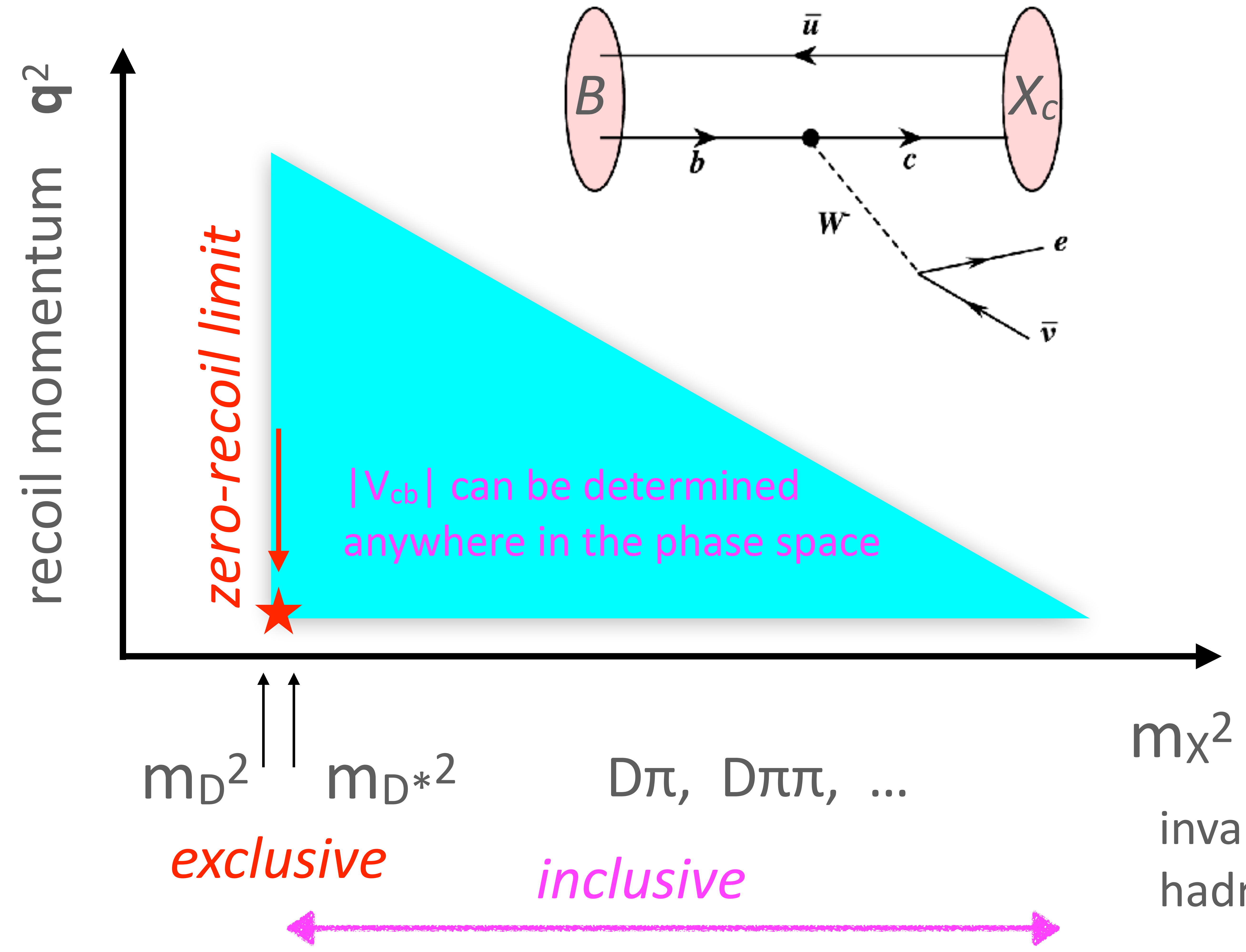
Inclusive semi-leptonic decays from lattice QCD

Shoji Hashimoto (KEK, SOKENDAI)

@ CKM 2021, WG2

Nov 23, 2021

Inclusive versus exclusive?



invariant mass of the hadronic system

Theoretically,

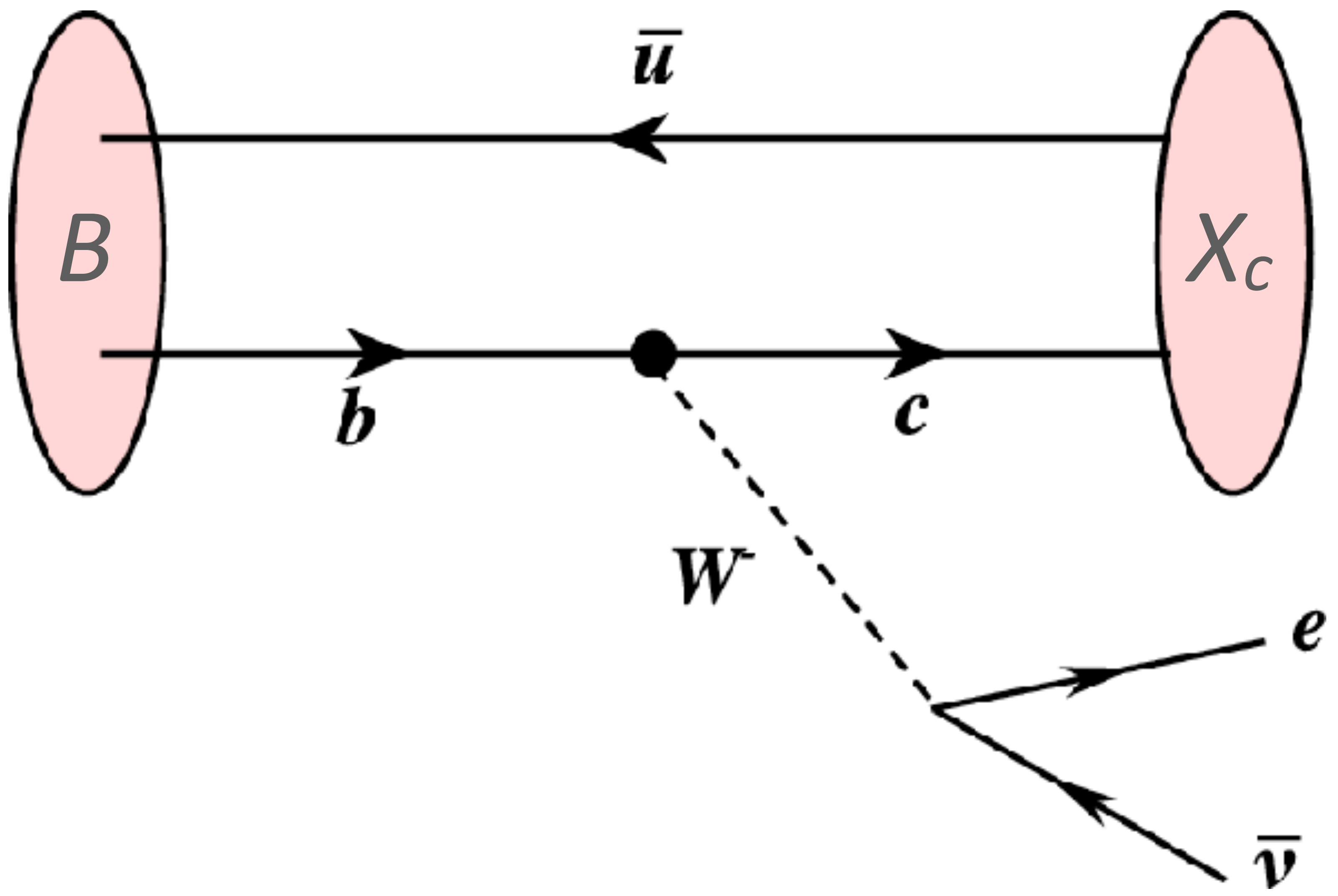
- Exclusive, with lattice FF
- Inclusive, with OPE

Each has its own pros & cons

Can't we unite them for better understanding?

