$B_{(s)} \to D_{(s)}^{(*)}$ and B_c Decays in Lattice QCD

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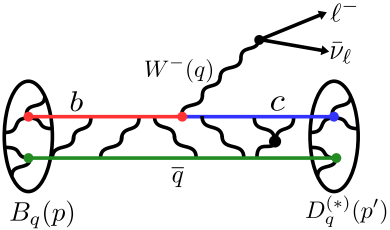
Background

Many interesting $B_{(s,c)}$ semileptonic decays currently under active investigation

- $B_c \to D_s \ell^+ \ell^-$ and $B_c \to D\ell\nu$[1]
- $B \to \pi/K$, $B_s \to K$ (see later talk by Andrew Lytle)
- Here, focus on $b \to c$ decays: $B_{(s)} \to D^{(*)}_{(s)} \ell\nu$, $B_c \to J/\psi \ell\nu$
  - Complementary determinations of V_{cb},
  - Comparison of observables sensitive to lepton flavor universality violation (LFUV) to experiment



Kinematic variables:

$$q^2 = (p - p')^2$$

$$w = \frac{p' \cdot p}{M_{B_q} M_{D_q^{(*)}}}$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

[1] 2111.06782

$P \to P$: $B_{(s)} \to D_{(s)}\ell\nu$

$$\frac{d\Gamma}{dq^2} = \mathcal{N}(q^2) \times \mathcal{G}^2(q^2)|V_{cb}|^2$$

where $\mathcal{G}(q^2)$ depends on nonperturbative QCD matrix elements, expressed in terms of form factors:

$$\mathcal{G}^2(q^2) = \left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_{(s)}}^2 |p_{D_{(s)}}|^2 f_+^{(s)2}(q^2) + \frac{3m_\ell^2}{8q^2}(M_{B_{(s)}}^2 - M_{D_{(s)}^2})^2 f_0^{(s)2}(q^2)$$

- Theoretical predictions boil down to computing form factors
  - 2 form factors within the Standard Model, only need $f_+^{(s)}$ for $\ell = e, \mu$
  - 1 additional tensor form factor for New Physics
- V_{cb} can be determined by comparing experimental value of $\eta_{\mathrm{EW}}\mathcal{G}(q^2_{\max})|V_{cb}|$ to lattice calculations of $\mathcal{G}(q^2_{\max})$
- Measurements of $R(D_{(s)})$ provide sensitivity to LFUV

$$R(D_{(s)}) = \frac{\Gamma\left(B_{(s)} \to D_{(s)}\tau\nu_\tau\right)}{\Gamma\left(B_{(s)} \to D_{(s)}\mu\nu_\mu\right)}$$

$P \to V$: $B_{(s)} \to D^*_{(s)}\ell\nu$, $B_c \to J/\psi\ell\nu$

Pseudoscalar to vector decay has more complicated structure in the SM:

$$\frac{d\Gamma}{dq^2} = \chi(q^2) \times \mathcal{F}^2(q^2)|V_{cb}|^2$$

$$\mathcal{F}^2(q^2) = \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\left(H_+^2(q^2) + H_-^2(q^2) + H_0^2(q^2)\right) + \frac{3m_\ell^2}{2q^2}H_t^2(q^2)\right]$$

Helicity amplitudes expressed in terms of form factors

$$\left\{H_+(q^2), H_-(q^2), H_0(q^2)\right\} \leftrightarrow \left\{A_1(q^2), A_2(q^2), V(q^2)\right\}$$
$$H_t(q^2) \propto A_0(q^2)$$

