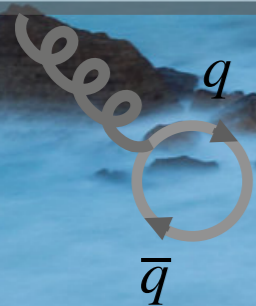
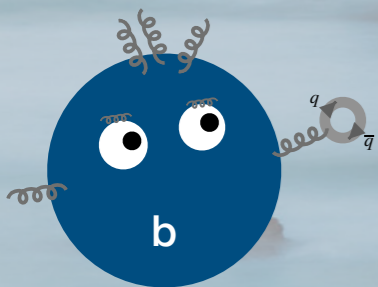


# Inclusive and Exclusive $|V_{xb}|$ Discrepancies

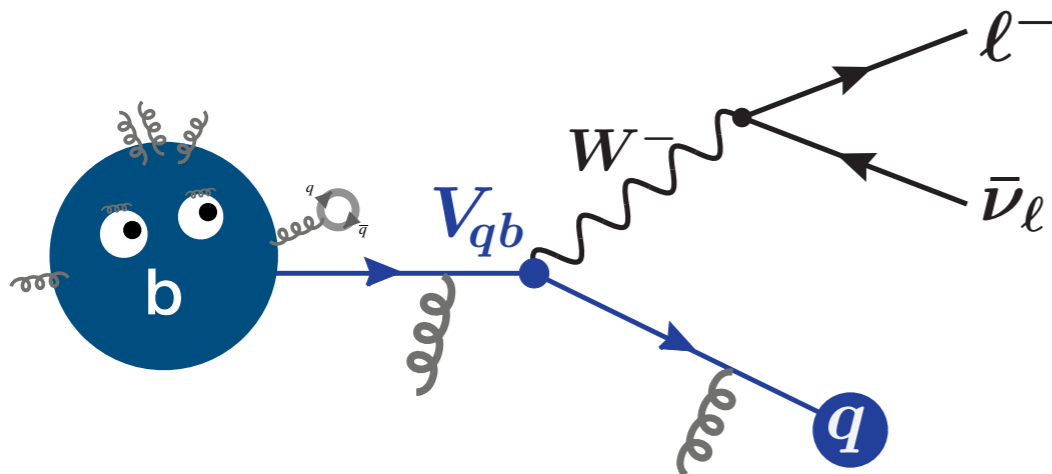
— An attempt of a guided diagnostic tour —

[florian.bernlochner@uni-bonn.de](mailto:florian.bernlochner@uni-bonn.de)

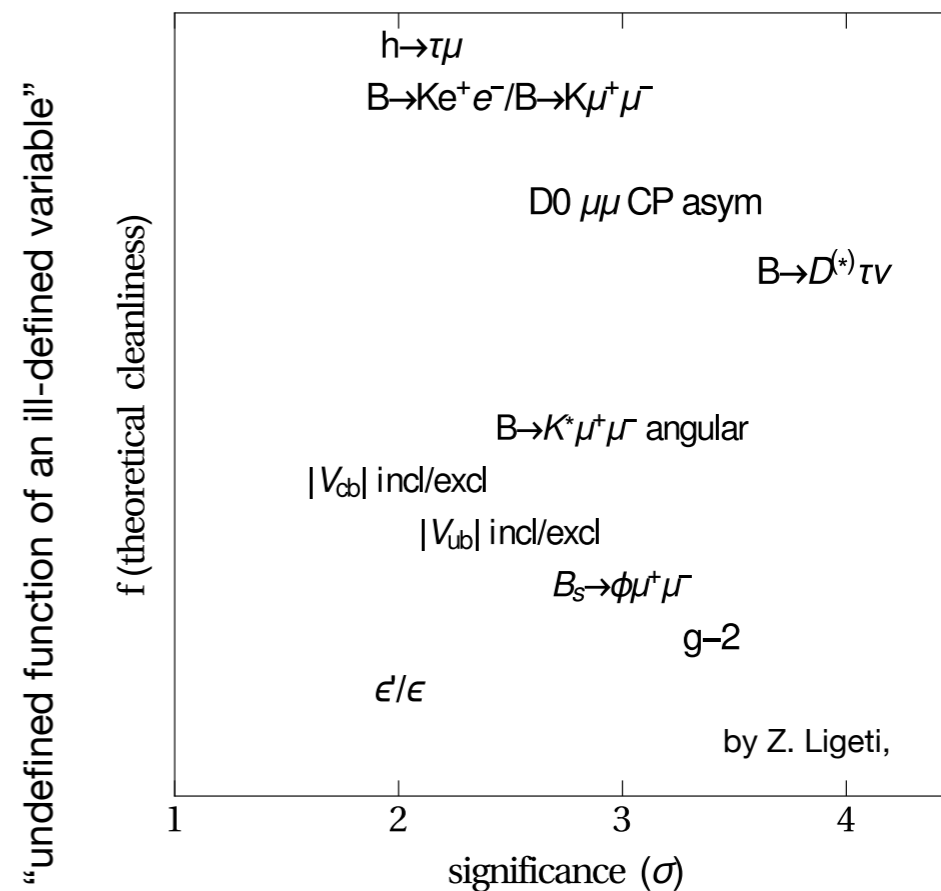
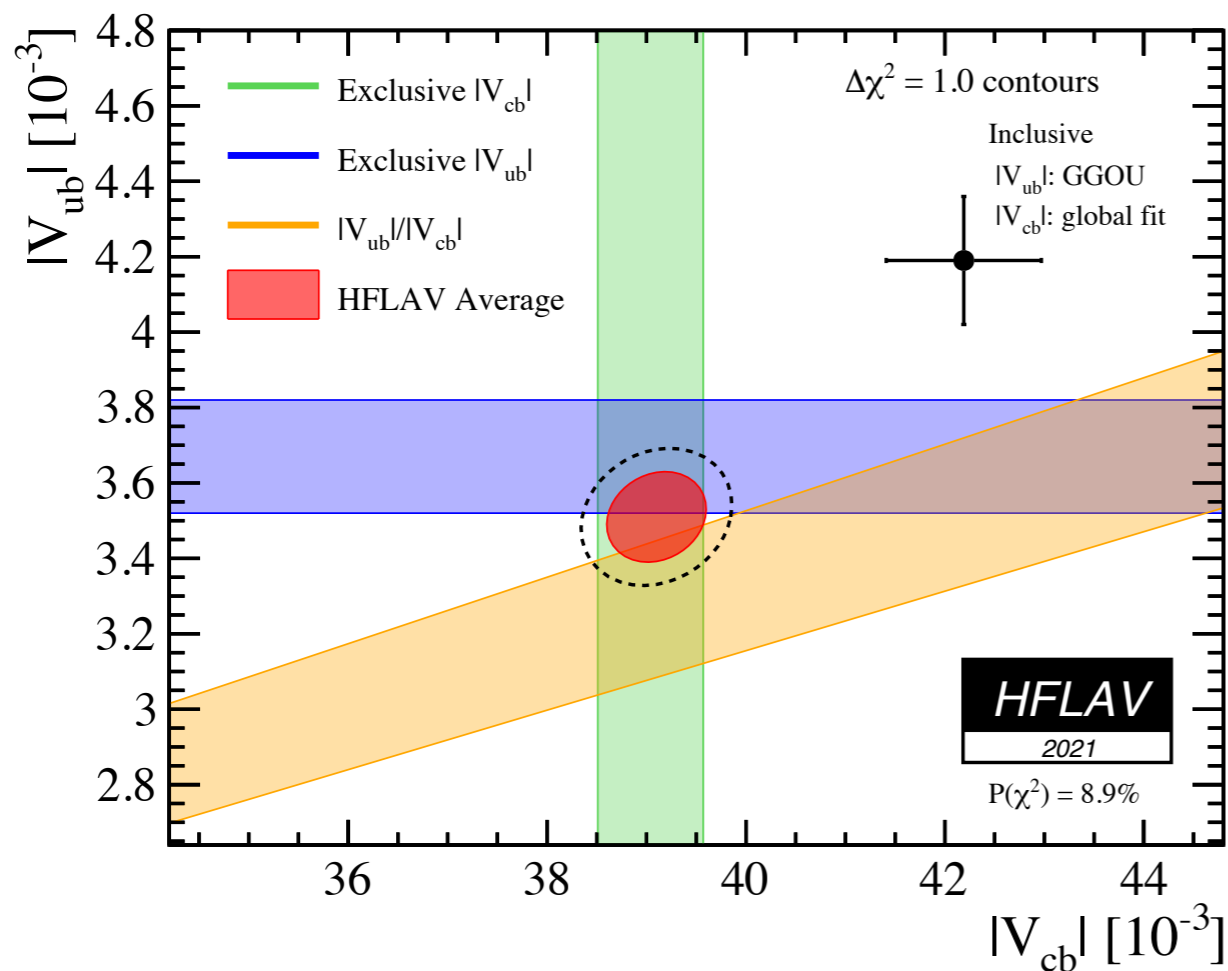
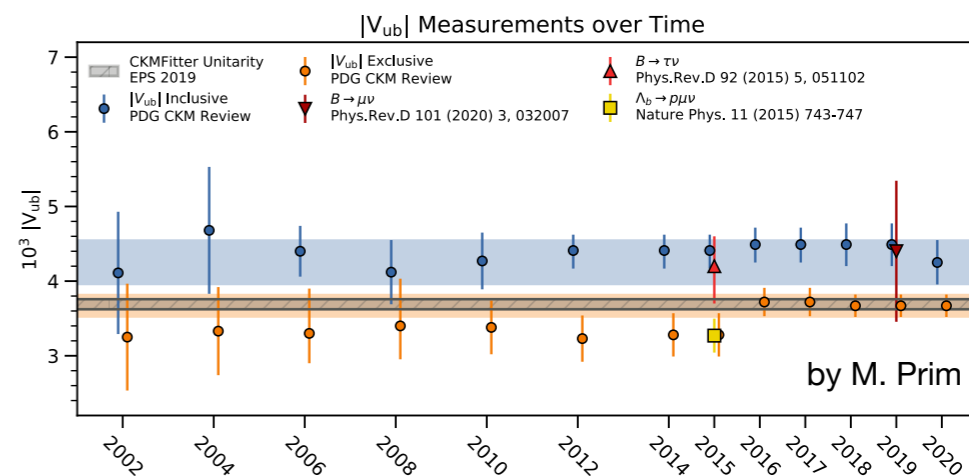


# The $|V_{xb}|$ puzzle

He may look cute, but that might be deceiving...



... Long-standing discrepancy since about a decade

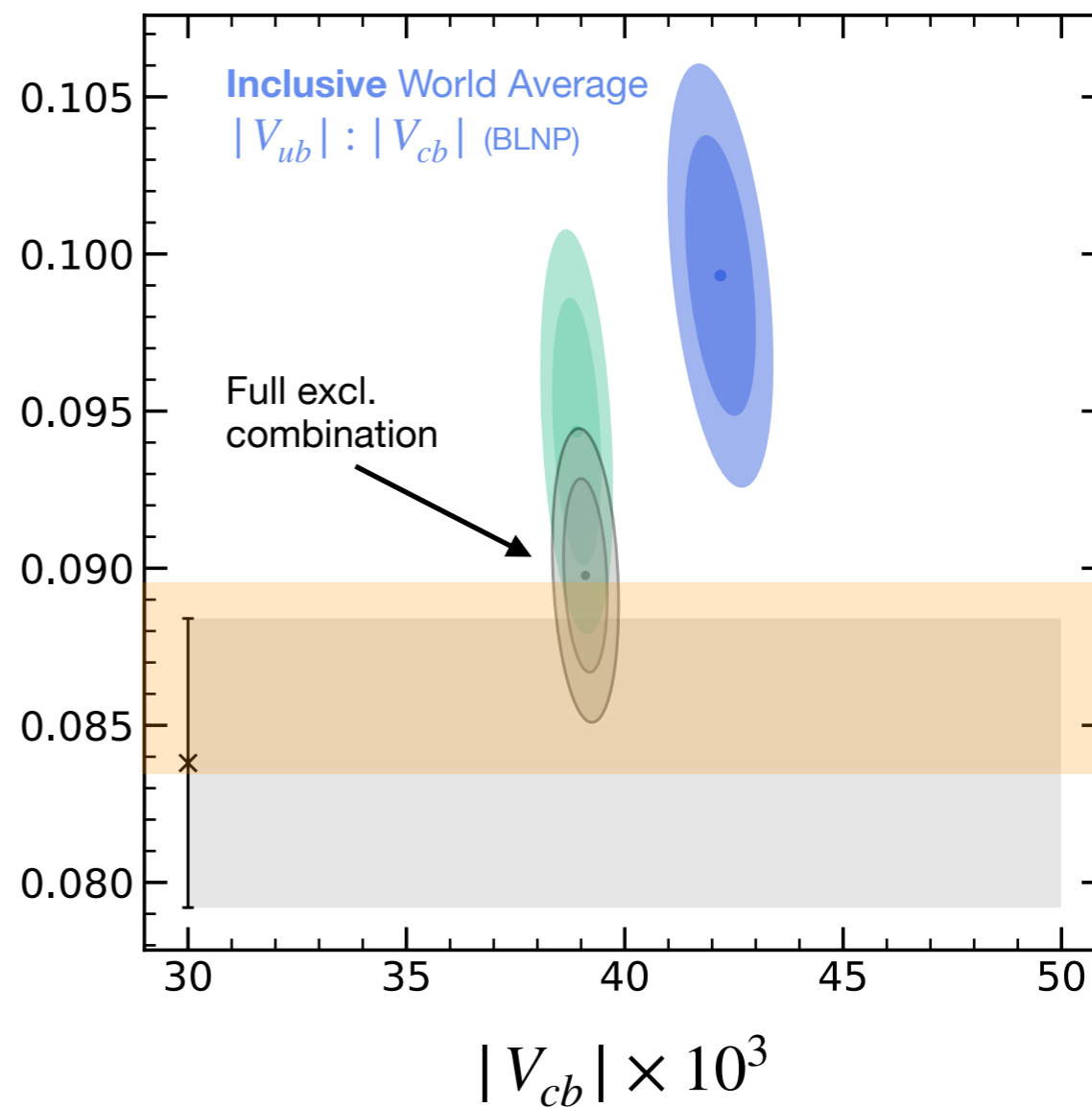
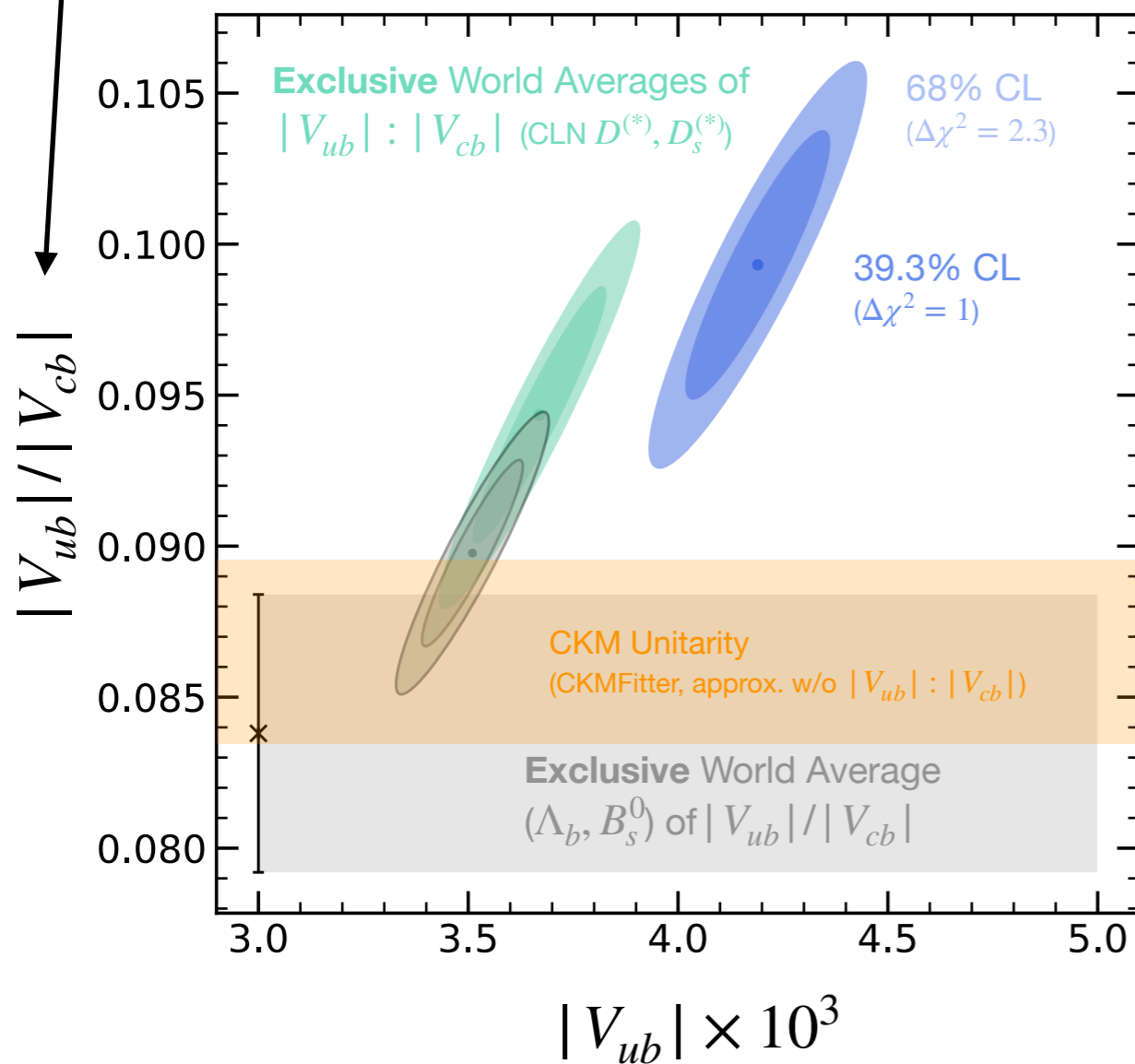
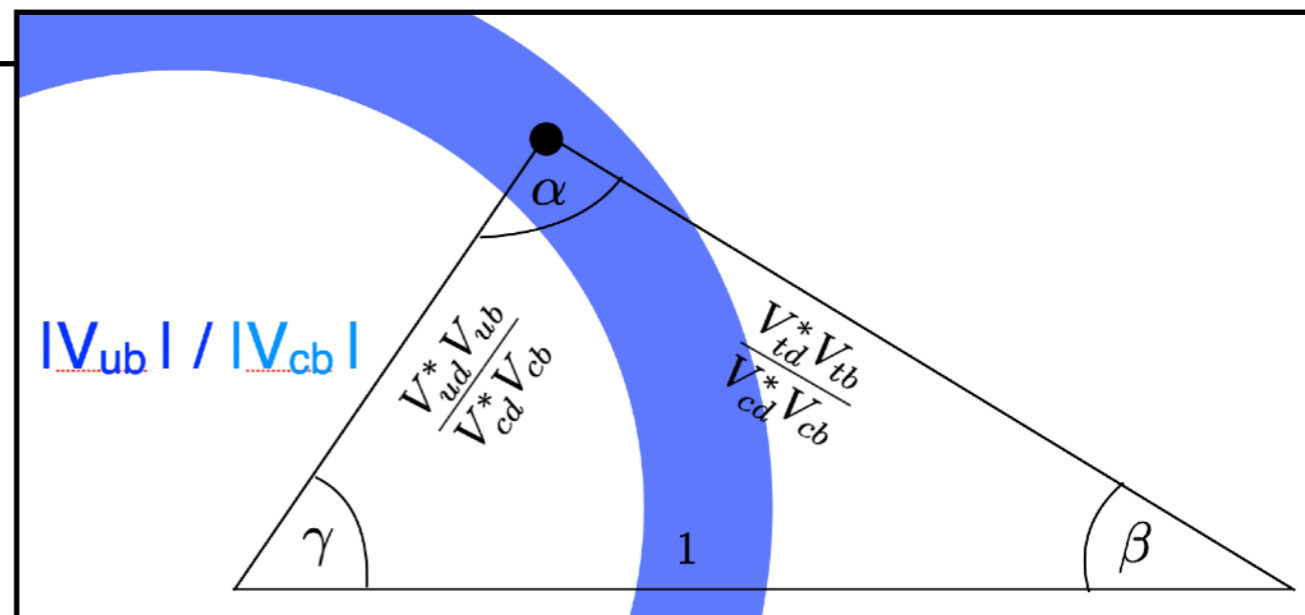


# Maybe some change of perspective helps?

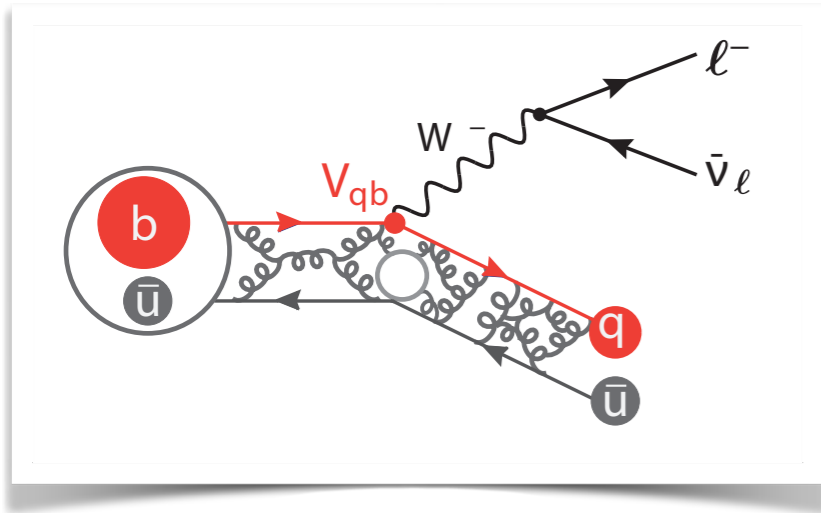
For global CKM fits we care about

$$|V_{ub}| / |V_{cb}|$$

Well, a little. But still troublesome..



“red thread”



Inclusive  $|V_{ub}|$

$$\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

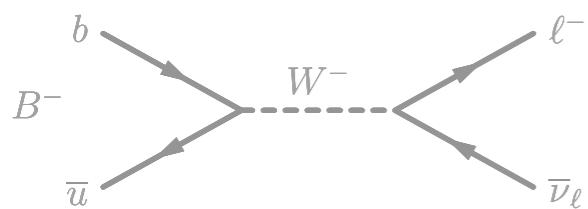
Inclusive  $|V_{cb}|$

$$\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[ \Gamma(b \rightarrow q \ell \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \dots \right]$$

‘Leptonic’  $|V_{ub}|$



$$\mathcal{B} \propto |V_{ub}|^2 f_B^2 m_\ell^2$$

B-Meson decay constant

Exclusive  $|V_{ub}|$

$$\bar{B} \rightarrow \pi \ell \bar{\nu}_\ell, \Lambda_b \rightarrow p \mu \bar{\nu}_\mu$$

A brief tour with personal highlights and thoughts

Exclusive  $|V_{cb}|$

$$\bar{B} \rightarrow D \ell \bar{\nu}_\ell, \bar{B} \rightarrow D^* \ell \bar{\nu}_\ell$$

$$B_s \rightarrow D_s^{(*)} \mu \bar{\nu}_\mu$$

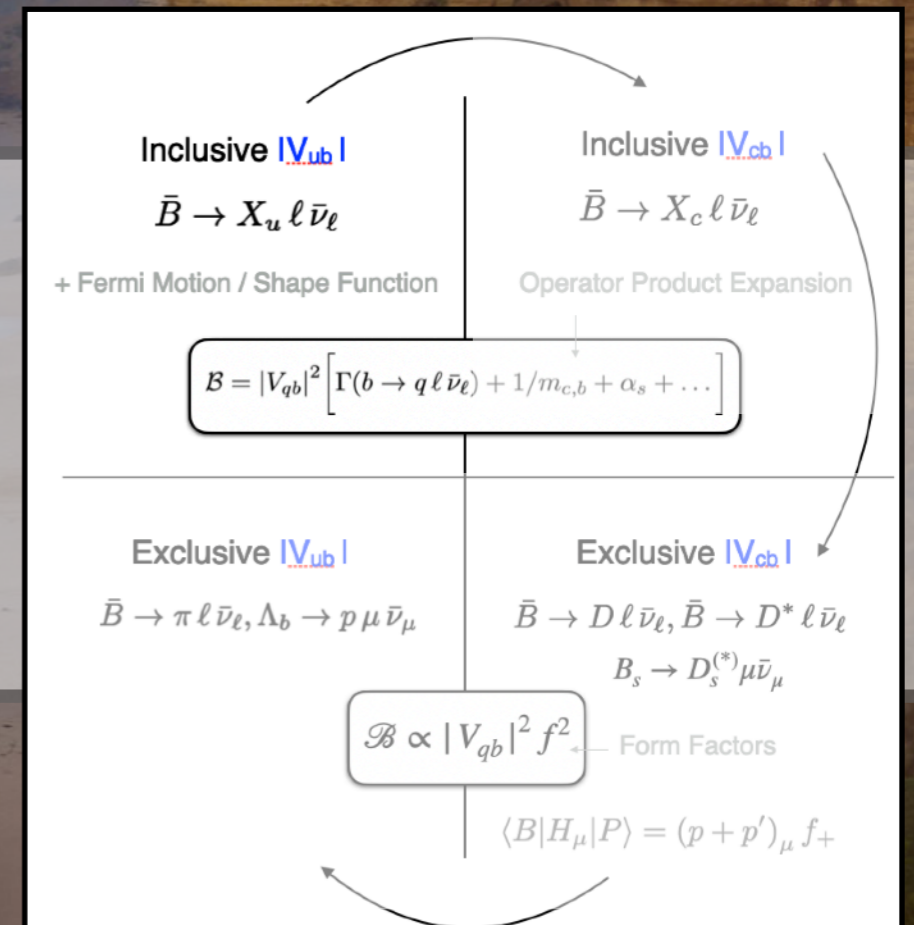
$$\mathcal{B} \propto |V_{qb}|^2 f^2$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu f_+$$

# Incl. $|V_{ub}|$

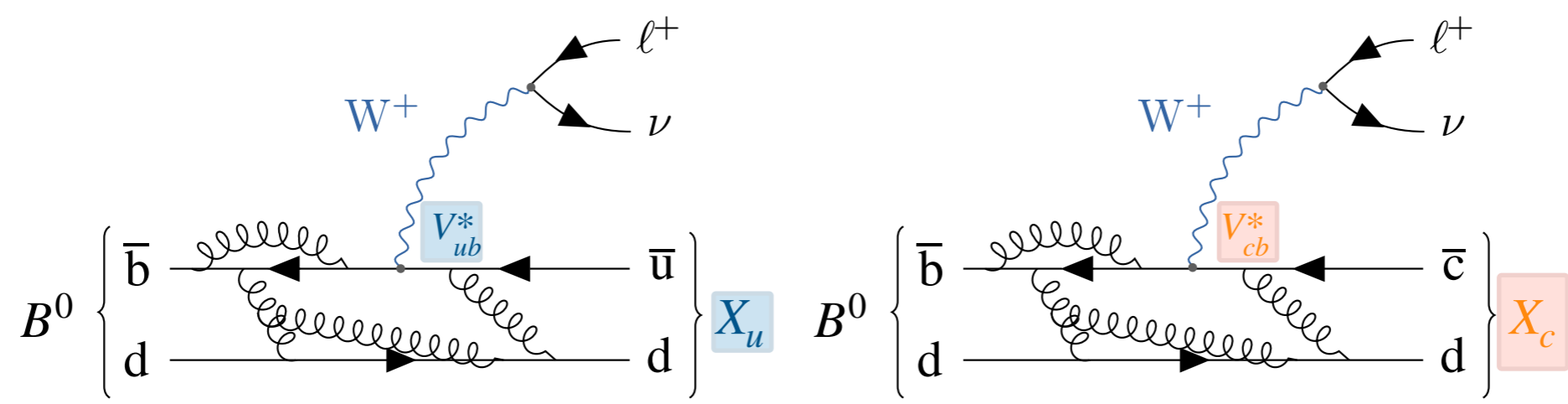
- ▶ Caveats on current (recent) results
- ▶ Era of Differential measurements



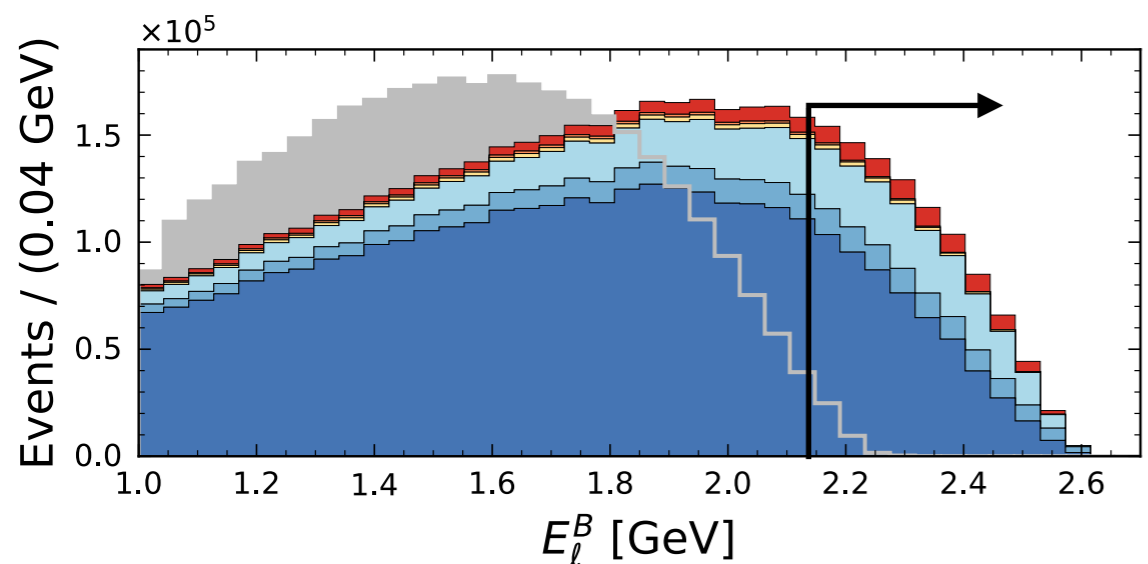
# State-of-the-Art

- x  $\mathcal{O}(100)$  more abundant
- Very similar **signature**:
  - high momentum lepton, hadronic system
- Clear separation only in corners of phase space
  - high  $E_\ell$ , low  $M_X$

Measuring  $|V_{ub}|$  is **hard** due to  $B \rightarrow X_c \ell \bar{\nu}_\ell$



The experimenter's dilemma illustrated with  $E_\ell^B$ :

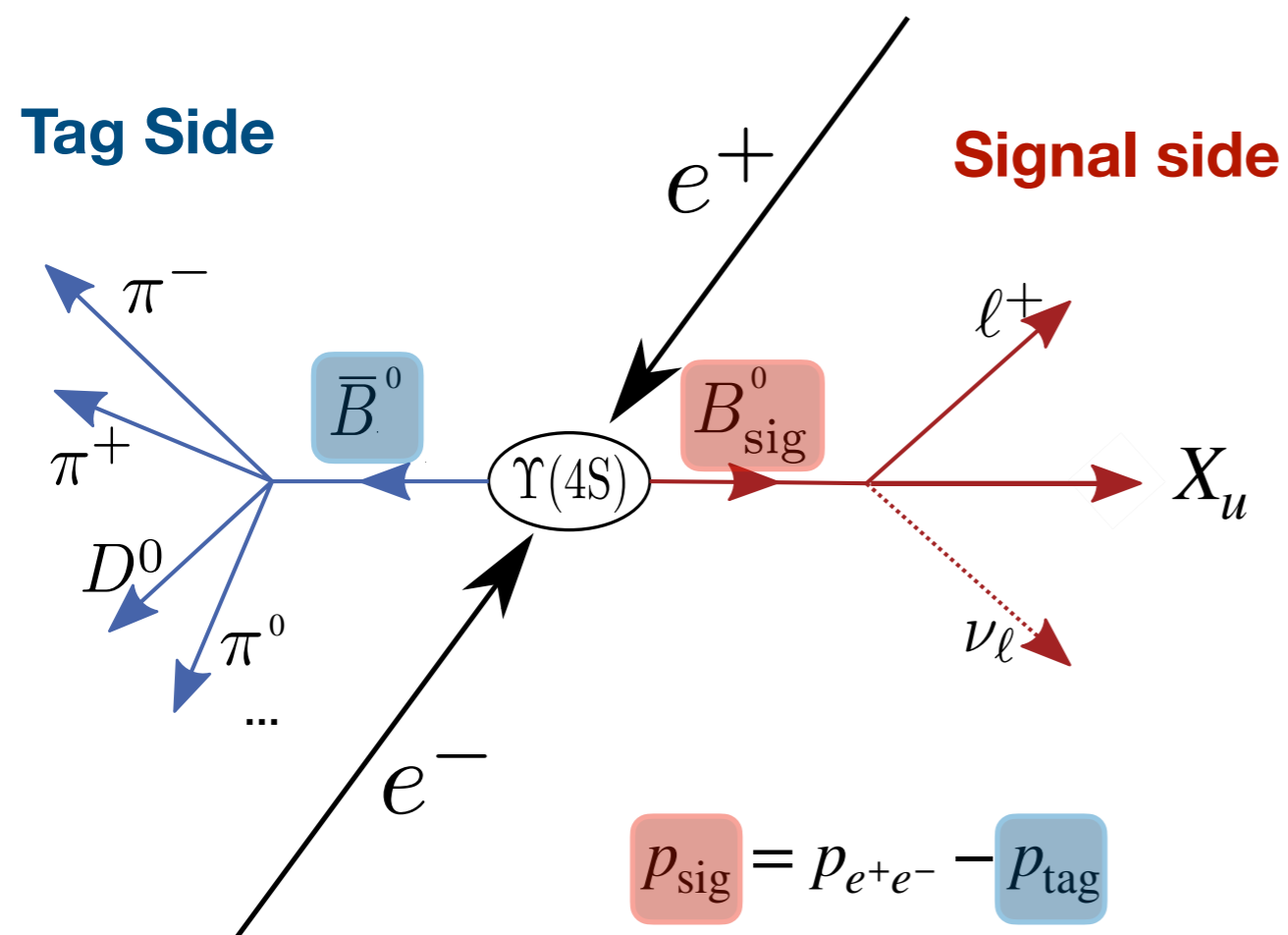


$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \rightarrow X_u \ell \bar{\nu}_\ell)}{\tau \Delta\Gamma(B \rightarrow X_u \ell \bar{\nu}_\ell)}}$$

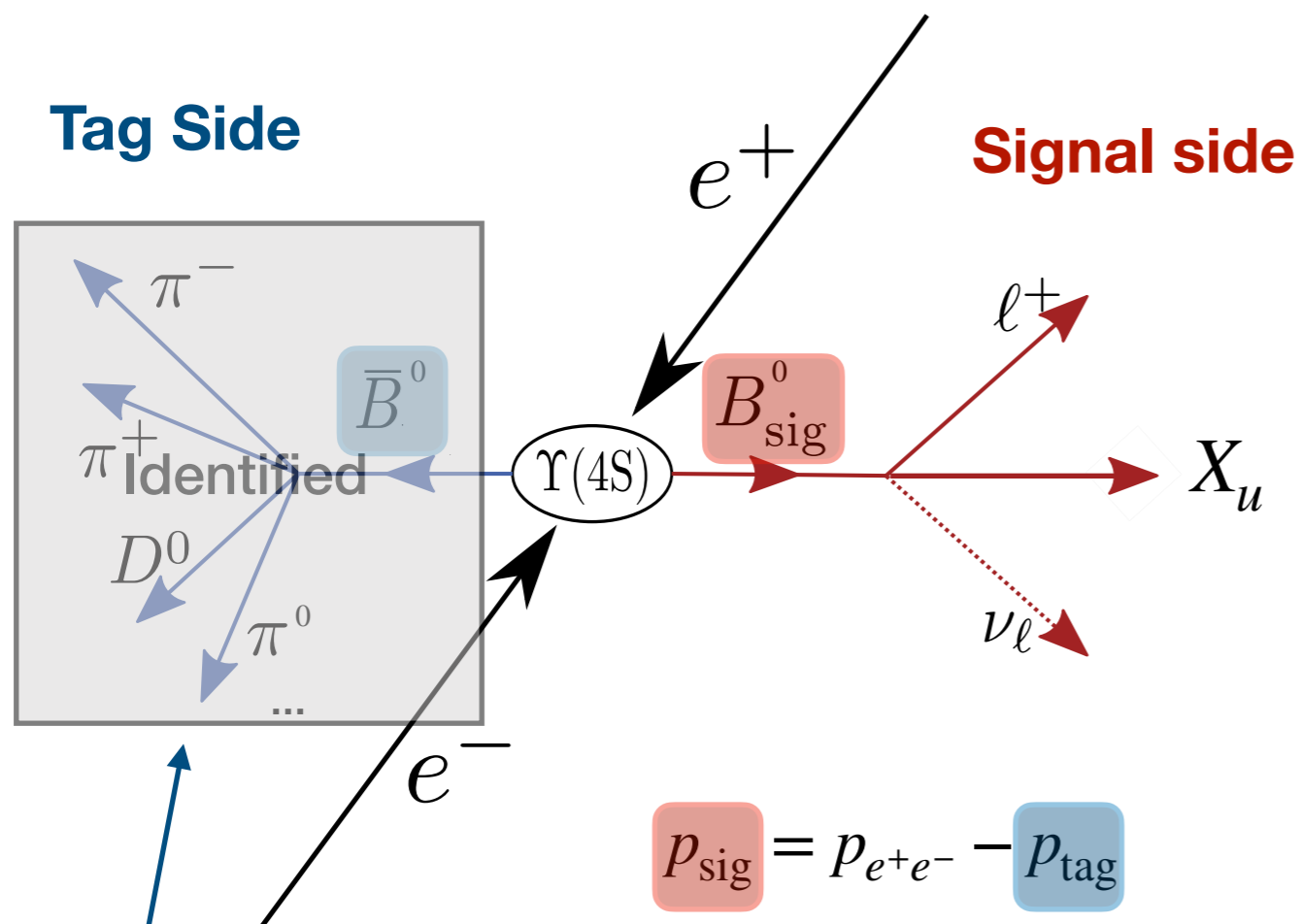
Theory error gets large  
Experimental uncert. small  
→ high cut

Theory error gets small  
Experimental uncert. large  
← low cut

# Hadronic Tagging

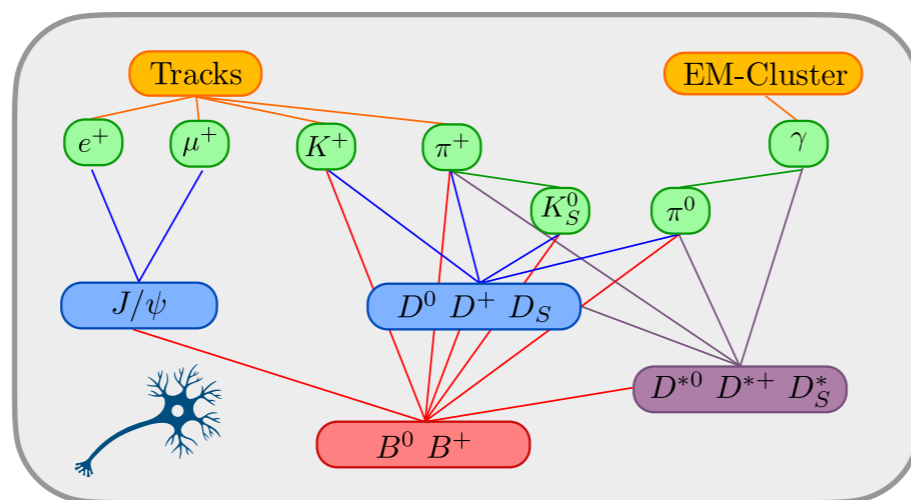


# Hadronic Tagging



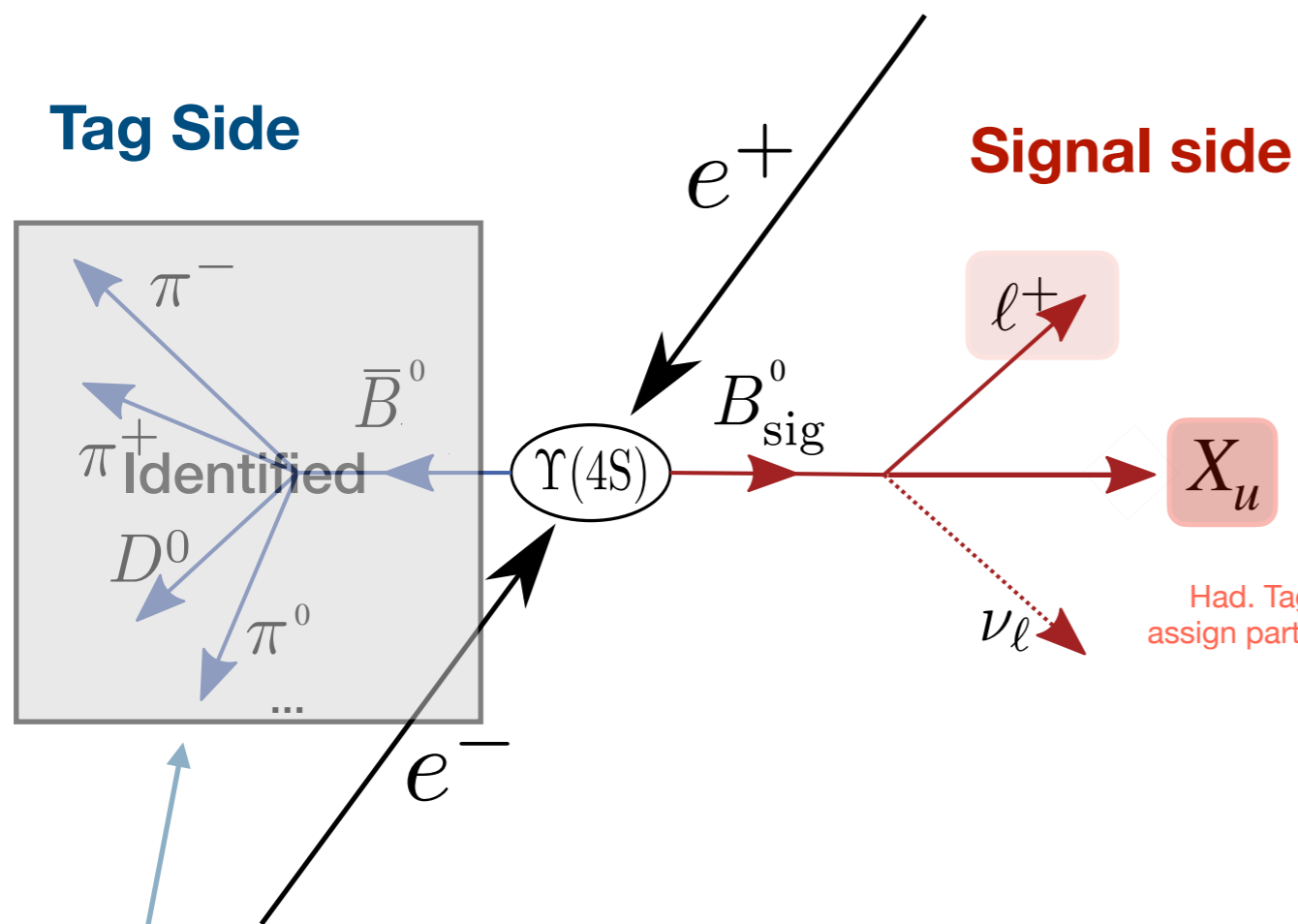
Candidates reconstructed with **hierarchical** approach & **neural networks** in **hadronic modes**