(= better controlled systematics)

Inclusive semi-leptonic B decays



Inclusive: sum over all final states,
can be computed using PT (or OPE);
a number of NP MEs involved

A new method to compute the “sum” in LQCD

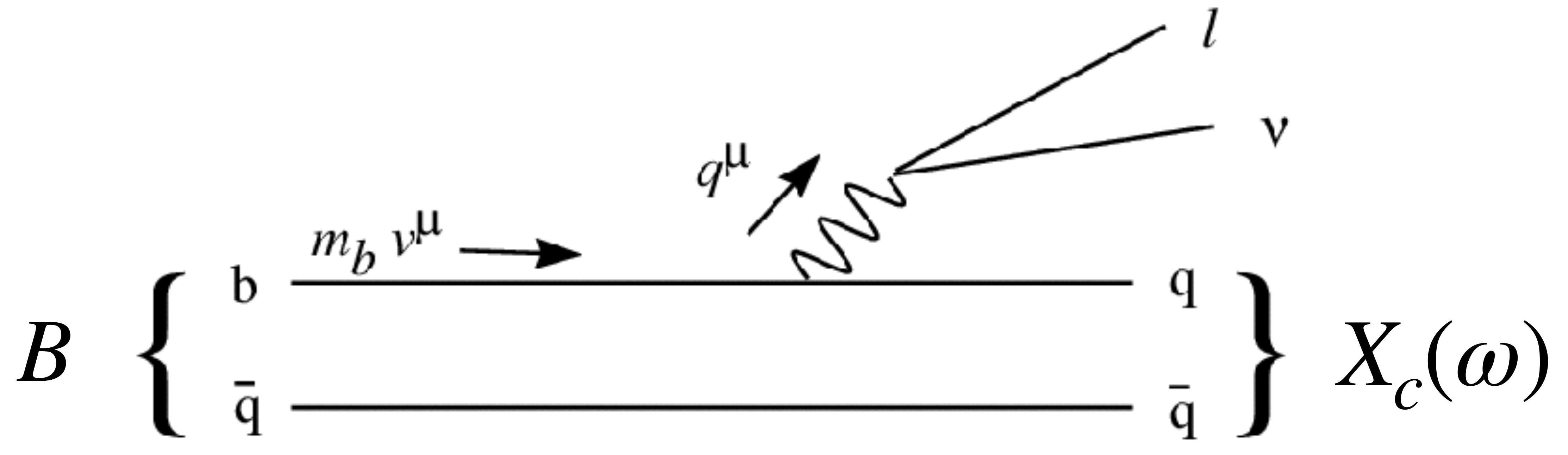
Gambino and SH, arXiv:2005.13730

from the forward-Compton amplitude.

$$\langle B(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q};t) \, \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$

all possible states contribute

Inclusive rate



Differential decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^2 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

$$\rightarrow \langle B(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q};t)\ \delta(\omega - \hat{H})\ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$

"spectral function"

Decay rate:

$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

known kinematical factor

Sum over states = energy integral

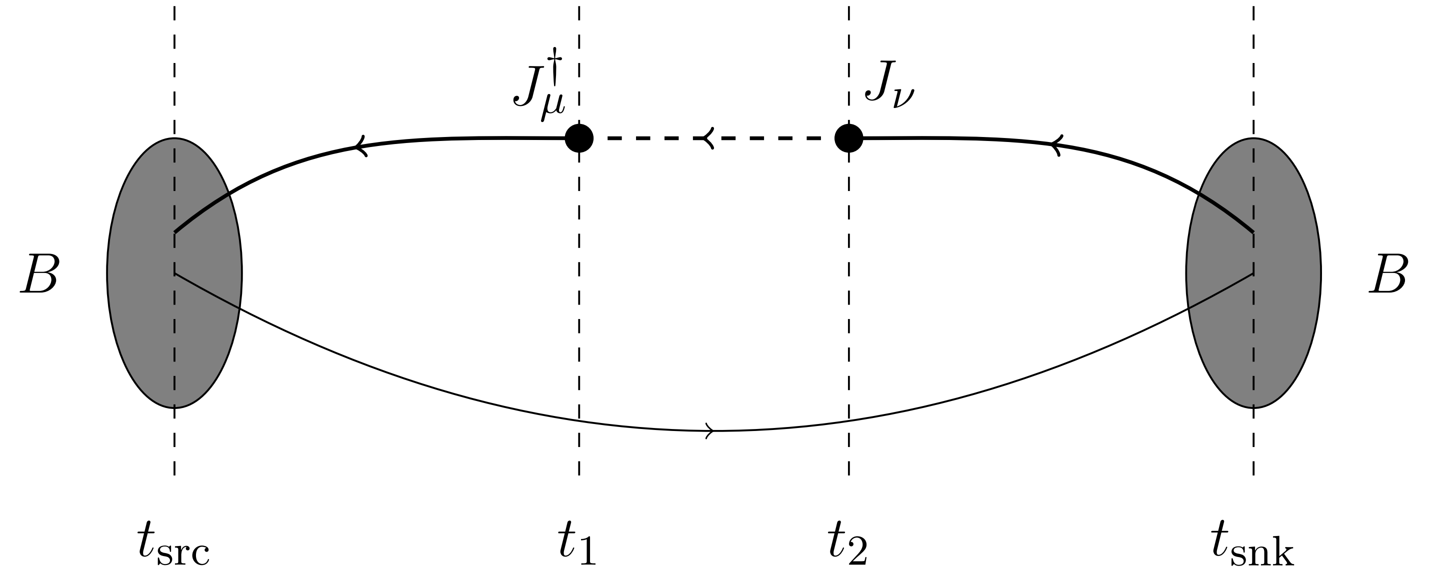
$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

$$= \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})K(\hat{H};\boldsymbol{q}^2)\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

“smeared spectral function”

Lattice Compton amplitude:

$$\langle B(\mathbf{0})|\tilde{J}^\dagger_\mu(-\boldsymbol{q};t)\ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle \longrightarrow \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})e^{-\hat{H}t}\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

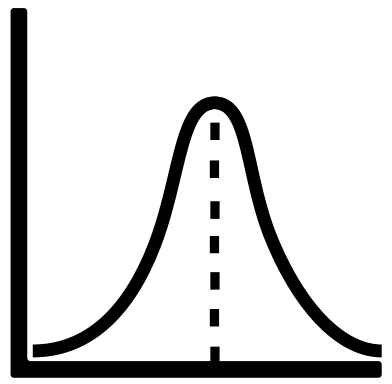


“smeared” in a different way depending on t

Approximation?

$$K(\hat{H}) = k_0 + k_1 e^{-\hat{H}} + k_2 e^{-2\hat{H}} + \cdots + k_N e^{-k_N \hat{H}}$$

Possible?

- No, if $K(\omega) = \delta(\omega - E)$. Corresponds to the full spectral function = Famous ill-posed problem.
- More chance if $K(\omega)$ is a smooth function, like 

- Backus-Gilbert (and its variants) or Chebyshev approx.

