QED CORRECTIONS: OPEN CHALLENGES

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MOTIVATION

- *Flavor physics is a precision physics laboratory*
- *Chances are high that we are already seeing signals of new physics*

To claim a discovery, we need a proper understanding of the SM

QED effects deserve proper scrutiny

- *Large "LFV logs" need to be under control*
- *QED violates isospin symmetry*
- *Proper factorization of QED effects is rather new and still under development*

HISTORICAL LOOK AT QED IR EFFECTS

Photons are massless → problems in IR

↓

Observables need to be sufficiently inclusive

F. Bloch, A. Nordsieck, 1937

T. Kinoshita, 1962

T. Lee, M. Nauenberg, 1964

$$\Gamma_{\text{phys}}(B \to f) = \sum_{n=0}^{\infty} \Gamma(B \to f + n\gamma) \Big|_{\sum_n E_n^\gamma \leq \Delta E}$$

↙

YFS (ultra-soft factorization)

D. Yennie, S. Frautschi, H. Suura, 1961

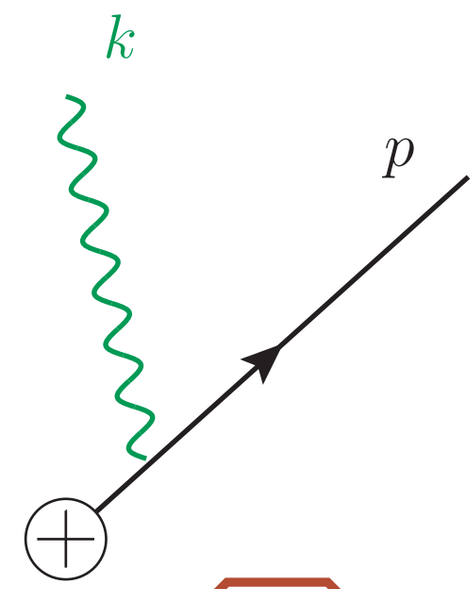
$$\Gamma_{\text{phys}}(B \to f) = \omega(\Delta E, \Lambda) \times \Gamma_{\text{non-rad}}(B \to f; \Lambda)$$

ULTRA-SOFT APPROXIMATION

S. Weinberg, 1964

Eikonal current

$$J^{\mu}_{\text{eik}} = eQ\frac{p^{\mu}}{p \cdot k}$$



Assumes photon momentum is smaller than any scale

$\Delta E \ll \Lambda_{\text{QCD}}$: point-like mesons

$\Delta E \ll m_{\ell}$: be careful with electrons

$\Lambda = M_B$ makes no sense

Ultra-soft corrections exponentiate

$$\omega(\Delta E, \Lambda) = \left(\frac{\Delta E}{\Lambda}\right)^{\frac{\alpha}{\pi}\sum_{i,j} Q_i Q_j f(\beta_{ij})}$$

WHERE DO THE IR QED LOGS COME FROM?

$$J^{\mu}_{\text{eik}} = eQ\frac{p^{\mu}}{p \cdot k} \qquad p = (E_p, \vec{p}) \qquad p^2 = m^2$$

Singularities of eikonal propagator

$$p \cdot k = E_k E_p (1 - \beta \cos\theta) \qquad \beta = |\vec{p}|/E_p = \sqrt{E_p^2 - m^2}/E_p$$

$E_k \to 0$ *Soft singularity* → $\ln \frac{\Delta E}{\Lambda}$

$\beta \to 1$, $m \to 0$ *Collinear singularity* → $\ln \frac{m}{\Lambda}$

CAN THERE BE LARGE UNACCOUNTED LFV QED CORRECTIONS?

Computing QED corrections in point-like approximation is easy.