**1104 decay cascades** used with an **efficiency** of **0.28% / 0.18%** for  $B^\pm$  and  $B^0/\bar{B}^0$

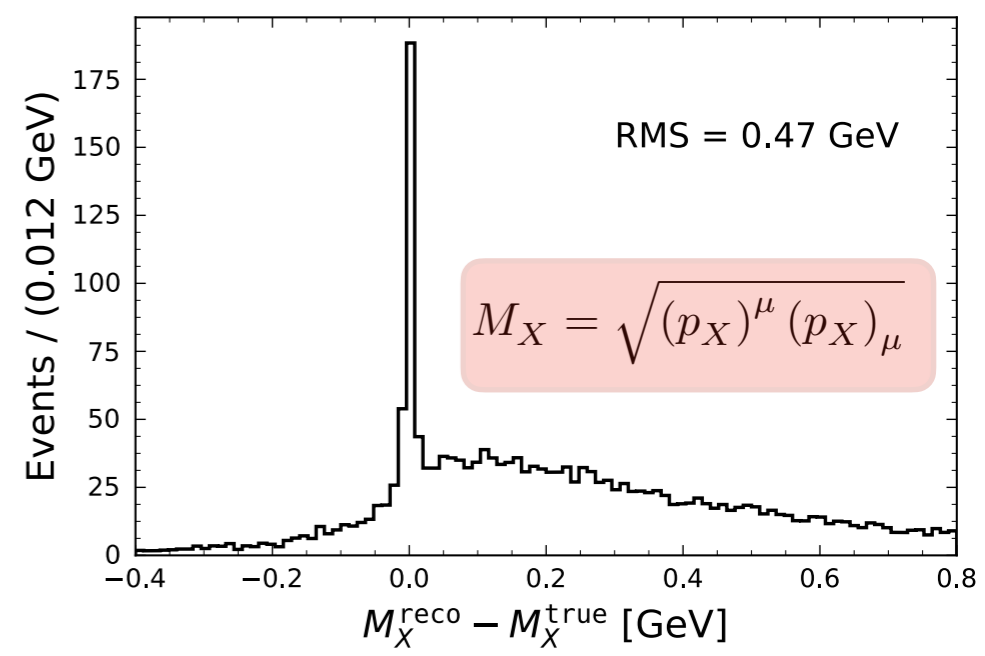




# Hadronic Tagging

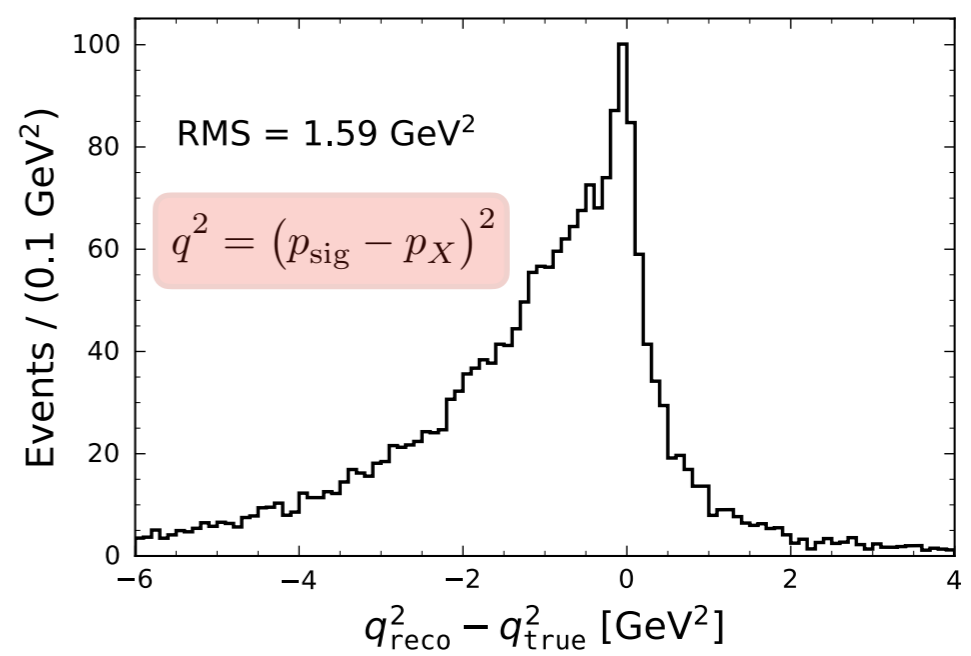
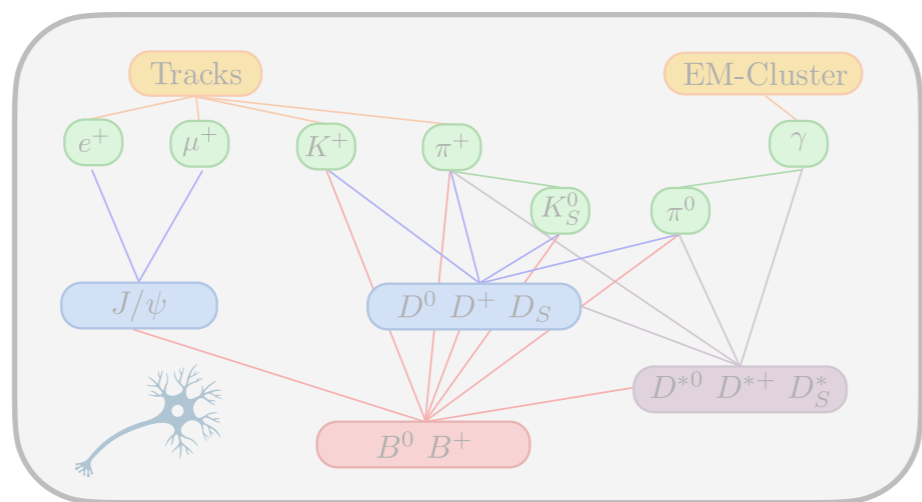


$$p_X = \sum_i \left( \sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

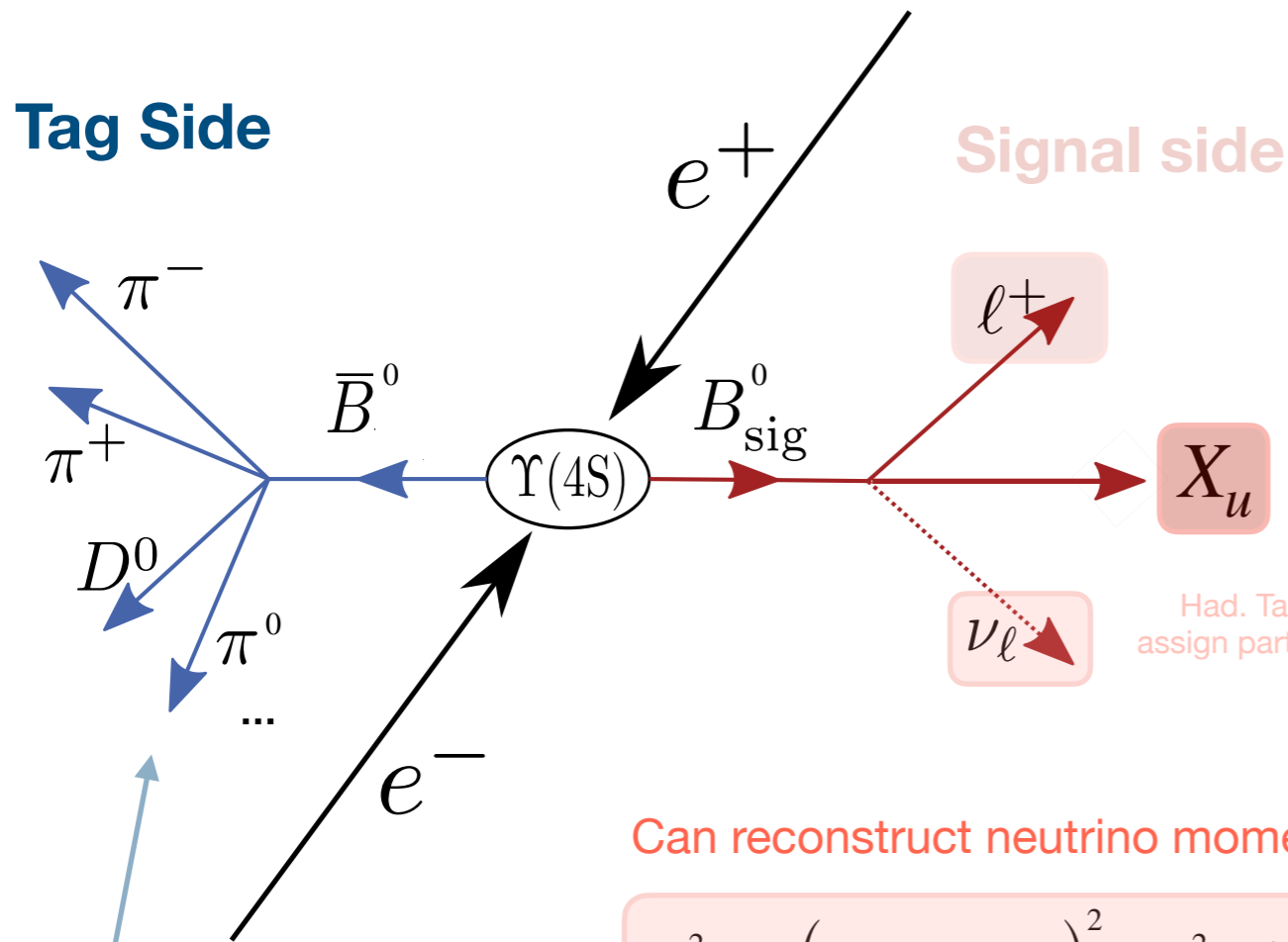


Candidates reconstructed with **hierarchical** approach & **neural networks** in **hadronic modes**

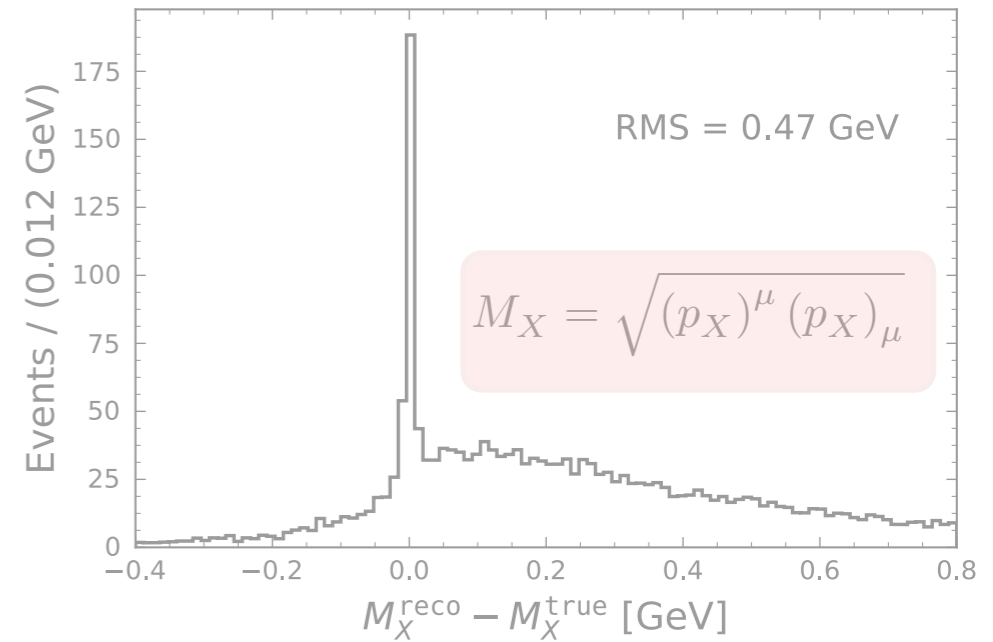
**1104 decay cascades** used with an **efficiency** of **0.28% / 0.18%** for  $B^\pm$  and  $B^0/\bar{B}^0$



# Hadronic Tagging



$$p_X = \sum_i \left( \sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

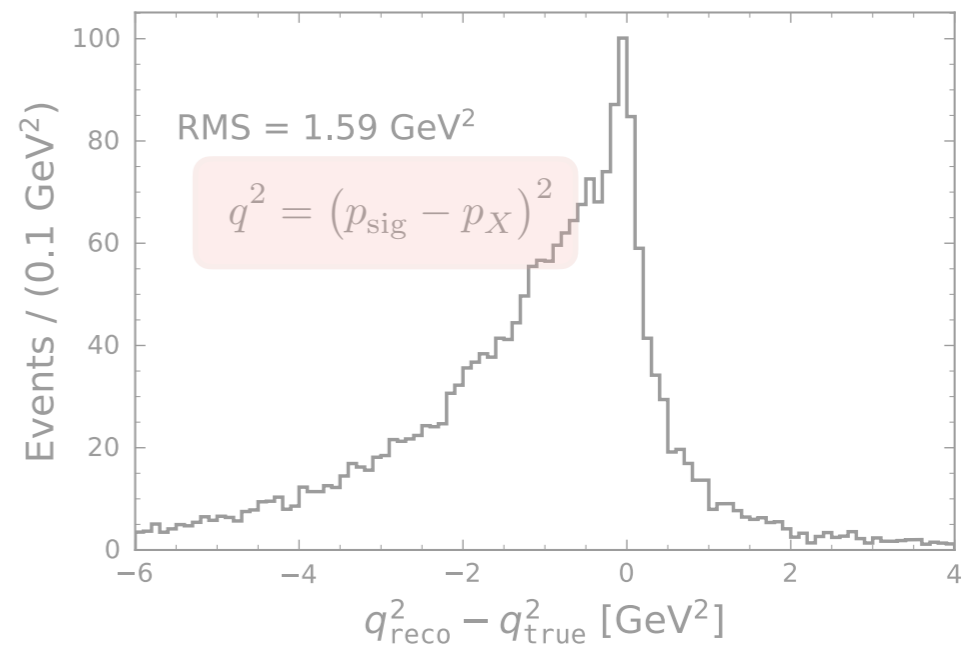
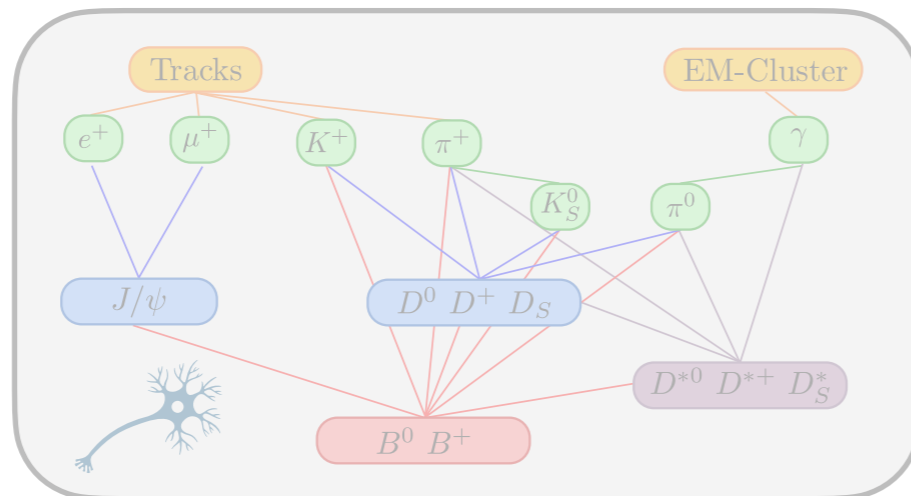


Can reconstruct neutrino momentum:

$$m_{\text{miss}}^2 = (p_{\text{sig}} - p_X - p_\ell)^2 \approx m_\nu^2 = 0 \text{ GeV}^2$$

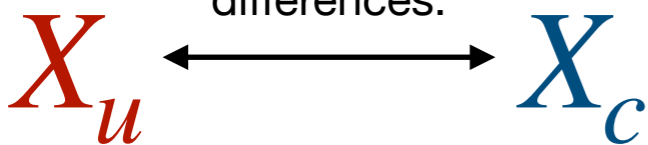
Candidates reconstructed with hierarchical approach & neural networks in hadronic modes

1104 decay cascades used with an efficiency of 0.28% / 0.18% for  $B^\pm$  and  $B^0/\bar{B}^0$



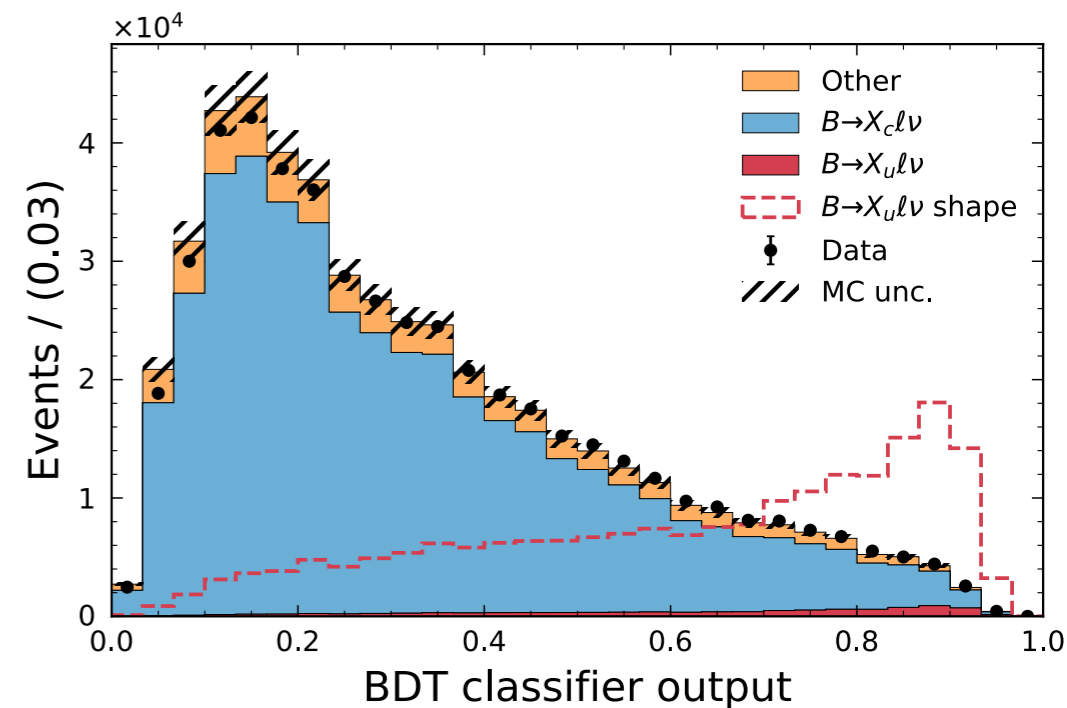
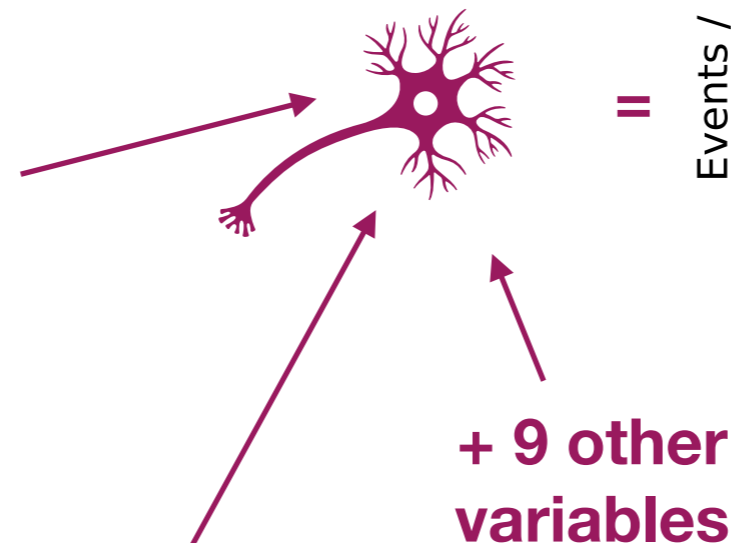
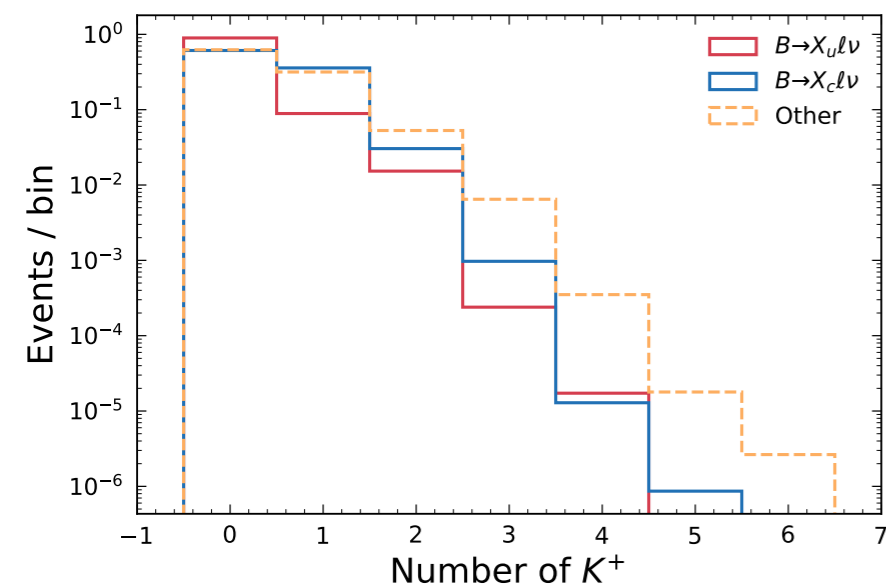
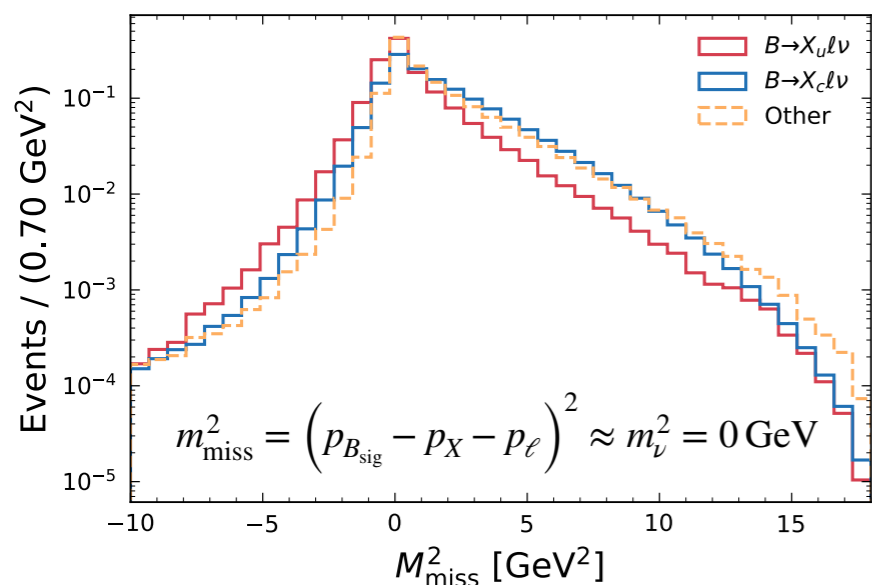
# Multivariate Sledgehammer

Can exploit that there are differences:



**Higher multiplicity**  
Often come with charged and neutral **Kaons**  
**D\* decays (slow pions)**  
(Slightly lower  $E_e$ )

**Direct cuts on  $m_X, E_\ell$  problematic**  
(i.e. direct shape-function dependence)

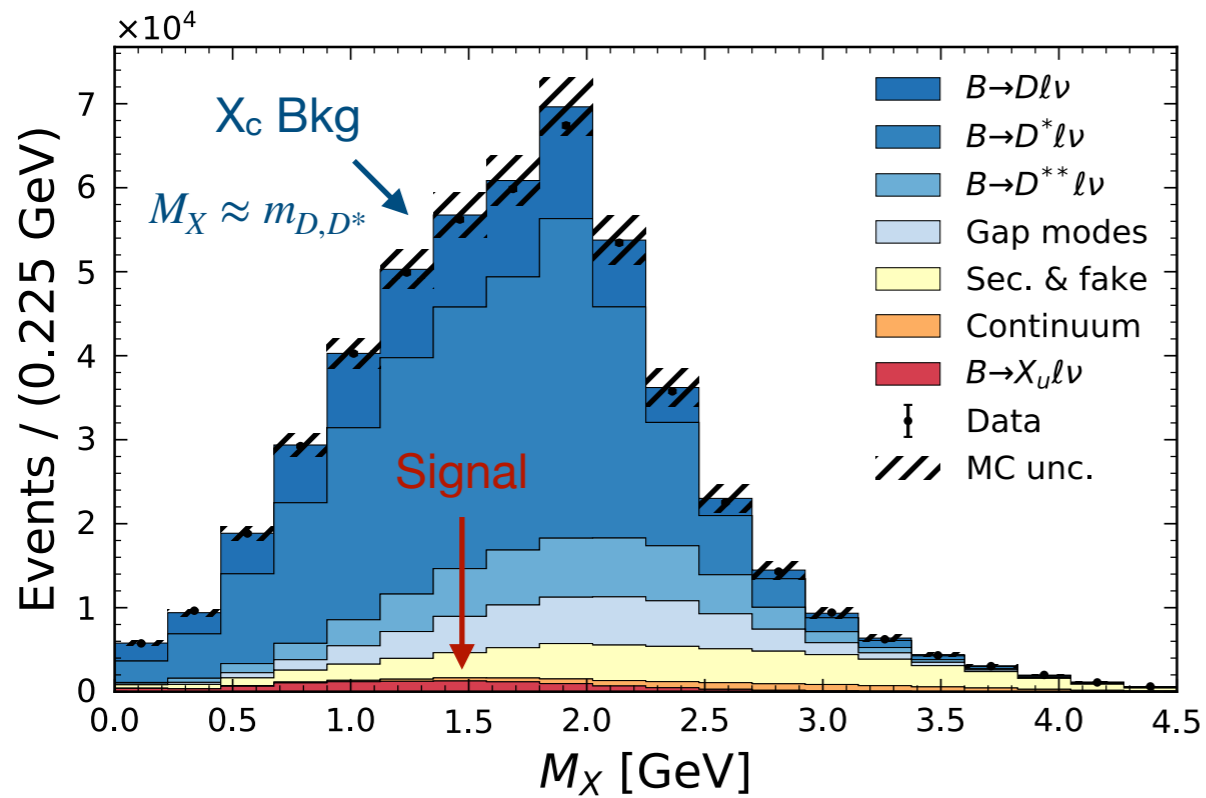


Can reject **98.7%** of  $X_c$

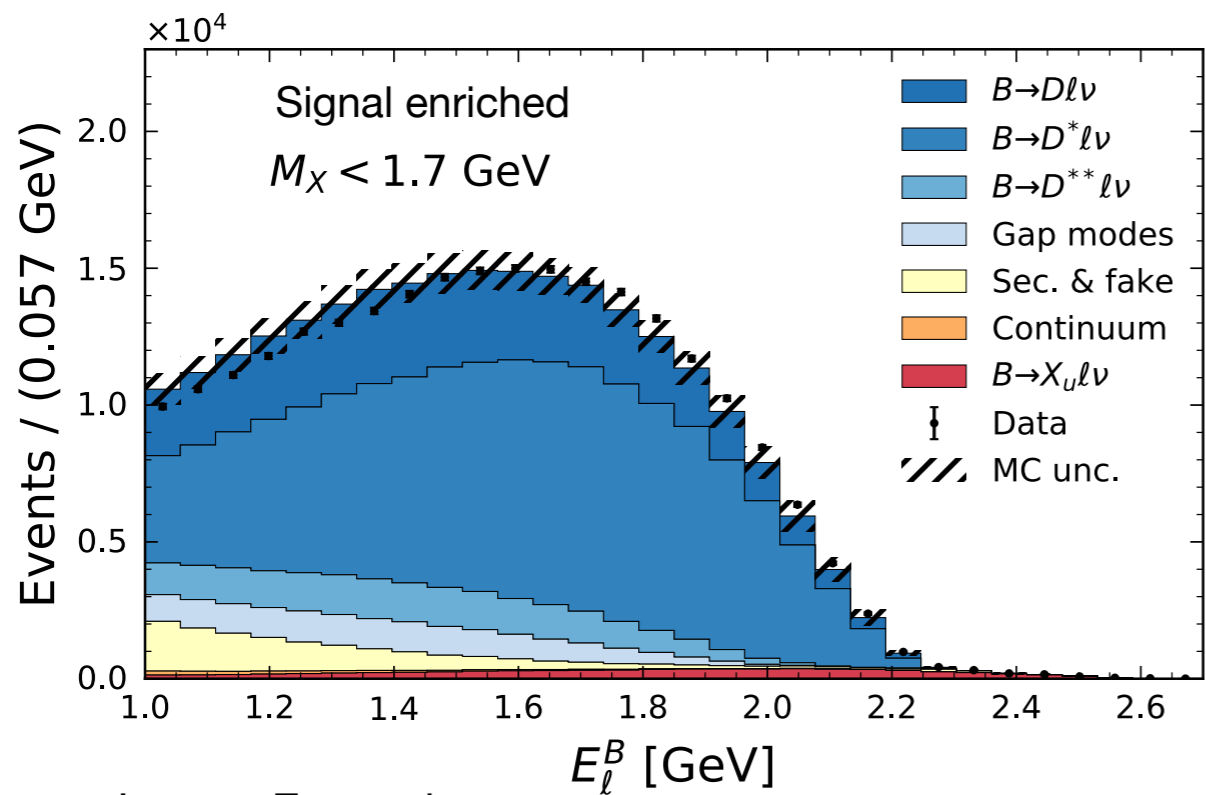
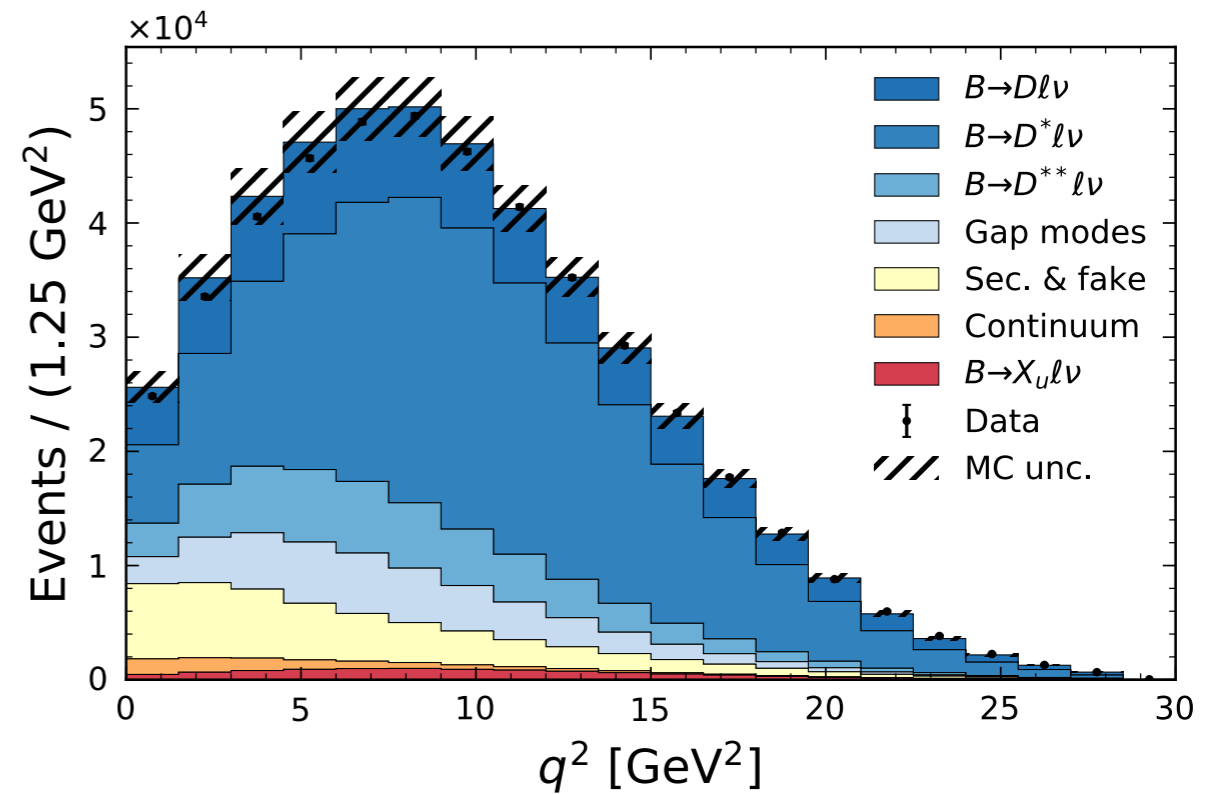
Selection	$B \rightarrow X_u \ell^+ \nu_\ell$	$B \rightarrow X_c \ell^+ \nu_\ell$	Data
$M_{bc} > 5.27 \text{ GeV}$	84.8%	83.8%	80.2%
$\mathcal{O}_{\text{BDT}} > 0.85$	18.5%	1.3%	1.6%
$\mathcal{O}_{\text{BDT}} > 0.83$	21.9%	1.7%	2.1%
$\mathcal{O}_{\text{BDT}} > 0.87$	14.5%	0.9%	1.1%

... and retain **18.5%** of  $X_u$

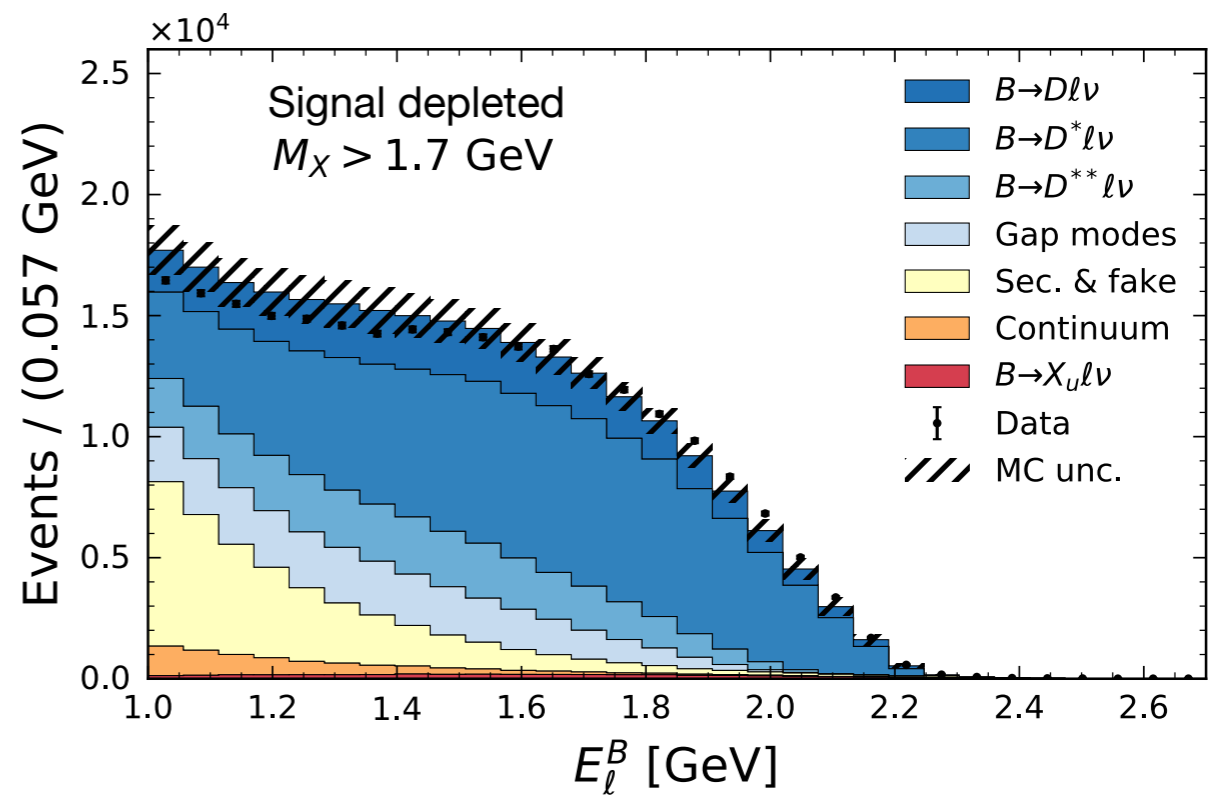
Hadronic Mass  $M_X = \sqrt{p_X^2}$



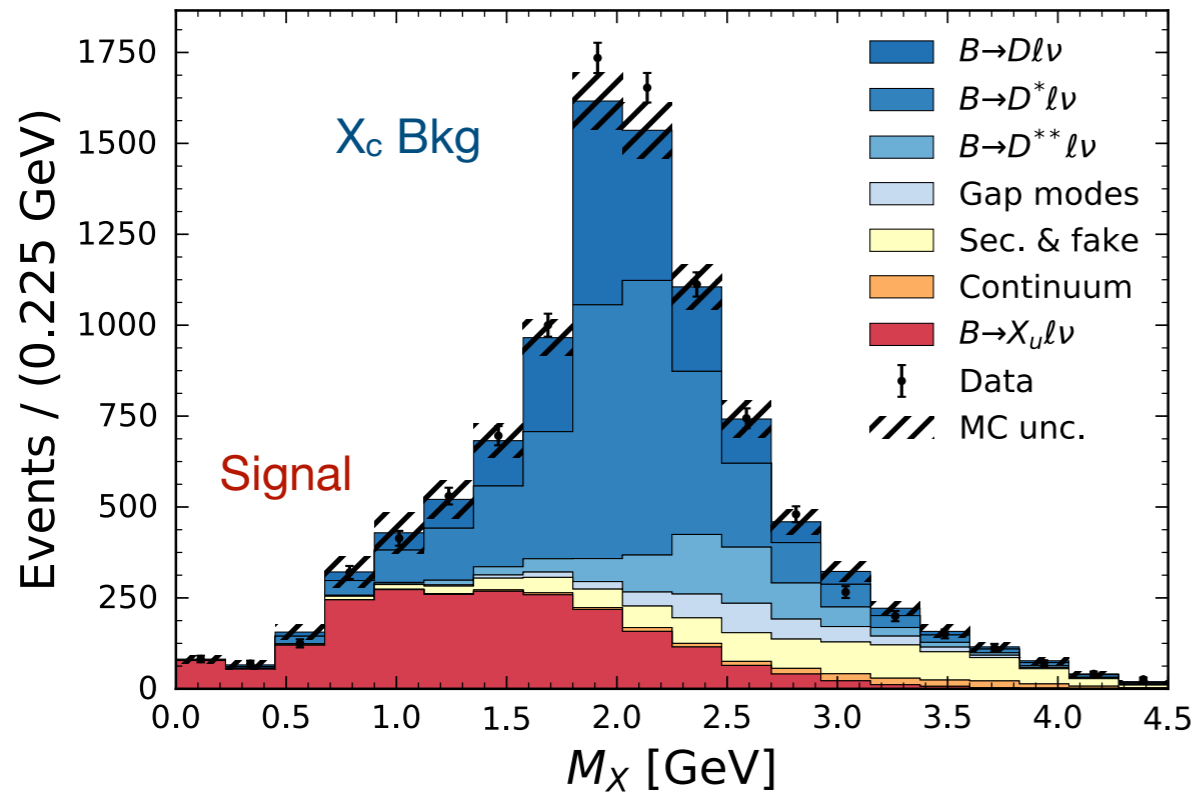
Four-momentum transfer squared  $q^2 = (p_B - p_X)^2$



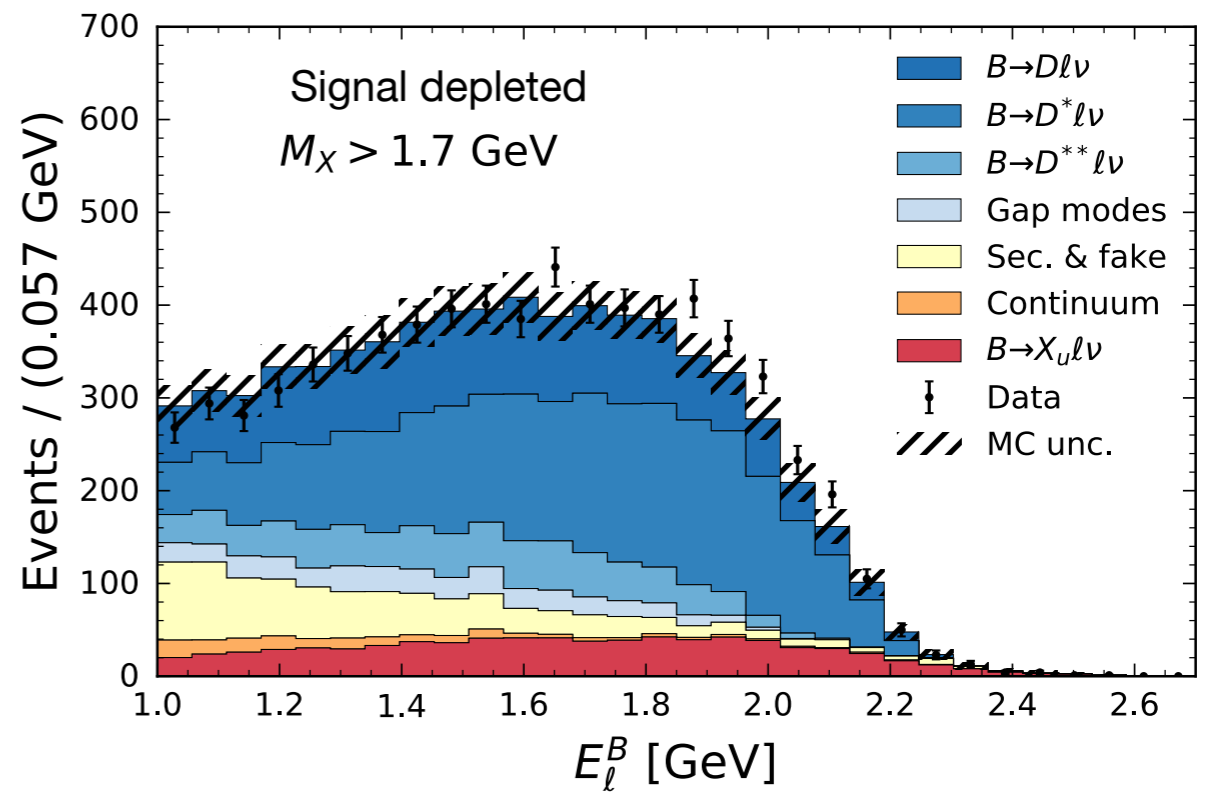
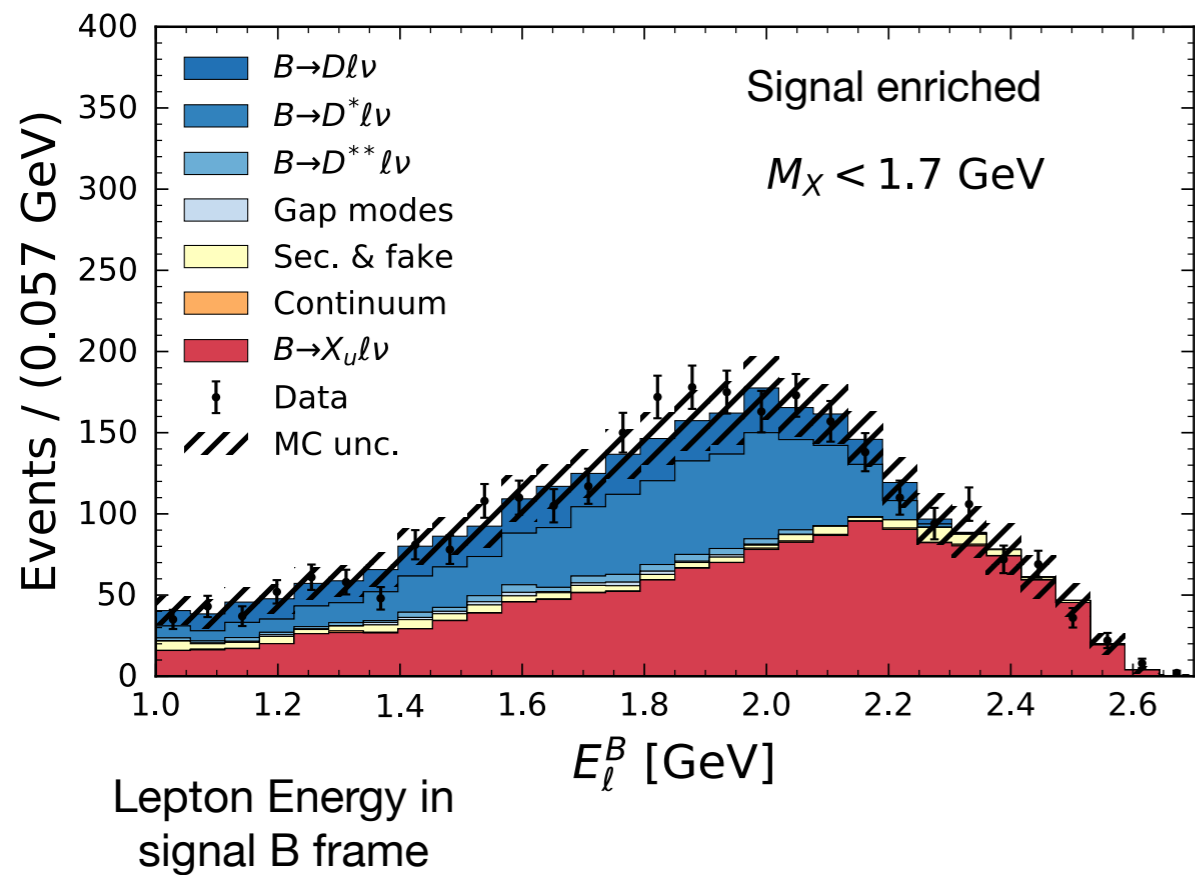
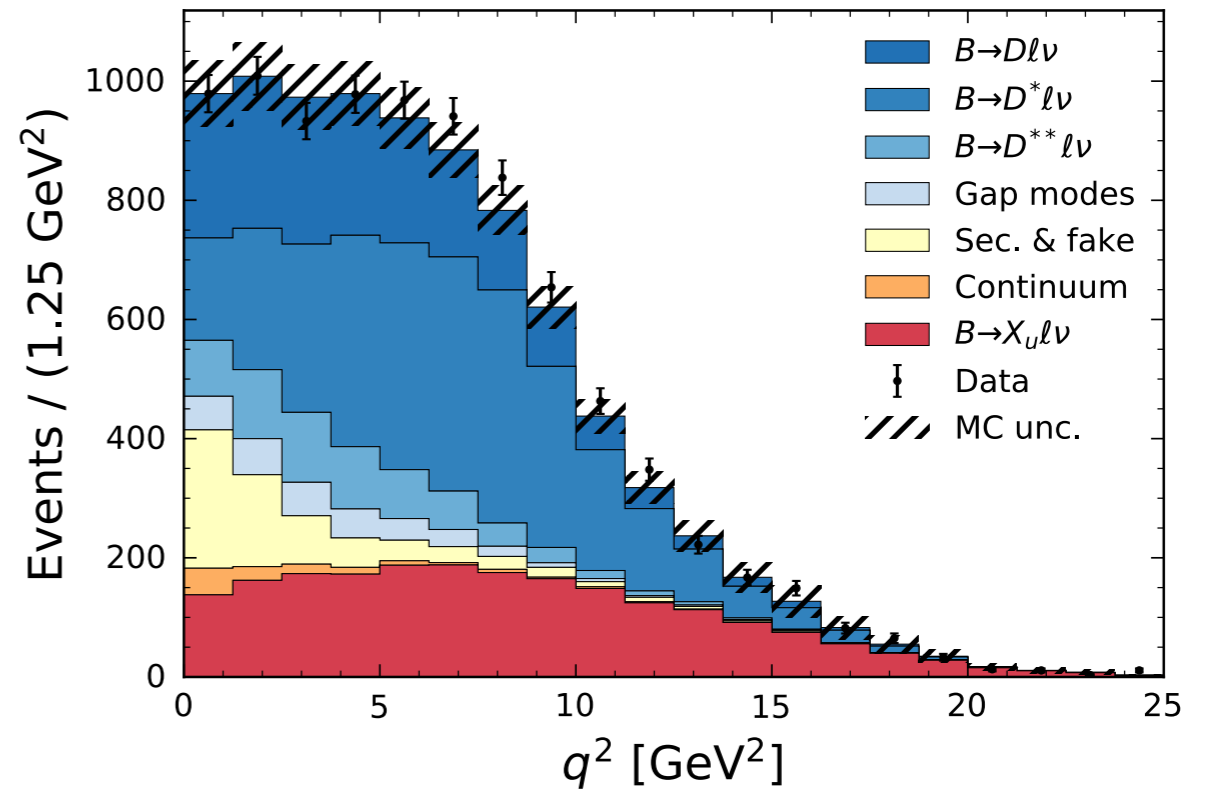
Lepton Energy in  
signal B frame



Hadronic Mass  $M_X = \sqrt{p_X^2}$



Four-momentum transfer squared  $q^2 = (p_B - p_X)^2$



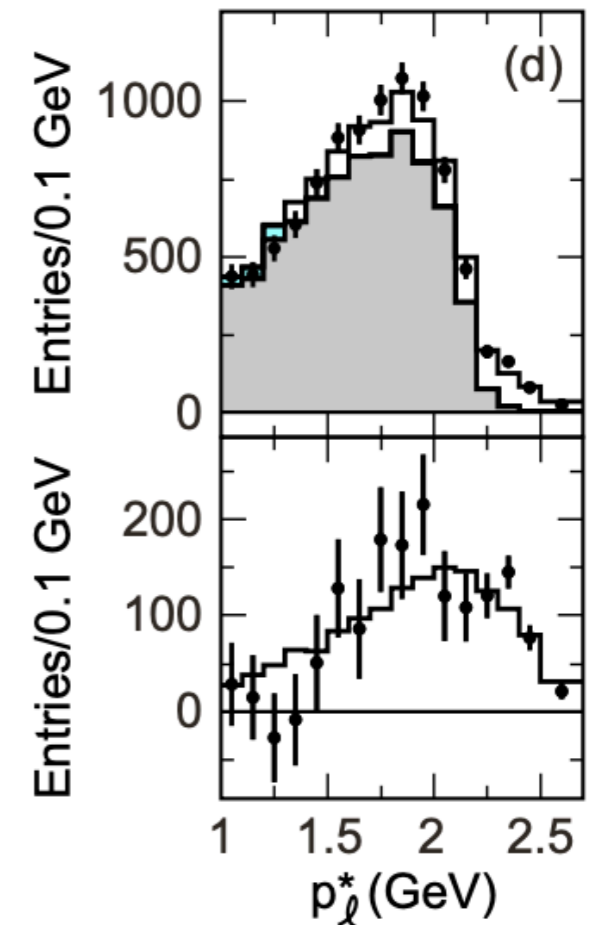
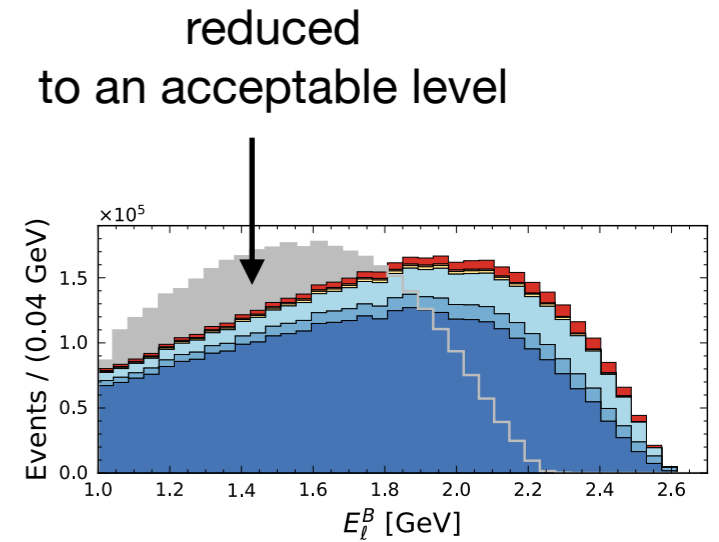
Ok, but what's the problem?