Hansen, Meyer, Robaina, arXiv:1704.08993
Hansen, Rupo, Tantalo, arXiv:1903.06476

Bailas, Ishikawa, SH, arXiv:2001.11779

Chebyshev approx:

Bailas, Ishikawa, SH, arXiv:2001.11779

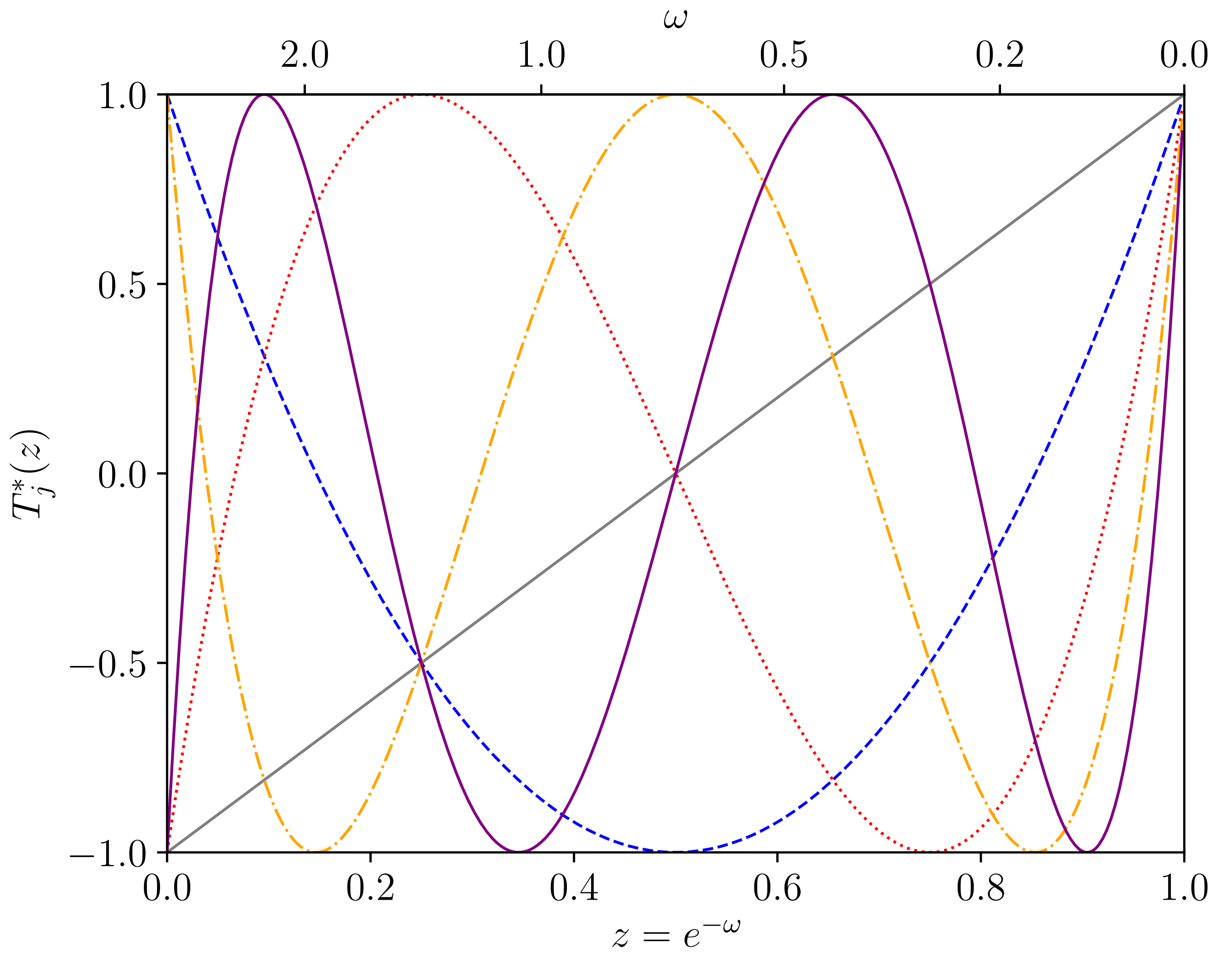
$$K(\omega) \simeq \frac{c_0}{2} + \sum_{j=1}^{N} c_j^* T_j^*(e^{-\omega})$$

(shifted) Chebyshev polynomials

$$T_0^*(x) = 1$$
$$T_1^*(x) = 2x - 1$$
$$T_2^*(x) = 8x^2 - 8x + 1$$
$$\vdots$$
$$T_{j+1}^*(x) = 2(2x-1)T_j^*(x) - T_{j-1}^*(x)$$

each term corresponds to the correlator, because $x = e^{-H}$

- Constraint $|T_j^*(e^{-\omega})| \leq 1$ stabilizes the expansion.
- Higher orders are suppressed when the coefficients are. It is the case for smooth function $K(\omega)$



"Best" approximation can be obtained with

$$c_j^* = \frac{2}{\pi}\int_0^{\pi} d\theta\, S\left(-\ln\frac{1+\cos\theta}{2}\right)\cos(j\theta)$$

