$B \to \pi/K$ and $B_s \to K$ decays from LQCD

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Intro & Motivation

• Semileptonic decays are a rich source of information for determining CKM matrix elements.

• Lattice data a critical source of input for testing the CKM paradigm.

• With new experimental and theoretical data on the horizon, these are interesting times!

Fig. courtesy M. Bona
Outline

• Intro & Motivation

• Review of lattice results (cf FLAG review 2111.09849)
  ▶ $B \to \pi$
  ▶ $B_s \to K$
  ▶ $B \to K$

• FNAL/MILC all-HISQ semileptonic decays
  ▶ Calculation framework
  ▶ Preliminary results

• Summary & Outlook
\[ B \rightarrow \pi \ell \nu \]
\[ B \rightarrow \pi \ell \nu \]

- Process gives the most precise determination of \(|V_{ub}|\).

- Challenges:
  - Simulating at physical pion masses / quantifying chiral behavior
  - Large pion recoil, lattice results best near \(q_{\text{max}}^2\)

- Particularly difficult calculation requiring significant time/resources.

- Expt’l reconstruction best at low \(q^2\), resulting in model dependence to bridge low–high \(q^2\).
$B \to \pi \ell \nu$ - Lattice results

Three published results quoted by FLAG, from 2015 or earlier.

- hep-lat/0601021 HPQCD, NRQCD $b$ on $n_f = 2 + 1$ staggered

- 1501.05373 RBC/UKQCD, RHQ $b$ on $n_f = 2 + 1$ domain wall

- 1503.07839 FNAL/MILC, Fermilab $b$ on $n_f = 2 + 1$ staggered (asqtad)
\[ B \rightarrow \pi \ell \nu - \text{In progress/proceedings} \]

- **1912.09946** RBC/UKQCD extend to a third lattice spacing \( a^{-1} \approx 2.77 \text{ GeV} \) w/ lightest pion mass \( m_{\pi} \approx 234 \text{ MeV}. \)

- **1912.02409** JLQCD (new calculation) Moebius domain-wall three lattice spacings \( a^{-1} \approx 2.4, 3.6, 4.5 \text{ GeV} \), lightest \( m_{\pi} \approx 225 \text{ MeV}. \ am_h < 0.7 \rightarrow m_h \approx 2.44m_c \)

- **1912.13358** FNAL/MILC Fermilab \( b \) on staggered HISQ, successor calculation to 1503.07839 asqtad.

- **2111.05184** FNAL/MILC all-HISQ calculation (see part II)
$B \rightarrow \pi \ell \nu$ - Takeaways

- Three published calculations (2015 and earlier) from HPQCD, RBC/UKQCD, and FNAL/MILC.

- Differing treatments of $b$ and different discretizations, overall agreement in continuum among these.

- Combining results FLAG obtains: $|V_{ub}| = 3.73(14) \times 10^{-3}$

- Several in progress calculations - new lattice spacings (RBC/UKQCD), new ensembles (FNAL/MILC), new groups (JLQCD) → expect new results in coming years.
$B_s \rightarrow K\ell\nu$
$B_s \rightarrow K\ell\nu$

- Computationally similar to $B \rightarrow \pi$ (spectator $l \rightarrow s$).

- Kaon mass easier to access $\rightarrow$ Systematics related to chiral extrapolation smaller

- $B_s \rightarrow K\mu\nu$ measured for the first time by LHCb! 2012.05143

- Normalized by $B_s \rightarrow D_s\mu\nu$, gives new constraint on $|V_{ub}|/|V_{cb}|$, competitive with constraint from baryon decay.
**$B_s \rightarrow K \ell \nu$ - Lattice results**

- **1501.05373** RBC/UKQCD, RHQ $b$ on $n_f = 2 + 1$ domain wall
- **1406.2279,1808.09285** HPQCD, NRQCD $b$ on $n_f = 2 + 1$ staggered (asqtad)
- **1901.02561** FNAL/MILC, Fermilab $b$ on $n_f = 2 + 1$ staggered (asqtad) + ratio with $B_s \rightarrow D_s$
$B_s \rightarrow K \ell \nu$ - In progress/proceedings

- 1912.09946, 2012.04323 RBC/UKQCD extend to a third lattice spacing $a^{-1} \approx 2.77$ GeV w/ lightest pion mass $m_\pi \approx 234$ MeV.

