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XI International Workshop on
CKM Unitarity Triangle
Melbourne, Australia, November 22-26, 2021

Exclusive B-meson semileptonic decays from unitarity and lattice QCD

in collaboration with:

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outline of the talk

* **the Dispersion Matrix approach:** an attractive way to implement unitarity and lattice QCD calculations in the analysis of exclusive semileptonic decays of hadrons [PRD '21 (2105.02497), PRD '21 (2105.07851)]
* results for $B \to D^{(*)} \ell \nu_\ell$ decays: extraction of $|V_{cb}|$ and theoretical determination of $R(D^{(*)})$ [2105.08674, 2109.15248]
* preliminary results for $|V_{ub}|$ from $B \to \pi \ell \nu_\ell$ decays [in preparation]

motivations

* two critical issues in semileptonic $B \to D^{(*)}\ell\nu_\ell$ decays

- exclusive/inclusive $|V_{cb}|$ puzzle:

<u>exclusive</u> (FLAG '21): $|V_{cb}|(BGL) \cdot 10^3 = 39.36\ (68)$

<u>inclusive</u> (HFLAV '19): $|V_{cb}| \cdot 10^3 = 42.00\ (65)$

difference of $\sim 2.8\ \sigma$

$|V_{cb}| \cdot 10^3 = 42.16\ (50)$

(Bordone et al. 2107.00604)

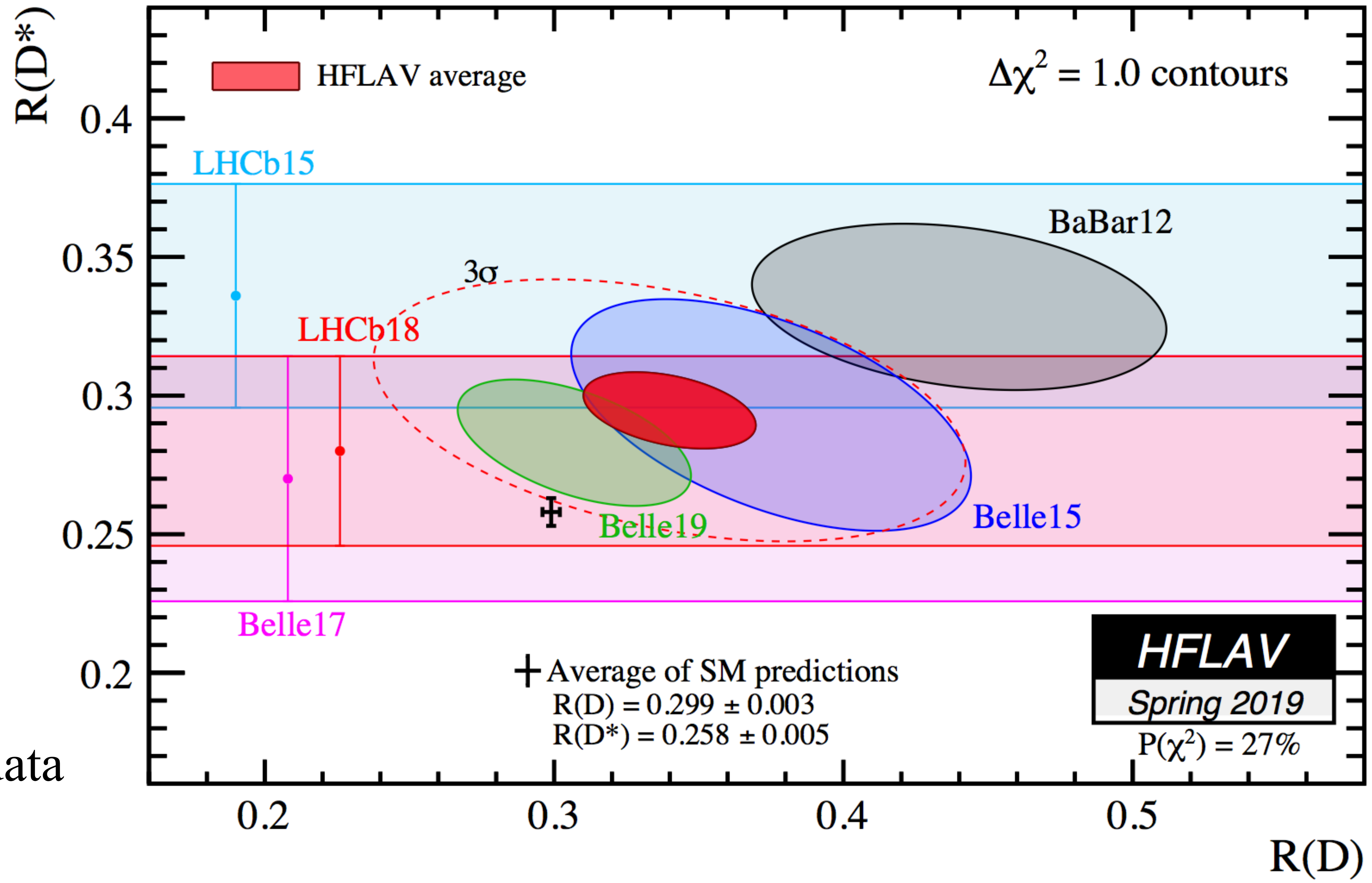
- $R(D^{(*)})$ anomalies:

$$R(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)}$$

$$R(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu_\ell)} \qquad \ell = e, \mu$$

differences of $\sim 3.1\sigma$ between exp.'s and "SM"

" " = mix of theoretical calculations and experimental data



aim of the talk

to show the relevant, attractive features of the Dispersion Matrix (DM) approach (arXiv:2105.02497), which is a rigorously model-independent tool for describing the hadronic form factors (FFs) in their whole kinematical range

- entirely based on first principles (i.e. lattice QCD simulations of 2- and 3-point Euclidean correlators)
- independent on any assumption about the momentum dependence of the FFs
- proper treatment of the uncertainties related to the unitarity (and kinematical) bounds
- applicable to theoretical calculations of the FFs, but also to experimental data

no mixing among theoretical calculations and experimental data to describe the shape of the FFs

* applied to $D \to K \ell \nu_\ell$ decays as a benchmark case [2105.02497]
* results for the $B \to D^{(*)} \ell \nu_\ell$ decays: extraction of $|V_{cb}|$ and theoretical determination of $R(D^{(*)})$ [2105.08674, 2109.15248]
* preliminary results for $|V_{ub}|$ from $B \to \pi \ell \nu_\ell$ decays [in preparation]

BGL approach

(Boyd, Grinstein and Lebed '95-'97)

* the hadronic form factors corresponding to definite spin-parity can be represented as an expansion, originating from unitarity, analyticity and crossing symmetry, in terms of the conformal variable z ($|z| \leq 1$)

$$f_+(q^2) = \frac{1}{\sqrt{\chi_{1^-}(q_0^2)}} \frac{1}{\phi_+(z(q^2), q_0^2)\; P_+(z(q^2))} \sum_{n=0}^{\infty} a_n\, z^n(q^2)$$

$$z(t) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$t_0 \to t_-$

$t_\pm \equiv (m_B \pm m_D)^2$

$\phi_+(z(q^2), q_0^2)$ = kinematical function (q_0^2 = auxiliary quantity)

$P_+(z(q^2))$ = Blaschke factor including resonances below the pair-production threshold t_+

$\chi_{1^-}(q_0^2)$ = transverse vector susceptibility $\equiv \frac{1}{2} \frac{\partial^2}{\partial (q_0^2)^2} \left[q_0^2\, \Pi_{1^-}(q_0^2) \right] = \frac{1}{\pi} \int_0^\infty ds \frac{s\, \mathrm{Im}\Pi_{1^-}(s)}{(s - q_0^2)^3}$

calculable nonperturbatively from appropriate 2-point lattice correlators (see arXiv:2105.07851)

unitarity constraint: $\sum_{n=0}^{\infty} a_n^2 \leq 1$

a test case

* FNAL/MILC synthetic data (arXiv:2105.14019) for the form factor g(w) of the $B \to D^* \ell \nu_\ell$ decay

w	g(w)
1.03	**0.372 (14)**
1.10	**0.331 (13)**
1.17	**0.291 (17)**

correlation matrix

1	**0.928**	**0.657**
	1	**0.832**
		1

$\chi_{1^-}(0) = 0.000513\ \text{GeV}^{-2}$ and B_c^* resonances from Gambino et al.: 1707.09509

* multivariate (Gaussian) distribution with 10^5 events

BGL linear fit: $a_0^2 + a_1^2 \leq 1$ for 100% of events

BGL quadratic fit: $a_0^2 + a_1^2 + a_2^2 \leq 1$ for 12% of events

note: input data exactly reproduced for each event

lattice data are OK, but unitarity is not built-in

<u>subtle issue</u>

a truncated BGL fit might be distorted by events which do not fulfill unitarity

*** need of a unitarity check independent of the parameterization ***

does it exist ? **Yes !**

Dispersion Matrix (DM) approach

PRD ’21 (2105.02497)

