Theory review of $B \to D^{(*)}\ell\nu$ decays

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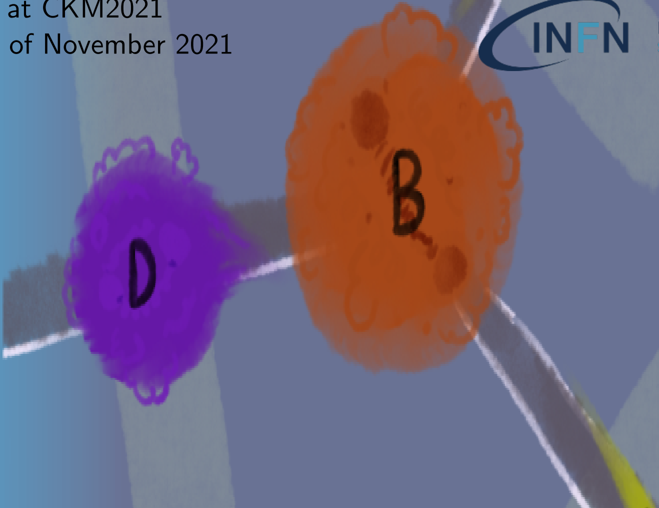
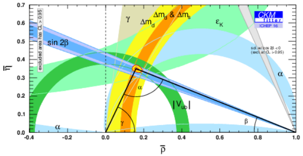
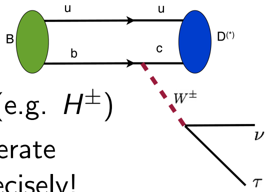


Importance of (semi-)leptonic hadron decays



In the Standard Model:

- Tree-level, $\sim |V_{ij}|^2 G_F^2\,\mathrm{FF}^2$
- Determination of $|V_{ij}|$ $(6(+1)/9)$
- Lepton-flavour universal W couplings!



Beyond the Standard Model:

- Leptonic decays $\sim m_l^2$
  - ➥ large relative NP influence possible (e.g. $H^\pm$)
- NP in semi-leptonic decays small/moderate
  - ➥ Need to understand the SM very precisely!

Key advantages:

- Large rates
- Minimal hadronic input $\Rightarrow$ systematically improvable
- Differential distributions $\Rightarrow$ large set of observables

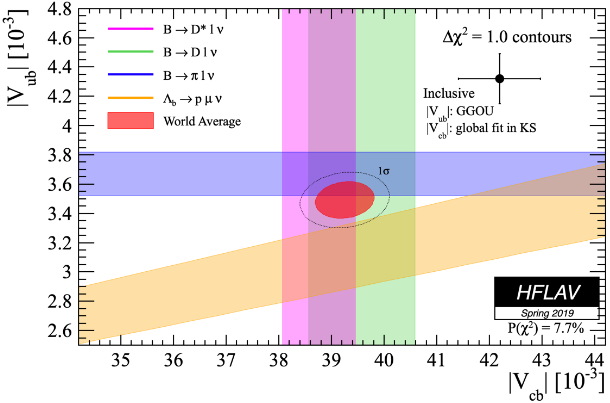
Puzzling V_{cb} results

The V_{cb} puzzle has been around for 20+ years...

- $\sim 3\sigma$ between exclusive (mostly $B \to D^* \ell\nu$) and inclusive V_{cb}
- Inclusive determination: includes $\mathcal{O}(1/m_b^3, \alpha_s/m_b^2, \alpha_s^3)$
  - ➥ Excellent theoretical control, $|V_{cb}| = (42.2 \pm 0.5) \times 10^{-3}$

    [Bordone+'21,Fael+'20,'21]

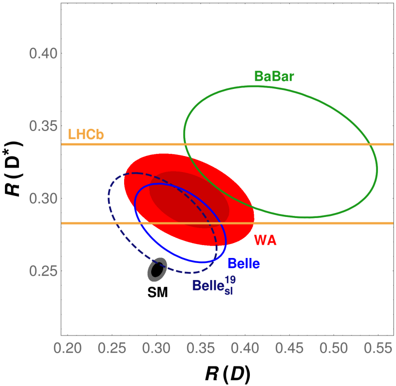
    [Talks by Vos, Gambino, Fael, Schönwald, Hashimoto, Van Tonder]
- Exclusive determinations: $B \to D^{(*)}\ell\nu$, using CLN (→ later)



