Nonleptonic B decays

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Outline

- Introduction to nonleptonic decays
- Direct CP asymmetries to NLO
- Puzzles in tree-level color-allowed decays
- QCD factorisation and flavour symmetries
- Conclusion

Not covered: 3-body decays [see talk by D. Torres Machado, also Zou et al. 2003.03754; Mannel, Olschewsky, Vos 2003.12053; Fan, Wang 2006.08223; Li et al. 2007.13629; Cheng, Chiang, Chua 2011.03201, 2011.07468; Virto, Vos, TH 2007.08881; Lü et al. 2107.11079; Chai et al. 2109.00664; Bediaga et al. 2109.01625; ...]

CPV in mixing, lifetimes [see talks by V. Shtabovenko, L. Vale Silva and in WG4 on Tuesday, also Lenz, Rauh et al. 1711.02100, 1904.00940, 1909.11087, 1911.07856; Nierste et al. 1709.02160, 2006.13227, 2106.05979; ...]
Anatomy of nonleptonic $B$ decays

- Generic structure of amplitude for $B$ decays
  \[
  A(B \to f) = \sum_i \left[ \lambda_{\text{CKM}} \times C \times \langle f|O|B\rangle_{\text{QCD+QED}} \right]_i
  \]

- Interplay between
  - Wilson coefficients $C$ in $H_{\text{eff}}$, known to NNLL in SM
  [Bobeth,Misiak,Urban’99;Misiak,Steinhauser’04,Gorbahn,Haisch’04;Gorbahn,Haisch,Misiak’05;Czakon,Haisch,Misiak’06]
  - CKM factors $\lambda_{\text{CKM}}$. Hierarchy of CKM elements, weak phase
  - Hadronic matrix elements $\langle f|O|B\rangle$. Can contain strong phases.

- Interplay offers rich and interesting phenomenology for $B$ decays
  - Plethora of data, numerous observables
  - Test of CKM mechanism and indirect search for New Physics

- BUT: Challenging QCD dynamics in hadronic matrix elements.
  Effects from many different scales !!
Leptonic decays

\[ \langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B^- (p) \rangle = i f_B \, p^\mu \]
Exclusive $B$ decays, generalities

- **Leptonic decays**

$$
\langle 0|\bar{u}\gamma^\mu\gamma_5 b|B^-(p)\rangle = i f_B p^\mu
$$

- **Semi-leptonic decays**

$$
\langle D(p')|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = F_+(q^2) \left[ (p + p')^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu \right] + F_0(q^2) \frac{m_B^2 - m_D^2}{q^2} q^\mu
$$
Exclusive $B$ decays, generalities

- **Leptonic decays**
  \[
  \langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B^- (p) \rangle = i f_B \ p^\mu
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  \]

- **Non-leptonic decays**
  \[
  \langle \pi^- D^+ | Q_i | \bar{B} \rangle \simeq m_B^2 f_{M2} F_{+ \rightarrow D}^B (m_\pi^2) \times \int_0^1 du \ T_i^L(u) \phi_\pi(u)
  \]
Theory approaches based on factorisation

- Disentangle long and short distances

QCD Factorisation

- Systematic framework to all orders in $\alpha_s$ and leading power in $\Lambda/m_b$
- Problems with factorisation of power suppressed and annihilation contributions. Endpoint divergences.
- Countless pheno applications


[Beneke,Neubert’03; Cheng,Yang’08; Cheng,Chua’09; Bell,Pilipp’09; Beneke,Li,TH’09; Bobeth,Gorbahn,Vickers’14; Bell,Beneke,Li,TH’15’20]

[Beneke,Böer,Toelsted/Finauri,Vos’20’21, ...]
Theory approaches

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PQCD

- Based on $k_T$-factorisation. Organises amplitude differently
- Generates larger strong phases. Avoids endpoint divergences.
- Discussion of theoretical uncertainties difficult since no complete NLO ($O(\alpha_s^2)$) analysis available
- Also countless pheno applications
More theory approaches

- **Flavour symmetries:**
  
  Isospin, U-Spin ($d \leftrightarrow s$), V-Spin ($u \leftrightarrow s$), Flavour SU(3)

  [Zeppenfeld'81]