* reappraisal and improvement of the method originally proposed by Bourrely et al. NPB ’81 and Lellouch in NPB ’96

$$\mathscr{M} = \begin{pmatrix} <\phi f|\phi f> & <\phi f|g_t> & <\phi f|g_{t_1}> & \dots & <\phi f|g_{t_N}> \\ <g_t|\phi f> & <g_t|g_t> & <g_t|g_{t_1}> & \dots & <g_t|g_{t_N}> \\ <g_{t_1}|\phi f> & <g_{t_1}|g_t> & <g_{t_1}|g_{t_1}> & \dots & <g_{t_1}|g_{t_N}> \\ \dots & \dots & \dots & \dots & \dots \\ <g_{t_N}|\phi f> & <g_{t_N}|g_t> & <g_{t_N}|g_{t_1}> & \dots & <g_{t_N}|g_{t_N}> \end{pmatrix}$$

inner product: $<g|h> \equiv \dfrac{1}{2\pi i}\displaystyle\int_{|z|=1} \frac{dz}{z}\, \overline{g}(z)\, h(z)$

$$g_t(z) \equiv \frac{1}{1-\overline{z}(t)\, z}$$

$$<g_t|\phi f> \equiv \phi(z, q_0^2) f(z) \qquad <g_t|g_{t_m}> = \frac{1}{1-\overline{z}(t_m)\, z(t)}$$

$t_1, t_2, \dots, t_N$ are the N values of the squared 4-momentum transfer where the form factor *f* has been computed and t is its value where we want to compute *f*(t)

$$\text{unitarity bound:} \quad <\phi f|\phi f> \equiv \frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{z}\,|\phi(z, q_0^2) f(z)|^2 \leq \chi(q_0^2)$$

in the case of interest $z_i \equiv z(t_i)$ and $\phi_i f_i \equiv \phi(z_i, q_0^2) f(t_i)$ are real numbers and the positivity of the inner product implies:

$$\det[\overline{\mathscr{M}}] = \begin{vmatrix} \chi(q_0^2) & \phi f & \phi_1 f_1 & \dots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-z z_1} & \dots & \frac{1}{1-z z_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \dots & \frac{1}{1-z_1 z_N} \\ \dots & \dots & \dots & \dots & \dots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \dots & \frac{1}{1-z_N^2} \end{vmatrix} \geq 0$$

* the explicit solution is a band of values: $\beta - \sqrt{\gamma} \leq f(z) \leq \beta + \sqrt{\gamma}$

$$\beta = \frac{1}{d(z)\,\phi(z)} \sum_{j=1}^{N} f_j \phi_j d_j \frac{1 - z_j^2}{z - z_j} \qquad \gamma = \frac{1}{d^2(z)\,\phi^2(z)} \frac{1}{1 - z^2} \left[\chi - \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

χ, f_i : nonperturbative input quantities, $\phi(z), d(z), \phi_i, d_i$: kinematical coefficients depending on z_i

* unitarity is satisfied when $\gamma \geq 0$, which implies: $\chi \geq \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}$

*** this is the parameterization-independent unitarity test of the set of input data $\{f_j\}$ ***

* important feature: when $z \to z_j$ one has $\beta \to f_j$ and $\gamma \to 0$, i.e. the DM band collapses to f_j for $z = z_j$

for any given set of input data the DM approach reproduces exactly the known data and it allows to extrapolate the form factor in the whole kinematical range in a parameterization-independent way providing a band of values representing the results of all possible BGL fits satisfying unitarity and passing through the known points (important for estimating uncertainties)

* the DM band represents a uniform distribution which is combined with the multivariate distribution of the input data $\{f_j\}$ to generate the final band for $f(z)$

* kinematical constraint(s) can be easily and rigorously implemented in the DM approach (see for details arXiv:2105.02497)

nonperturbative determination of the susceptibilities

* lattice QCD simulations can provide a first-principle determination of the unitarity bounds (arXiv:2105.02497)

time-momentum representation (Q = Euclidean 4-momentum)

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^+}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^+}(t)\ ,$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^-}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^-}(t)\ ,$$

$$\chi_{0^-}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^-}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^-}(t)\ ,$$

$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^+}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^+}(t)\ ,$$

2-point Euclidean correlation functions

$$C_{0^+}(t) = \widetilde{Z}_V^2 \int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0 q_2(x)\ \bar{q}_2(0)\gamma_0 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^-}(t) = \widetilde{Z}_V^2\ \frac{1}{3}\sum_{j=1}^{3}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j q_2(x)\ \bar{q}_2(0)\gamma_j q_1(0)\right]|0\rangle\ ,$$

$$C_{0^-}(t) = \widetilde{Z}_A^2 \int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_0\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^+}(t) = \widetilde{Z}_A^2\ \frac{1}{3}\sum_{j=1}^{3}\int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_j\gamma_5 q_1(0)\right]|0\rangle\ ,$$

* in arXiv:2105.02497 and arXiv:2105.07851 we have calculated the χ's for the $c \to s$ and $b \to c$ transitions using the $N_f = 2+1+1$ gauge ensembles generated by ETMC

$b \to c$

- subtraction of discretization effects evaluated in perturbation theory at order $\mathcal{O}(\alpha_s^0)$
- implementation of WI for the 0^+ and 0^- channels to avoid exactly contact terms
- use of the ETMC ratio method (hep-lat/0909.3187) to reach the physical b-quark point

$b \to c$ transition

channel	nonPT	with GS subtr.	NNLO PT	with GS subtr.
0^+ [10^{-3}]	**7.58 (59)**	**—**	**6.204 (81)**	**—**
1^- [10^{-4} GeV^{-2}]	**6.72 (41)**	**5.88 (44)**	**6.486 (48)**	**5.131 (48)**
0^- [10^{-2}]	**2.58 (17)**	**2.19 (19)**	**2.41**	**1.94**
1^+ [10^{-4} GeV^{-2}]	**4.69 (30)**	**—**	**3.894**	**—**

nonperturbative: arXiv:2105.07851

GS = ground state

perturbative

Bigi, Gambino PRD ’16
Bigi, Gambino, Schacht PLB ’17
Bigi, Gambino, Schacht JHEP ’17

* differences with NNLO PT ~ 4% for 1^-, ~7% for 0^-, ~20 % for 0^+ and 1^+

$c \to s$ transition (arXiv:2105.02497)

channel	nonPT	with GS subtr.
0^+ [10^{-2}]	**0.929 (64)**	**0.433 (133)**
1^- [10^{-3} GeV^{-2}]	**7.88 (41)**	**4.19 (36)**
0^- [10^{-2}]	**2.48 (15)**	**0.942 (91)**
1^+ [10^{-3} GeV^{-2}]	**4.89 (29)**	**3.74 (56)**

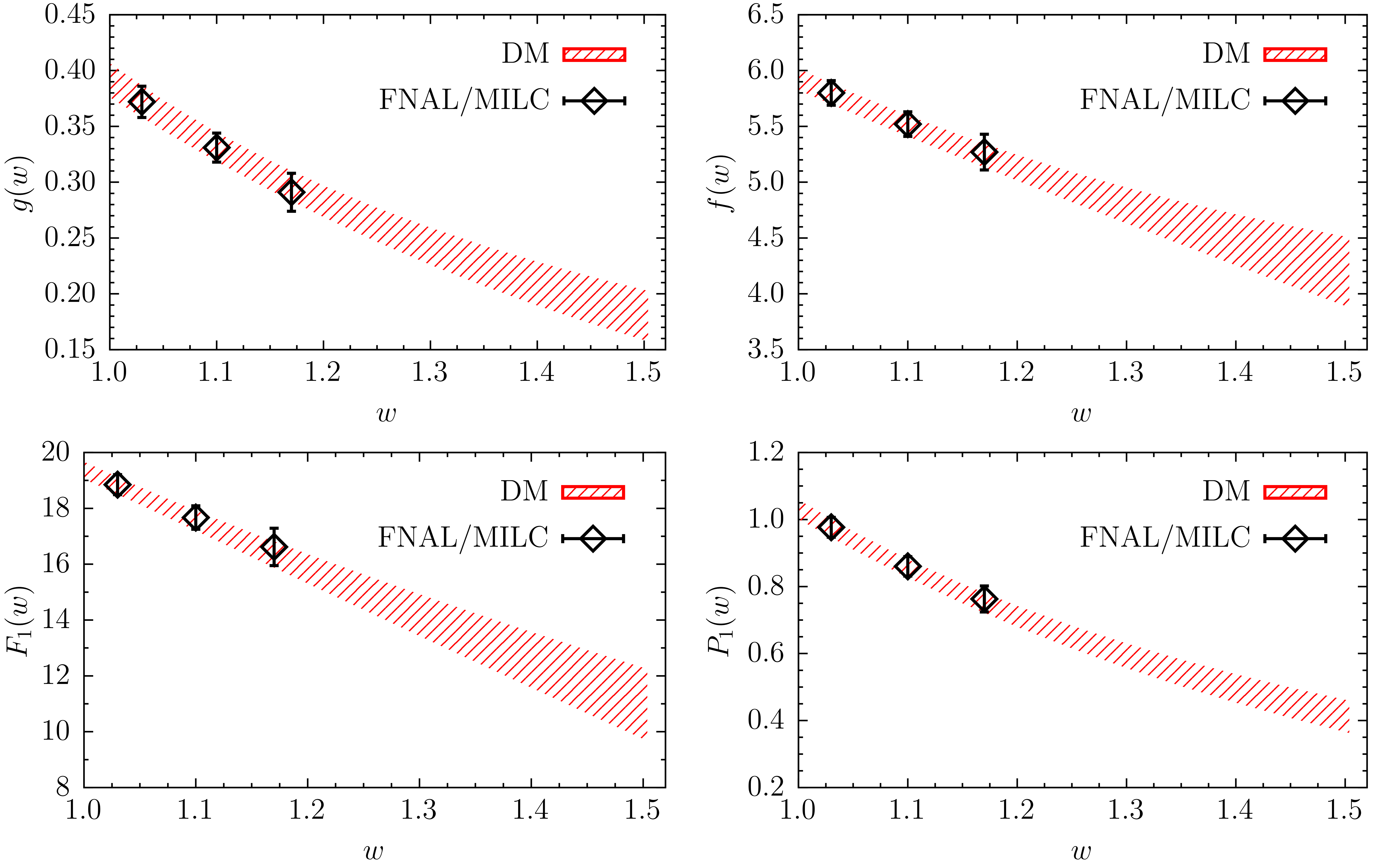
$b \to \ell$ transition (paper in preparation)

