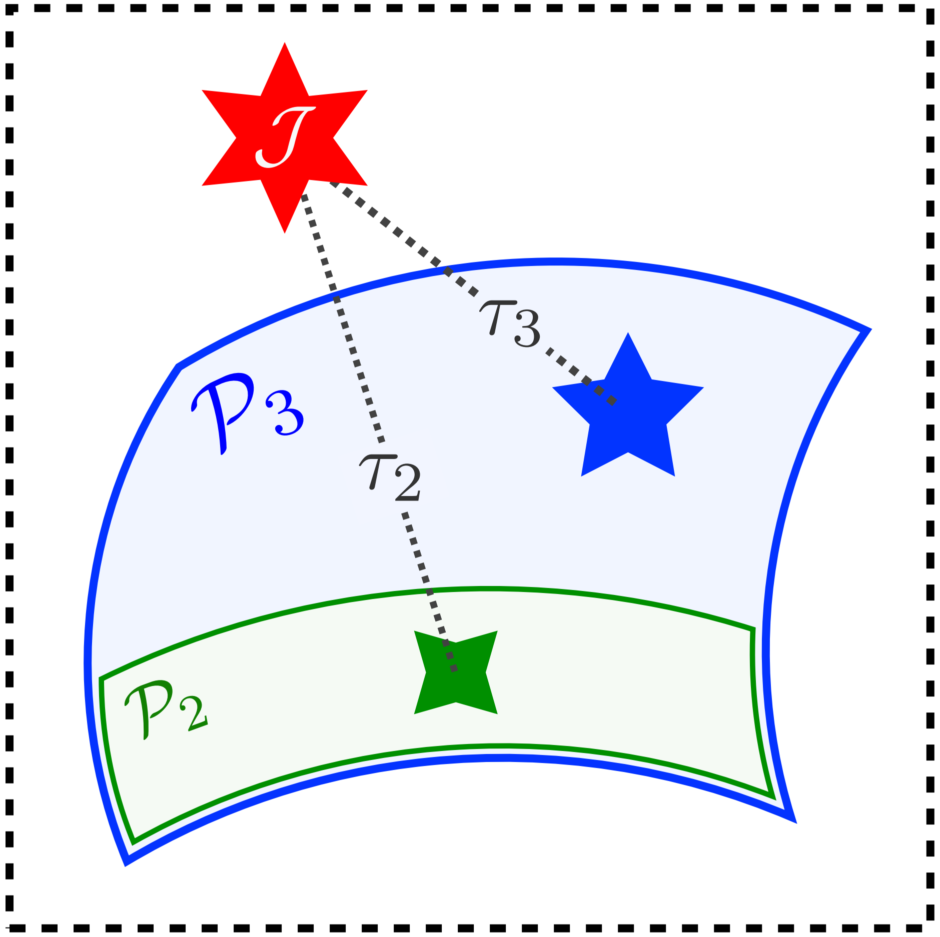
[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
based on Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

N-subjettiness = Point to Manifold EMD

…is secretly an optimal transport problem

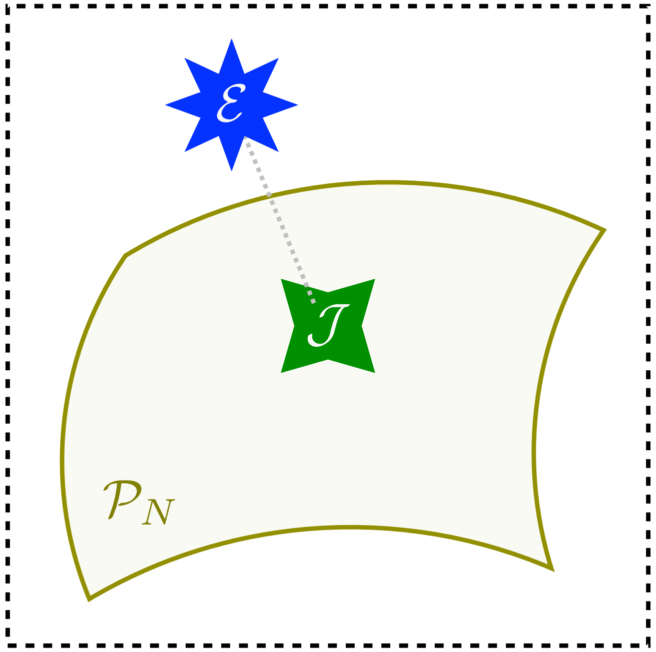
$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \mathrm{EMD}(\mathcal{J}, \mathcal{J}')$$





[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
rephrased in the language of Komiske, Metodiev, JDT, PRL 2019]