"best" = maximal deviation is minimal

Kernel to approximate

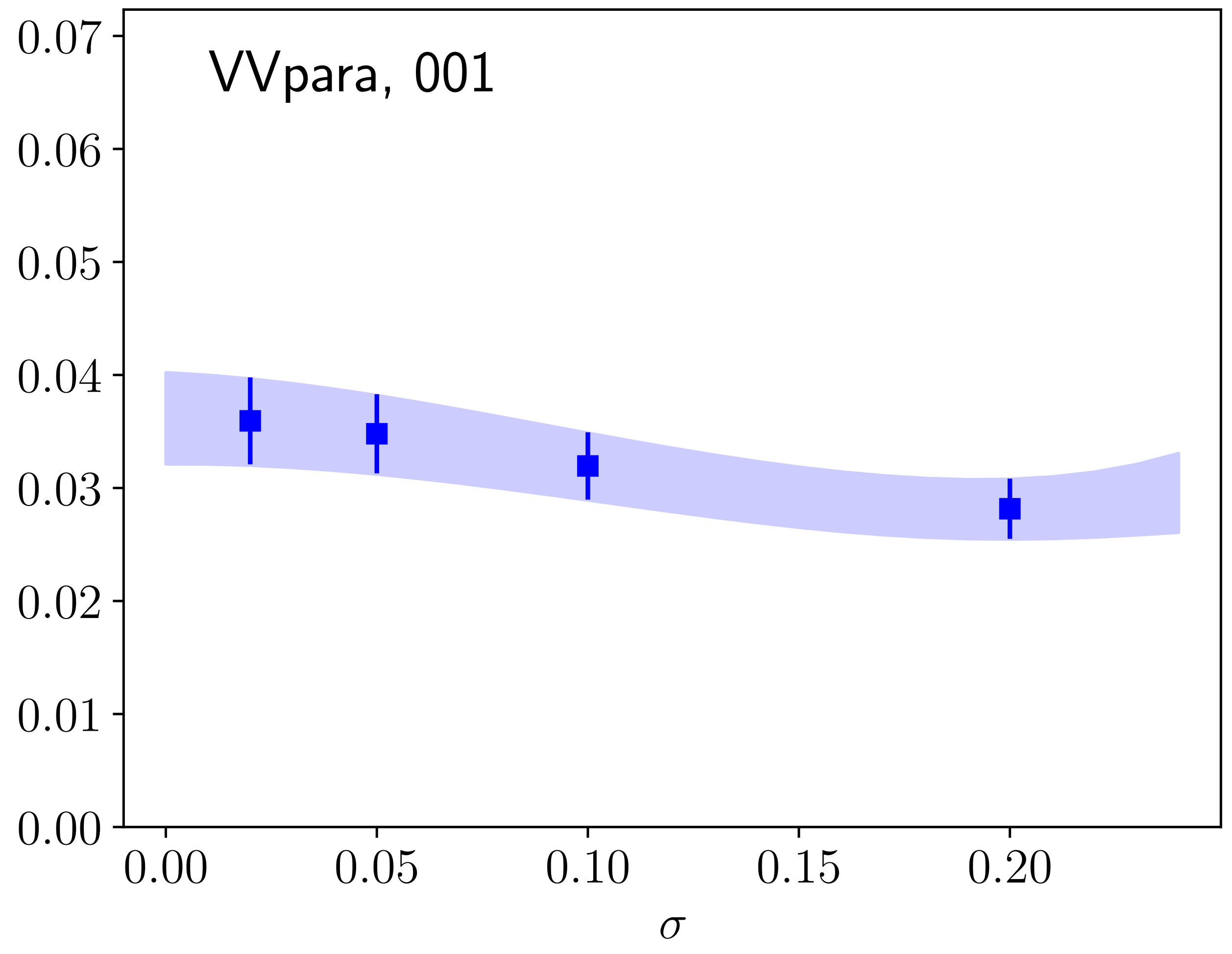
To implement the upper limit of integ

$$K(\omega) \sim e^{2\omega t_0}(m_B - \omega)^l \theta(m_B - |\mathbf{q}| - \omega)$$

