POSITIVE BOUNDARIES OF CONSISTENT EFTS

Working Group:

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An onslaught of activity in honing the EFT knife for precision study

- Particles of different spins and quantum numbers interact linearly with the SM and generate EFT effects at tree-level
- Heavy particles interact such that resulting EFT arises only at loop-level
- Strongly coupled scenarios where the Wilson coefficients controlled by symmetry
Scattering Amplitudes For All Masses and Spins
NCTS-TH-1714
References | BibTeX | LaTeX(X) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 90 records 50+

On-shell heavy particle effective theories
PTEP 2020, 28, 053C02

New Selection Rules from Angular Momentum Conservation

The electroweak effective field theory from on-shell amplitudes

2-, 3- and 4-Body Decays in the Constructive Standard Model

Coupled-channel approach in hadron-hadron scattering

The Rise of SMEFT On-shell Amplitudes
Published in JHEP 1809 (2018) 068

Standard Model Effective Field Theory from On-shell Amplitudes
Published in Phys. Rev. D100 (2019), 016003

Constructing effective field theories via their harmonics
Published in Phys. Rev. D100 (2019), 016005

Effective Field Theory Amplitudes the On-Shell Way: Scalar and Vector Couplings to Gluons

Constructive standard model
• Are there model independent constraints on the EFT itself?
• Through the EFT, can we say anything about the new physics standard model couplings?
At the origin, we find our effective theory description

\[
\int dx^4 \frac{1}{2} \phi \Box \phi + a_0 \phi^4 + a_1 (\partial \phi)^2 \phi^2 + a_2 (\partial \phi)^4 + \cdots
\]

We have a polynomial at the origin, with the Taylor coefficients identified with the Wilson coefficients of the EFT

\[
M^{IR}(s, t) = \{\text{massless poles}\} + \sum_{k,q} g_{k,q} s^{k-q} t^q ,
\]
A **Unitary** theory in the UV tell us that $M(s,t) \leq s^2$ at large $s$.

\[ I = \frac{i}{2\pi} \int_{\infty} ds \frac{ds}{s^{n+1}} M(s,0) = 0 \] for $n>1$

The vanishing of the contour tell us that the Taylor coefficients of the EFT is completely controlled by the discontinuity.

\[ M(s,0) = g_0 + g_1 s + g_2 s^2 + g_3 s^3 \cdots \]

\[ g_n = \int_{s_0}^\infty \frac{Dis[M(s,0)]}{s^{n+1}} = \int_{s_0}^\infty \frac{\sigma}{s^n} > 0 \]
All coefficients must be definite positive! This puts constraints on the EFT coupling.

Dimension eight operators for neutral diboson processes

\[
\mathcal{O}_{\psi_B}^{(8)} = -\frac{1}{4} \left( i\bar{\psi} \gamma^\rho D^\nu \psi + \text{h.c.} \right) B_{\mu\nu} B^\mu_{\rho}
\]

\[
\mathcal{O}_{\psi_W}^{(8)} = -\frac{1}{4} \left( i\bar{\psi} \gamma^\rho D^\nu \psi + \text{h.c.} \right) W^a_{\mu\nu} W^a_{\rho}
\]

\[
\mathcal{O}_{\psi_H}^{(8)} = \frac{1}{2} \left( i\bar{\psi} \gamma^{\mu} D^\nu \psi + \text{h.c.} \right) D_\mu H^\dagger D_\nu H
\]

\[
A_{QZ_L\to QZ_L}(s, t = 0) = \left( c_{QH}^{(8)} \mp \hat{c}_{QH}^{(8)} \right) \frac{s^2}{\Lambda^4},
\]

\[
A_{QV\to QV}(s, t = 0) = c_{QV}^{(8)} \pm \frac{s^2}{\Lambda^4}.
\]
All coefficients must be definite positive! This puts constraints on the EFT coupling