- Theoretical predictions more difficult for vector meson final state:
  - 4 form factors within the Standard Model
  - 3 additional tensor form factor for New Physics
- V_{cb} - compare experimental value of $\eta_{\mathrm{EW}}\mathcal{F}(q^2_{\mathrm{max}})|V_{cb}|$ to lattice calculations of $\mathcal{F}(q^2_{\mathrm{max}})$
  - preferred over $B_{(s)} \to D_{(s)}$ due to favorable kinematics near zero-recoil.
- $R(D^*)$
  - Sensitive to LFUV
  - Theory for $R(D^*)$ relies on experimental fits + HQET for A_0
  - On the lattice, typically use unphysically heavy pions and treat $D^* \to D\pi$ resonance using $\chi\mathrm{PT}$
- Lattice calculation of $B_s \to D_s^*$ and $B_c \to J/\psi$ FFs easier
  - Computational cost of propagators for $c < s << u/d$
  - J/ψ and D_s^* are 'gold-plated'

Overview of Lattice Results

- SM FFs for $B \to D\ell\nu$ available away from zero recoil[2]
- SM FFs for $B_s \to D_s\ell\nu$ now available across the full kinematic range, tensor FF available close to zero-recoil, with work also ongoing[3]
- SM FFs for $B \to D^*\ell\nu$ recently became available from Fermilab-MILC away from zero-recoil[4], with lattice calculations also underway by JLQCD as well as HPQCD.
- SM FFs for $B_s \to D_s^*\ell\nu$ and $B_c \to J/\psi\ell\nu$ available across full kinematic range from HPQCD[5]

[2] e.g. 1503.07237,1505.03925
[3] 1906.00701,1310.5238,2110.10061
[4] 2105.14019
[5] 2105.11433

Current Results

	Lattice only	Lattice+Exp[6]	Experiment	Tension
$R(D)$	0.293(4)[7]	0.299(3)	0.340(30)	1.4σ
$R(D^*)$	0.265(13)	0.2483(13)	0.295(14)	3.3σ
$R(D_s)$	0.299(5)	–	–	–
$R(D_s^*)$	0.244(8)	–	–	–
$R(J/\psi)$	0.258(4)	–	0.71(25)[8]	1.8σ

HFLAV average, Fermilab-MILC, HPQCD.

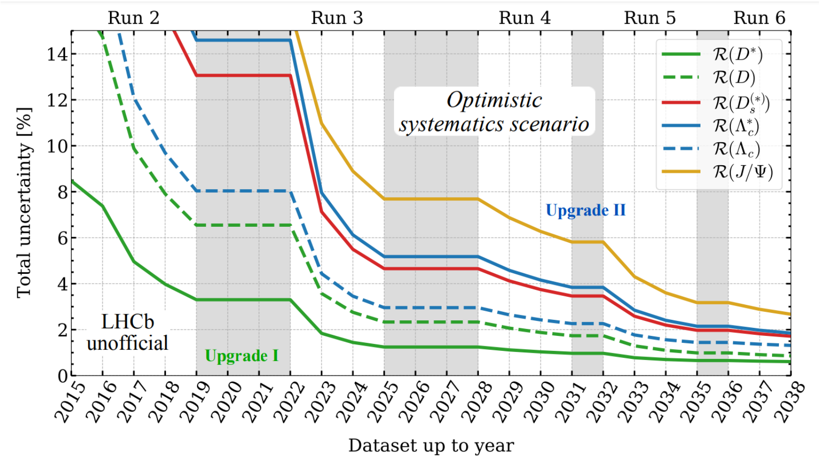
	V_{cb}	
$B \to D$	$39.58(94)_{\rm exp}(37)_{\rm th} \times 10^{-3}$	HFLAV
$B \to D^*$	$38.76(42)_{\rm exp}(55)_{\rm th} \times 10^{-3}$	
$B_s \to D_s^{(*)}$	$42.3(1.2)_{\rm exp}(1.2)_{\rm th} \times 10^{-3}$	LHCb (2001.03225)
$B \to X_c \ell\nu$	$42.16(51) \times 10^{-3}$	Bordone et al.(2107.00604)

[6] Assumes new physics only possible in semitauonic mode

[7] FLAG review

[8] LHCb-1711.05623

Experimental Outlook



- Need precise SM form factors across full kinematic range
  - Resolve discrepancy between inclusive and exclusive determinations of V_{cb}
  - Make first principles predictions for $R(D^*_{(s)})$ independent of experimental measurements
- Need tensor form factors to disentangle possible new physics effects