How far are we off by ignoring proper treatment of structure dependent terms?

cf. G. Isidori, S. Nabeebaccus, R. Zwicky, 2020

$$\varepsilon^*_\mu \mathscr{A}_\mu = \varepsilon^*_\mu \left(J^\mu_{\text{eik}} + (\mathscr{A}^\mu - J^\mu_{\text{eik}}) \right) \qquad J^\mu_{\text{eik}} = eQ \frac{p^\mu_\ell}{p_\ell \cdot k}$$

$$\left| \mathscr{A}^*_\mu \mathscr{A}^\mu \right| = J^\mu_{\text{eik}} J^*_{\text{eik}\,\mu} + (\mathscr{A}^\mu - J^\mu_{\text{eik}})(\mathscr{A}^*_\mu - J^*_{\text{eik}\,\mu}) + 2\Re \left[J^\mu_{\text{eik}} \mathscr{A}^*_\mu \right]$$

✓ ✓ $\mathscr{O}(m_\ell^2)$

Collinear photon $k = p_\ell + \mathscr{O}(m_\ell^2)$

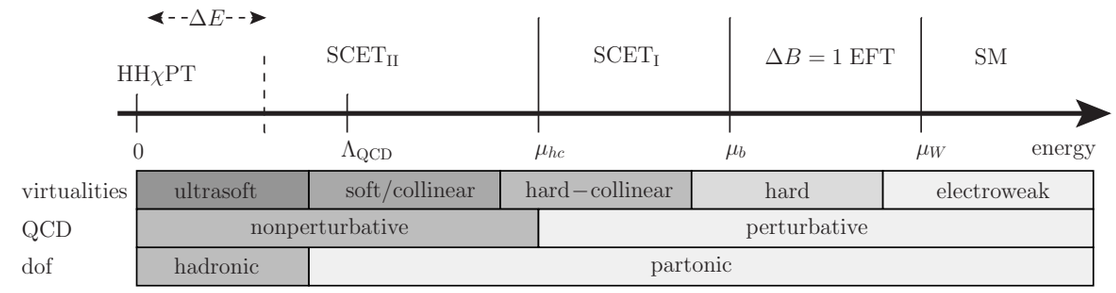
$$k_\mu \mathscr{A}^\mu = 0$$

STOP

What is in the denominator? $\dfrac{m_\ell^2}{?}$

CAN THERE BE LARGE UNACCOUNTED LFV QED CORRECTIONS?

$$\Delta^{\ell}_{\rm QED} \sim \mathcal{O}(\alpha)\mathcal{O}\left(\frac{m_\ell^2}{\Lambda^2}\right)\mathcal{O}\left(\ln\frac{m_\ell^2}{\ldots}\right)$$



As we progress through the scales, logs can grow and become large $\ln\frac{m_\ell}{\ldots} + \ldots + \ln\frac{\ldots}{M_B}$

But Λ is fixed: depends on the order in the EFT power expansion

For a point-like meson theory $\Lambda_{\rm QCD}$ is the cut-off

$$\Delta^{R}_{\rm QED} = \frac{1+\Delta^{\mu}_{\rm QED}}{1+\Delta^{e}_{\rm QED}} \sim \frac{1+\mathcal{O}(\alpha)\mathcal{O}\left(\frac{m_\mu^2}{\Lambda^2_{\rm QCD}}\right)\mathcal{O}\left(\ln\frac{m_\mu^2}{\ldots}\right)}{1+\mathcal{O}(\alpha)\mathcal{O}\left(\frac{m_e^2}{\Lambda^2_{\rm QCD}}\right)\mathcal{O}\left(\ln\frac{m_e^2}{\ldots}\right)} = 1+\mathcal{O}(\alpha)\mathcal{O}\left(\ln\frac{m_\mu^2}{\ldots}\right)$$

Worst case scenario: additional few % uncertainty from structure dependent corrections!

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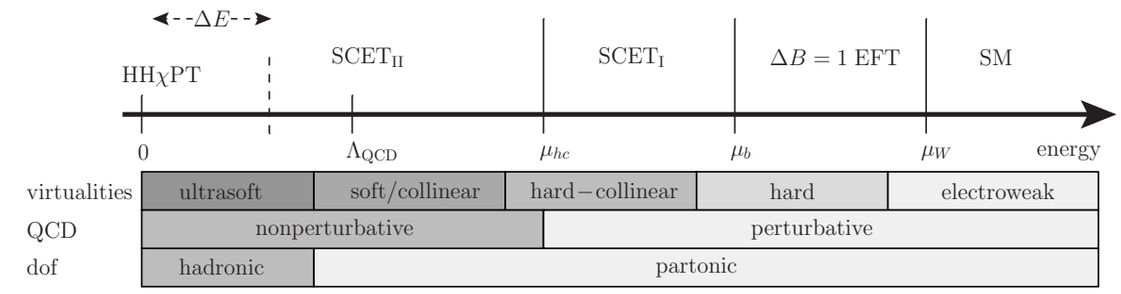

