Quark mixing matrix from mass matrices with the maximum number of texture zeros

Work in collaboration with E. Rojas, based on Phys.Rev.D 104 (2021) 7, 075009, e-Print: 1511.08858 [hep-ph].

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June 16, 2022

COMHEP 6th ComHEP: Colombian Meeting on High Energy Physics

29 de noviembre de 2021 to 3 de diciembre de 2021

CKM mixings from mass matrices with five texture zeros

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(Received 19 May 2021; revised 15 September 2021; accepted 17 September 2021; published 7 October 2021)

In this work we carry out an exhaustive study to find quark mass matrices in the Standard Model (SM), with the maximum number of texture zeros consistent with the experimental data. We found four viable configurations of five texture zeros that adjust the quark masses, the mixing angles and the *CP* violation phase, with deviations below 1σ level respect to the current SM best fit values. One of the most important aspects of this work is an economic procedure to find the texture zeros: we resort to the weak basis transformation method, which, as we will show, exhaustively search every possible configuration. We report various leading order relations between the mixing angles and the quark masses for each case.

DOI: 10.1103/PhysRevD.104.075009

I. INTRODUCTION

In the Standard Model (SM), the quark mass matrices come from the interaction between the Higgs boson and the SM fermions. After the spontaneous breaking of the SM gauge symmetry we obtain

$$-\mathcal{L}_M = \bar{u}_R M_u u_L + \bar{d}_R M_d d_L + \text{H.c.}, \tag{1.1}$$

where M_u and M_d are arbitrary, 3×3 quark mass matrices containing thirty-six (36) real parameters, which cannot be fully determined from the ten (10) physical observables that they must account for: six (6) quark masses, three (3) flavor mixing angles, and one (1) charge-parity (*CP*) violating phase. However, in models like the SM (or its extensions) where the right fields are singlets under the gauge group, it is always possible to choose a suitable basis for the right quarks, such that by using *the polar decomposition theorem* of the matrix algebra, the mass matrices of type "up" and "down" became Hermitian [1–6].

$$M_u^\dagger = M_u, \quad \text{and} \quad M_d^\dagger = M_d. \tag{1.2}$$

Additionally, for Hermitian quark mass matrices, you can make a unitary transformation acting simultaneously on the up-type and down-type quark mass matrices, leaving the

$$M_u \to M_u' = U^\dagger M_u U, \qquad M_d \to M_d' = U^\dagger M_d U, \tag{1.3}$$

where U is an arbitrary unitary matrix that preserves the hermiticity of the mass matrices and leaving the physical quantities invariant, in particular, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. This common unitary transformation applied to M_u and M_d, in Eq. (1.3), is known as a "weak basis" (WB) transformation [1,7–10]. As it was shown in [3,11], for a given set of quark masses, mixing angles and the *CP*-violating phase, all the mass matrices consistent with these experimental values are unitarily equivalent. This result can be used to calculate the maximum number of texture zeros, since it guarantees that by using WB transformations it is possible to reach all physical and nonphysical zeros consistent with the data [3,7]. Through a WB transformation, it is possible to rewrite the quark mass matrices as follows [3,7,11,12]:

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix},$$
$$M_d = V D_d V^\dagger, \tag{1.4a}$$

or

Overview

Abstract

- We carry out an exhaustive study to find quark mass matrices in the Standard Model (SM), with the maximum number of texture zeros consistent with the experimental data.

- We found four viable configurations of five texture zeros that adjust the quark masses, the mixing angles and the CP violation phase, with deviations below 1 level respect to the current SM best fit values.

- One of the most important aspects of this work is an economic procedure to find the texture zeros: we resort to the weak basis transformation method, which, as we will show, exhaustively search every possible configuration.

- We report various leading order relations between the mixing angles and the quark masses for each case

Introduction

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- It is always possible to choose a suitable basis for the right quarks, such that by using the *the polar decomposition theorem* of the matrix algebra, the mass matrices of type "up" and "down" became hermitian.

$$M_u^\dagger = M_u, \quad \text{and} \quad M_d^\dagger = M_d.$$

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or

$$M_u = V^\dagger D_u V,$$

$$M_d = D_d = \begin{pmatrix} \lambda_{1d} & 0 & 0 \\ 0 & \lambda_{2d} & 0 \\ 0 & 0 & \lambda_{3d} \end{pmatrix},$$

where $V = U_u^\dagger U_d$ is the CKM mixing matrix, U_u and U_d are the diagonalization matrices for the mass matrices M_u and M_d, respectively. The parameters λ_{iq} $(i = 1, 2, 3)$ are the quark mass matrix eigenvalues for up-type $(q = u)$ and down-type $(q = d)$ quarks. λ_{iq} can be positive or negative and obey the hierarchy

$$|\lambda_{1q}| \ll |\lambda_{2q}| \ll |\lambda_{3q}|.$$

Introduction

- Weinberg and Fritzsch introduced texture-zeros into the mass matrices with a dual purpose, first of all, to obtain self-consistent relationships between the quark masses and the flavor mixing parameters that can be experimentally verified.

