$B_{(s)} \rightarrow D_{(s)}^{(*)}$ and $B_c$ Decays in Lattice QCD

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Background

Many interesting $B_{(s,c)}$ semileptonic decays currently under active investigation

- $B_c \to D_s \ell^+ \ell^-$ and $B_c \to D \ell \nu$
- $B \to \pi/K, B_s \to K$ (see later talk by Andrew Lytle)
- Here, focus on $b \to c$ decays: $B_{(s)} \to D_{(s)}^{(*)} \ell \nu, B_c \to J/\psi \ell \nu$
  - Complementary determinations of $V_{cb}$,
  - Comparison of observables sensitive to lepton flavor universality violation (LFUV) to experiment

Kinematic variables:

$q^2 = (p - p')^2$

$w = \frac{p' \cdot p}{M_{B_q} M_{D_q}^{(*)}}$

$z = \frac{\sqrt{t+ - q^2} - \sqrt{t+ - t_0}}{\sqrt{t+ - q^2} + \sqrt{t+ - t_0}}$
\[ \frac{d\Gamma}{dq^2} = \mathcal{N}(q^2) \times \mathcal{G}^2(q^2)|V_{cb}|^2 \]

where \( \mathcal{G}(q^2) \) depends on nonperturbative QCD matrix elements, expressed in terms of form factors:

\[ \mathcal{G}^2(q^2) = \left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B(s)}^2 |p_{D(s)}|^2 f_+^{(s)}(q^2) + \frac{3 m_\ell^2}{8q^2} (M_{B(s)}^2 - M_{D(s)}^2)^2 f_0^{(s)}(q^2) \]

- Theoretical predictions boil down to computing form factors
  - 2 form factors within the Standard Model, only need \( f_+^{(s)} \) for \( \ell = e, \mu \)
  - 1 additional tensor form factor for New Physics
- \( V_{cb} \) can be determined by comparing experimental value of \( \eta_{EW} \mathcal{G}(q_{max}^2)|V_{cb}| \) to lattice calculations of \( \mathcal{G}(q_{max}^2) \)
- Measurements of \( R(D(s)) \) provide sensitivity to LFUV

\[ R(D(s)) = \frac{\Gamma (B(s) \rightarrow D(s)\tau\nu_\tau)}{\Gamma (B(s) \rightarrow D(s)\mu\nu_\mu)} \]
\[ P \rightarrow V: \ B(s) \rightarrow D_s^* \ell \nu, \ B_c \rightarrow J/\psi \ell \nu \]

Pseudoscalar to vector decay has more complicated structure in the SM:

\[
\frac{d\Gamma}{dq^2} = \chi(q^2) \times F^2(q^2)|V_{cb}|^2
\]

\[
F^2(q^2) = \left[ \left( 1 + \frac{m_{\ell}^2}{2q^2} \right) (H_+^2(q^2) + H_-(q^2) + H_0^2(q^2)) + \frac{3m_{\ell}^2}{2q^2} H_t^2(q^2) \right]
\]

Helicity amplitudes expressed in terms of form factors

\[
\{ H_+(q^2), H_-(q^2), H_0(q^2) \} \leftrightarrow \{ A_1(q^2), A_2(q^2), V(q^2) \}
\]

\[ H_t(q^2) \propto A_0(q^2) \]
Theoretical predictions more difficult for vector meson final state:
- 4 form factors within the Standard Model
- 3 additional tensor form factor for New Physics

\[ V_{cb} \] - compare experimental value of \( \eta_{EW} \mathcal{F}(q_{\text{max}}^2)|V_{cb}| \) to lattice calculations of \( \mathcal{F}(q_{\text{max}}^2) \)
- preferred over \( B(s) \rightarrow D(s) \) due to favorable kinematics near zero-recoil.

\( R(D^*) \)
- Sensitive to LFUV
- Theory for \( R(D^*) \) relies on experimental fits + HQET for \( A_0 \)
- On the lattice, typically use unphysically heavy pions and treat \( D^* \rightarrow D\pi \) resonance using \( \chi \)PT

Lattice calculation of \( B_s \rightarrow D_s^* \) and \( B_c \rightarrow J/\psi \) FFs easier
- Computational cost of propagators for \( c < s << u/d \)
- \( J/\psi \) and \( D_s^* \) are ‘gold-plated’
Overview of Lattice Results

- SM FFs for $B \rightarrow D \ell \nu$ available away from zero recoil\(^2\)
- SM FFs for $B_s \rightarrow D_s \ell \nu$ now available across the full kinematic range, tensor FF available close to zero-recoil, with work also ongoing\(^3\)
- SM FFs for $B \rightarrow D^* \ell \nu$ recently became available from Fermilab-MILC away from zero-recoil\(^4\), with lattice calculations also underway by JLQCD as well as HPQCD.
- SM FFs for $B_s \rightarrow D^*_s \ell \nu$ and $B_c \rightarrow J/\psi \ell \nu$ available across full kinematic range from HPQCD\(^5\)