channel	nonPT	with GS subtr.
0^+ [10^{-2}]	**2.04 (20)**	**—**
1^- [10^{-4} GeV^{-2}]	**4.88 (1.16)**	**4.45 (1.16)**
0^- [10^{-2}]	**2.34 (13)**	**—**
1^+ [10^{-4} GeV^{-2}]	**4.65 (1.02)**	**—**

form factors for $B \to D^* \ell \nu_\ell$ decays

(arXiv:2109.15248)

* lattice QCD form factors from FNAL/MILC arXiv:2105.14019: synthetic data points at 3 (small) values of the recoil w
* nonperturbative susceptibilities from arXiv:2105.07851 (resonances from Bigi et al., arXiv:1707.09509)



unitarity bounds: χ_{1^-} for g, χ_{1^+} for f and $\mathcal{F}_1$, χ_{0^-} for P_1

unitarity + two kinematical constraints

$$w = 1 : \qquad \mathcal{F}_1(1) = m_B(1-r)f(1)$$

$$w = w_{max} : \qquad P_1(w_{max}) = \frac{\mathcal{F}_1(w_{max})}{m_B^2(1+w_{max})(1-r)\sqrt{r}}$$

extrapolation at maximum recoil

$$f(w_{max}) = 4.19 \pm 0.31 \ ,$$

$$g(w_{max}) = 0.180 \pm 0.023 \ ,$$

$$\mathcal{F}_1(w_{max}) = 11.0 \pm 1.3 \ ,$$

$$P_1(w_{max}) = 0.411 \pm 0.048 \ .$$

LCSR: $\mathcal{F}_1(w_{max}) = 16.0 \pm 2.1$ (arXiv:1811.00983)

experimental data for $B \to D^* \ell \nu_\ell$ decays

- two sets of data from Belle collaboration arXiv:1702.01521 and arXiv:1809.03290
- four different differential decay rates $d\Gamma/dx$ where $x = \{w, \cos\theta_v, \cos\theta_\ell, \chi\}$: 10 bins for each variable

total of 80 data points

*** we do not mix theoretical calculations with experimental data to describe the shape of the FFs ***

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}} \qquad i = 1, \ldots, N_{bins}$$

* issue with the covariance matrix $C_{ij}^{exp.}$ of the Belle data: $\Gamma^{exp.} \equiv \sum_{i=1}^{10} \left(\frac{d\Gamma}{dx}\right)_i^{exp.}$ should be the same for all the variables x

(see D'Agostini, arXiv: 2001.07562)

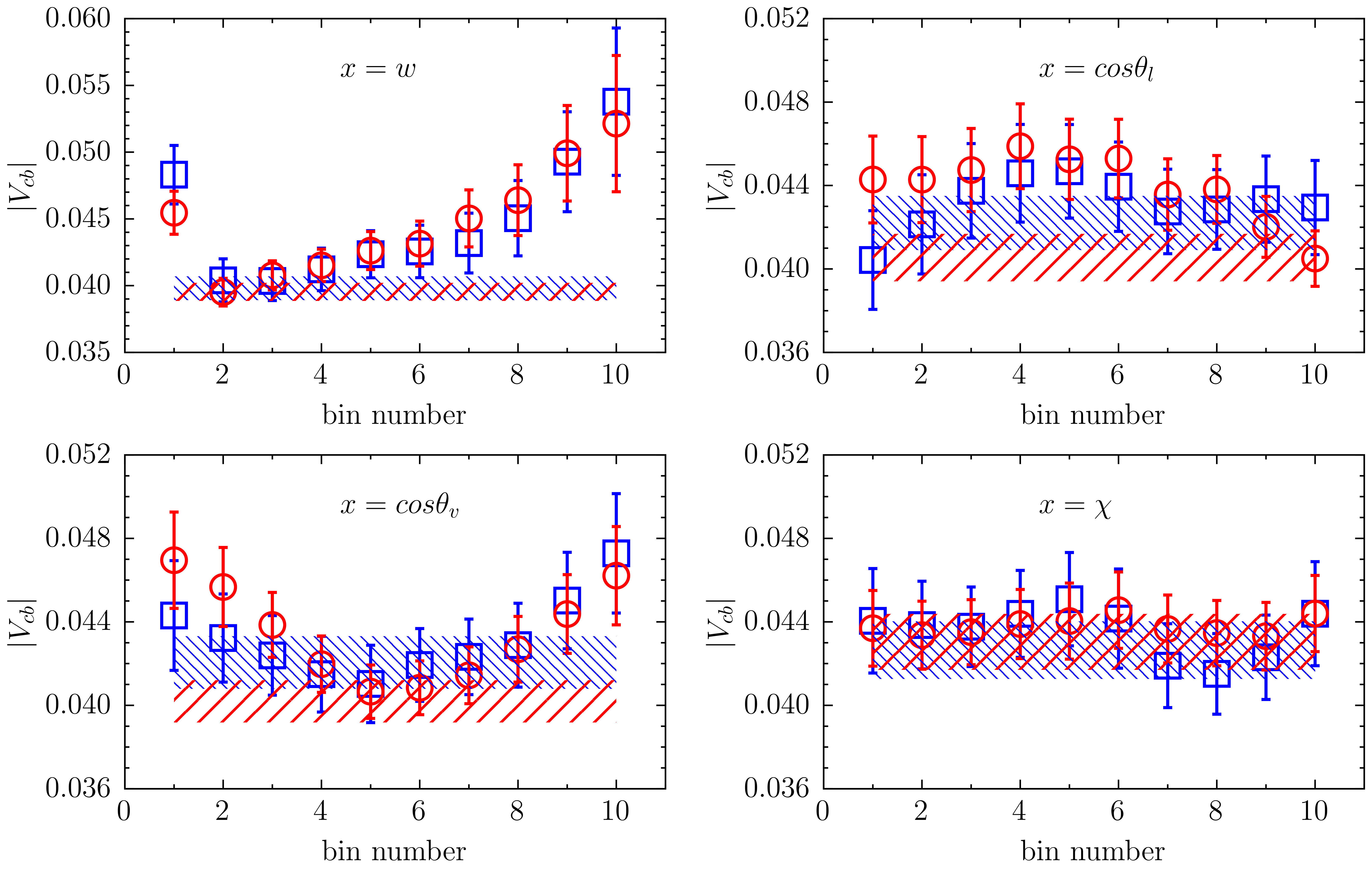
- we recover the above property by evaluating the correlation matrix of the experimental ratios

$$\frac{1}{\Gamma^{exp.}} \left(\frac{d\Gamma}{dx}\right)_i^{exp.}$$

and by considering the new covariance matrix of the experimental data given by (see arXiv:2105.08674)

$$\widetilde{C}_{ij}^{exp.} \to \rho_{ij}^{ratios} \sqrt{C_{ii}^{exp.} C_{jj}^{exp.}}$$

extraction of |V$_{cb}$| from $B \to D^* \ell \nu_\ell$ decays



blue data: Belle 1702.01521

red data: Belle 1809.03290

bands are (correlated) weighted averages

$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} ,$$

$$\sigma^2_{|V_{cb}|} = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} ,$$

	experiment	$\lvert V_{cb}\rvert(x=w)$	$\lvert V_{cb}\rvert(x=\cos\theta_l)$	$\lvert V_{cb}\rvert(x=\cos\theta_v)$	$\lvert V_{cb}\rvert(x=\chi)$
Belle 1702.01521	Ref. [11]	0.0398 (9)	0.0422 (13)	0.0421 (13)	0.0426 (14)
Belle 1809.03290	Ref. [12]	0.0395 (7)	0.0405 (11)	0.0402 (10)	0.0430 (13)

averaging procedure

$$\mu_x = \frac{1}{N}\sum_{k=1}^{N} x_k \ ,$$

$$\sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N}\sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k-\mu_x)^2,$$

$$|V_{cb}|_{excl.}\cdot 10^3 = 41.3 \pm 1.7$$

with the original covariance of Belle data

$$|V_{cb}|_{excl.}\cdot 10^3 = 40.0 \pm 2.6$$

$|V_{cb}|_{excl.}\cdot 10^3 = 39.6^{+1.1}_{-1.0}$ Gambino et al., arXiv:1905.08209