More Fun with N-particle Manifolds



N-jettiness

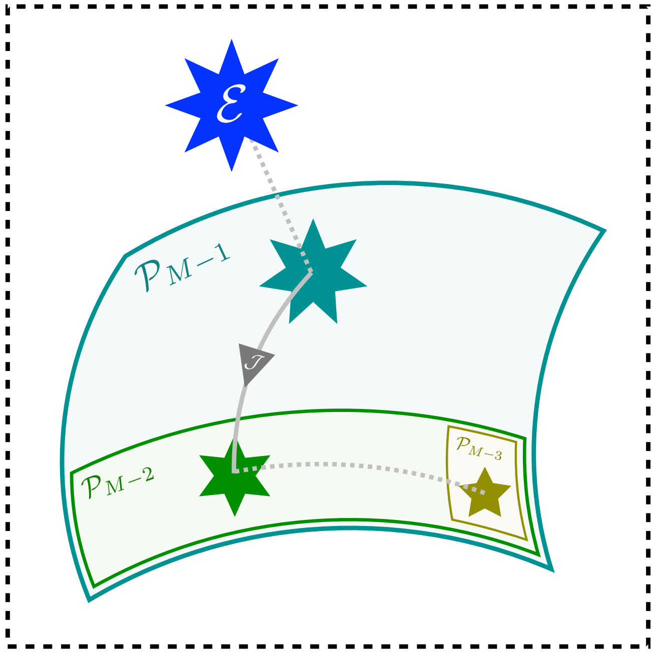
Distance of closest approach to N-particle manifold

[Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

Exclusive Cone Jet Finding

Point of closest approach on N-particle manifold

[Stewart, Tackmann, JDT, Vermilion, Wilkason, JHEP 2015]



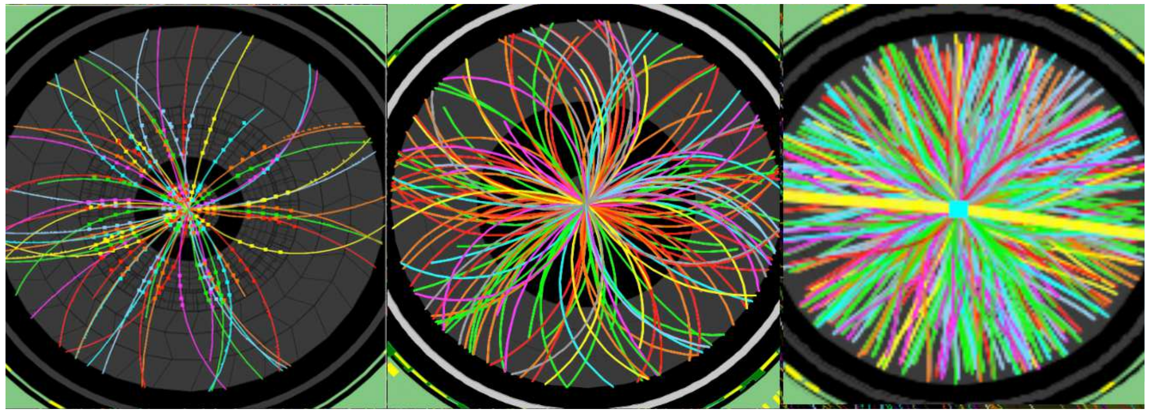
Sequential Jet Recombination

Iteratively stepping between various N-particle manifolds

[Catani, Dokshitzer, Seymour, Webber, NPB 1993; Ellis, Soper, PRD 1993]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Wobisch, Wengler, arXiv 1999]
[Butterworth, Couchman, Cox, Waugh, CPC 2003; Larkoski, Neill, JDT, JHEP 2014]

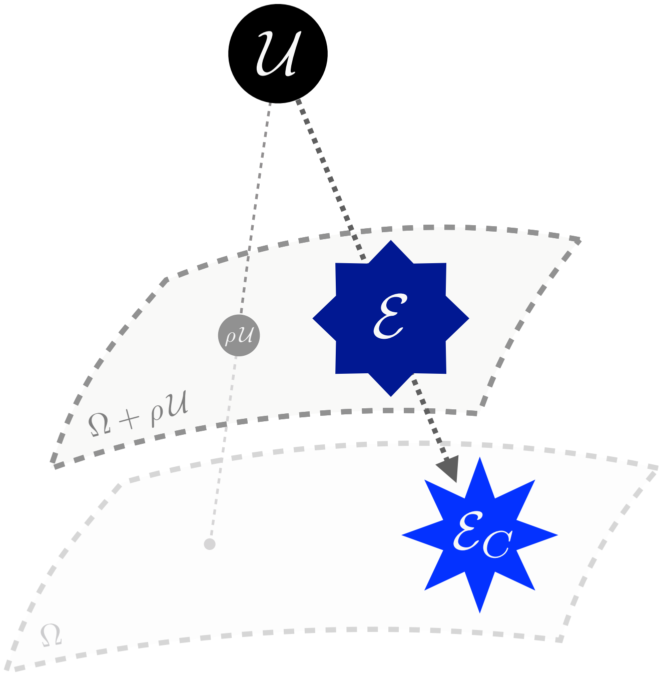
[Komiske, Metodiev, JDT, arXiv 2020]