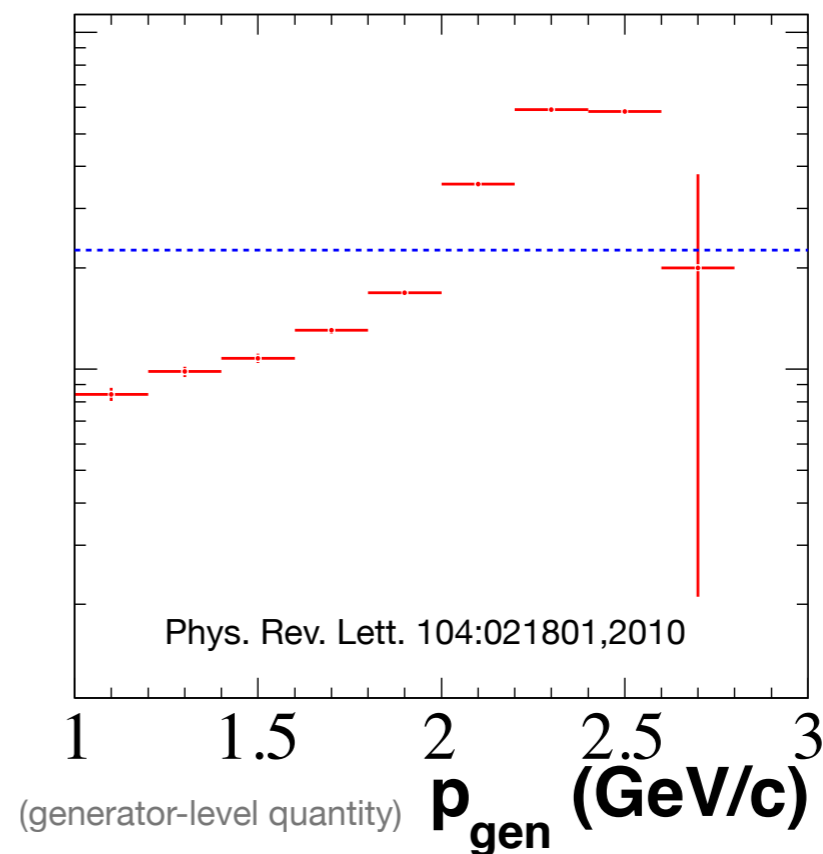
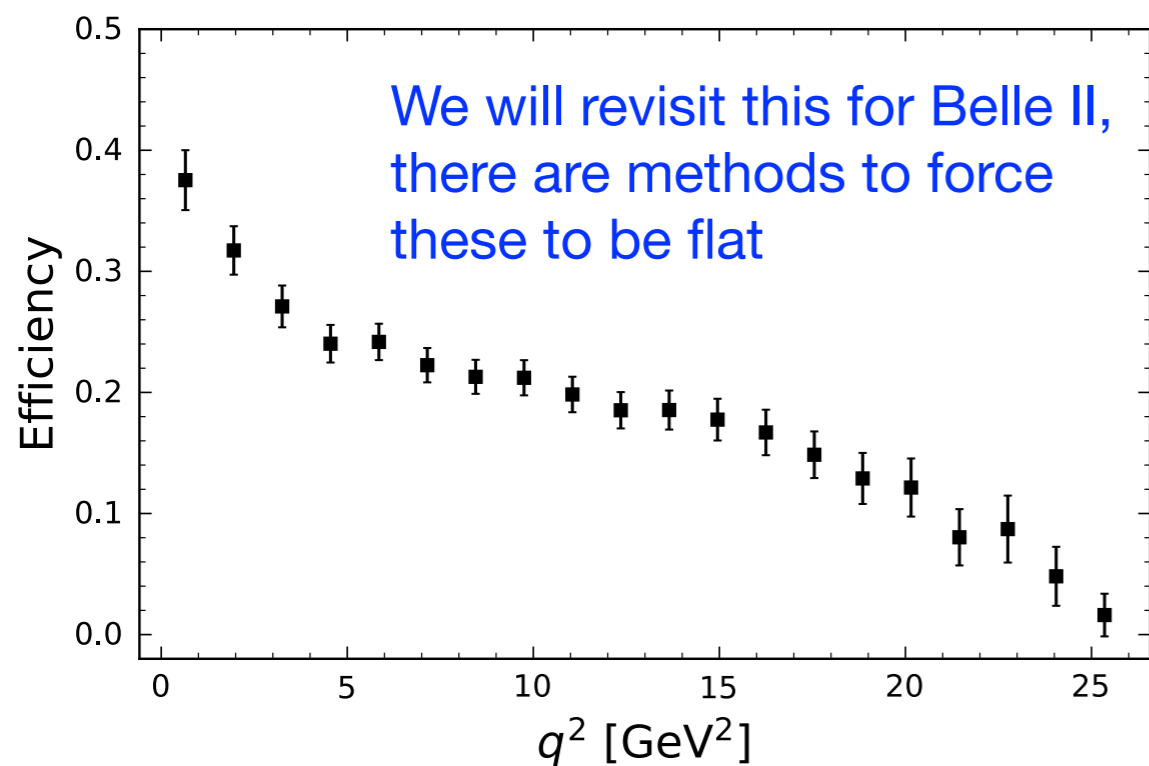
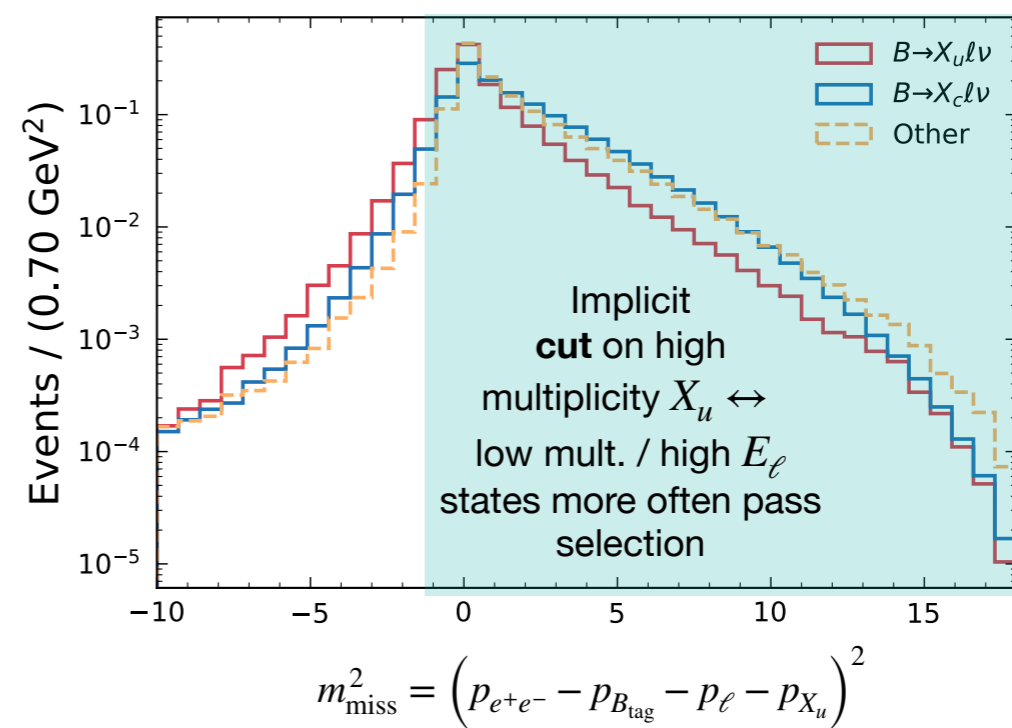
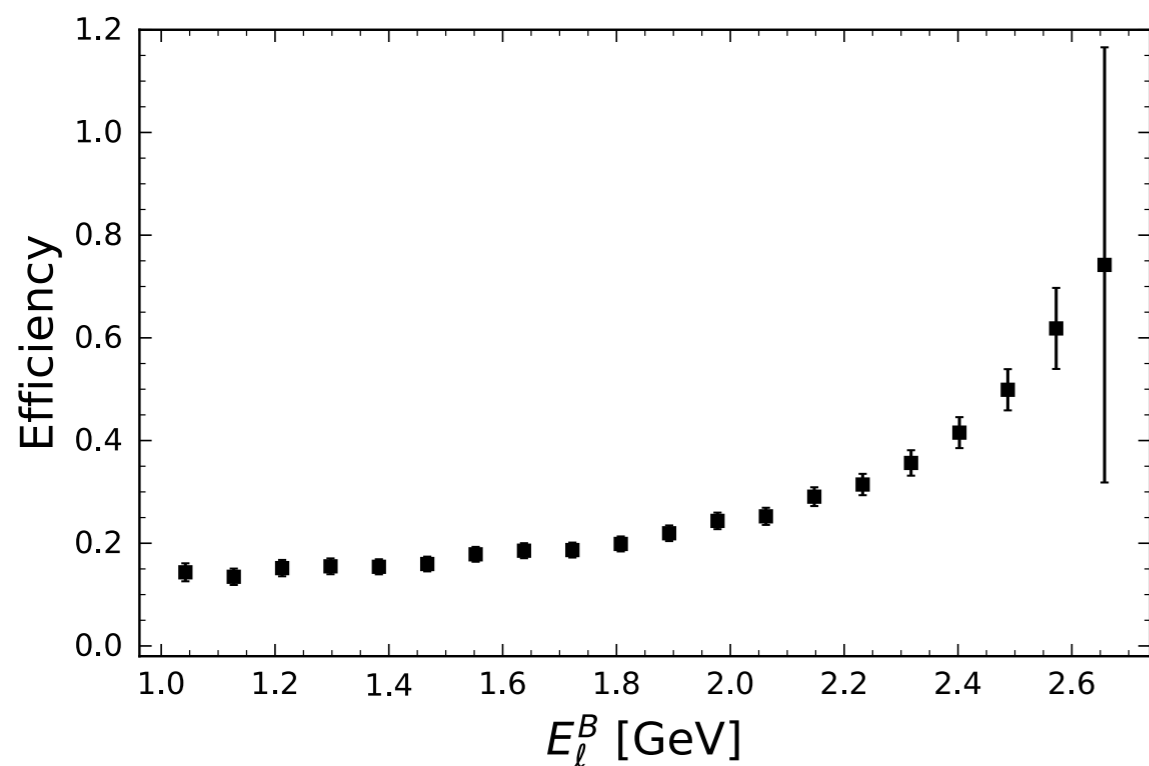
### Abstract

We present the partial branching fraction for inclusive charmless semileptonic  $B$  decays and the corresponding value of the CKM matrix element  $|V_{ub}|$ , using a multivariate analysis method to access  $\sim 90\%$  of the  $B \rightarrow X_u \ell \nu$  phase space. This approach dramatically reduces the theoretical uncertainties from the  $b$ -quark mass and non-perturbative QCD compared to all previous inclusive measurements. The results are based on a sample of 657 million  $B\bar{B}$  pairs collected with the Belle detector. We find that  $\Delta\mathcal{B}(B \rightarrow X_u \ell \nu; p_\ell^{*B} > 1.0 \text{ GeV}/c) = 1.963 \times (1 \pm 0.088_{\text{stat.}} \pm 0.081_{\text{sys.}}) \times 10^{-3}$ . Corresponding values of  $|V_{ub}|$  are extracted using several theoretical calculations.

We report measurements of partial branching fractions for inclusive charmless semileptonic  $B$  decays  $\bar{B} \rightarrow X_u \ell \bar{\nu}$ , and the determination of the CKM matrix element  $|V_{ub}|$ . The analysis is based on a sample of 467 million  $\Upsilon(4S) \rightarrow B\bar{B}$  decays recorded with the BABAR detector at the PEP-II  $e^+e^-$  storage rings. We select events in which the decay of one of the  $B$  mesons is fully reconstructed and an electron or a muon signals the semileptonic decay of the other  $B$  meson. We measure partial branching fractions  $\Delta\mathcal{B}$  in several restricted regions of phase space and determine the CKM element  $|V_{ub}|$  based on different QCD predictions. For decays with a charged lepton momentum  $p_\ell^* > 1.0 \text{ GeV}$  in the  $B$  meson rest frame, we obtain  $\Delta\mathcal{B} = (1.80 \pm 0.13_{\text{stat.}} \pm 0.15_{\text{sys.}} \pm 0.02_{\text{theo.}}) \times 10^{-3}$  from a fit to the two-dimensional  $M_X - q^2$  distribution. Here,  $M_X$  refers to the invariant mass of the final state hadron  $X$  and  $q^2$  is the invariant mass squared of the charged lepton and neutrino. From this measurement we extract  $|V_{ub}| = (4.33 \pm 0.24_{\text{exp.}} \pm 0.15_{\text{theo.}}) \times 10^{-3}$  as the arithmetic average of four results obtained from four different QCD predictions of the partial rate. We separately determine partial branching fractions for  $\bar{B}^0$  and  $B^-$  decays and derive a limit on the isospin breaking in  $\bar{B} \rightarrow X_u \ell \bar{\nu}$  decays.

Comes at a cost





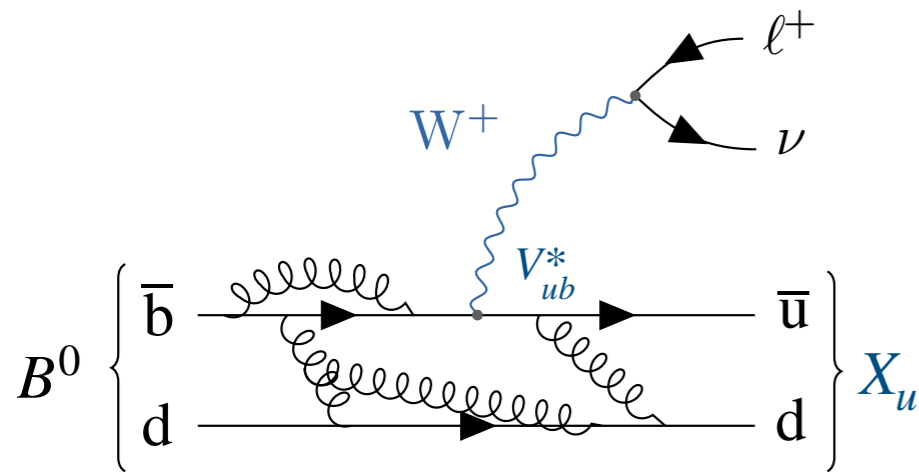
(reconstructed quantities)

(generator-level quantity)

# $B \rightarrow X_u \ell \bar{\nu}_\ell$ MC: Going Hybrid

Many measurements target inclusive decays

- ▶  $B \rightarrow X_u \ell \bar{\nu}_\ell$  with  $X_u \in [\pi, \rho, \omega, \eta, \eta', \text{non-resonant decays}, \dots]$
- ▶  $B \rightarrow X_s \gamma$  or  $B \rightarrow X_s \ell \ell$  with  $X_s \in [K^*, K\pi, \text{non-resonant}, \dots]$



Simulated as mix of **exclusive** & **inclusive** processes

**Inclusive:** Simulate  $X$  system with kinematic properties following (N)NLO calculation w/ non-perturbative QCD input (e.g. from auxiliary measurements)

**Hadronized with Pythia / JETSET**

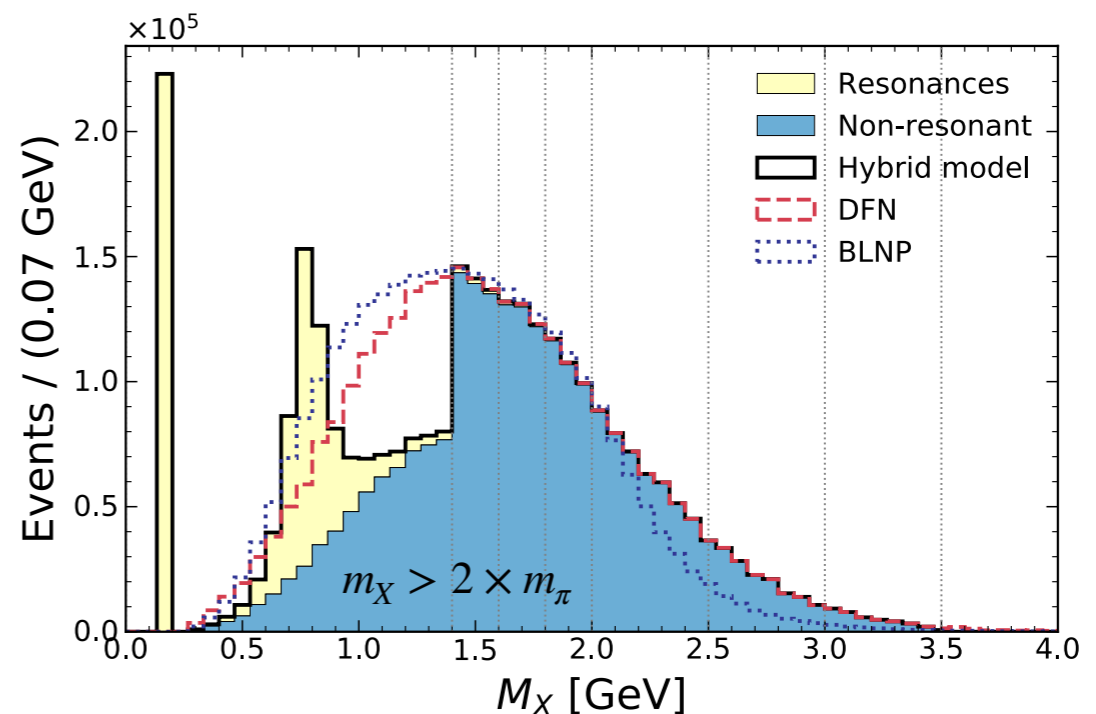
Hybrid Approach: Originally proposed by Phys. Rev. D **41**, 1496

$$\Delta\mathcal{B}_{ijk}^{\text{incl}} = \Delta\mathcal{B}_{ijk}^{\text{excl}} + w_{ijk} \times \Delta\mathcal{B}_{ijk}^{\text{incl}},$$

$$q^2 = [0, 2.5, 5, 7.5, 10, 12.5, 15, 20, 25] \text{ GeV}^2,$$

$$E_\ell^B = [0, 0.5, 1, 1.25, 1.5, 1.75, 2, 2.25, 3] \text{ GeV},$$

$$M_X = [0, 1.4, 1.6, 1.8, 2, 2.5, 3, 3.5] \text{ GeV}.$$

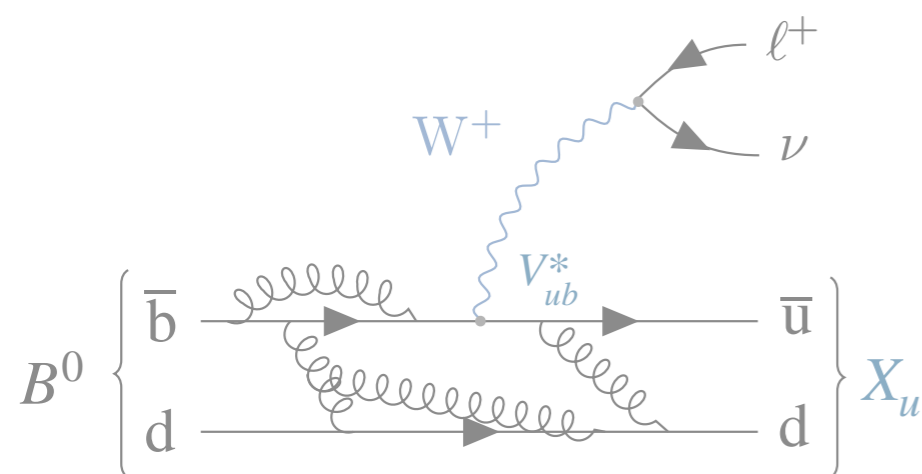




# $B \rightarrow X_u \ell \bar{\nu}_\ell$ MC: Going Hybrid

Many measurements target inclusive decays

- ▶  $B \rightarrow X_u \ell \bar{\nu}_\ell$  with  $X_u \in [\pi, \rho, \omega, \eta, \eta', \text{non-resonant decays}, \dots]$
- ▶  $B \rightarrow X_s \gamma$  or  $B \rightarrow X_s \ell \ell$  with  $X_s \in [K^*, K\pi, \text{non-resonant}, \dots]$



Simulated as mix of **exclusive** & **inclusive** processes

**Inclusive:** Simulate  $X$  system with kinematic properties following (N)NLO calculation w/ non-perturbative QCD input (e.g. from auxiliary measurements)

**Hadronized with Pythia / JETSET**

Hybrid Approach: Originally proposed by Phys. Rev. D **41**, 1496

Assessment of uncertainties:

- \* Change excl. FFs and incl. admixture
- \* Changes to underlying model parameters
- \* Vary **Pythia / JETSET** parameters that affect e.g. **scalar vs. vector final states, suppression of s quark production,  $\eta$ -meson suppression**

mesonUDvector, mesonSvector, mesonCvector	for the relative production ratio vector/pseudoscalar for light (u, d), s and c mesons
probStoUD	the suppression of s quark production relative to u or d production.
probQQtoQ	parameter for the suppression of diquark production relative to quark production, i.e. of baryon relative to meson production
etaSup	the $\eta$ -meson suppression

Estimated by variations of **underlying theory assumptions** and **Hybrid model** parameters used to determine (and correct for) selection efficiencies

Phase space restriction	$M_X - q^2$
Data statistical uncertainty	7.1
MC statistical uncertainty	1.1
Track efficiency	0.7
Photon efficiency	1.0
$\pi^0$ efficiency	0.9
Particle identification	2.3
$K_L$ production/detection	1.6
$K_S$ production/detection	1.2
Shape function parameters	5.4
Shape function form	1.5
Exclusive $\bar{B} \rightarrow X_u \ell \bar{\nu}$	1.9
$s\bar{s}$ production	2.7
$B$ semileptonic branching ratio	1.0
$D$ decays	1.1
$B \rightarrow D \ell \nu$ form factor	0.4
$B \rightarrow D^* \ell \nu$ form factor	0.7
$B \rightarrow D^{**} \ell \nu$ form factor	0.9
$B \rightarrow D^{**}$ reweighting	1.9
$m_{ES}$ background subtraction	1.9
combinatorial backg.	1.0
Total semileptonic BF	1.4
Total systematic uncertainty	8.4
Total experimental uncertainty	11.0

$p_\ell^{*B} > 1.0 \text{ GeV}$	$\Delta\mathcal{B}/\mathcal{B} (\%)$
$\mathcal{B}(D^{(*)} \ell \nu)$	1.2
$(D^{(*)} \ell \nu)$ form factors	1.2
$\mathcal{B}(D^{**} e \nu)$ & form factors	0.2
$B \rightarrow X_u \ell \nu$ (SF)	3.6
$B \rightarrow X_u \ell \nu$ ( $g \rightarrow s\bar{s}$ )	1.5
$\mathcal{B}(B \rightarrow \pi/\rho/\omega \ell \nu)$	2.3
$\mathcal{B}(B \rightarrow \eta, \eta' \ell \nu)$	3.2
$\mathcal{B}(B \rightarrow X_u \ell \nu)$ un-meas.	2.9
Cont./Comb.	1.8
Sec./Fakes/Fit.	1.0
PID/Reconstruction	3.1
BDT	3.1
Systematics	8.1
Statistics	8.8

Tables from Phys. Rev. Lett. 104:021801,2010,  
Phys.Rev. D86 (2012) 032004, Phys. Rev. D  
104, 012008 (2021)

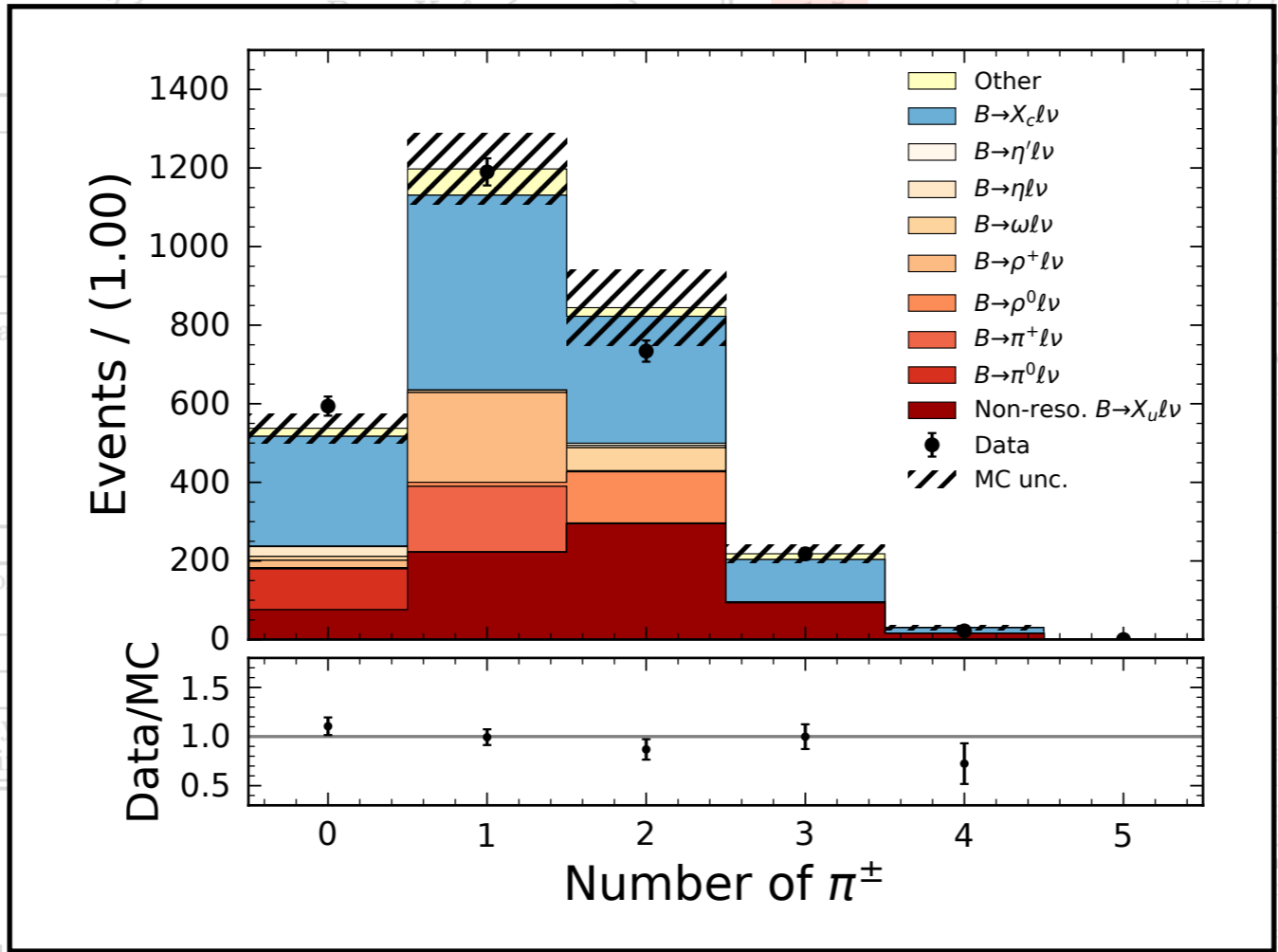
Phase-space region	$E_\ell^B > 1 \text{ GeV}$
Fit variable(s)	$(M_X : q^2 \text{ fit})$
<b>Additive uncertainties</b>	
$B \rightarrow X_u \ell^+ \nu_\ell$ modeling	
$B \rightarrow \pi \ell^+ \nu_\ell$ FFs	0.4
$B \rightarrow \rho \ell^+ \nu_\ell$ FFs	0.7
$B \rightarrow \omega \ell^+ \nu_\ell$ FFs	0.8
$B \rightarrow \eta \ell^+ \nu_\ell$ FFs	0.3
$B \rightarrow \eta' \ell^+ \nu_\ell$ FFs	1.6
$\mathcal{B}(B \rightarrow \pi \ell^+ \nu_\ell)$	0.2
$\mathcal{B}(B \rightarrow \rho \ell^+ \nu_\ell)$	0.4
$\mathcal{B}(B \rightarrow \omega \ell^+ \nu_\ell)$	0.1
$\mathcal{B}(B \rightarrow \eta \ell^+ \nu_\ell)$	<0.1
$\mathcal{B}(B \rightarrow \eta' \ell^+ \nu_\ell)$	<0.1
$\mathcal{B}(B \rightarrow X_u \ell^+ \nu)$	2.1
DFN parameters	5.0
Hybrid model	3.1
$B \rightarrow X_c \ell^+ \nu_\ell$ modeling	
$B \rightarrow D \ell^+ \nu_\ell$ FFs	<0.1
$B \rightarrow D^* \ell^+ \nu_\ell$ FFs	1.1
$B \rightarrow D^{**} \ell^+ \nu_\ell$ FFs	0.4
$\mathcal{B}(B \rightarrow D \ell^+ \nu_\ell)$	0.2
$\mathcal{B}(B \rightarrow D^* \ell^+ \nu_\ell)$	0.2
$\mathcal{B}(B \rightarrow D^{**} \ell^+ \nu_\ell)$	0.5
Gap modeling	1.0
MC statistics	1.6
Tracking efficiency	0.4
$\mathcal{L}_{\ell\text{ID}}$ shape	1.2
$\mathcal{L}_{K/\pi\text{ID}}$ shape	1.0
$D \rightarrow X \ell \nu_\ell$	0.1
$\pi_s$ efficiency	0.1
<b>Multiplicative uncertainties</b>	
$B \rightarrow X_u \ell^+ \nu_\ell$ modeling	
$B \rightarrow \pi \ell^+ \nu_\ell$ FFs	0.2
$B \rightarrow \rho \ell^+ \nu_\ell$ FFs	0.6
$B \rightarrow \omega \ell^+ \nu_\ell$ FFs	1.1
$B \rightarrow \eta \ell^+ \nu_\ell$ FFs	0.2
$B \rightarrow \eta' \ell^+ \nu_\ell$ FFs	0.2
$\mathcal{B}(B \rightarrow \pi \ell^+ \nu_\ell)$	0.3
$\mathcal{B}(B \rightarrow \rho \ell^+ \nu_\ell)$	0.4
$\mathcal{B}(B \rightarrow \omega \ell^+ \nu_\ell)$	<0.1
$\mathcal{B}(B \rightarrow \eta \ell^+ \nu_\ell)$	<0.1
$\mathcal{B}(B \rightarrow \eta' \ell^+ \nu_\ell)$	0.1
$\mathcal{B}(B \rightarrow X_u \ell^+ \nu)$	3.8
DFN parameters	3.6
Hybrid model	2.8
$\pi^+$ multiplicity	1.7
$\gamma_s$ ( $s\bar{s}$ fragmentation)	0.8
$\mathcal{L}_{\ell\text{ID}}$ efficiency	1.5
$\mathcal{L}_{K/\pi\text{ID}}$ efficiency	0.7
$N_{B\bar{B}}$	1.3
Tracking efficiency	0.9
Tagging calibration	3.6
<b>Total syst. uncertainty</b>	<b>10.4</b>

Estimated by variations of **underlying theory assumptions** and **Hybrid model** parameters used to determine (and correct for) selection efficiencies

Phase space restriction	$M_X - q^2$
Data statistical uncertainty	7.1
MC statistical uncertainty	1.1
Track efficiency	0.7
Photon efficiency	1.0
$\pi^0$ efficiency	0.9
Particle identification	2.2
$K_L$ production/detection	
$K_S$ production/detection	
Shape function parameters	
Shape function form	
Exclusive $\bar{B} \rightarrow X_u \ell \bar{\nu}$	
$s\bar{s}$ production	
$B$ semileptonic branching ratios	
$D$ decays	
$B \rightarrow D \ell \nu$ form factor	
$B \rightarrow D^* \ell \nu$ form factor	
$B \rightarrow D^{**} \ell \nu$ form factor	
$B \rightarrow D^{**}$ reweighting	
$m_{ES}$ background subtraction	
combinatorial backg.	
Total semileptonic BF	
Total systematic uncertainty	
Total experimental uncertainty	

**Can we do better?**  
**Can data teach us more?**

$p_\ell^{*  }$	
$B(D^{(*)} \ell \nu)$	1.2
$(D^{(*)} \ell \nu)$ form factors	1.2
$B(D^{**} \ell \nu)$ & form factors	0.2
$B \rightarrow X_u \ell \nu$ (SF)	3.6

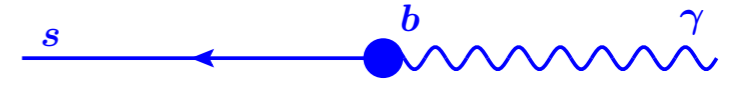


Phase-space region	$E_\ell^B > 1 \text{ GeV}$
Fit variable(s)	$(M_X : q^2 \text{ fit})$
<b>Additive uncertainties</b>	
$B \rightarrow X_u \ell^+ \nu_\ell$ modeling	
$B \rightarrow \pi \ell^+ \nu_\ell$ FFs	0.4
$B \rightarrow \rho \ell^+ \nu_\ell$ FFs	0.7
$B \rightarrow \omega \ell^+ \nu_\ell$ FFs	0.8
$B \rightarrow \eta \ell^+ \nu_\ell$ FFs	0.3
$B \rightarrow \eta' \ell^+ \nu_\ell$ FFs	1.6
$B(B \rightarrow \pi \ell^+ \nu_\ell)$	0.2
$B(B \rightarrow \rho \ell^+ \nu_\ell)$	0.4
$B(B \rightarrow \omega \ell^+ \nu_\ell)$	0.1
$B(B \rightarrow \eta \ell^+ \nu_\ell)$	<0.1
$B(B \rightarrow \eta' \ell^+ \nu_\ell)$	<0.1
$B(B \rightarrow X_u \ell^+ \nu)$	2.1
DFN parameters	5.0
Hybrid model	3.1
$B \rightarrow X_c \ell^+ \nu_\ell$ modeling	
$B \rightarrow D \ell^+ \nu_\ell$ FFs	<0.1
$B \rightarrow D^* \ell^+ \nu_\ell$ FFs	1.1
$\ell^+ \nu_\ell$ FFs	0.4
$\ell^+ \nu_\ell$	0.2
$\ell^+ \nu_\ell$	0.2
$\ell^+ \nu_\ell$	0.5
Modeling	1.0
Efficiency	1.6
Efficiency	0.4
Efficiency	1.2
Efficiency	1.0
Efficiency	0.1
Efficiency	0.1
<b>Relative uncertainties</b>	
$\nu_\ell$ modeling	0.2
$\nu_\ell$ FFs	0.6
$\nu_\ell$ FFs	1.1
$\nu_\ell$ FFs	0.2
$\nu_\ell$ FFs	0.2
$\ell^+ \nu_\ell$	0.3
$\ell^+ \nu_\ell$	0.4
$\ell^+ \nu_\ell$	<0.1
$\ell^+ \nu_\ell$	<0.1
$\ell^+ \nu_\ell$	0.1
$\ell^+ \nu_\ell$	3.8
Parameters	3.6
Model	2.8
Efficiency	1.7
Fragmentation	0.8
Efficiency	1.5
Efficiency	0.7
Efficiency	1.3
Efficiency	0.9
Tagging calibration	3.6
<b>Total syst. uncertainty</b>	<b>10.4</b>

Tables from Phys. Rev. Lett. 104:021801, 2010,  
 Phys.Rev. D86 (2012) 032004, Phys. Rev. D  
 104, 012008 (2021)

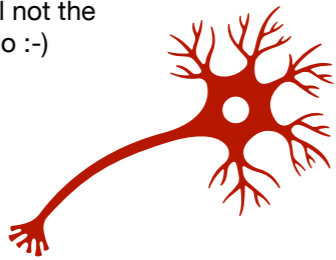
# Going differential

Focus on experimental **most sensitive region** (e.g. high  $E_\ell^B$ )



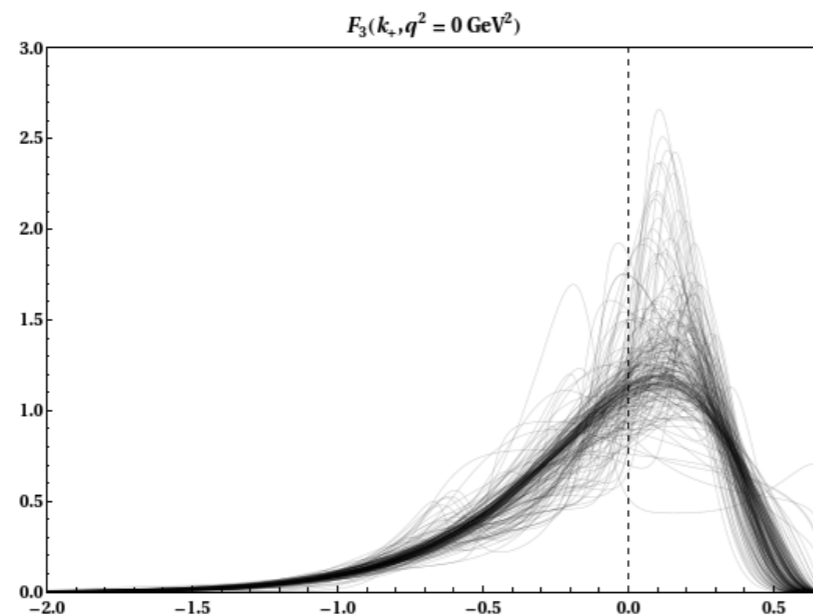
Determine Shape-Function in a data-driven way

Note: this is still not the  
NNVub Logo :-)



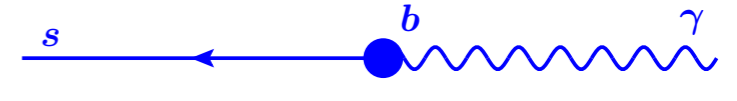
**NNVub**

P. Gambino, K. Healey, C. Mondino,  
Phys. Rev. D 94, 014031 (2016),  
[arXiv:1604.07598]



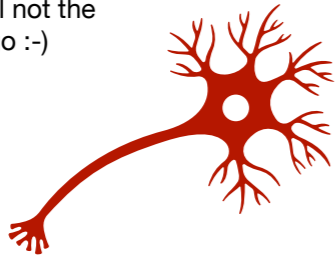
# Going differential

Focus on experimental **most sensitive region** (e.g. high  $E_\ell^B$ )



Determine Shape-Function in a data-driven way

Note: this is still not the  
NNVub Logo :-)

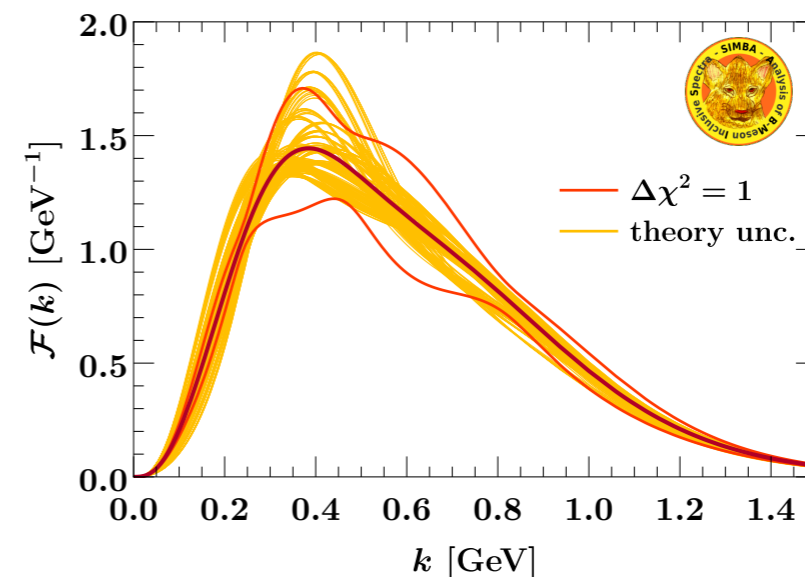
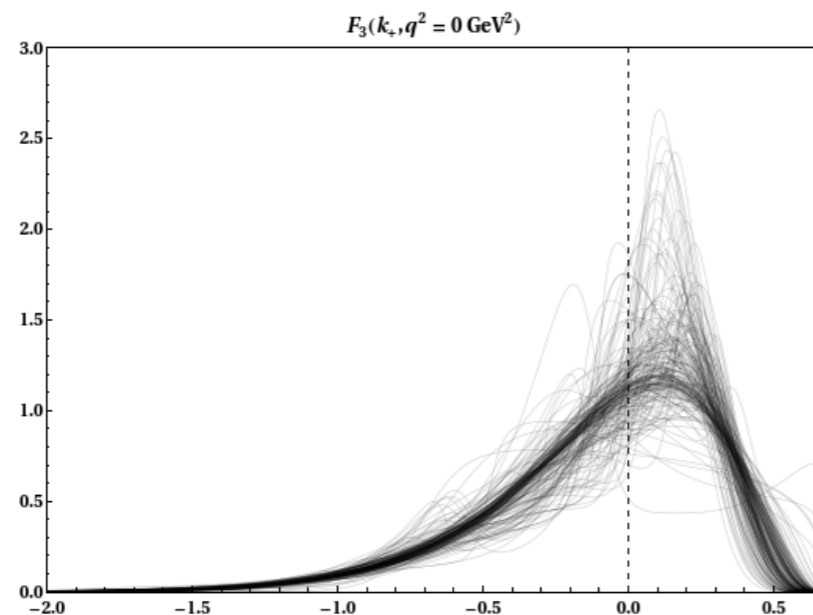


**NNVub**

P. Gambino, K. Healey, C. Mondino,  
Phys. Rev. D 94, 014031 (2016),  
[arXiv:1604.07598]



F. Bernlochner, H. Lacker, Z. Ligeti, I.  
Stewart, F. Tackmann, K. Tackmann  
Phys. Rev. Lett. 127, 102001 (2021)  
[arXiv:2007.04320]



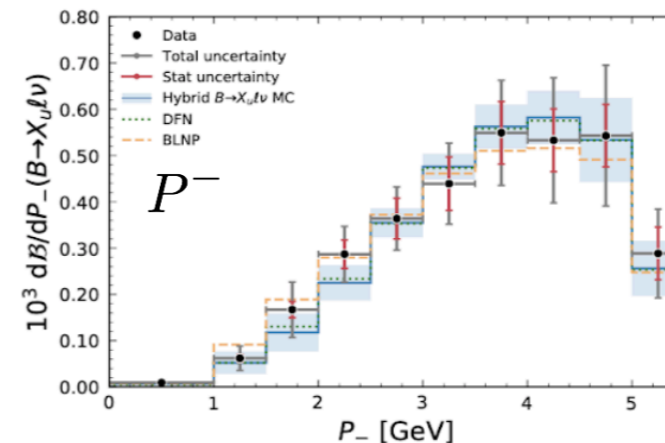
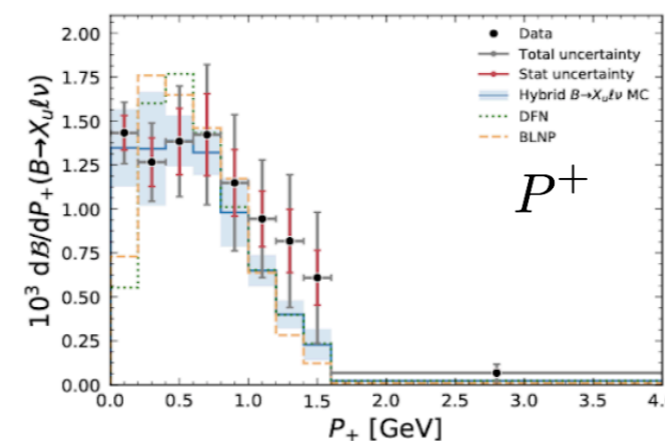
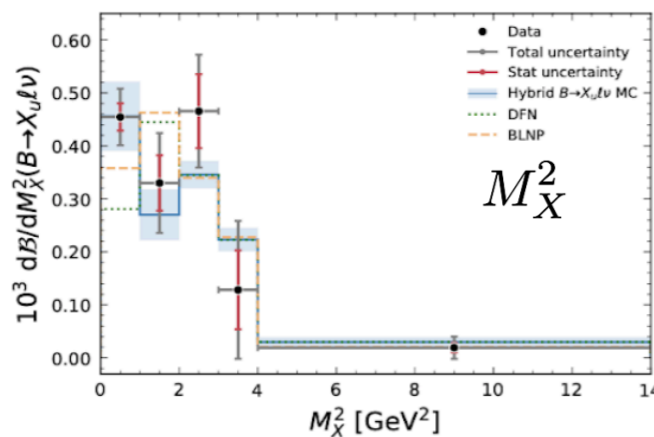
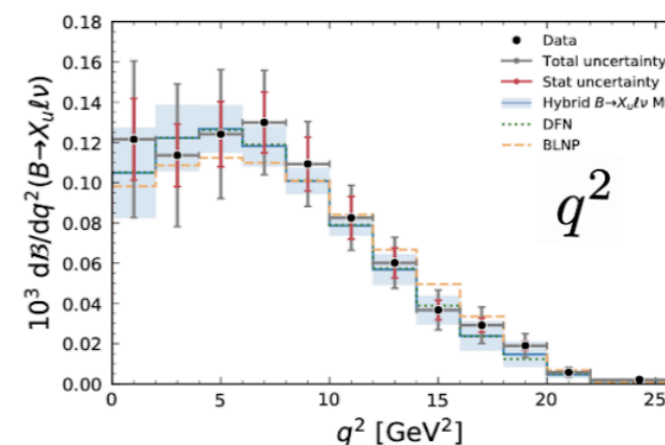
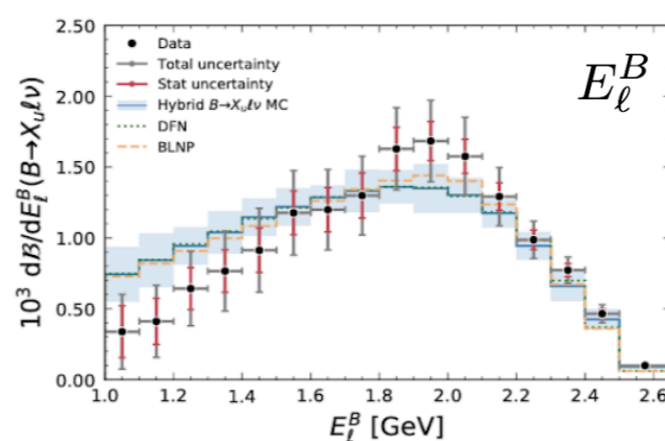
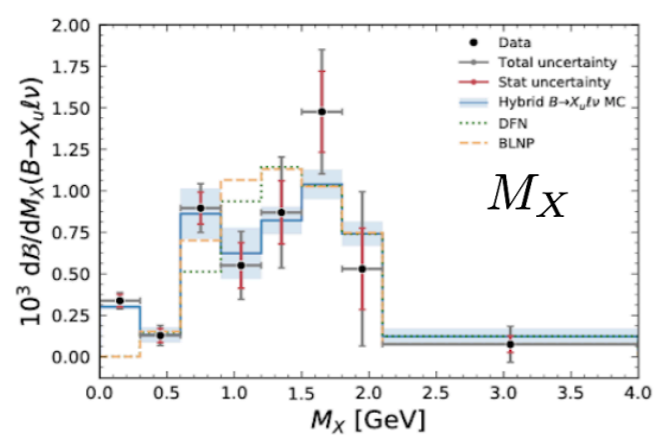
~ Momentum Distribution of b-Quark in B Meson