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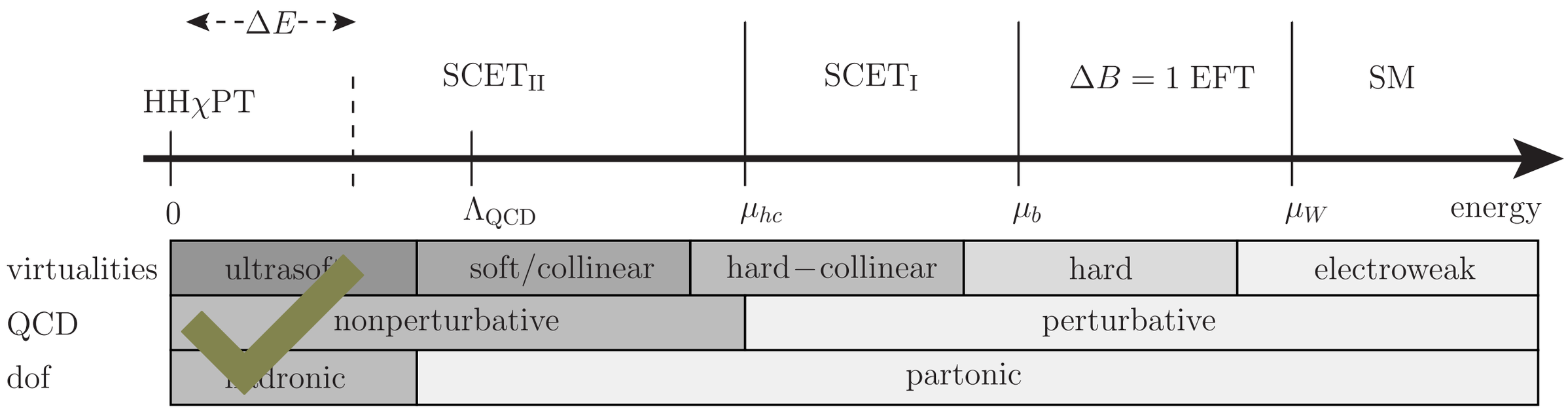
$$\frac{m_e^2}{\Lambda^2_{\rm QCD}} \ll \frac{m_\mu^2}{\Lambda^2_{\rm QCD}} \sim 1$$

Worst case scenario: additional few % uncertainty from structure dependent corrections!

EFT APPROACH

M. Beneke, C. Bobeth and R. Szafron, 2019



At low energies, the photon does not resolve mesons: scalar QED valid up to $\Lambda_{\rm QCD}$