Pileup Mitigation



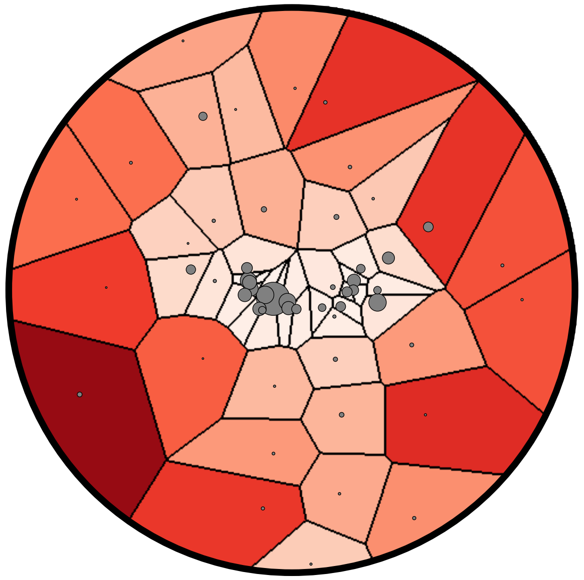
[see review in Soyez, PR 2019]

Uniform event contamination from overlapping proton-proton collisions



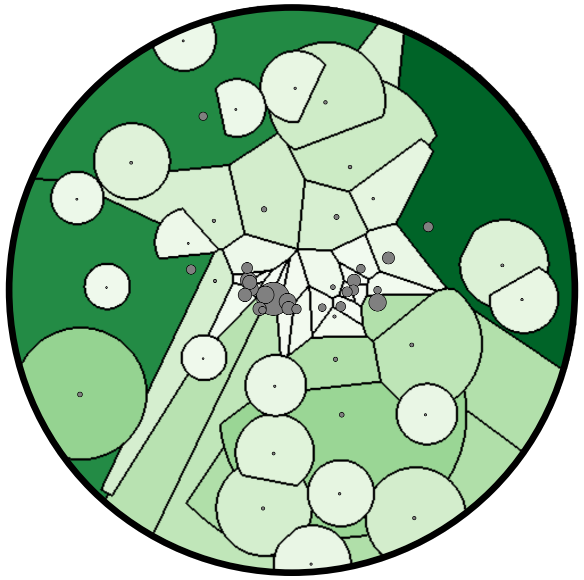
Pileup Mitigation:
"Move away" from uniform event

Voronoi



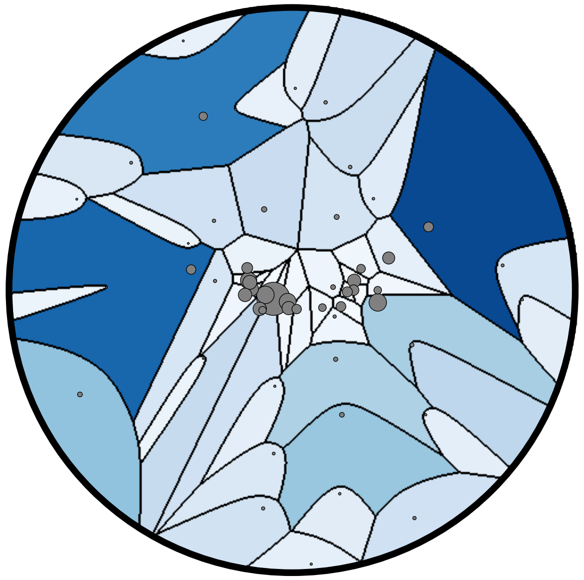
[Cacciari, Salam, Soyez, JHEP 2008]

Constituent Subtraction



[Berta, Spousta, Miller, Leitner, JHEP 2014]

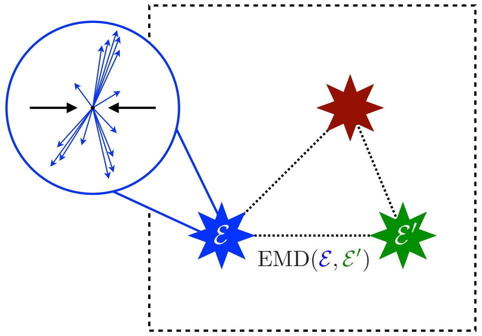
Apollonius



[Komiske, Metodiev, JDT, arXiv 2020]

Pause

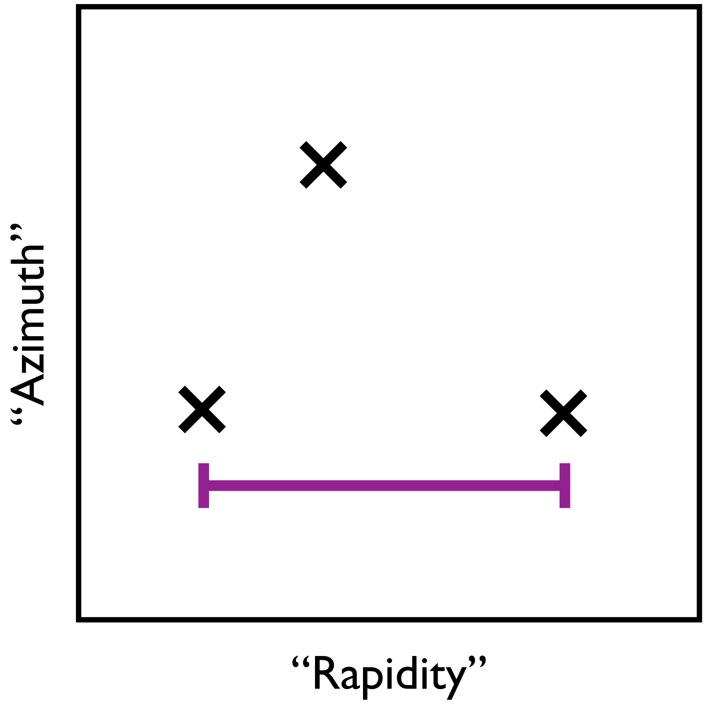
What can be Geometrized?