## Differential Spectra of $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$

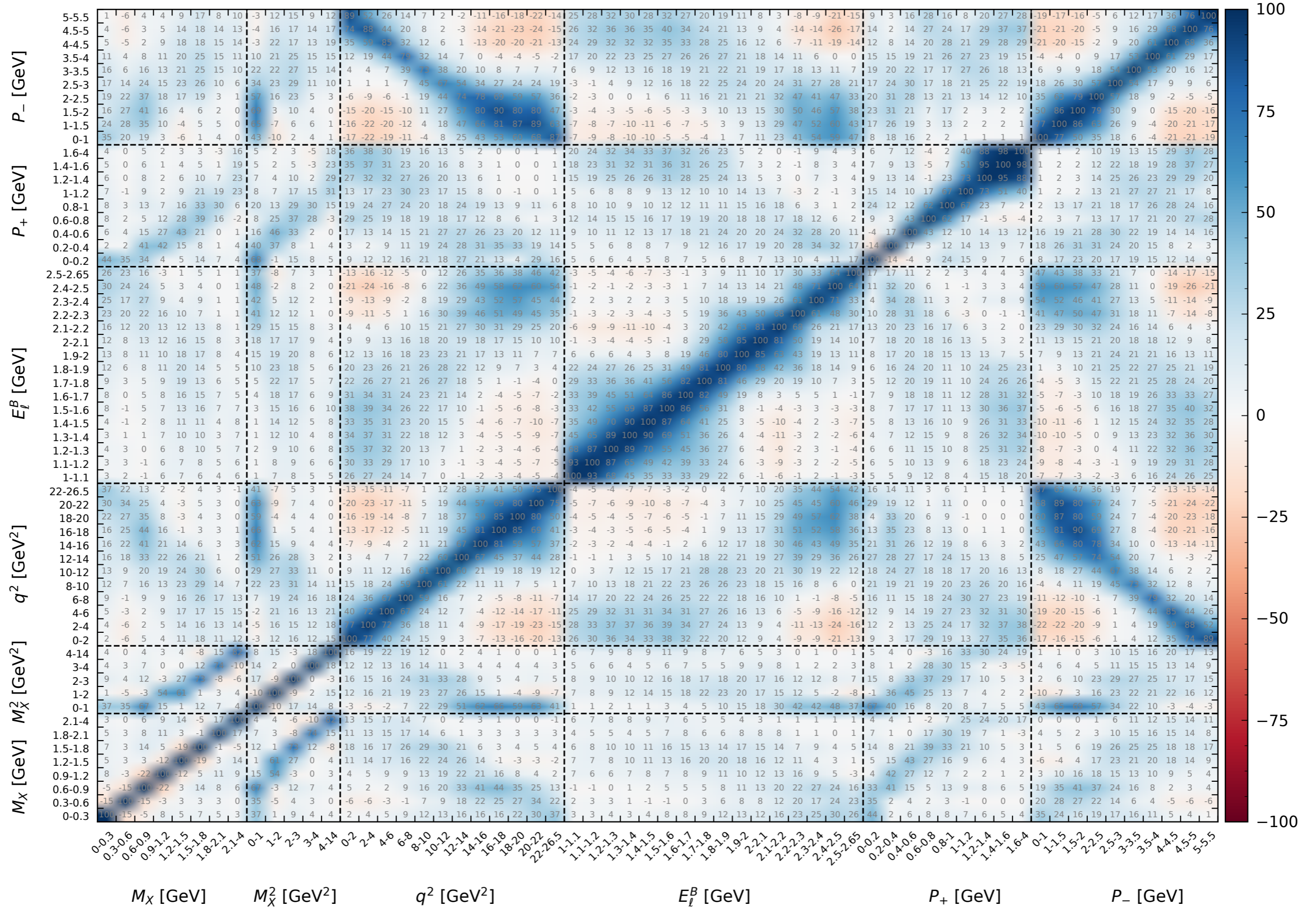
# 20

arXiv:2107.13855,  
accepted by PRL

- **Convert** unfolded yield to  $\Delta\mathcal{B}$  in each bin considering reco. efficiency & acceptance
- **First measurement** of differential branching fractions in the  $E_\ell^B > 1$  GeV region of phase space
- Necessary input for future **model-independent** determinations of  $|V_{ub}|$  (e.g. NNvub, SIMBA)



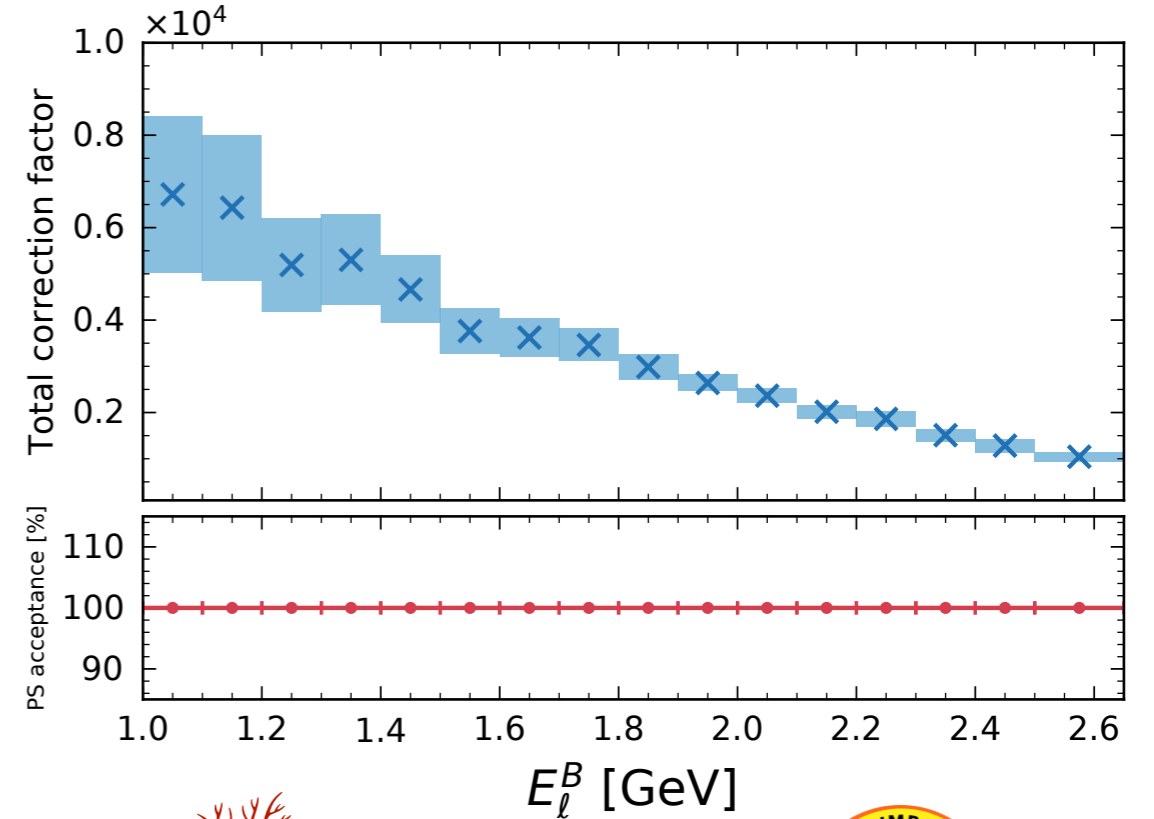
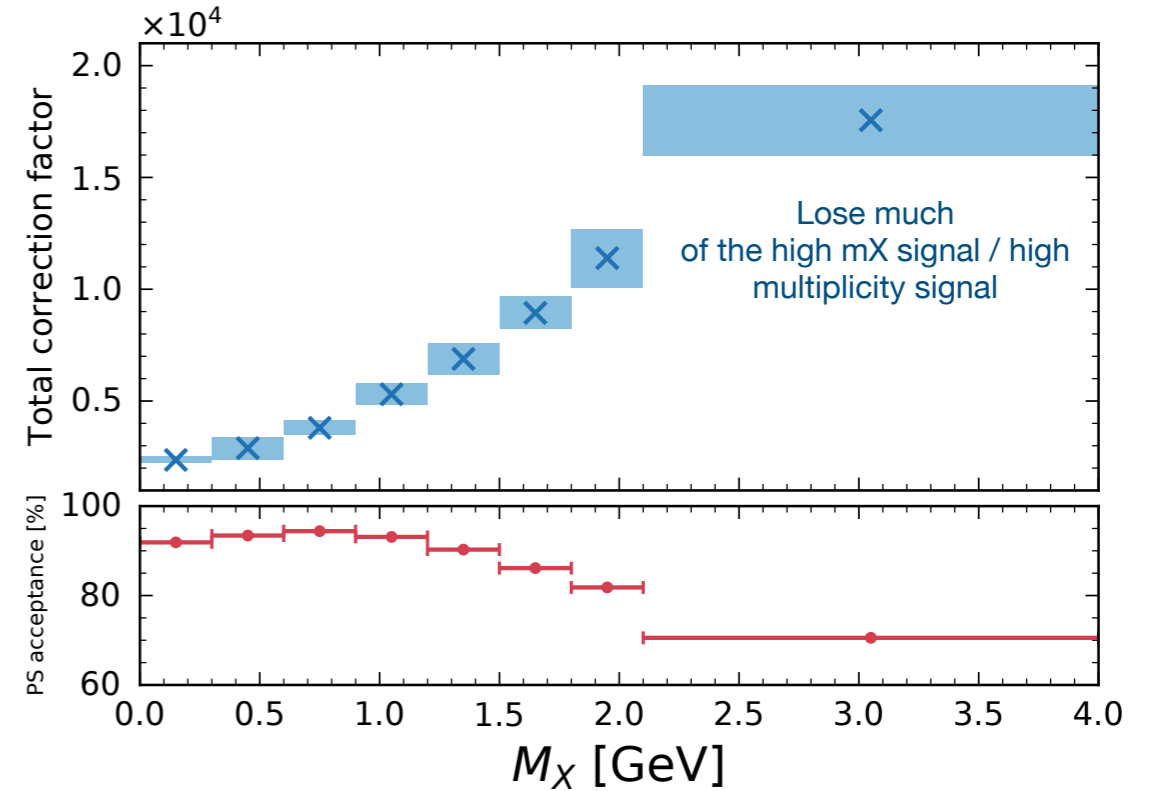
# Full experimental correlations



# $M_X$

$M_X$ [GeV]	0-0.3	0.3-0.6	0.6-0.9	0.9-1.2	1.2-1.5	1.5-1.8	1.8-2.1	2.1-4.0
Tracking efficiency	0.55	0.56	0.82	0.86	0.95	1.05	1.15	1.19
Tagging calibration	3.69	3.69	3.65	3.64	3.64	3.57	3.79	3.66
Slow pion efficiency	0.00	0.07	0.04	0.05	0.04	0.04	0.06	0.04
$K_S^0$	0.04	0.05	0.04	0.02	0.04	0.03	0.02	0.05
$e$ ID	0.72	0.83	0.74	0.69	0.73	0.74	0.94	1.22
$\mu$ ID	1.59	1.25	1.34	1.29	1.44	1.35	1.09	0.70
$K/\pi$ ID	0.39	0.67	0.68	0.74	0.81	1.02	1.27	1.24
$\mathcal{B}(B \rightarrow X_u \ell \nu)$	0.18	0.44	0.07	0.59	0.82	0.69	0.73	0.46
$\mathcal{B}(B \rightarrow \pi \ell \nu)$	0.42	0.45	0.45	0.14	0.05	0.04	0.05	0.05
$\mathcal{B}(B \rightarrow \rho \ell \nu)$	0.42	1.00	0.61	0.56	0.33	0.16	0.22	0.15
$\mathcal{B}(B \rightarrow \omega \ell \nu)$	0.42	0.39	0.65	0.12	0.11	0.06	0.11	0.10
$\mathcal{B}(B \rightarrow \eta \ell \nu)$	0.41	1.16	0.46	0.11	0.06	0.03	0.03	0.14
$\mathcal{B}(B \rightarrow \eta' \ell \nu)$	0.42	0.39	0.46	0.24	0.30	0.03	0.14	0.11
$B \rightarrow \pi \ell \nu$ FF	0.98	3.08	1.52	0.53	1.05	0.37	0.36	0.38
$B \rightarrow \rho \ell \nu$ FF	2.77	8.54	3.96	2.94	1.65	0.59	0.83	0.89
$B \rightarrow \omega \ell \nu$ FF	2.40	9.71	1.10	0.90	1.41	0.70	0.65	1.32
$B \rightarrow \eta \ell \nu$ FF	0.71	3.58	0.09	0.09	0.51	0.28	0.27	0.07
$B \rightarrow \eta' \ell \nu$ FF	0.69	3.65	0.16	0.27	0.48	0.29	0.32	0.15
Hybrid model	0.21	5.86	5.08	4.01	0.50	1.97	2.02	6.13
DFN parameters	0.18	3.66	1.01	1.38	1.64	0.87	0.50	1.35
$\gamma_s$	0.47	4.17	2.36	3.98	3.08	4.10	9.31	3.60
$\pi^+$ multiplicity modeling	0.57	0.42	0.45	4.15	7.98	4.78	3.98	2.34
$N_{B\bar{B}}$	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
Background subtraction	5.97	26.93	8.23	25.15	29.65	16.80	73.36	126.64
MC stat. (migration matrix)	4.04	11.22	3.54	6.85	4.30	4.71	6.85	8.22
Total syst. uncertainty	9.36	33.77	12.32	27.56	31.62	19.21	74.55	127.23
Total stat. uncertainty	11.11	32.64	10.77	24.99	21.88	16.54	46.24	66.76
Total uncertainty	14.53	46.97	16.36	37.20	38.45	25.35	87.73	143.68

background dominated region



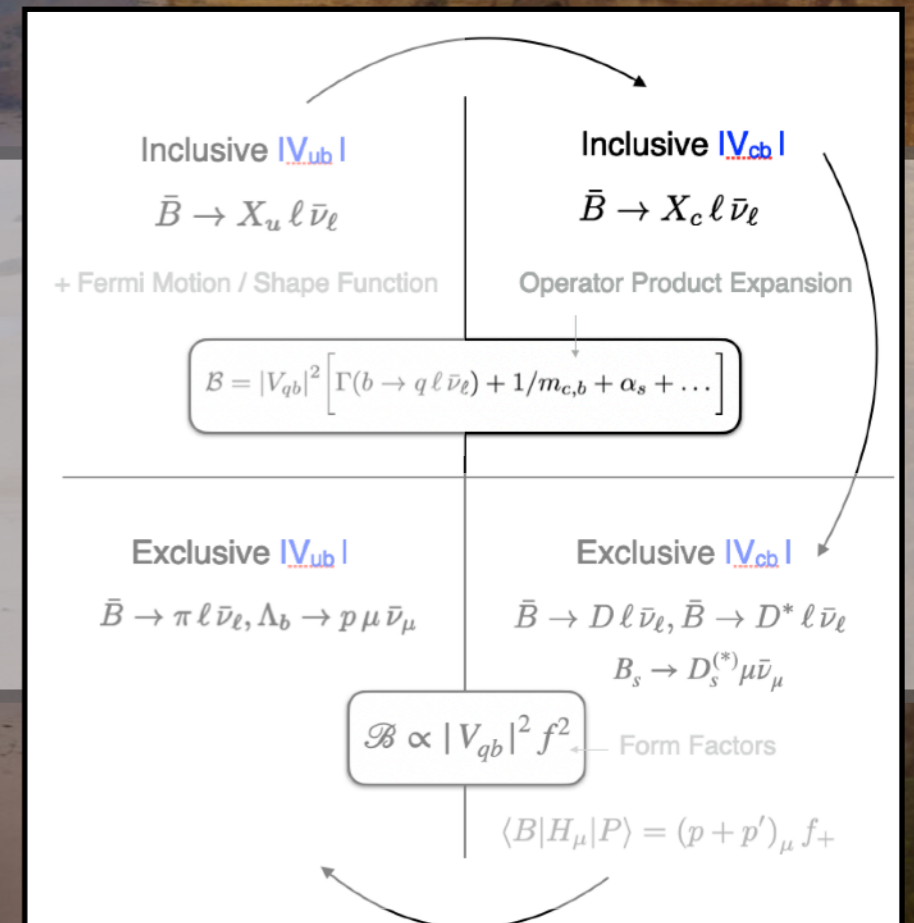
It will be exciting to get first fits with these done..



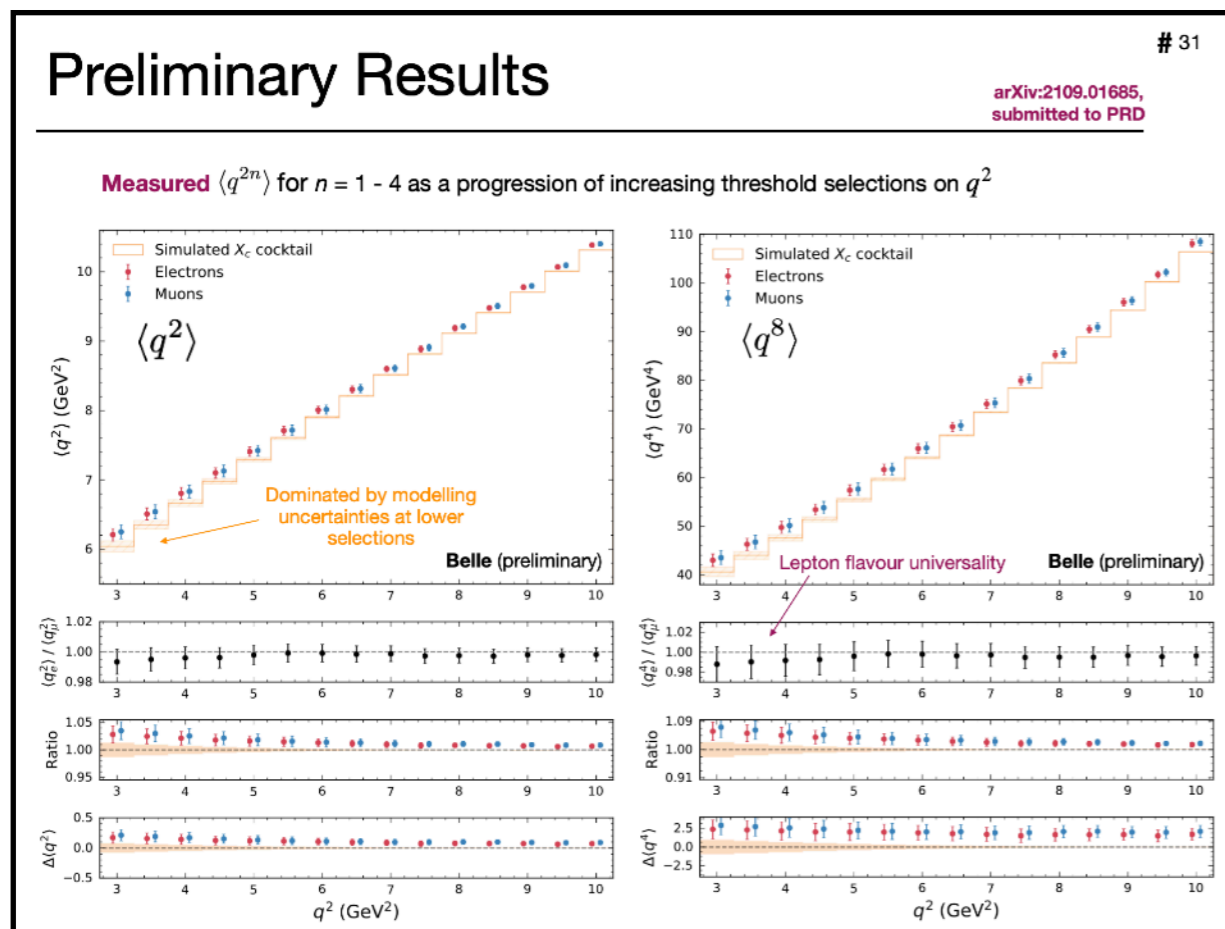


# Incl. $|V_{cb}|$

- ▶ Systematic covariances of inputs
- ▶ Theory correlations
- ▶ Role of lattice QCD



# New results from Belle (soon also from Belle II)



With Belle II (& LHCb via SEM-techniques?) we should **systematically remeasure** properties & actively investigate if other observables could be helpful

Experiment	Hadron moments $\langle M^n_X \rangle$	Lepton moments $\langle E^n_\ell \rangle$	References
BaBar	n=2 c=0.9,1.1,1.3,1.5 n=4 c=0.8,1.0,1.2,1.4 n=6 c=0.9,1.3 [1]	n=0 c=0.6,1.2,1.5 n=1 c=0.6,0.8,1.0,1.2,1.5 n=2 c=0.6,1.0,1.5 n=3 c=0.8,1.2 [1,2]	[1] <a href="#">Phys.Rev. D81 (2010) 032003</a> [2] <a href="#">Phys.Rev. D69 (2004) 111104</a>
Belle	n=2 c=0.7,1.1,1.3,1.5 n=4 c=0.7,0.9,1.3 [3]	n=0 c=0.6,1.4 n=1 c=1.0,1.4 n=2 c=0.6,1.4 n=3 c=0.8,1.2 [4]	[3] <a href="#">Phys.Rev. D75 (2007) 032005</a> [4] <a href="#">Phys.Rev. D75 (2007) 032001</a>
CDF	n=2 c=0.7 n=4 c=0.7 [5]	.	[5] <a href="#">Phys.Rev. D71 (2005) 051103</a>
CLEO	n=2 c=1.0,1.5 n=4 c=1.0,1.5 [6]	.	[6] <a href="#">Phys.Rev. D70 (2004) 032002</a>
DELPHI	n=2 c=0.0 n=4 c=0.0 n=6 c=0.0 [7]	n=1 c=0.0 n=2 c=0.0 n=3 c=0.0 [7]	[7] <a href="#">Eur.Phys.J. C45 (2006) 35-59</a>

We learnt a fair bit in the last decade and should evaluate, how this propagates into our measurements, e.g.

- ▶  $B \rightarrow X_c \ell \bar{\nu}_\ell$  composition and modeling
- ▶ Revisit most important systematic uncertainties, e.g. modelling of detector resolution
- ▶ New experimental techniques

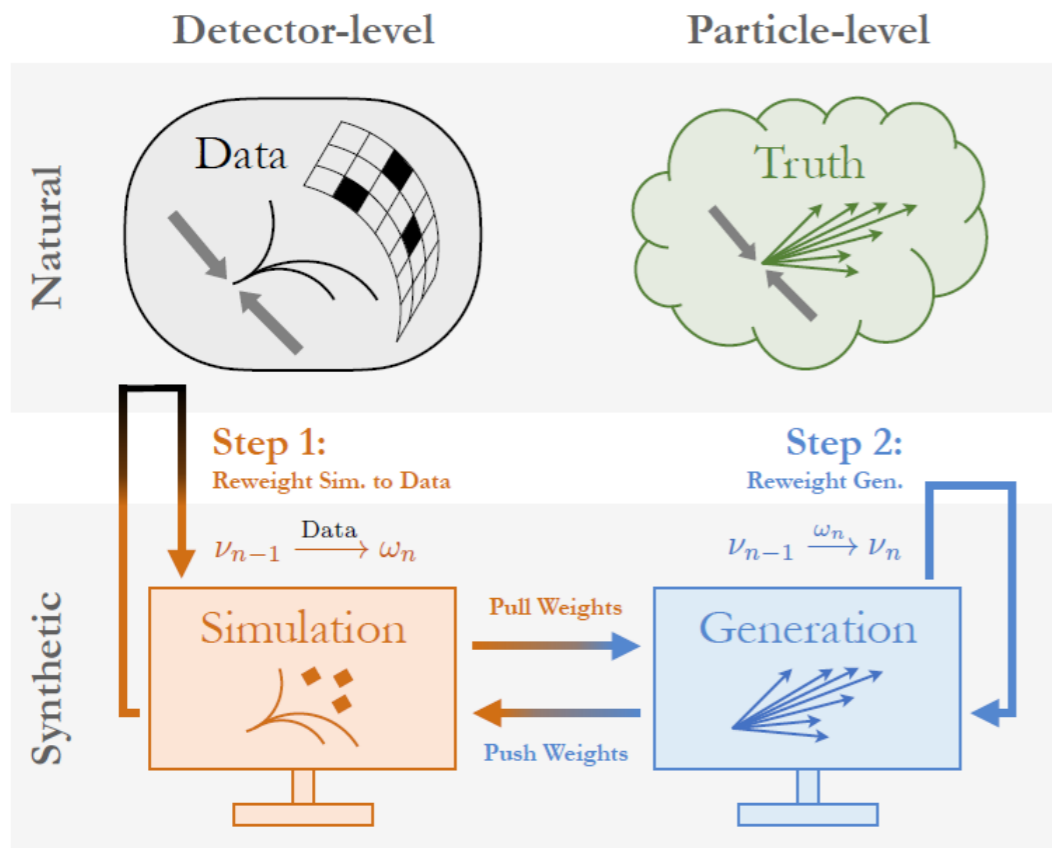
Subset of currently used measurements in global fits from HFLAV

# Let's get ambitious

Figure right: Lukas Reinartz, FB

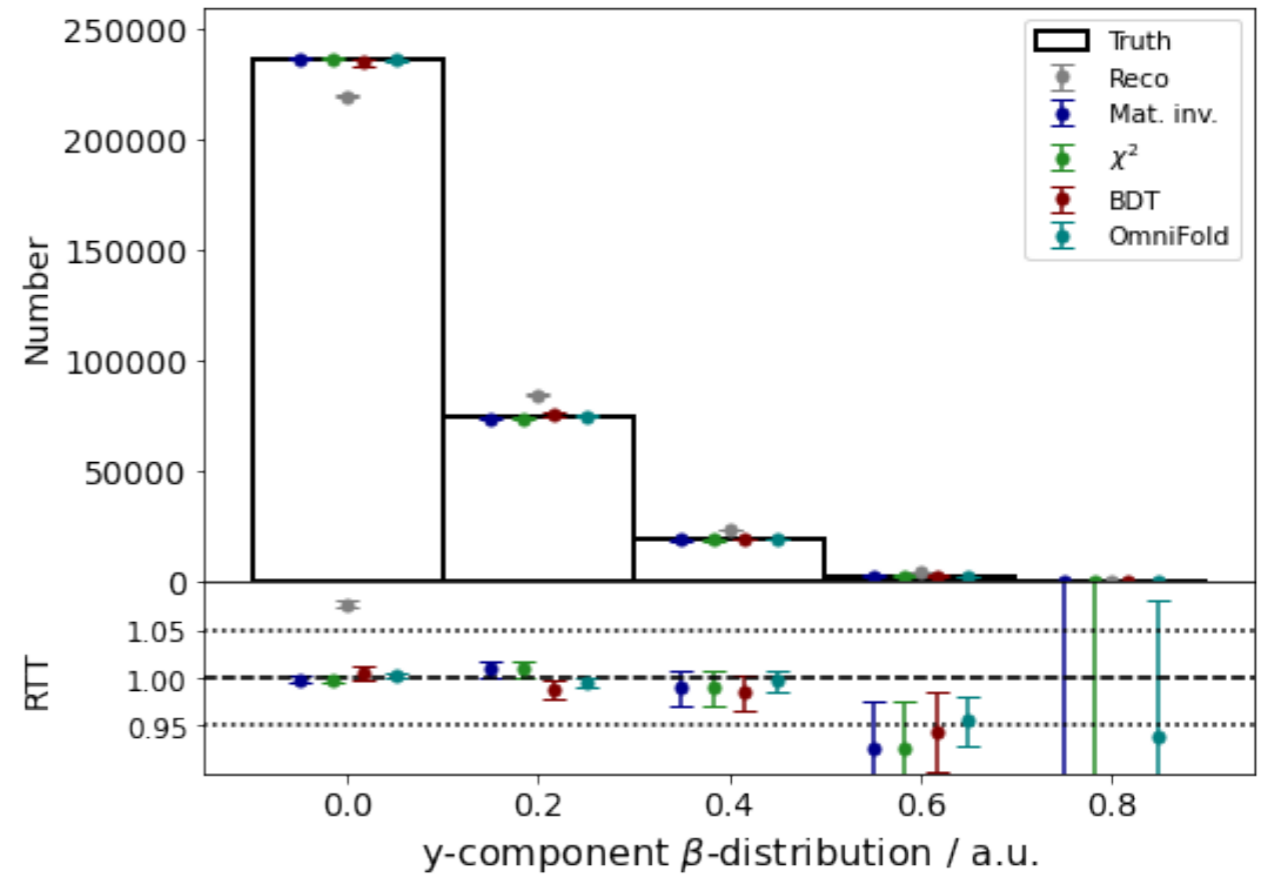
## OmniFold: A Method to Simultaneously Unfold All Observables

Phys. Rev. Lett. 124, 182001 (2020)



### 6D Example:

(3D Gaussian, 2D beta distribution  
1 exponential, all smeared with gaussian resolution)



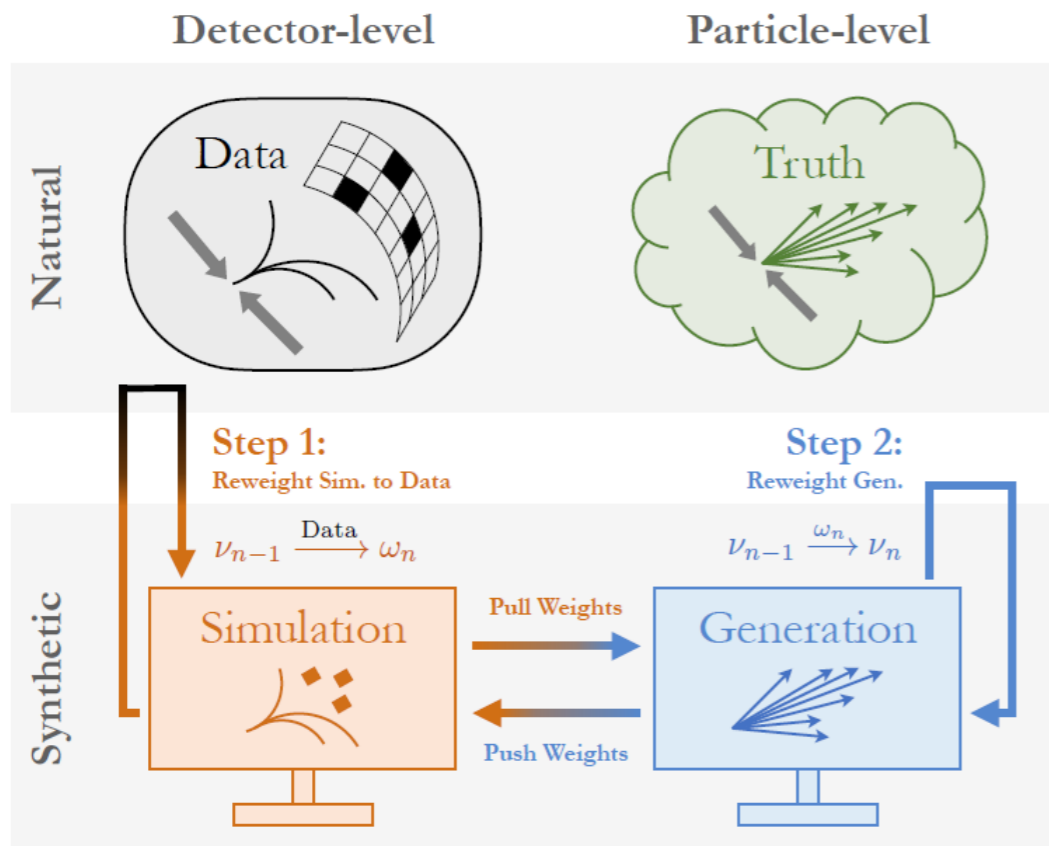
based on training a deep network to unfold detector distortions and effects

# Let's get ambitious

Figure right: Lukas Reinartz, FB

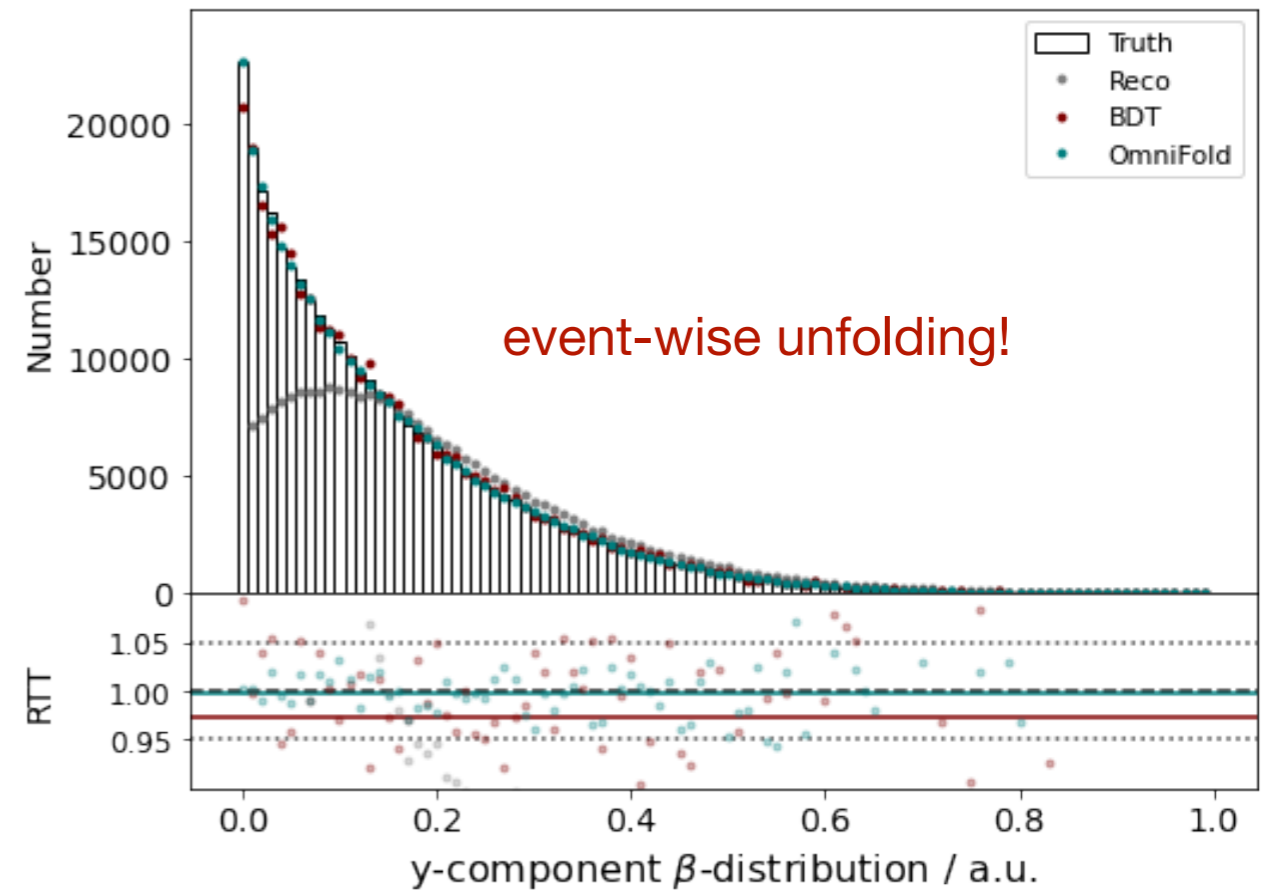
## OmniFold: A Method to Simultaneously Unfold All Observables

Phys. Rev. Lett. 124, 182001 (2020)



## 6D Example:

(3D Gaussian, 2D beta distribution  
1 exponential, all smeared with gaussian resolution)



based on training a deep network to unfold detector distortions and effects

# Theory Correlations

$|V_{cb}|$  “extraction formula”:

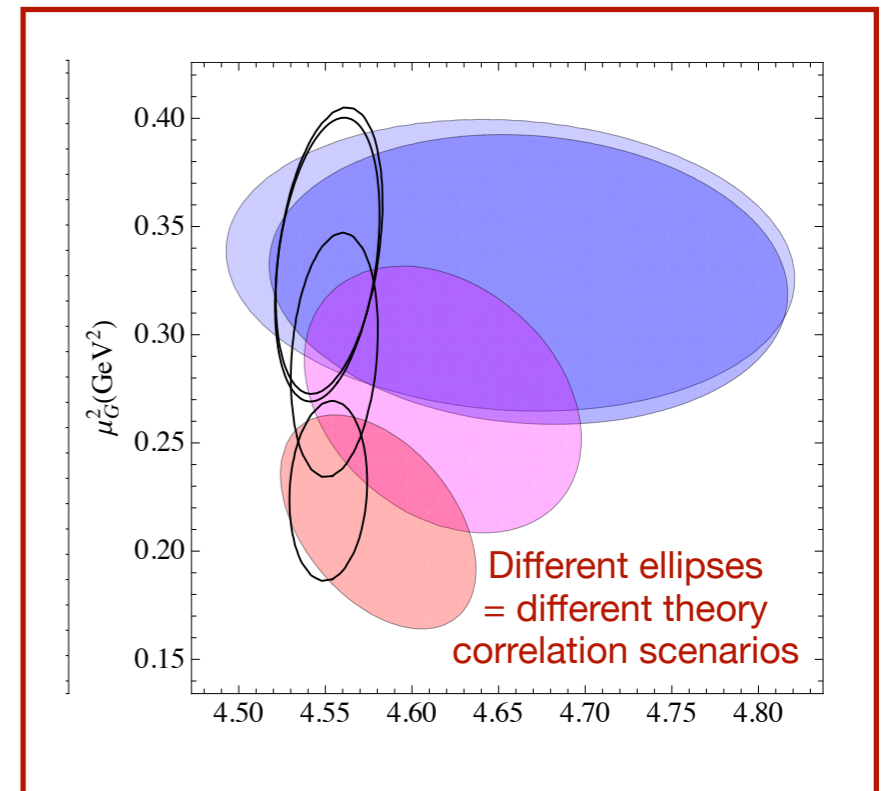
$$C = C_{\text{exp}} + C_{\text{theo}}$$

Introduce theory correlations between moments of various cuts and orders

$$\chi^2(|V_{cb}|, \vec{\mu}_{\text{HQE}}) = \left( \overset{\text{measured moments}}{\mathcal{M}} - \overset{\text{predicted moments}}{\mathcal{P}(\vec{\mu}_{\text{HQE}})} \right) C^{-1} \left( \mathcal{M} - \mathcal{P}(\vec{\mu}_{\text{HQE}}) \right) + \left( \mathcal{B} - |V_{cb}|^2 \tau_B \Gamma(\vec{\mu}_{\text{HQE}}) \right)^2 / \sigma_{\mathcal{B}}^2$$

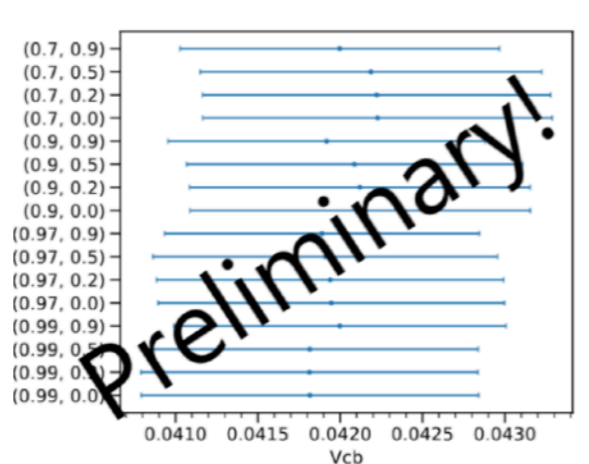
Precise choice of correlations entering  $C_{\text{theo}}$  introduce a large dependence on HQE parameters

$|V_{cb}|$  seems not to care so much



P. Gambino, C. Schwanda  
Phys. Rev. D 89, 014022 (2014)

- Flexible theory covariance matrix:
  - $V_{cb}$  rather insensitive dominated by  $B \rightarrow X_c \ell \nu$  branchingratio
  - HQE parameters sensitive



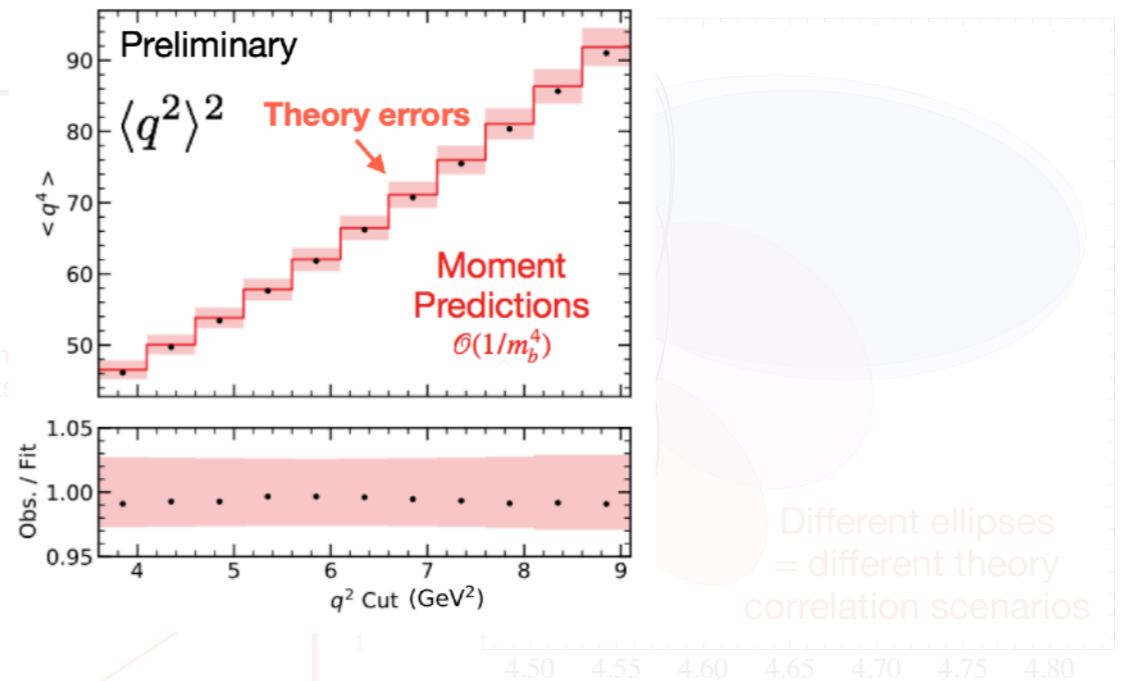
Keri's talk on Monday

# Theory Correlations

Could e.g. theory nuisance parameters help?

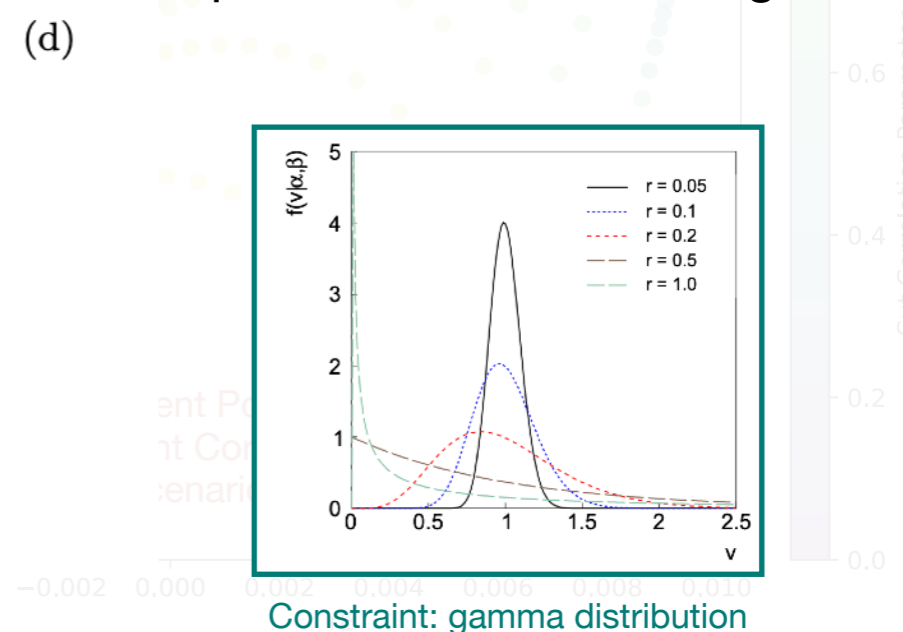
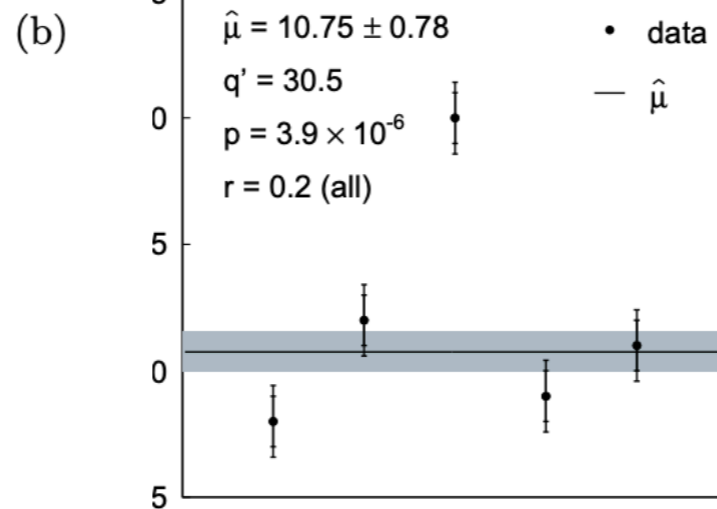
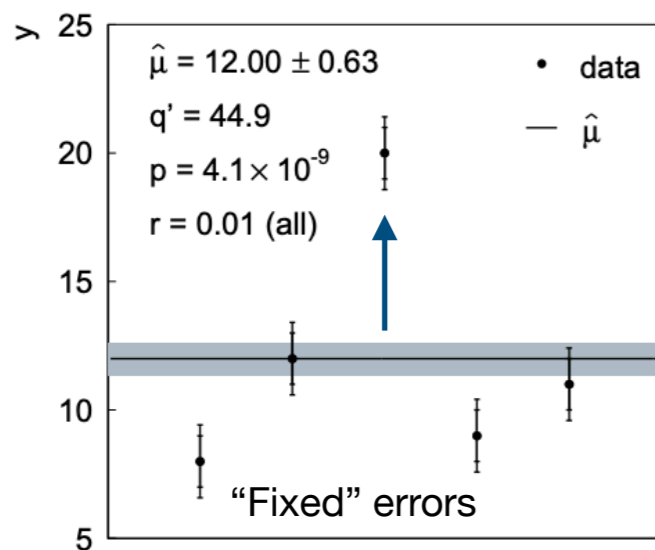
The data is so precise, it can tell us which theory variations are **too conservative**

Can we **parametrize our ignorance**?