  [e.g. Savage,Wise'89; Gronau,Hernandez,London,Rosner'95; Chiang,Gronau,Rosner'08; Chiang,Zhou'06'08; Grossman,Ligeti,Robinson'13]

  [Cheng,Chiang,Kuo'14'16, Hsiao,Chang,He'15; . . . ]

  - Only few a priori assumptions about scales needed
  - Implementation of symmetry breaking difficult

  [Jung,Mannel'09; Cheng,Chiang'12; Grossman,Robinson'12]
More theory approaches

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Combination:

Recently: factorization-assisted topological-amplitude approach (FAT)

Many more analysis
Theory approaches

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- Combination:
  - Recently: factorization-assisted topological-amplitude approach (FAT)
    - [Li,Lü,Yu’12; Li,Lü,Qin,Yu’13; Wang,Zhang,Li,Lü’17]
  - Many more analysis
    - [Ali,Kramer,Lü’98; Descotes-Genon,Matias,Virto’06; Ciuchini,Silvestrini et al.; Nandi,Soni’10; ]
    - [Fleischer et al.’99+; Fleischer,Jaarsma,Malami,Vos’16+; Datta,London,Imbeault’03,’12; Cheng,Chua’09; Tetlalmatzi,TH’21 . . . ]

- Dalitz plot analysis. Applied to 3-body decays. Important for phenomenology.
  - Mostly data-driven, but also QCD-based predictions possible
    - [Kränkl,Mannel,Virto’15; Klein,Mannel,Virto,Vos’17]
  - Also Flavour-symmetry analyses
    - [e.g. Bhattacharya,Gronau,Imbeault,London,Rosner’14; Bhattacharya,London’15; Bediaga,Magalhaes et al. . . . ]
Amplitude in the limit $m_b \gg \Lambda_{\text{QCD}}$

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 F_+^{B \rightarrow M_1}(0) f_{M_2} \int_0^1 du \ T_I^I(u) \ \phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$

$$+ f_B f_{M_1} f_{M_2} \int_0^\infty d\omega \int_0^1 dv du \ T_{II}^I(\omega, v, u) \ \phi_B(\omega) \ \phi_{M_1}(v) \ \phi_{M_2}(u) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

\(T_{I,II}^I\): Hard scattering kernels, perturbatively calculable

\(F_+ : B \rightarrow M\) form factor

\(f_i : \text{decay constants}\)

\(\phi_i : \text{light-cone distribution amplitudes}\)

Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$
Classification of amplitudes

- $\alpha_1$: colour-allowed tree amplitude

- $\alpha_2$: colour-suppressed tree amplitude

- $\alpha_{u,c}^4$: QCD penguin amplitudes

\[
\sqrt{2} \left\langle \pi^- \pi^0 \mid H_{\text{eff}} \mid B^- \right\rangle = A_{\pi\pi} \lambda_u \left[ \alpha_1(\pi\pi) + \alpha_2(\pi\pi) \right] \\
\left\langle \pi^+ \pi^- \mid H_{\text{eff}} \mid \bar{B}^0 \right\rangle = A_{\pi\pi} \left\{ \lambda_u \left[ \alpha_1(\pi\pi) + \alpha_u^4(\pi\pi) \right] + \lambda_c \alpha_c^4(\pi\pi) \right\} \\
- \left\langle \pi^0 \pi^0 \mid H_{\text{eff}} \mid \bar{B}^0 \right\rangle = A_{\pi\pi} \left\{ \lambda_u \left[ \alpha_2(\pi\pi) - \alpha_u^4(\pi\pi) \right] - \lambda_c \alpha_c^4(\pi\pi) \right\} \\
\left\langle \pi^- \bar{K}^0 \mid H_{\text{eff}} \mid B^- \right\rangle = A_{\pi\bar{K}} \left[ \lambda_u^{(s)} \alpha_u^4 + \lambda_c^{(s)} \alpha_c^4 \right] \\
\left\langle \pi^+ K^- \mid H_{\text{eff}} \mid \bar{B}^0 \right\rangle = A_{\pi\bar{K}} \left[ \lambda_u^{(s)} (\alpha_1 + \alpha_u^4) + \lambda_c^{(s)} \alpha_c^4 \right] \\
\]

Tree amplitudes $\alpha_1$ and $\alpha_2$ known analytically to NNLO

[Bell’07’09; Beneke, Li, TH’09]

[Beneke, Neubert’03]
Direct CP asymmetries to NLO require QCD penguin amplitudes $\alpha_{4}^{u,c}$ at NNLO

Complicated calculation: $\mathcal{O}(100)$ diagrams, two loops, two scales, ...