Quartic gauge boson couplings

\[ O_{S,0} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi] \]
\[ O_{S,1} = [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi] \]
\[ O_{S,2} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi] \]
\[ O_{M,0} = \text{Tr} \left[ \hat{W}_{\mu \nu} \hat{W}^{\mu \nu} \right] \times [(D_\beta \Phi)^\dagger D^\beta \Phi] \]
\[ O_{M,1} = \text{Tr} \left[ \hat{W}_{\mu \nu} \hat{W}^{\nu \beta} \right] \times [(D_\beta \Phi)^\dagger D^\mu \Phi] \]
\[ O_{M,2} = \hat{B}_{\mu \nu} \hat{B}^{\mu \nu} \times [(D_\beta \Phi)^\dagger D^\beta \Phi] \]
\[ O_{M,3} = \hat{B}_{\mu \nu} \hat{B}^{\nu \beta} \times [(D_\beta \Phi)^\dagger D^\mu \Phi] \]
\[ O_{M,4} = (D_\mu \Phi)^\dagger \hat{W}_{\beta \nu} D^\mu \Phi \times \hat{B}^{\beta \nu} \]
\[ O_{M,5} = (D_\mu \Phi)^\dagger \hat{W}_{\beta \nu} D^\nu \Phi \times \hat{B}^{\beta \mu} \]
\[ O_{M,7} = (D_\mu \Phi)^\dagger \hat{W}_{\beta \nu} \hat{W}^{\beta \mu} D^\nu \Phi \]

\[ O_{T,0} = \text{Tr} \left[ \hat{W}_{\mu \nu} \hat{W}^{\mu \nu} \right] \times \text{Tr} \left[ \hat{W}_{\alpha \beta} \hat{W}^{\alpha \beta} \right] \]
\[ O_{T,1} = \text{Tr} \left[ \hat{W}_{\alpha \nu} \hat{W}^{\mu \beta} \right] \times \text{Tr} \left[ \hat{W}_{\mu \beta} \hat{W}^{\alpha \nu} \right] \]
\[ O_{T,2} = \text{Tr} \left[ \hat{W}_{\alpha \mu} \hat{W}^{\mu \beta} \right] \times \text{Tr} \left[ \hat{W}_{\beta \nu} \hat{W}^{\nu \alpha} \right] \]
\[ O_{T,5} = \text{Tr} \left[ \hat{W}_{\mu \nu} \hat{W}^{\mu \nu} \right] \times \hat{B}_{\alpha \beta} \hat{B}^{\alpha \beta} \]
\[ O_{T,6} = \text{Tr} \left[ \hat{W}_{\alpha \nu} \hat{W}^{\mu \beta} \right] \times \hat{B}_{\mu \beta} \hat{B}^{\alpha \nu} \]
\[ O_{T,7} = \text{Tr} \left[ \hat{W}_{\alpha \mu} \hat{W}^{\mu \beta} \right] \times \hat{B}_{\beta \nu} \hat{B}^{\nu \alpha} \]
\[ O_{T,8} = \hat{B}_{\mu \nu} \hat{B}^{\mu \nu} \times \hat{B}_{\alpha \beta} \hat{B}^{\alpha \beta} \]
\[ O_{T,9} = \hat{B}_{\alpha \mu} \hat{B}^{\mu \beta} \times \hat{B}_{\beta \nu} \hat{B}^{\nu \alpha} \]
All coefficients must be definite positive! This puts constraints on the EFT coupling

Quartic gauge boson couplings

Chen Zen, Shuang-Yong Zhou
1808.00010
$g_1 > 0, \; g_2 > 0, \; g_3 > 0, \; g_4 > 0, \ldots$

But this is just the tip of the "iceberg"
A Unitary theory in the UV tells us that $M(s,t) \leq s^2$ at large $s$.

$$I = \frac{i}{2\pi} \int_{\infty}^\infty \frac{ds}{s^{n+1}} M(s,t)$$

$= 0$ for $n > 1$

The vanishing of the contour tells us that the Taylor coefficients of the EFT is completely controlled by the residue and discontinuity (Arkani-hamed, Huang, Huang)

$$M^{IR}(s,t) = \text{massless poles} + \sum_{k,q} g_{k,q} s^{k-q} t^q,$$

$$\sum_q g_{k,q} t^q = \left( \sum_a \frac{p_a G_a^\alpha (1 + \frac{t}{m_a^2})}{(m_a^2)^{k-q+1}} + \sum_b \int ds' p_{b,\ell}(s') \frac{G^{\alpha}(1 + \frac{t}{s'})}{(s')^{k-q+1}} + \{u\} \right)$$
What about higher order in $t$?

\[
\sum_q g_{k,q} t^q = \left( \sum_a p_a G_{\ell_a}^\alpha \frac{1 + 2 \frac{t}{m_a^2}}{(m_a^2)^{k-q+1}} + \sum_b \int ds' p_{b,\ell}(s') \frac{G_{\ell}^\alpha(1 + 2 \frac{t}{s'})}{(s')^{k-q+1}} + \{u\} \right)
\]

\[
g_{k,q} = \sum_a p_a \frac{2^q u_{\ell_a,k,q}^\alpha}{(m_a^2)^{k+1}} \quad p_a > 0
\]

\[
G_{\ell}^\alpha(1 + \delta) = \sum_q v_{\ell,q}^\alpha \delta^q.
\]

The couplings must sit inside the convex hull of cyclic polytope!
Through the EFT, can we say anything about the new physics standard model couplings?