- 1912.13358 FNAL/MILC Fermilab $b$ on staggered HISQ, successor calculation to 1901.02561 asqtad.

- 2111.05184 FNAL/MILC all-HISQ calculation (see part II)
• New measurements from LHCb combined with lattice form factor predictions give new CKM constraint on $|V_{ub}|/|V_{cb}|$:

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.0946(30)_{\text{stat}}(25)_{\text{syst}}(13)_{D_s}^{(68)}_{\text{FF}}$$

• $B_s \to K \ell \nu$ at low $q^2$ needed from lattice!

• Total uncertainty can be reduced from the lattice side.
$B \rightarrow K\ell\ell$
**$B \to K\ell\ell$**

- FCNC interaction potentially sensitive to new physics.
- $b \to s$ transition forbidden at tree level – considerably more complicated effective Hamiltonian.
- Most important contributions still come from $\langle K|\mathcal{O}|B\rangle$ matrix elements, with $\mathcal{O} = V, T \to f_{0}(q^2), f_{+}(q^2), f_{T}(q^2)$.
$B \rightarrow K$ - Lattice results

- 1306.2384, 1306.0434 HPQCD, NRQCD $b$ on $n_f = 2 + 1$ staggered (asqtad), HISQ valence

- 1509.06235, 1510.02349 FNAL/MILC, Fermilab $b$ on $n_f = 2 + 1$ staggered (asqtad)
$B \rightarrow K\ell\ell$ - In progress/proceedings

- HPQCD all-HISQ calculation Lattice 2021
  https://indico.cern.ch/event/1006302/contributions/4375436/

- 1912.13358 FNAL/MILC Fermilab $b$ on staggered HISQ, successor calculation to 1509.06235 asqtad.

- 2111.05184 FNAL/MILC all-HISQ calculation (see part II)
$B \rightarrow K\ell\ell$ - Takeaways

- Important process sensitive to new physics.
- Lattice calculations from HPQCD (2013) and FNAL-MILC (2015), some tension in the tensor form factor.
- Additional contributions from lattice needed!
FNAL-MILC all-HISQ semileptonic decays
Carleton DeTar
Elvira Gámiz
Steve Gottlieb
William Jay
Aida El-Khadra
Andreas Kronfeld
Jim Simone
Alejandro Vaquero
Heavy quarks

Treatment of \(c\) and especially \(b\) quarks challenging in lattice simulation due to lattice artifacts which grow as \((am_h)^n\)

- May use an effective theory framework to handle the \(b\) quark.
  - Fermilab method, RHQ, OK, NRQCD
  - Pros: Solves problem w/ \(am_h\) artifacts.
  - Cons: Requires matching, can still have \(ap\) artifacts.

- Also possible to use relativistic fermion provided \(a\) is sufficiently small \(am_c \ll 1, am_b < 1\).
  - Use improved actions e.g. \(O(a^2) \rightarrow O(\alpha_s a^2)\)
  - Pros: Absolutely normalised current, straightforward continuum extrap.
  - Cons: Numerically expensive, extrapolate \(m_h \rightarrow m_b\).
allhisq simulations

• Here we simulate *all* quarks with the HISQ action.

• Unified treatment for wide range of $B_{(s)}$ (and $D_{(s)}$) to pseudoscalar transitions
  ▶ $B_{(s)} \rightarrow D_{(s)}$
  ▶ $B_{(s)} \rightarrow K$
  ▶ $B \rightarrow \pi$

• Ensembles with (HISQ) sea quarks down to physical at each lattice spacing.

• Enables correlated studies of $f_f$ ratios.

See our 2021 Lattice proceeding for more details! 2111.05184
• HISQ fermion action.
  ▶ Discretization errors begin at $\mathcal{O}(\alpha_s a^2)$.
  ▶ Designed for simulating heavy quarks ($m_c$ and higher at current lattice spacings).