Recently, also QED corrections became available
Direct CP asymmetries in percent.
Errors are CKM and hadronic, respectively.

<table>
<thead>
<tr>
<th></th>
<th>NLO</th>
<th>NNLO</th>
<th>NNLO + LD</th>
<th>Exp</th>
</tr>
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<tbody>
<tr>
<td>$\pi^- \bar{K}^0$</td>
<td>$0.71^{+0.13+0.21}_{-0.14} - 0.19$</td>
<td>$0.77^{+0.14+0.23}_{-0.15} - 0.22$</td>
<td>$0.10^{+0.02+1.24}_{-0.02} - 0.27$</td>
<td>$-1.7 \pm 1.6$</td>
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<td>$\pi^0 K^-$</td>
<td>$9.42^{+1.77+1.87}_{-1.76} - 1.88$</td>
<td>$10.18^{+1.91+2.03}_{-1.90} - 2.62$</td>
<td>$-1.17^{+0.22+20.00}_{-0.22} - 6.62$</td>
<td>$4.0 \pm 2.1$</td>
</tr>
<tr>
<td>$\pi^+ K^-$</td>
<td>$7.25^{+1.36+2.13}_{-1.36} - 2.58$</td>
<td>$8.08^{+1.52+2.52}_{-1.51} - 2.65$</td>
<td>$-3.23^{+0.61+19.17}_{-0.61} - 3.36$</td>
<td>$-8.2 \pm 0.6$</td>
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<td>$\pi^0 \bar{K}^0$</td>
<td>$-4.27^{+0.83+1.48}_{-0.77} - 2.23$</td>
<td>$-4.33^{+0.84+3.29}_{-0.78} - 2.32$</td>
<td>$-1.41^{+0.27+5.54}_{-0.25} - 6.10$</td>
<td>$1 \pm 10$</td>
</tr>
<tr>
<td>$\delta(\pi \bar{K})$</td>
<td>$2.17^{+0.40+1.39}_{-0.40} - 0.74$</td>
<td>$2.10^{+0.39+1.40}_{-0.39} - 2.86$</td>
<td>$2.07^{+0.39+2.76}_{-0.39} - 4.55$</td>
<td>$12.2 \pm 2.2$</td>
</tr>
<tr>
<td>$\Delta(\pi \bar{K})$</td>
<td>$-1.15^{+0.21+0.55}_{-0.22} - 0.84$</td>
<td>$-0.88^{+0.16+1.31}_{-0.17} - 0.91$</td>
<td>$-0.48^{+0.09+1.09}_{-0.09} - 1.15$</td>
<td>$-14 \pm 11$</td>
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</table>

\[
\delta(\pi \bar{K}) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)
\]

\[
\Delta(\pi \bar{K}) = A_{CP}(\pi^+ K^-) + \frac{\Gamma_{\pi^- \bar{K}^0}}{\Gamma_{\pi^+ K^-}} A_{CP}(\pi^- \bar{K}^0) - \frac{2\Gamma_{\pi^0 K^-}}{\Gamma_{\pi^+ K^-}} A_{CP}(\pi^0 K^-) - \frac{2\Gamma_{\pi^0 K^0}}{\Gamma_{\pi^+ K^-}} A_{CP}(\pi^0 \bar{K}^0)
\]

[For QED corrections see Beneke,Böer,Finauri/Toelstede,Vos’20’21 and Keri Vos’ talk on Monday]

[For a first number on $A_{CP}(\mathcal{B}^0 \rightarrow \pi^0 K^0)$ see 2104.14871 + talk by Hazra]
Determine $b$-quark fragmentation fractions $f_s/f_d$ from hadronic two-body decays into heavy-light final states