IRC Safety, Observables,
Jet Algorithms, Pileup Mitigation

How far down does this rabbit hole go?

Direction Space



× = Direction

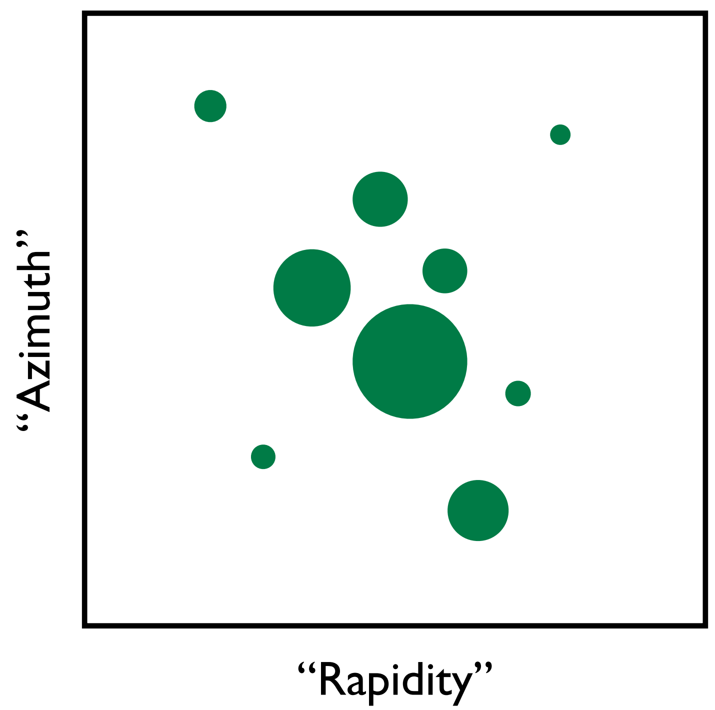
$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

= Angular Distance

$$\theta_{ij} = \sqrt{2 n_i^\mu n_{j\mu}}$$

(for massless particles)

Direction Space Distribution



= Weighted Direction

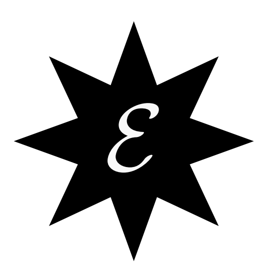
$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$w_i = E_i$$

= Angular Distance

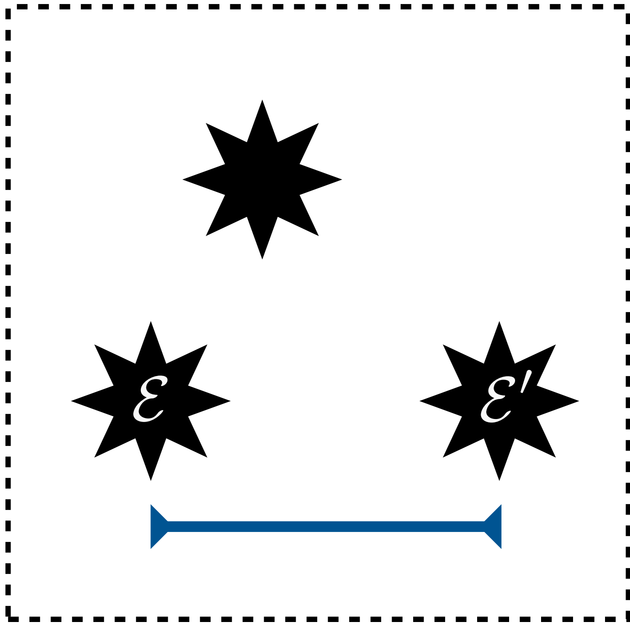
$$\theta_{ij} = \sqrt{2 n_i^\mu n_{j\mu}}$$

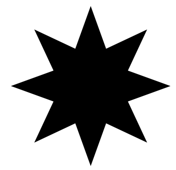
(for massless particles)



