# $B \to D^{(*)} \ell \nu$ semileptonic decay form factors in LQCD

KEK, SOKENDAI

Takashi Kaneko (JLQCD Collaboration )

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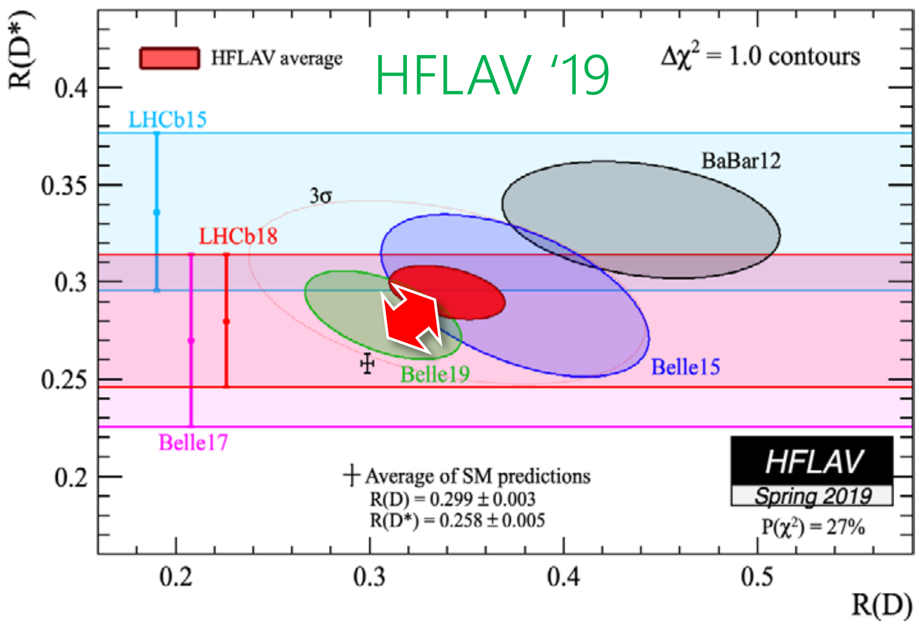
# introduction

hint of new physics ?

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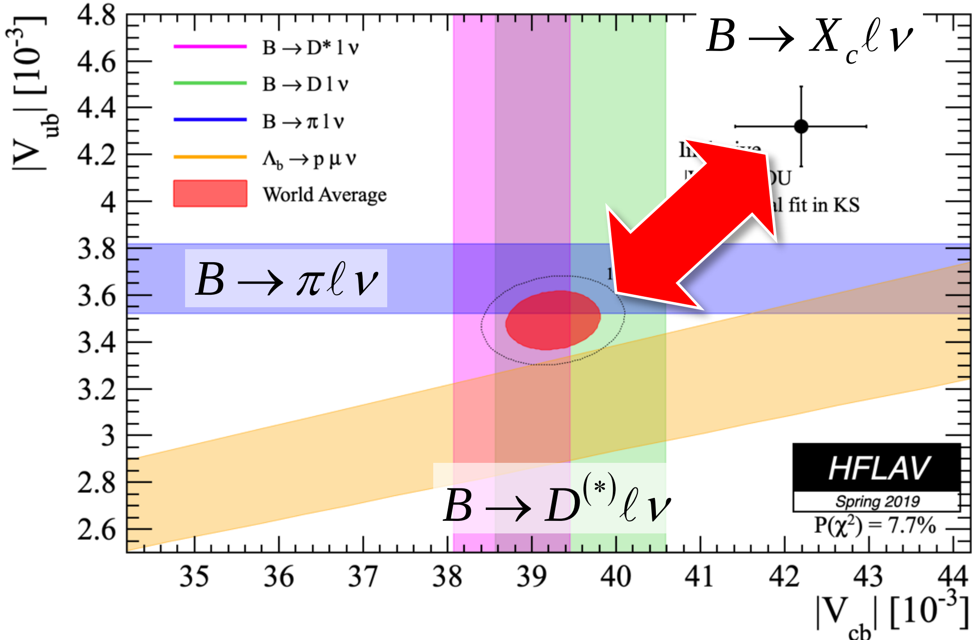
$|V_{cb}|$ tension

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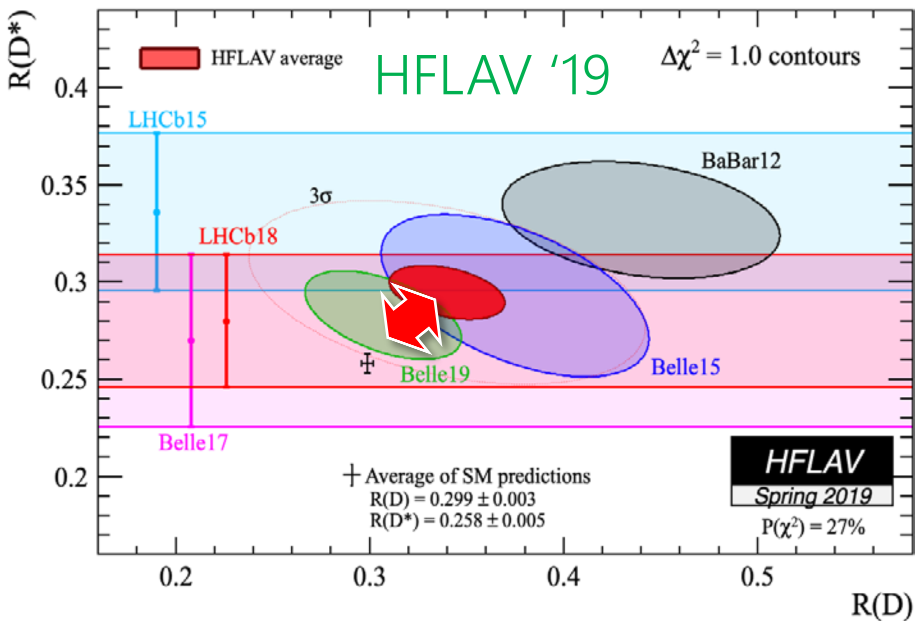


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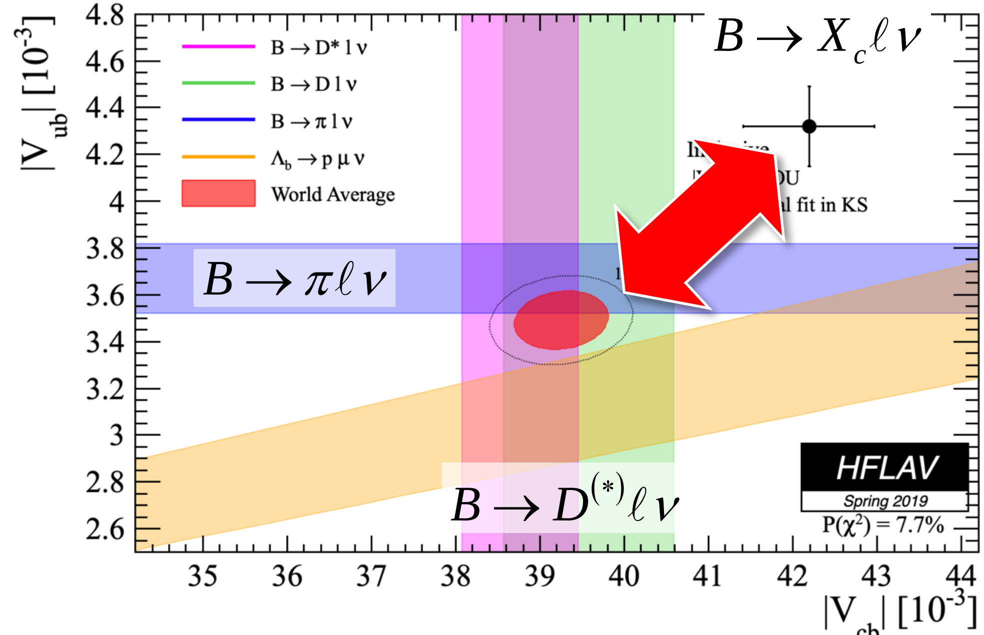


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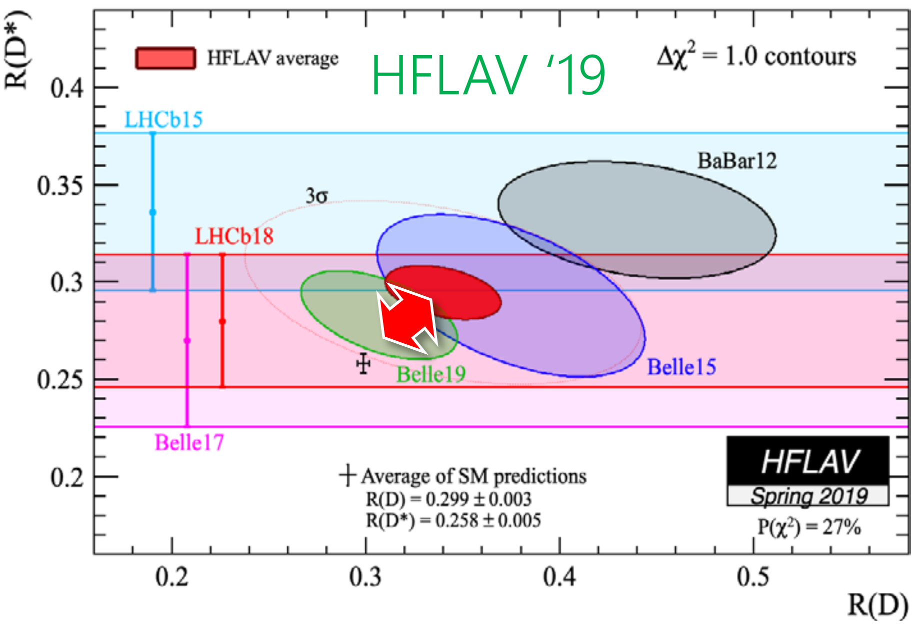


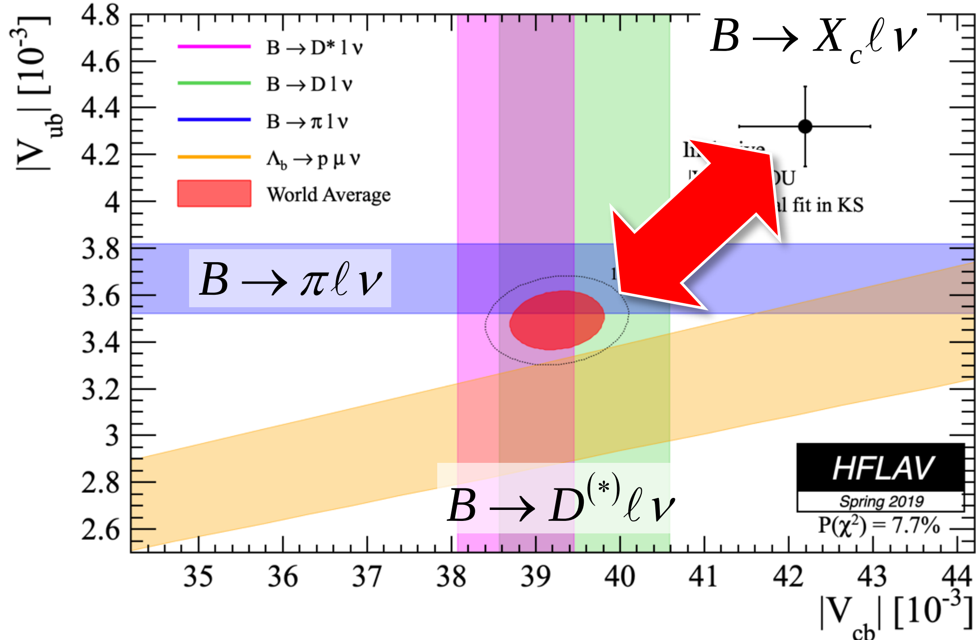


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need deeper understanding of th. and/or exp't uncertainties

– theory side : form factors (FFs) describing non-perturbative QCD effects

# $B \to D^{(*)} \ell \nu$ form factors

"relativistic" convention

$$\langle D(p')|V^\mu|\bar{B}(p)\rangle = f_+(p+p')^\mu + f_-(p-p')^\mu$$

$$\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle = ig\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p'_\beta p_\gamma,$$

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- 2 FFs for $B \to D\ell\nu$ ; 4 FFs for $B \to D^*\ell\nu$ w/ $\varepsilon_{D^*} p_{D^*} = 0$
- function of $q^2 = t = (p - p')^2$ and $w = v\,v'$ w/ $v^{(\prime)} = p\,/M_{B(D(*))}$

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two independent calculations w/ very different systematics

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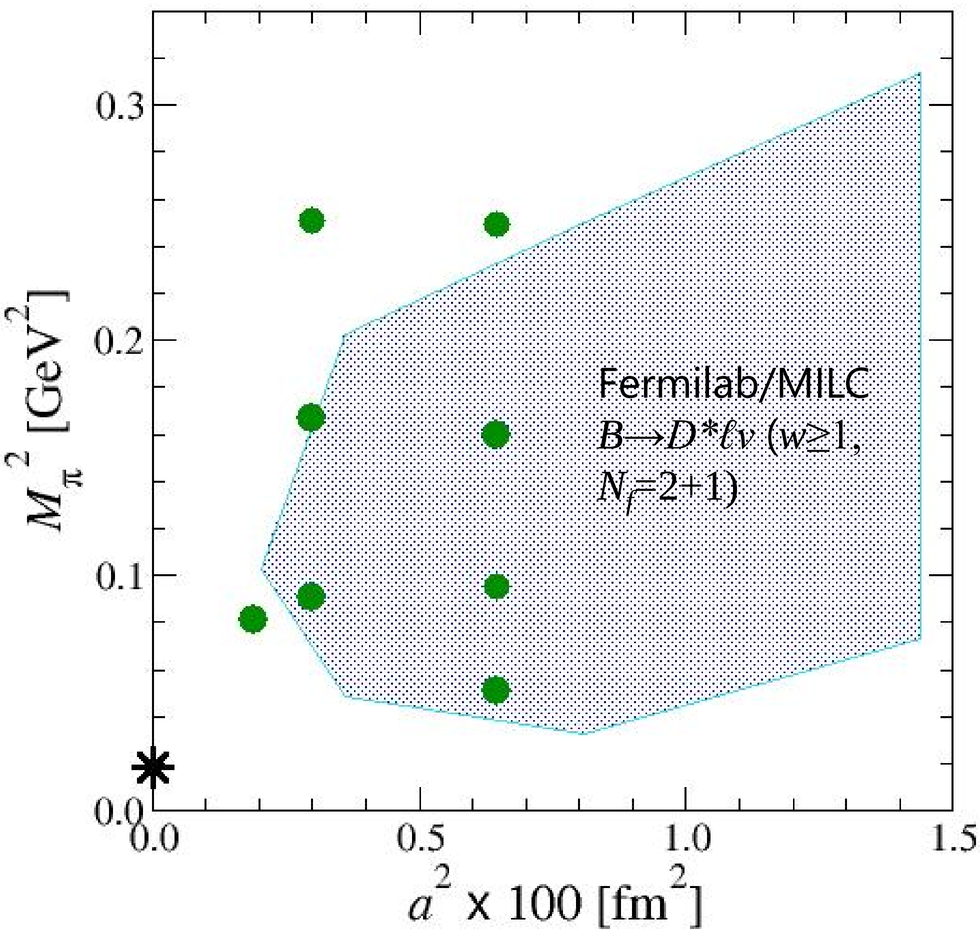
this talk : $B \to D^* \ell \nu$, JLQCD's update & comparison w/ Fermilab/MILC