Requires ratio

$$\mathcal{R}_{s/d}^{P(V)} \equiv \frac{\mathcal{B}(\bar{B}_s^0 \to D_s^{(*)}+\pi^-)}{\mathcal{B}(\bar{B}_s^0 \to D^{(*)}+K^-)}$$
Two-body heavy-light final states

- Determine $b$-quark fragmentation fractions $f_s/f_d$ from hadronic two-body decays into heavy-light final states

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\[
\mathcal{R}_{s/d}^{P(V)} \equiv \frac{\mathcal{B}(\bar{B}_s \to D_s^{(*)+}\pi^-)}{\mathcal{B}(\bar{B}^0 \to D^{(*)+}K^-)}
\]

- QCD factorization for $\bar{B}_q \to D_q^{(*)+}L^-$ decays

\[
\langle D_q^{(*)+}L^- | Q_i | \bar{B}_q^0 \rangle = \sum_j F_{ij}^{\bar{B}_q \to D_q^{(*)}} (M_L^2) \int_0^1 du T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)
\]

- Particularly clean: Only colour-allowed tree amplitude
  - No colour-suppressed tree amplitude, no penguins
  - Spectator scattering and weak annihilation power suppressed
  - Weak annihilation absent if all final-state flavours distinct
    - as in $\bar{B}_s^0 \to D_s^+\pi^-$ and $\bar{B}^0 \to D^+K^-$ but not in $\bar{B}^0 \to D^+\pi^-$
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$$\langle D_q^{(*)+} L^- | Q_i | \bar{B}_q^0 \rangle = \sum_j F_{j}^{\bar{B}_q^0 \to D_q^{(*)+}} (M_L^2) \int_0^1 du T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{QCD} \sqrt{m_b}}{m_b}\right)$$

- Particularly clean: Only colour-allowed tree amplitude
  - No colour-suppressed tree amplitude, no penguins
  - Spectator scattering and weak annihilation power suppressed
  - Weak annihilation absent if all final-state flavours distinct
    - as in $\bar{B}_s^0 \to D_s^+ \pi^-$ and $\bar{B}^0 \to D^+ K^-$ but not in $\bar{B}^0 \to D^+ \pi^-$
  - Hard function known to $\mathcal{O}(\alpha_s^2)$
  - Form factors from recent precision study

[Sources: Bordone, Gubernari, Jung, van Dyk, TH'20; Beneke, Buchalla, Neubert, Sachrajda'99-'04; Kränkl, Li, TH'16; Bordone, Gubernari, Jung, van Dyk'19]
Power corrections arise from several effects

- Higher twist effects to the light-meson LCDA
- Hard-collinear gluon emission from the spectator quark $q$
- Hard-collinear gluon emission from the heavy quarks $b$ and $c$
- Soft-gluon exchange between $B \rightarrow D$ and light-meson system
Subleading power

Power corrections arise from several effects

- Higher twist effects to the light-meson LCDA
- Hard-collinear gluon emission from the spectator quark $q$
- Hard-collinear gluon emission from the heavy quarks $b$ and $c$
- Soft-gluon exchange between $B \rightarrow D$ and light-meson system

Estimate of total size of power corrections

$$\left. R_{s/d}^P \right|_{NLP} / \left. R_{s/d}^P \right|_{LP} - 1 \approx -1.7\%$$

$$\left. R_{s/d}^V \right|_{NLP} / \left. R_{s/d}^V \right|_{LP} - 1 \approx -1.7\%$$