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\(^2\) e.g. 1503.07237, 1505.03925
\(^3\) 1906.00701, 1310.5238, 2110.10061
\(^4\) 2105.14019
\(^5\) 2105.11433
### Current Results

<table>
<thead>
<tr>
<th></th>
<th>Lattice only</th>
<th>Lattice+Exp(^6)</th>
<th>Experiment</th>
<th>Tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R(D))</td>
<td>0.293(4)(^7)</td>
<td>0.299(3)</td>
<td>0.340(30)</td>
<td>1.4(\sigma)</td>
</tr>
<tr>
<td>(R(D^*))</td>
<td>0.265(13)</td>
<td>0.2483(13)</td>
<td>0.295(14)</td>
<td>3.3(\sigma)</td>
</tr>
<tr>
<td>(R(D_s))</td>
<td>0.299(5)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(R(D_s^*))</td>
<td>0.244(8)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(R(J/\psi))</td>
<td>0.258(4)</td>
<td>–</td>
<td>0.71(25)(^8)</td>
<td>1.8(\sigma)</td>
</tr>
</tbody>
</table>

HFLAV average, Fermilab-MILC, HPQCD.

<table>
<thead>
<tr>
<th></th>
<th>(V_{cb})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B \to D)</td>
<td>39.58(94)(<em>{\text{exp}})(</em>{37})(_{\text{th}} \times 10^{-3}) HFLAV</td>
</tr>
<tr>
<td>(B \to D^*)</td>
<td>38.76(42)(<em>{\text{exp}})(</em>{55})(_{\text{th}} \times 10^{-3})</td>
</tr>
<tr>
<td>(B_s \to D_s^*)</td>
<td>42.3(1.2)(<em>{\text{exp}})(</em>{1.2})(_{\text{th}} \times 10^{-3}) LHCb (2001.03225)</td>
</tr>
<tr>
<td>(B \to X_{c} \ell\nu)</td>
<td>42.16(51) (_{\times 10^{-3}}) Bordone et al.(2107.00604)</td>
</tr>
</tbody>
</table>

\(^6\)Assumes new physics only possible in semitauonic mode

\(^7\)FLAG review

\(^8\)LHCb-1711.05623
Experimental Outlook

![Graph showing optimistic systematic scenario for different datasets up to 2038.](image-url)
Need precise SM form factors across full kinematic range
  - Resolve discrepancy between inclusive and exclusive determinations of $V_{cb}$
  - Make first principles predictions for $R(D^*_s)$ independent of experimental measurements

Need tensor form factors to disentangle possible new physics effects
In the standard model $\mathcal{F}(q^2)$ is a simple function of the form factors, $A_1(q^2)$, $A_0(q^2)$, $A_2(q^2)$ and $V(q^2)$, defined in terms of matrix elements. For example, for $B_s \to D_s^* \ell \nu$:

\[
\langle D_s^*(p', \lambda) | \bar{c} \gamma^\mu b | B_s^0(p) \rangle = \frac{2i V(q^2)}{M_{B_s} + M_{D_s^*}} \epsilon^{\mu \nu \rho \sigma} \epsilon^*(p', \lambda) p'_\rho p_\sigma \\
\langle D_s^*(p', \lambda) | \bar{c} \gamma^\mu \gamma^5 b | B_s^0(p) \rangle = 2M_{D_s^*} A_0(q^2) \frac{\epsilon^*(p', \lambda) \cdot q}{q^2} q^\mu \\
+ (M_{B_s} + M_{D_s^*}) A_1(q^2) \left[ \epsilon^*_{\mu \lambda} (p', \lambda) - \frac{\epsilon^*(p', \lambda) \cdot q}{q^2} q^\mu \right] \\
- A_2(q^2) \frac{\epsilon^*(p', \lambda) \cdot q}{M_{B_s} + M_{D_s^*}} \left[ p^\mu + p'^{\mu} - \frac{M_{B_s}^2 - M_{D_s^*}^2}{q^2} q^\mu \right]
\]
Form Factors Across the Full $q^2$ Range with Lattice QCD\textsuperscript{9}

- Use HISQ action for all quarks - fully relativistic, small discretisation effects, nonperturbatively normalised currents
- Compute form factors at multiple daughter momenta, using multiple heavy masses ranging up to close to the physical mass
- Fit the form factor data including $am_h$ discretisation effects, physical heavy mass dependence, and lattice spacing dependence
  - Here we first convert to $z$ space, e.g.

$$P(q^2) \times A_1(q^2) = \sum_{n=0}^{3} a_n z^n(q^2) N_n$$

$$a_n = \sum_{j,k,l=0}^{3} b_n^{jkl} \left( \frac{2\Lambda_{\text{QCD}}}{M_{\eta_h}} \right)^j \left( \frac{am^\text{val}_c}{\pi} \right)^{2k} \left( \frac{am^\text{val}_h}{\pi} \right)^{2l}$$

\textsuperscript{9}Bs \rightarrow D_s^*:2105.11433, B_c \rightarrow J/\psi :2007.06957
$P(q^2) \times A_1$ for $B_c \rightarrow J/\psi$, plotted in $z$ space, showing the physical continuum form factor as a blue band.
We use the second generation MILC HISQ gauge configurations with $u/d$, $s$ and $c$ quarks in the sea.