# gauge ensembles




similar simulation parameters

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- $M_\pi \gtrsim$ 200 MeV $\Leftrightarrow$ $D^* \to D\pi$
- $M_\pi L \gtrsim 4$
  + JLQCD : two $L$'s to check FVEs

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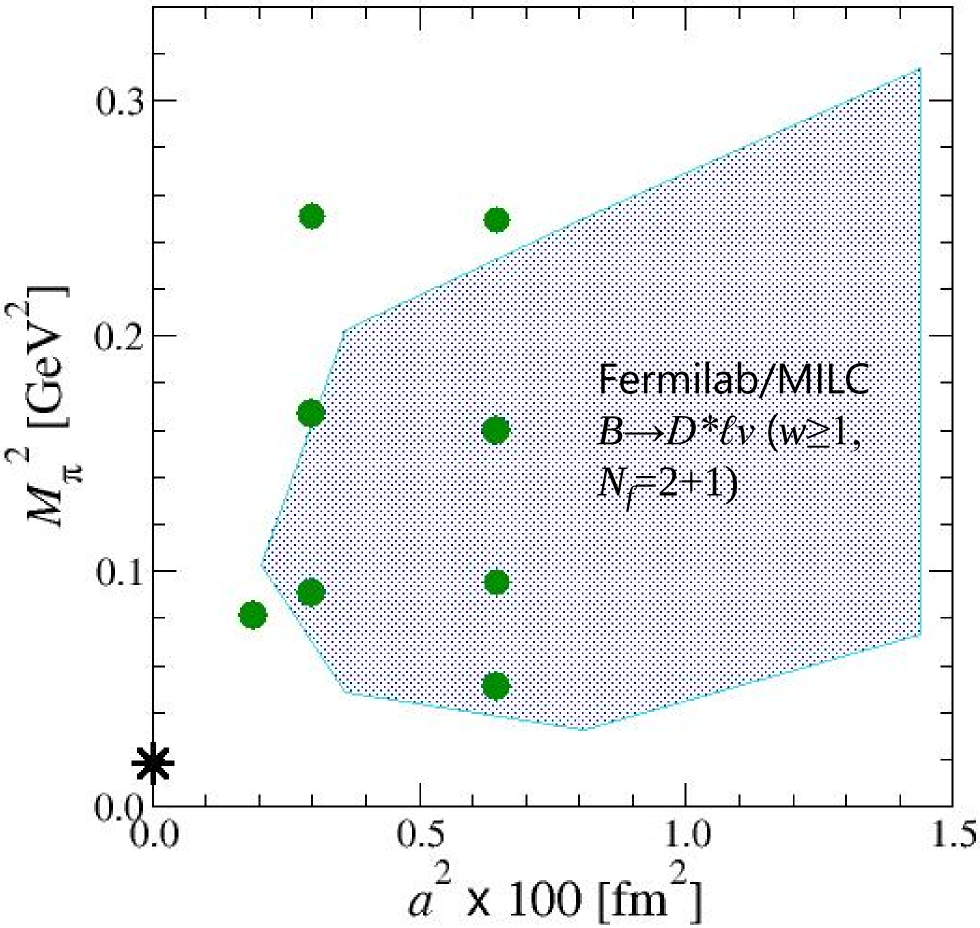


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Fermilab/MILC 2105.14019

Asqtad staggered light quarks

- fast : high statistics
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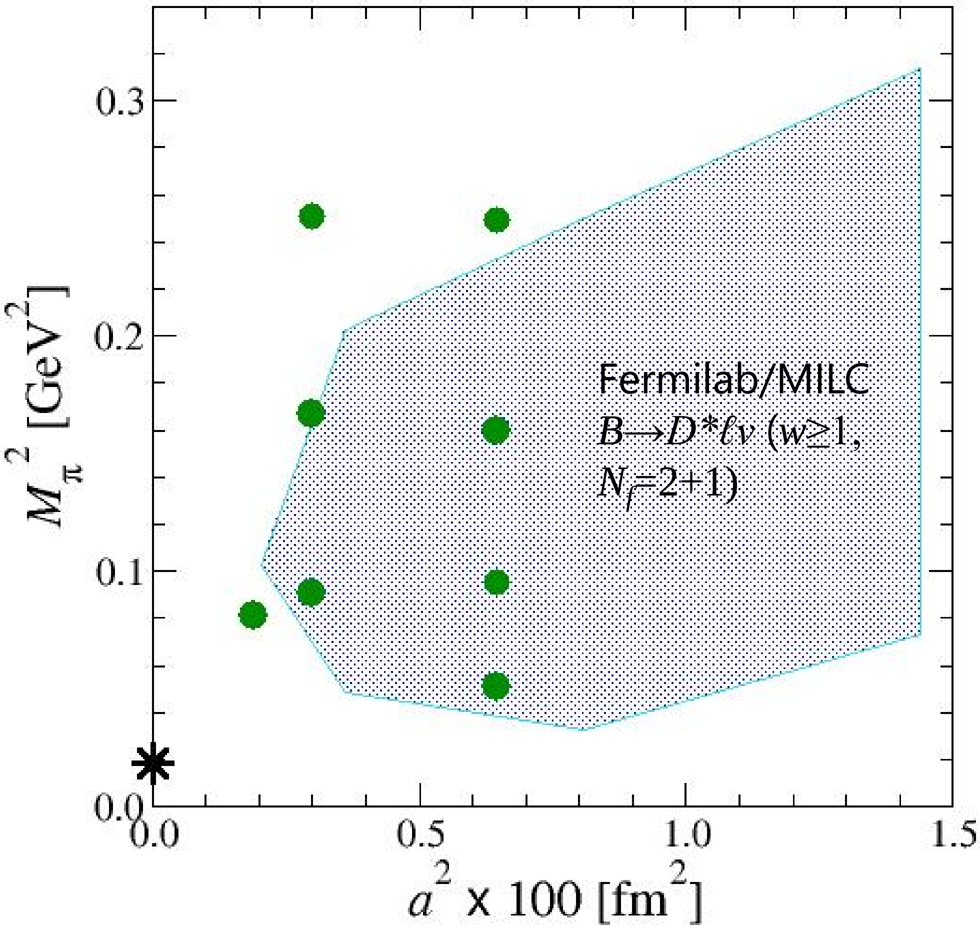


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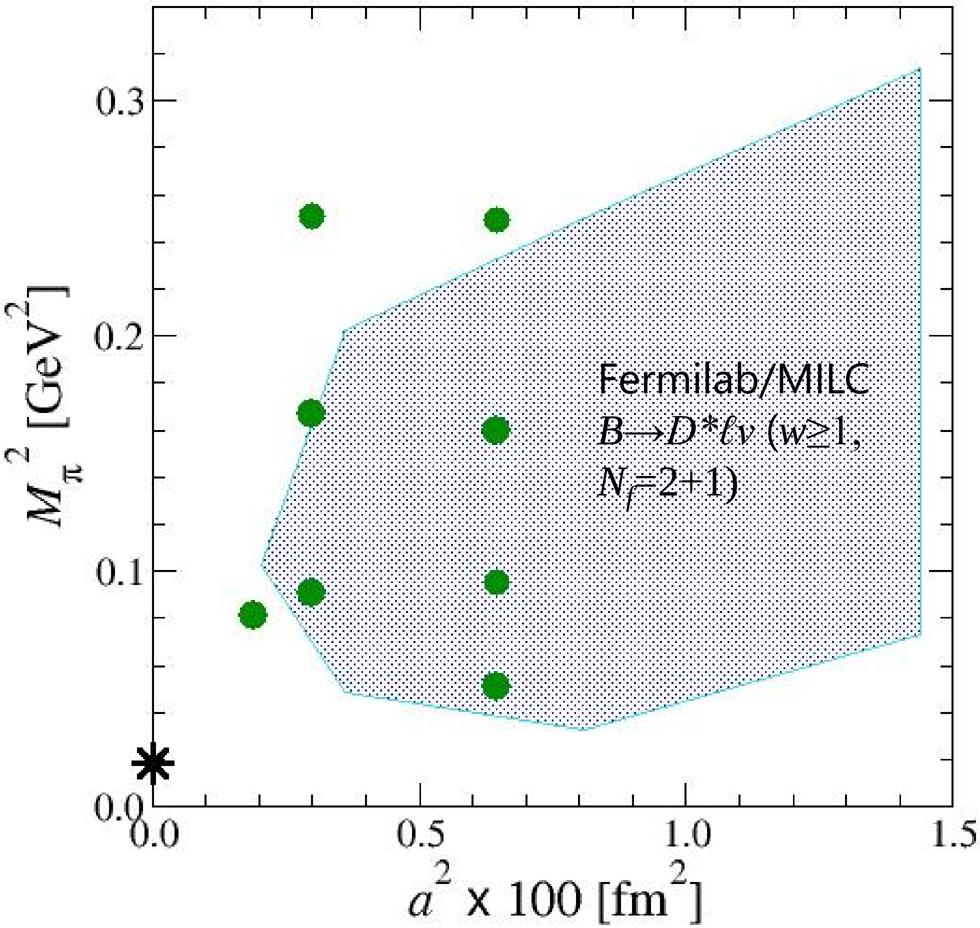


$a^{-1} \lesssim m_b \Rightarrow$ need a careful treatment of heavy quarks

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$c$ and $b$ quarks in EFT approach

- Fermilab interpretation of Wilson action
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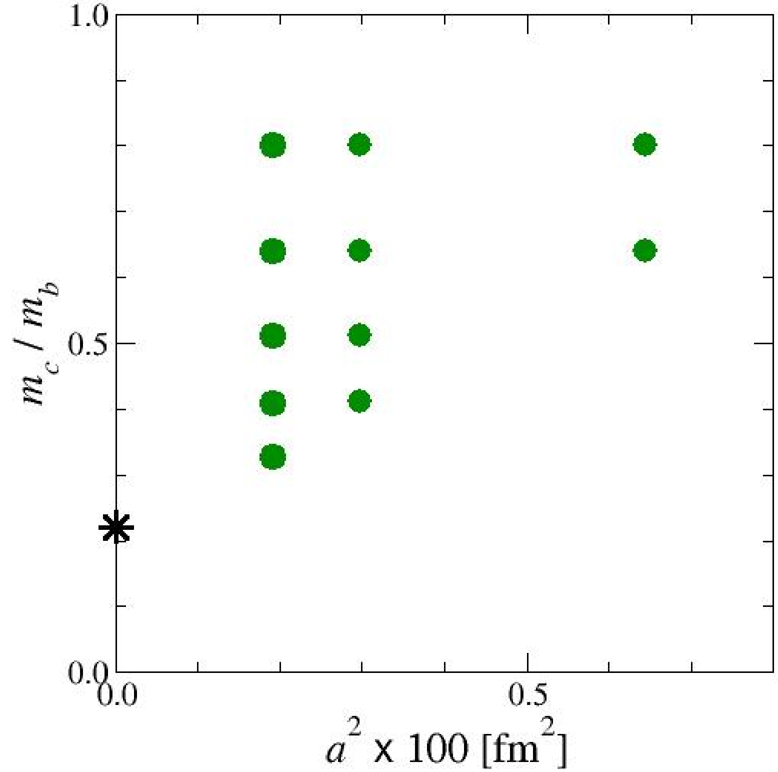

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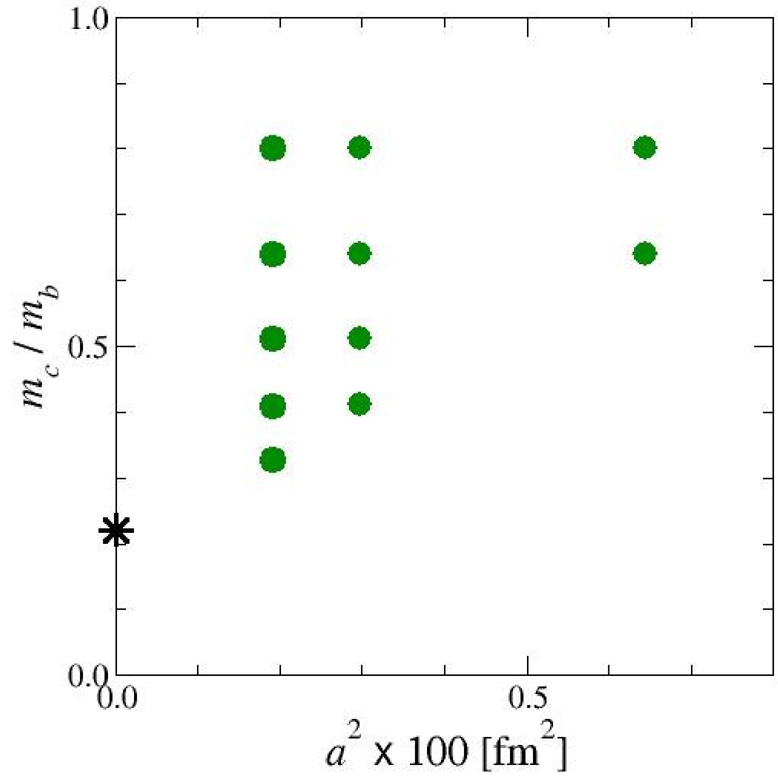