kinematical factor

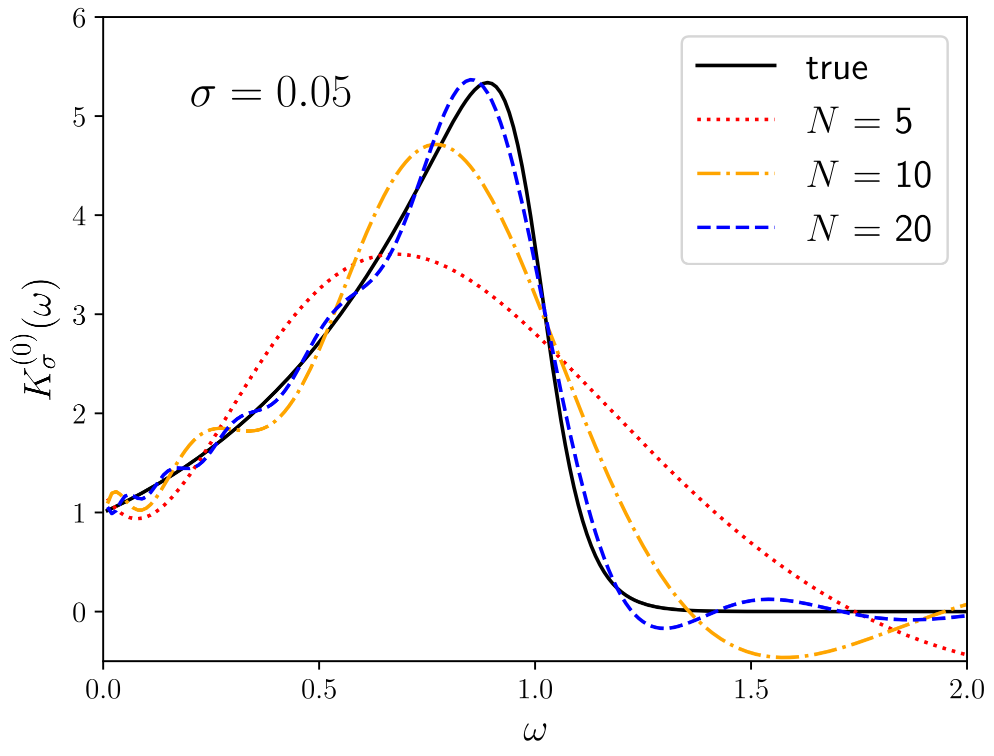
Smear by “sigmoid” with a width σ

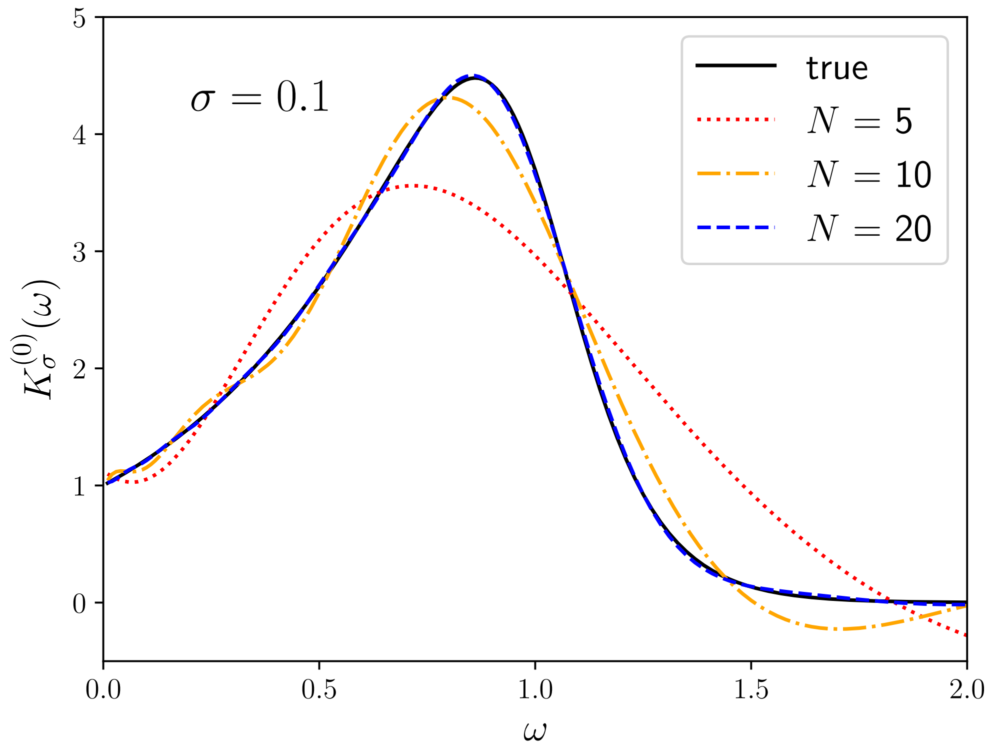
Need to take a limit of σ→ 0

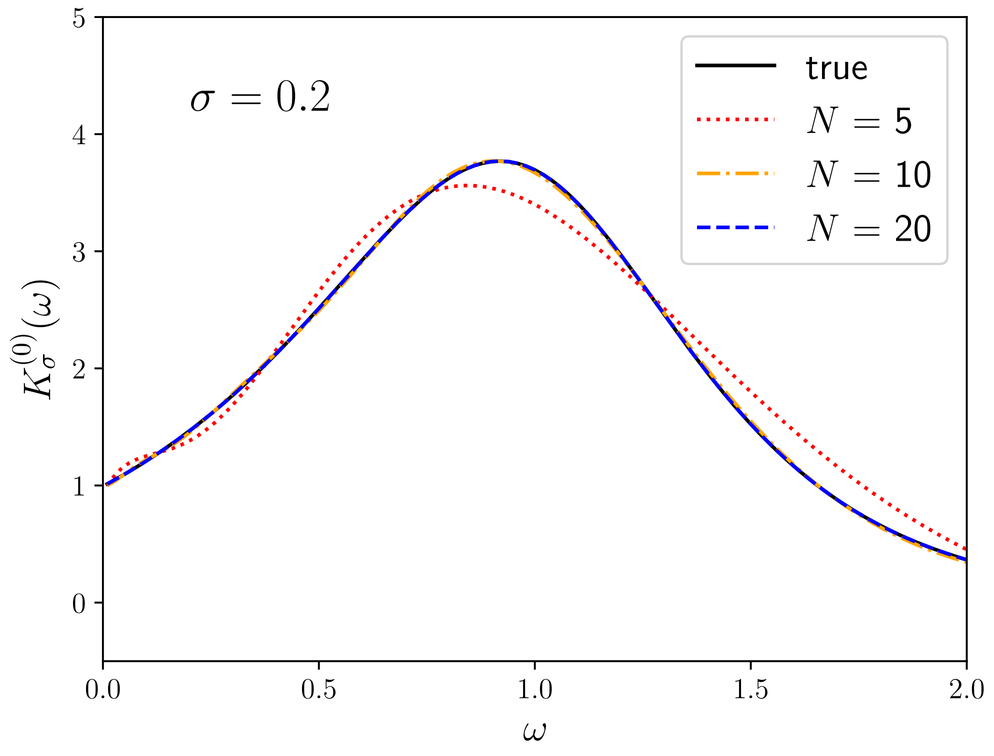


narrow

wide

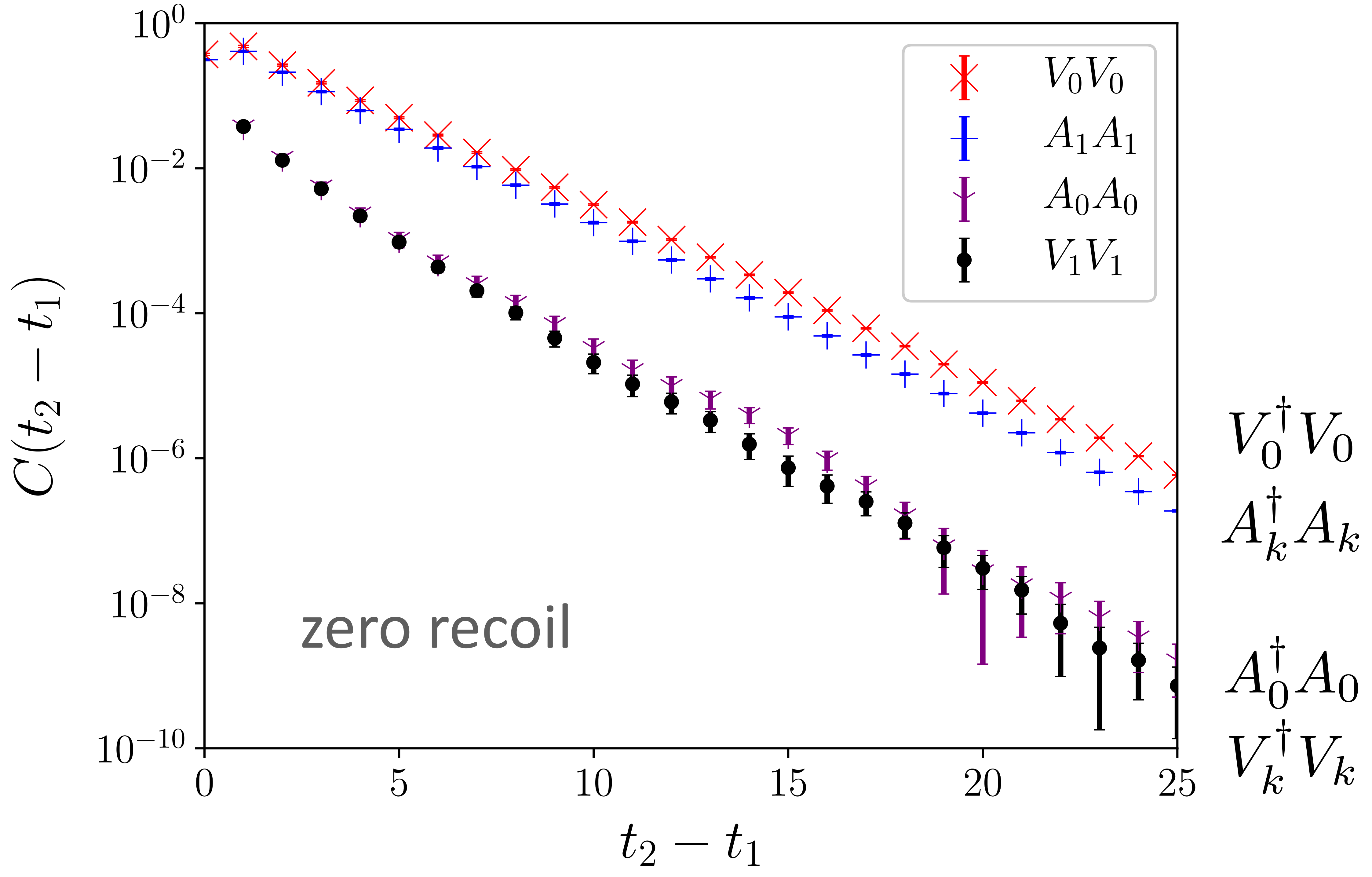






Compton amplitude

$$\langle B(\mathbf{0})|\tilde{J}^{\dagger}_{\mu}(-\boldsymbol{q};t)\ \ \tilde{J}_{\nu}(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$



Pilot lattice computation [JLQCD setup]

- On a lattice of 48^3x96 at $1/a$ = 3.6 GeV
- Strange spectator quark
- physical charm quark mass
- (unphysically) light b quark ~ 2.7 GeV
- 100 configs x 4 src

S-wave (D and D*)

- Very well approximated by a single-exp = no sign of excited state contrib.

P-wave (D**'s)