- Supports the picture of these decays being very clean
### Results

<table>
<thead>
<tr>
<th>source scenario</th>
<th>PDG</th>
<th>our fit (w/ QCDF, no $f_s/f_d$) ratios only</th>
<th>$\mathcal{B}(U(3))$</th>
<th>QCDF prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$/dof</td>
<td>—</td>
<td>—</td>
<td>4.6/6</td>
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</tr>
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<td>$\mathcal{B}(B^0 \to D^+_s \pi^-)$</td>
<td>3.00 ± 0.23</td>
<td>3.11$^{+0.21}<em>{-0.19}$ 3.20$^{+0.20}</em>{-0.26}$</td>
<td>4.42 ± 0.21</td>
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<td>$\mathcal{B}(B^0 \to D^+_s K^-)$</td>
<td>0.186 ± 0.020</td>
<td>0.227 ± 0.012 0.226 ± 0.012</td>
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<td>$\mathcal{B}(B^0 \to D^+ \pi^-)$</td>
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<td>2.0 ± 0.5</td>
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<td>4.3$^{+0.9}_{-0.8}$</td>
<td></td>
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<td>$\mathcal{R}_{s/d}$</td>
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<td>9.4 ± 2.5</td>
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<td>$\mathcal{R}_{s/d}^V$</td>
<td>0.66 ± 0.16</td>
<td>0.81$^{+0.12}<em>{-0.11}$ 0.76$^{+0.11}</em>{-0.10}$</td>
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<td>1.14 ± 0.15</td>
<td>0.97 ± 0.06 0.95 ± 0.07</td>
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<td>$(f_s/f_d)^7$</td>
<td>—</td>
<td>0.261$^{+0.018}<em>{-0.016}$ 0.252$^{+0.015}</em>{-0.015}$</td>
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- **BR discrepancies**
  - $\bar{B}_s^0 \to D_s^+ \pi^- \to 4\sigma$
  - $\bar{B}_s^0 \to D^+ K^- \to 5\sigma$
  - $\bar{B}_s^0 \to D_s^{*+} \pi^- \to 2\sigma$
  - $\bar{B}_s^0 \to D_s^{*+} K^- \to 3\sigma$

- **Ratios OK**
## Results

### BR discrepancies

- \( \bar{B}_s^0 \to D_s^+ \pi^- \) → 4σ
- \( \bar{B}_s^0 \to D_s^+ K^- \) → 5σ
- \( \bar{B}_s^0 \to D_s^{*-} \pi^- \) → 2σ
- \( \bar{B}_s^0 \to D_s^{*-} K^- \) → 3σ

### Ratios OK

### Table

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### Plot

[Plot courtesy of N. Skidmore]
Potential explanations

- Universal non-factorizable contributions of $\mathcal{O}(-15 - 20\%)$ to amplitude?
- QED corrections
  - Ease the tension, but are not large enough
- Experimental issues?
- Shift or larger uncertainties in the input (CKM) parameters?
- Rescattering effects are also too small
- BSM physics?
- Combination thereof?
Interpretation

- Potential explanations
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Mini-Workshop on Colour Allowed Non-Leptonic Tree-Level Decays

25 March 2021 to 1 April 2021
Europe/Berlin timezone

https://indico.scc.kit.edu/event/2352/
Further developments, BSM

- Tension can be partially explained by a left-handed $W'$ model, compatible with other flavor and collider bounds

[Iguro, Kitahara'20; see Iguro's talk]

- New tensor structures

[Cai, Deng, Li, Yang'21]

- Some of them can explain the data at the $1\sigma$-level

- Also model-dependent analysis, e.g. with colorless charged scalar

[Bordone, Greljo, Marzocca'21]

- Combine with dijet searches

[Consider mediators with various $SU(3)\times SU(2)\times U(1)$ quantum numbers]
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- Tension can be partially explained by a left-handed $W'$ model, compatible with other flavor and collider bounds
  
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[Iguro, Kitahara'20; see Iguro's talk]

[Cai, Deng, Li, Yang'21]
Further developments, BSM

- Combine nonleptonics and lifetimes

\[ \frac{\tau_{B_s}}{\tau_{B_d}} \]

\[ \delta(\tau_{B_s}/\tau_{B_d}) = 0.0020 \]

\[ \delta(\tau_{B_s}/\tau_{B_d}) = 0.0010 \]

Current status

\[ \frac{\tau_{B_s}}{\tau_{B_d}} \]

\[ \delta(\tau_{B_s}/\tau_{B_d}) \text{ Theor.} = \delta(\tau_{B_s}/\tau_{B_d}) \text{ Exp.} = \delta(\tau_{B_s}/\tau_{B_d}) \]

New-physics searches using \( B_0^s \rightarrow D^{\pm} s K^{\mp} \) [Fleischer, Malami'21; see Malami’s talk this session]

Flavour-specific nonleptonic decays are sensitive to CP violation in \( B_0^0 (s) - \bar{B}_0^0 (s) \) mixing [Gershon, Lenz, Rusov, Skidmore'21; see Rusov’s talk]

BSM contributions to nonleptonic decay amplitudes could give significant enhancements to flavour-specific CP asymmetries

T. Huber
Further developments, BSM

- Combine nonleptonics and lifetimes

![Graph showing Re $\Delta C^s_{1,cc} (M_W)$ vs. Im $\Delta C^s_{1,cc} (M_W)$ with $\tau_{B_s}/\tau_{B_d}$ on the x-axis.]