- On the other hand, the discrete (or continuous) flavor symmetries hidden in such textures may finally provide clues on the origin of the energy scales in the quark sector of the SM as residual symmetries of a more fundamental symmetry at high energies.

Introduction

- Hermitian quark mass matrices with six texture zeros were introduced in what is currently known as the Fritzsch type, where the mass matrices, M_u, and M_d, have the same texture ("up-down" parallel) each with three zeros. This type of ansatz was ruled out due to the large value of the mass of the top quark, since that for this case the CKM element $|V_{cb}|$ is in tension with the experimental data. Furthermore, for reasonable values of the current quark masses m_u and m_c, the expected magnitude for $|V_{ub}/V_{cb}| = \sqrt{m_u/m_c} \approx 0.05$ is too small in comparison with the experimental value ($|V_{ub}/V_{cb}|_{\text{exp.}} \approx 0.09$
- In this sense, one of the difficulties of working with texture zeros is keeping the predictions for V_{us} and V_{cd} right, and simultaneously reproducing the ratios V_{ub}/V_{cb} and V_{td}/V_{ts}, i.e.,

$$\left|\frac{V_{ub}}{V_{cb}}\right|_{\text{exp.}} = 0.0861 \pm 0.0027,$$
$$\left|\frac{V_{td}}{V_{ts}}\right|_{\text{exp.}} = 0.2107 \pm 0.0044.$$

Five-zero textures

TABLE I. Mass matrix patterns with three texture-zeros. We are considering two cases, depending on the number of zeros in the diagonal (one or two texture zeros). It is not necessary to include phases.

Permutation matrices	Pattern with two zeros on the diagonal $(p_i \quad M_q \quad p_i^T)$	Pattern with one zero on the diagonal $(p_i \quad M_q \quad p_i^T)$
$p_1 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert\xi_q\rvert & 0 \\ \lvert\xi_q\rvert & 0 & \lvert\beta_q\rvert \\ 0 & \lvert\beta_q\rvert & \alpha_q \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert\xi_q\rvert & 0 \\ \lvert\xi_q\rvert & \gamma_q & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_2 = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \lvert\xi_q\rvert \\ 0 & \alpha_q & \lvert\beta_q\rvert \\ \lvert\xi_q\rvert & \lvert\beta_q\rvert & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \lvert\xi_q\rvert \\ 0 & \alpha_q & 0 \\ \lvert\xi_q\rvert & 0 & \gamma_q \end{pmatrix}$
$p_3 = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & \lvert\beta_q\rvert & 0 \\ \lvert\beta_q\rvert & 0 & \lvert\xi_q\rvert \\ 0 & \lvert\xi_q\rvert & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & \gamma_q & \lvert\xi_q\rvert \\ 0 & \lvert\xi_q\rvert & 0 \end{pmatrix}$
$p_4 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert\xi_q\rvert & \lvert\beta_q\rvert \\ \lvert\xi_q\rvert & 0 & 0 \\ \lvert\beta_q\rvert & 0 & \alpha_q \end{pmatrix}$	$\begin{pmatrix} \lvert\gamma_q\rvert & \lvert\xi_q\rvert & 0 \\ \lvert\xi_q\rvert & 0 & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_5 = \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & \lvert\beta_q\rvert \\ 0 & 0 & \lvert\xi_q\rvert \\ \lvert\beta_q\rvert & \lvert\xi_q\rvert & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & 0 & \lvert\xi_q\rvert \\ 0 & \lvert\xi_q\rvert & \gamma_q \end{pmatrix}$
$p_6 = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert\beta_q\rvert & \lvert\xi_q\rvert \\ \lvert\beta_q\rvert & \alpha_q & 0 \\ \lvert\xi_q\rvert & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \gamma_q & 0 & \lvert\xi_q\rvert \\ 0 & \alpha_q & 0 \\ \lvert\xi_q\rvert & 0 & 0 \end{pmatrix}$

Five-zero textures

$$M'_u = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} = p_2 \cdot \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix} \cdot p_2^T ,$$

$$M'_d = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix} = p_2 \cdot \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} \cdot p_2^T ,$$