$$\frac{1}{\Lambda_{\rm QCD}^2}(D^\mu B)^\dagger F_{\mu\nu} D^\nu K$$

$q^\mu \sim m_e$ ✓

$q^\mu \sim m_\mu$ ✗

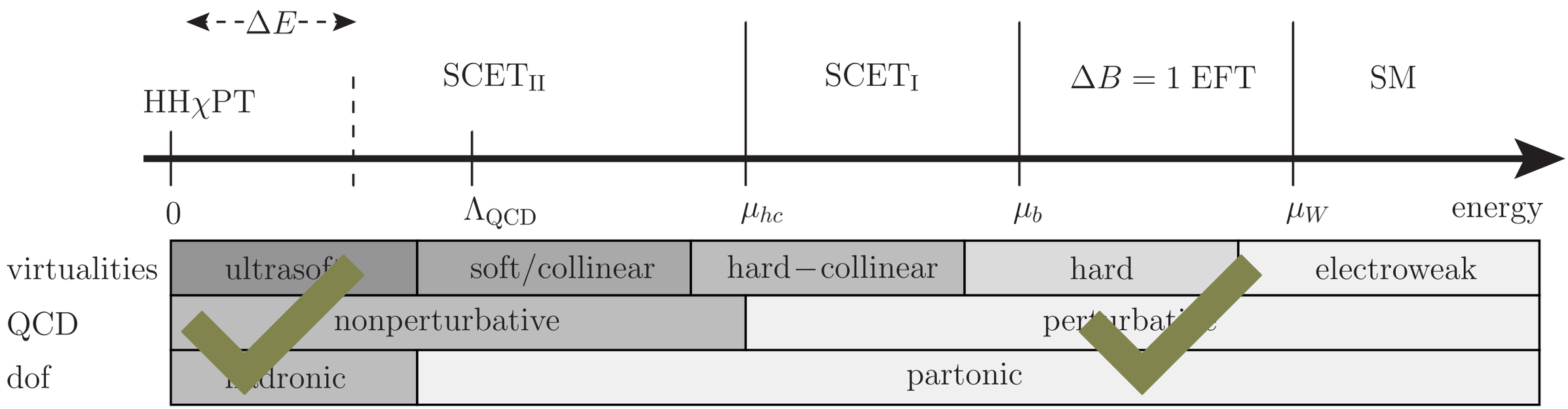
STOP

Collinear logs for muon have virtuality $q^2 \sim m_\mu^2 \sim \Lambda_{\rm QCD}^2$

For muons - these terms must be resumed

EFT APPROACH

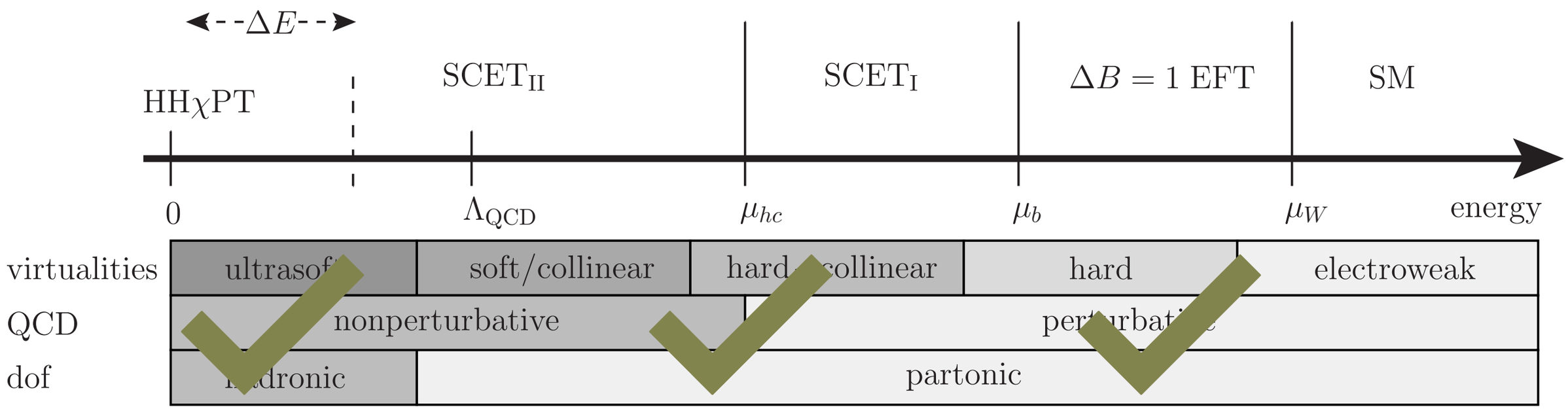
M. Beneke, C. Bobeth and R. Szafron, 2019



Above m_b, partonic picture works well — QED corrections can be easily included in the short-distance Wilson coefficients

EFT APPROACH

M. Beneke, C. Bobeth and R. Szafron, 2019



Intermediate scales:

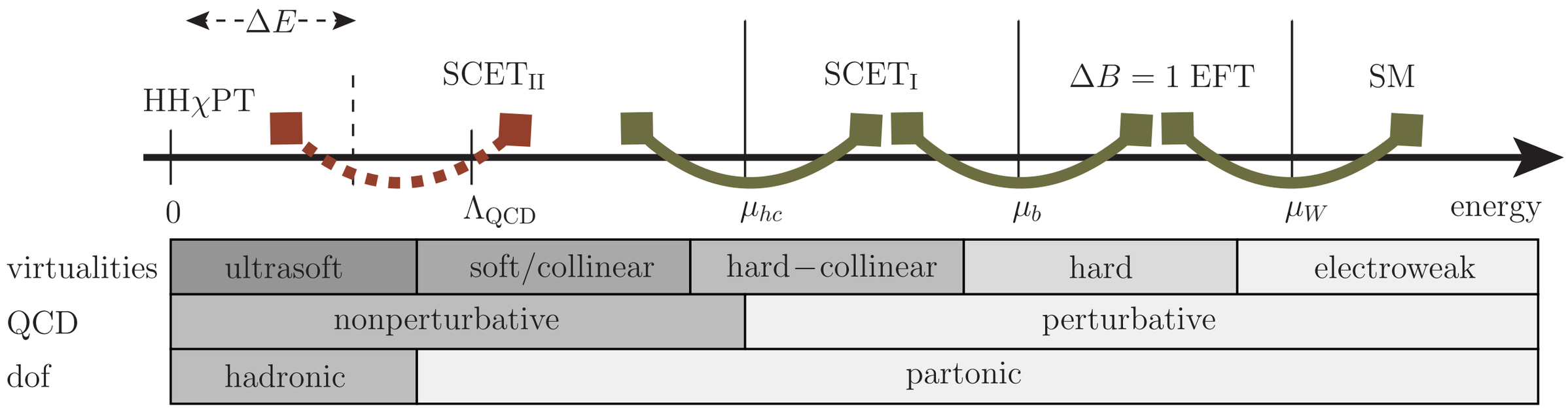
- *Soft* $\sim \Lambda_{QCD}$
- *Collinear* $\sim m_\ell$, $\ell = \mu$
- *Hard-collinear* $\sim \sqrt{m_b \Lambda_{\rm QCD}}$

Soft-collinear EFT (SCET)

Non-local EFT expansion, necessary to describe dynamics at distances $1/\Lambda_{\rm QCD}$

EFT APPROACH

M. Beneke, C. Bobeth and R. Szafron, 2019



Matching: perturbative above $\Lambda_{\rm QCD}$

Well understood, performed in the partonic picture

Matching: non-perturbative between point-like meson theory and SCET

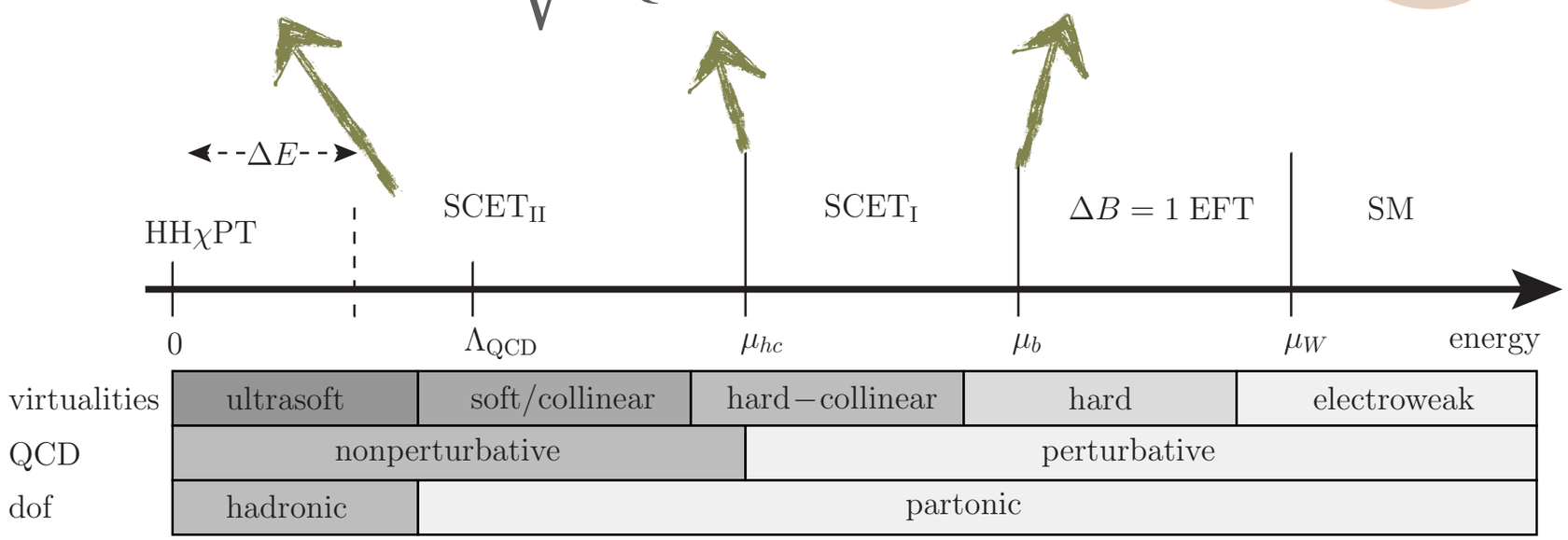
Challenge 1: matching has to be defined in terms of mesonic states