$b \to c$ Pseudoscalar to Vector Form Factors

In the standard model $\mathcal{F}(q^2)$ is a simple function of the form factors, $A_1(q^2)$, $A_0(q^2)$, $A_2(q^2)$ and $V(q^2)$, defined in terms of matrix elements. For example, for $B_s \to D_s^* \ell\nu$:

$$\langle D_s^*(p',\lambda)|\bar{c}\gamma^\mu b|B_s^0(p)\rangle = \frac{2iV(q^2)}{M_{B_s}+M_{D_s^*}}\varepsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu(p',\lambda)p'_\rho p_\sigma$$

$$\langle D_s^*(p',\lambda)|\bar{c}\gamma^\mu\gamma^5 b|B_s^0(p)\rangle = 2M_{D_s^*}A_0(q^2)\frac{\epsilon^*(p',\lambda)\cdot q}{q^2}q^\mu$$

$$+\,(M_{B_s}+M_{D_s^*})A_1(q^2)\Big[\epsilon^{*\mu}(p',\lambda) - \frac{\epsilon^*(p',\lambda)\cdot q}{q^2}q^\mu\Big]$$

$$-\,A_2(q^2)\frac{\epsilon^*(p',\lambda)\cdot q}{M_{B_s}+M_{D_s^*}}\Big[p^\mu + p'^\mu - \frac{M_{B_s}^2 - M_{D_s^*}^2}{q^2}q^\mu\Big]$$

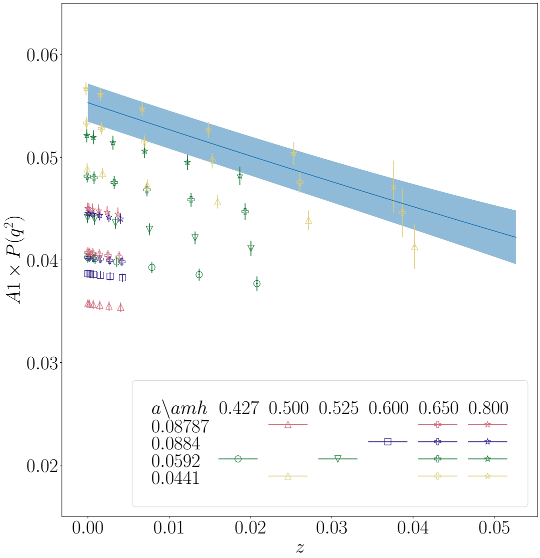
Form Factors Across the Full q^2 Range with Lattice QCD[9]

- Use HISQ action for all quarks - fully relativistic, small discretisation effects, nonperturbatively normalised currents
- Compute form factors at multiple daughter momenta, using multiple heavy masses ranging up to close to the physical mass
- Fit the form factor data including am_h discretisation effects, physical heavy mass dependence, and lattice spacing dependence
  - Here we first convert to z space, e.g.

$$P(q^2) \times A_1(q^2) = \sum_{n=0}^{3} a_n z^n(q^2)\mathcal{N}_n$$

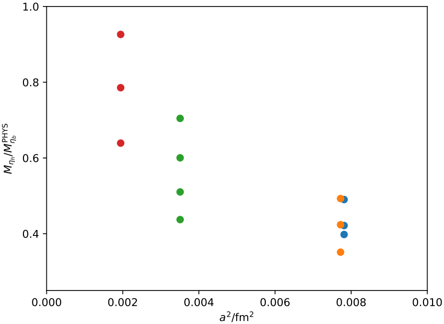
$$a_n = \sum_{j,k,l=0}^{3} b_n^{jkl} \left(\frac{2\Lambda_{\mathrm{QCD}}}{M_{\eta_h}}\right)^j \left(\frac{am_c^{\mathrm{val}}}{\pi}\right)^{2k} \left(\frac{am_h^{\mathrm{val}}}{\pi}\right)^{2l}$$