Usually we fix the size of theory errors, but are we certain we know them as precisely as we think we do? I.e. are they themselves uncertain?

Glen Cowan, Eur. Phys. J. C (2019) 79:133



Size or errors are **constrained**, but floating parameter in the average

Constraint: gamma distribution

# Experimental correlations

Phys. Rev. D81:032003, 2010

$C_{\text{exp}}$

Many of the measurement are systematically limited

(with the notable exception of some high cut moments)

i.e. systematic correlations are important

**Source-wise correlations well motivated scheme**

Sum over independent uncertainties / eigendirections

$$C_{\text{exp}} = C_{\text{stat}} + \sum_i^{\text{Source}} C_i$$

Independent error source has a fully correlated effect over all measured quantities

$$C_i = \vec{\sigma}_i \otimes \vec{\sigma}_i$$

Note that there are other experimental errors that need a more careful treatment, e.g. Lepton ID (one can use replicas as there might be decorrelation effects across measured quantities)

$$(C_i)_{km} = (\sigma_i)_k (\sigma_i)_m \rho_{km}$$

BABAR-PUB-09/004  
SLAC-PUB-13735

## Measurement and Interpretation of Moments in Inclusive Semileptonic Decays $\bar{B} \rightarrow X_c \ell^- \bar{\nu}$

B. Aubert, Y. Karyotakis, J. P. Lees, V. Poireau, E. Prencipe, X. Prudent, and V. Tisserand  
*Laboratoire d'Annecy-le-Vieux de Physique des Particules (LAPP),  
Université de Savoie, CNRS/IN2P3, F-74941 Annecy-Le-Vieux, France*

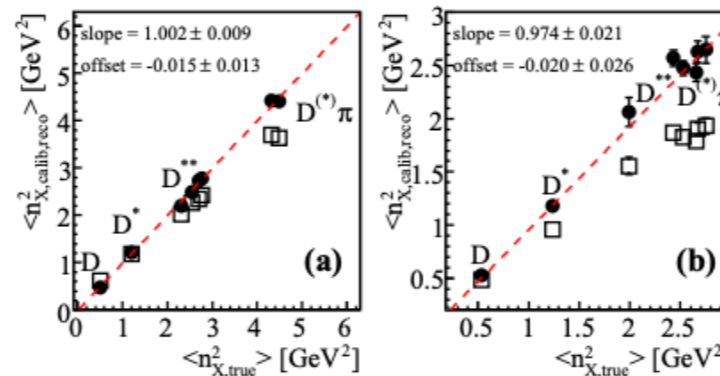


FIG. 7: Example of the calibration verification procedure for different minimum lepton momenta (a)  $p_{\ell,\text{min}}^* = 0.8 \text{ GeV}/c$  and (b)  $p_{\ell,\text{min}}^* = 1.7 \text{ GeV}/c$ . Moments  $\langle n_X^2 \rangle$  of exclusive modes on simulated events before ( $\square$ ) and after ( $\bullet$ ) calibration are plotted against the true moments for each mode. The dotted line shows the result of a fit to the calibrated moments, the resulting parameters are given.

The bias correction factors  $\mathcal{C}(p_{\ell}^*, k)$ , depending on the minimum lepton momentum and the order of the extracted moments, are determined by MC simulations; they combine the two factors  $\mathcal{C}_{\text{cal}}$  and  $\mathcal{C}_{\text{true}}$  as described in Section IV B.

Varying the branching fractions of the exclusive signal modes in the MC simulation has, in agreement with the mass-moment studies, a very small impact on the measured combined moments. Also, no significant variations of the results are observed when splitting the data sample into the same subsamples as for the mass moments.

### D. Results

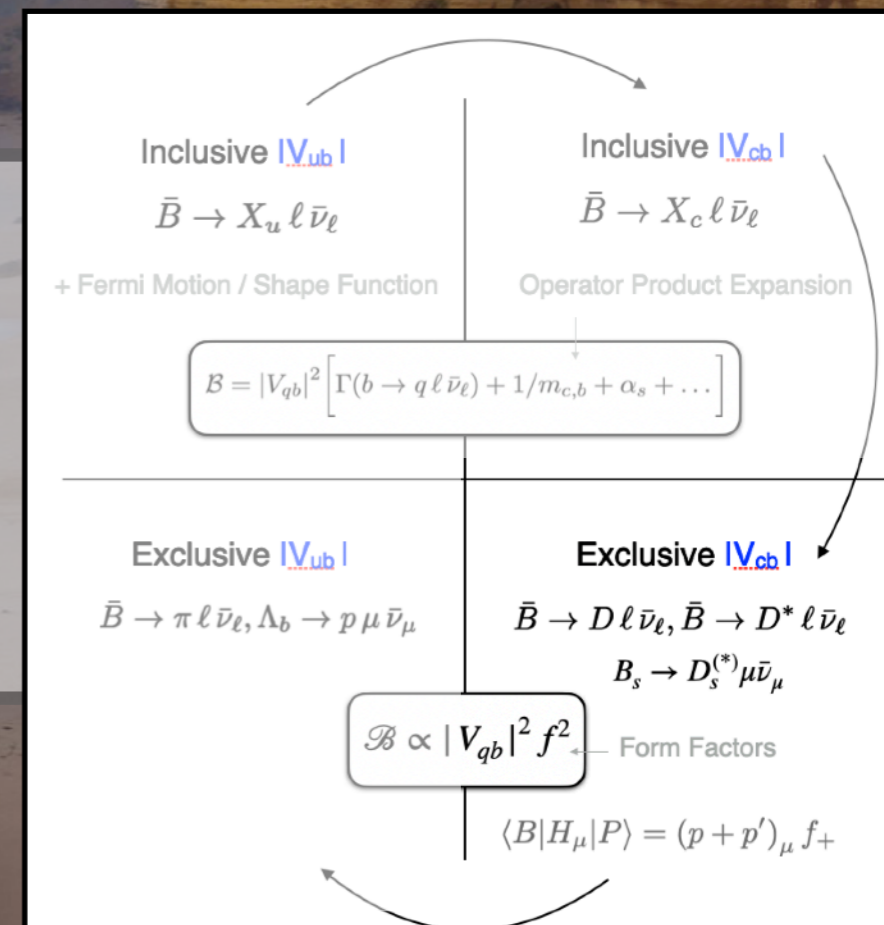
Figure 8 shows the results for the moments  $\langle n_X^2 \rangle$ ,  $\langle n_X^4 \rangle$ , and  $\langle n_X^6 \rangle$  as a function of the minimum lepton momentum  $p_{\ell,\text{min}}^*$ . The moments are highly correlated due to the overlapping data samples. The full numerical results and the statistical and the estimated systematic uncertainties are given in Table A.III. The systematic covariance matrix for the moments of different order and with different cuts on  $p_{\ell,\text{min}}^*$  is built using statistical correlations. This correlation matrix for the moments is given in the EPAPS document [43].

A clear dependence on the minimum lepton momentum is observed for all moments, due to the increasing contributions from higher-mass final states with decreasing lepton momentum. In most cases we obtain systematic uncertainties slightly exceeding the statistical uncertainty.

We should check the impact of **alternative approaches**

# Excl. $|V_{cb}|$

- ▶ 4 x 1D: Beware
- ▶ Recent Lattice QCD results
- ▶ New from LHCb & Belle II





# 4 x 1D Fits

See also Bordone, Jung, Van Dyk [arXiv:1908.09398]

$$B \rightarrow D^*[\rightarrow D\pi]\ell\bar{\nu}_\ell$$

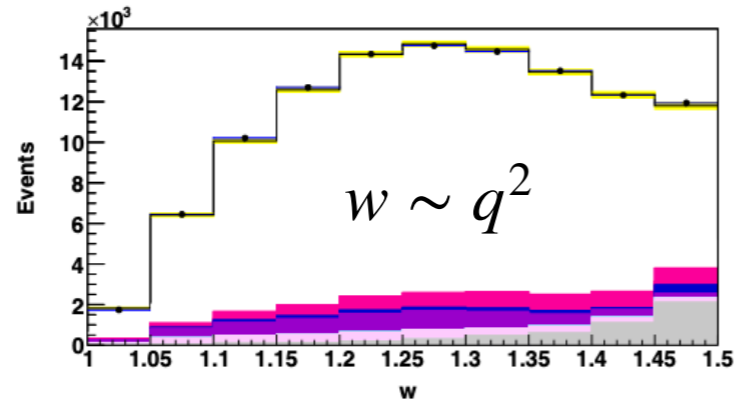
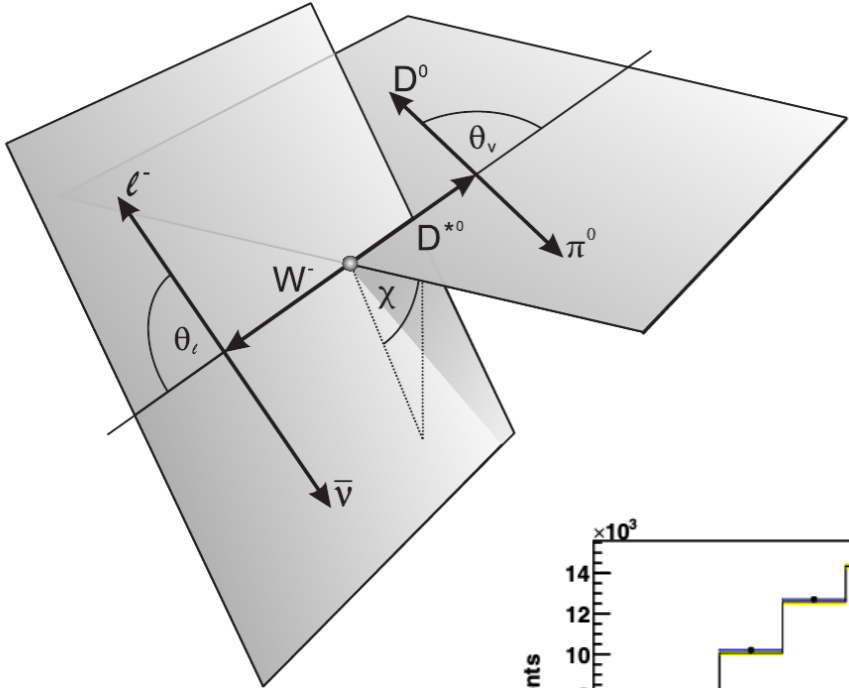
$$JC_{40 \times 40}J^T = C_{4 \times 4}$$

$$J = \begin{pmatrix} 1,1,1,1,1,1,1,1,1,1 \\ 1,1,1,1,1,1,1,1,1,1 \\ 1,1,1,1,1,1,1,1,1,1 \\ 1,1,1,1,1,1,1,1,1,1 \end{pmatrix}$$

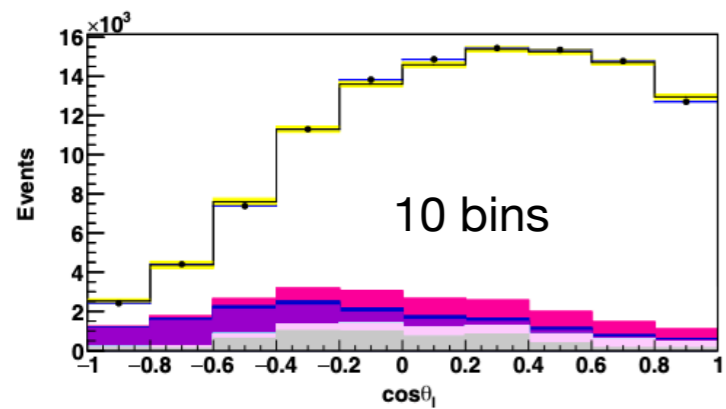
If there would be no *background*,  $C_{4 \times 4}$  should be **singular**

Statistical correlation matrix of all 40 bins  
 $C_{40 \times 40} \rightarrow C_{4 \times 4}$   
 Sum all 10 bins

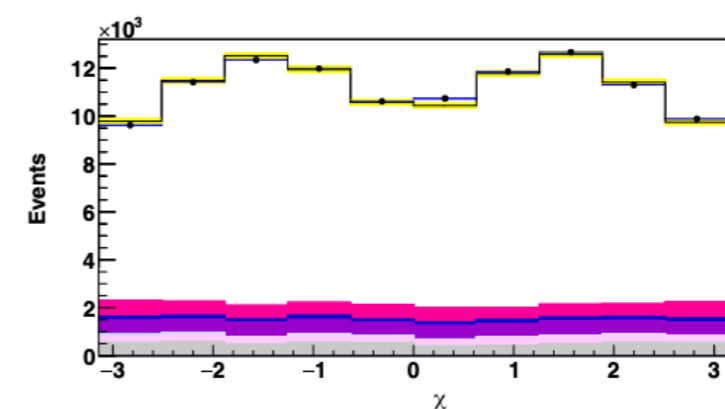
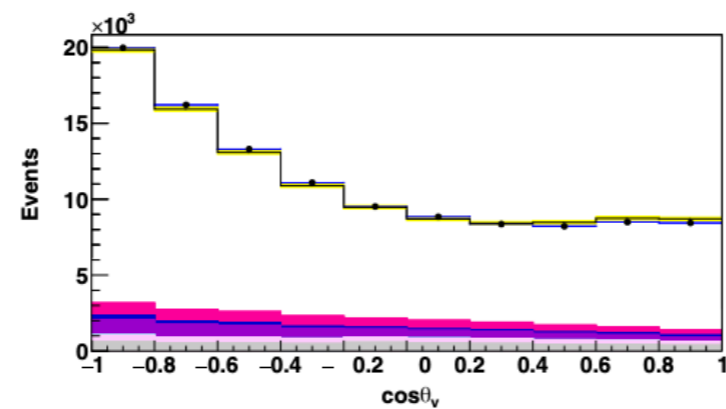
i.e. if you setup your  $|V_{cb}|$  extraction by fitting all 40 bins, this could create problems

$$\chi^2(|V_{cb}|, \vec{\mu}_{FF}) = \left( \Delta\mathcal{B} - |V_{cb}|^2 \tau_B \Gamma(\vec{\mu}_{FF}) \right) C_{40 \times 40}^{-1} \left( \Delta\mathcal{B} - |V_{cb}|^2 \tau_B \Gamma(\vec{\mu}_{FF}) \right)$$


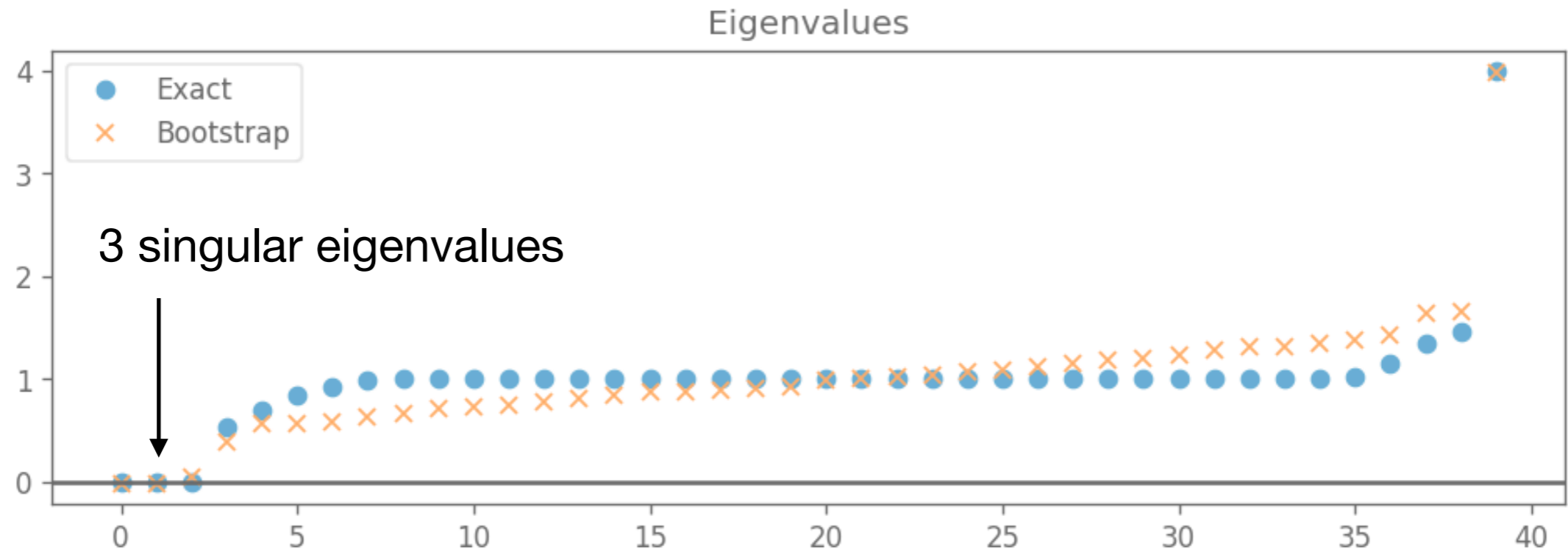
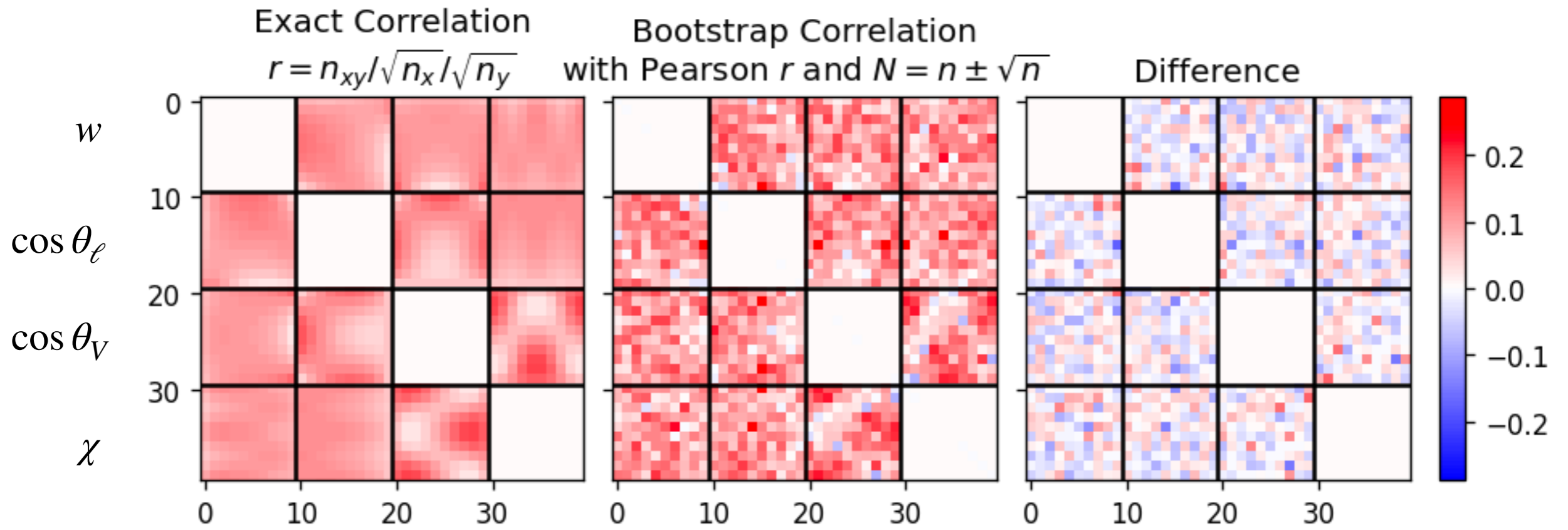
Projection of same Events



Helicity Angles



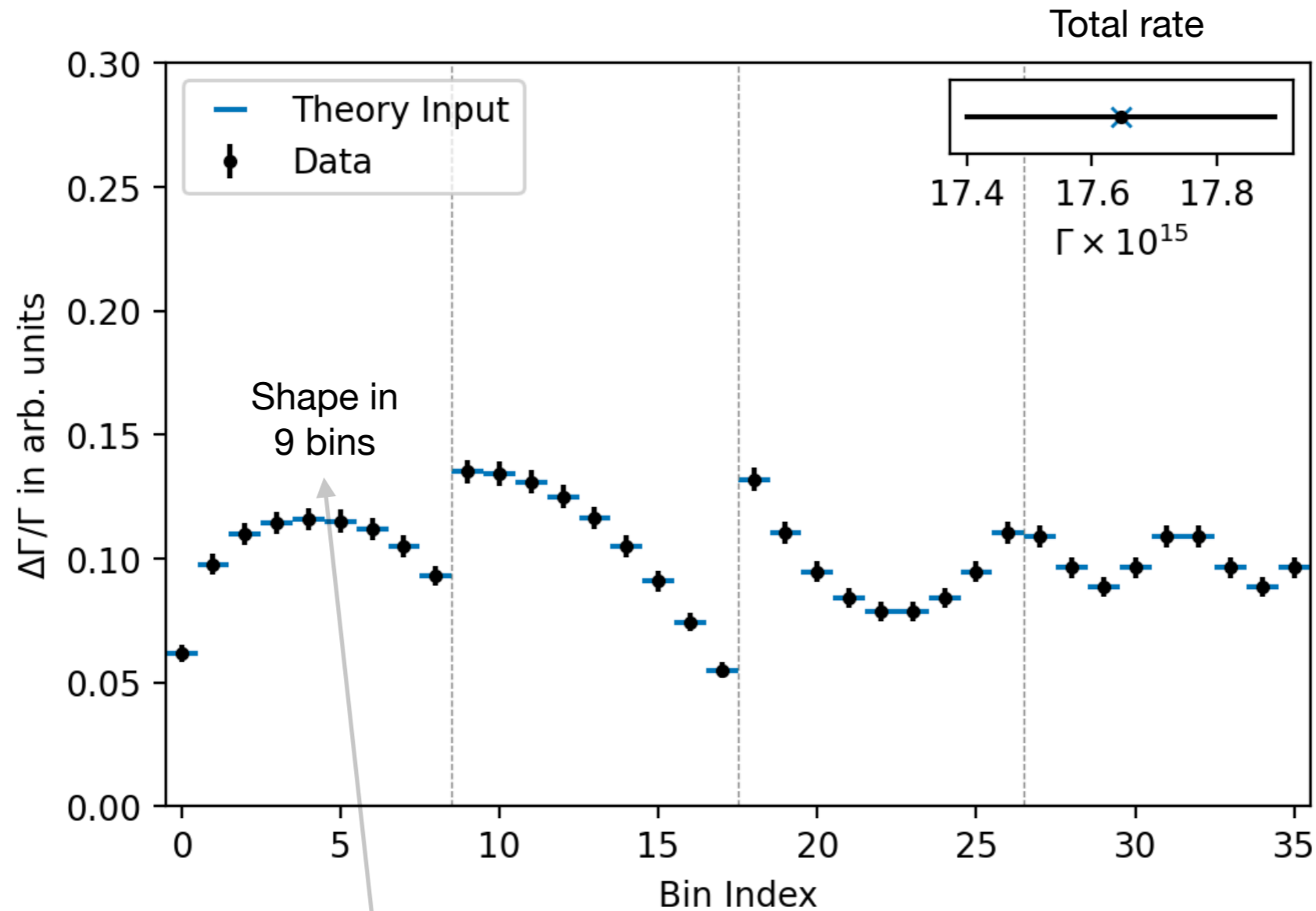
Toy example:



## Toy example:

Elegant Solution: Fit the total rate and 4 shapes

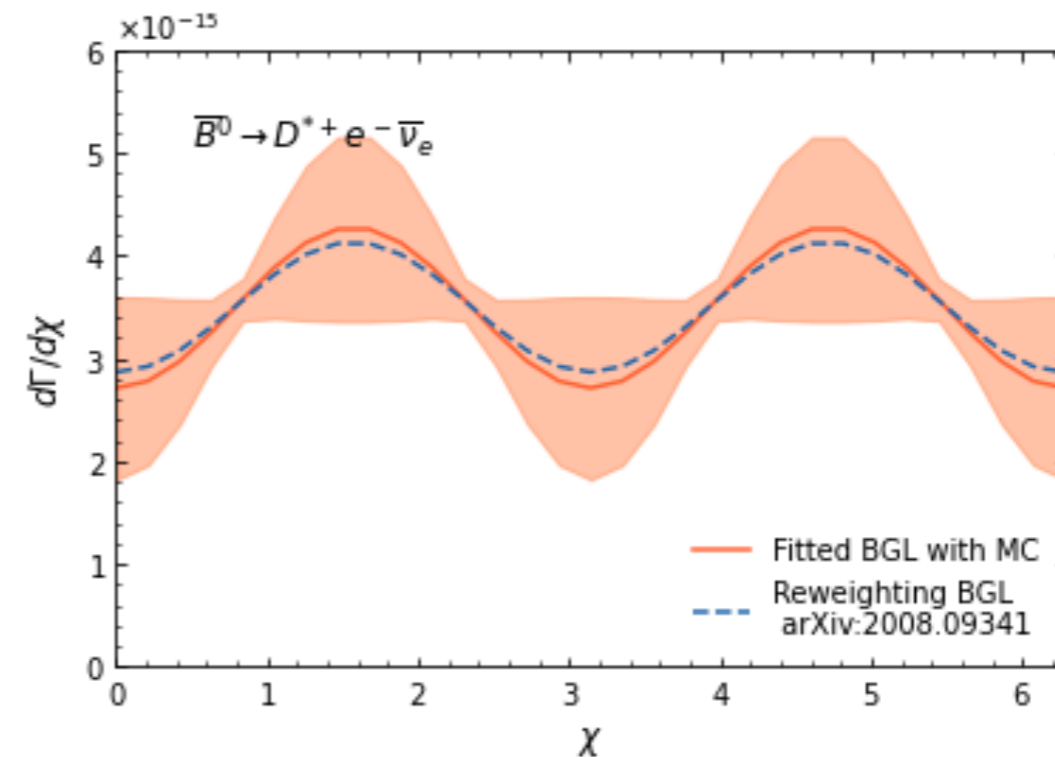
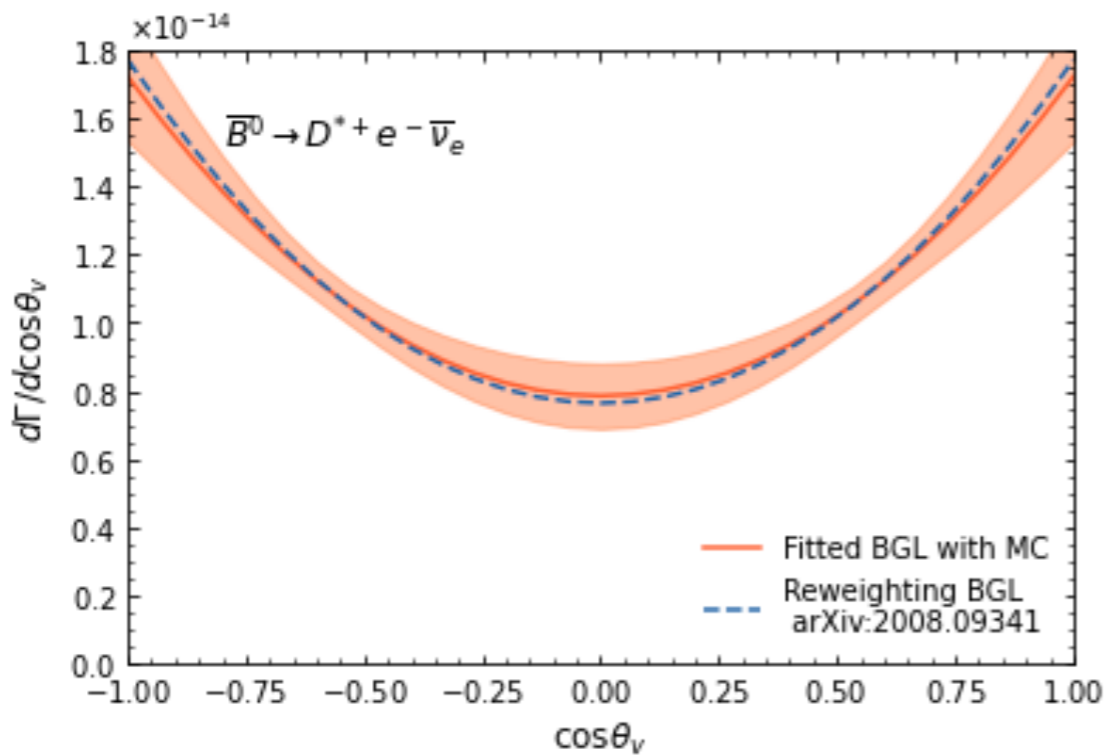
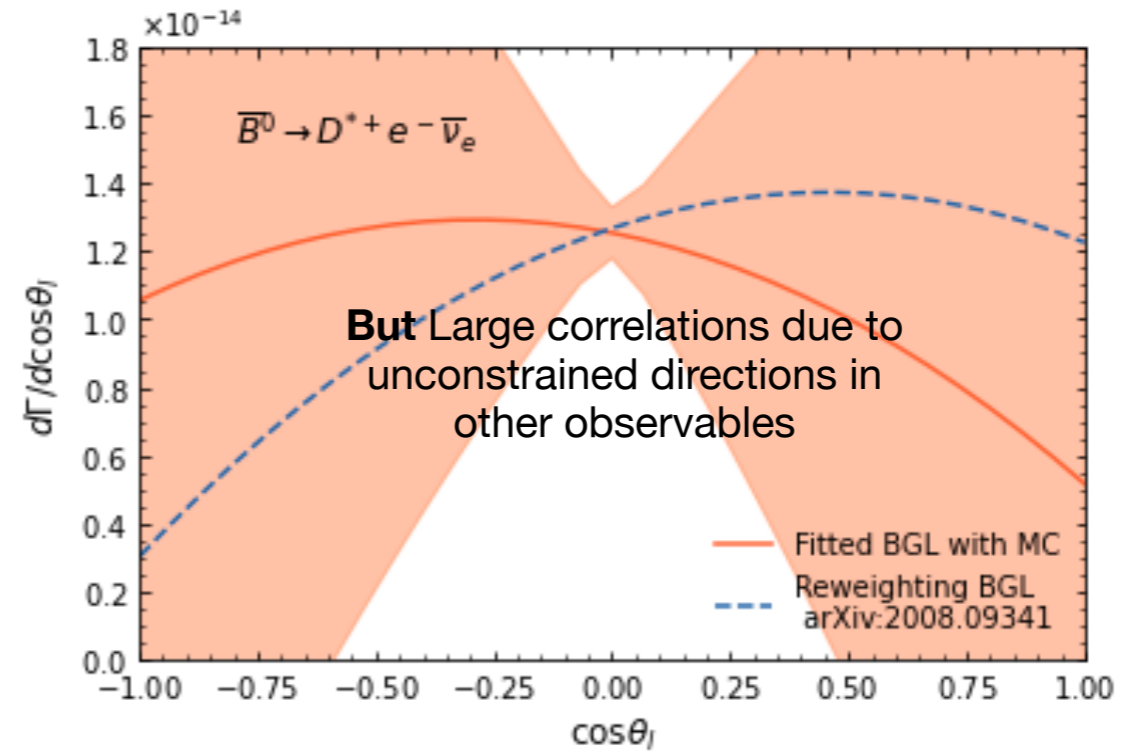
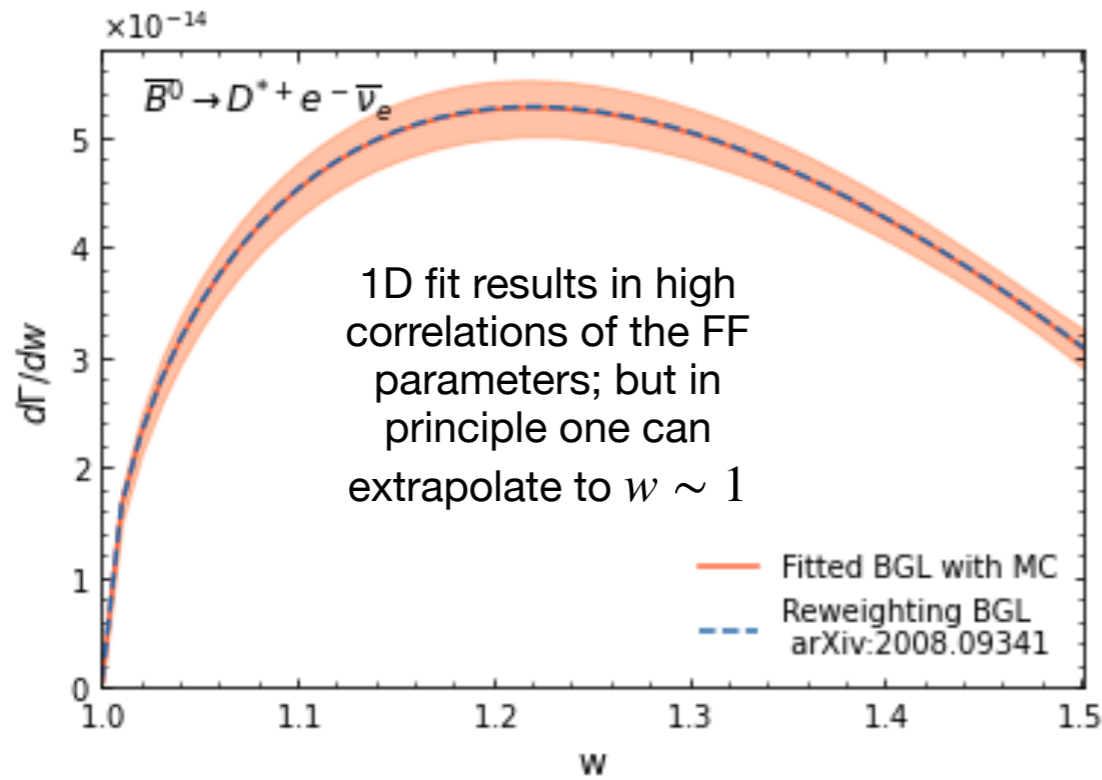
$$\chi^2(|V_{cb}|, \vec{\mu}_{\text{FF}}) = \left( \Delta\mathcal{B}/\mathcal{B} - \Delta\Gamma(\vec{\mu}_{\text{FF}})/\Gamma(\vec{\mu}_{\text{FF}}) \right) C_{36 \times 36}^{-1} \left( \Delta\mathcal{B}/\mathcal{B} - \Delta\Gamma(\vec{\mu}_{\text{FF}})/\Gamma(\vec{\mu}_{\text{FF}}) \right) + \left( \mathcal{B} - |V_{cb}|^2 \tau_B \Gamma(\vec{\mu}_{\text{FF}}) \right)^2 / \sigma_{\mathcal{B}}^2$$



(last bin is fully determined by 1 - sum of all other bins)

# 4 x 1D or 1D or 4D?

Chaoyi Lyu, FB



The future might be unbinned...

**Un-binned Angular Analysis of  $B \rightarrow D^* \ell \nu_\ell$  and the Right-handed Current**

Z.R. Huang\* and E. Kou†

*Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France*

C.D. Lü‡ and R.Y. Tang§

*Institute of High Energy Physics,  
Chinese Academy of Sciences, Beijing 100049, China*

*and  
School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*

(Dated: June 29, 2021)

Experimentally challenging (solved e.g. by Phys. Rev. Lett. 123, 091801 (2019))

BABAR-PUB-19/001  
SLAC-PUB-17420

**Extraction of form factors from a four-dimensional angular analysis of  $\bar{B} \rightarrow D^* \ell^- \bar{\nu}_\ell$**

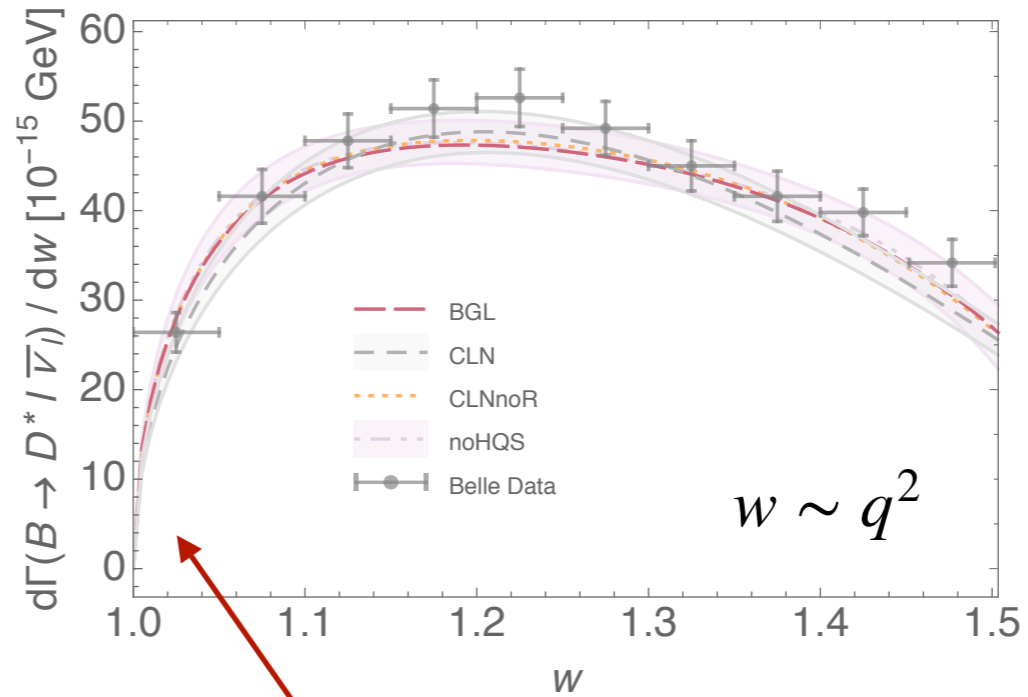
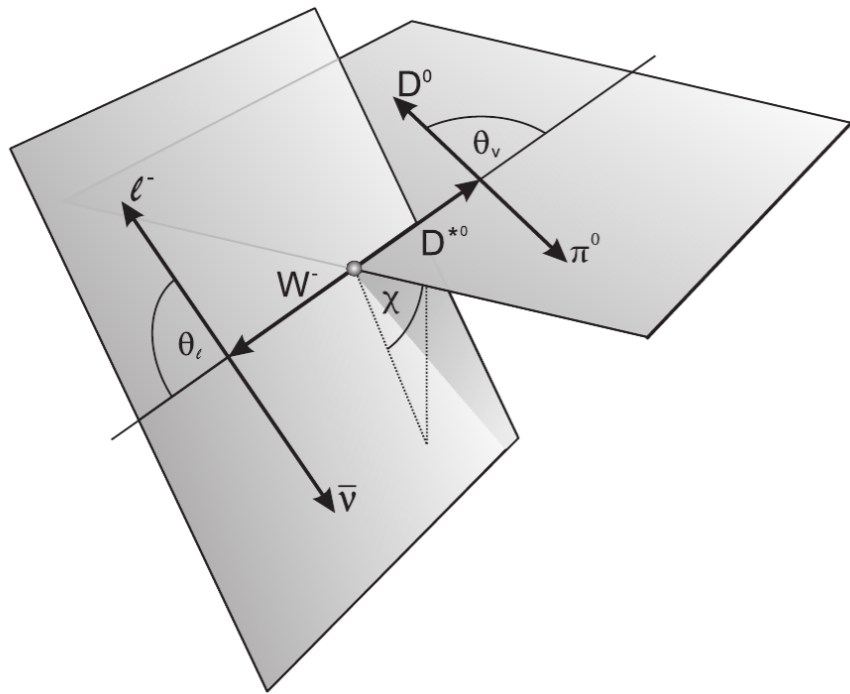
J. P. Lees,<sup>1</sup> V. Poireau,<sup>1</sup> V. Tisserand,<sup>1</sup> E. Grauges,<sup>2</sup> A. Palano,<sup>3</sup> G. Eigen,<sup>4</sup> D. N. Brown,<sup>5</sup> Yu. G. Kolomensky,<sup>5</sup>  
M. Fritsch,<sup>6</sup> H. Koch,<sup>6</sup> T. Schroeder,<sup>6</sup> C. Hearty<sup>ab,7</sup>, T. S. Mattison<sup>b,7</sup>, J. A. McKenna<sup>b,7</sup>, R. Y. So<sup>b,7</sup>,  
V. E. Blinov<sup>abc,8</sup>, A. R. Buzykaev<sup>a,8</sup>, V. P. Druzhinin<sup>ab,8</sup>, V. B. Golubev<sup>ab,8</sup>, E. A. Kozyrev<sup>ab,8</sup>, E. A. Kravchenko<sup>ab,8</sup>,  
A. P. Onuchin<sup>abc,8</sup>, S. I. Serednyakov<sup>ab,8</sup>, Yu. I. Skovpen<sup>ab,8</sup>, E. P. Solodov<sup>ab,8</sup>, K. Yu. Todyshev<sup>ab,8</sup>, A. J. Lankford,<sup>9</sup>  
B. Dey,<sup>10</sup> J. W. Gary,<sup>10</sup> O. Long,<sup>10</sup> A. M. Eisner,<sup>11</sup> W. S. Lockman,<sup>11</sup> W. Panduro Vazquez,<sup>11</sup> D. S. Chao,<sup>12</sup>  
C. H. Cheng,<sup>12</sup> B. Echenard,<sup>12</sup> K. T. Flood,<sup>12</sup> D. G. Hitlin,<sup>12</sup> J. Kim,<sup>12</sup> Y. Li,<sup>12</sup> T. S. Miyashita,<sup>12</sup>  
P. Ongmongkolkul,<sup>12</sup> F. C. Porter,<sup>12</sup> M. Röhrken,<sup>12</sup> Z. Huard,<sup>13</sup> B. T. Meadows,<sup>13</sup> B. G. Pushpawela,<sup>13</sup>  
M. D. Sokoloff,<sup>13</sup> L. Sun,<sup>13,\*</sup> J. G. Smith,<sup>14</sup> S. R. Wagner,<sup>14</sup> D. Bernard,<sup>15</sup> M. Verderi,<sup>15</sup> D. Bettoni<sup>a,16</sup>,  
C. Bozzi<sup>a,16</sup>, R. Calabrese<sup>ab,16</sup>, G. Cibinetto<sup>ab,16</sup>, E. Fioravanti<sup>ab,16</sup>, I. Garzia<sup>ab,16</sup>, E. Luppi<sup>ab,16</sup>, V. Santoro<sup>a,16</sup>,  
A. Calcaterra,<sup>17</sup> R. de Sangro,<sup>17</sup> G. Finocchiaro,<sup>17</sup> S. Martellotti,<sup>17</sup> P. Patteri,<sup>17</sup> I. M. Peruzzi,<sup>17</sup> M. Piccolo,<sup>17</sup>

2019

Largest challenge: how to make this data accessible to others?  
(also cf. Omnifold)

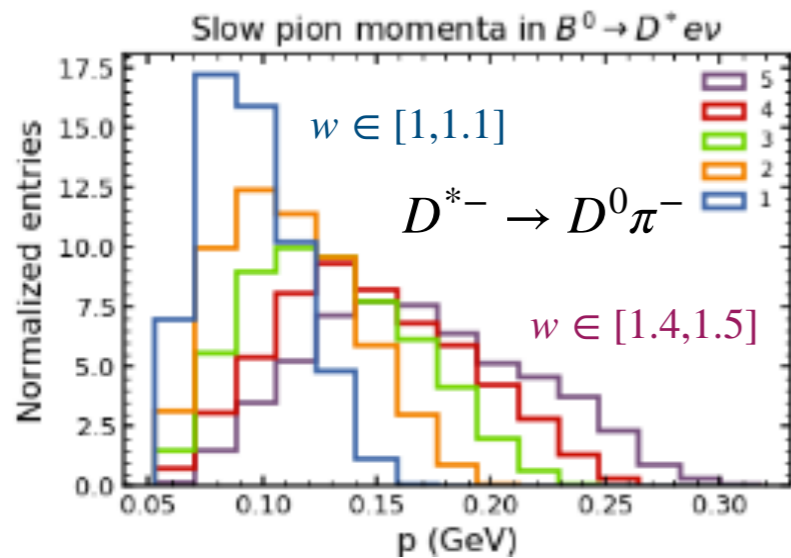
# Slow pions

$$B \rightarrow D^*[\rightarrow D\pi]\ell\bar{\nu}_\ell$$

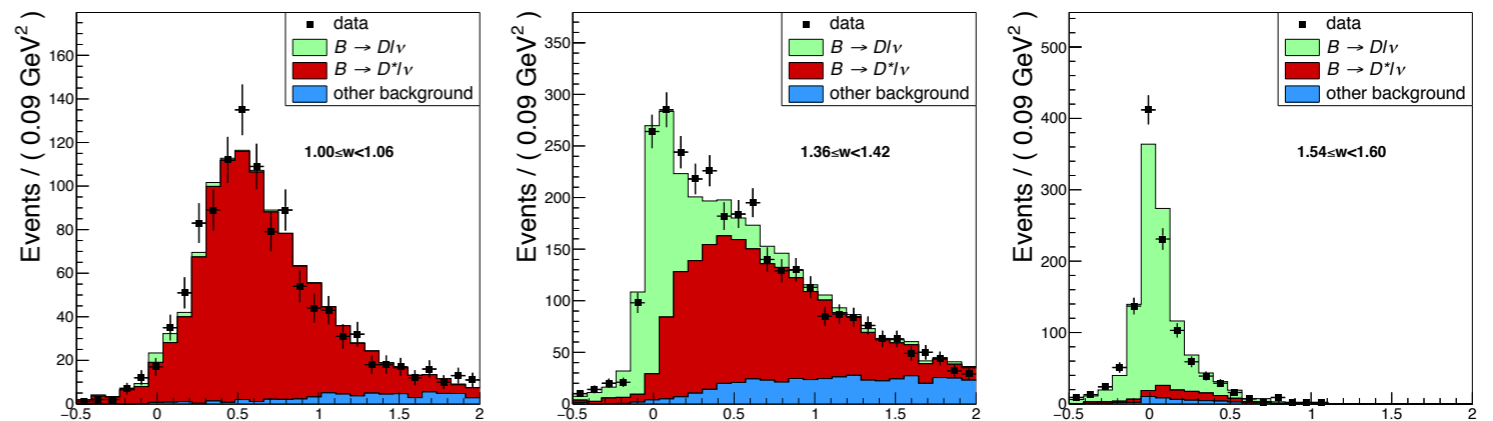


To determine  $|V_{cb}|$  we extrapolate to rate near  $w \sim 1$

Experimentally very challenging region:



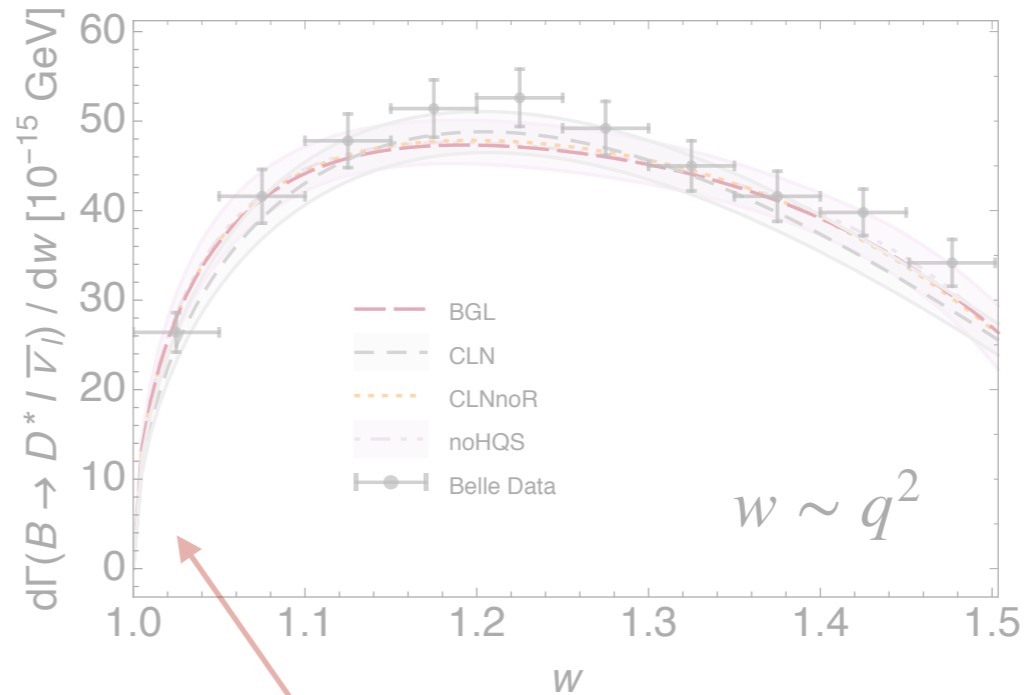
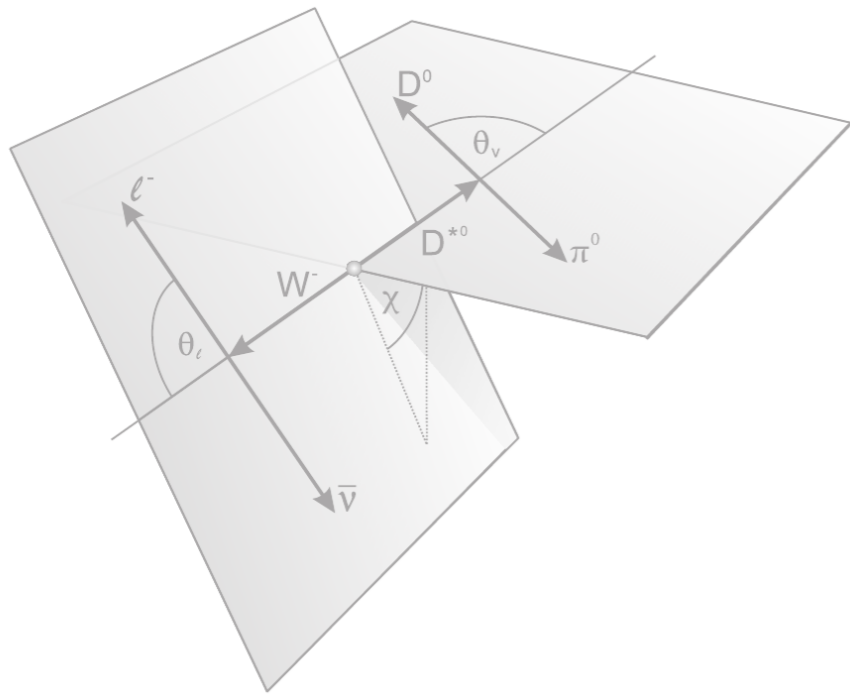
We see many of these events as down-feed in e.g.  $B^+ \rightarrow D^0 e \bar{\nu}_e$



# Slow pions

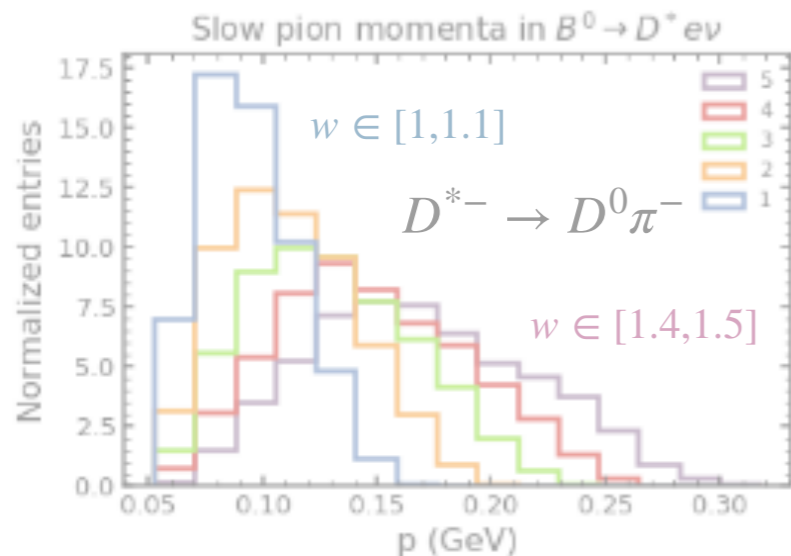
This has been exploited e.g. by the BaBar global analysis  $|V_{cb}|$  [Phys. Rev. D79:012002, 2009]

$$B \rightarrow D^*[\rightarrow D\pi]\ell\bar{\nu}_\ell$$

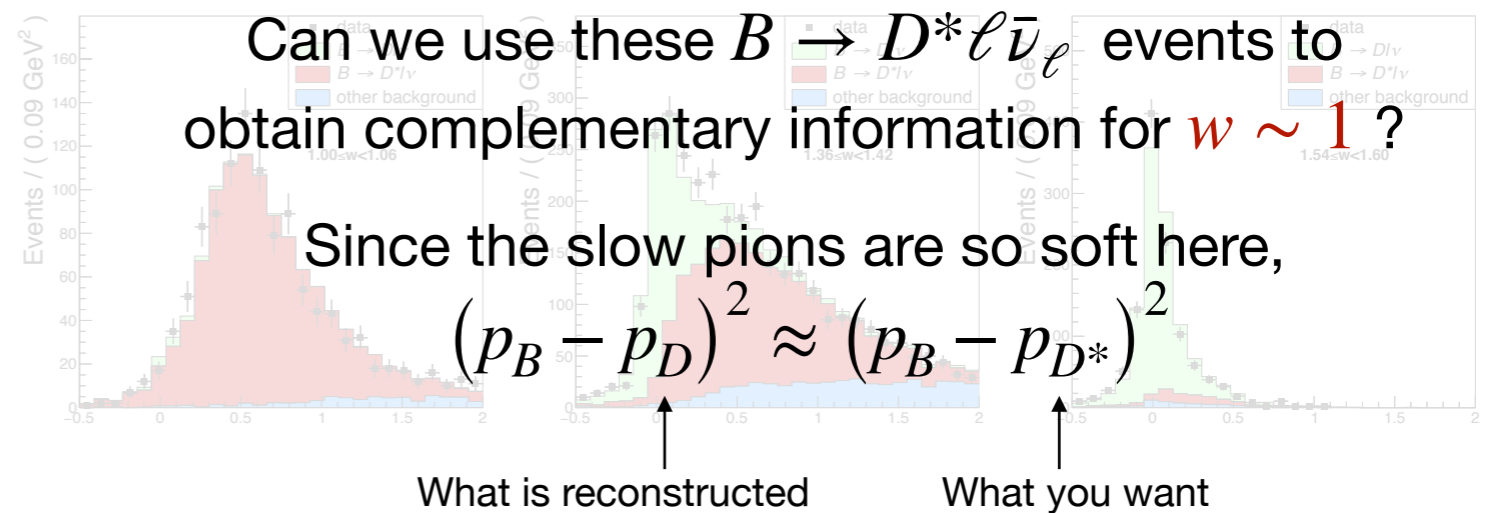


To determine  $|V_{cb}|$  we extrapolate to rate near  $w \sim 1$

Experimentally very challenging region:



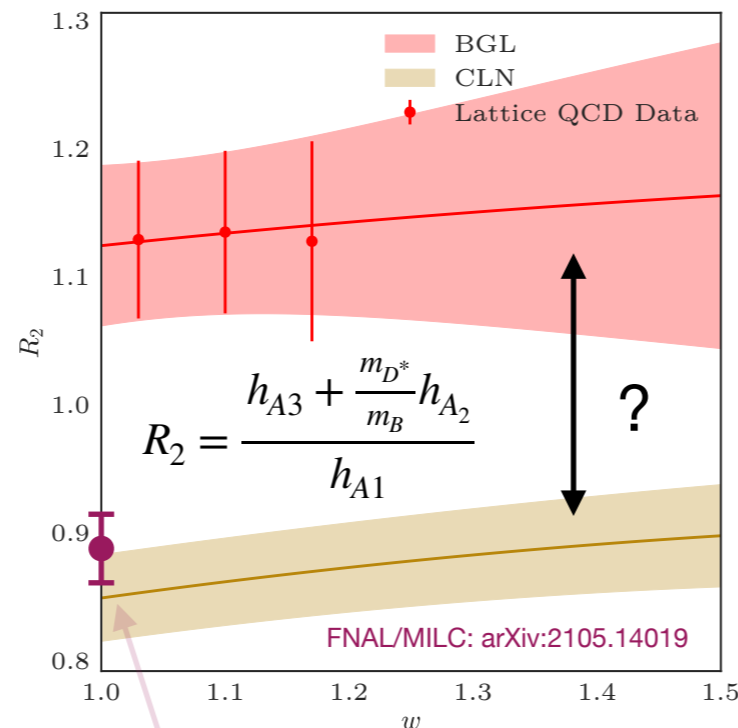
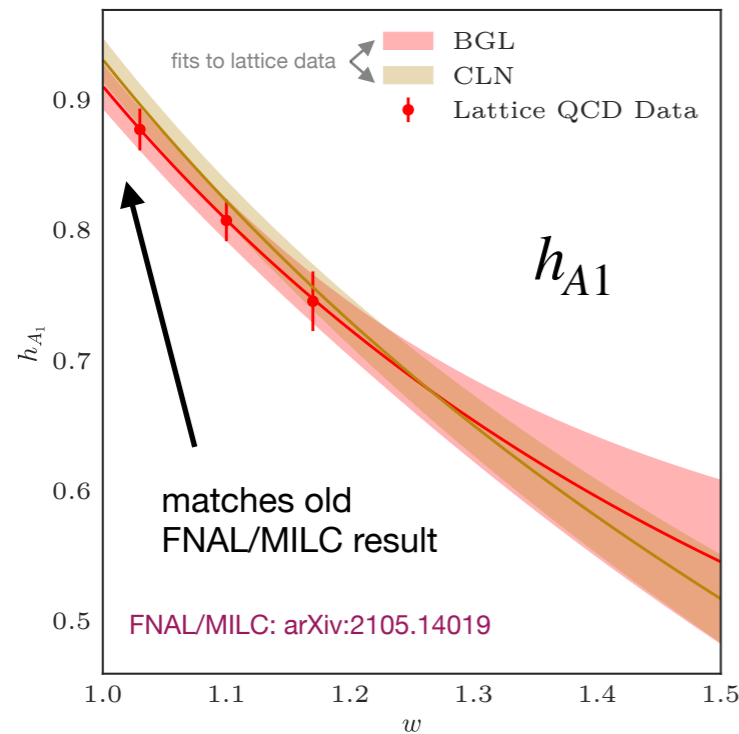
We see many of these events as down-feed in e.g.  $B^+ \rightarrow D^0 e \bar{\nu}_e$



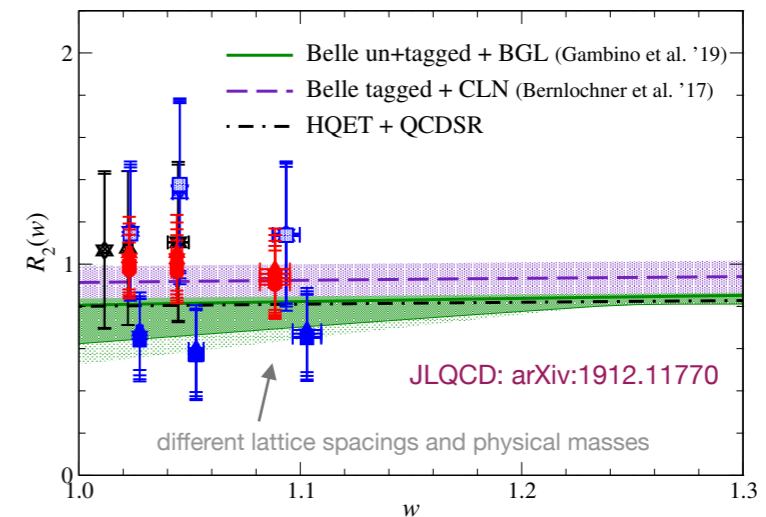
# FNAL $D^*$ Lattice Results

See also BGL fit to BaBar Data  
 Phys. Rev. Lett. 123, 091801 (2019)  
 and studies of Belle untagged  $D^*$  spectrum  
 Phys. Rev. D 103, 073005 (2021)

**New** results from **FNAL/MILC** on  $B \rightarrow D^*$  form factors [FNAL/MILC: arXiv:2105.14019]



**Intriguing deviation from HQET expectation!**



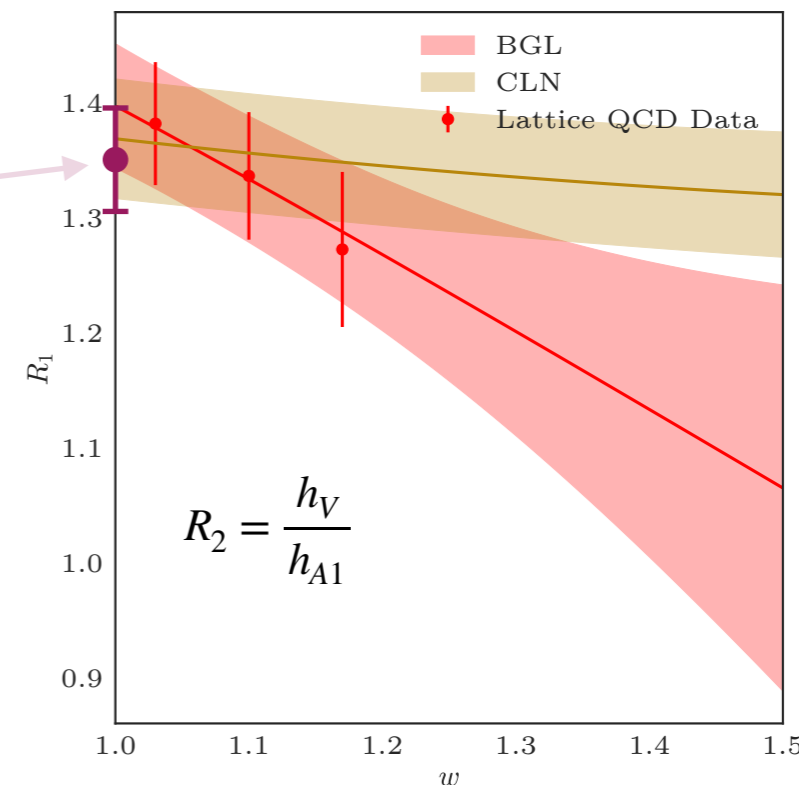
HQET prediction: Based on Phys. Rev. D 95, 115008 (2017) fit results

$$R_1(1) = 1.34 - 0.12\eta(1) \approx 1.35 \pm 0.05$$

$$R_2(1) = 0.98 - 0.42\eta(1) - 0.54\hat{\chi}_2(1) \approx 0.89 \pm 0.03$$

To explain FNAL/MILC need negative  $\eta(1)$ ,  $\hat{\chi}_2(1)$

$$\eta(1) \approx -0.5 \quad \hat{\chi}_2(1) \approx -0.4$$

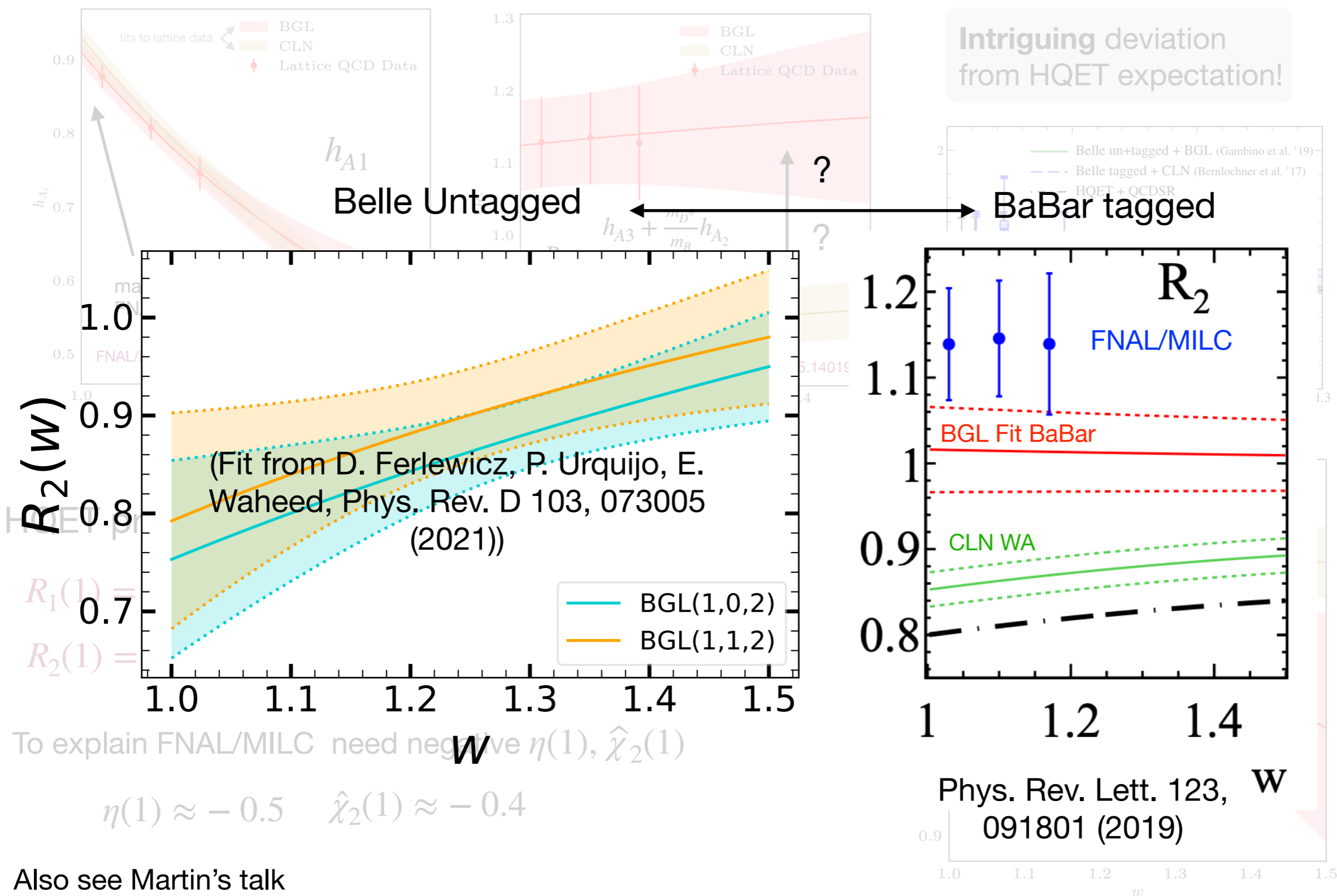




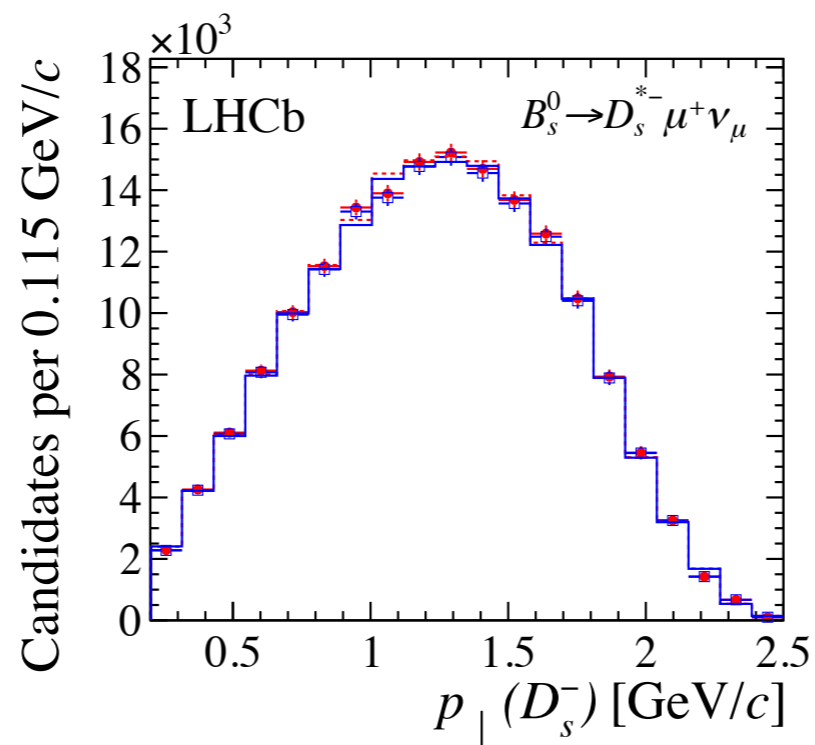
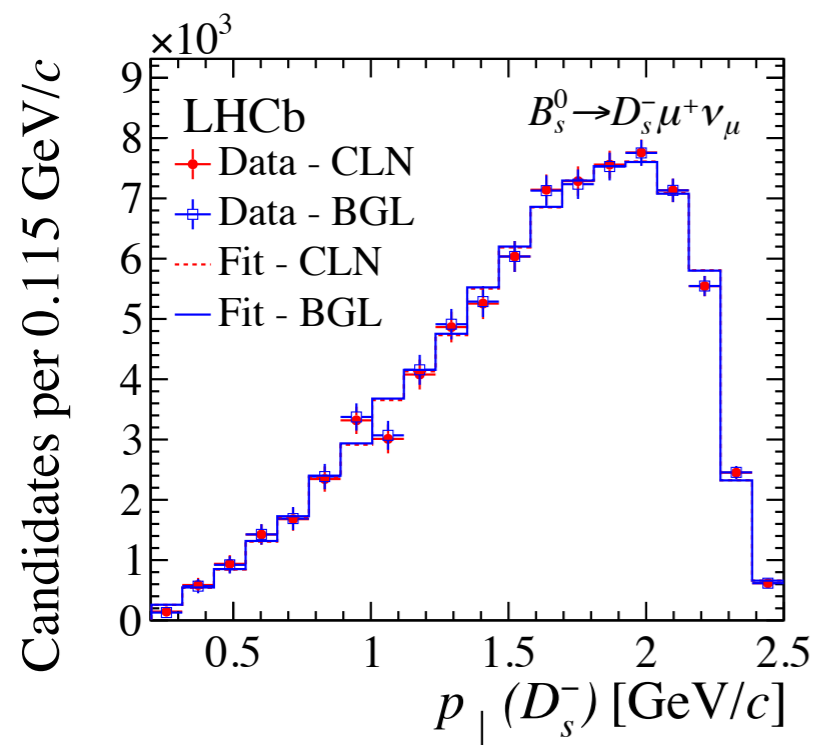
# FNAL $D^*$ Lattice Results

See also BGL fit to BaBar Data  
 Phys. Rev. Lett. 123, 091801 (2019)  
 and studies of Belle untagged  $D^*$  spectrum  
 Phys. Rev. D 103, 073005 (2021)

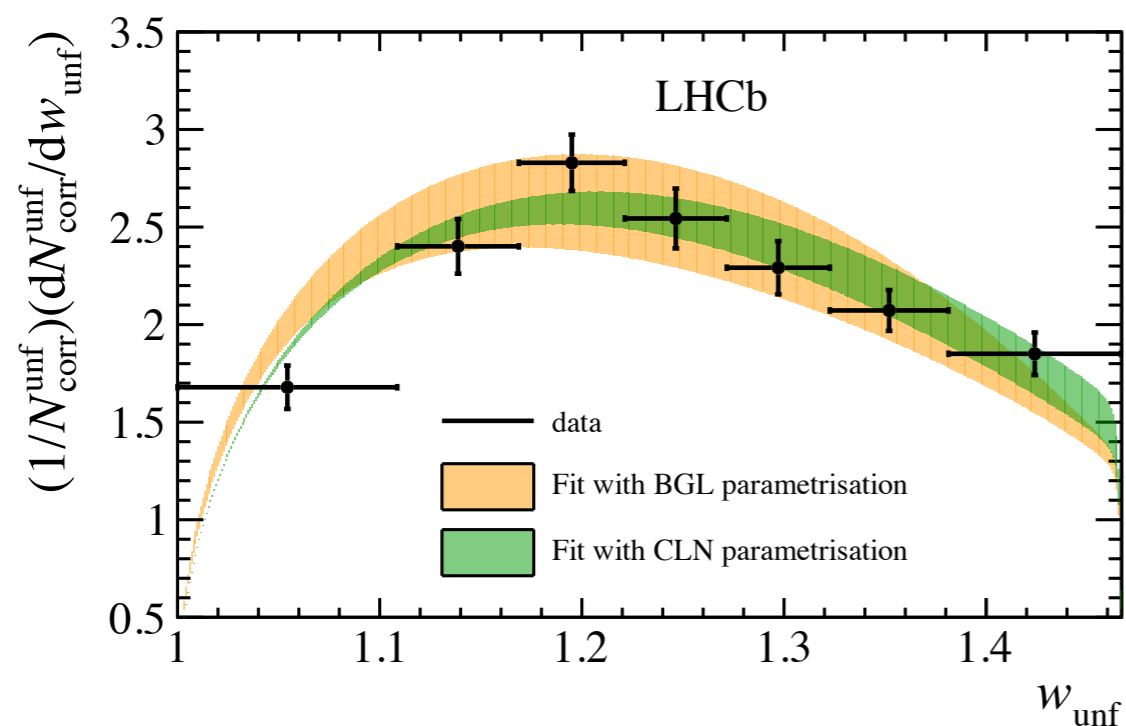
New results from FNAL/MILC on  $B \rightarrow D^*$  form factors [FNAL/MILC: arXiv:2105.14019]



Also see Martin's talk



Parameter	Value
$ V_{cb}  [10^{-3}]$	$41.4 \pm 0.6$ (stat) $\pm 1.2$ (ext)
$\mathcal{G}(0)$	$1.102 \pm 0.034$ (stat) $\pm 0.004$ (ext)
$\rho^2(D_s^-)$	$1.27 \pm 0.05$ (stat) $\pm 0.00$ (ext)
$\rho^2(D_s^{*-})$	$1.23 \pm 0.17$ (stat) $\pm 0.01$ (ext)
$R_1(1)$	$1.34 \pm 0.25$ (stat) $\pm 0.02$ (ext)
$R_2(1)$	$0.83 \pm 0.16$ (stat) $\pm 0.01$ (ext)



CLN fit

Unfolded fit  $\rho^2 = 1.16 \pm 0.05 \pm 0.07$

Unfolded fit with massless leptons  $\rho^2 = 1.17 \pm 0.05 \pm 0.07$

Folded fit  $\rho^2 = 1.14 \pm 0.04 \pm 0.07$

BGL fit

Unfolded fit  $a_1^f = -0.005 \pm 0.034 \pm 0.046$

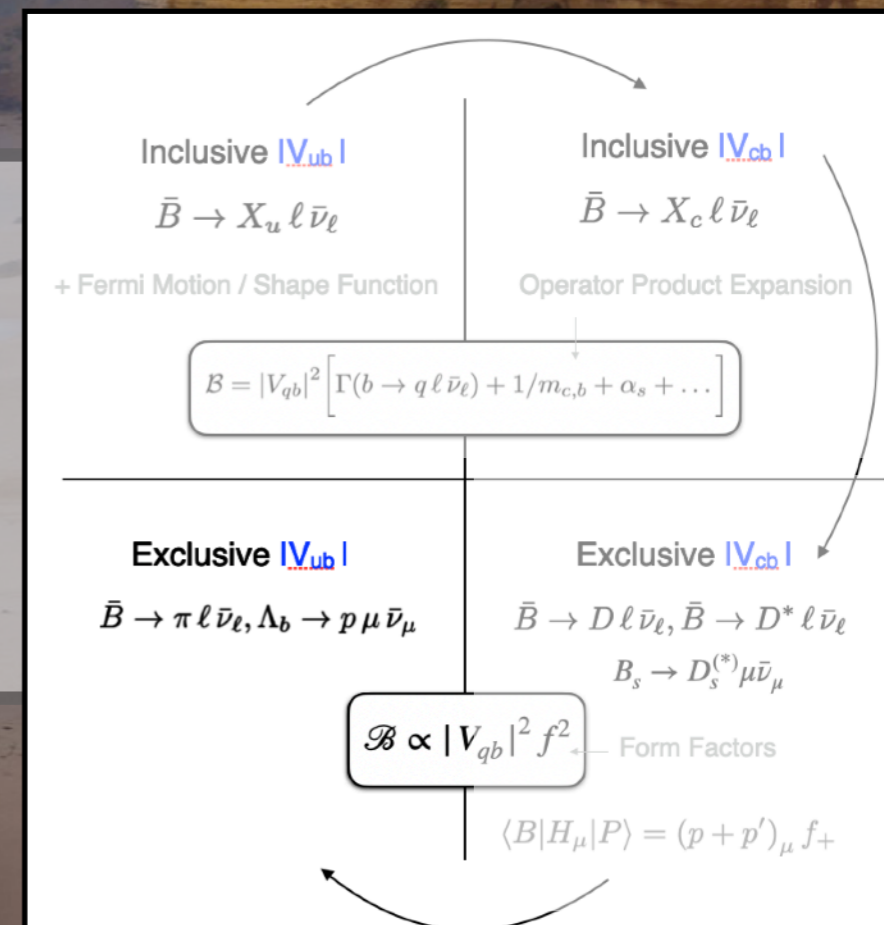
$a_2^f = 1.00^{+0.00+0.00}_{-0.19-0.38}$

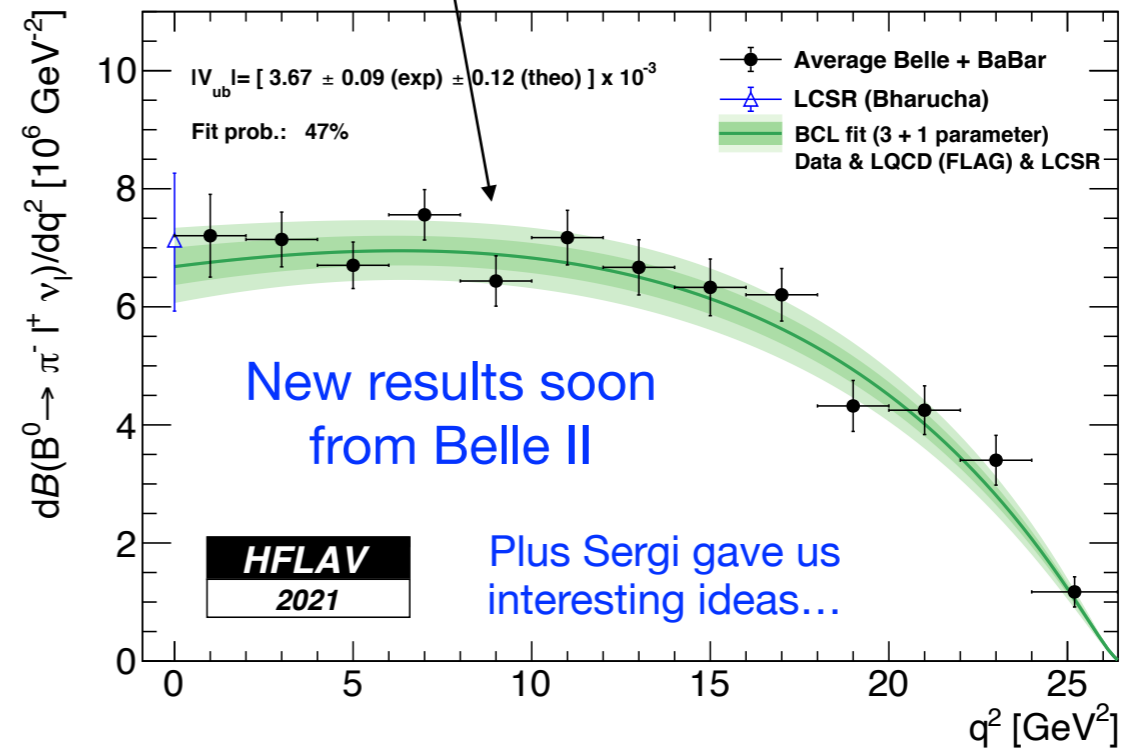
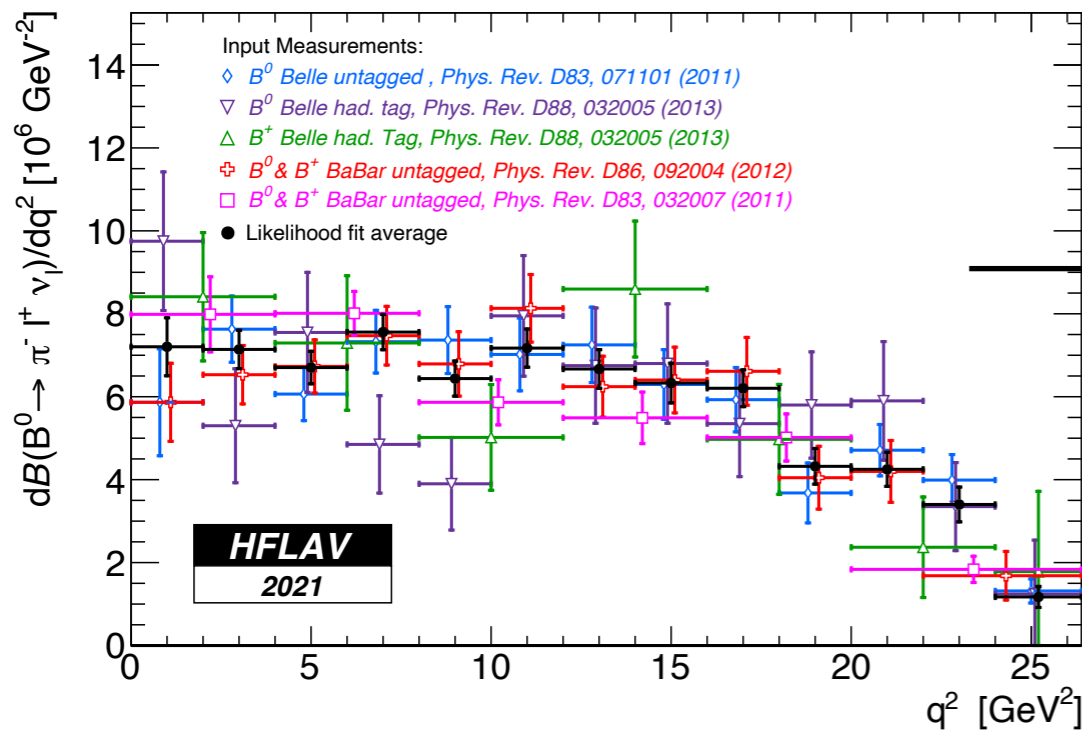
Folded fit  $a_1^f = 0.039 \pm 0.029 \pm 0.046$

$a_2^f = 1.00^{+0.00+0.00}_{-0.13-0.34}$

# Excl. $|V_{ub}|$

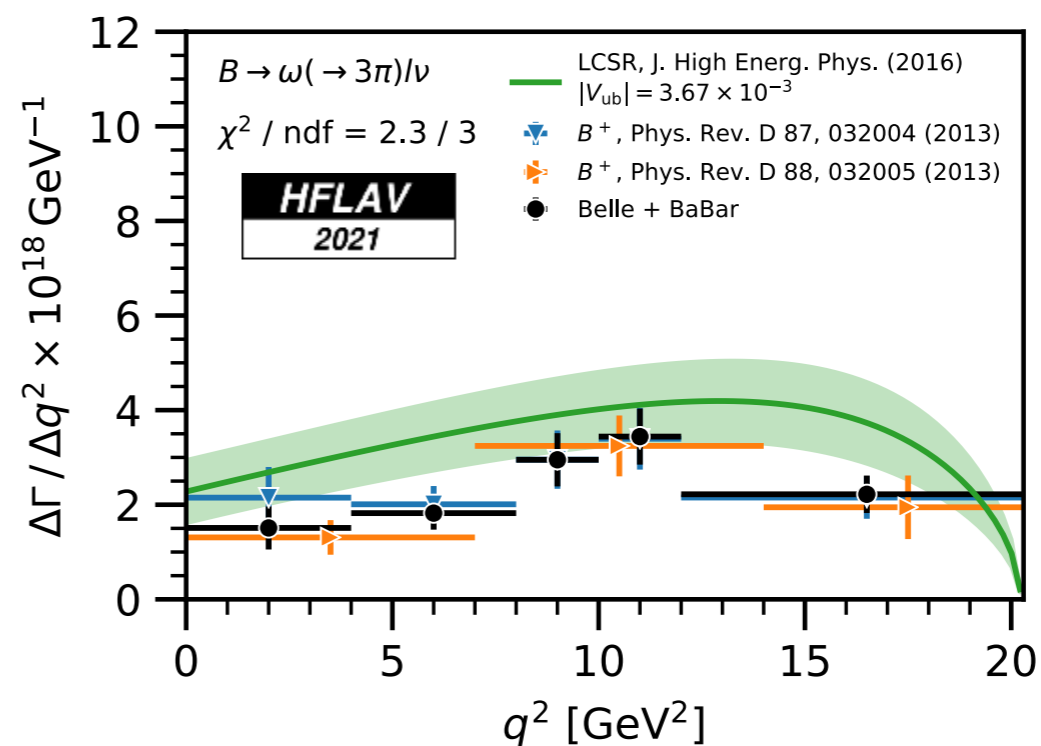
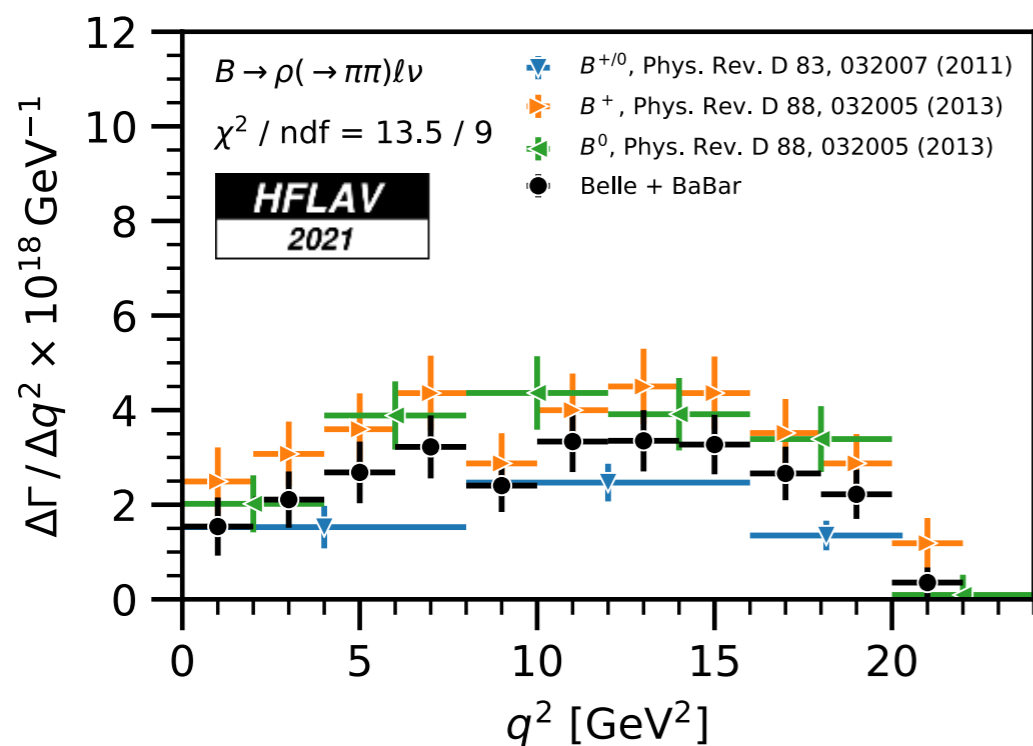
- ▶ State-of-the-Art
- ▶ New averages from HFLAV
- ▶ News from LHCb