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independent calc.s w/ very different systematics ⇒ solid prediction

# continuum + chiral extrap. - JLQCD

NLO HMChPT (Randall-Wise '92, Savage '01) + polynomial corrections

$$\frac{h_{A_1}(w)}{\eta_{A_1}} = c + \frac{g_{D^*D\pi}^2}{16\pi^2 f_\pi^2} \Delta_c^2 b_{\log} \bar{F}_{\log}\left(M_\pi, \Delta_c, \Lambda_\chi\right)$$

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- singular correlation matrix ⇒ SVD cut, shrinkage ⇔ Fermilab/MILC
- one-loop radiative correction $\eta_X$ is explicitly included (Neubert '92)
- $g_{D^*D\pi}$ = 0.53(8) (Fermilab/MILC '14) ⇒ small systematic error
- ξ - expansion : better convergence for light quark obs. (JLQCD '08)
- $O((w-1)/m_b)$ for $h_{A1}$, $h_+$ ⇔ Luke's theorem '90 ; include $O(1/m_b^2)$

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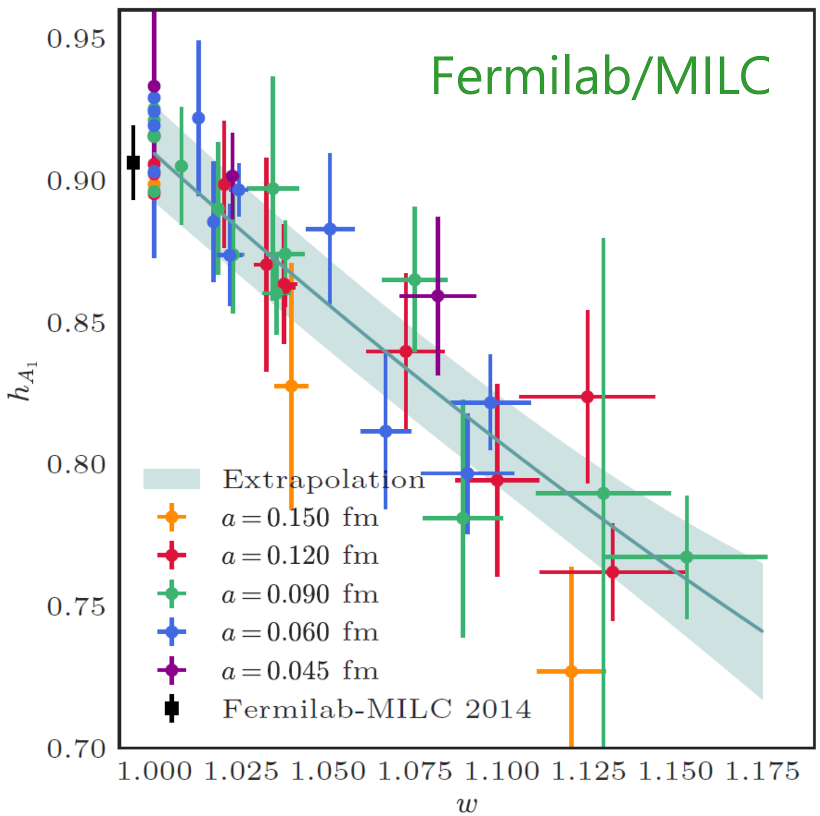
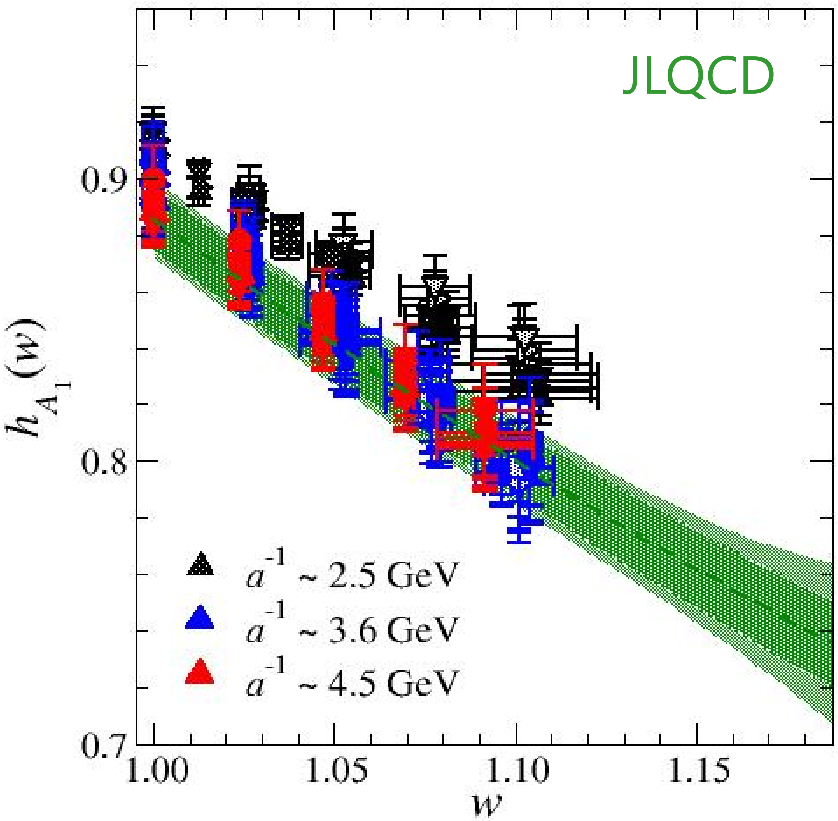
## FF comparison : $h_{A1}$



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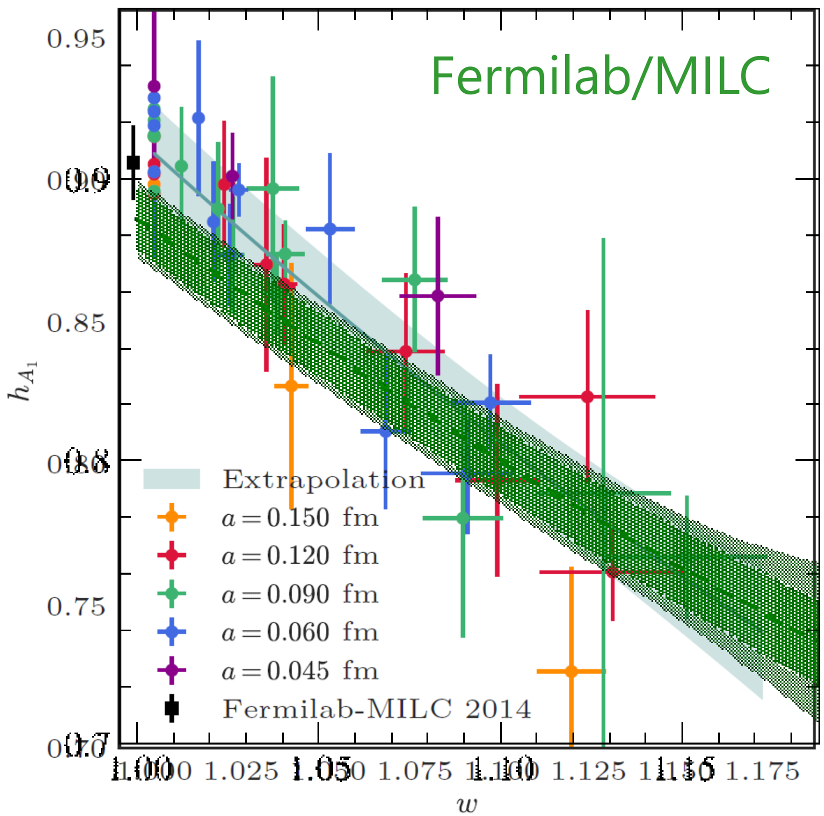


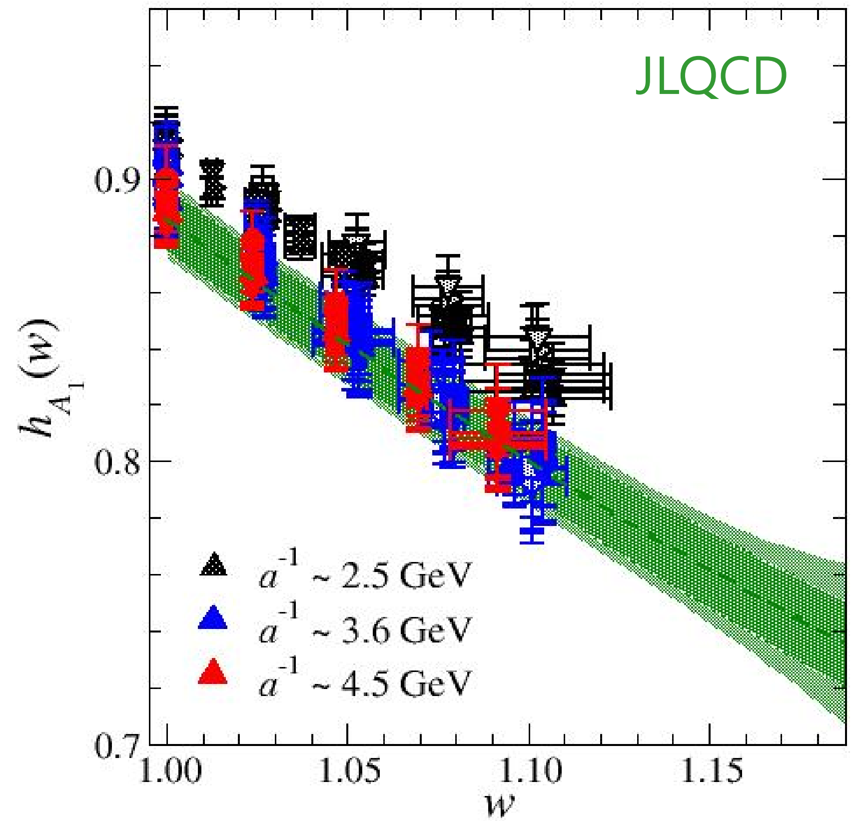


- reasonable consistency in spite of very different systematics
- JLQCD: slightly narrower $w$, significant $a \neq 0$, slightly gentle slope (?)

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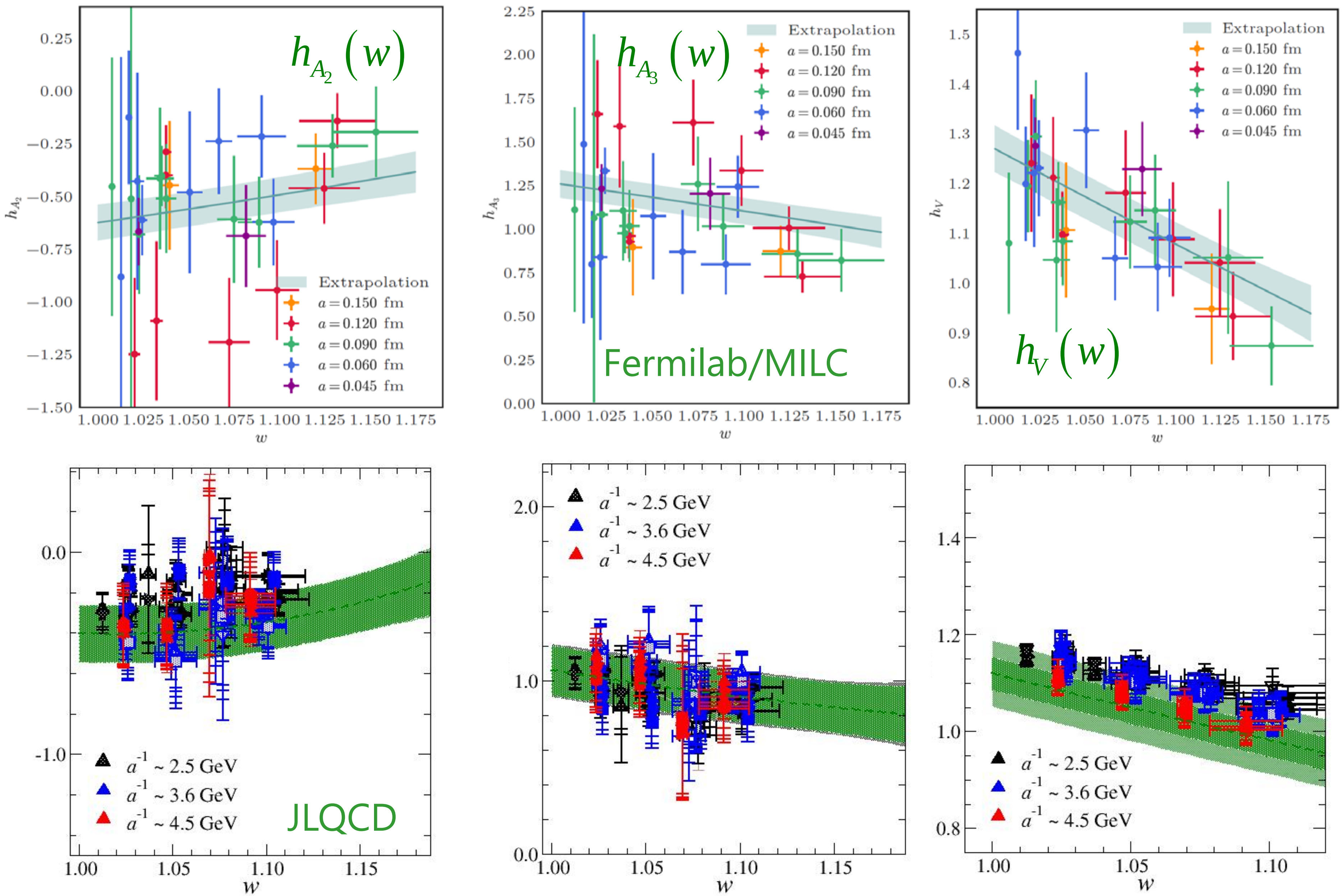