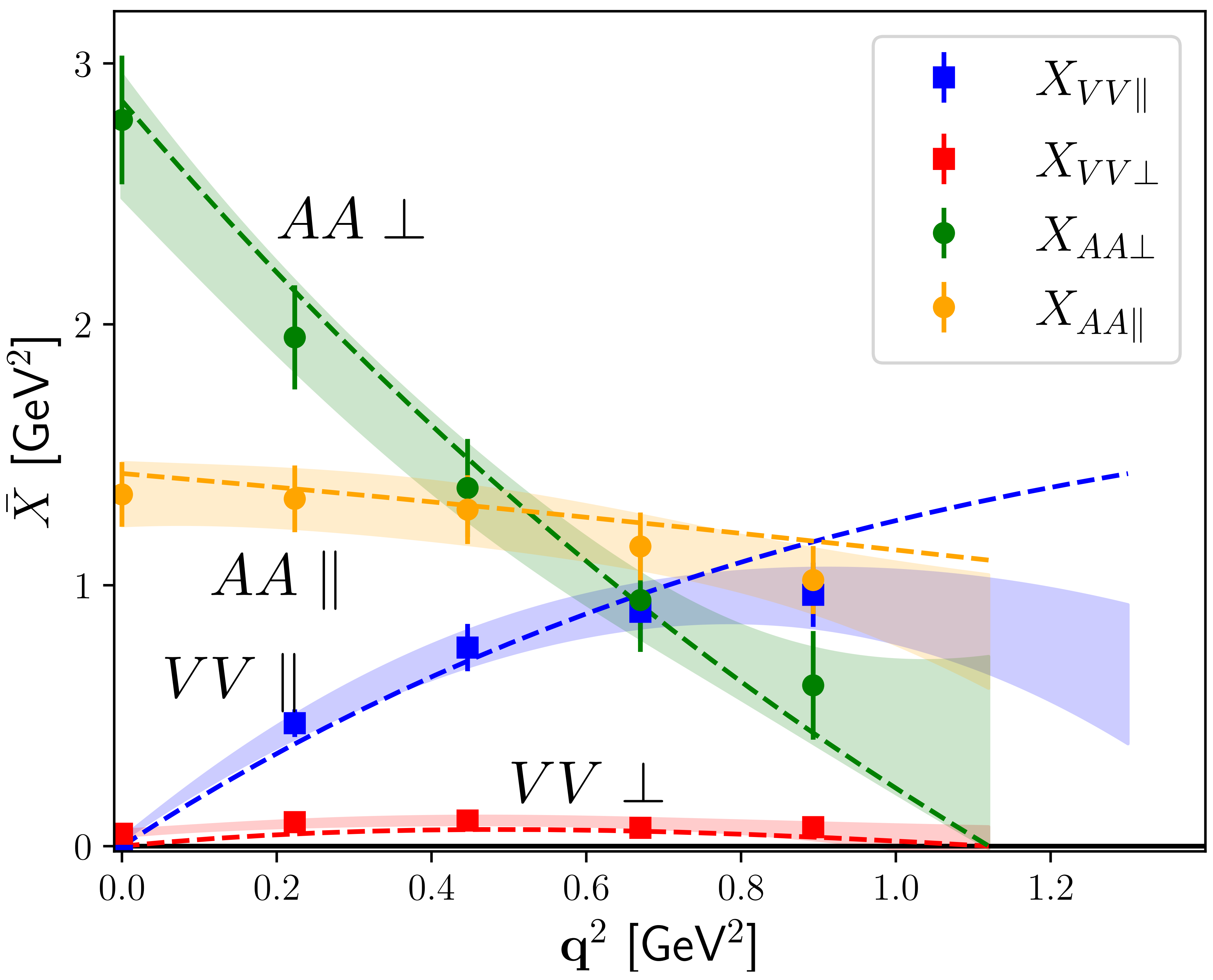
- Small : no wave function overlap of excited states when m_b=m_c and zero recoil

Inclusive decay rate

- Breakdown to individual channels: VV and AA; parallel and perp with respect to the recoil momentum
- Compared to exclusive contributions estimated from $B \to D^{(*)}$ form factors (dashed line), that are separately calculated.

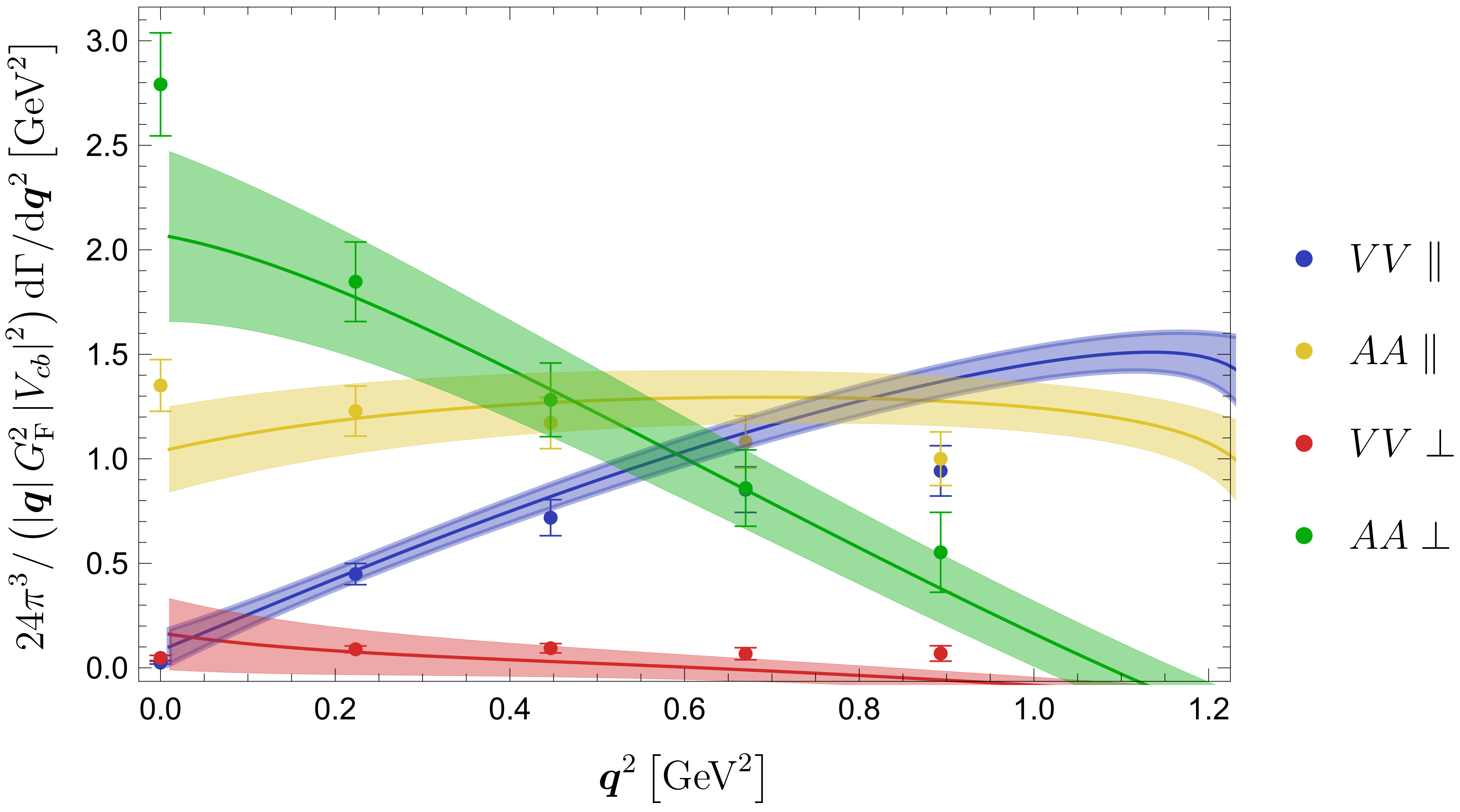
  - $VV_{||}$ dominated by $B \to D$
  - All others by $B \to D^*$