$|V_{cb}|_{excl.}\cdot 10^3 = 39.56^{+1.04}_{-1.06}$ Jaiswal et al., arXiv:2002.05726

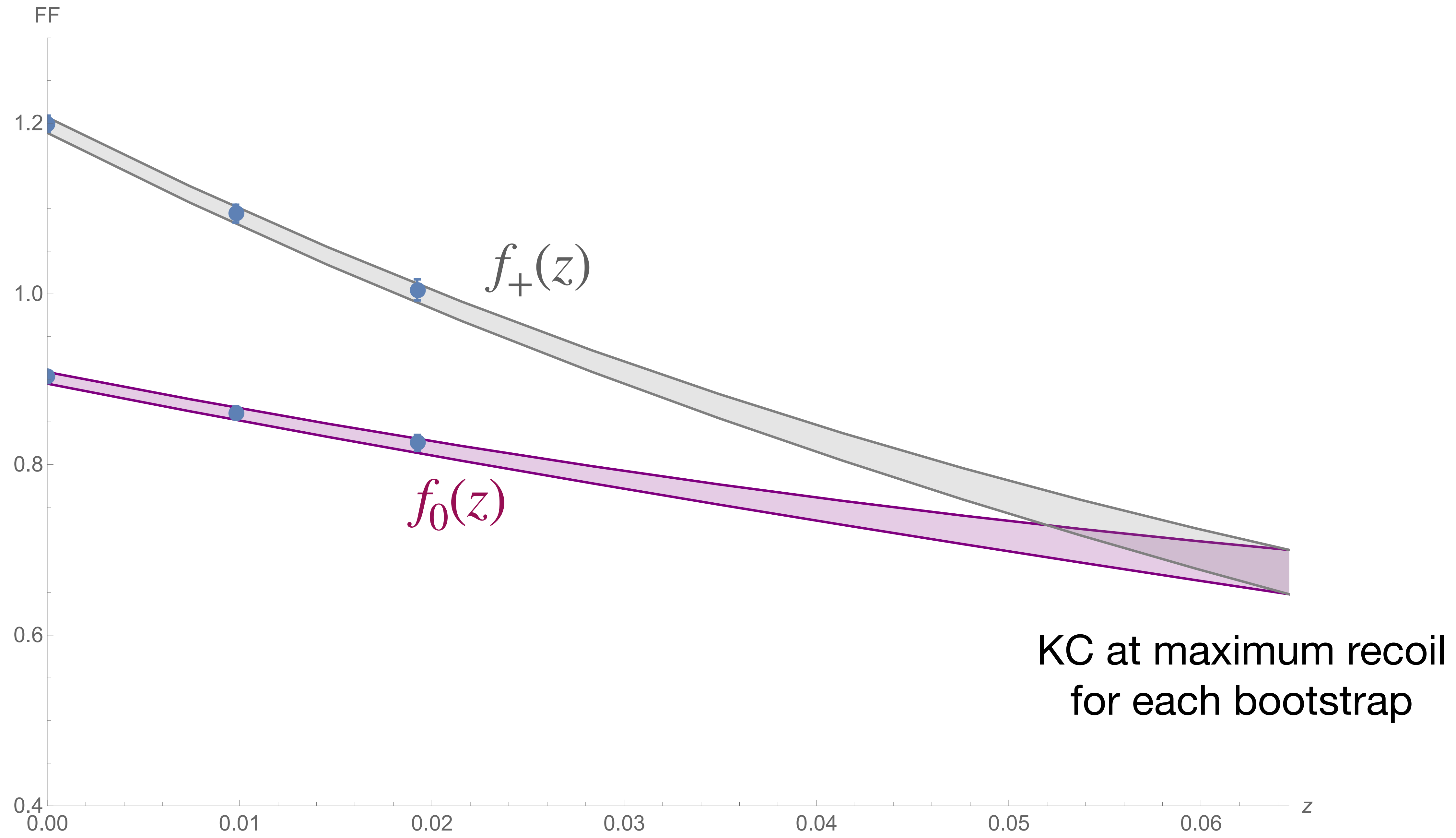
$|V_{cb}|_{excl.}\cdot 10^3 = 38.86 \pm 0.88$ FLAG '21, arXiv:2111.09849

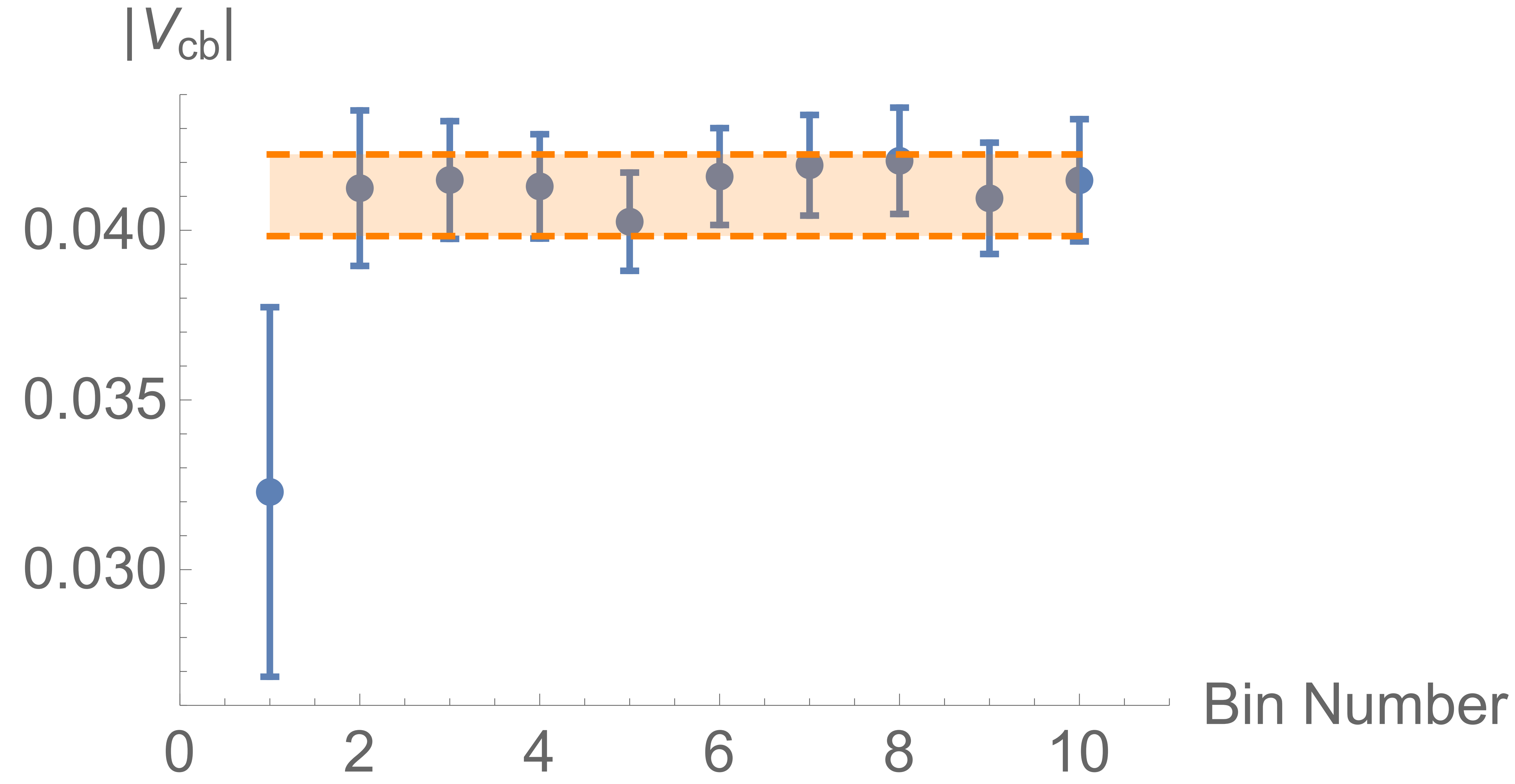
the use of exp. data to describe the shape of the FFs leads to smaller errors, but the use of truncated BGL fits does not guarantee that the final error is not underestimated