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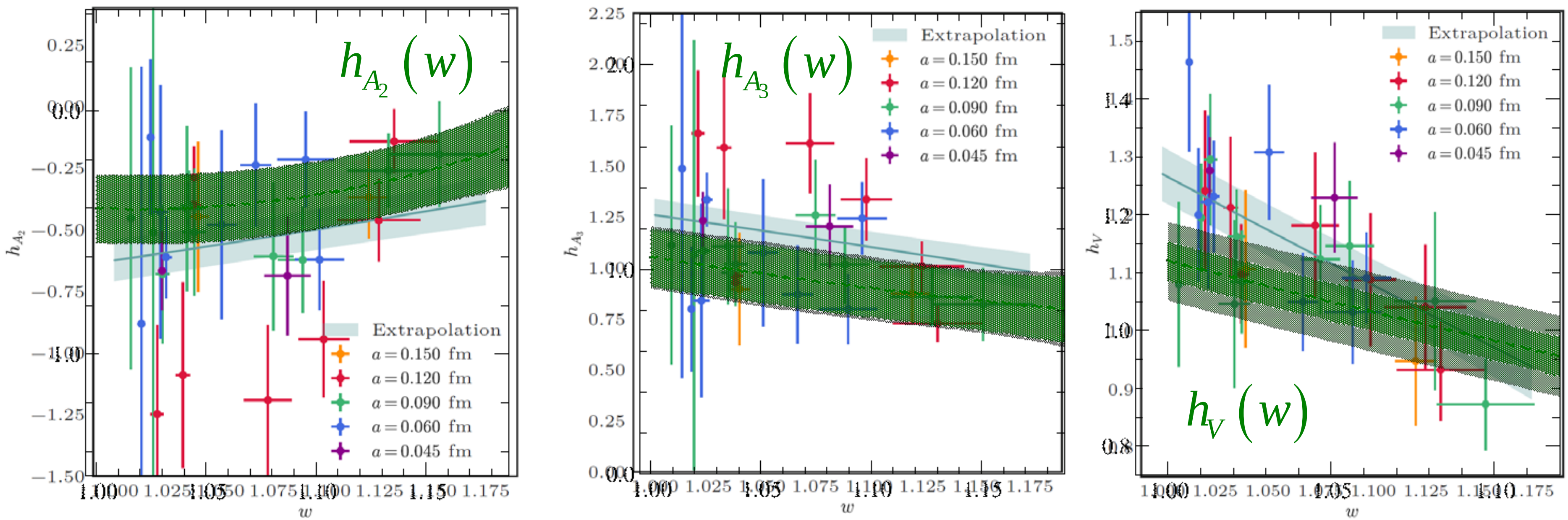
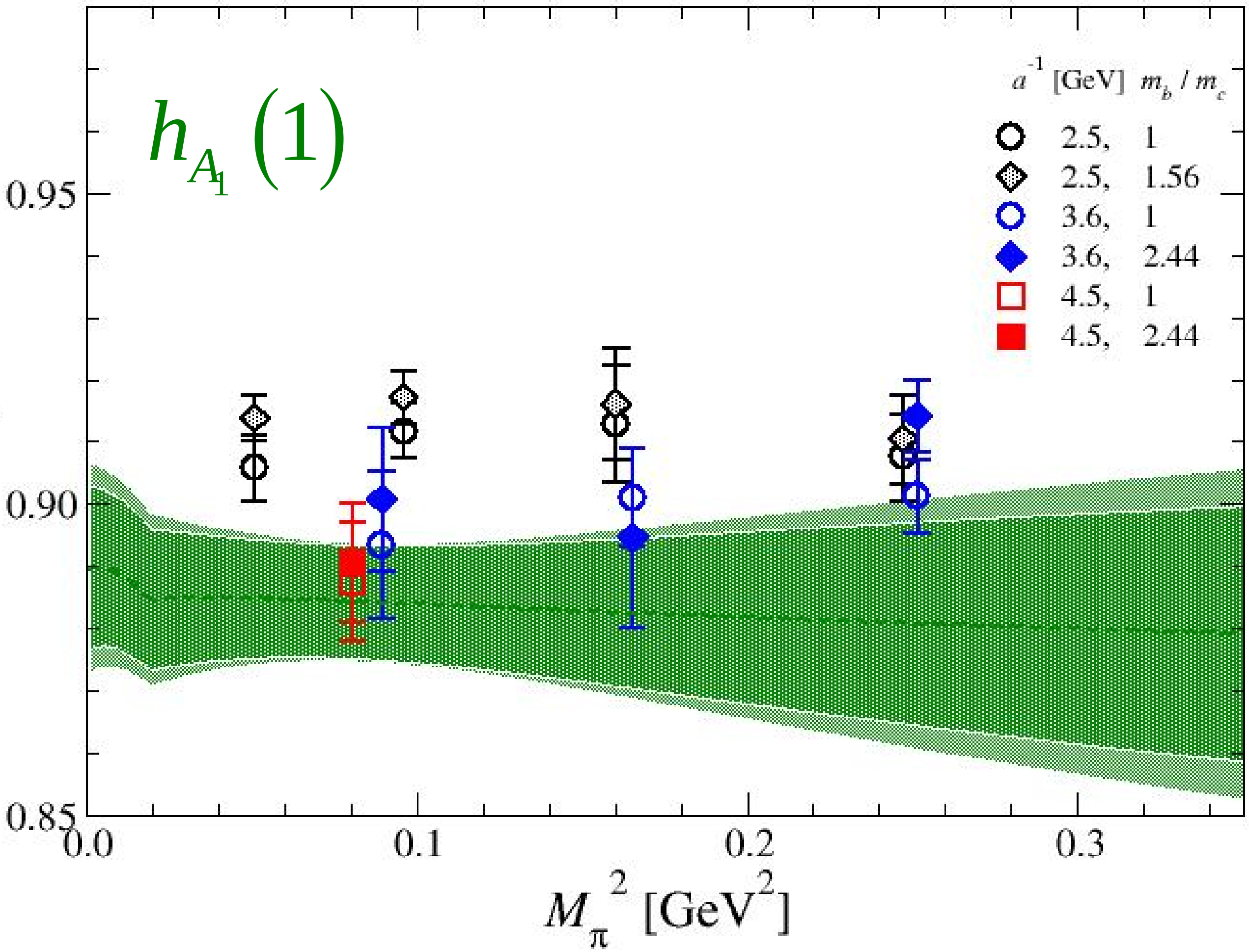



- reasonable consistency also for these FFs
- JLQCD: slightly gentle slope for $h_V$ ?

# $M_\pi$ dependence




- mild dependence
  - suppressed log
  - no valence $\pi$
- similar for other $w$, FFs

w/ $H^{(Q)}$, $\pi$

$$\sqrt{\Delta^2 - M_\pi^2}\,\ln\left[\frac{\Delta + \sqrt{\Delta^2 - M_\pi^2}}{\Delta - \sqrt{\Delta^2 - M_\pi^2}}\right]$$

$$\Delta = M_{D*} - M_D$$

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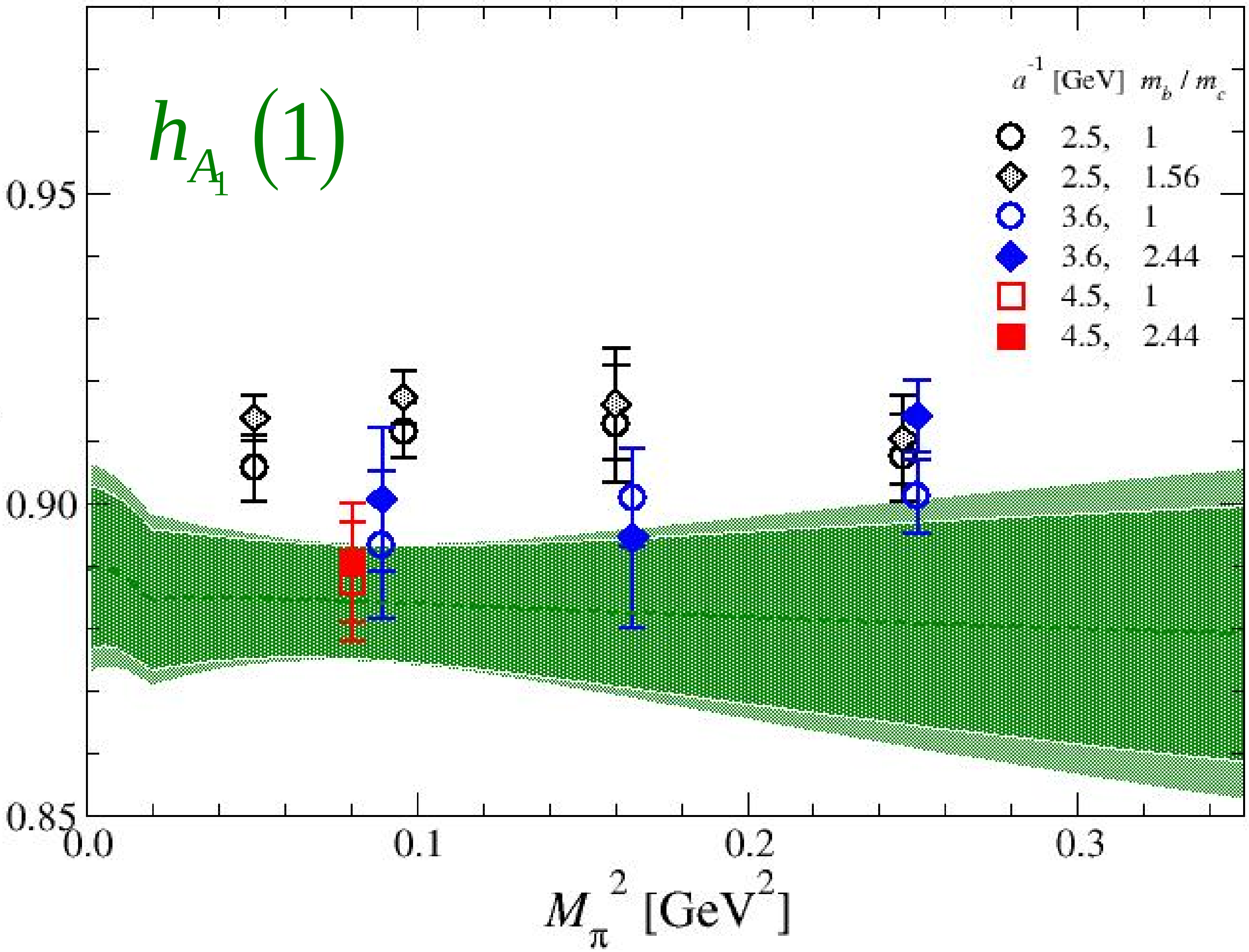


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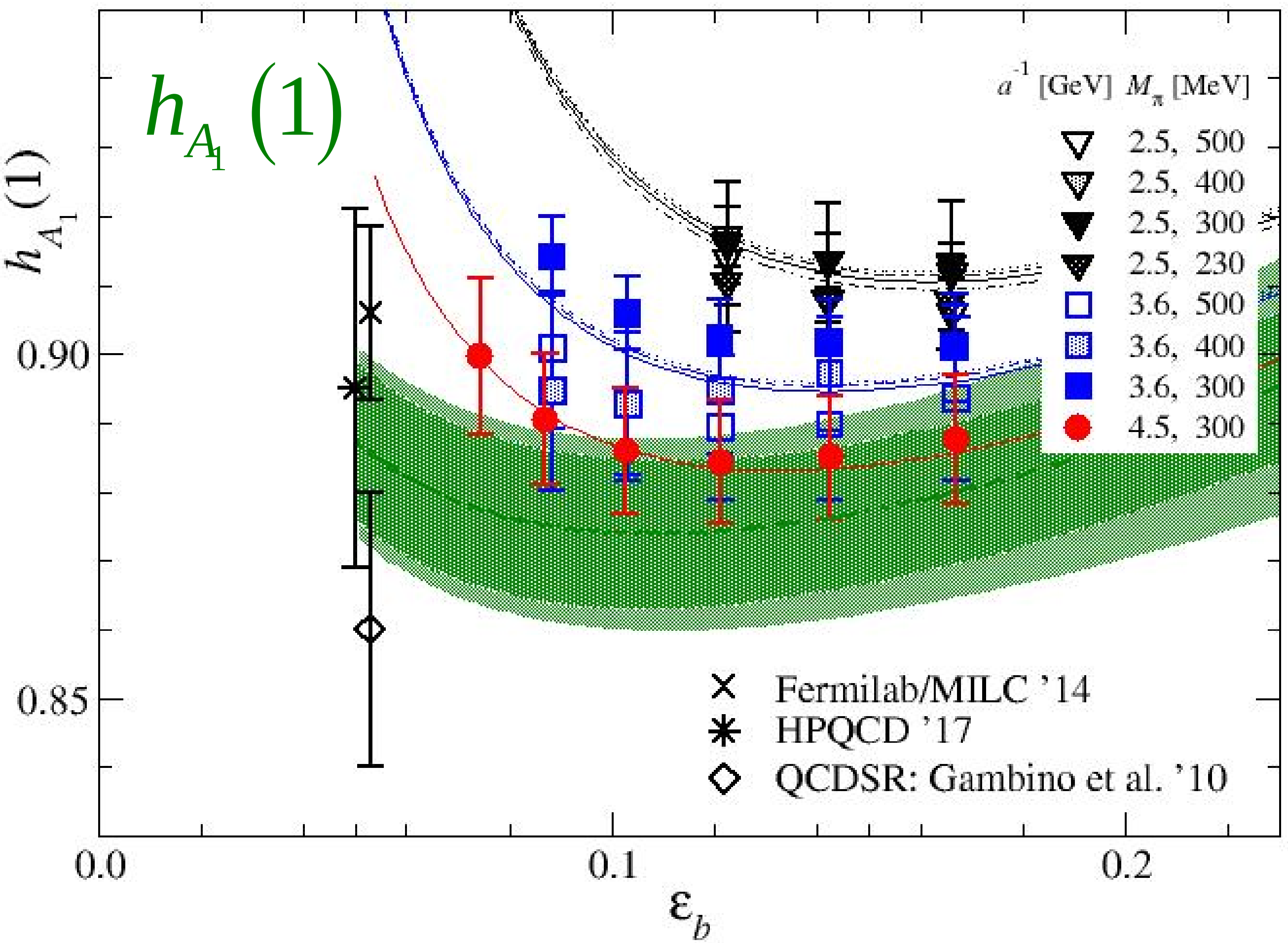
- $D^* \rightarrow D\pi$ $\Rightarrow$ concave structure < statistical accuracy
- mild dependence $\Rightarrow$ chiral extrapolation under control

# $m_b$ and $a$ dependences




$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b} = \frac{\bar{\Lambda}}{M_{\eta_b}}$$

$\bar{\Lambda} = 0.5\text{GeV}$

- two $a \neq 0$ effects
  - $(a\Lambda)^{2n}$, $(am_c)^{2n}$
  - $(am_b)^{2n}$
- consistency w/ QCDSR ?