differential decay rate / |**q**|



Comparison with OPE

Gambino, SH, Machler, arXiv:2111.02833



OPE at $O(\alpha_s)$, $O(1/m_b^3)$ with

- physical charm mass
- m_b to reproduce B_s mass

Gambino, Melis, Simula, arXiv:1704.06105

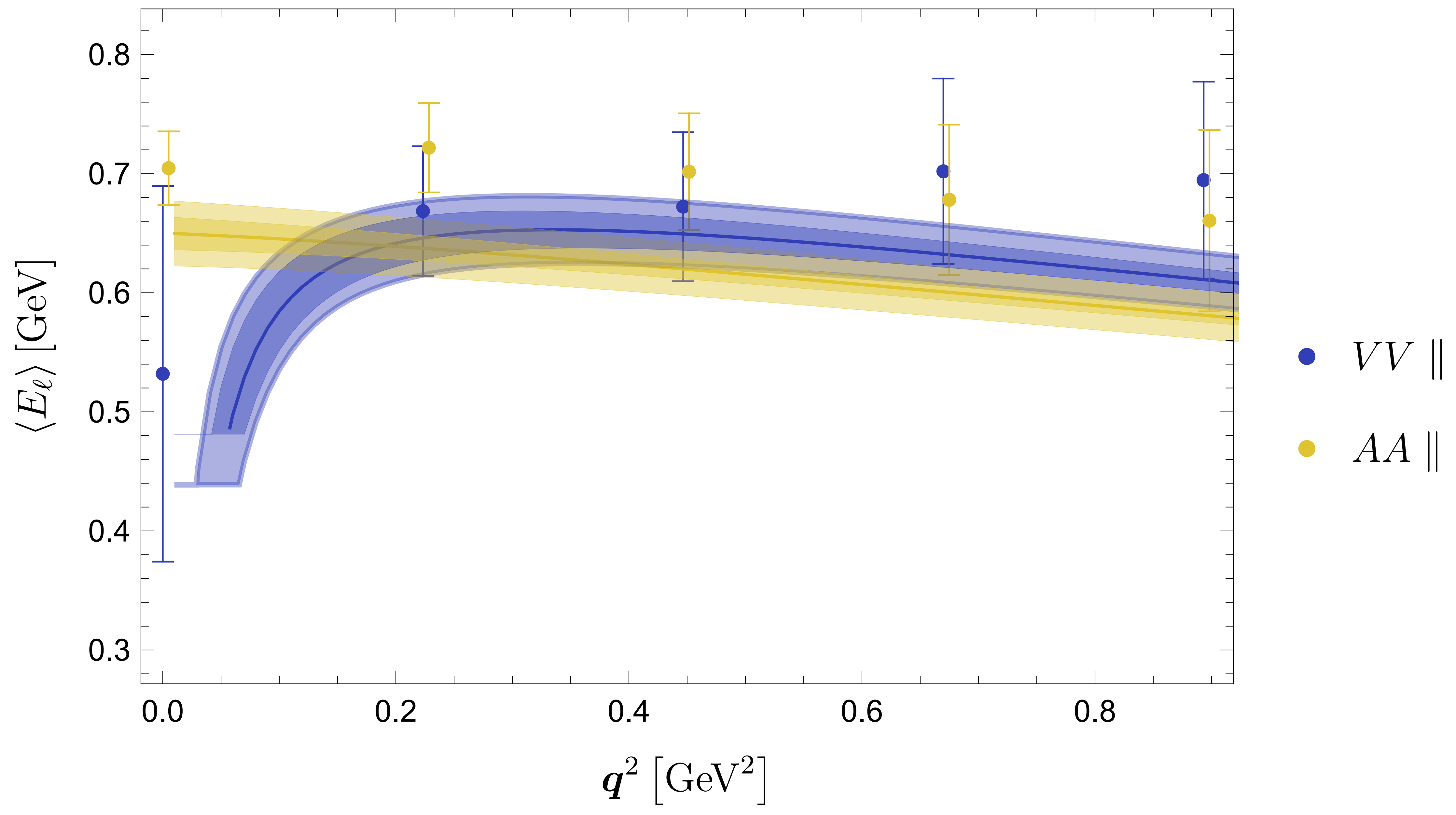
- MEs from fits of exp't; allowing 15% or 25% uncertainty (for those of $1/m_b^2$ and $1/m_b^3$)
- $\alpha_s = 0.32(1)$

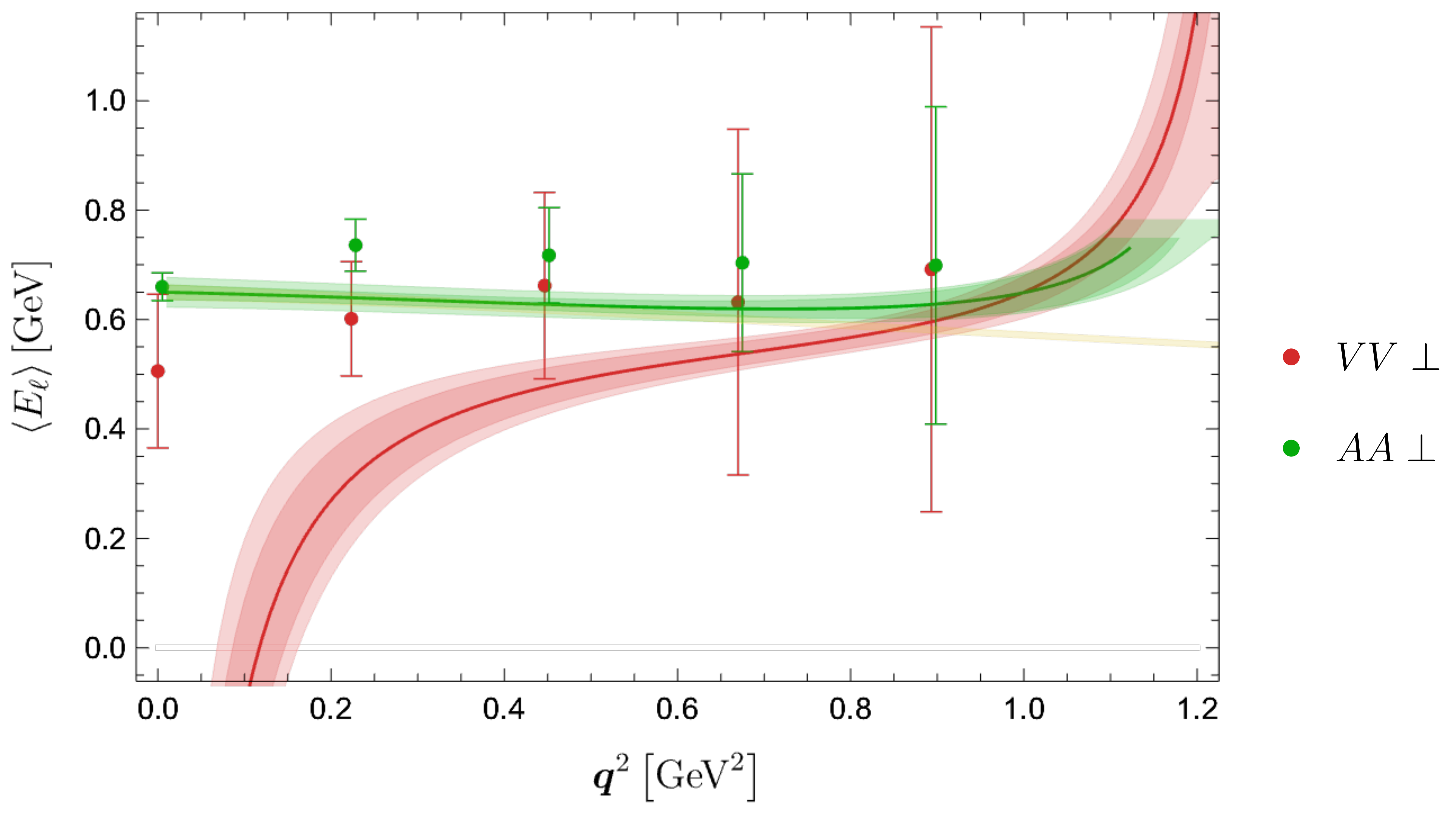
Reasonable agreement observed. Further analysis to study the consistency between OPE and lattice.