Yes! Through the EFT of an EFT
Now let’s consider the EFT for generic QFTs involving gravity. At low energies we have

\[ \mathcal{L} = -\frac{1}{4} F^2 + \sum_{i=0} c_i D^{2i} F^4 \]

with the coefficients parametrized as:

\[ c_i \left( \frac{qg}{m}, M_{pl} \right) \]

The Wilson coefficients are functions of the charge, gauge coupling and mass. It is more natural to parameterize in dimensionless ratio

\[ z = \frac{gq M_{pl}}{m} \]

with the coefficients parametrized as:

\[ c_i \left( \frac{qg}{m}, M_{pl} \right) = \frac{1}{M_{pl}^4} \left( \alpha_i z^0 + \beta_i z^2 + \gamma_i z^4 \right) \frac{1}{m^{2i}} \]

\[ \alpha \beta \gamma \] are calculable coefficients. Could unitarity constrain \( z \)??
Expanding in $1/m$, in the forward limit we have the EFT description

$$M_4(s, 0)|_{s/m^2 \ll 1} = c_0 \frac{s^3/2}{M_{pl}^2} + \sum_{i,n} \frac{(c_{n,4}z_i^4 + c_{n,2}z_i^2 + c_{n,0})}{m_{i}^{2n-3}M_{pl}^2} s^n$$

(10)

With exact expression for EFT Taylor coefficients to all order in derivatives

$$c_{n,4} = \frac{(n^2 + n + 1)}{2^{2n+4}(n+2)(n+1)(2n+1)\pi}$$

if $n = 2$

$$c_{n,2} = \frac{1}{3840\pi} \frac{(n+1)}{2^{2n+7}(2n+3)(2n+5)\pi}$$

if $n > 2$

$$c_{n,0} = \frac{1}{1280\pi} \frac{(n/2+1)(n+1)}{2^{2n+7}(2n+1)(2n+3)(2n+5)\pi}$$

if $n = 2$

$$c_{n,0} = \frac{1}{1920\pi} \frac{(n+1)}{2^{2n+9}(2n+1)(2n+3)(2n+5)\pi}$$

if $n > 2$

Scalars

Fermions

We can now analyze the sign of Det[$K_n$]
We can now analyze the sign of $\text{Det}[K_n]$.

In general $\text{Det}[K_n]>0$, imposes $0<z<a$ and $b<z$. For example the positivity of $\text{det}[K_{206}]$ imposes $0 <|z| < 1.02$, or $|z| > 34.82$.

The asymptotic behavior of $(a,b)$ is very different.
However, in three-dimensions, we have the special feature that the Wilson coefficients are dominated by the largest $z$ and the smallest $m$!

\[
c_i \left( \frac{q q}{m}, M_{pl} \right) = \frac{1}{m M_{pl}^2} \left( \alpha_i z^0 + \beta_i z^2 + \gamma_i z^4 \right) \frac{1}{m_{2i}}
\]

We can consider adding other neutral states, with $\beta = m_e/m_0$

\[
g_n = \frac{c_{n,4} z^4 + c_{n,2} z^2 + (1 + \beta^{2n-3}) c_{n,0}}{m_{e_n}^{2n-3} M_{pl}^2}
\]

The presence of light neutral states alleviate the tension of having large $z$.
The presence of light neutral states alleviate the tension of having large $z$.

In general we would like to understand the analytic behavior of

$$0 < |z| \leq a_{asym} (\beta).$$

$$a_{asym} (0) = 1.01, \quad a_{asym} \left( \frac{1}{4} \right) = 1.14,$$
$$a_{asym} \left( \frac{5}{4} \right) = 1.52, \quad a_{asym} \left( \frac{9}{4} \right) = 1.84.$$
GENERALIZATION TO 4D

With extended Higgs sector, the new physics (strongly coupled) induce with suitable cut off (set by the Landau poles!)

\[ H^\dagger \psi \bar{\psi} DH \]

We will get bounds on Yukawa couplings to standard model fermion mass!
HAPPY SILVER JUBILEE