$|V_{cb}|_{incl.}\cdot 10^3 = 42.16 \pm 0.50$ (Bordone et al: arXiv:2107.00604)

extraction of $|V_{cb}|$ from $B \to D\ell\nu_\ell$ decays

* lattice QCD form factors from FNAL/MILC (arXiv:1503.07237): synthetic data points at 3 (small) values of the recoil
* experimental data from Belle collaboration in 10 bins (arXiv:1510.03657)





$$|V_{cb}|_{excl.} \cdot 10^3 = 41.0 \pm 1.2$$

nice consistency with $B \to D^*$

$$|V_{cb}|_{excl.} \cdot 10^3 = 40.49 \pm 0.97$$ Gambino et al., arXiv:1606.08030

$$|V_{cb}|_{excl.} \cdot 10^3 = 41.0 \pm 1.1$$ Jaiswal et al., arXiv:1707.09977

$$|V_{cb}|_{excl.} \cdot 10^3 = 40.0 \pm 1.0$$ FLAG '21, arXiv:2111.09849

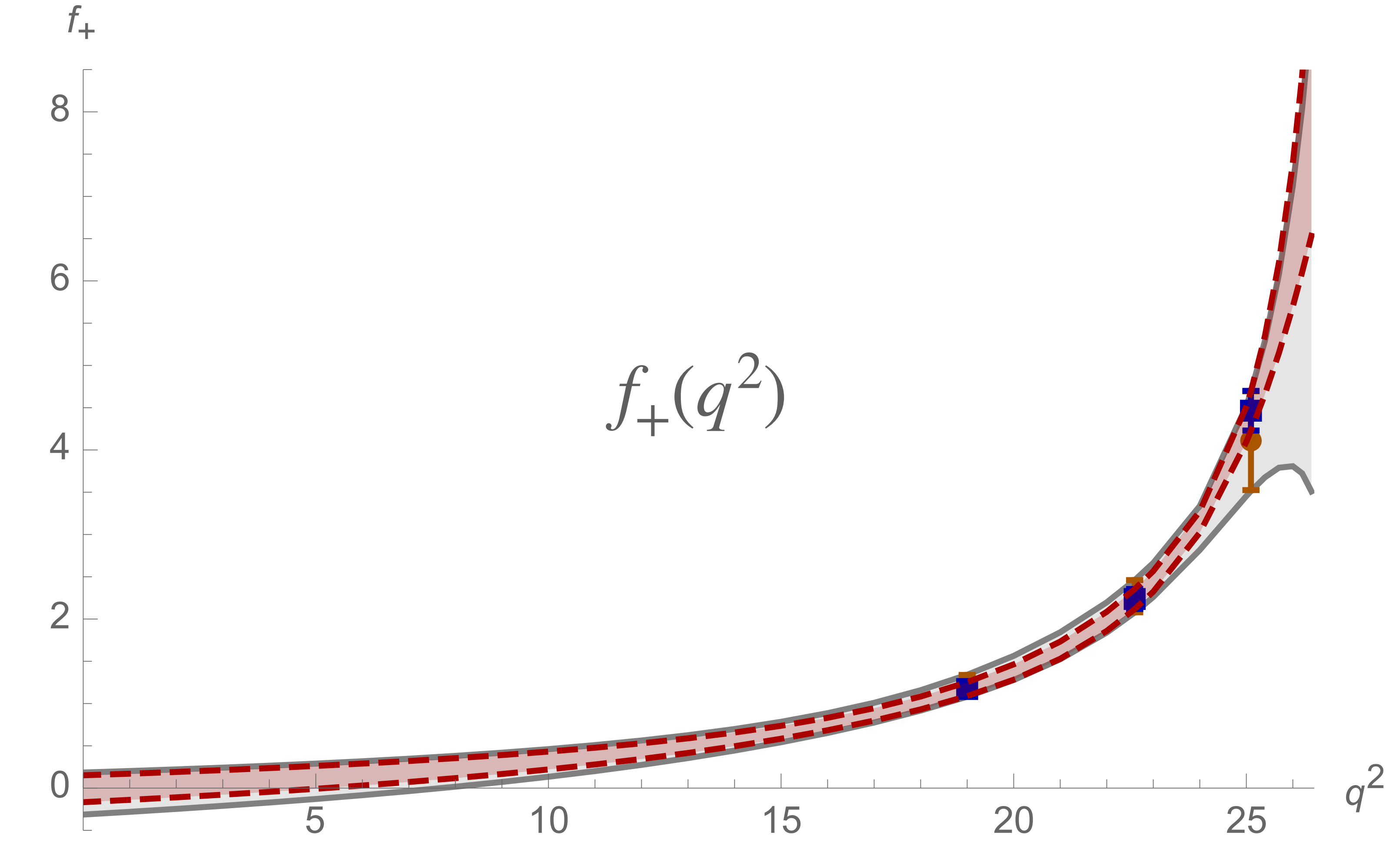
$R(D)$, $R(D^*)$ and polarization observables

observable	DM	experiment	difference
$R(D)$	$0.289\,(8)$	$0.340\,(27)\,(13)$	$\simeq 1.6\,\sigma$
$R(D^*)$	$0.269\,(8)$	$0.295\,(11)\quad(8)$	$\simeq 1.6\,\sigma$
$P_\tau(D^*)$	$-0.52\,(1)$	$-0.38\,(51)\,(^{+21}_{-16})$	
$F_L(D^*)$	$0.42\,(1)$	$0.60\,(8)\,(4)$	$\simeq 2.0\,\sigma$

*** pure theoretical and parameterization-independent determinations within the DM approach ***

extraction of $|V_{ub}|$ from $B \to \pi \ell \nu_\ell$ decays

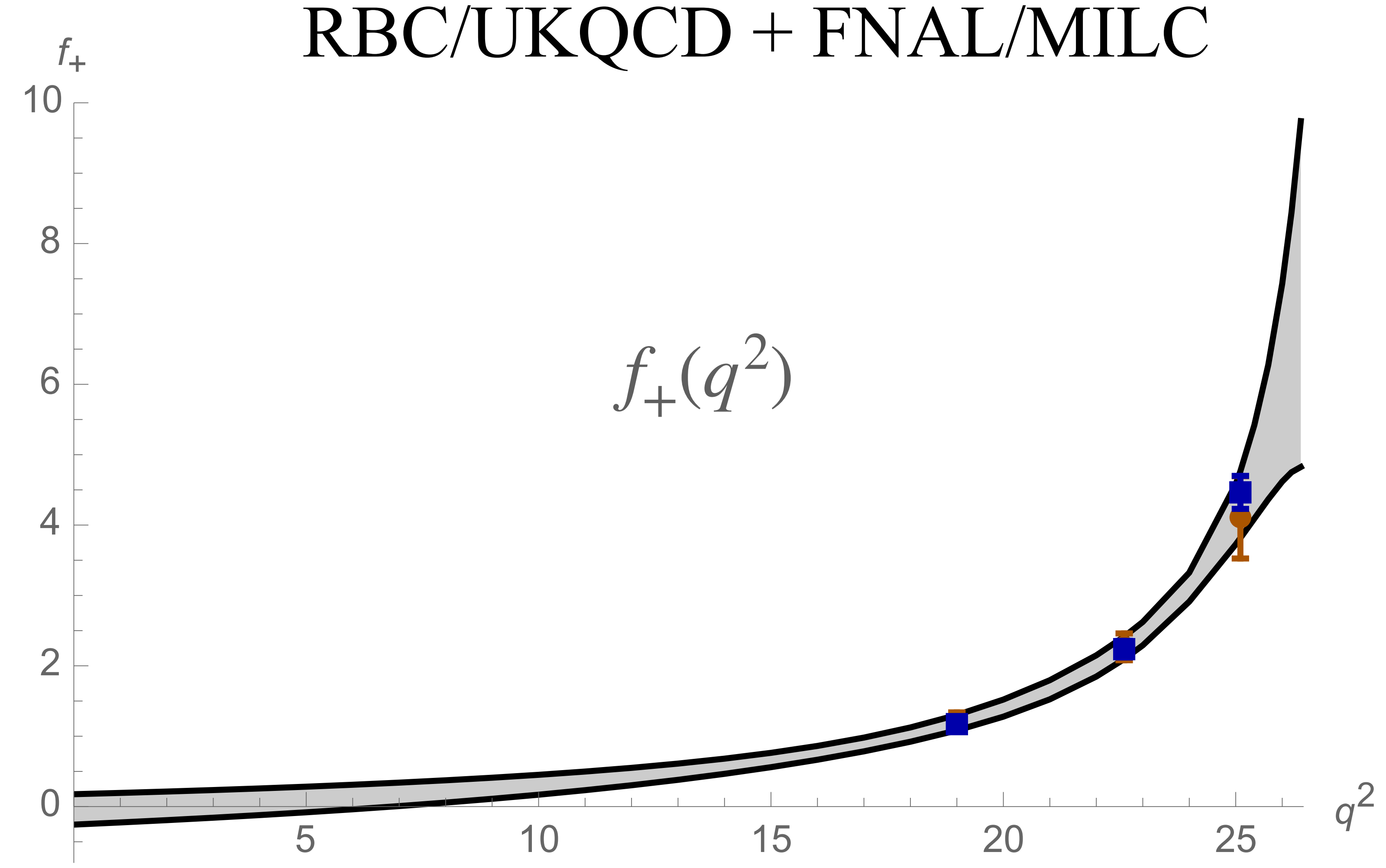
* lattice QCD form factors from RBC/UKQCD (arXiv:1501.05363) and FNAL/MILC (arXiv:1503.07839): two sets of synthetic data points at 3 (large) values of q^2 (19.0, 22.6, 25.1 GeV^2) and their combination
* nonperturbative susceptibilities (paper in preparation)