Lepton-non-Universality in $b \to c\tau\nu$

$$R(X) \equiv \frac{\mathrm{Br}(B \to X\tau\nu)}{\mathrm{Br}(B \to X\ell\nu)}$$

- Partial cancellation of uncertainties
- ➥ Precise predictions (and measurements)



contours: 68% CL
filled: 95(68)% CL

- $R(D^{(*)})$: BaBar, Belle, LHCb
  - ➥ average $\sim 3-4\sigma$ from SM

More flavour $b \to c\tau\nu$ observables:

- τ-polarization ($\tau \to$ had) [1608.06391]
- $B_c \to J/\psi\tau\nu$ [1711.05623] : huge
- Differential rates from Belle, BaBar
- Total width of B_c
- $b \to X_c\tau\nu$ by LEP
- D^* polarization (Belle)

Note: only 1 result $\geq 3\sigma$ from SM

[Talks by Grinstein,Watanabe,Cheaib]

Form factors: basics

Form Factors (FFs) parametrize fundamental mismatch:

Theory (e.g. SM) for partons (quarks)
vs.
Experiment with hadrons

$$\left\langle D_q^{(*)}(p')|\bar{c}\gamma^\mu b|\bar{B}_q(p)\right\rangle = (p+p')^\mu f_+^q(q^2) + (p-p')^\mu f_-^q(q^2)\,,\; q^2 = (p-p')^2$$

Most general matrix element parametrization, given symmetries:
Lorentz symmetry plus P- and T-symmetry of QCD
$f_\pm(q^2)$: real, scalar functions of one kinematic variable

How to obtain these functions?

- Calculable w/ non-perturbative methods (Lattice, LCSR,...)
  Precision?
- Measurable e.g. in semileptonic transitions
  Normalization? Suppressed FFs? NP?

q^2 dependence

- q^2 range can be large, e.g. $q^2 \in [0, 12]\ \mathrm{GeV}^2$ in $B \to D$
- Calculations give usually one or few points
- ➥ Knowledge of functional dependence on q^2 cruical
- This is where discussions start...

Give as much information as possible independent of this choice!

In the following: discuss BGL and HQE ($\to$ CLN) parametrizations

q^2 dependence usually rewritten via conformal transformation:

$$z\left(t = q^2, t_0\right) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$t_+ = (M_{B_q} + M_{D_q^{(*)}})^2$: pair-production threshold

$t_0 < t_+$: free parameter for which $z(t_0, t_0) = 0$

Usually $|z| \ll 1$, e.g. $|z| \leq 0.06$ for semileptonic $B \to D$ decays

➥ Good expansion parameter

The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

1. Consider analytical structure, make poles and cuts explicit
2. Without poles or cuts, the rest can be Taylor-expanded in z
3. Apply QCD properties (unitarity, crossing symmetry)
   ➥ dispersion relation
4. Calculate partonic part perturbatively (+condensates)

Result:

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n [z(t, t_0)]^n \,.$$

- a_n: real coefficients, the only unknowns
- $P(t)$: Blaschke factor(s), information on poles below t_+
- $\phi(t)$: Outer function, chosen such that $\sum_{n=0}^{\infty} a_n^2 \leq 1$

➥ Series in z with bounded coefficients (each $|a_n| \leq 1$)!

➥ Uncertainty related to truncation is calculable!

$B \to D\ell\nu$

$B \to D\ell\nu$, aka "What it should look like":

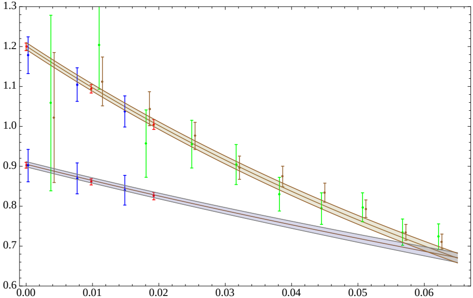
- Excellent agreement between experiments [BaBar'09,Belle'16]
- Excellent agreement between two lattice determinations [FNAL/MILC'15,HPQCD'16]

➥ Lattice data contradict CLN parametrization!
(Not HQE@1/m, discussed later)

- BGL fit [Bigi/Gambino'16] :

$$|V_{cb}| = 40.5(10) \times 10^{-3} \qquad R(D) = 0.299(3)\,.$$

See also [Jaiswal+,Berlochner+'17,MJ/Straub'18,Bordone/MJ/vanDyk'19]



$f_{+,0}(z)$, inputs:

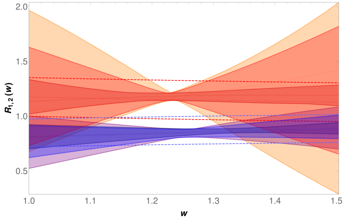
- FNAL/MILC'15
- HPQCD'16
- BaBar'09
- Belle'16

$V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

Belle'17+'18 provide FF-independent data for 4 single-differential rates

Analysis of these data with BGL form factors:

- Datasets roughly compatible
- d'Agostini bias + syst. important
- All FFs to z^2 to include uncertainties
  ➥ 50% increased uncertainties
- 2018: no parametrization dependence



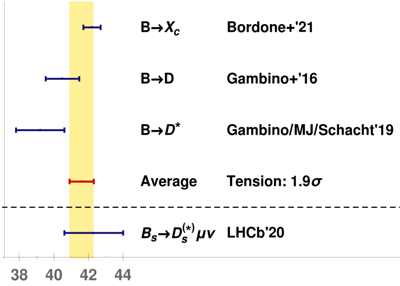
$$|V_{cb}^{D^*}| = 39.6^{+1.1}_{-1.0}\,[39.2^{+1.4}_{-1.2}] \times 10^{-3}$$

$$R(D^*) = 0.254^{+0.007}_{-0.006}\,[0.253^{+0.007}_{-0.006}]$$

In brackets: 2018 only

Updating the $|V_{cb}|$ puzzle:

- Tension 1.9σ (larger $\delta V_{cb}^{B\to D^*}$)
- $B_s \to D_s^{(*)}$ reduces tension further
- $V_{cb}^{B\to D^*}$ vs. $V_{cb}^{\rm incl}$ still problematic



See also [Bigi+,Bernlocher+,Grinstein+'17,Jaiswal+'17'19,MJ/Straub'18,Bordone+'19/'20]

HQE parametrization

HQE parametrization uses additional information compared to BGL

➥ Heavy-Quark Expansion (HQE)

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
  ➥ Parameter reduction, necessary for NP analyses!

CLN parametrization [Caprini+'97] :
HQE to order $1/m_{b,c}, \alpha_s$ plus (approx.) constraints from unitarity
[Bernlochner/Ligeti/Papucci/Robinson'17] : identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in $B \to D$ and $B \to D^*$)
Dealt with by varying calculable (@$1/m_{b,c}$) parameters, e.g. $h_{A_1}(1)$
➥ Not a systematic expansion in $1/m_{b,c}$ anymore!
➥ Related uncertainty remains $\mathcal{O}[\Lambda^2/(2m_c)^2] \sim 5\%$, insufficient

Solution: Include systematically $1/m_c^2$ corrections
[Bordone/MJ/vDyk'19,Bordone/Gubernari/MJ/vDyk'20] ,using [Falk/Neubert'92]

Theory determination of $b \to c$ Form Factors

[Bordone/MJ/vanDyk'19,Bordone/Gubernari/MJ/vanDyk'20]

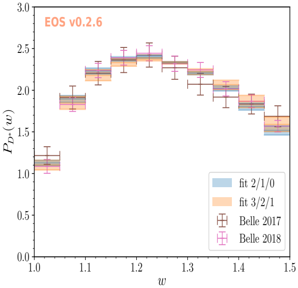
To determine general NP, FF shapes needed from theory!

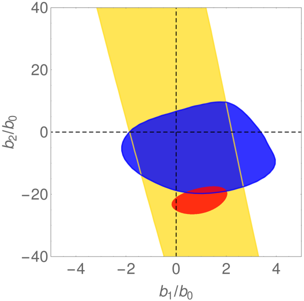
Fit to all $B \to D^{(*)}$ FFs, using lattice, LCSR, QCDSR and unitarity

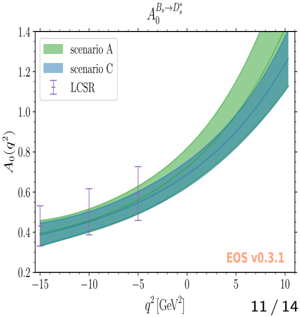
[CLN,BGL,HPQCD'15'17,FNAL/MILC'14'15,Gubernari+'18,Ligeti+'92'93]

k/l/m order in z for leading/subleading/subsubleading IW functions

- 2/1/0 works, but only 3/2/1 captures uncertainties
- Consistent V_{cb} value from Belle'17+'18
- Predictions for diff. rates, perfectly confirmed by data
- Explicit inclusion of $B_s \to D_s^{(*)}$: improvement for all FFs



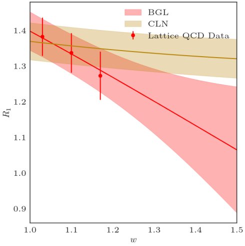


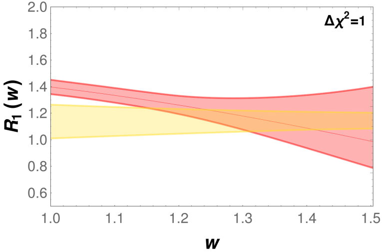


Comparison with new lattice calculations

Major improvement: $B \to D^*_{(s)}$ FFs@$w > 1$!