In practice: we can resum logs but non-logarithmic corrections are incomplete

MODERN PICTURE

Modern understanding of EFTs forces us to abandon the point-like approximation

$$\ln\frac{m_e}{\Lambda_{\rm QCD}} + \ln\frac{\Lambda_{\rm QCD}}{\sqrt{\Lambda_{\rm QCD} m_b}} + \ln\frac{\sqrt{\Lambda_{\rm QCD} m_b}}{m_b} = \ln\frac{m_e}{m_b}$$



mode	relative scaling	absolute scaling	virtuality k^2
hard	$(1,1,1)$	(m_b, m_b, m_b)	m_b^2
hard-collinear	$(1,\lambda,\lambda^2)$	$(m_b, \sqrt{m_b\Lambda_{\rm QCD}}, \Lambda_{\rm QCD})$	$m_b\Lambda_{\rm QCD}$
anti-hard-collinear	$(\lambda^2,\lambda,1)$	$(\Lambda_{\rm QCD}, \sqrt{m_b\Lambda_{\rm QCD}}, m_b)$	$m_b\Lambda_{\rm QCD}$
collinear	$(1,\lambda^2,\lambda^4)$	$(m_b, m_\mu, m_\mu^2/m_b)$	m_μ^2
anticollinear	$(\lambda^4,\lambda^2,1)$	$(m_\mu^2/m_b, m_\mu, m_b)$	m_μ^2
soft	$(\lambda^2,\lambda^2,\lambda^2)$	$(\Lambda_{\rm QCD}, \Lambda_{\rm QCD}, \Lambda_{\rm QCD})$	$\Lambda_{\rm QCD}^2$

M. Beneke, C. Bobeth and R. Szafron, 2019

FIRST PROPER TREATMENT: $B_s \to \mu^+\mu^-$

M. Beneke, C. Bobeth and R. Szafron, 2017

Dangerous logs have been identified in $B_s \to \mu^+\mu^-$

$$
\begin{aligned}
i\mathcal{A} &= m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5\ell \\
&+ \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell m_B f_{B_q} \mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \times \Bigg\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2) \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln\frac{m_b\omega}{m_\ell^2} + \ln\frac{u}{1-u} \right] \\
&- Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln^2\frac{m_b\omega}{m_\ell^2} - 2\ln\frac{m_b\omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \Bigg\} + \dots ,
\end{aligned}
$$

** helicity suppressed SM amplitude*

In this case

- *Scale* $1/\Lambda_{\rm QCD}$ *appears in the result*
- *Structure dependent logs appear between collinear scale and hard-collinear scale*
- *Double logs appear*

Note: the effect is non-local — cannot be captured by local expansion below $\Lambda_{\rm QCD}$

!