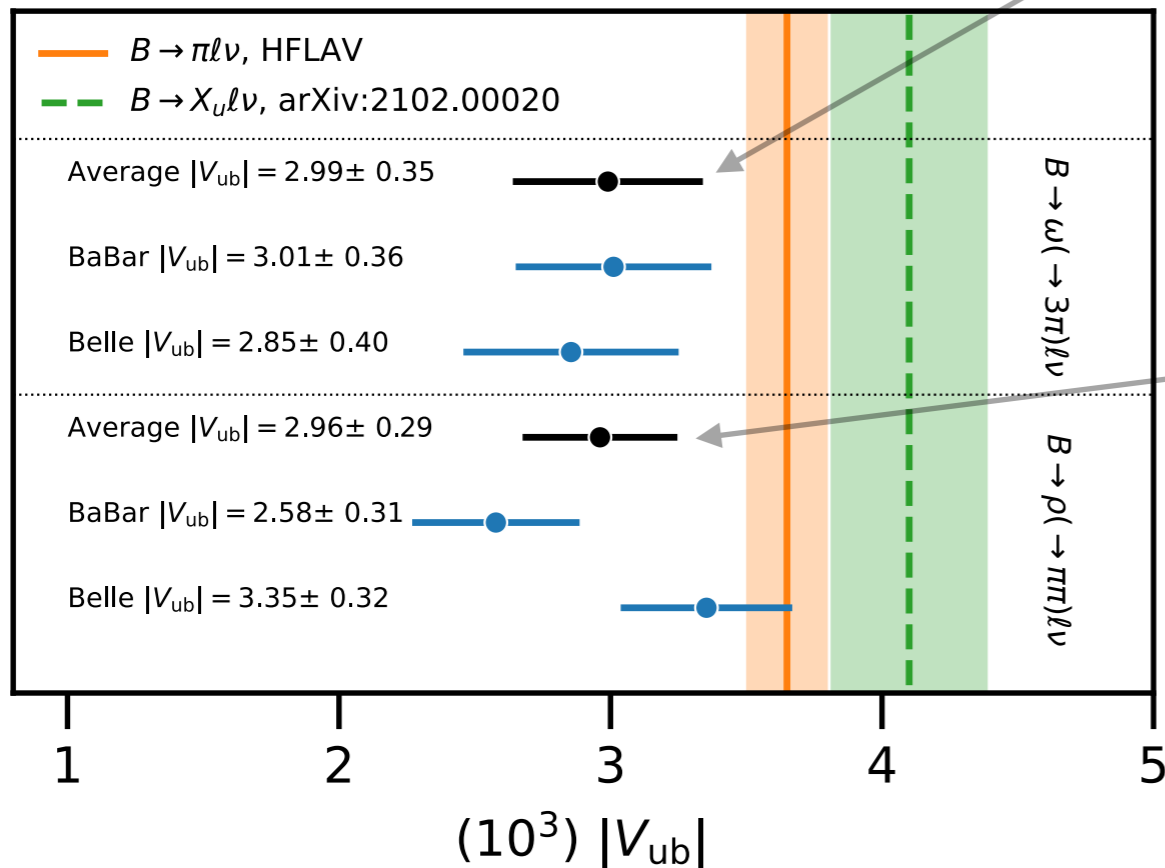
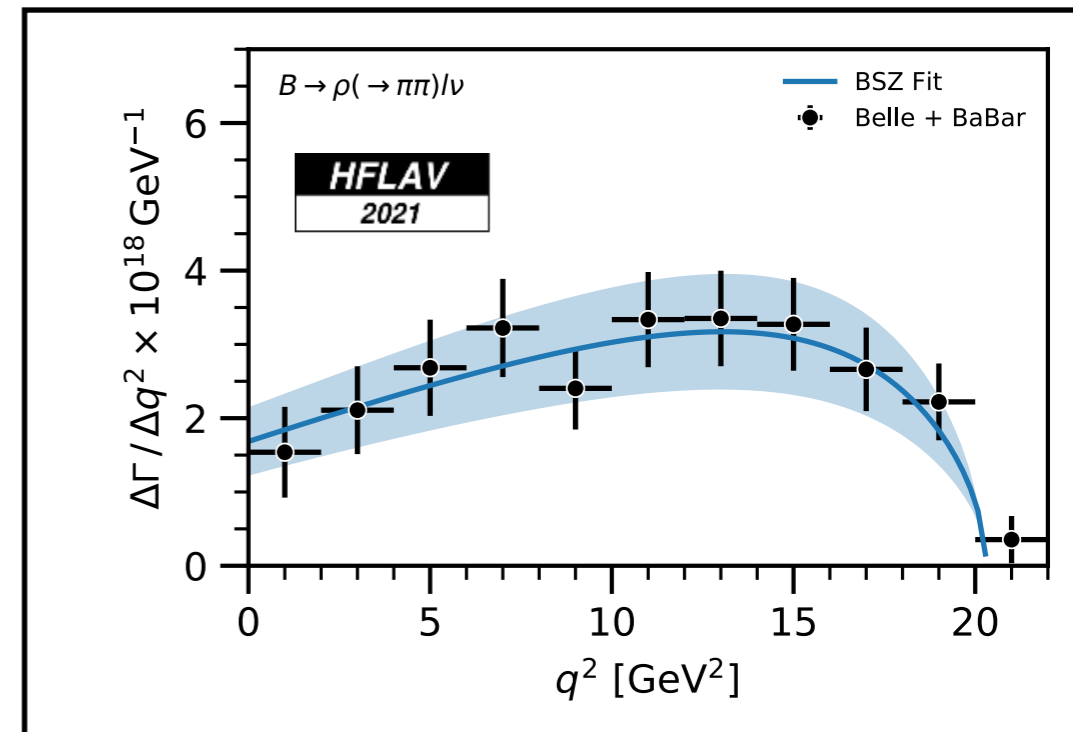
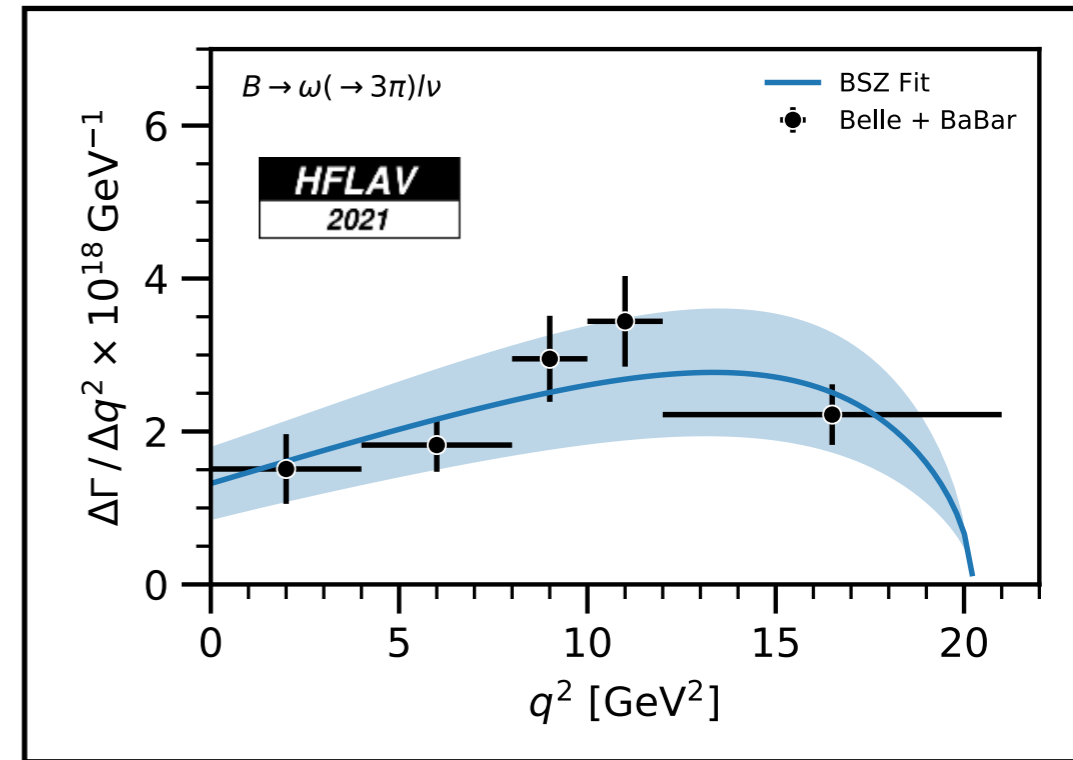
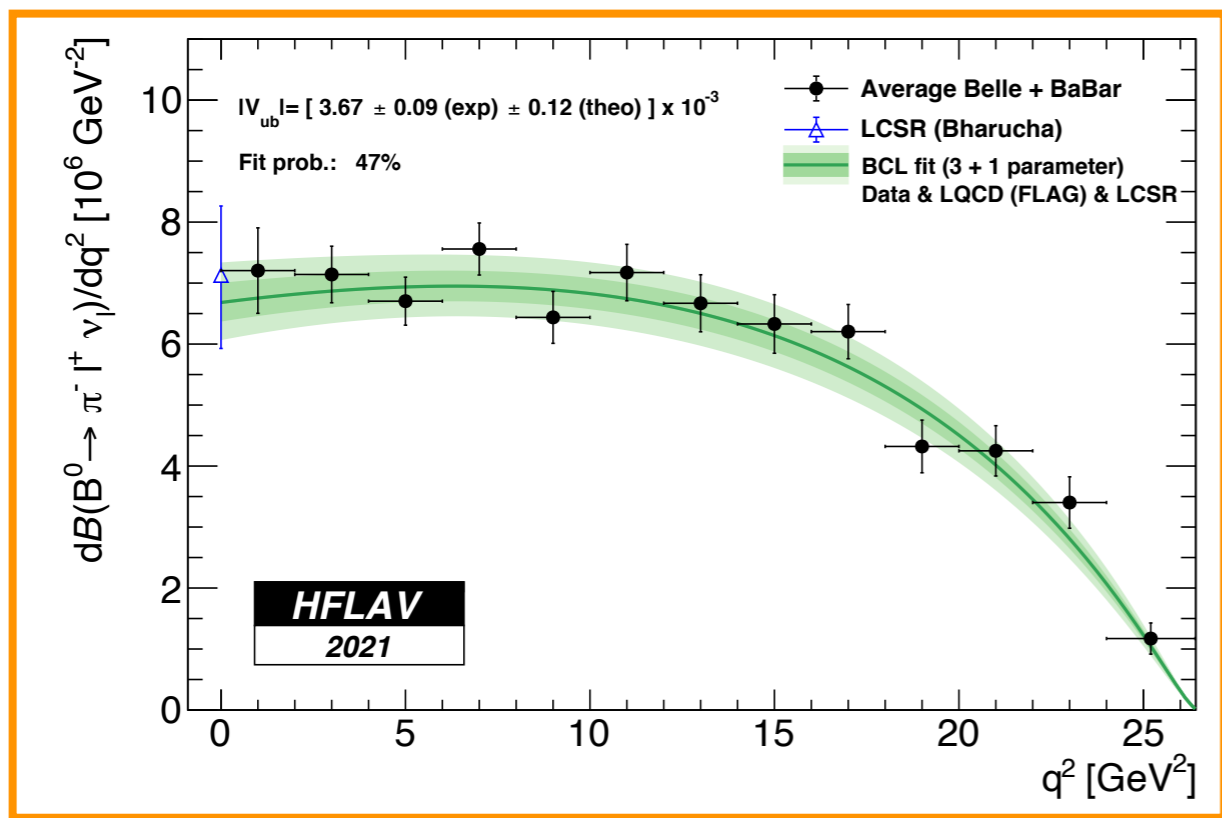




Now also available for  $B \rightarrow \rho/\omega \ell \bar{\nu}_\ell$ :

Plan to release public code for all of these



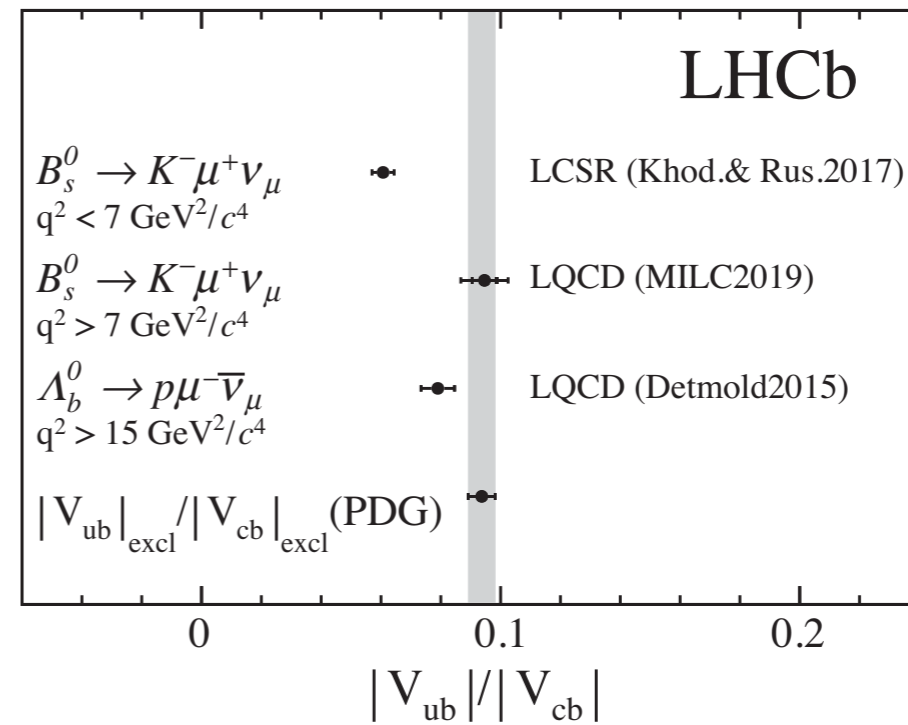
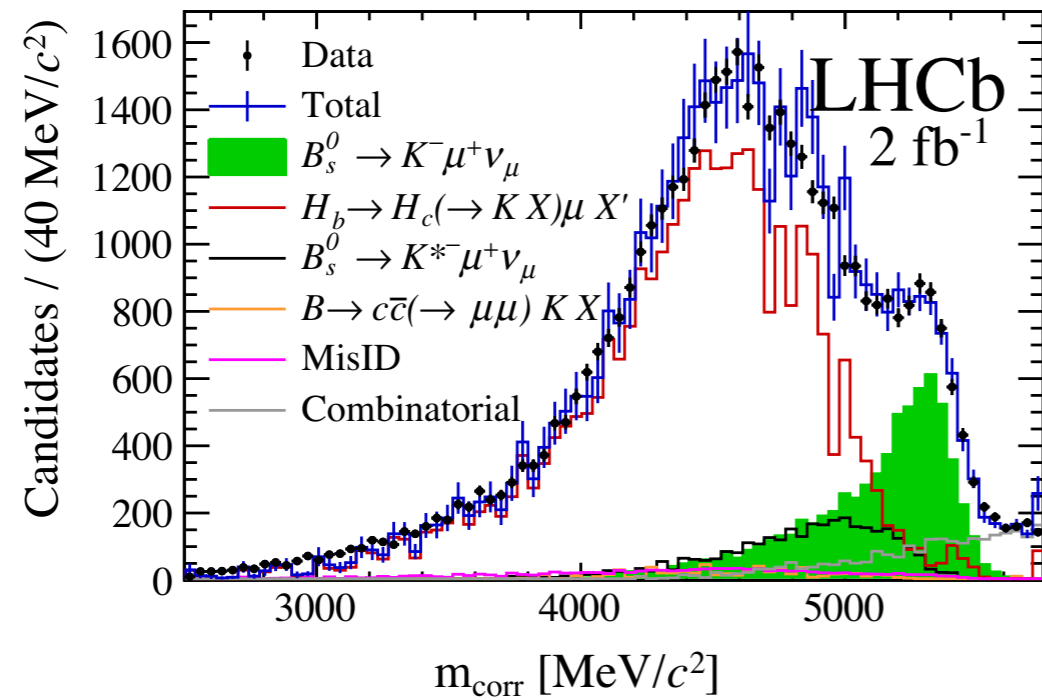


$\rho/\omega$  prefer much lower values of  $|V_{ub}|$  ...

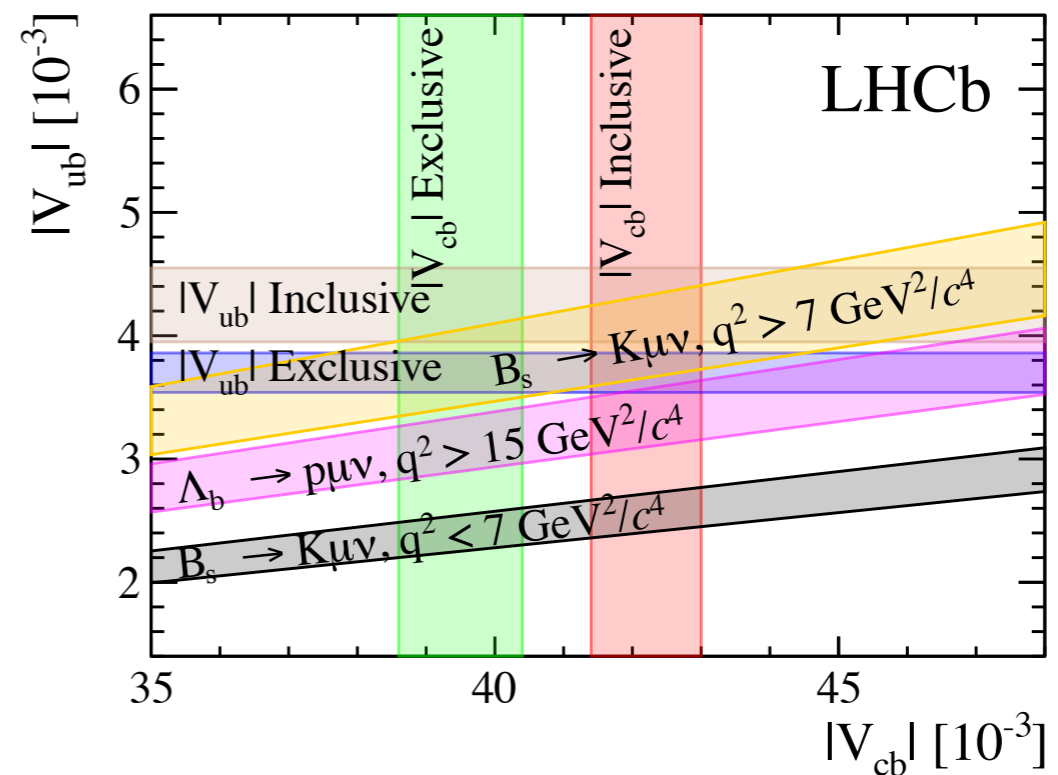
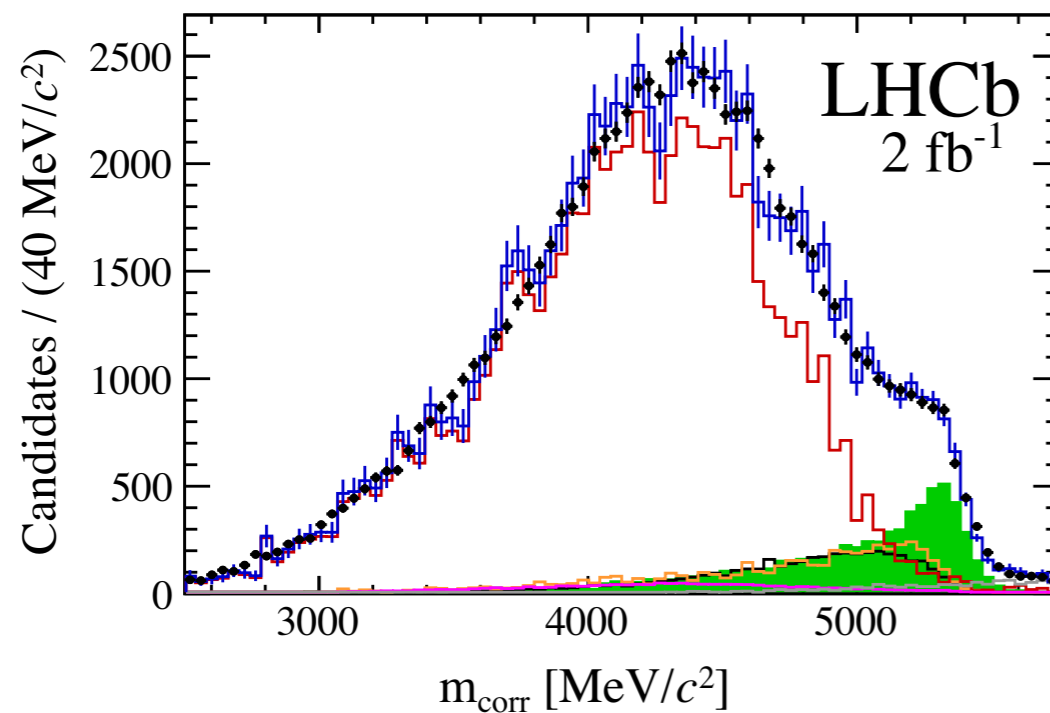
# Again: Great results from LHCb

Phys. Rev. Lett. 126, 081804 (2021)

$$q^2 < 7 \text{ GeV}^2$$



$$q^2 > 7 \text{ GeV}^2$$





# Quo vadis?

- ▶ Where should and where can we go?

# Some closing thoughts

---

Number of exciting developments are happening:

**Belle II** has many exciting results in the making (stay tuned for Spring / Summer 2022)

→ Will revisit all inclusive and exclusive results of the B-factory era

We need to start an era  
of more accessible data

**LHCb** has evolved into a semileptonic results machine

→ Adds new perspectives and results from baryons, maybe also inclusive determinations?

**Lattice QCD** made impressive leaps forward

→ Some are puzzling and need to be understood; lattice role in understanding inclusive decays has just started and is exciting

**LCSR** for many decays available

→ Interesting orthogonal information for phase-space regions plus only input for decays for which we have no lattice information



# Some closing thoughts

Number of exciting developments are happening:

**Belle II** has many exciting results in the making (stay tuned for Spring / Summer 2022)

→ Will revisit all inclusive and exclusive results of the B-factory era

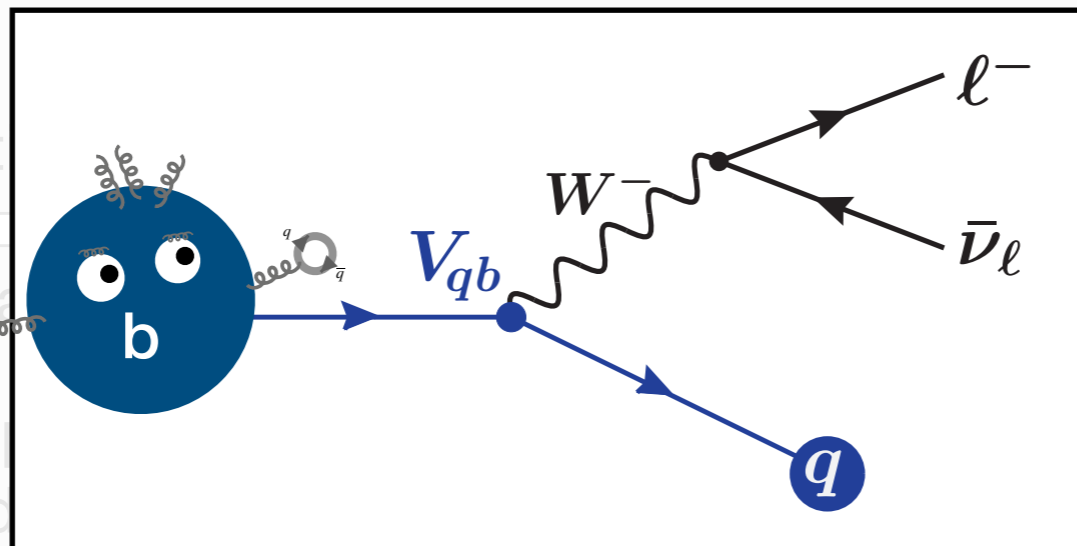
He may look cute, but that  
might be deceiving...

**LHCb** has evolved into a high precision results machine

→ Adds new determinations, maybe also inclusive

**Lattice QCD** makes

→ Some are understanding the lattice role in and is exciting



**LCSR** for many decays available

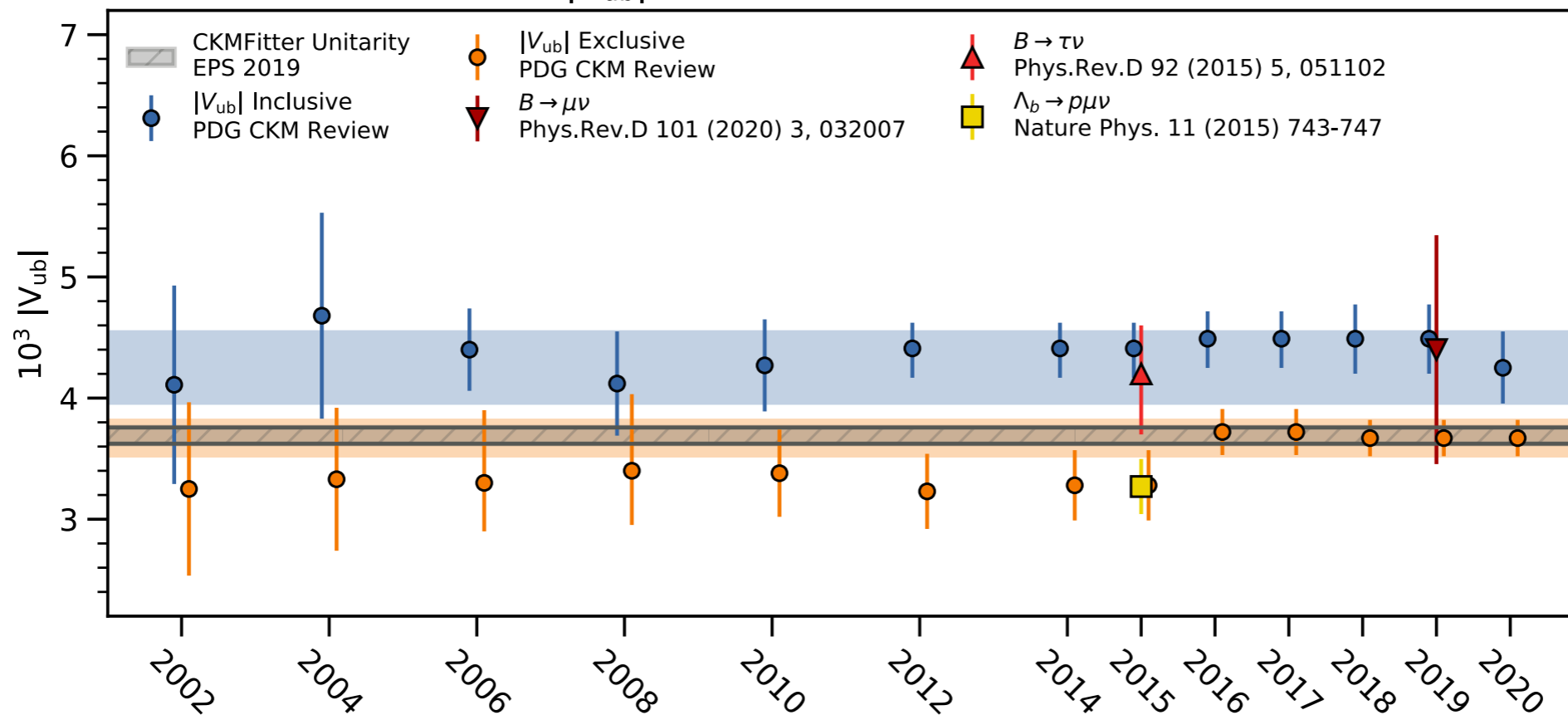
... but we are hot on his trail

→ Interesting orthogonal information for phase-space regions plus only input for decays for which we have no lattice information

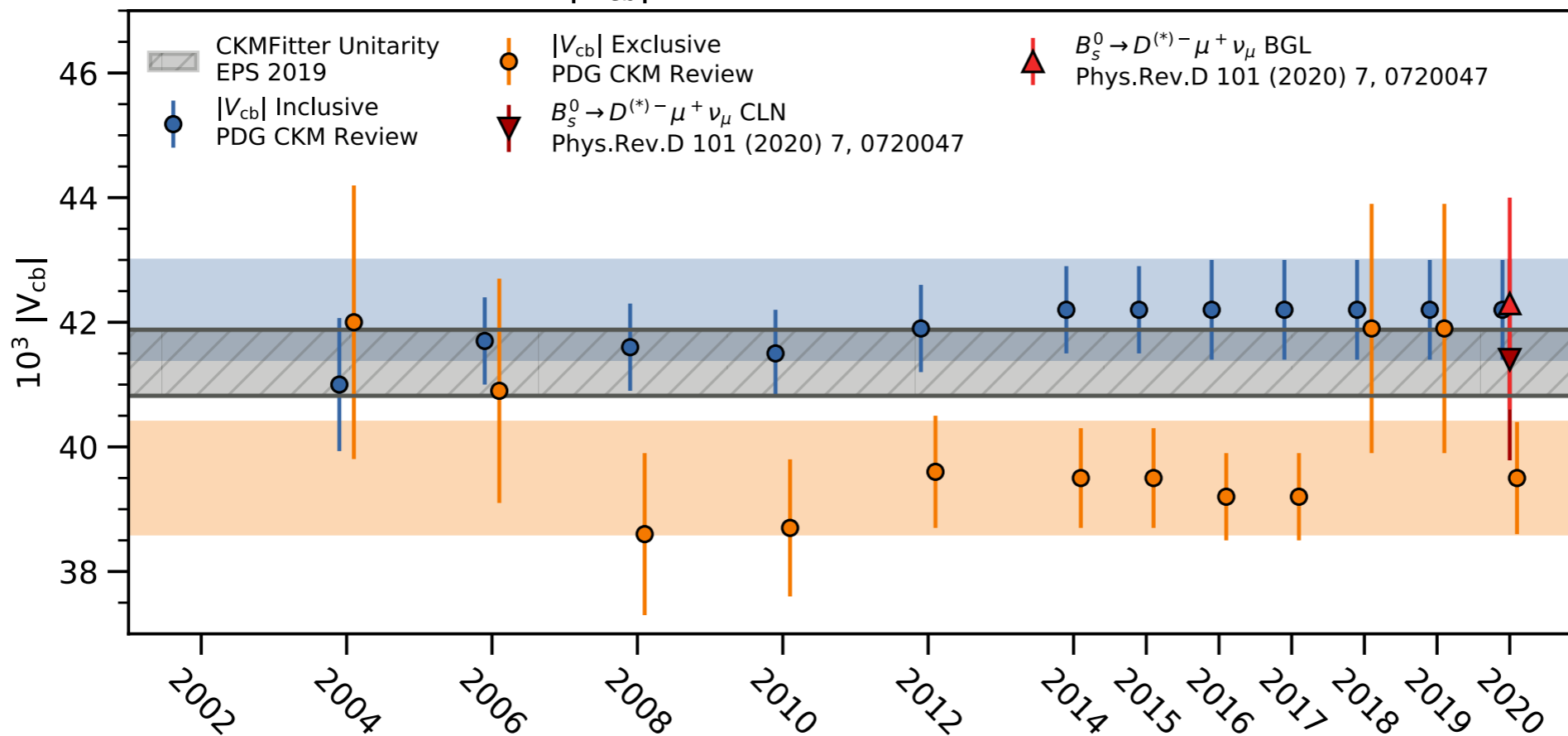


# More Material

### $|V_{ub}|$ Measurements over Time



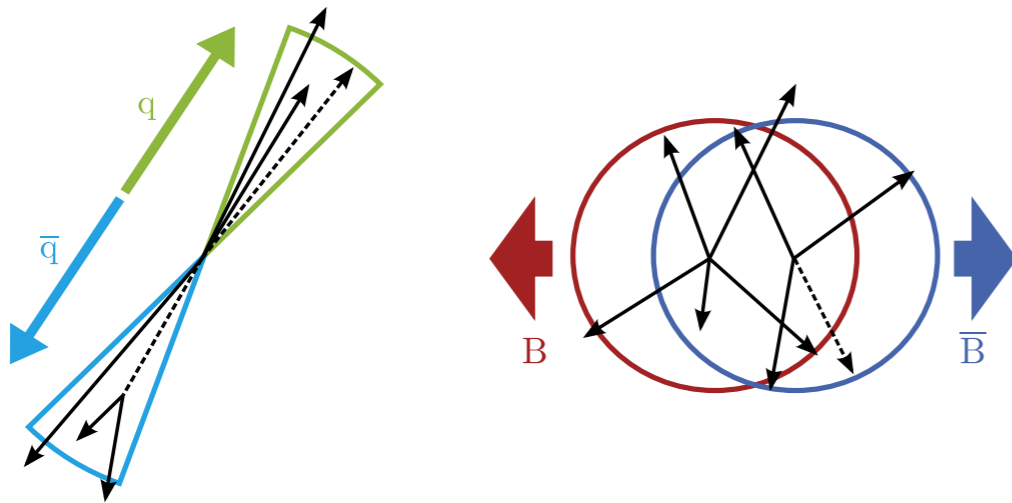
### $|V_{cb}|$ Measurements over Time



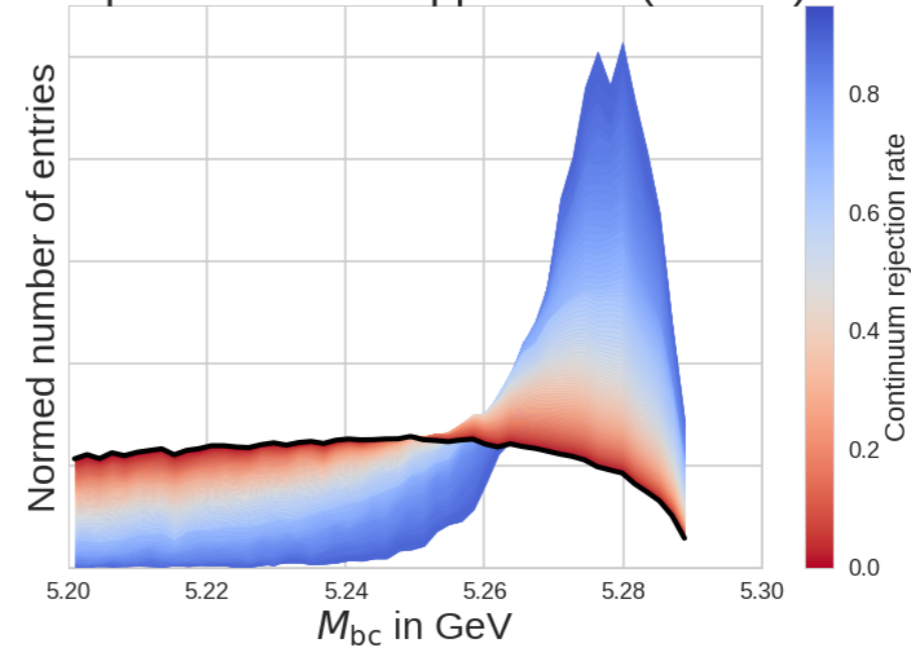
# Future directions:

$$M_{bc} = \sqrt{E_{\text{beam}}^2 - p_B^2}$$

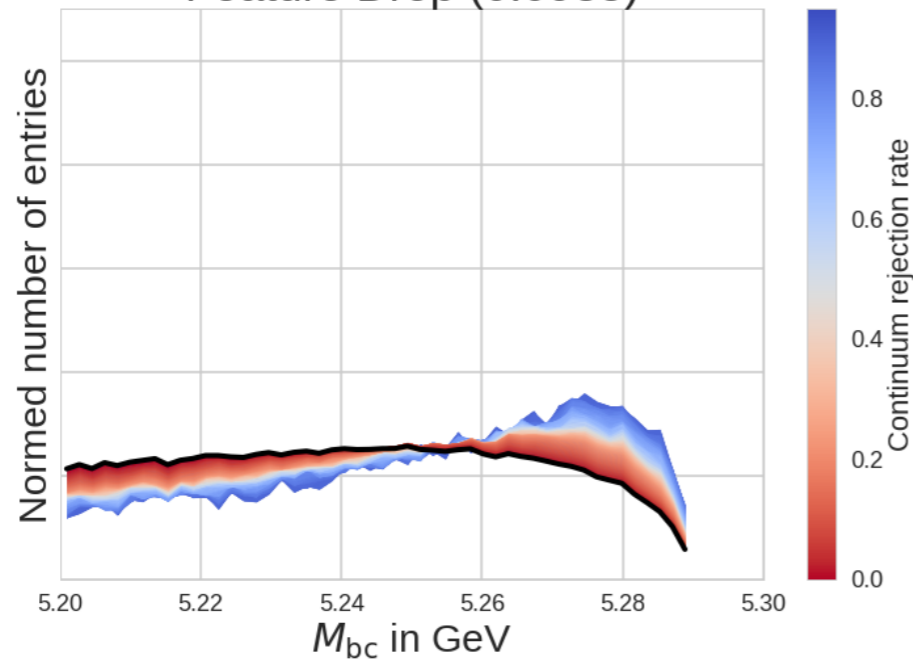
## Avoid Efficiency shaping



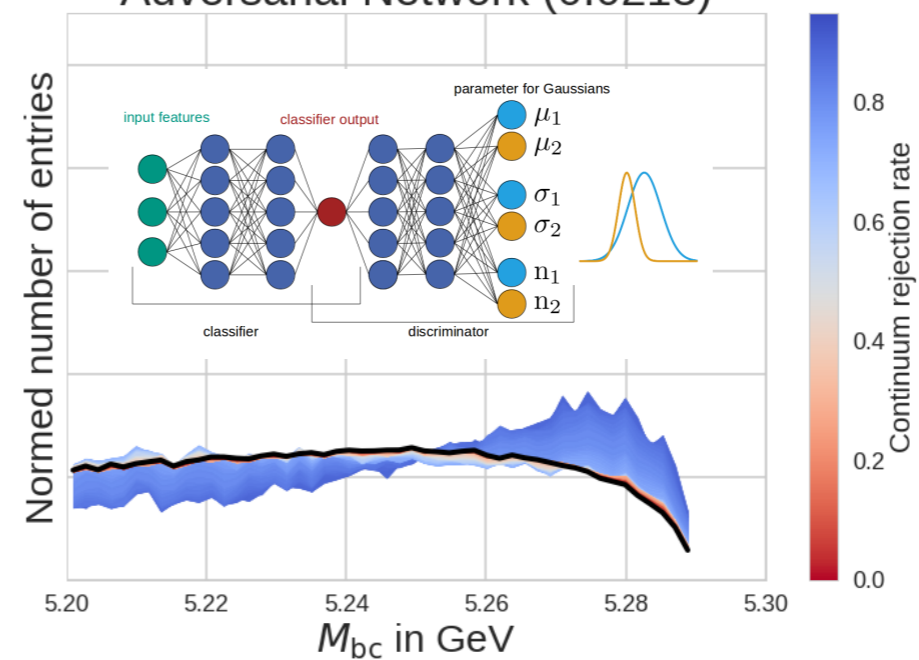
## Deep Continuum Suppression (0.2013)



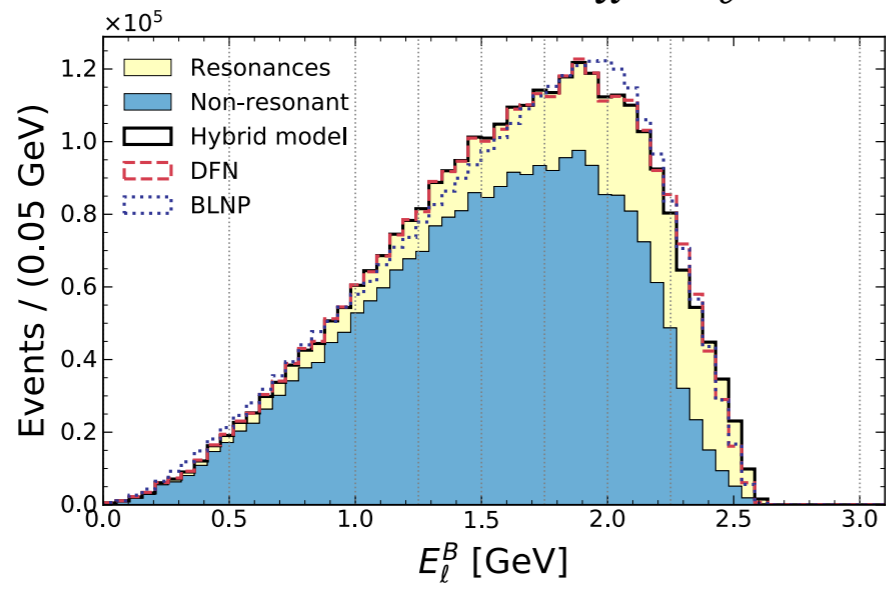
## Feature Drop (0.0935)



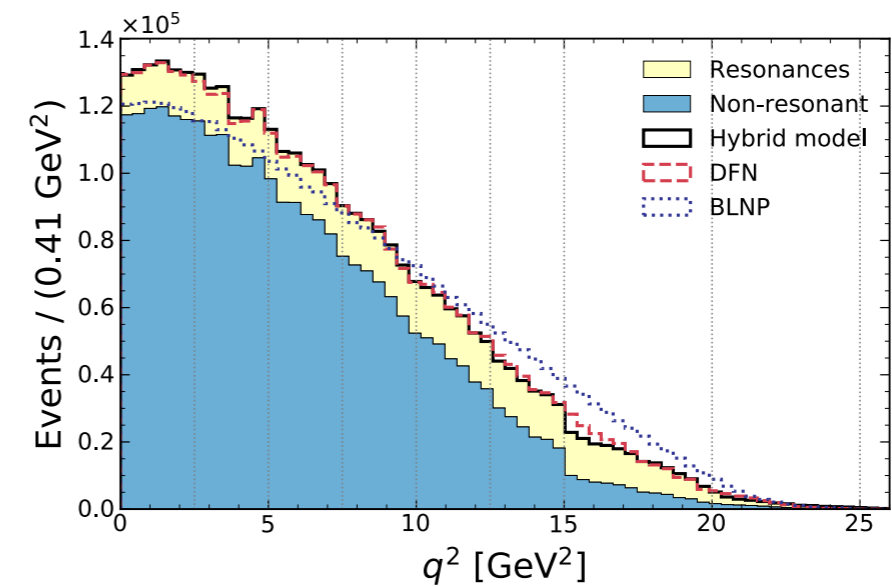
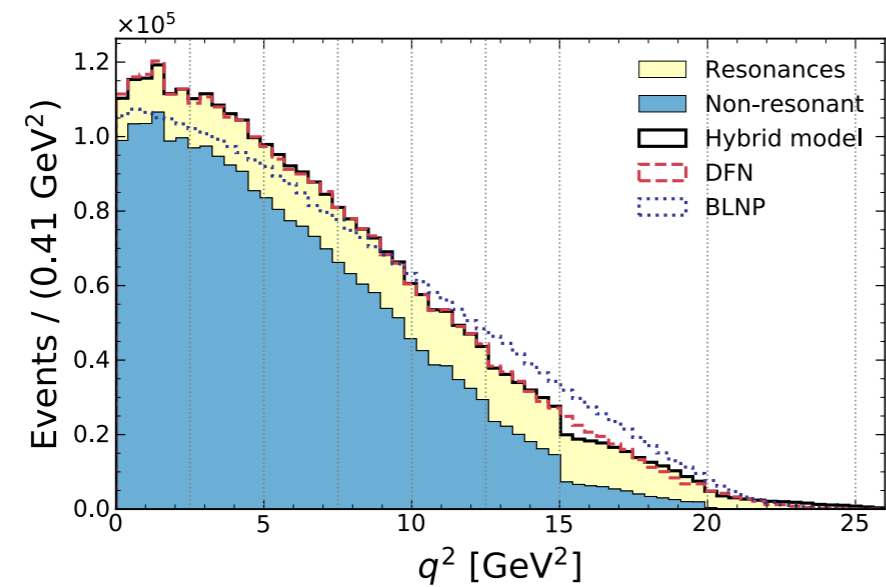
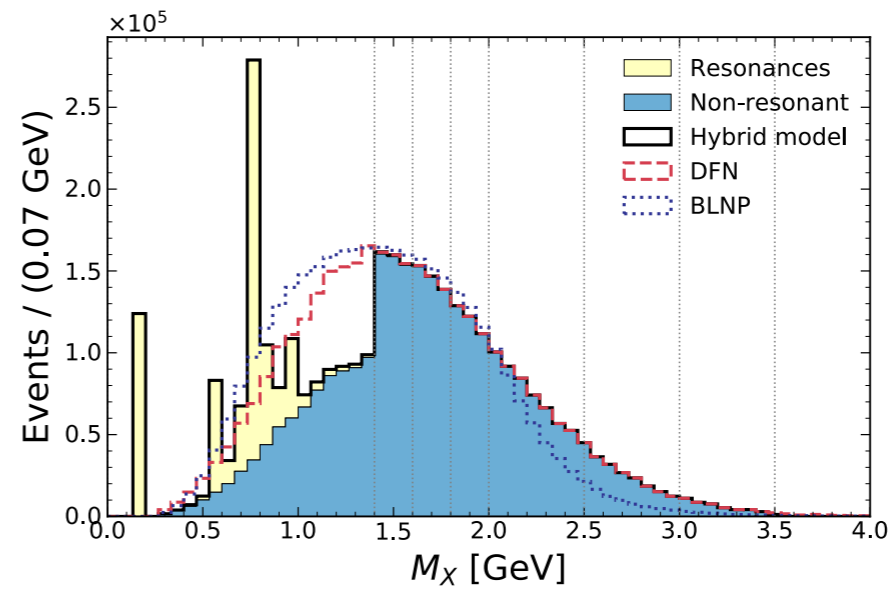
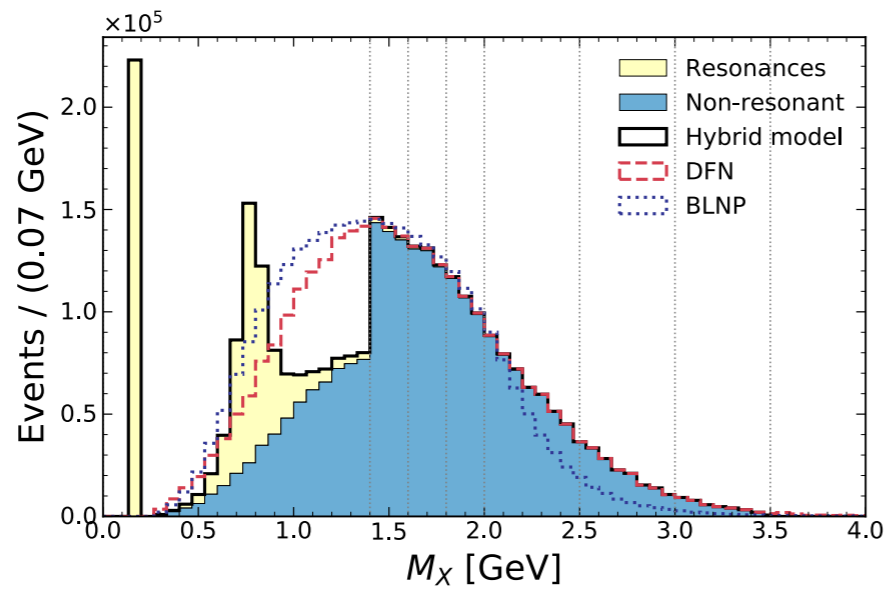
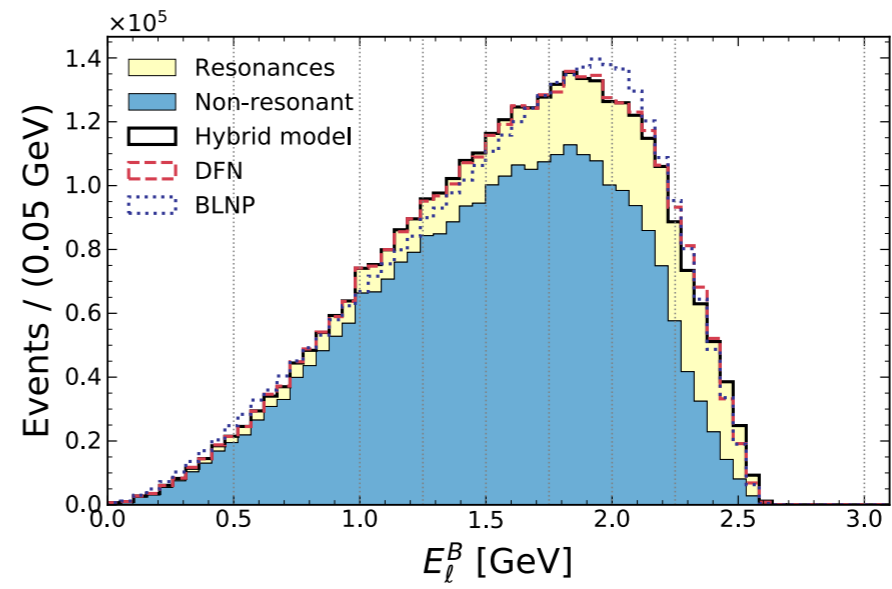
## Adversarial Network (0.0213)



$$B^0 \rightarrow X_u \ell \bar{\nu}_\ell$$



$$B^+ \rightarrow X_u \ell \bar{\nu}_\ell$$



# $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$ modelling

- **Update** excl. branching ratios to PDG 2020 and the masses and widths of  $D^{**}$  decays
- **Generate** additional MC samples to fill the **gap** between the exclusive & inclusive measurement (assign 100% BR uncertainty in systematics covariance matrix)

BR		$B^+$	$B^0$
$B \rightarrow X_c \ell^+ \nu_\ell$			
$B \rightarrow D \ell^+ \nu_\ell$	<b>D, D*</b>	$(2.5 \pm 0.1) \times 10^{-2}$	$(2.3 \pm 0.1) \times 10^{-2}$
$B \rightarrow D^* \ell^+ \nu_\ell$		$(5.4 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$		$(0.420 \pm 0.075) \times 10^{-2}$	$(0.390 \pm 0.069) \times 10^{-2}$
$(\leftrightarrow D\pi)$			
$B \rightarrow D_1^* \ell^+ \nu_\ell$		$(0.423 \pm 0.083) \times 10^{-2}$	$(0.394 \pm 0.077) \times 10^{-2}$
$(\leftrightarrow D^*\pi)$			
$B \rightarrow D_1 \ell^+ \nu_\ell$	<b>D**</b>	$(0.422 \pm 0.027) \times 10^{-2}$	$(0.392 \pm 0.025) \times 10^{-2}$
$(\leftrightarrow D^*\pi)$			
$B \rightarrow D_2^* \ell^+ \nu_\ell$		$(0.116 \pm 0.011) \times 10^{-2}$	$(0.107 \pm 0.010) \times 10^{-2}$
$(\leftrightarrow D^*\pi)$			
$B \rightarrow D_2^* \ell^+ \nu_\ell$		$(0.178 \pm 0.024) \times 10^{-2}$	$(0.165 \pm 0.022) \times 10^{-2}$
$(\leftrightarrow D\pi)$			
$\rho(D_2^* \rightarrow D^*\pi, D_2^* \rightarrow D\pi) = 0.693$			
$B \rightarrow D_1 \ell^+ \nu_\ell$	<b>Gap</b>	$(0.242 \pm 0.100) \times 10^{-2}$	$(0.225 \pm 0.093) \times 10^{-2}$
$(\leftrightarrow D\pi\pi)$			
$B \rightarrow D\pi\pi \ell^+ \nu_\ell$		$(0.06 \pm 0.06) \times 10^{-2}$	$(0.06 \pm 0.06) \times 10^{-2}$
$B \rightarrow D^*\pi\pi \ell^+ \nu_\ell$		$(0.216 \pm 0.102) \times 10^{-2}$	$(0.201 \pm 0.095) \times 10^{-2}$
$B \rightarrow D\eta \ell^+ \nu_\ell$		$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$B \rightarrow D^*\eta \ell^+ \nu_\ell$		$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$B \rightarrow X_c \ell^+ \nu_\ell$		$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$