Further test with moments

Gambino, SH, Machler, arXiv:2111.02833

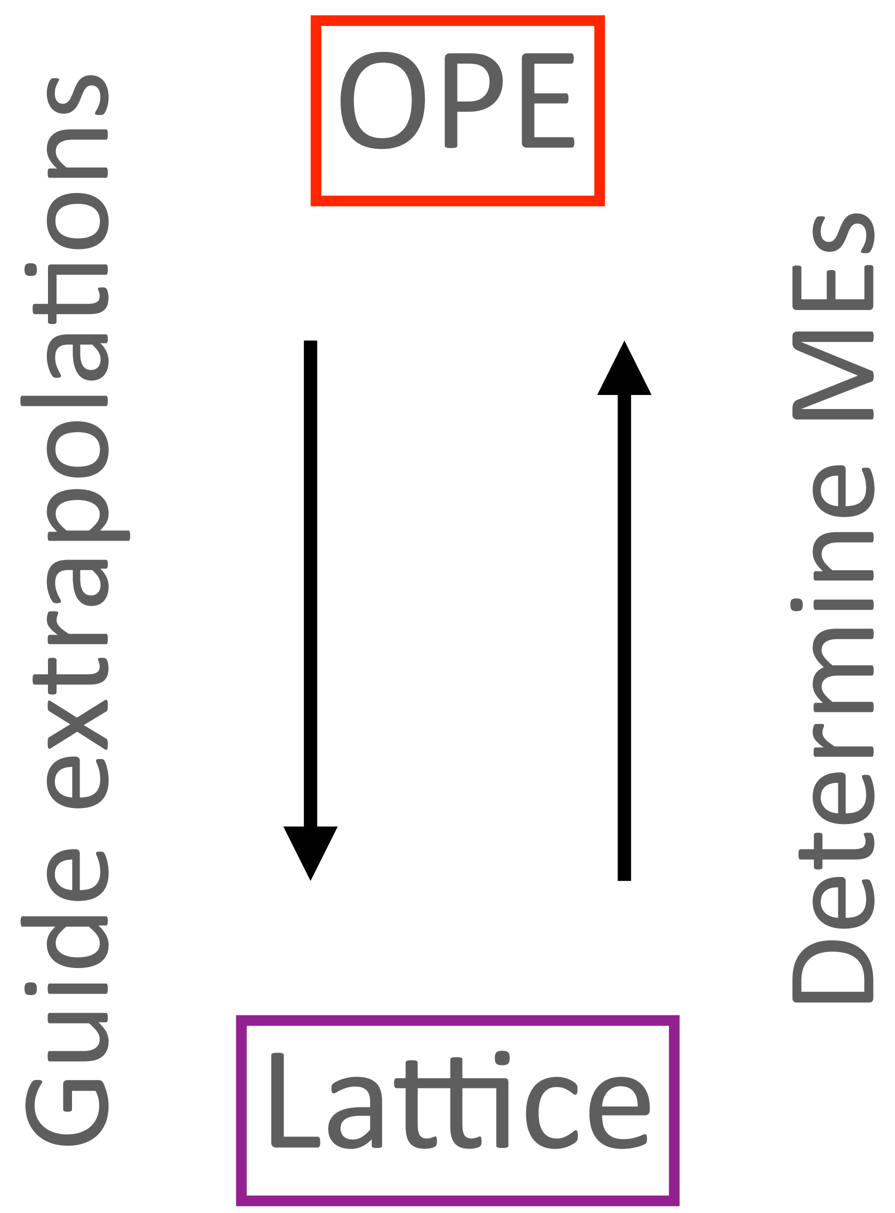
e.g. Lepton energy moment <El>





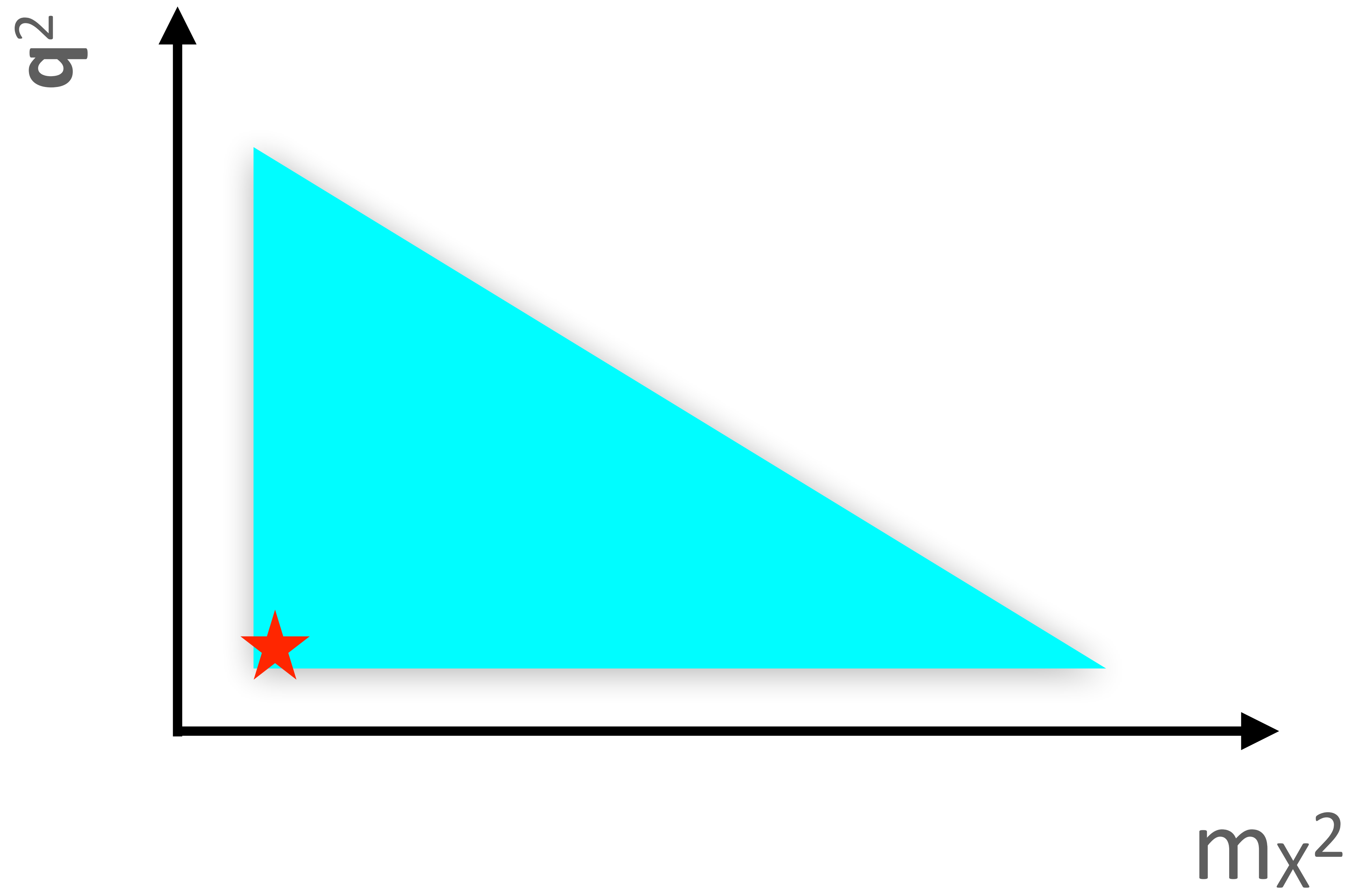
Good agreement in general.

Outlook: OPE vs Lattice



- Help each other
  - More cross-checks with moments; eventually determine the MEs necessary in OPE.
  - Lattice limited to small **q**^{2}'s. Need extrapolation.
- Also, should extend towards…
  - Physical b quark mass; light quarks
  - Crosscheck against exp't with D decays
  - Formulation of b→u

Outlook: incl. vs excl. puzzle



- Framework to compute inclusive decay rate on the lattice is now available. The energy integral can be reconstructed from Euclidean lattice correlators.
- Comparison with OPE will elucidate any inconsistency on the theory side. An initial result is encouraging.
- Use the phase space more widely to better control the systematic errors? Collaboration among exp, pheno, lattice would be crucial.