= Event

Event Space



 = Event

$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$

 = EMD

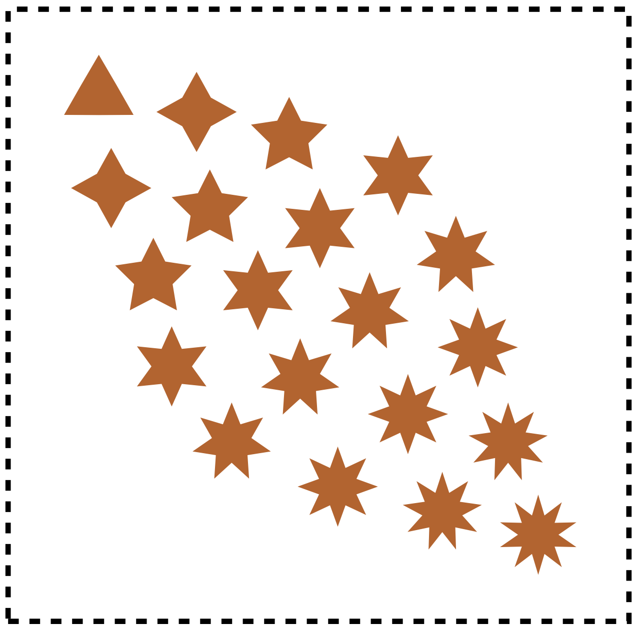
Energy
Mover's Distance

$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

(for equal total energy)

[Komiske, Metodiev, JDT, PRL 2019]

Event Space Distribution



= Weighted Event $\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$

$w_a = \sigma_a$

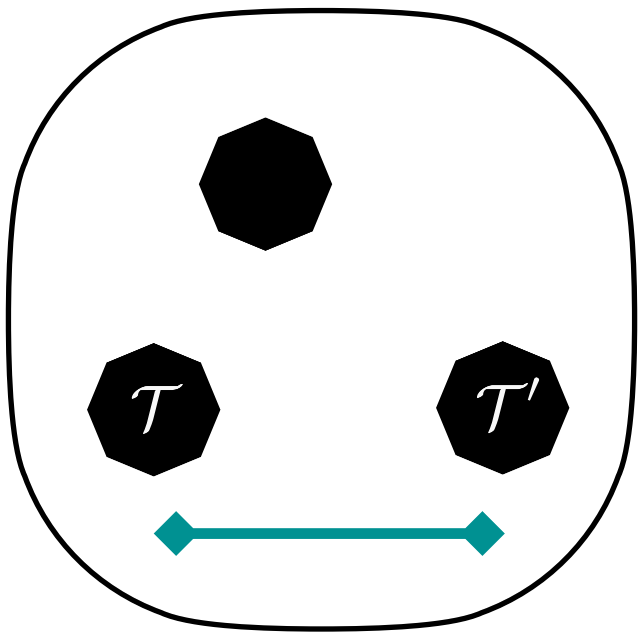