Logs depend on muon mass

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TOWARDS A SYSTEMATIC TREATMENT OF QED

Charmless decays: $B \to \pi^+\pi^-$

M. Beneke, P. Boer, J. Toelstede, K. Vos, 2020

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = i m_B^2 \left\{ \mathcal{F}_{Q_2}^{BM_1}(0) \int_0^1 du\, T_{i,Q_2}^{\mathrm{I}}(u) \mathscr{F}_{M_2} \Phi_{M_2}(u) \right.$$
$$\left. + \int_{-\infty}^{\infty} d\omega \int_0^1 du\, dv\, T_{i,\otimes}^{\mathrm{II}}(u,v,\omega) \mathscr{F}_{M_1} \Phi_{M_1}(v) \mathscr{F}_{M_2} \Phi_{M_2}(u) \mathscr{F}_{B,\otimes} \Phi_{B,\otimes}(\omega) \right\}$$

Factorization formula retains the pure QCD form, but the hadronic matrix elements need to be generalized. They become process-dependent.

$$\Delta(\pi K) \equiv A_{\mathrm{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 K^-)$$
$$- \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{\mathrm{QCD}} + \delta\Delta(\pi K)$$

$\Delta(\pi K)^{\mathrm{QCD}} = (0.5 \pm 1.1)\,\%$ $\qquad$ $\delta\Delta(\pi K) = -\,0.41\,\%$

G. Bell, M. Beneke, T. Huber, X. Li, 2015

TOWARDS SYSTEMATIC TREATMENT OF QED

M. Beneke, P. Boer, G. Finauri, K. Vos, 2021

Heavy to heavy decays: $B \to D^+ L^-, \quad B \to D^+ \ell^- \bar{\nu}_\ell$

$$R_L^{(0),(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+} L^-)(\Delta E)}{d\Gamma^{(0)}(\bar{B}_d \to D^{(*)+} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}}$$

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$R_L^{(*)}$	LO	QCD NNLO	$+\delta_{\rm QED}$	$+\delta_{\rm U}\,(\delta_{\rm U}^{(0)})$
R_π	0.969 ± 0.021	$1.078^{+0.045}_{-0.042}$	$1.069^{+0.045}_{-0.041}$	$1.074^{+0.046}_{-0.043}\,(1.003^{+0.042}_{-0.039})$
R_π^*	0.962 ± 0.021	$1.069^{+0.045}_{-0.041}$	$1.059^{+0.045}_{-0.041}$	$1.065^{+0.047}_{-0.042}\,(0.996^{+0.043}_{-0.039})$
$R_K \cdot 10^2$	7.47 ± 0.07	$8.28^{+0.27}_{-0.26}$	$8.21^{+0.27}_{-0.26}$	$8.44^{+0.29}_{-0.28}\,(7.88^{+0.26}_{-0.25})$
$R_K^* \cdot 10^2$	6.81 ± 0.16	$7.54^{+0.31}_{-0.29}$	$7.47^{+0.30}_{-0.29}$	$7.68^{+0.32}_{-0.30}\,(7.19^{+0.29}_{-0.28})$

LIGHT-CONE DISTRIBUTION AMPLITUDES IN QED

M. Beneke, C. Bobeth and R. Szafron, 2019 *M. Beneke, P. Boer, J. Toelstede, K. Vos, 2021*

Soft Matrix element:

$$\frac{\langle 0|\overline{q}_s(vn_-)Y(vn_-,0)\not{n}_-\gamma_5\, h_v(0)\, Y_+^\dagger(0)Y_-(0)|\overline{B}_q(p)\rangle}{\langle 0|\left[Y_+^\dagger\, Y_-\right](0)|0\rangle}$$

$$\equiv i m_{B_q}\int_0^\infty d\omega\, e^{-i\omega v}\,\mathscr{F}_{B_q}\Phi_+(\omega)$$