  Gambino-Mannel-Uraltsev '10

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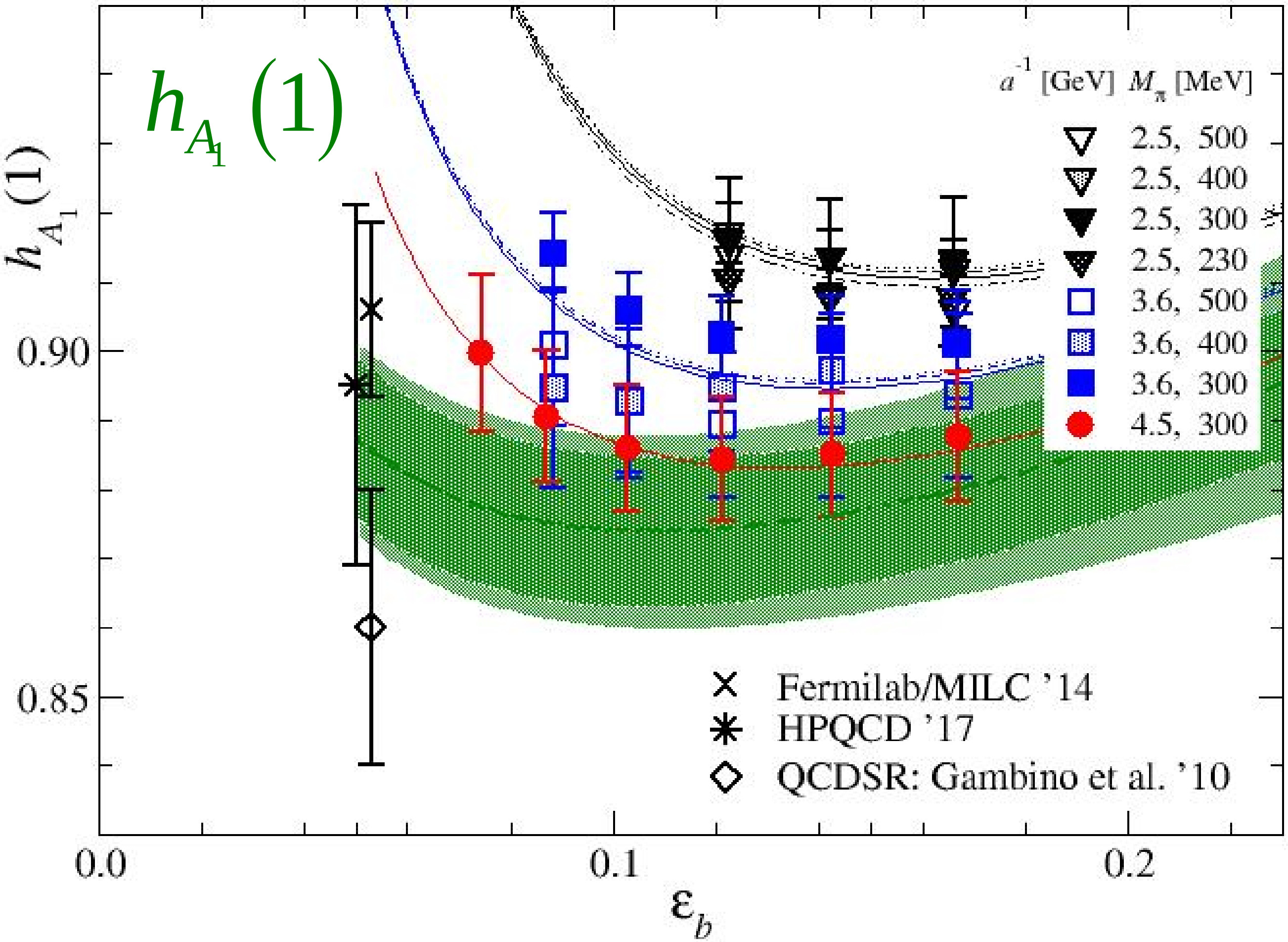


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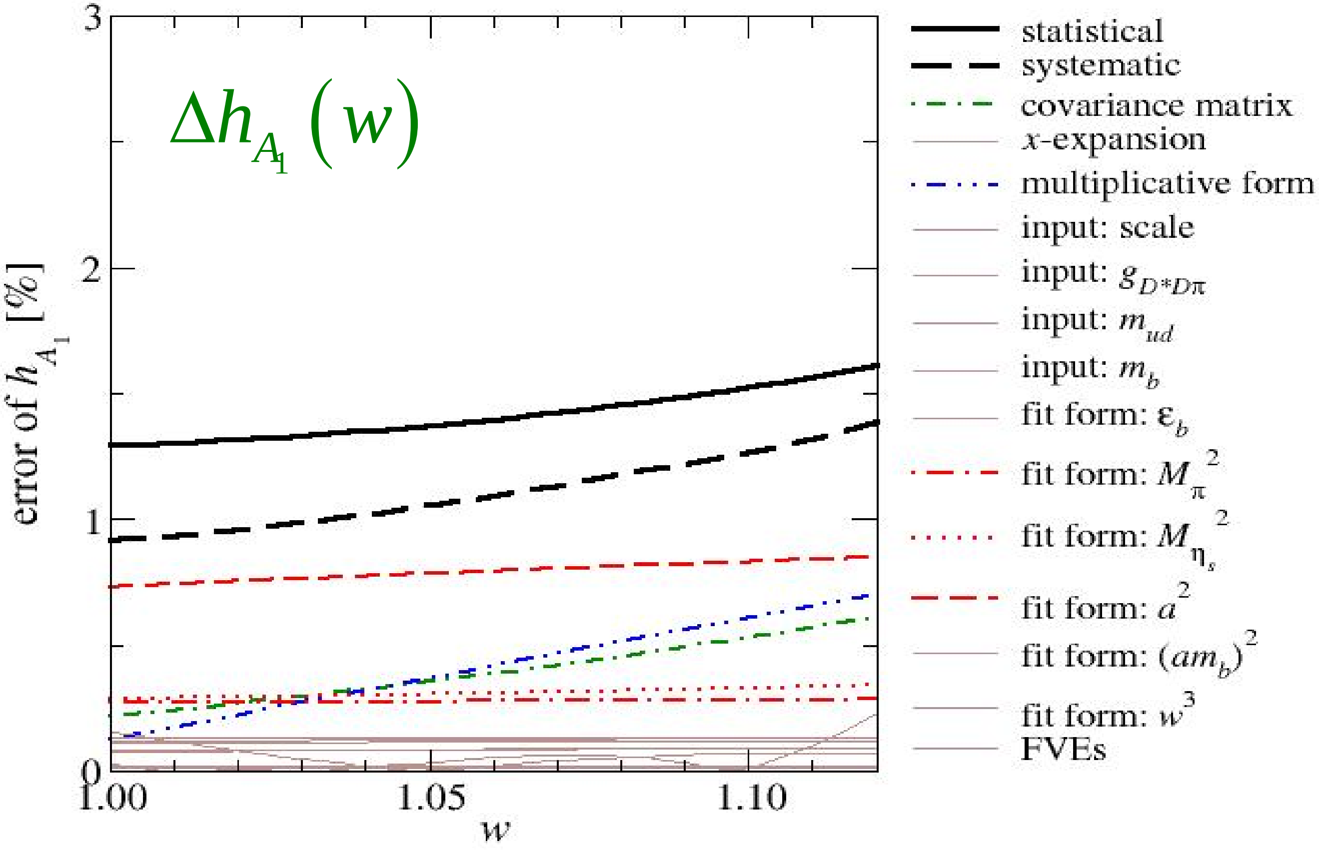
  Gambino-Mannel-Uraltsev '10

- turned out to be a few % effects
- reasonably controlled extrapolation in $\varepsilon_b$ and $a$ $\Leftrightarrow$ smaller $a$ ?

# uncertainties

JLQCD (individual error)



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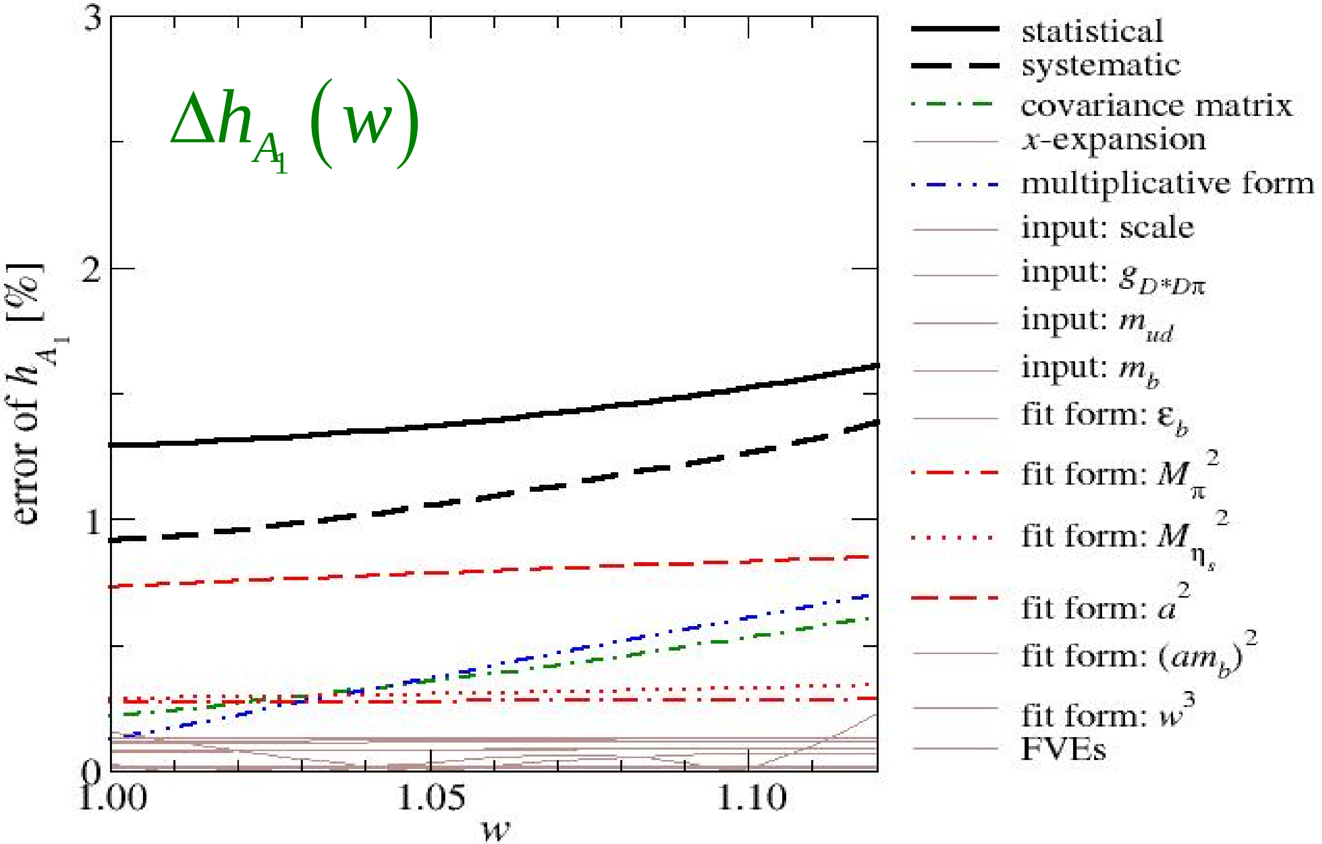