= EMD $\mathrm{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$

Energy
Mover's Distance

(for equal total energy)

$\mathcal{T}$ = Theory

Theory Space



= Theory

$$\mathcal{T}(\mathcal{E}) = \sum_a \sigma_a \, \delta(\mathcal{E} - \mathcal{E}_a)$$

= ΣMD

Cross-Section
Mover's Distance

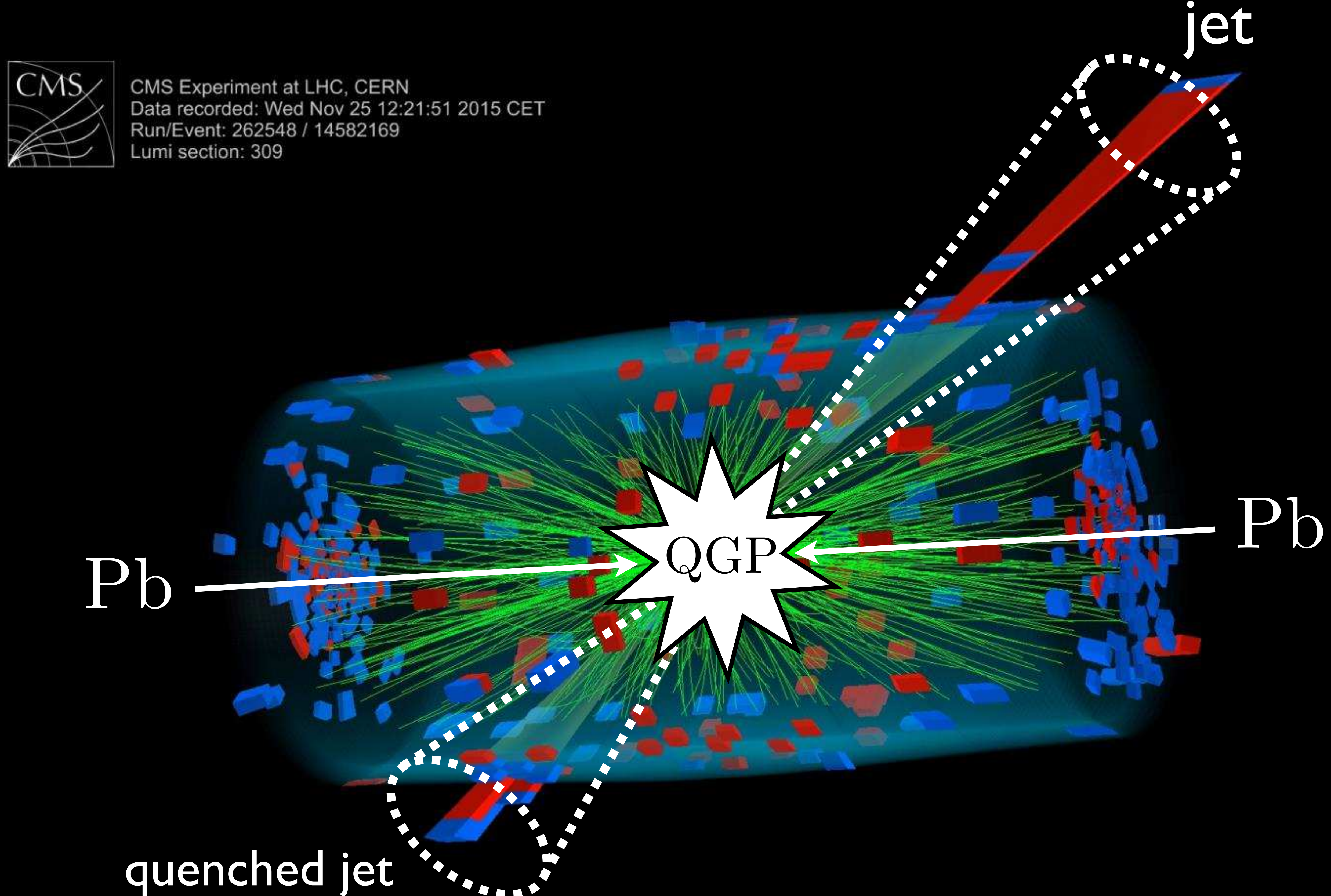
$$\Sigma\mathrm{MD}(\mathcal{T}, \mathcal{T}') = \min_{\{\mathcal{F}\}} \sum_a \sum_b \mathcal{F}_{ab} \mathrm{EMD}(\mathcal{E}_a, \mathcal{E}'_b)$$