The subset of configurations we use include physical $u/d$ quark masses, and have small lattice spacings allowing us to come very close to the physical $b$ mass.
$B_c \rightarrow J/\psi$ Results - 2007.06956, 2007.06957

\[ R(J/\psi) = 0.2582(38) \]
\[ \Gamma(B_c^- \rightarrow J/\psi \mu^- \bar{\nu}_\mu) / \eta_{EW}^2 |V_{cb}|^2 = 1.73(12) \times 10^{13} \text{s}^{-1} \]

- Experimental results for $B_c \rightarrow J/\psi$ are currently much less precise than our lattice results, but expect this to improve in future.
- In addition to $R(J/\psi)$, other observables and ratios may be constructed with high precision from our form factor results
  - Can study the effect of NP couplings - full details in 2007.06956
$B_s \rightarrow D_s^*$ Results - 2105.11433

\[
R(D_s^*) = 0.244(8)_{\text{latt}}(4)_{\text{EM}}
\]

\[
\Gamma(B_s^0 \rightarrow D_{s^-}^* \mu^+ \nu_{\mu})/\eta_{\text{EW}}^2 |V_{cb}|^2 = 2.06(21) \times 10^{13} \text{s}^{-1}
\]
$R(D_{s}^*)$, $V_{cb}$ ...

Many new lattice predictions for $B_s \rightarrow D_{s}^*$ quantities:

<table>
<thead>
<tr>
<th></th>
<th>This work</th>
<th>Exp.$^{10}$</th>
<th>$B \rightarrow D^*$ $^{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(B_s^0 \rightarrow D_s^- \mu^+ \nu_{\mu})$</td>
<td>0.444(49)</td>
<td>0.464(45)</td>
<td>0.457(23)</td>
</tr>
<tr>
<td>$\Gamma(B_s^0 \rightarrow D_{s}^{*-} \mu^+ \nu_{\mu})$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R(D_{s}^*)$</td>
<td>0.244(8)</td>
<td>–</td>
<td>0.2483(13)</td>
</tr>
<tr>
<td>$F_L$</td>
<td>0.448(22)</td>
<td>–</td>
<td>0.464(10)</td>
</tr>
<tr>
<td>$A_{\lambda_{\tau}} = -P_{\tau}$</td>
<td>0.514(18)</td>
<td>–</td>
<td>0.496(15)</td>
</tr>
</tbody>
</table>

► Can also infer a total experimental rate $\Gamma$ from LHCb analysis of $V_{cb}$ in 2001.03225, we can use this with our results to give a value of $V_{cb}$

$$|V_{cb}| = 43.0(2.1) \times 10^{-3}$$

► Consistent with the result using lattice data only at zero-recoil.

$^{10}$LHCb 2001.03225  
$^{11}$HFLAV 1909.12524,Bordone et. al 1908.09398
$B_s \rightarrow D_s^*$ Shape

We can compare the binned experimental results\textsuperscript{12} for the $B_s \rightarrow D_s^*$ shape to our results

\[ \chi^2 / \text{dof} = 1.8 \]

\textsuperscript{12}LHCb:2003.08453
Summary

- Published lattice results for $B_c \rightarrow J/\psi$ form factors, corresponding experimental measurements are currently imprecise.
  - Experimental results for $B_c \rightarrow J/\psi$ decays are expected to become more precise

- Results for the $B_s \rightarrow D_s^*$ form factors now on arXiv
  - Model independent determinations of $R(D_s^*)$ and other observables
  - Model independent determination of $|V_{cb}|$, though ideally would use experimental results directly

- Work on $B \rightarrow D^*$, including Tensor form factors, now underway

Thanks for listening!
Backup Slides
$B_s \to D_s^*$ Shape Parameters

In the CLN parameterisation, the shape of the decay for massive leptons in the SM is fully described by the four parameters $\rho^2$, $R_1(1)$, $R_2(1)$ and $R_0(1)$, with $\rho^2$, $R_1(1)$, $R_2(1)$ determined from experiment and $R_0(1)$ known to NLO in HQET\textsuperscript{13}

\begin{itemize}
  \item LHCb $B_s \to D_s^*$
  \item LHCb $B_s \to D_s^*$
  \item HFLAV $B \to D^*$
  \item HQET $B \to D^*$
  \item HPQCD $B_s \to D_s^*$
\end{itemize}

\begin{itemize}
  \item $\rho^2(1)$
  \item $R_1(1)$
  \item $R_2(1)$
  \item $R_0(1)$
\end{itemize}

Our results are broadly consistent with the measured values of $\rho^2$, $R_1(1)$ and $R_2(1)$ for $B_s \to D_s^*$, and with the NLO HQET value of $R_0(1)$.

Comparison to Fermilab-MILC $B \rightarrow D^*$ Form Factors

Comparison done in helicity-basis: $\chi^2$/dof = 1.8