correlation matrix
extrapolation form

# uncertainties

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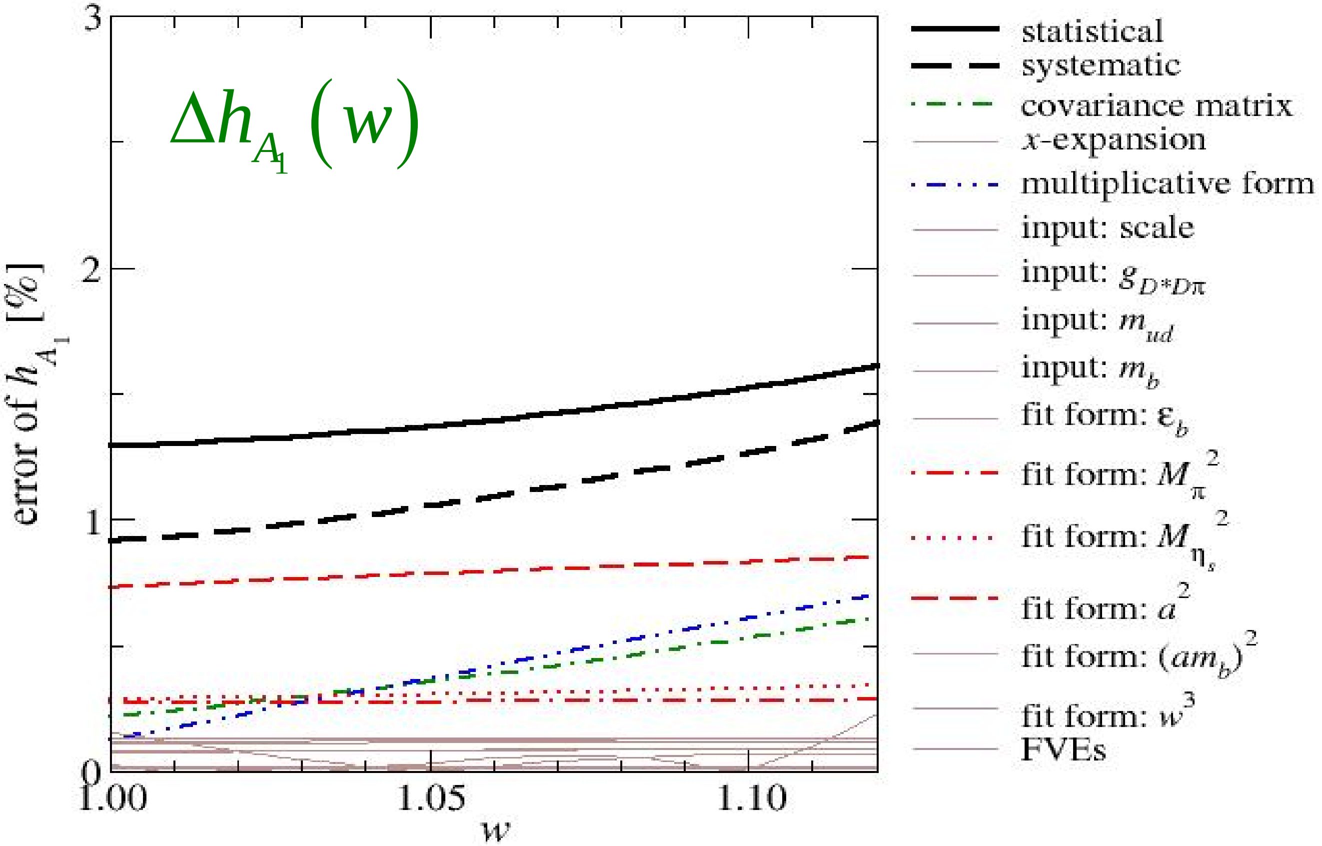
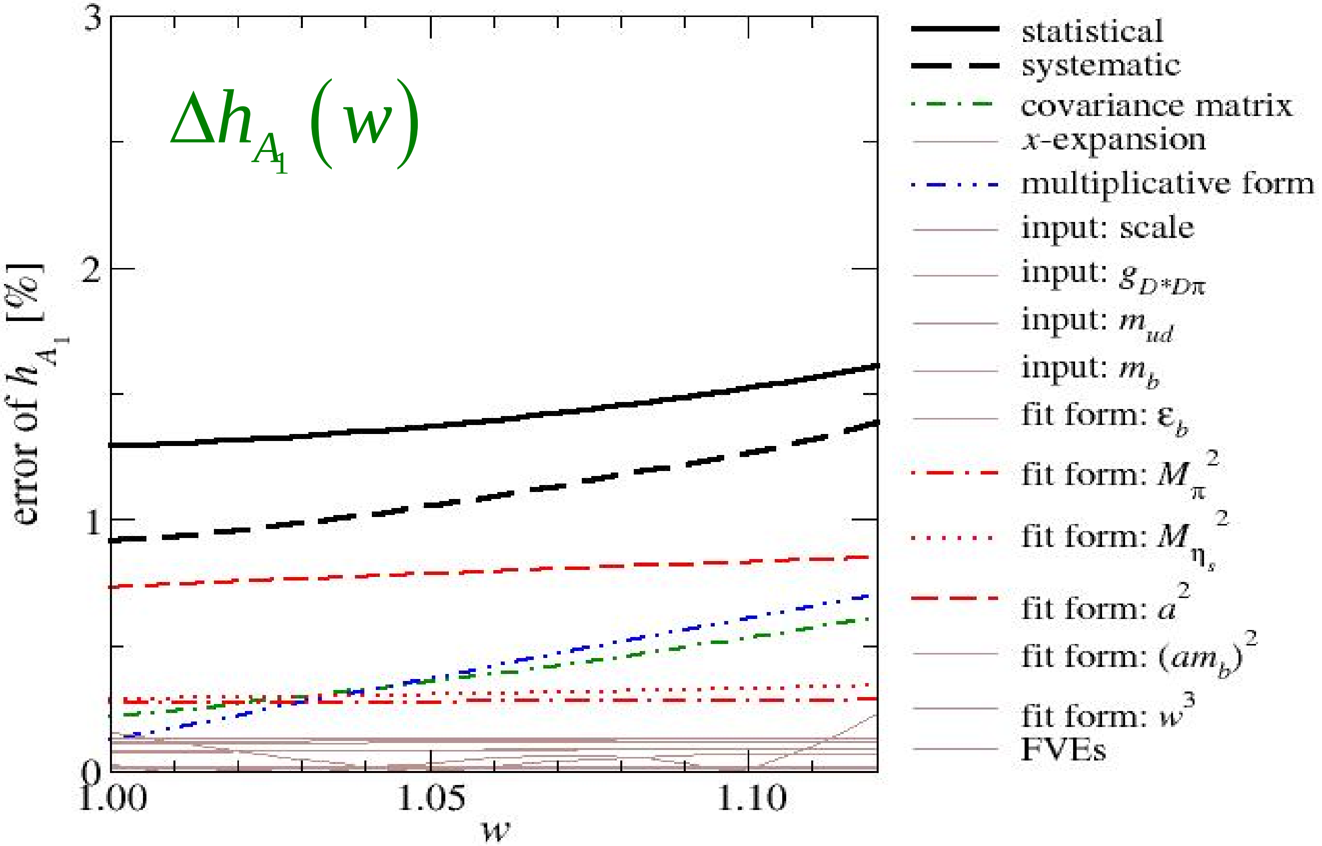



correlation matrix
extrapolation form

inputs
$a$, $m_{q'}$, $g_{D^*D\pi}$

# uncertainties

JLQCD (individual error)

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add higher order
remove poorly determined term

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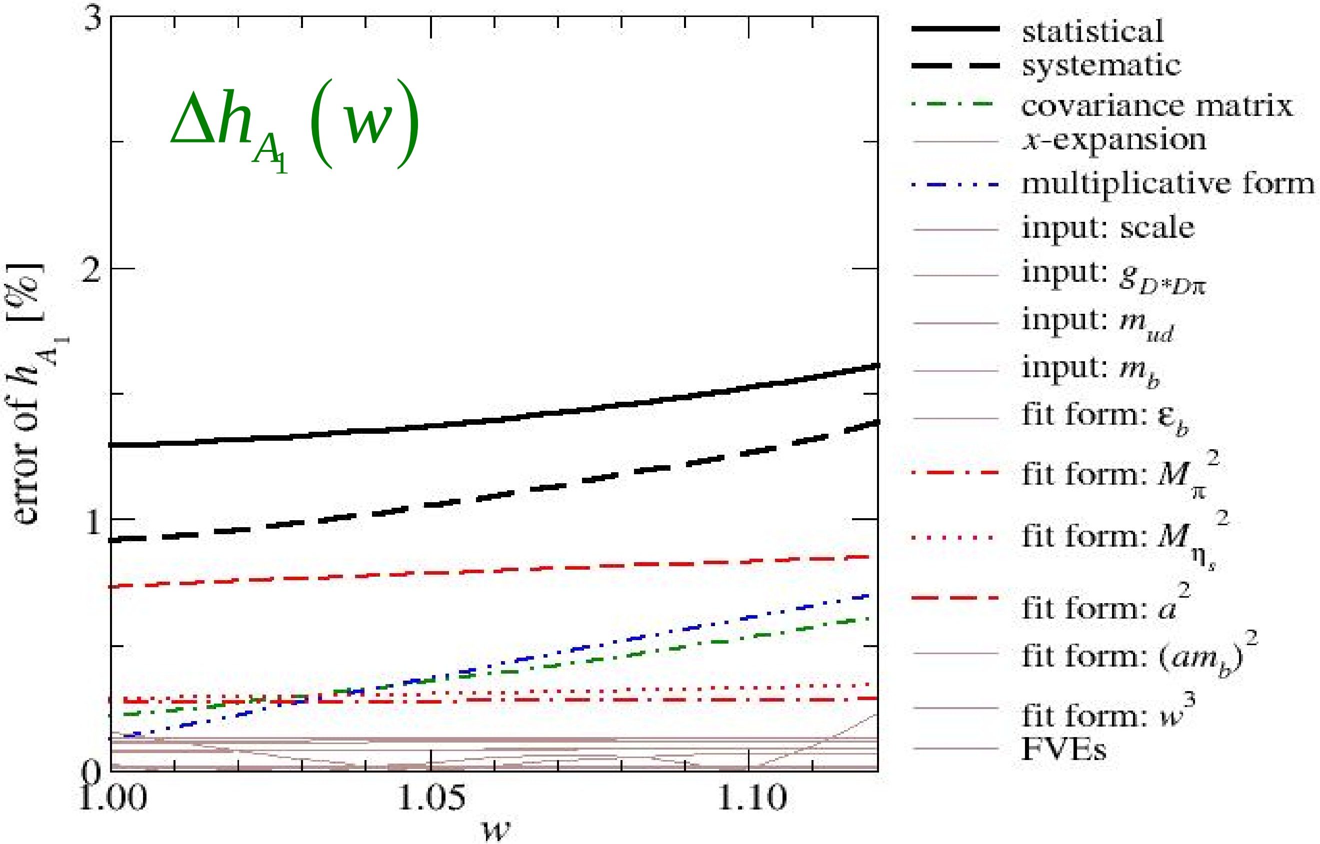


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- small by $2L$'s

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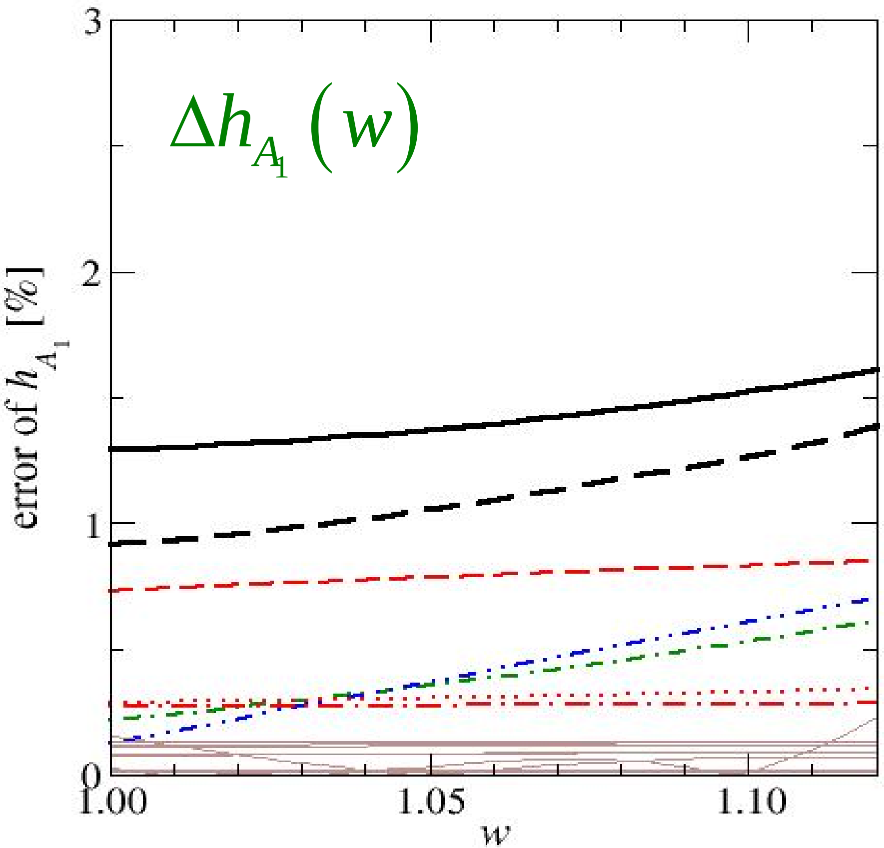
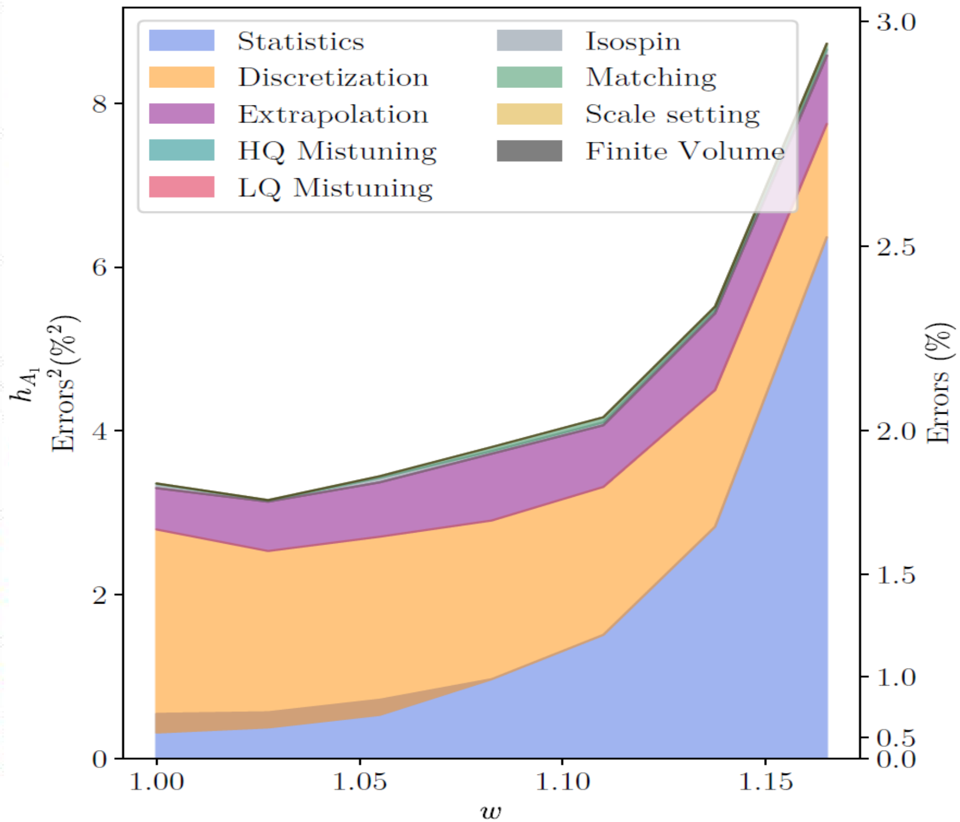



Fermilab/MILC (accumulated error)