[HPQCD'21,FNAL/MILC'21,JLQCD prel., talks by Kaneko,Harrison]





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- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel

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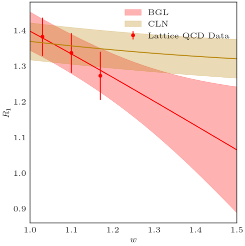

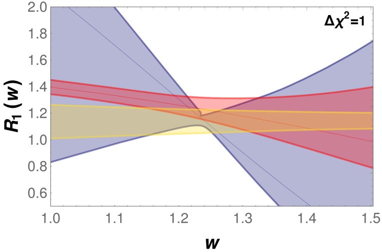

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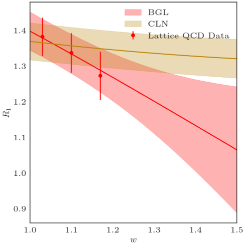

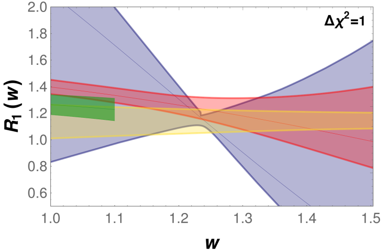

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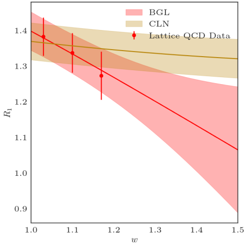

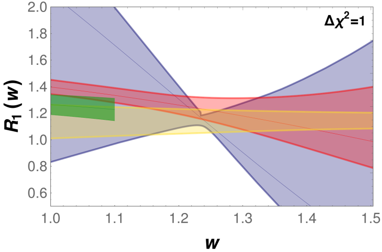

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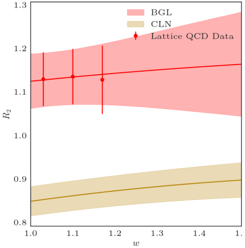

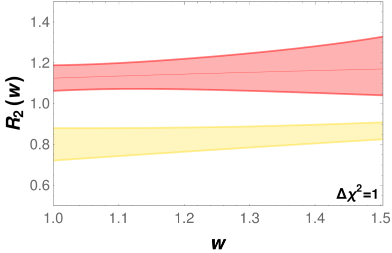

- Deviation wrt previous FFs

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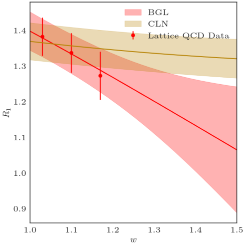

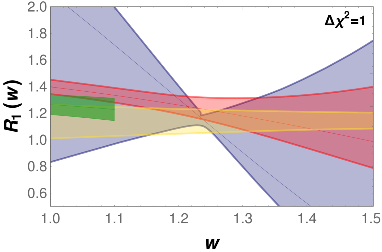

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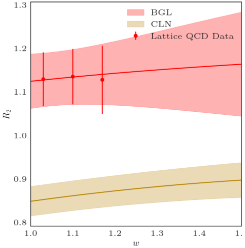

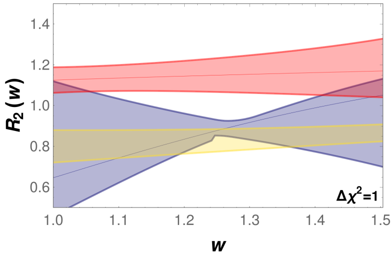

- Deviation wrt previous FFs
- Deviation wrt experiment

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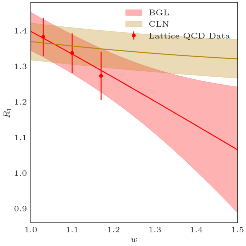