(for equal total xsec)

A distance between theories!

(e.g. EMD : N-jettiness :: ΣMD : k-eventiness)

[Komiske, Metodiev, JDT, arXiv 2020]



CMS
CMS Experiment at LHC, CERN
Data recorded: Wed Nov 25 12:21:51 2015 CET
Run/Event: 262548 / 14582169
Lumi section: 309
jet
Pb
QGP
Pb
quenched jet

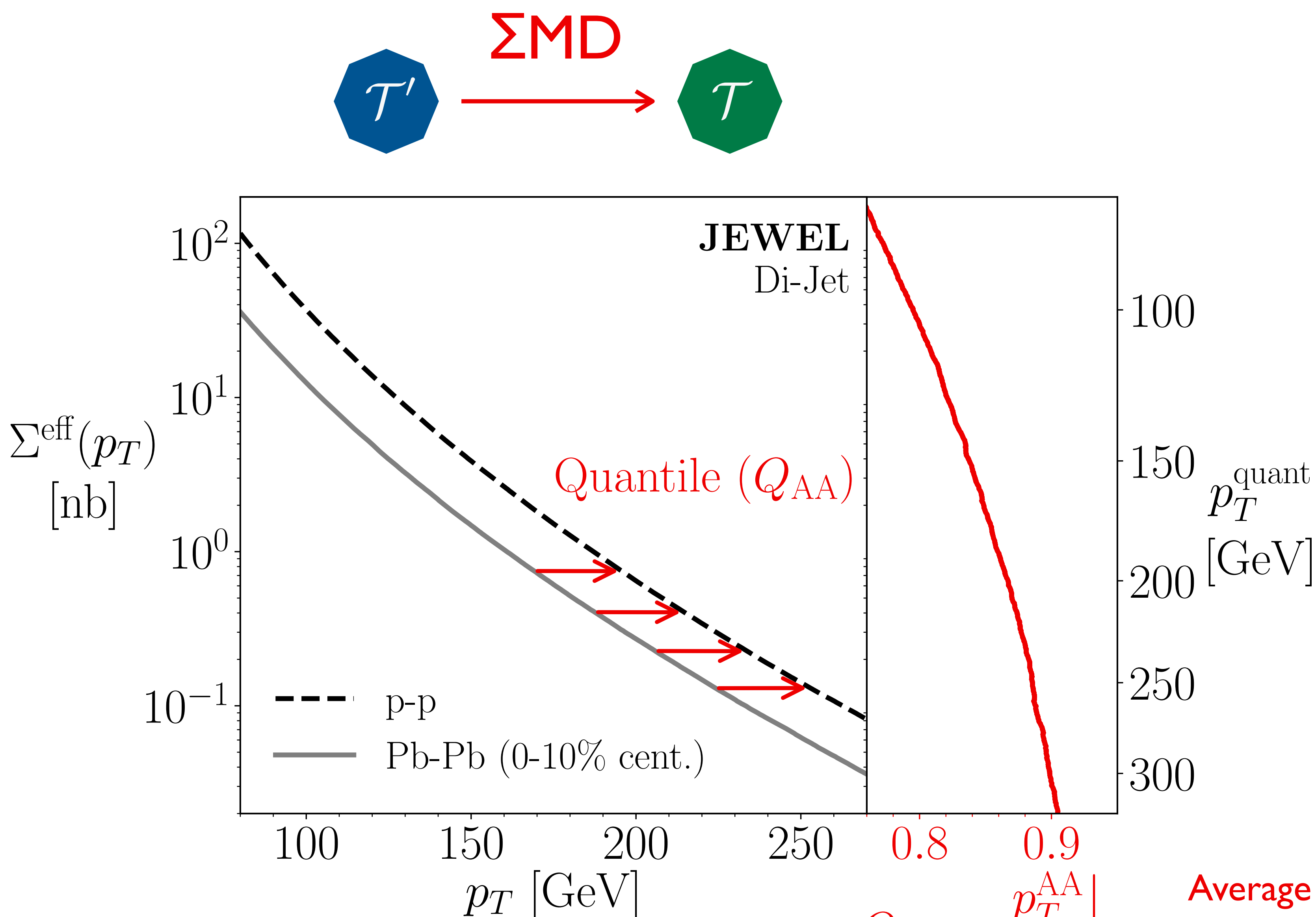
Theory Prime:
In-Medium QCD

Theory:
Vacuum QCD

Optimal transportation plan defines mapping between in-medium jets and vacuum jets!

Jet Quenching via Quantile Matching

Equivalent to following a geodesic in theory space (!)

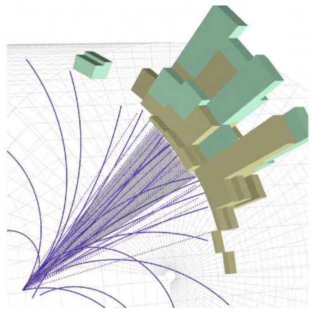


$$Q_{\mathrm{AA}} = \left.\frac{p_T^{\mathrm{AA}}}{p_T^{\mathrm{pp}}}\right|_{\Sigma^{\mathrm{eff}}}$$

Average p_T loss per jet

[Brewer, Milhano, JDT, PRL 2019]

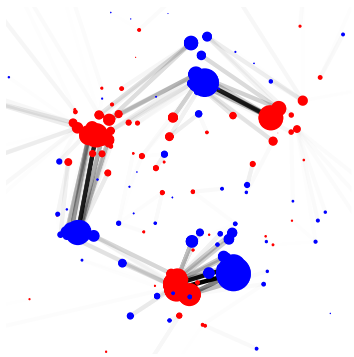
Summary



What is a Collider Event?

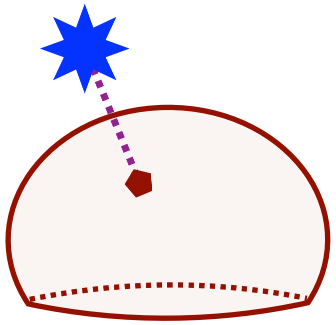
An unordered set of particles that describes the energy flow away from the collision point

(ask me about ML!)



When are Events Similar?

When they are close in the geometric space triangulated by the energy mover's distance



What can be Geometrized?

Many concepts/techniques in quantum field theory and collider physics from the last half century

Fin

Backup Slides

Aside: Machine Learning for Jets

"ML4Jets"
NYU, January 2020

Symmetry: $\mathcal{J} = \{\underbrace{\vec{p}_1, \vec{p}_2, \vec{p}_3, \ldots, \vec{p}_N}_{\text{Unordered, Variable Length Set (QM!)}}\}$

Safety: $\vec{p} = \{E, \hat{n}_x, \hat{n}_y, \hat{n}_z\}$

Energy weighting (QFT!)