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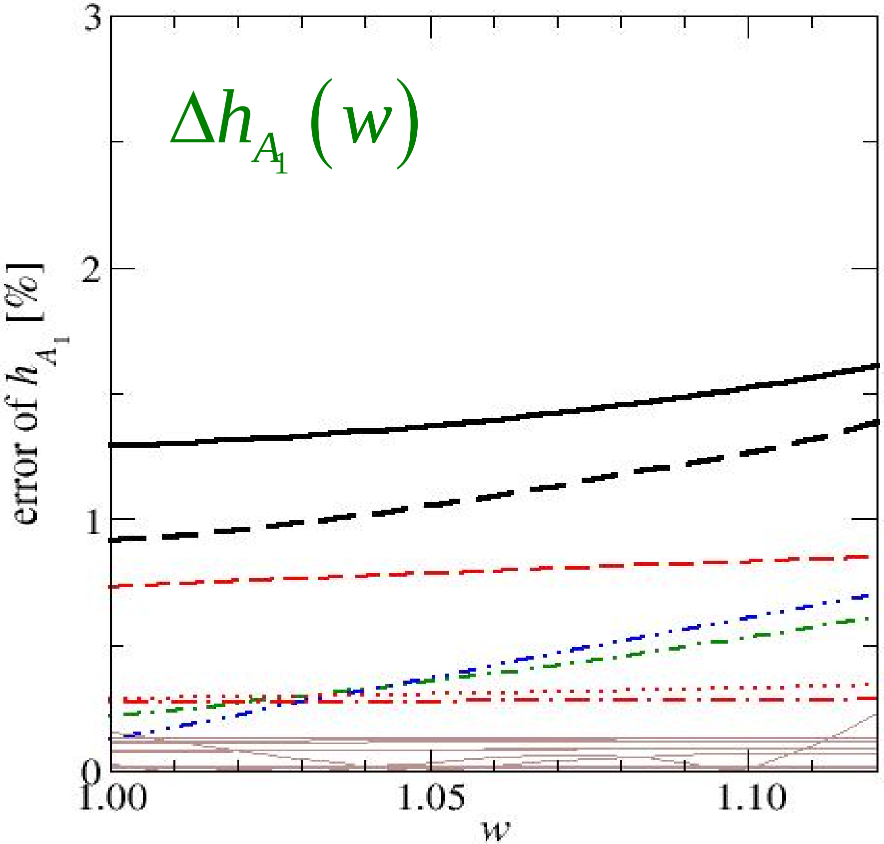


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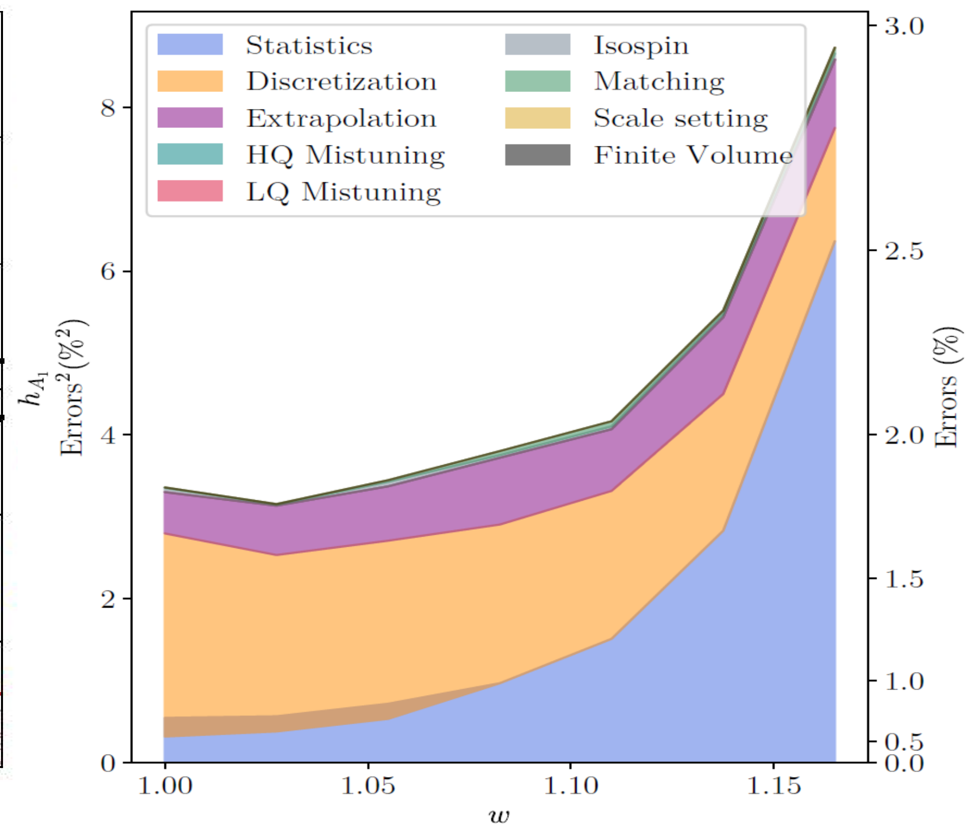
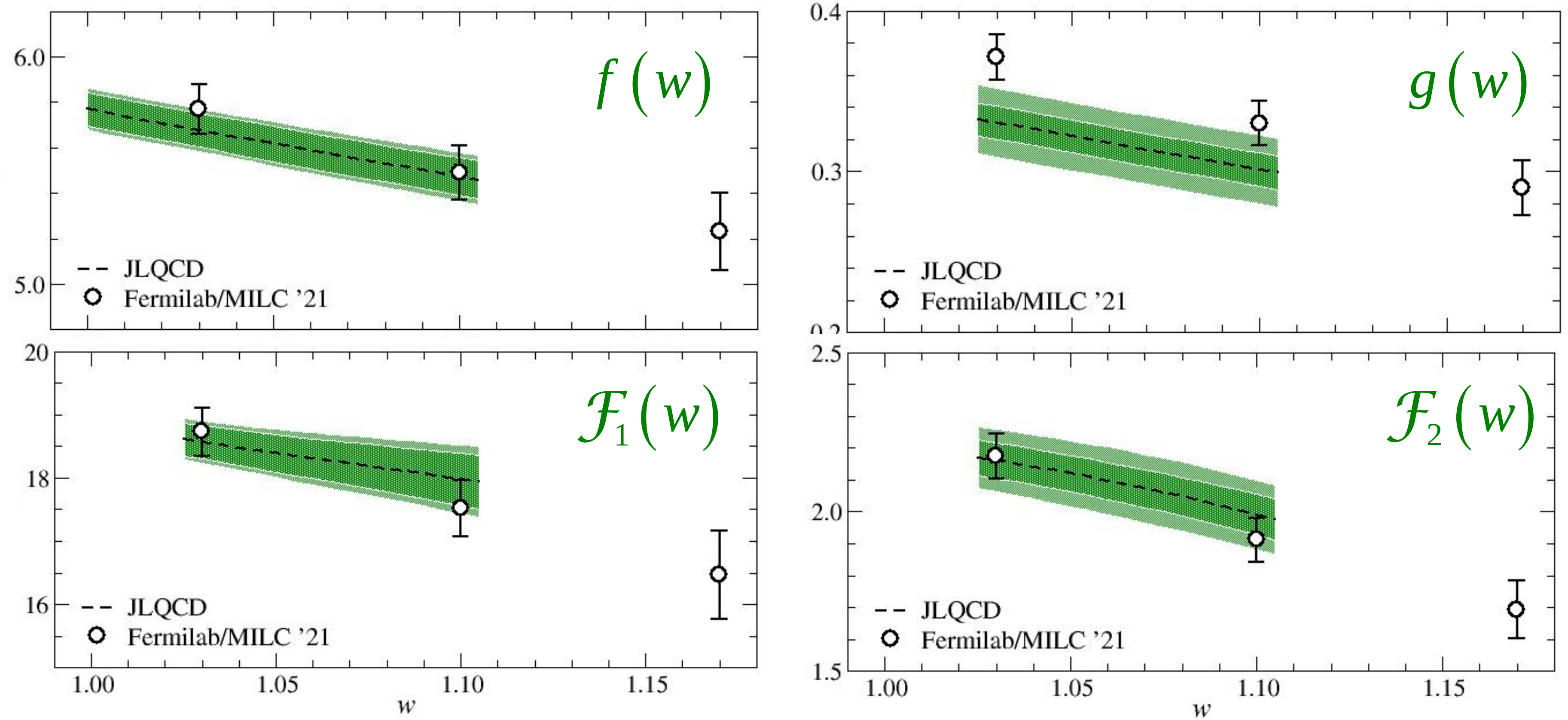



- $h_{A1}$ : largest uncertainties – statistics and discretization – but 1-2 %

  mild $a$, $m_q$ dependences $\Rightarrow$ $O(1)$ or less $c_X$ $\Rightarrow$ controlled extrapolation
- other FFs : larger and more dominant statistical error

# BGL parameterization of FFs

synthetic data @ $a$=0 and physical $m_q$'s



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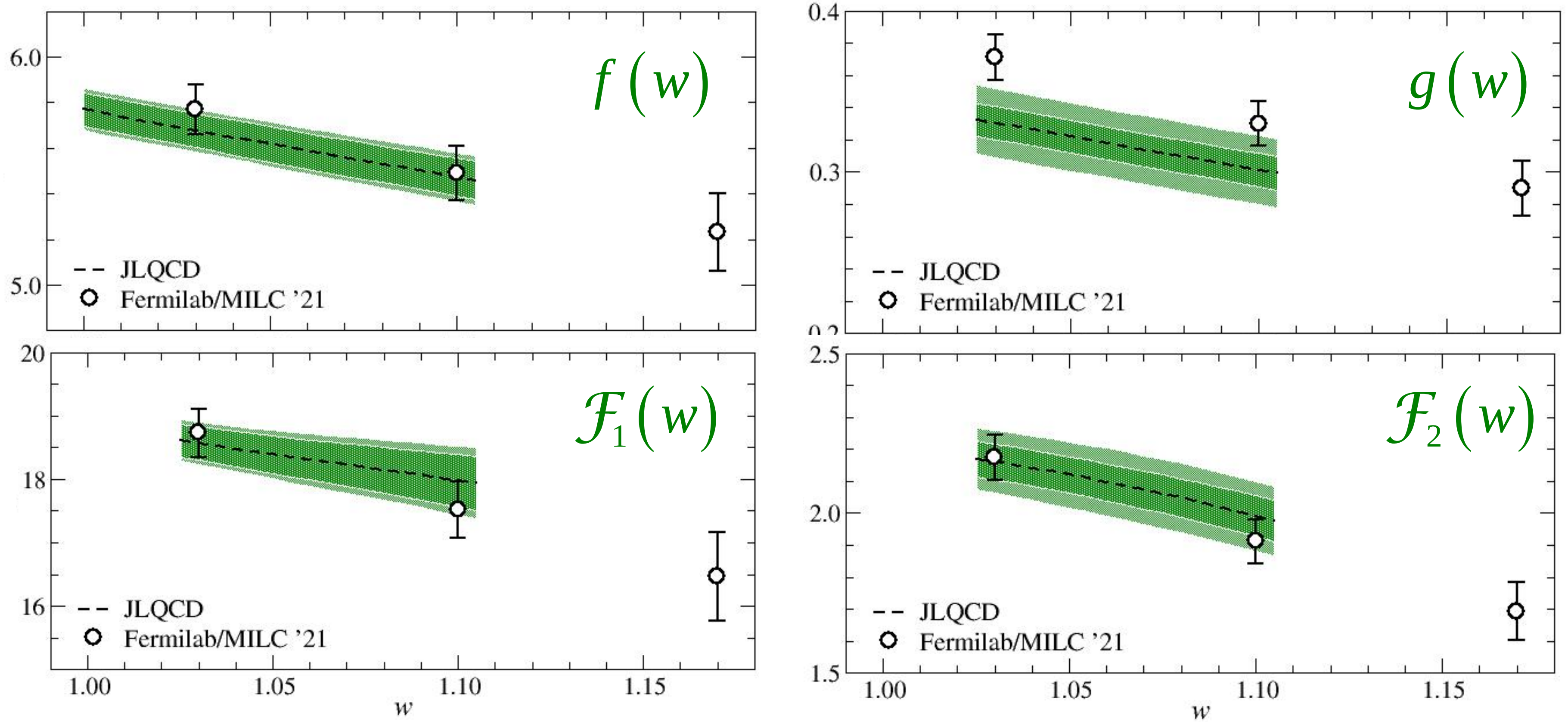
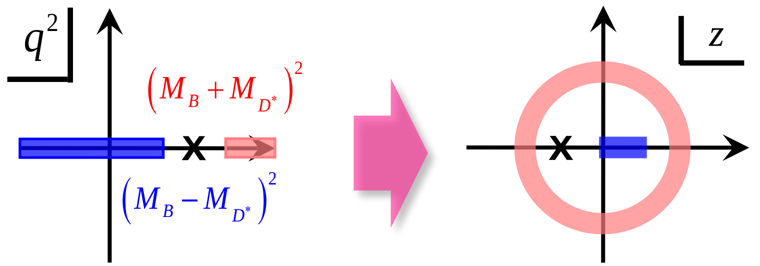



- relativistic FFs from HQET FFs
- Fermilab/MILC : w = 1.03, 1.07, 1.17 (more data available on arXiv)
- JLQCD : w = 1.000, 1.050, 1.100 ( $f$ ), 1.030, 1.065, 1.100 (other FFs)

# BGL parameterization of FFs

Boyd-Grinstein-Lebed parameterization '97

$$F(w) = \frac{1}{P_F(z)\phi_F(z)} \sum_n^{n_F} a_n^F z^n, \quad z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$