In QED, they depend on directions and charges of particles

QED effects change the endpoint behavior of the LCDA

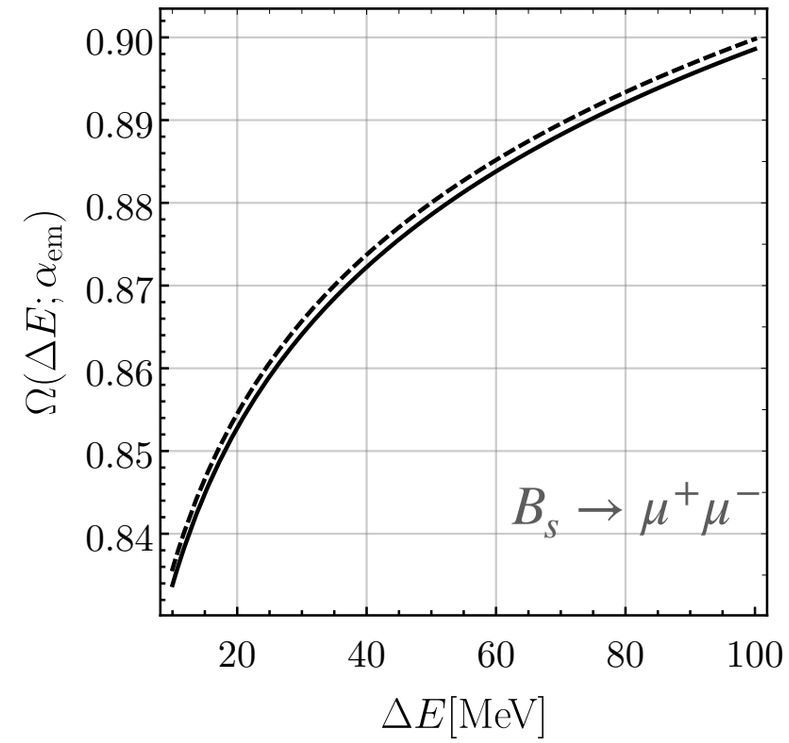
For pions, QED effects and the QCD NLL are at the % level — similar size as the uncertainties from the lattice input values

We can compute their evolution but not the initial conditions

Challenge 2: Non-perturbative matrix elements in QCD×QED

ULTRA-SOFT EFFECTS

Structure-dependent corrections can be large, but ultra-soft effects are guaranteed to be non-negligible



Experiments uses the PHOTOS, which neglects radiation from charged initial state particles and other important effects P. Golonka, Z. Was, 2005

Challenge 3:Flavor physics needs dedicated Monte Carlo for QED

See also G. Isidori, S. Nabeebaccus, R. Zwicky, 2020

We need to learn a lesson from QCD parton showers: Monte Carlo needs to be properly matched with fixed order computations

SUMMARY: MUONS AND ELECTRONS

Collinear loops with muons and electrons require different treatment

For light-light transitions, structure dependent terms are small: everything happens around soft scale, collinear loops do not resolve mesons

G. D'Ambrosio and G. Isidori, 1994

G. D'Ambrosio, G. Ecker, G. Isidori, and H. Neufeld, 1997

For pure hadronic, or heavy-heavy decays also no unusually large effects

M. Beneke, P. Boer, J. Toelstede, K. Vos, 2020

M. Beneke, P. Boer, G. Finauri, K. Vos, 2021

However, large "LFV" corrections for semi-leptonic heavy-light decays are not excluded and even plausible in certain regions of the phase space

SUMMARY: OPEN CHALLENGES

Theory

1. *Non-perturbative matching between point-like EFT and microscopic description*
2. *Non-perturbative soft matrix elements in QCD×QED*
3. *Dedicated Monte Carlo for QED compatible with EFT description above* $\Lambda_{\rm QCD}$
4. *Going beyond leading power in* $1/M_B$ *expansion*

Phenomenology

1. *Structure dependent corrections for semi-leptonic heavy-to-light and* $R_K^{(*)}$ *ratios*
2. *Lattice evaluation of QED corrections for heavy mesons*

$$\ln\frac{m_\mu}{\Delta E}\sim 2.5; \qquad \ln\frac{m_B}{m_\mu}\sim 4; \qquad \ln\frac{m_B}{\Lambda_{\rm QCD}}\sim 3; \qquad \ldots$$