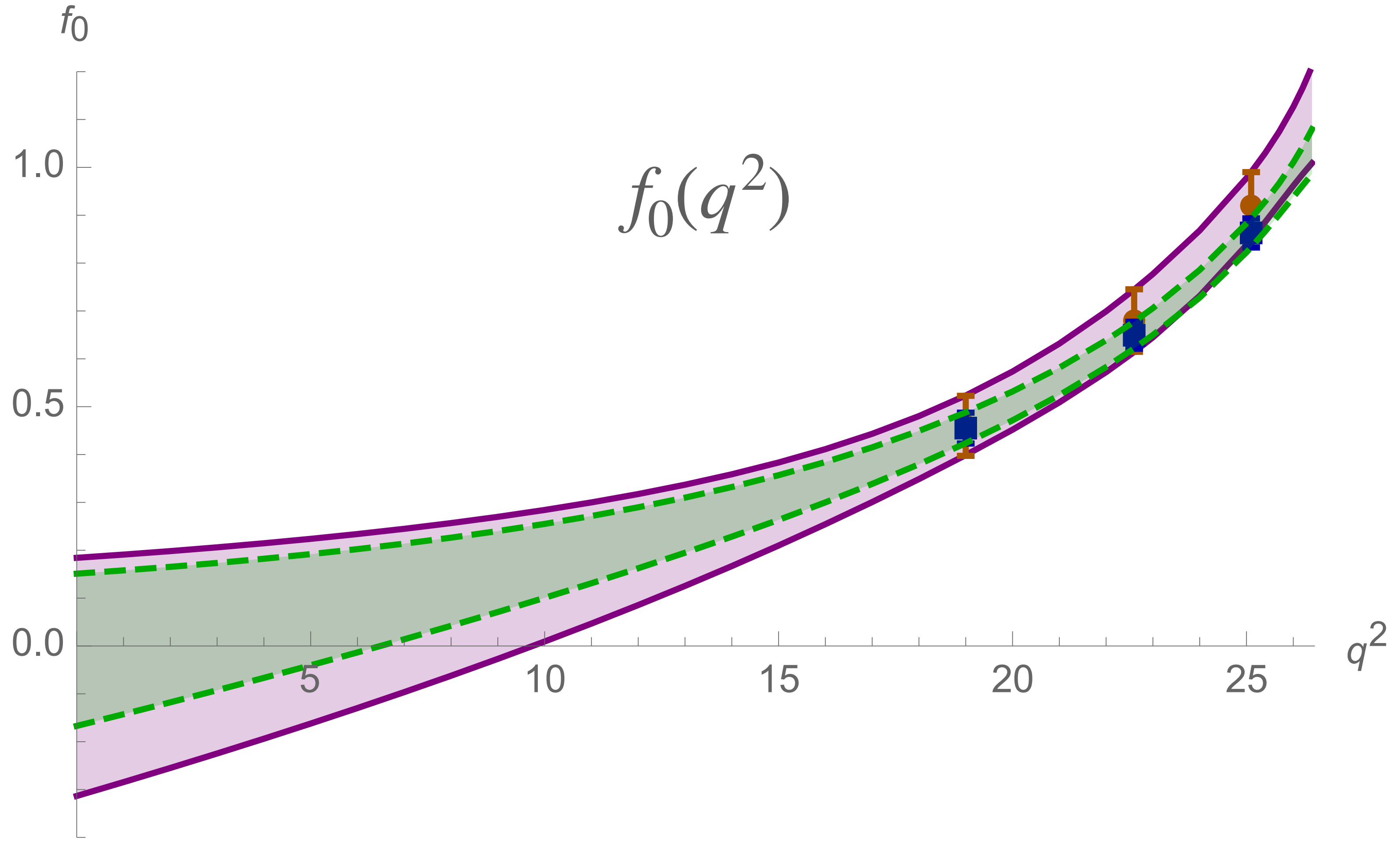


red dots: RBC/UKQCD
blue squares: FNAL/MILC

DM bands
solid: RBC/UKQCD
dashed: FNAL/MILC

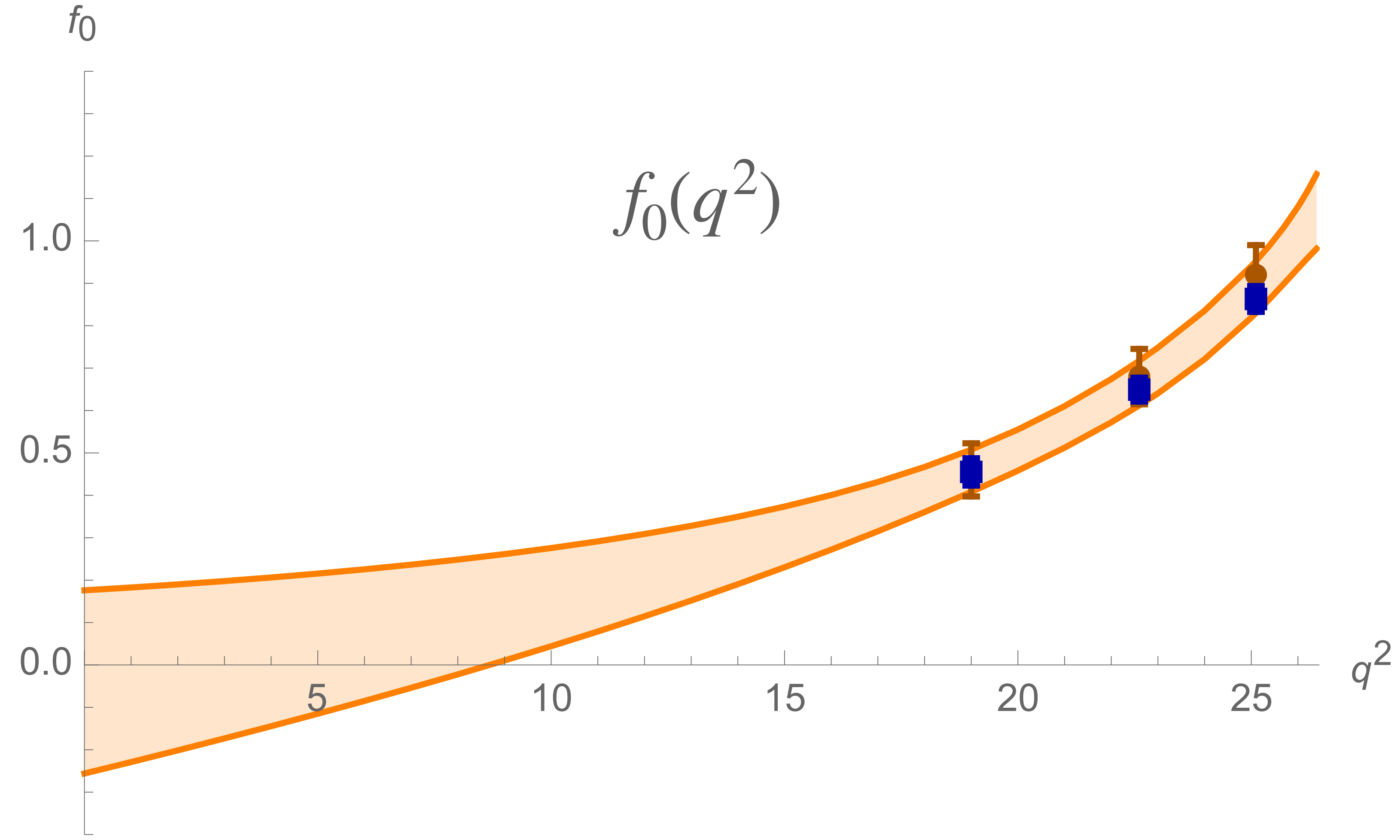
RBC/UKQCD + FNAL/MILC





	f(0)
RBC/UKQCD	-0.06 ±0.25
FNAL/MILC	-0.01 ±0.16
combined	-0.04±0.22
LCSR	0.28±0.03