⇩

Properly specified problem ⇨

Many (labeled) examples $\{\mathcal{J}\}$ ⇨

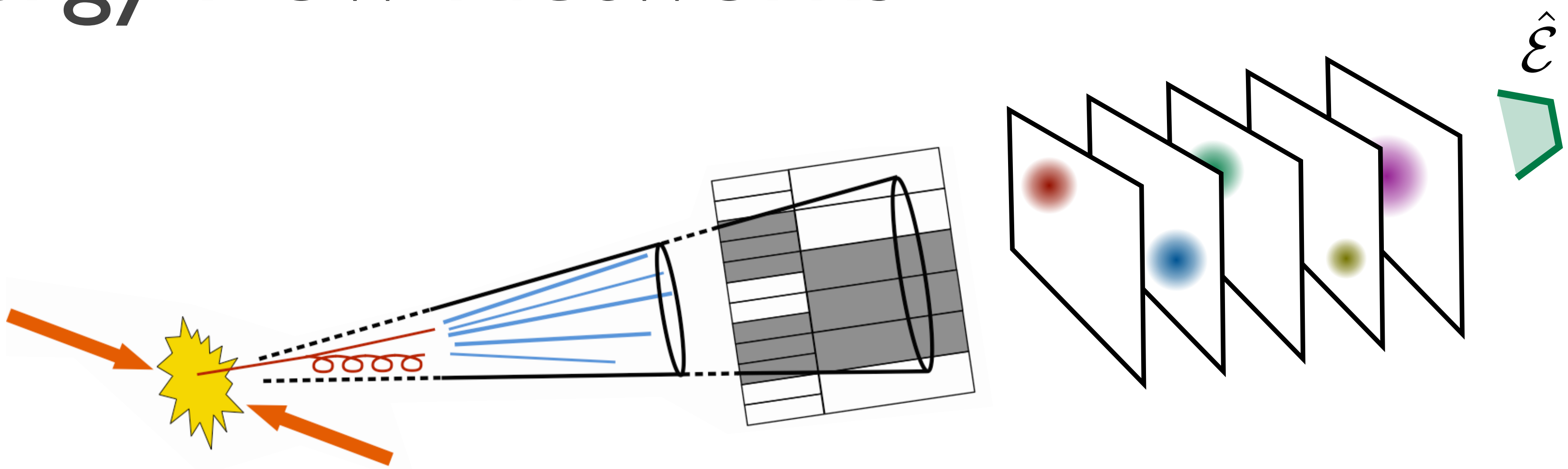
The Machine

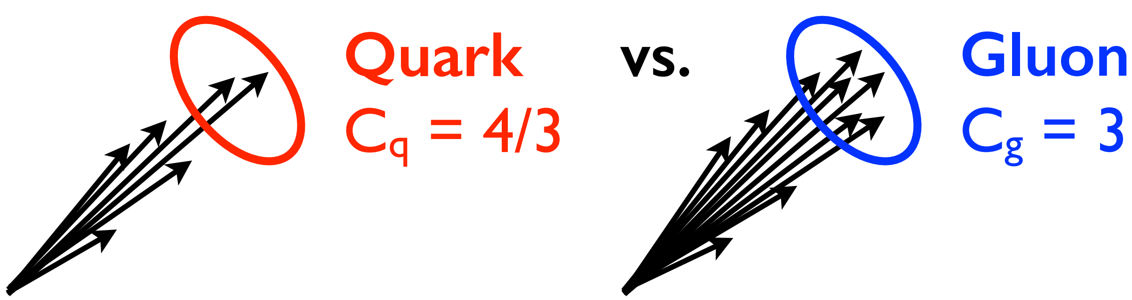
⇨ **Solution** $S(\mathcal{J})$

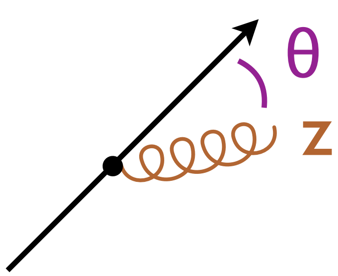
⇨ **Verification** $V(\mathcal{J})$

Check that answer is physically sensible

E.g. Energy Flow Networks



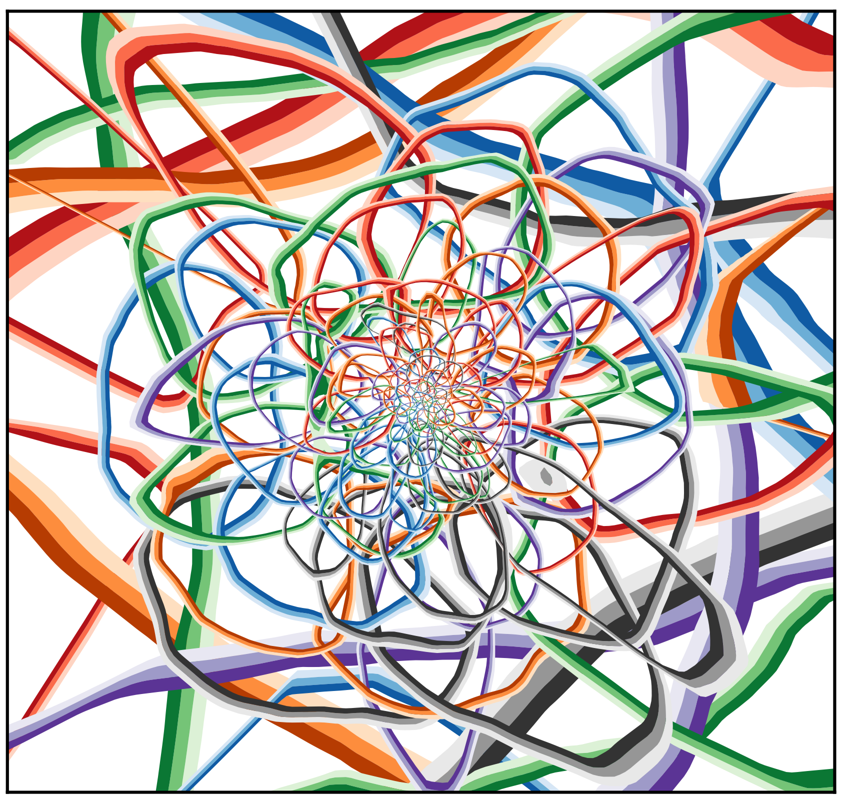




AP splitting probability:

$$\mathrm{d}P_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}z}{z}$$

Learning QCD singularities!



[Komiske, Metodiev, JDT, JHEP 2019; see also Komiske, Metodiev, JDT, JHEP 2018, PRD 2020; special case of Zaheer, Kottur, Ravanbakhsh, Poczos, Salakhutdinov, Smola, NIPS 2017]

Additional Travel Documentation