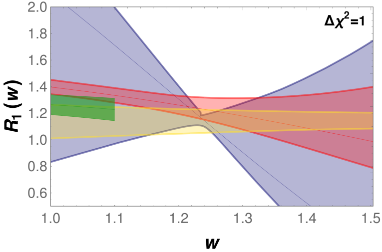

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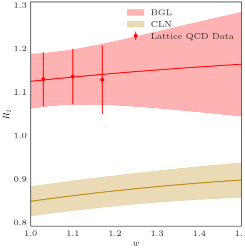

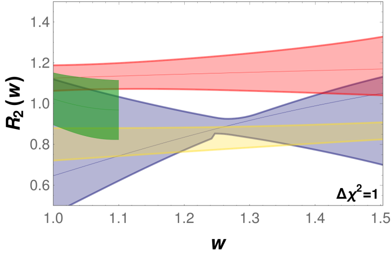

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- JLQCD "diplomatic"

➡ Requires further investigation! Combination difficult.

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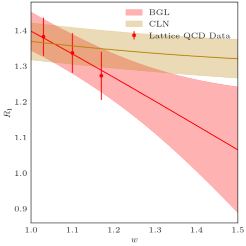

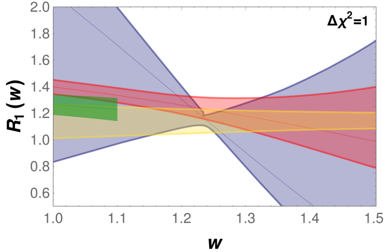

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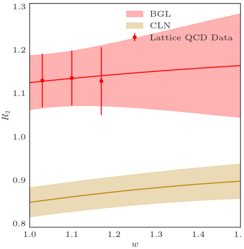

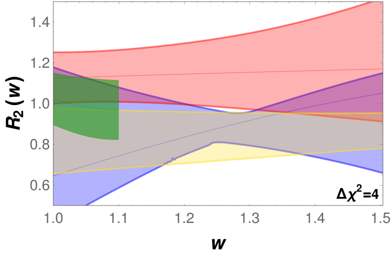

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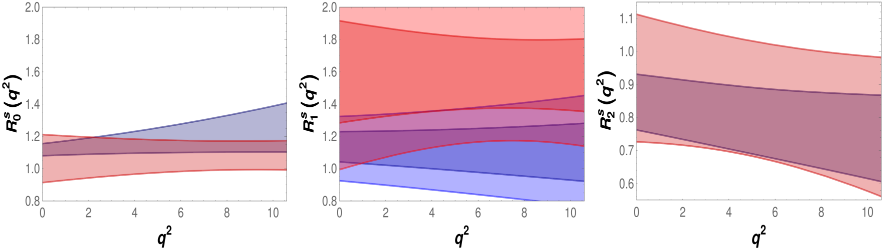
Comparison to HPQCD results for $B_s \to D^*_s$:

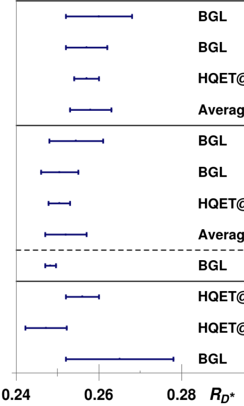


- Overall very good compatibility
- Slight tension in $R_1^s(w)$, below 2σ
- ➥ Combination with previous results wip

Points of discussion regarding presentation of lattice results:

- Priors on theory parameters
- Inclusion (or not) of unitarity constraints

Overview over predictions for $R(D^*)$



Value	Method	Input Theo	Input Exp	Reference
	BGL	Lattice, HQET	Belle'17	Bigi et al.'17
	BGL	Lattice, HQET	Belle'17	Jaiswal et al.'17
	HQET@$1/m_c,\alpha_s$	Lattice, QCDSR	Belle'17	Bernlochner et al.'17
	Average			HFLAV'19
	BGL	Lattice, HQET	Belle'17'18	Gambino et al.'19
	BGL	Lattice, HQET	Belle'18	Jaiswal et al.'20
	HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	Belle'17'18	Bordone et al.'20
	Average			HFLAV'21
	BGL	Lattice	Belle'18, Babar'19	Vaquero et al.'21v2
	HQET@$1/m_c,\alpha_s$	Lattice, QCDSR	---	Bernlochner et al.'17
	HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	---	Bordone et al.'20
	BGL	Lattice	---	Vaquero et al.'21v2

Lattice $B \to D^*$: $h_{A_1}(w=1)$ [FNAL/MILC'14,HPQCD'17] , [FNAL/MILC'21]

Other lattice: $f_{+,0}^{B\to D}(q^2)$ [MILC,HPQCD'15]

QCDSR: [Ligeti/Neubert/Nir'93,'94] , LCSR: [Gubernari/Kokulu/vDyk'18]

Consistent SM predictions!