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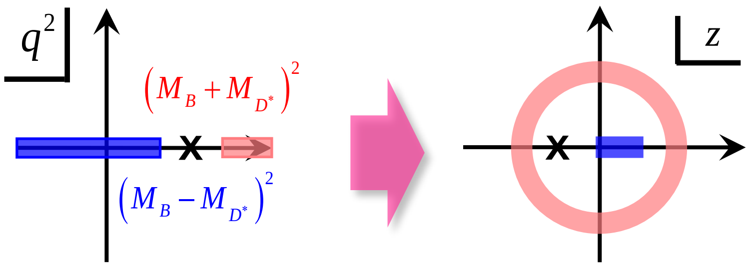
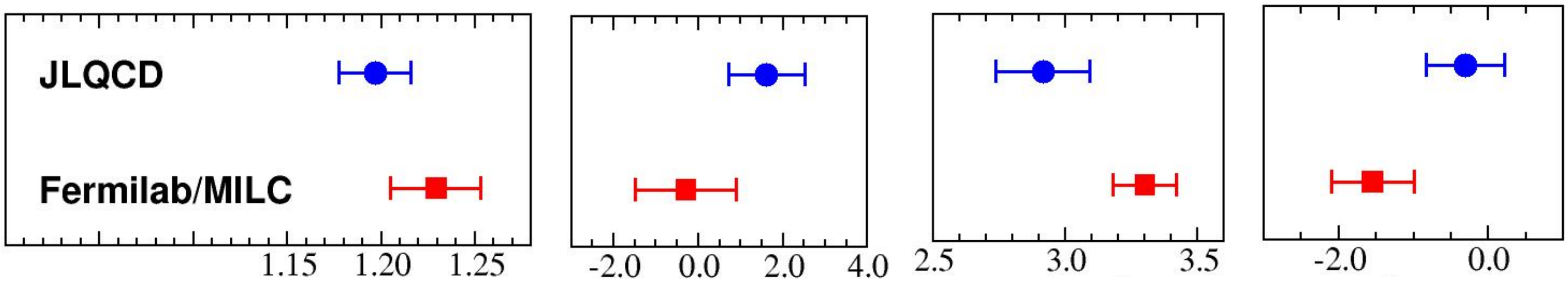



- JLQCD : $w$ = [1.0, 1.1], $q^2$ = [13.0, 10.7] $\longrightarrow$ $z$ =[0.000, 0.012]
- Baschke factors $P_F$, outer functions $\phi_F$ : same as Bigi-Gambino-Schacht '17

  $\Leftrightarrow$ hadronic susceptibilities $\chi_T$ from lattice QCD : Martinelli et al. '21
- w/ a kinematical constraint $\mathcal{F}_1(1) = (M_B\text{-}M_{D^*})\, f(1)$
- JLQCD : w/o unitarity constraint
- employ quadratic fits : $n_g$, $n_f$, $n_{\mathcal{F}1}$, $n_{\mathcal{F}2}$ = 2
- JLQCD : preliminary

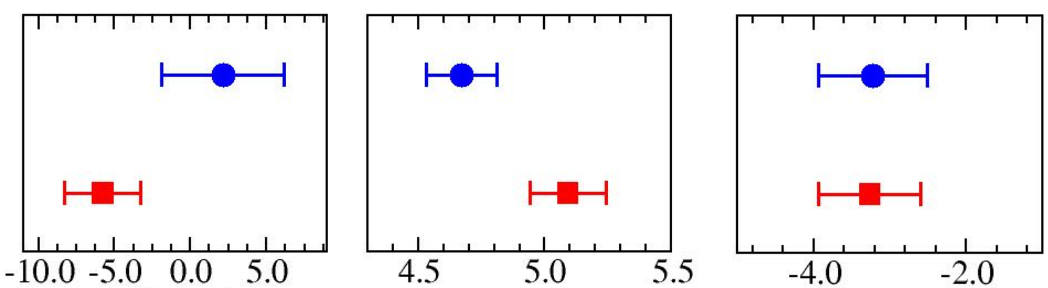
# BGL parametrization of FFs

coefficients $a_0^f \times 10^2$ $a_1^f \times 10^2$ $a_0^g \times 10^2$ $a_1^g \times 10^2$




- consistent within 2σ or so in spite of very different systematics
- JLQCD tends to favor smaller normalization and slope for $f$ and $g$ ($h_{A1}$ and $h_V$)
- JLQCD's analysis together w/ experimental data in progress

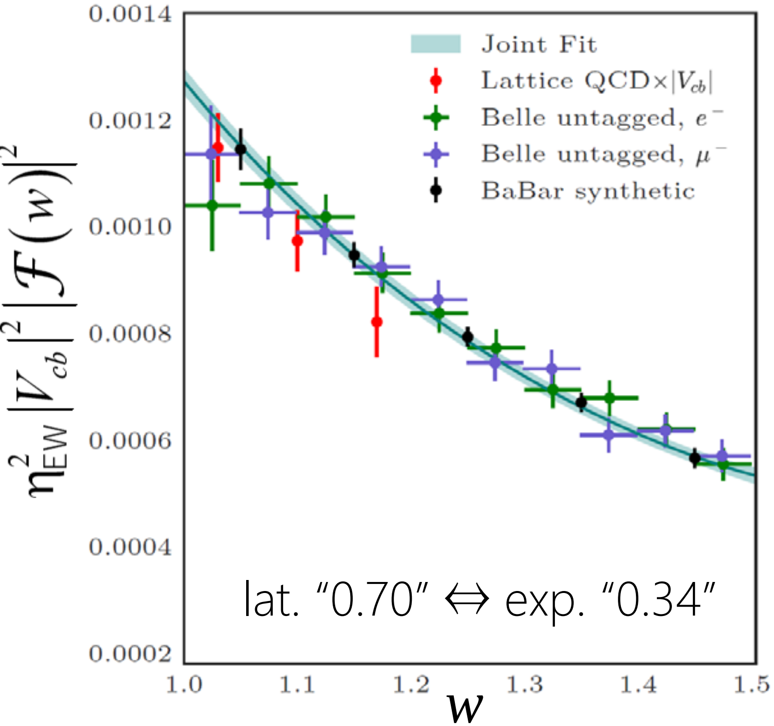
$a_1^{\mathcal{F}_1} \times 10^3$ $a_0^{\mathcal{F}_2} \times 10^2$ $a_1^{\mathcal{F}_2} \times 10$




- ≈ 2σ consistency also for $\mathcal{F}_1$ and $\mathcal{F}_2$
- data for $\mathcal{F}_2$ available $\Rightarrow B \to D^{(*)}\tau\nu$, $R(D^*)$

# Fermilab/MILC : impact on SM test

simultaneous fit to lat.+exp. data




$|V_{cb}| = 38.57(0.78) \times 10^3$

- consistent w/ previous exclusive calc.

  ⇒ $|V_{cb}|$ tension still remains ...

- slight tension in slope b/w lat. & exp.??

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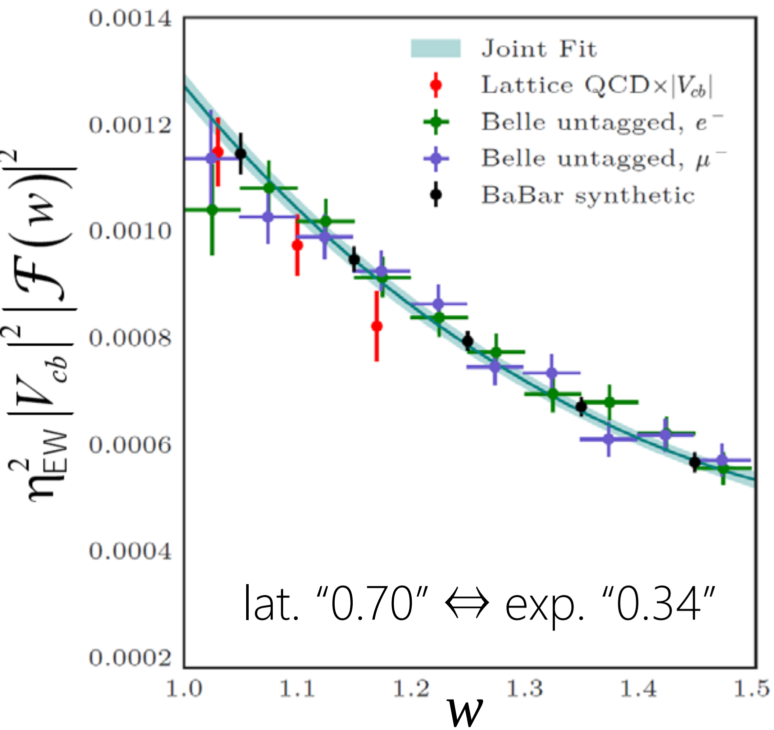


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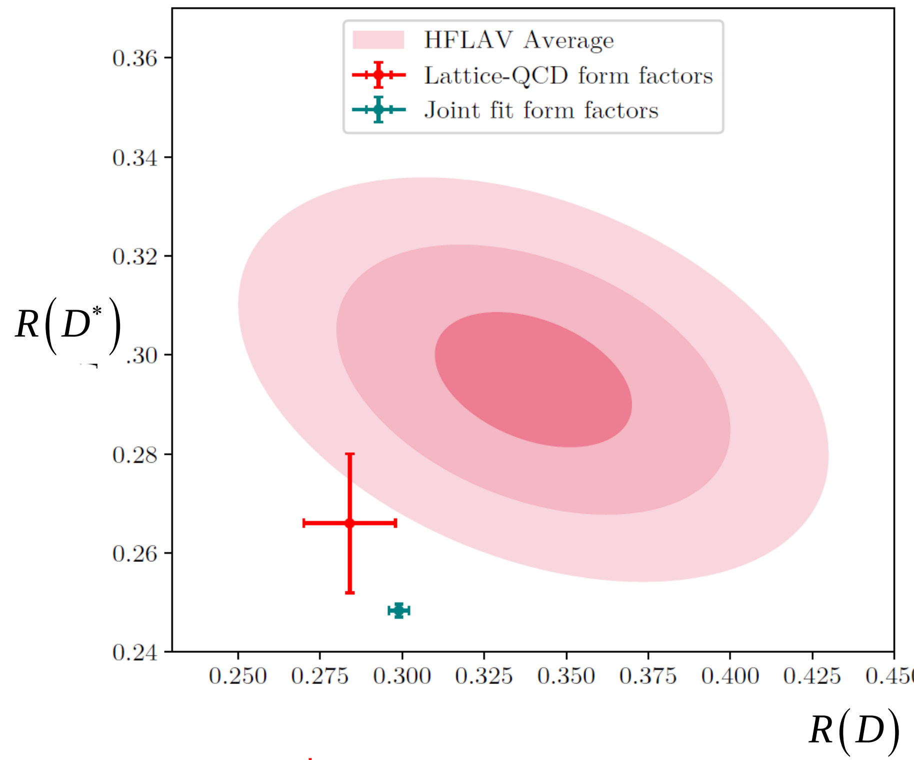
$R(D^{(*)})$ from lattice QCD, and + exp.

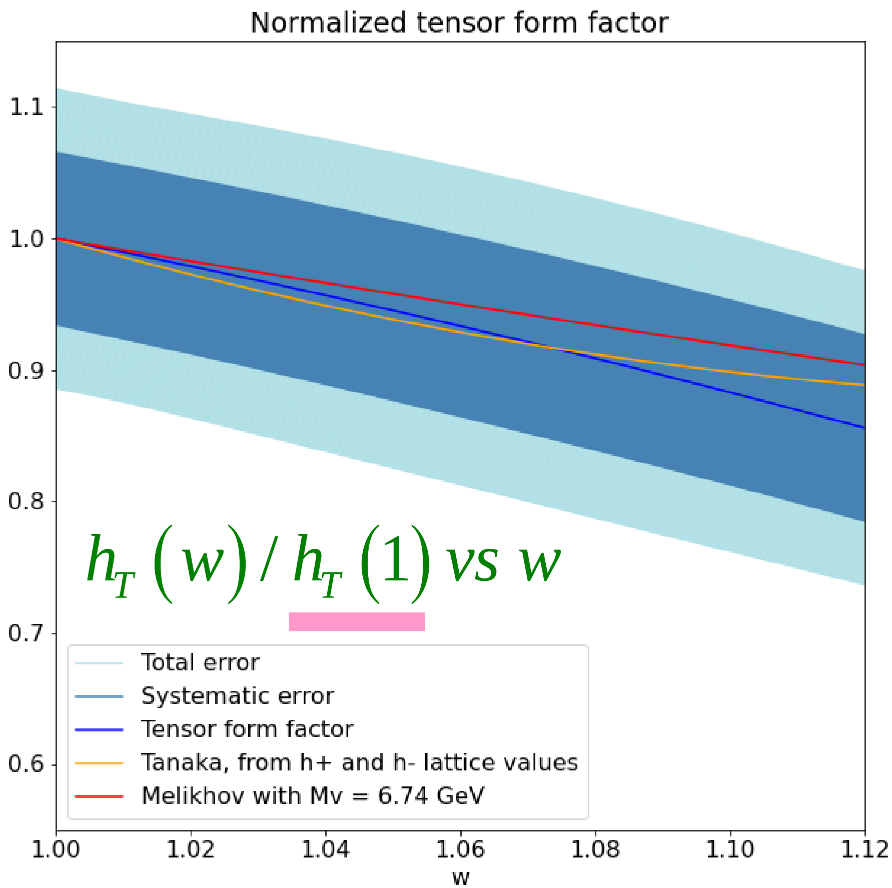



$R(D^*) = 0.266(14)$

- purely theoretical estimate
- *c.f.* w/ exp. 0.2484(13)

# beyond SM

$B \to D\ell\nu$ tensor FF (M. Faur [Paris ENS, internship] + Kou + JLQCD)




$$\langle D(p') | T_{\mu\nu} | B(p) \rangle = i \left( v'^{\mu} v^{\nu} - v'^{\nu} v^{\mu} \right) h_T(w)$$

- ✓ extraction of tensor FF
- ✓ continuum chiral extrapolation
- ✓ systematic uncertainties
- renormalization in progress

T. Ishikawa @ Lattice

- 10% stat. and 10% sys. errors
- consistent w/ phenomenology

useful input for BSM interpretation of B anomalies

# Summary

recent progress on $B \to D^{(*)} \ell \nu$ FFs from lattice QCD

- two independent calculations of $B \to D^* \ell \nu$ FFs @ $w > 1$
  - Fermilab/MILC 2105.14019;  JLQCD on-going
  - very different systematics : EFT and relativistic approaches
  - mild $a$, $m_q$ dependences $\Rightarrow$ controlled extrapolation
  - $\approx 2\sigma$ consistency in FFs
  - can be improved
    - Fermillab/MILC : more realistic $N_f$=4 ensembles
    - JLQCD : higher statistics / finer lattices

- BSM FFs for NP model interpretation of B anomalies
  - JLQCD $B \to D \ell \nu$ tenor FF  / expect more for the future