arXiv:2102.07233

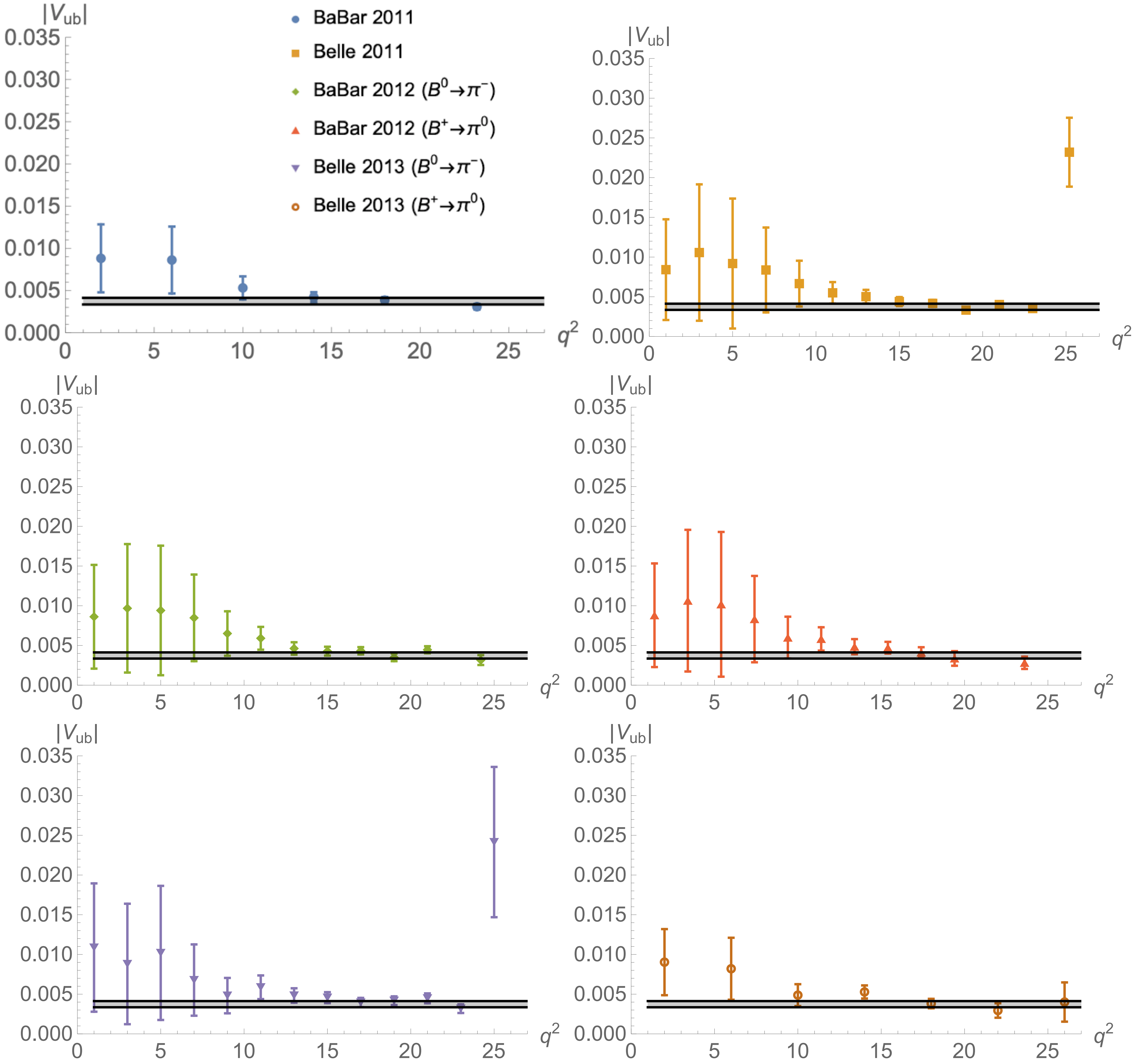


experimental data for $B \to \pi \ell \nu_\ell$ decays

* six sets of data from Belle and BaBar collaborations:

BaBar 2011, Belle 2011, BaBar 2012 ($B^0 \to \pi^-$), BaBar 2012 ($B^+ \to \pi^0$), Belle 2013 ($B^0 \to \pi^-$), Belle 2013 ($B^+ \to \pi^0$)

*** we do not mix theoretical calculations with experimental data to describe the shape of the FFs ***



bands are (correlated) weighted averages

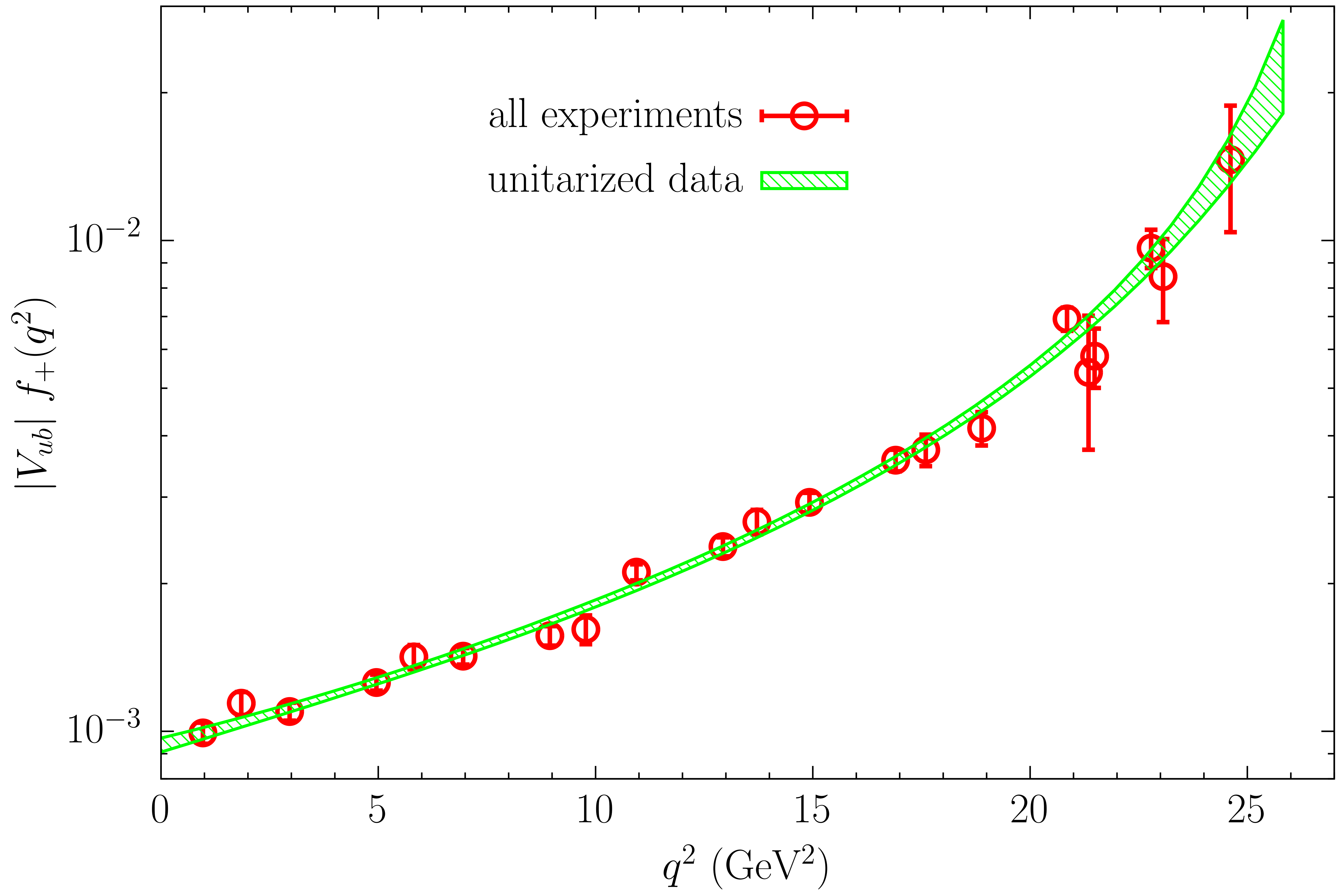
$$|V_{ub}|_n = \frac{\sum_{i,j} (\mathbf{C}^{-1})_{ij} |V_{ub}|_j}{\sum_{i,j} (\mathbf{C}^{-1})_{ij}}, \qquad \sigma^2_{|V_{ub}|_n} = \frac{1}{\sum_{i,j} (\mathbf{C}^{-1})_{ij}},$$

after averaging over the six exp.'s

input data	$\lvert V_{ub} \rvert$ x10^3
RBC/UKQCD	3.46 (46)
FNAL/MILC	3.74 (39)
combined	3.58 (43)
exclusive (FLAG '21)	3.74 (17)
inclusive (PDG/HFLAV)	4.32 (29)
inclusive (Belle '21)	4.10 (28)

a new strategy: unitarization of the data

* construct the experimental values of $|V_{ub} f_+(q_i^2)| = \sqrt{\Delta\Gamma_i / z_i}$ (z_i = kinematical coefficient in the i-th bin)
* apply the DM method on the data points $|V_{ub} f_+(q_i^2)|$ using the unitarity bound $|V_{ub}|^2 \chi_{1^-}(0)$ with an initial guess for $|V_{ub}|$
* determine $|V_{ub}|$ using the theoretical DM bands and iterate the procedure until consistency for $|V_{ub}|$ is reached



we still keep separate the theoretical calculations and the experimental data for describing the shape of the FFs

$$|V_{ub}|_{DM} \cdot 10^3 = 3.88 \pm 0.32$$

$$|V_{ub}|_{incl.} \cdot 10^3 = 4.32 \pm 0.29$$

difference of $\approx 1\sigma$

$|V_{ub}|_{excl.} \cdot 10^3 = 3.74 \pm 0.17$ (FLAG '21)

the use of exp. data to describe the shape of the FFs leads to a much smaller error (0.17), but the use of truncated BCL fits does not guarantee that the final error is not underestimated

Conclusions

* the Dispersion Matrix approach is an attractive tool to implement unitarity and lattice QCD calculations in the analysis of exclusive semileptonic decays of hadrons. The main features are:

  - it does not rely on any assumption about the momentum dependence of the hadronic form factors
  - it can be based entirely on first principles using lattice determinations both of the relevant form factors and of the dispersive bounds (the susceptibilities) from appropriate 2-point and 3-point (Euclidean) correlation functions
  - it allows to implement unitarity and kinematical constraints in a rigorous and parameterization-independent way
  - it predicts band of values that are equivalent to the infinite number of BGL (or BCL) fits satisfying unitarity and kinematical constraints and reproducing exactly a given set of data points
  - it can be applied to any exclusive semileptonic decay of hadrons

* we have applied the DM approach to $D \to K \ell \nu_\ell$ decays (2105.02497), to $B \to D^{(*)} \ell \nu_\ell$ decays (2105.08674, 2109.15248), to $B \to \pi \ell \nu_\ell$ and $B_s \to K \ell \nu_\ell$ decays (in preparation) and to $B_s \to D_s^{(*)} \ell \nu_\ell$ decays (in progress)