• Symanzik-improved gauge action, takes into account $\mathcal{O}(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]

• Multiple lattice spacings down to $\sim 0.042$ (now 0.03) fm.

• Effects of $u/d$, $s$, and $c$ quarks in the sea.

• Multiple light-quark input parameters down to physical pion mass.
  ▶ Chiral fits.
  ▶ Reduce statistical errors.
MILC ensemble parameters

\[ \approx a^2 (\text{fm}^2) \]

\[ M_\pi (\text{MeV}) \]

- $0.0$ (0.03)$^2$
- $0.042)^2$
- $0.06)^2$
- $0.09)^2$
- $0.12)^2$
- $0.15)^2$

\[ 135 \ 200 \ 300 \ M_\pi \ (\text{MeV}) \]
all hisq $b$

- Use a heavy valence mass $h$ as a proxy for the $b$ quark.

- Work at a range of $m_h$, with $am_c < am_h \lesssim 1$ on each ensemble. On sufficiently fine ensembles, $m_h$ is near to $m_b$ (e.g. $m_b$ at $am_h \approx 0.65$ on $a = 0.03$ fm).

- Map out physical dependence on $m_h$, remove discretisation effects $\sim (am_h)^{2n}$ using information from several ensembles. Extrapolate results $a^2 \to 0, m_h \to m_b$. 
Preliminary results
Two point functions

Consider $B_{(s)} \rightarrow K$ decays for $a = 0.06 \text{ fm}$, $m_l/m_s = 0.1$.

- Compute $H_{(s)}$ mesons at a range of $am_H$ values:

- Compute $K$ mesons for a range of momenta:
Three point functions

- Generate three-point functions for scalar, vector, and tensor current insertions, $\langle K(T) J(t) H^\dagger(s)(0) \rangle$.
- Fit simultaneously with two-point functions to extract the matrix elements of interest \( \rightarrow \langle K | J | H(s) \rangle \)

Scalar form factor extracted directly from scalar current:

$$f_0^{(s)}(q^2) = \frac{m_b - m_{s(u)}}{M^2_{H(s)} - M^2_K} \langle K | S | H(s) \rangle$$
\[ B_s \rightarrow K: f_0(q^2) \]

- \( a = 0.088 \) fm: \( m_h = 1.5, 2, 2.5 \, m_c \)
- \( a = 0.057 \) fm: \( m_h = 2, 2.5, 3 \, m_c \)
- Good precision out to \( p = 300 \)
$B \rightarrow K: \ f_0(q^2)$

- $a = 0.088 \text{ fm}: \ m_h = 1.5, 2, 2.5 \text{ } m_c$
- $a = 0.057 \text{ fm}: \ m_h = 2, 2.5, 3 \text{ } m_c$
- Increasing stat error at large momentum.
$B_s \to K$ at zero-recoil

- Good statistical control ($a = 0.042$ fm stats still limited).
- Small disc. effects even for $am \gtrsim 1$. 
**B → K** at zero-recoil

- Good statistical control \((a = 0.042 \text{ fm stats still limited})\).
- Small disc. effects even for \(am \gtrsim 1\).
- Add’l structure consists of light quark, heavy quark, and lattice spacing dependences.
• Unified treatment for range of semileptonic decays.

• HISQ action used for all quarks.

• Good statistical precision (percent-level or less) achieved.

• Small discretization effects.

• Will permit interpolation in both $m_l$ and $m_h$. 
• Reviewed lattice results for \( B \to \pi/K \) and \( B_s \to K \)

• Each of these processes plays an important role in constraining the flavor sector.
  - \( B \to \pi \): gives precision \( |V_{ub}| \)
  - \( B_s \to K \): lattice + new LHCb result constrains \( |V_{ub}|/|V_{cb}| \)
  - \( B \to K \): FCNC process sensitive to new physics, current tensions w/ SM predictions.

• Good quality results from lattice community, with more on the way.
Thank you!