Further improvement expected from lattice

Flavour universality in $B \to D^*(e,\mu)\nu$

[Bobeth/Bordone/Gubernari/MJ/vDyk'21]

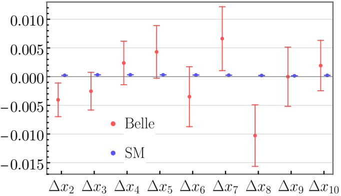
So far: Belle'18 data used in SM fits, flavour-averaged

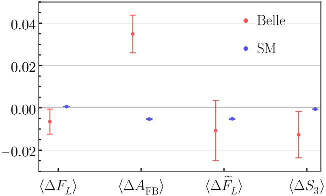
However: Bins 40×40 covariances given separately for $\ell = e, \mu$

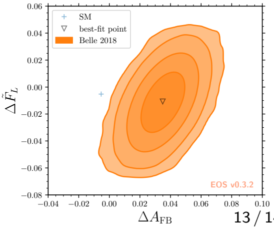
- Belle'18: $R_{e/\mu}(D^*) = 1.01 \pm 0.01 \pm 0.03$
- What can we learn about flavour-non-universality? $\to$ 2 issues:
  1. $e - \mu$ correlations not given $\to$ constructable from Belle'18
  2. 3 bins linearly dependent, but covariances not singular

Two-step analysis:

1. Extract 2×4 angular observables for 2×30 angular bins
   - Model-independent description including NP!
2. Compare with SM predictions, using FFs@$1/m_c^2$ [Bordone+'19]
   - $\sim 4\sigma$ discrepancy in $\Delta A_{\rm FB} = A_{\rm FB}^{\mu} - A_{\rm FB}^{e}$







Conclusions

Form factors essential ingredients in precision-flavour physics!

- q^2 dependence critical $\rightarrow$ need FF-independent data
- ➥ Inclusion of higher-order (theory) uncertainties important
- BGL: model-independent, truncation uncertainty limited
- ➥ $B \rightarrow D^*$: Reduced V_{cb} puzzle, somewhat lower $R(D^*)$ prediction
- Theory determinations for NP required $\rightarrow$ HQE to relate FFs
- $\mathcal{O}(1/m_c)$ not good enough for precision analyses
- ➥ First analysis at $1/m_c^2$ provides all $B \rightarrow D^{(*)}$ FFs
- ➥ V_{cb} consistent w/ BGL
- First LQCD analyses in $B \rightarrow D^*$ and $B_s \rightarrow D_s^*$ @ finite recoil
- ➥ Tension with experiment as well as other theory inputs
- LFU-violation in $b \rightarrow c\ell\nu$@$\sim 4\sigma$!
- ➥ Experimental issues? NP?

Central lesson: experiment and theory need to work closely together!

Theory determination of $b \to c$ Form Factors

SM: BGL fit to data + FF normalization $\to |V_{cb}|$

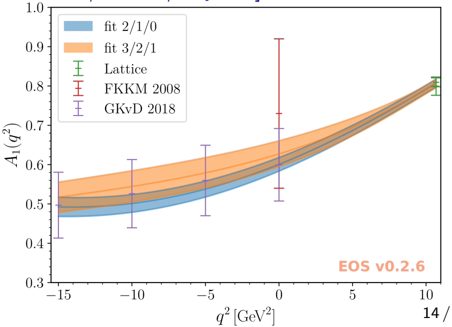
NP: can affect the q^2-dependence, introduces additional FFs

➥ To determine general NP, FF shapes needed from theory

[MJ/Straub'18,Bordone/MJ/vDyk'19] used all available theory input:

- Unitarity bounds (using results from [CLN, BGL])
  - ➥ non-trivial $1/m$ vs. z expansions
- LQCD for $f_{+,0}(q^2)$ $(B \to D)$, $h_{A_1}(q^2_{\rm max})$ $(B \to D^*)$ [HPQCD'15,'17,Fermilab/MILC'14,'15]
- LCSR for all FFs (mod f_T) [Gubernari/Kokulu/vDyk'18]
- QCDSR results for $1/m$ IW functions [Ligeti+'92'93]
- HQET expansion to $\mathcal{O}(\alpha_s, 1/m_b, 1/m_c^2)$