Texture-zero patterns

We consider the most general case of a symmetric mass matrix with two texture zeros

$$M_q = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & \gamma_q & |\beta_q| \\ 0 & |\beta_q| & \alpha_q \end{pmatrix},$$

The mass matrix M_q can be diagonalized using the transformation

$$U_q^\dagger M_q U_q = D_q = \begin{pmatrix} \lambda_{1q} & & \\ & \lambda_{2q} & \\ & & \lambda_{3q} \end{pmatrix},$$

Note that γ_q, $|\beta_q|$ and $|\xi_q|$ can be expressed in terms of α_q and the λ_{iq}'s.

$$\gamma_q = \lambda_{1q} + \lambda_{2q} + \lambda_{3q} - \alpha_q, \tag{1}$$

$$|\beta_q| = \sqrt{\frac{(\alpha_q - \lambda_{1q})(\alpha_q - \lambda_{2q})(\lambda_{3q} - \alpha_q)}{\alpha_q}}, \tag{2}$$

$$|\xi_q| = \sqrt{\frac{-\lambda_{1q}\lambda_{2q}\lambda_{3q}}{\alpha_q}}. \tag{3}$$

$$\text{If } \lambda_{1q} < 0, \lambda_{2q} > 0 \text{ and } \lambda_{3q} > 0 \implies |\lambda_{2q}| \leq \alpha_q \leq |\lambda_{3q}|$$
$$\text{If } \lambda_{1q} > 0, \lambda_{2q} < 0 \text{ and } \lambda_{3q} > 0 \implies |\lambda_{1q}| \leq \alpha_q \leq |\lambda_{3q}|.$$
$$\text{If } \lambda_{1q} > 0, \lambda_{2q} > 0 \text{ and } \lambda_{3q} < 0 \implies |\lambda_{1q}| \leq \alpha_q \leq |\lambda_{2q}|.$$

Texture-zero patterns

$$U_q = \begin{pmatrix} e^{i\theta_1}\frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{2q}\lambda_{3q}(\alpha_q-\lambda_{1q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & e^{i\theta_2}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}\lambda_{3q}(\lambda_{2q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{1q}\lambda_{2q}(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ -e^{i\theta_1}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}(\lambda_{1q}-\alpha_q)}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & e^{i\theta_2}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{2q})}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{3q}(\lambda_{3q}-\alpha_q)}{(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ e^{i\theta_1}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}(\alpha_q-\lambda_{2q})(\alpha_q-\lambda_{3q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{i\theta_2}\frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{2q}(\alpha_q-\lambda_{1q})(\lambda_{3q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\alpha_q-\lambda_{1q})(\alpha_q-\lambda_{2q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \end{pmatrix}$$

where we have included additional phases (non-physical) to adjust the CKM mixing matrix to the usual convention, as shown in the reference. It is not necessary to include a phase in the third column, as it can be absorbed by the remaining phases.

Texture-zero patterns

$$M'_u = U_u(D_u)U_u^\dagger = \begin{pmatrix} 0 & |\xi_u| & 0 \\ |\xi_u| & \gamma_u & |\beta_u| \\ 0 & |\beta_u| & \alpha_u \end{pmatrix}, \tag{1a}$$

$$M'_d = U_u\left(VD_dV^\dagger\right)U_u^\dagger\,, \tag{1b}$$

- As we have already mentioned, if we want a pattern of three zeros in the mass matrix M'_u, with two zeros on the diagonal, that is, with $\gamma_u = 0$, it is necessary to make $\alpha_u = \lambda_{1u} + \lambda_{2u} + \lambda_{3u}$ according to (1). From (4) this configuration is only possible for $\lambda_{1u}, \lambda_{3u} > 0$ and $\lambda_{2u} < 0$.
- To find two additional texture zeros in the inputs of the mass matrix, we adjust the free parameters θ_1 and θ_2 of the diagonalization matrix
- On the other hand, if we want three zeros for the mass matrix M'_u, but with a single zero on the diagonal, it is necessary to set $|\beta_u| = 0$. To achieve this we have three possibilities (from Eq. (2)): $\alpha_u = \lambda_{1u}$, or $\alpha_u = \lambda_{2u}$, or $\alpha_u = \lambda_{3u}$.
- In each of these cases, one of the remaining λ_{iu}'s must be negative, which gives a total of six different possibilities.

Texture-zero patterns

TABLE II. Patterns for quark mass matrices with five texture zeros. The Wolfenstein parameters for the CKM mixing matrix and the quark masses are reproduced with deviations below 1σ level. In the last column $P_A = \frac{A_{\mathrm{WB}} - A_{\mathrm{PDG}}}{\Delta A}$, where A_{WB} and A_{PDG} are the values for A from the WB transformation and the PDG best fit, respectively. ΔA is the uncertainty for A reported in the PDG.