[9] $B_s \to D_s^*$:2105.11433, $B_c \to J/\psi$:2007.06957



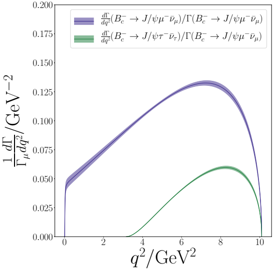
$P(q^2) \times A_1$ for $B_c \to J/\psi$, plotted in z space, showing the physical continuuum form factor as a blue band

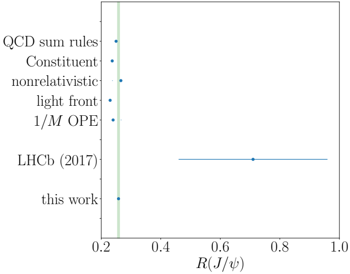
- We use the second generation MILC HISQ gauge configurations with u/d, s and c quarks in the sea.



- The subset of configurations we use include physical u/d quark masses, and have small lattice spacings allowing us to come very close to the physical b mass.

$B_c \to J/\psi$ Results - 2007.06956, 2007.06957



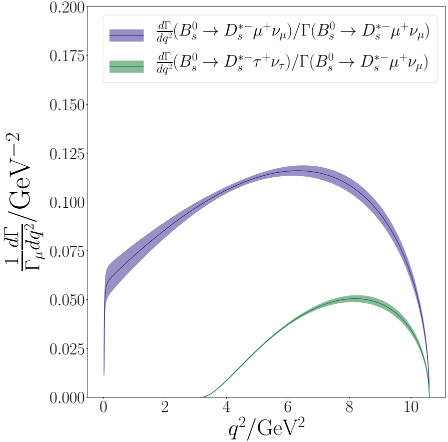


$$R(J/\psi) = 0.2582(38)$$

$$\Gamma(B_c^- \to J/\psi\mu^-\bar{\nu}_\mu)/\eta_{\mathrm{EW}}^2|V_{cb}|^2 = 1.73(12) \times 10^{13} s^{-1}$$

- Experimental results for $B_c \to J/\psi$ are currently much less precise than our lattice results, but expect this to improve in future.
- In addition to $R(J/\psi)$, other observables and ratios may be constructed with high precision from our form factor results
  - Can study the effect of NP couplings - full details in 2007.06956

$B_s \to D_s^*$ Results - 2105.11433



$$R(D_s^*) = 0.244(8)_{\mathrm{latt}}(4)_{\mathrm{EM}}$$

$$\Gamma(B_s^0 \to D_s^{*-}\mu^+\nu_\mu)/\eta_{\mathrm{EW}}^2|V_{cb}|^2 = 2.06(21) \times 10^{13} s^{-1}$$

$R(D_s^*)$, V_{cb}...

Many new lattice predictions for $B_s \to D_s^*$ quantities:

	This work	Exp.[10]	$B \to D^*$ [11]
$\frac{\Gamma(B_s^0 \to D_s^- \mu^+ \nu_\mu)}{\Gamma(B_s^0 \to D_s^{*-} \mu^+ \nu_\mu)}$	0.444(49)	0.464(45)	0.457(23)
$R(D^*_{(s)})$	0.244(8)	–	0.2483(13)
F_L	0.448(22)	–	0.464(10)
$\mathcal{A}_{\lambda_\tau} = -P_\tau$	0.514(18)	–	0.496(15)

- Can also infer a total experimental rate Γ from LHCb analysis of V_{cb} in 2001.03225, we can use this with our results to give a value of V_{cb}

$$|V_{cb}| = 43.0(2.1) \times 10^{-3}$$

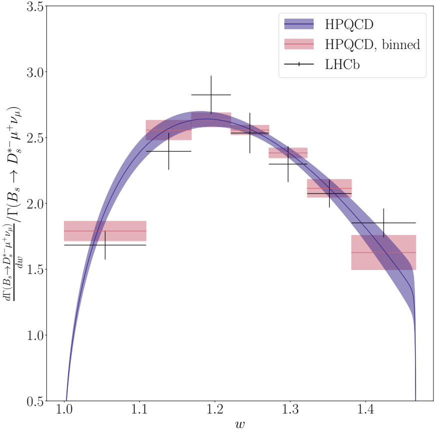
- Consistent with the result using lattice data only at zero-recoil.