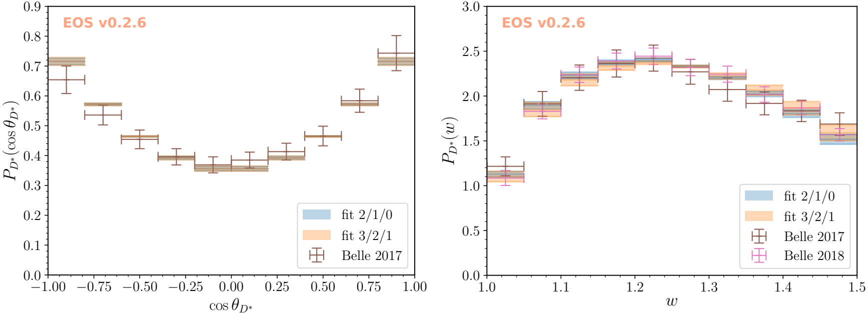
FFs under control;
$R(D^*) = 0.247(6)$
[Bordone/MJ/vDyk'19]



Robustness of the HQE expansion up to $1/m_c^2$

[Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:



- Fits 3/2/1 and 2/1/0 are theory-only fits(!)
- $k/l/m$ denotes orders in z at $\mathcal{O}(1, 1/m_c, 1/m_c^2)$
- w-distribution yields information on FF shape $\rightarrow V_{cb}$
- Angular distributions more strongly constrained by theory, only

➥ Predicted shapes perfectly confirmed by $B \rightarrow D^{(*)}\ell\nu$ data

➥ V_{cb} from Belle'17 compatible between HQE and BGL!

Robustness of the HQE expansion up to $1/m_c^2$

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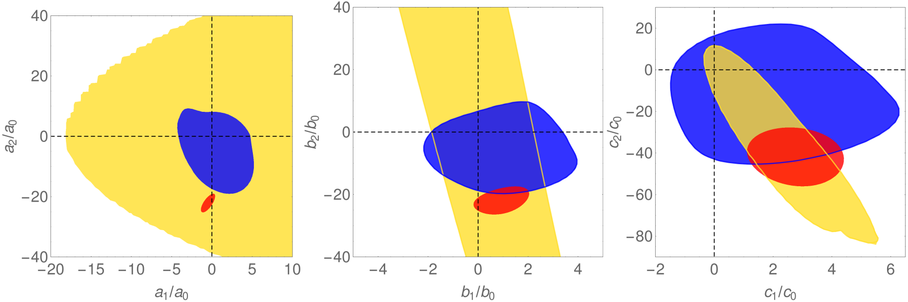


- $B \to D^*$ BGL coefficient ratios from:
  1. Data (Belle'17+'18) + weak unitarity (yellow)
  2. HQE theory fit 2/1/0 (red)
  3. HQE theory fit 3/2/1 (blue)

➥ Again compatibility of theory with data

➥ 2/1/0 underestimates the uncertainties massively

➥ For b_i, c_i ($\to f, \mathcal{F}_1$) data and theory complementary

Including $\bar{B}_s \to D_s^{(*)}$ Form Factors [Bordone/Gubernari/MJ/vDyk'20]

Dispersion relation *sums* over hadronic intermediate states

➥ Includes $B_s D_s^{(*)}$, included via SU(3) + conservative breaking

➥ Explicit treatment can improve also $\bar{B} \to D^{(*)}\ell\nu$

Experimental progress in $\bar{B}_s \to D_s^{(*)}\ell\nu$:
2 new LHCb measurements [2001.03225, 2003.08453]

Improved theory determinations required, especially for NP

We extend our $1/m_c^2$ analysis by including:

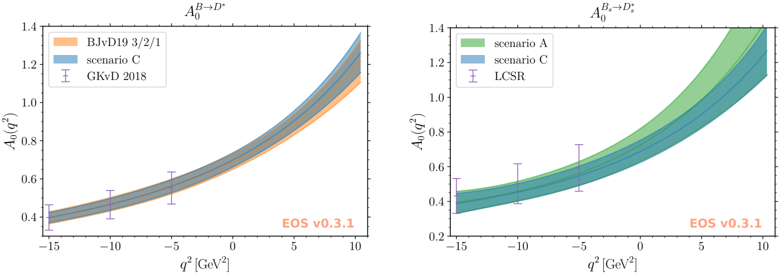
- Available lattice data:
  (2 $\bar{B}_s \to D_s$ FFs (q^2 dependent), 1 $\bar{B}_s \to D^*$ FF (only q^2_{max}))
- Adaptation of existing QCDSR results [Ligeti/Neubert/Nir'93'94] , including SU(3) breaking
- New LCSR results extending [Gubernari+'18] to B_s, including SU(3) breaking