* results for $B \to D^{(*)} \ell \nu_\ell$ decays:

$$|V_{cb}|_{DM} \cdot 10^3 = 41.0 \pm 1.2 \qquad (B \to D)$$
$$= 41.3 \pm 1.7 \qquad (B \to D^*)$$

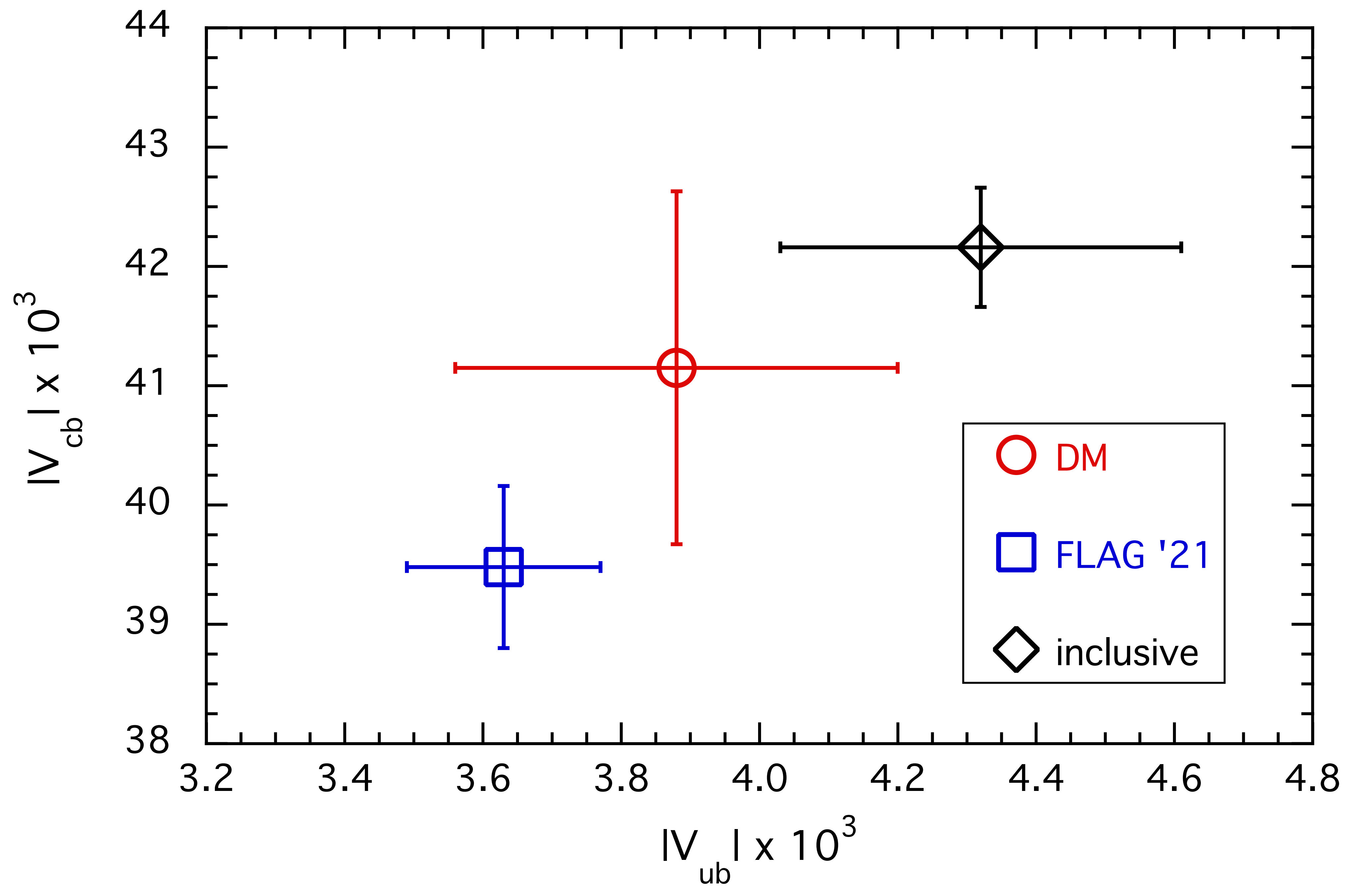
$$|V_{cb}|_{incl.} \cdot 10^3 = 42.16 \pm 0.50$$

differences $< 1\,\sigma$

	DM	experiment
R(D)	0.289 (8)	0.340 (27)
R(D*)	0.269 (8)	0.295 (11)

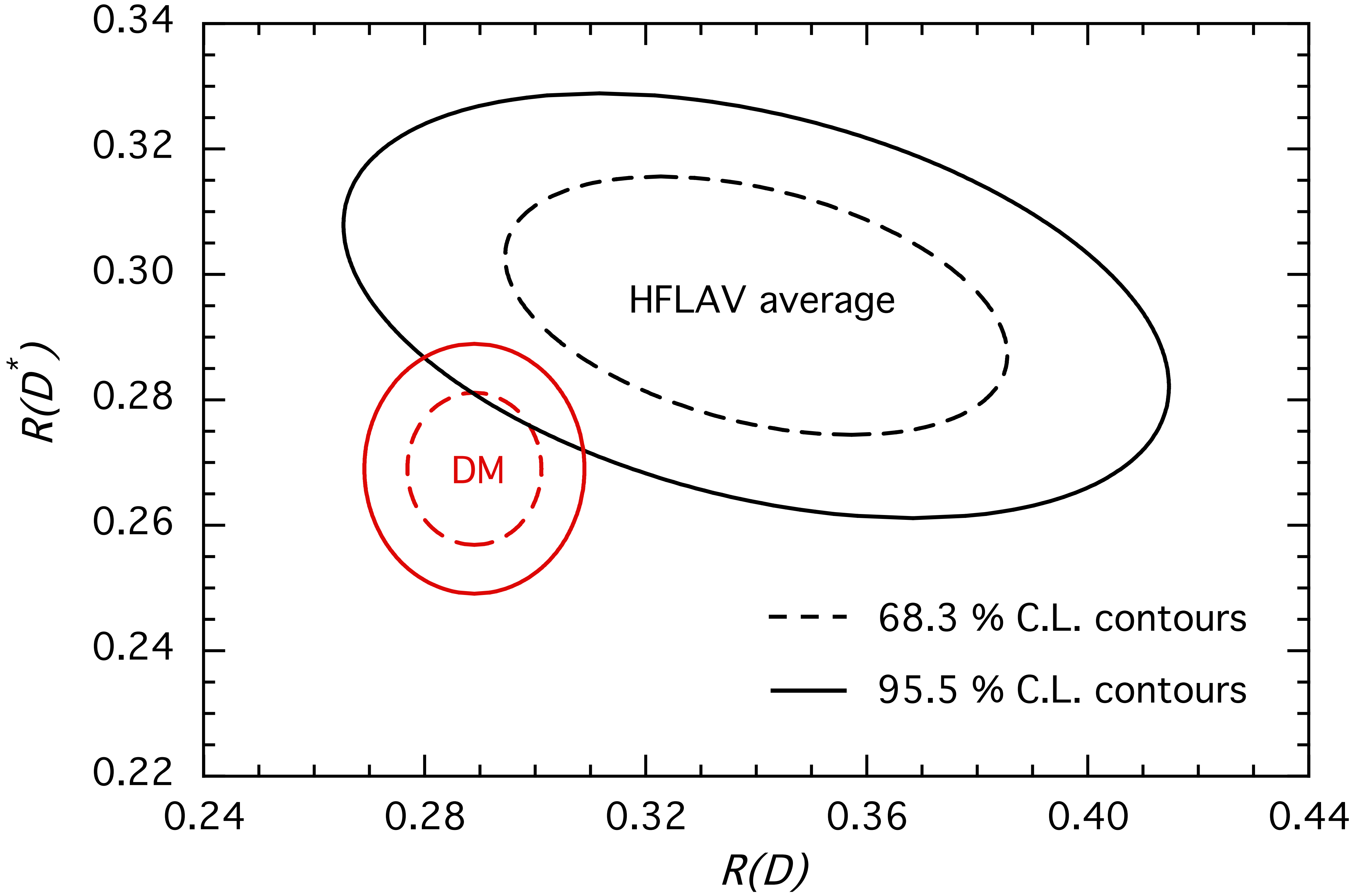
differences of $\approx 1.6\,\sigma$

* results for $B \to \pi \ell \nu_\ell$ decays: $|V_{ub}|_{DM} \cdot 10^3 = 3.88 \pm 0.32$ consistent with $|V_{ub}|_{incl.} \cdot 10^3 = 4.32 \pm 0.29$



	DM	FLAG '21	inclusive
$\lvert V_{ub}\rvert \cdot 10^3$	3.88 (32)	3.63 (14)	4.32 (29)
$\lvert V_{cb}\rvert \cdot 10^3$	41.15 (1.48)	39.48 (68)	42.16 (50)

average of $B \to D$ and $B \to D^*$



	DM	HFLAV '19
R(D)	0.289 (8)	0.340 (27) (13)
R(D*)	0.269 (8)	0.295 (11) (8)