[10] LHCb 2001.03225

[11] HFLAV 1909.12524,Bordone et. al 1908.09398

$B_s \to D_s^*$ Shape

We can compare the binned experimental results[12] for the $B_s \to D_s^*$ shape to our results



$$\chi^2/\text{dof} = 1.8$$

[12] LHCb:2003.08453

Summary

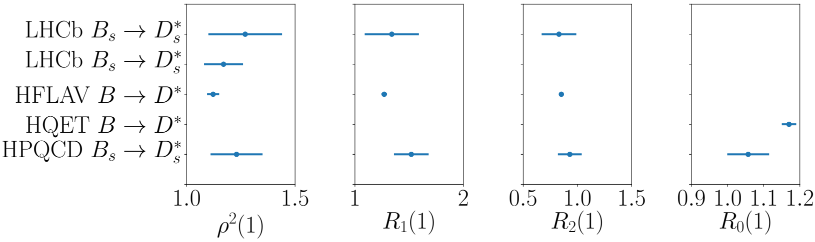
- Published lattice results for $B_c \to J/\psi$ form factors, corresponding experimental measurements are currently imprecise.
  - Experimental results for $B_c \to J/\psi$ decays are expected to become more precise
- Results for the $B_s \to D_s^*$ form factors now on arXiv
  - Model independent determinations of $R(D_s^*)$ and other observables
  - Model independent determination of $|V_{cb}|$, though ideally would use experimental results directly
- Work on $B \to D^*$, including Tensor form factors, now underway

Thanks for listening!

Backup Slides

$B_s \to D_s^*$ Shape Parameters

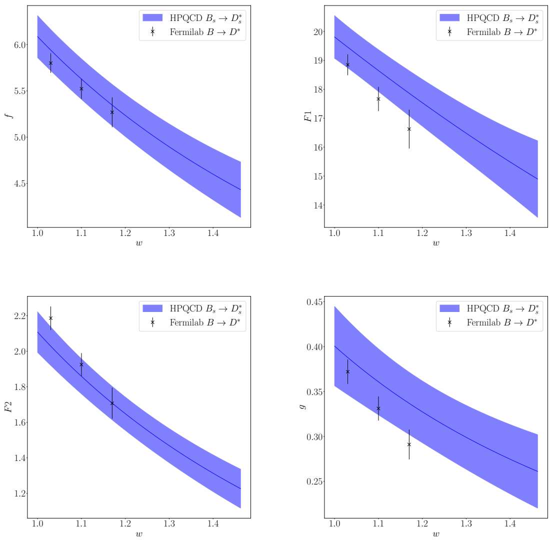
In the CLN parameterisation, the shape of the decay for massive leptons in the SM is fully described by the four parameters ρ^2, $R_1(1)$, $R_2(1)$ and $R_0(1)$, with ρ^2, $R_1(1)$, $R_2(1)$ determined from experiment and $R_0(1)$ known to NLO in HQET[13]



- Our results are broadly consistent with the measured values of ρ^2, $R_1(1)$ and $R_2(1)$ for $B_s \to D_s^*$, and with the NLO HQET value of $R_0(1)$.

[13] LHCb:2001.03225+2003.08453, HFLAV:1909.12524, HQET:1703.05330

Comparison to Fermilab-MILC $B \to D^*$ Form Factors



Comparison done in helicity-basis: $\chi^2/\mathrm{dof} = 1.8$