➥ Fully correlated fit to $\bar{B} \to D^{(*)}$, $\bar{B}_s \to D_s^{(*)}$ FFs

Including $\bar{B}_s \to D_s^{(*)}$ Form Factors, Results

We observe the following:

- Theory constraints fitted consistently in an HQE framework
- $\mathcal{O}(1/m_c^2)$ power corrections have $\mathcal{O}(1)$ coefficients
- No indication of sizable SU(3) breaking
- Slight influence of strengthened unitarity bounds
- Improved determination of $\bar{B}_s \to D_s^{(*)}$ FFs



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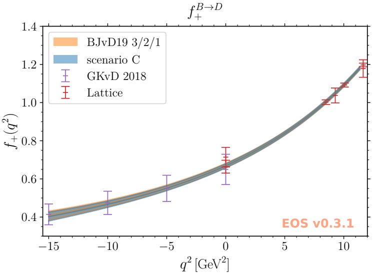

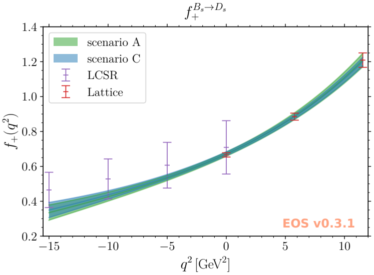

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- Improved determination of $\bar{B}_s \to D_s^{(*)}$ FFs

Theory-only predictions:

$$R(D) = 0.299(3) \qquad R(D^*) = 0.247(5)$$
$$R(D_s) = 0.297(3) \qquad R(D_s^*) = 0.245(8)$$

Theory+Experiment (Belle'17) predictions:

$$R(D) = 0.298(3) \qquad R(D^*) = 0.250(3)$$
$$R(D_s) = 0.297(3) \qquad R(D_s^*) = 0.247(8)$$

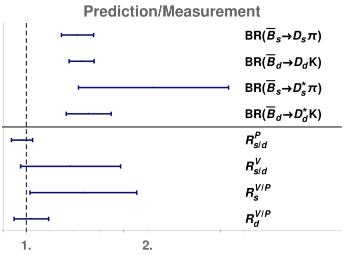
A puzzle in non-leptonic $b \to c$ transitions

[Bordone/Gubernari/Huber/MJ/vDyk'20]

FFs also of central importance in non-leptonic decays:

- Complicated in general, $B \to M_1 M_2$ dynamics
- Simplest cases: $\bar{B}_d \to D_d^{(*)} \bar{K}$ and $\bar{B}_s \to D_s^{(*)} \pi$ (5 diff. quarks)
  - Colour-allowed tree, $1/m_b^0$@$\mathcal{O}(\alpha_s^2)$ [Huber+'16] , factorizes at $1/m_b$
  - Amplitudes dominantly $\sim \bar{B}_q \to D_q^{(*)}$ FFs
  - Used to determine f_s/f_d at hadron colliders [Fleischer+'11]

Updated and extended calculation: tension of 4.4σ w.r.t. exp.!



- Large effect, $\sim -30\%$ for BRs
- Ratios of BRs ok
- QCDf uncertainty $\mathcal{O}(1/m_b^2, \alpha_s^3)$
- Data consistent (too few abs. BRs)
- NP? $\Delta_P \sim \Delta_V \sim -20\%$ possible
- We will learn something important!

Generalities regarding this anomaly

$\sim 15\%$ of a SM tree decay $\sim V_{cb}$: This is a huge effect!
➥ Need contribution of $\sim 5-10\%$ (w/ interference) or $\gtrsim 40\%$ (w/o interference) of SM

What do we do about this?

- Check the SM prediction!
  [→ Bigi+,Bordone+,Gambino+,Grinstein+,Bernlochner+]
  ➥ $\delta R(D^*)$ larger, anomaly remains
- Combined analysis of all $b \to c\tau\nu$ observables [100+ papers]
  ➥ First model discrimination
- Related indirect bounds (partly model-dependent)
  ➥ High p_T searches, lepton decays, LFV, EDMs, . . .
- Analyze flavour structure of potential NP contributions
  ➥ quark flavour structure, e.g. $b \to u$
  ➥ lepton flavour structure, e.g. $b \to c\ell(= e, \mu)\nu$