Case	Five-zero textures	Best fit values (MeV)	Negative eigenvalues	Pulls:				
				Wolfenstein parameters:	P_λ	P_A	P_ρ	P_η
				Up-type quark masses:	P_{m_u}	P_{m_c}	P_{m_t}	…
				Down-type quark masses:	P_{m_d}	P_{m_s}	P_{m_b}	…
I	$M_{Iu} = \begin{pmatrix} 0 & 0 & \xi_u \\ 0 & \alpha_u & \beta_u \\ \xi_u^* & \beta_u^* & \gamma_u \end{pmatrix}$	**a.** $\xi_u = -85.47 + 157.0i$, $\beta_u = 29580 + 5435i$, $\alpha_u = 6054$, $\gamma_u = 167200$ $\lvert\xi_d\rvert = 14.53$, $\lvert\beta_d\rvert = 442.5$, $\alpha_d = 2904$	$\lambda_{1u} < 0$ $\lambda_{2d} < 0$		−0.54 0.98 0.36	0.79 0.13 0.60	0.44 0.43 0.55	−0.81 … …
	$M_{Id} = \begin{pmatrix} 0 & \lvert\xi_d\rvert & 0 \\ \lvert\xi_d\rvert & 0 & \lvert\beta_d\rvert \\ 0 & \lvert\beta_d\rvert & \alpha_d \end{pmatrix}$	**b.** $\xi_u = 21.04–284.5i$, $\beta_u = 18950 + 5890i$, $\alpha_u = 1690$, $\gamma_u = 169000$, $\lvert\xi_d\rvert = 13.41$, $\lvert\beta_d\rvert = 392.6$, $\alpha_d = 2857$	$\lambda_{2u} < 0$ $\lambda_{2d} < 0$		−0.58 −0.28 0.68	−0.99 0.25 −0.26	−0.53 −0.69 0.000098	−0.73 … …
II	$M_{IIu} = \begin{pmatrix} 0 & 0 & \lvert\xi_u\rvert \\ 0 & \alpha_u & \lvert\beta_u\rvert \\ \lvert\xi_u\rvert & \lvert\beta_u\rvert & \gamma_u \end{pmatrix}$	**a.** $\lvert\xi_u\rvert = 431.5$, $\lvert\beta_u\rvert = 7251$, $\alpha_u = 957.9$, $\gamma_u = 171200$, $\xi_d = 4.316 + 14.26i$, $\gamma_d = 64.13$, $\alpha_d = 2969$	$\lambda_{1u} < 0$ $\lambda_{1d} < 0$		0.12 0.52 0.98	0.86 0.51 0.49	0.37 −0.47 0.53	0.89 … …
	$M_{IId} = \begin{pmatrix} 0 & \xi_d & 0 \\ \xi_d^* & \gamma_d & 0 \\ 0 & 0 & \alpha_d \end{pmatrix}$	**b.** $\lvert\xi_u\rvert = 426.3$, $\lvert\beta_u\rvert = 7336$, $\alpha_u = 868.1$, $\gamma_u = 172500$, $\xi_d = -4.152 - 13.81i$, $\gamma_d = -62.50$, $\alpha_d = 2916$	$\lambda_{1u} < 0$ $\lambda_{2d} < 0$		0.55 0.62 0.72	0.81 −0.97 0.38	0.85 0.63 0.052	0.96 … …

Summary and conclusions

1. Using the WB transformation method, we found configurations for the quark mass matrices with the maximum number of possible texture zeros.
2. Modulo permutations, only the configurations shown in Table, for mass matrices with one or two zeros in the diagonal, are possible.
3. From these patterns we obtained the cases I and II in Table, corresponding to the five-zero textures, which reproduce the quark masses, mixing angles and the CP violation phase, with deviations from the experimental values below 1σ level.
4. Additionally, our five-zero texture models reproduce the experimental quantities deviating from the experimental central value by at most $1\,\sigma$ level. First, for both cases I and II, we can verify that the relation $|V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$ is satisfied, while the relation $|V_{ub}/V_{cb}| \neq \sqrt{m_u/m_c}$ is not, as already indicated in previous works.
5. The case I is an original proposal which was not considered in the Fritzsch original work nor in later studies. Case II has been widely considered in the literature, but in our approach, we take a negative eigenvalue (which has not been considered previously) for the mass of the lightest down quark, that is, $\lambda_{1d} < 0$.
6. Here, it should be mentioned that, without losing generality, only one negative eigenvalue is necessary for each mass matrix.

THANK YOU!