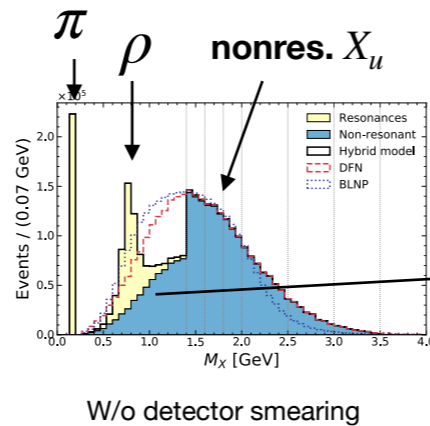
BR	$B^+$	$B^0$
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$(\leftrightarrow D\pi\pi)$		
$B \rightarrow D_1^* \ell^+ \nu_\ell$	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$(\leftrightarrow D\pi\pi)$		
$B \rightarrow D_0^* \pi\pi \ell^+ \nu_\ell$	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$(\leftrightarrow D^*\pi\pi)$		
$B \rightarrow D_1^* \pi\pi \ell^+ \nu_\ell$	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$(\leftrightarrow D^*\pi\pi)$		
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$(\leftrightarrow D\eta)$		
$B \rightarrow D_1^* \ell^+ \nu_\ell$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$(\leftrightarrow D^*\eta)$		

# Fit for partial BFs

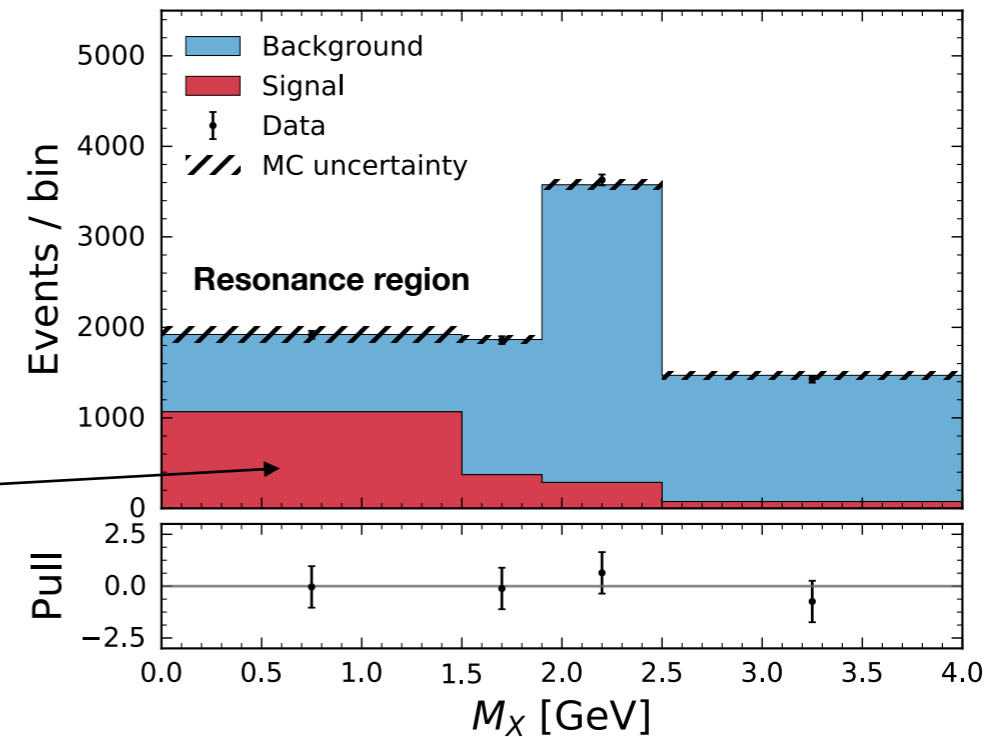
Subtraction of bkg in fit with coarse binning to minimize  $X_u$  modelling dependence  
(low  $m_X$ , high  $q^2$ )

$$\mathcal{L} = \prod_i^{\text{bins}} \mathcal{P}(n_i; \nu_i) \times \prod_k \mathcal{G}_k,$$

Signal and Bkg shape errors included in Fit via NPs



Projections of 2D fit in  $m_X : q^2$



Unfold measured yields to  
**3 phase-space** regions:

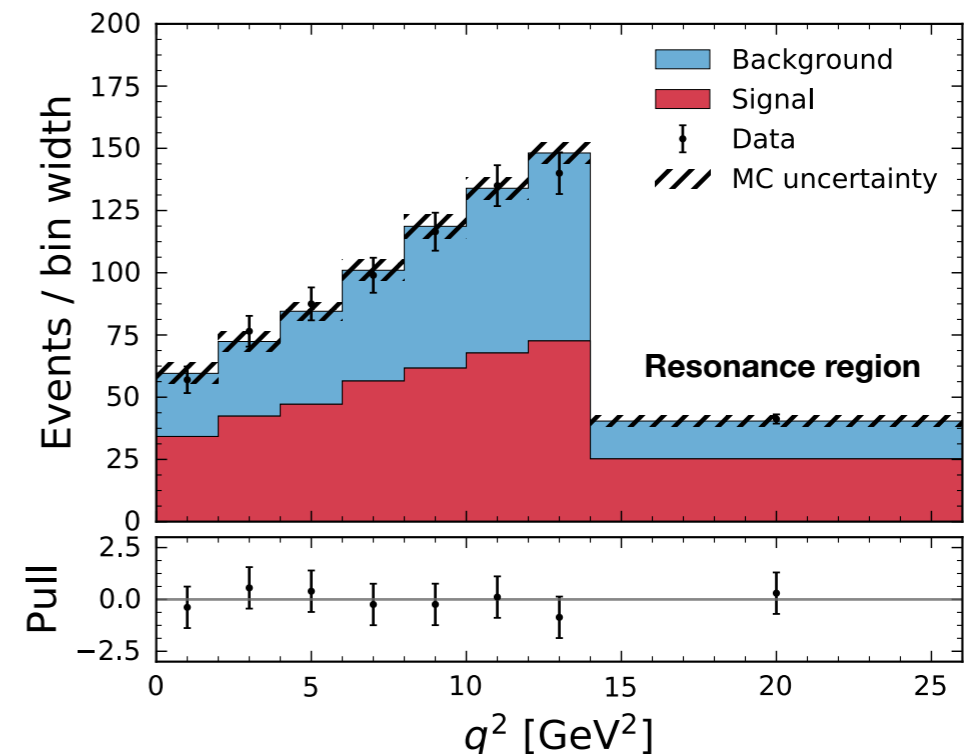
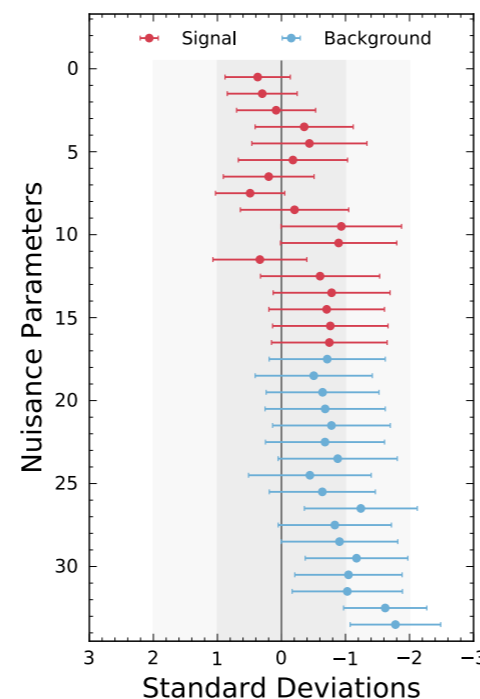
Phase-space region

$$M_X < 1.7 \text{ GeV}$$

$$M_X < 1.7 \text{ GeV}, q^2 > 8 \text{ GeV}^2$$

$$E_\ell^B > 1 \text{ GeV}$$

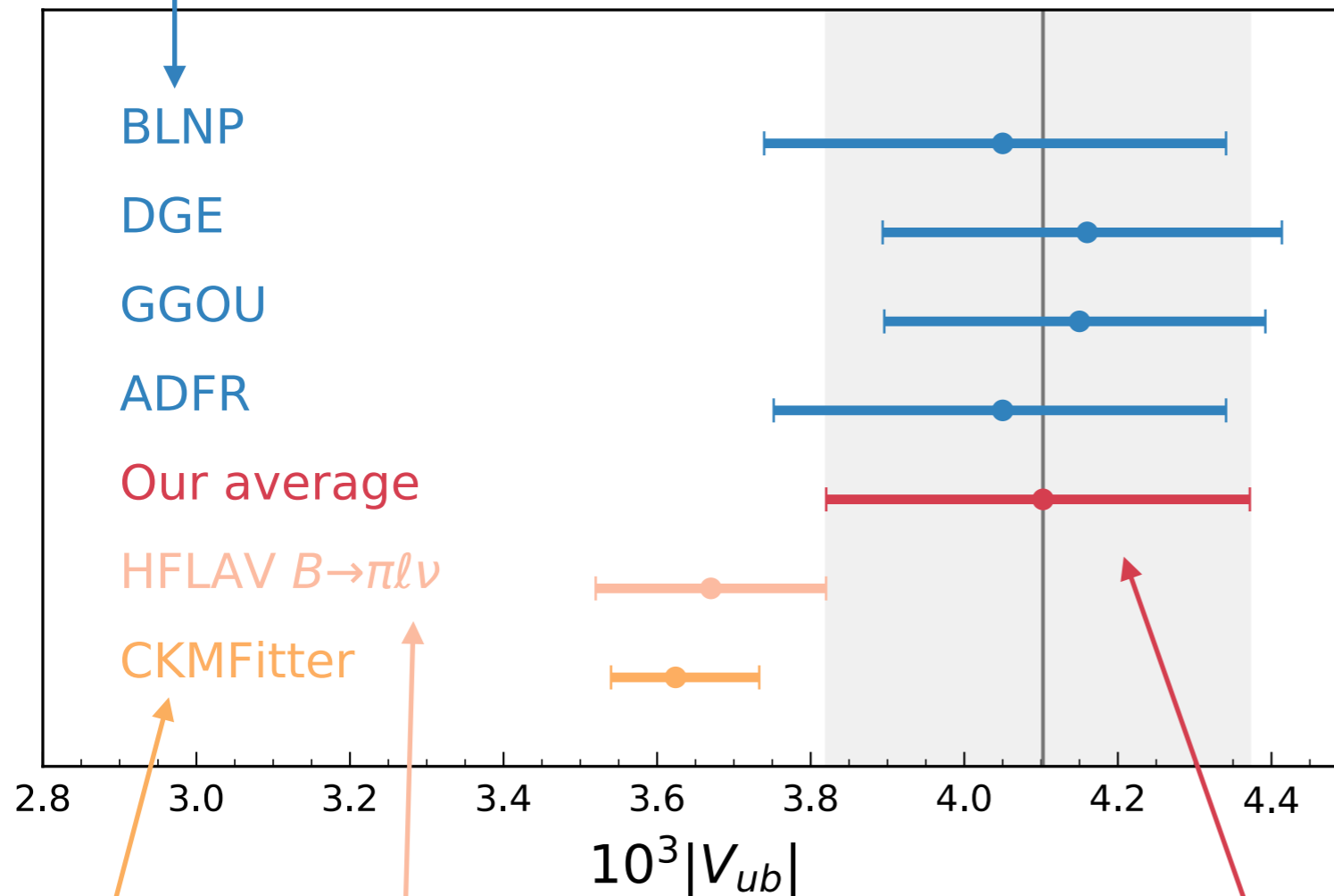
$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \rightarrow X_u \ell^+ \nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \rightarrow X_u \ell^+ \nu_\ell)}}$$



$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \rightarrow X_u \ell^+ \nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \rightarrow X_u \ell^+ \nu_\ell)}}$$

Fit kinematic distributions and measure **partial BF**

4 predictions of the partial rate



Exclusive Average for  $B \rightarrow \pi \ell \bar{\nu}_\ell$ :

$$|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$$

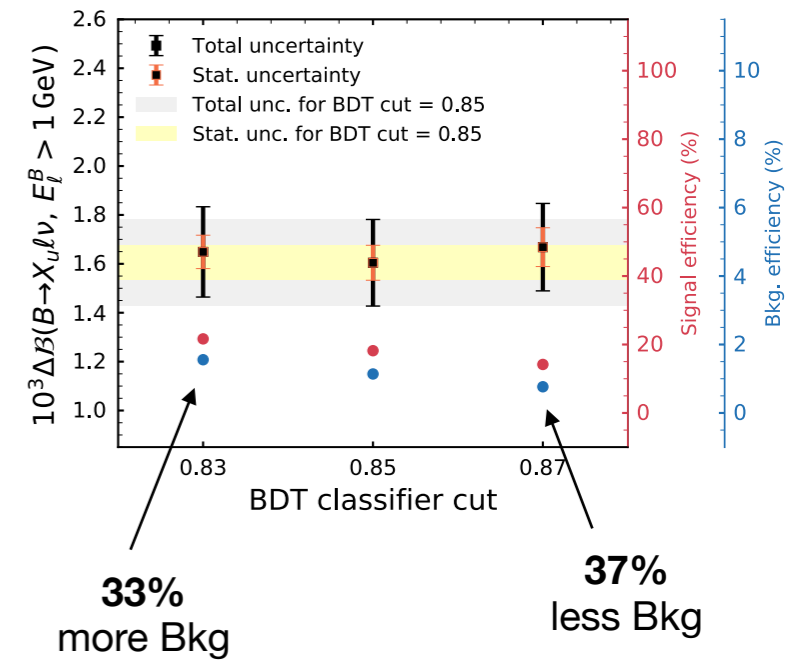
Arithmetic average:

$$|V_{ub}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$$

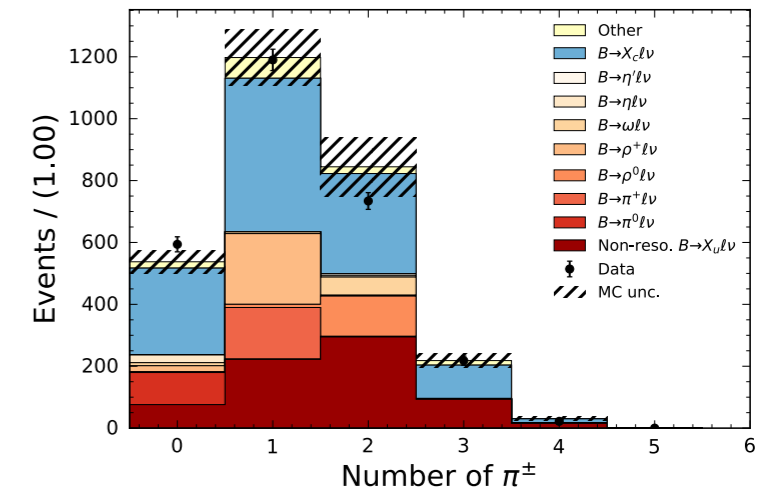
CKM Unitarity:

$$|V_{ub}| = (3.62^{+0.11}_{-0.08}) \times 10^{-3}$$

Stability as a function of BDT cut:



Post-fit  $N_{\pi^+}$  distribution:





# Into the tool shed: EvtGen & Pythia8

Many analyses need generic B-Meson decay samples

\* **Pythia8** hadronized modes make up ca. **48%** (!) of all simulated decays

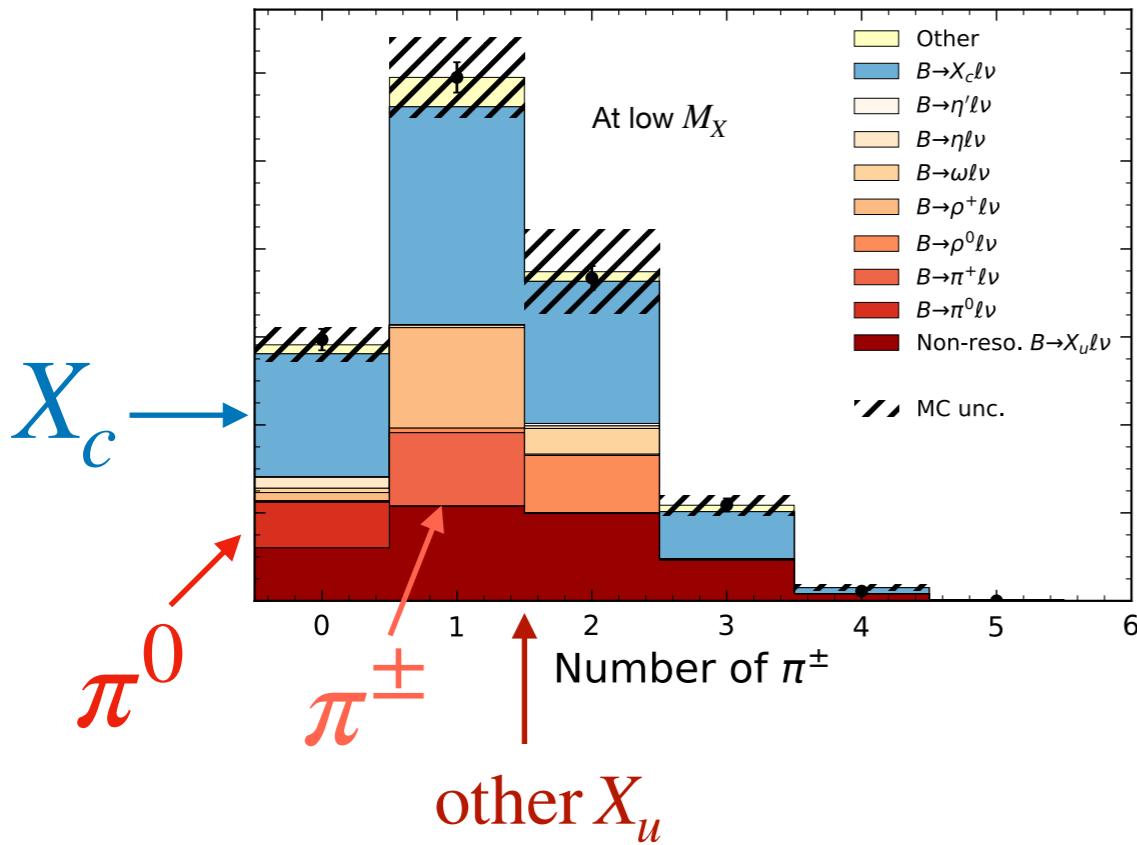
```
1594 # Lam_c X / Sigma_c X      4.0 %
1595 #
1596 0.010520663 anti-cd_0 ud_0      PYTHIA 23;
1597 0.021041421 anti-cd_1 ud_1      PYTHIA 23;
1598
1599 # Xi_c X      2.5%
1600 #
1601 0.002869298 anti-cs_0 ud_0      PYTHIA 23;
1602 0.005738595 anti-cs_1 ud_1      PYTHIA 23;
1603
1604 0.258091538 u      anti-d anti-c d      PYTHIA 48;
1605 0.043995612 u      anti-d anti-c d      PYTHIA 13;
1606 0.020084989 u      anti-s anti-c d      PYTHIA 13;
1607 0.017215691 u      anti-c anti-d d      PYTHIA 48;
1608 0.000860770 u      anti-c anti-s d      PYTHIA 48;
1609 #lange - try to crank up the psi production....
1610 0.070775534 c      anti-s anti-c d      PYTHIA 13;
1611 0.005738595 c      anti-d anti-c d      PYTHIA 13;
1612 0.002869298 u      anti-d anti-u d      PYTHIA 48;
1613 0.003825730 c      anti-s anti-u d      PYTHIA 48;
1614 # JGS 11/5/02 This and similar a few lines above have been divided by two
1615 # to solve a double-counting problem for this channel
1616 0.001960649 u      anti-u anti-d d      PYTHIA 48;
1617 0.000066973 d      anti-d anti-d d      PYTHIA 48;
1618 0.000086068 s      anti-s anti-d d      PYTHIA 48;
1619 0.002104095 u      anti-u anti-s d      PYTHIA 48;
1620 0.001721541 d      anti-d anti-s d      PYTHIA 48;
1621 0.001434649 s      anti-s anti-s d      PYTHIA 48;
1622 0.004782163 anti-s d      PYTHIA 32;
```

## Modes for Matrix Element Processing

Some decays can be treated better than what pure phase space allows, by reweighting with appropriate matrix element. This is signaled by a nonvanishing `meMode()` value for a decay mode in the `particle data table`. The list of allowed possibilities has been introduced, and most have been moved for better consistency. Here is the list of currently allowed `meMode()` codes:

- 0 : pure phase space of produced particles ("default"); input of partons is allowed and then the partonic content is hadronized
- 1 :  $\omega$  and  $\phi \rightarrow \pi^+ \pi^- \pi^0$
- 2 : polarization in  $V \rightarrow PS + PS$  ( $V$  = vector,  $PS$  = pseudoscalar), when  $V$  is produced by  $PS \rightarrow PS + V$  or  $F \rightarrow F + V$
- 11 : Dalitz decay into one particle, in addition to the lepton pair (also allowed to specify a quark-antiquark pair)
- 12 : Dalitz decay into two or more particles in addition to the lepton pair
- 13 : double Dalitz decay into two lepton pairs
- 21 : decay to phase space, but weight up `neutrino_tau` spectrum in `tau` decay
- 22 : weak decay; if there is a quark spectator system it collapses to one hadron; for leptonic/semileptonic decays
- 23 : as 22, but require at least three particles in decay
- 31 : decays of type  $B \rightarrow \gamma X$ , very primitive simulation where  $X$  is given in terms of its flavour content and the `gamma` spectrum is weighted up relative to pure phase space
- 42 - 50 : turn partons into a random number of hadrons, picked according to a Poissonian with average value `code`; new try with another multiplicity if the sum of daughter masses exceed the mother one
- 52 - 60 : as 42 - 50, with multiplicity between `code` - 50 and 10, but avoid already explicitly listed non-partonic channels
- 62 - 70 : as 42 - 50, but fixed multiplicity `code` - 60
- 72 - 80 : as 42 - 50, but fixed multiplicity `code` - 70, and avoid already explicitly listed non-partonic channels
- 91 : decay to  $q \bar{q}$  or  $g g$ , which should shower and hadronize
- 92 : decay onium to  $g g g$  or  $g g \gamma$  (with matrix element), which should shower and hadronize
- 93 : decay of colour singlet to  $q \bar{q}$  plus another singlet, flat in phase space (and arbitrarily ordered), where  $q$  is a quark
- 94 : same as 93, but weighted with  $V-A$  weak matrix element if the decay chain is of the type `neutrino larr;`
- 100 - : reserved for the description of partial widths of `resonances`

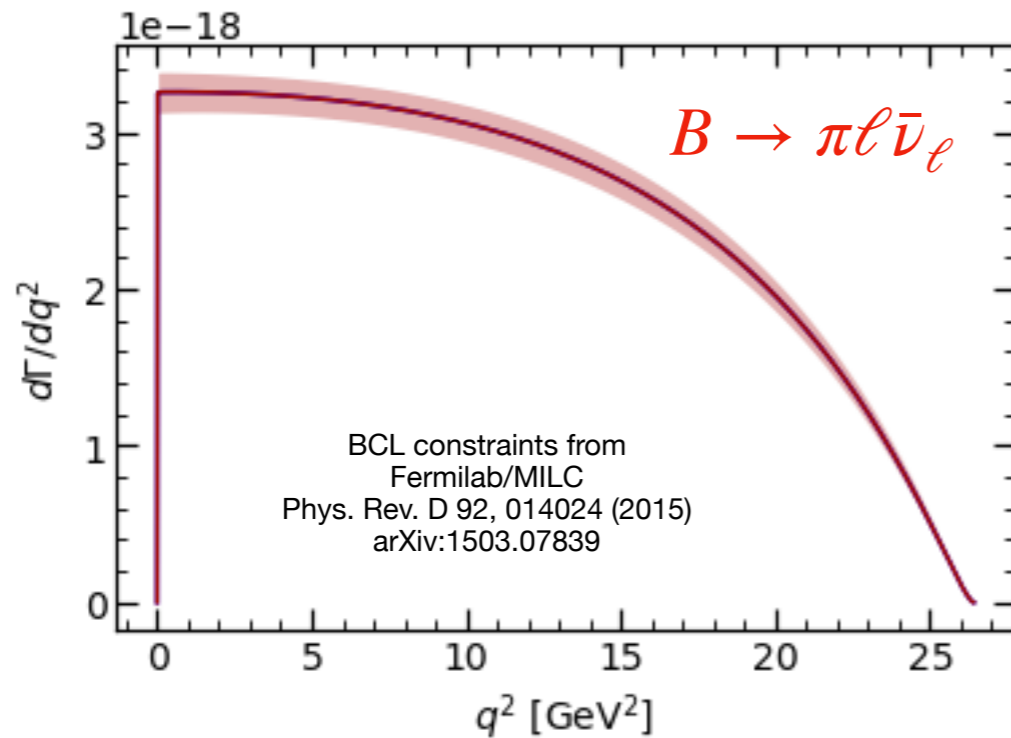
# Can we measure incl. and excl. $|V_{ub}|$ at the same time?



$$\Delta \mathcal{B}(B \rightarrow X_u \ell \bar{\nu}_\ell) = \Delta \mathcal{B}(B \rightarrow \pi \ell \bar{\nu}_\ell) + \Delta \mathcal{B}(B \rightarrow X_u^{\text{other}} \ell \bar{\nu}_\ell)$$

$$\mathcal{B}(B \rightarrow \pi \ell \bar{\nu}_\ell)$$

$$\frac{|V_{ub}^{\text{excl.}}|}{|V_{ub}^{\text{incl.}}|} = \frac{\sqrt{\frac{\mathcal{B}(B \rightarrow \pi \ell \bar{\nu}_\ell)}{\Gamma(B \rightarrow \pi \ell \bar{\nu}_\ell)}}}{\sqrt{\frac{\Delta \mathcal{B}(B \rightarrow X_u \ell \bar{\nu}_\ell)}{\Delta \Gamma(B \rightarrow X_u \ell \bar{\nu}_\ell)}}$$



Use  $q^2 : N_{\pi^\pm}$  to separate

$$B \rightarrow \pi^0 \ell \bar{\nu}_\ell$$

$$B \rightarrow \pi^\pm \ell \bar{\nu}_\ell$$

$$B \rightarrow X_u^{\text{other}} \ell \bar{\nu}_\ell$$

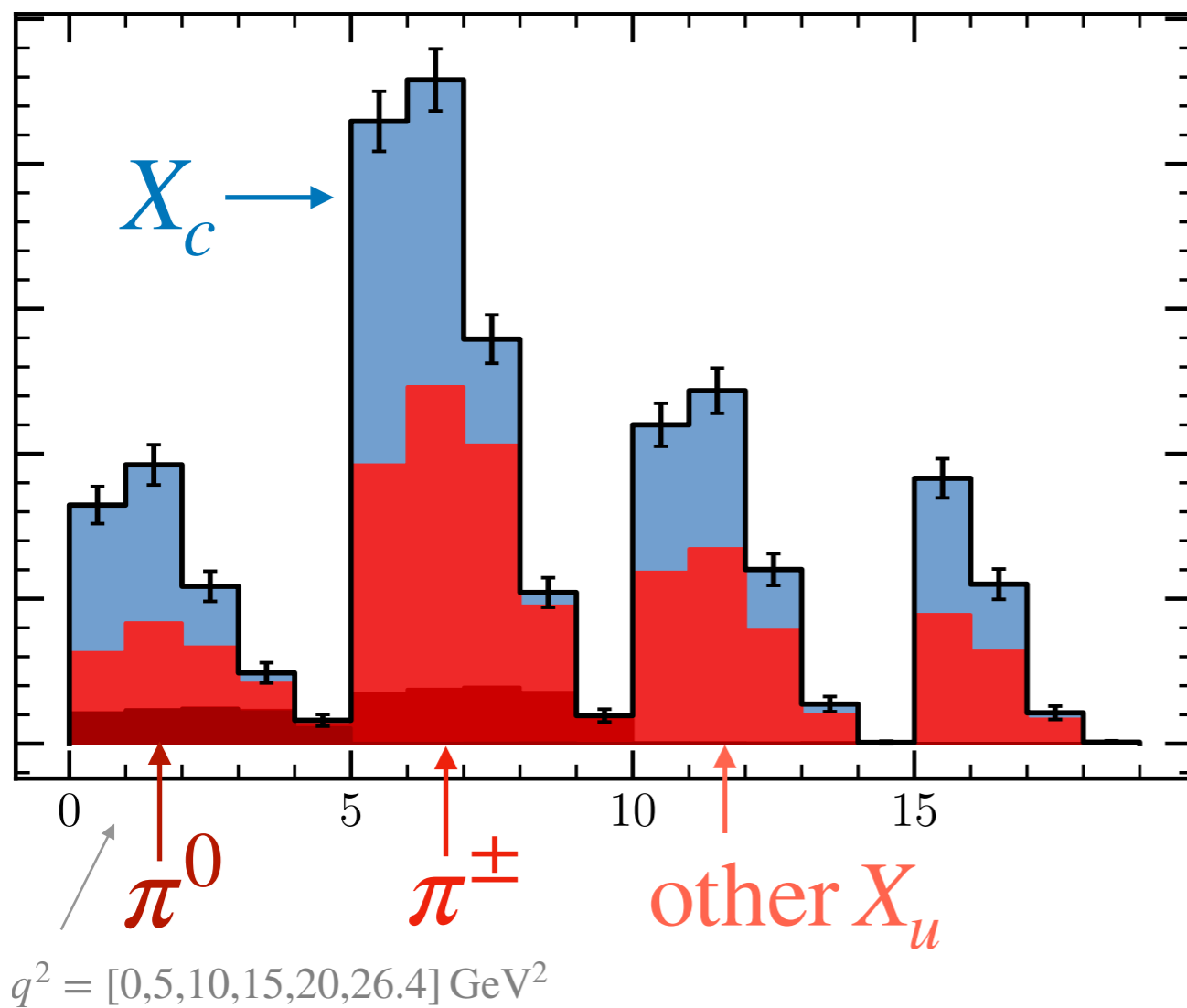
$$B \rightarrow X_c \ell \bar{\nu}_\ell + \text{other Bkg.}$$

# Asimov Fit

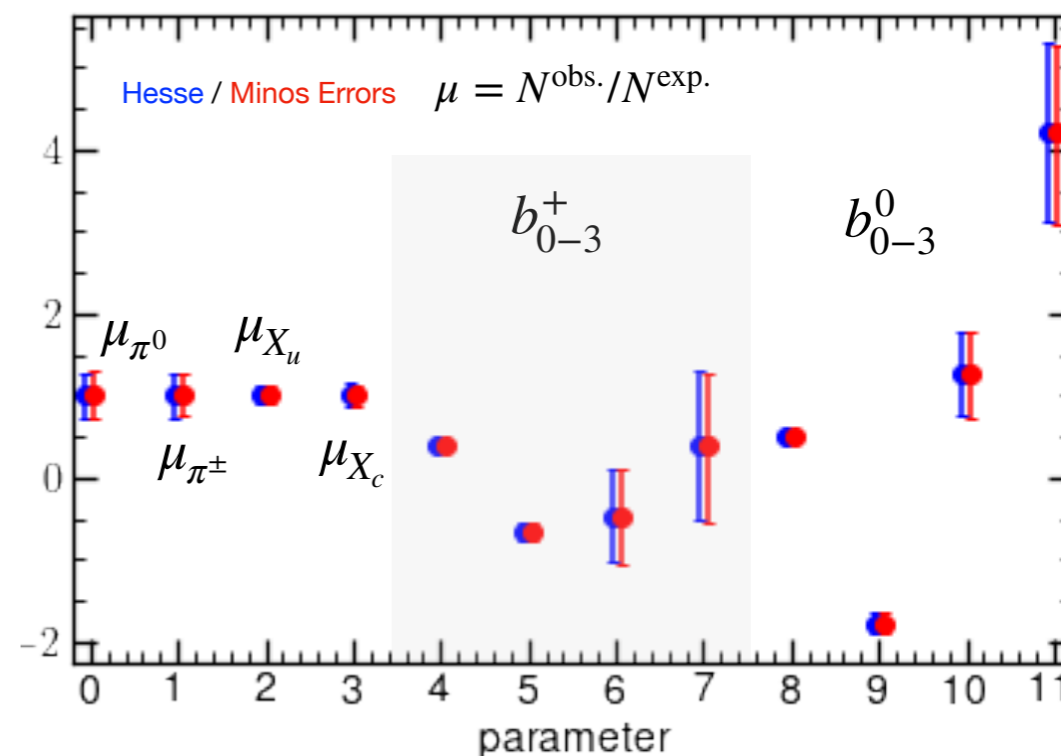
$$\mu_{\pi^0} = N_{\pi^0}^{\text{obs}} / N_{\pi^0}^{\text{MC}}$$

Fermilab/MILC  
Phys. Rev. D 92, 014024 (2015)  
arXiv:1503.07839

$N_{\pi^\pm} = 0$     $N_{\pi^\pm} = 1$     $N_{\pi^\pm} = 2$     $N_{\pi^\pm} \geq 3$



Fit Setup:  $\chi^2 = \chi_{\text{exp}}^2 + \chi_{\text{FNAL}}^2$



$\mu_{\pi^0}$	$\mu_{\pi^\pm}$	$\mu_{X_u}$	$\mu_{X_c}$	$b_{0-3}^+$			$b_{0-3}^0$					
1.00	0.51	0.39	-0.61	0.13	0.55	0.63	0.57	0.23	0.54	0.64	0.43	
0.51	1.00	0.10	-0.40	0.17	0.58	0.58	0.51	0.22	0.52	0.62	0.41	
0.39	0.10	1.00	-0.89	0.03	0.35	0.53	0.49	0.18	0.40	0.50	0.34	
-0.61	-0.40	-0.89	1.00	-0.09	-0.54	-0.70	-0.65	-0.25	-0.57	-0.69	-0.46	
0.13	0.17	0.03	-0.09	1.00	0.35	-0.10	-0.16	0.24	0.20	0.10	0.04	
0.55	0.58	0.35	-0.54	0.35	1.00	0.44	0.29	0.21	0.60	0.65	0.44	
0.63	0.58	0.53	-0.70	-0.10	0.44	1.00	0.97	0.29	0.66	0.87	0.62	
0.57	0.51	0.49	-0.65	-0.16	0.29	0.97	1.00	0.30	0.65	0.80	0.47	
0.23	0.22	0.18	-0.25	0.24	0.21	0.29	0.30	1.00	0.24	0.07	-0.10	
0.54	0.52	0.40	-0.57	0.20	0.60	0.66	0.65	0.24	1.00	0.60	0.08	
0.64	0.62	0.50	-0.69	0.10	0.65	0.87	0.80	0.07	0.60	1.00	0.70	
0.43	0.41	0.34	-0.46	0.04	0.44	0.62	0.47	-0.10	0.08	0.70	1.00	

Individual components seem to separate well in Asimov with made-up (but semi-realistic) distributions

# Combined Extractions

Interesting if heavy quark symmetry inspired Form Factors are used:

$$\hat{h}(w) = h(w)/\xi(w) \quad \leftarrow \text{Leading Isgur-Wise function}$$

$B \rightarrow D \ell \bar{\nu}_\ell$   
 $B \rightarrow D^* \ell \bar{\nu}_\ell$

$$\hat{h}_+ = 1 + \hat{\alpha}_s \left[ C_{V_1} + \frac{w+1}{2} (C_{V_2} + C_{V_3}) \right] + (\varepsilon_c + \varepsilon_b) \hat{L}_1,$$

$$\hat{h}_- = \hat{\alpha}_s \frac{w+1}{2} (C_{V_2} - C_{V_3}) + (\varepsilon_c - \varepsilon_b) \hat{L}_4,$$

$$\hat{h}_V = 1 + \hat{\alpha}_s C_{V_1} + \varepsilon_c (\hat{L}_2 - \hat{L}_5) + \varepsilon_b (\hat{L}_1 - \hat{L}_4),$$

$$\hat{h}_{A_1} = 1 + \hat{\alpha}_s C_{A_1} + \varepsilon_c \left( \hat{L}_2 - \hat{L}_5 \frac{w-1}{w+1} \right) + \varepsilon_b \left( \hat{L}_1 - \hat{L}_4 \frac{w-1}{w+1} \right),$$

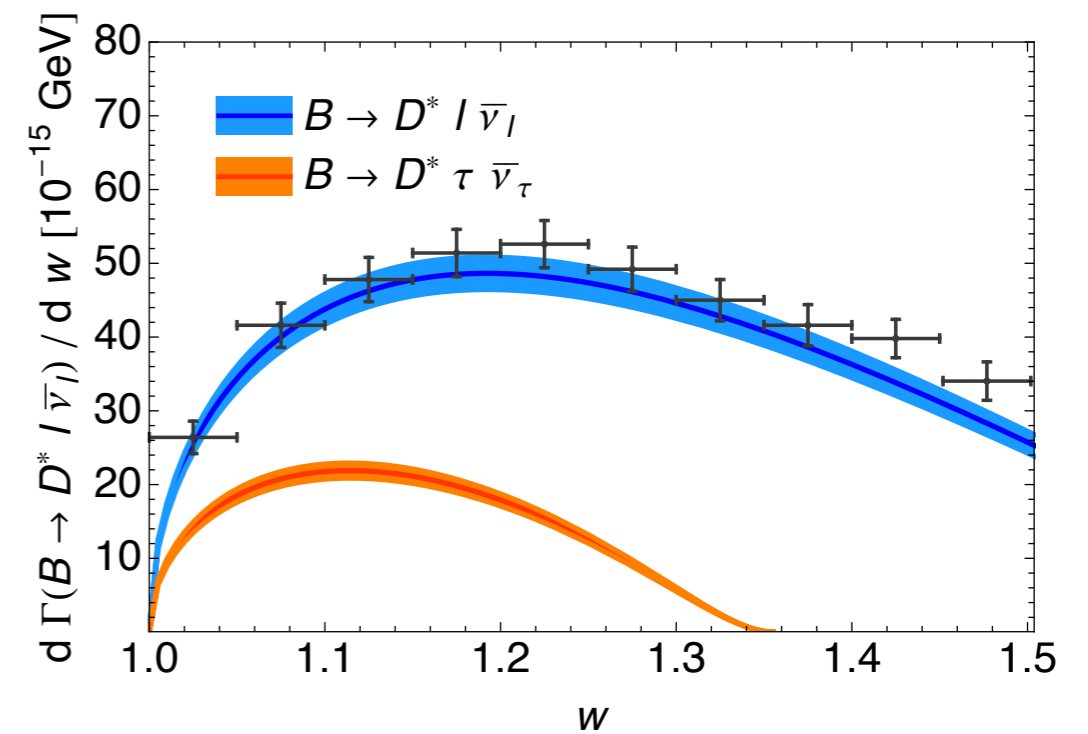
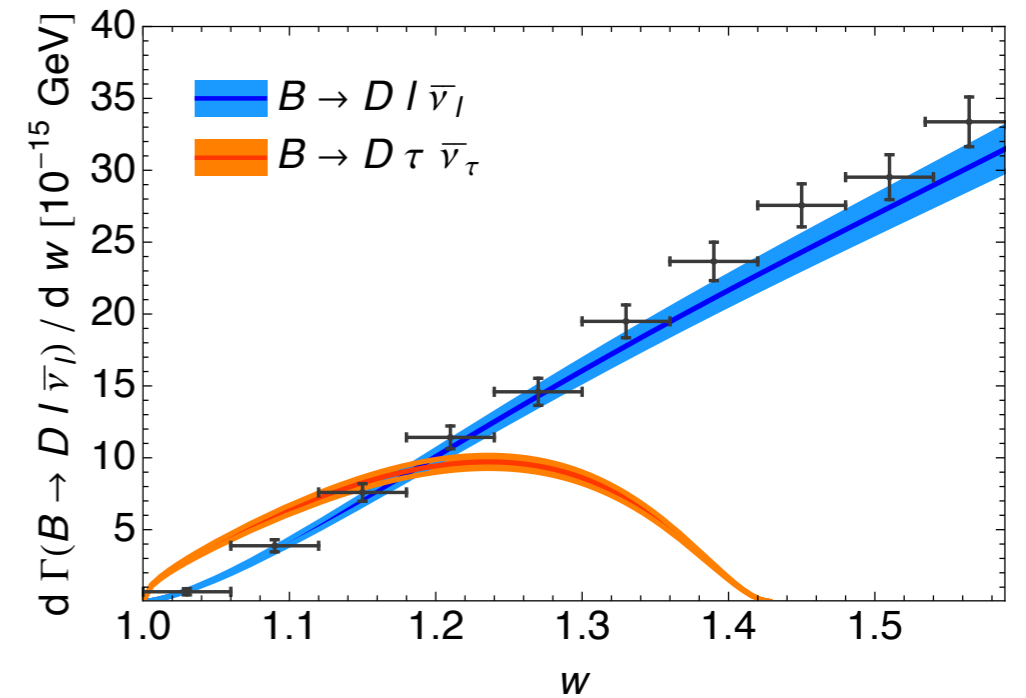
$$\hat{h}_{A_2} = \hat{\alpha}_s C_{A_2} + \varepsilon_c (\hat{L}_3 + \hat{L}_6),$$

$$\hat{h}_{A_3} = 1 + \hat{\alpha}_s (C_{A_1} + C_{A_3}) + \varepsilon_c (\hat{L}_2 - \hat{L}_3 + \hat{L}_6 - \hat{L}_5) + \varepsilon_b (\hat{L}_1 - \hat{L}_4),$$

This links dynamics of  
 $B \rightarrow D \ell \bar{\nu}_\ell$  &  $B \rightarrow D^* \ell \bar{\nu}_\ell$

Example fit for leading IW  
function and sub-leading  
parameters

$ V_{cb}  \times 10^3$	$38.8 \pm 1.2$
$\mathcal{G}(1)$	$1.055 \pm 0.008$
$\mathcal{F}(1)$	$0.904 \pm 0.012$
$\rho_*^2$	$1.17 \pm 0.12$
$\chi_2(1)$	$-0.26 \pm 0.26$
$\chi'_2(1)$	$0.21 \pm 0.38$
$\chi'_3(1)$	$0.02 \pm 0.07$
$\eta(1)$	$0.30 \pm 0.04$
$\eta'(1)$	0 (fixed)
$m_b^{1S}$ [GeV]	$4.70 \pm 0.05$
$\delta m_{bc}$ [GeV]	$3.40 \pm 0.02$



# LHCb Systematics

$$B_s \rightarrow K\mu\bar{\nu}_\mu$$

Uncertainty	All $q^2$	Low $q^2$	High $q^2$
Tracking	2.0	2.0	2.0
Trigger	1.4	1.2	1.6
Particle identification	1.0	1.0	1.0
$\sigma(m_{\text{corr}})$	0.5	0.5	0.5
Isolation	0.2	0.2	0.2
Charged BDT	0.6	0.6	0.6
Neutral BDT	1.1	1.1	1.1
$q^2$ migration	...	2.0	2.0
Efficiency	1.2	1.6	1.6
Fit template	+2.3 -2.9	+1.8 -2.4	+3.0 -3.4
Total	+4.0 -4.3	+4.3 -4.5	+5.0 -5.3

$$B_s \rightarrow D_s^{(*)}\mu\bar{\nu}_\mu$$

Source	Uncertainty															
	CLN parametrization						BGL parametrization									
	$ V_{cb} $ [ $10^{-3}$ ]	$\rho^2(D_s^-)$ [ $10^{-1}$ ]	$\mathcal{G}(0)$ [ $10^{-2}$ ]	$\rho^2(D_s^{*-})$ [ $10^{-1}$ ]	$R_1(1)$ [ $10^{-1}$ ]	$R_2(1)$ [ $10^{-1}$ ]	$ V_{cb} $ [ $10^{-3}$ ]	$d_1$ [ $10^{-2}$ ]	$d_2$ [ $10^{-1}$ ]	$\mathcal{G}(0)$ [ $10^{-2}$ ]	$b_1$ [ $10^{-1}$ ]	$c_1$ [ $10^{-3}$ ]	$a_0$ [ $10^{-2}$ ]	$a_1$ [ $10^{-1}$ ]	$\mathcal{R}$ [ $10^{-1}$ ]	$\mathcal{R}^*$ [ $10^{-1}$ ]
$f_s/f_d \times \mathcal{B}(D_s^- \rightarrow K^+K^-\pi^-)(\times\tau)$	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.4
$\mathcal{B}(D^- \rightarrow K^-K^+\pi^-)$	0.5	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.3
$\mathcal{B}(D^{*-} \rightarrow D^-X)$	0.2	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.1	0.0	0.2	0.0	0.3	-	0.2
$\mathcal{B}(B^0 \rightarrow D^-\mu^+\nu_\mu)$	0.4	0.0	0.3	0.1	0.2	0.1	0.5	0.1	0.0	0.1	0.1	0.4	0.1	0.7	-	-
$\mathcal{B}(B^0 \rightarrow D^{*-}\mu^+\nu_\mu)$	0.3	0.0	0.2	0.1	0.1	0.1	0.2	0.0	0.0	0.1	0.1	0.3	0.1	0.4	-	-
$m(B_s^0), m(D^{*-})$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-	-
$\eta_{\text{EW}}$	0.2	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-	-
$h_{A_1}(1)$	0.3	0.0	0.2	0.1	0.1	0.1	0.3	0.0	0.0	0.1	0.1	0.3	0.1	0.5	-	-
External inputs (ext)	1.2	0.0	0.4	0.1	0.2	0.1	1.2	0.1	0.0	0.1	0.1	0.6	0.1	0.8	0.5	0.5
$D_{(s)}^- \rightarrow K^+K^-\pi^-$ model	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.4
Background	0.4	0.3	2.2	0.5	0.9	0.7	0.1	0.5	0.2	2.3	0.7	2.0	0.5	2.0	0.4	0.6
Fit bias	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.2	0.4	0.2	0.4	0.0	0.0
Corrections to simulation	0.0	0.0	0.5	0.0	0.1	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0
Form-factor parametrization	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0	0.1
Experimental (syst)	0.9	0.3	2.2	0.5	0.9	0.7	0.9	0.5	0.2	2.3	0.7	2.1	0.5	2.0	0.6	0.7
Statistical (stat)	0.6	0.5	3.4	1.7	2.5	1.6	0.8	0.7	0.5	3.4	0.7	2.2	0.9	2.6	0.5	0.5