- New-physics searches using $B^0_s \rightarrow D^+_s K^\pm$

- Flavour-specific nonleptonic decays are sensitive to CP violation in $B^0_{(s)} - \bar{B}^0_{(s)}$ mixing
  - BSM contributions to nonleptonic decay amplitudes could give significant enhancements to flavour-specific CP asymmetries.

$$A_{fs}^q = \frac{\Gamma (\bar{B}_q(t) \rightarrow f) - \Gamma (B_q(t) \rightarrow f)}{\Gamma (\bar{B}_q(t) \rightarrow f) + \Gamma (B_q(t) \rightarrow f)}$$

[Tetlalmatzi, Lenz’19]

[Fleischer, Malami’21; see Malami’s talk this session]

[Gershon, Lenz, Rusov, Skidmore’21; see Rusov’s talk]
The amplitudes for $B \rightarrow PP$ ($P$ a pseudoscalar meson) can be expressed as

$$ \mathcal{A} = i \frac{G_F}{\sqrt{2}} [\mathcal{T} + \mathcal{P}] $$

$\mathcal{T}$: Tree sub-amplitudes.  $\mathcal{P}$: Penguin sub-amplitudes.

Topological decomposition of the sub-amplitudes [He,Wang'18]

$$ \mathcal{T}^{TDA} = T B_i(M)^i_j \bar{H}^{lj}_k(M)^l_k + C B_i(M)^i_j \bar{H}^{lj}_k(M)^l_k + A B_i \bar{H}^{il}_j(M)^j_k(M)^l_k $$

$$ + E B_i \bar{H}^{li}_j (M)^j_k (M)^l_k + T_{ES} B_i \bar{H}^{ij}_l (M)^l_j (M)^k_k + T_{AS} B_i \bar{H}^{ji}_l (M)^l_j (M)^k_k $$

$$ + T_{S} B_i (M)^i_j \bar{H}^{lj}_l (M)^l_k (M)^k_j + T_{PA} B_i \bar{H}^{li}_l (M)^l_j (M)^k_j + T_{P} B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l $$

$$ + T_{SS} B_i \bar{H}^{li}_l (M)^j_j (M)^k_k $$

(Insert diagrams here)
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$$+ T_S B_i (M)^i_j \bar{H}^j l (M)^k_l + T_{PA} B_i \bar{H}^i j l (M)^j_k (M)^l_k + T_P B_i (M)^i_j (M)^j_k \bar{H}^l k$$

$$+ T_{SS} B_i \bar{H}^i l j (M)^k_j (M)^k_l$$

$(B_i) = (B^+, B_d^0, B_s^0)$

$$(M^i_j) = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0_q}{\sqrt{2}} + \frac{\eta^0_{q'}}{\sqrt{2}} \\
\pi^+ \\
\frac{\pi^-}{\sqrt{2}} + \frac{\eta^-_q}{\sqrt{2}} + \frac{\eta^-_{q'}}{\sqrt{2}} \\
K^+ \\
\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0_q}{\sqrt{2}} + \frac{\eta^0_{q'}}{\sqrt{2}} \\
K^0 \\
\eta_s + \eta^s_{q'}
\end{pmatrix}$$
QCD-factorization and flavour symmetries

- The amplitudes for $B \to PP$ ($P$ a pseudoscalar meson) can be expressed as

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$$\mathcal{T}^{TDA} = T B_i(M)^i_j \bar{H}^{jl}_{k} (M)^{k}_l + C B_i(M)^i_j \bar{H}^{lj}_{k} (M)^{k}_l + A B_i \bar{H}^{il}_{j} (M)^{j}_k (M)^{k}_l$$

$$+ E B_i \bar{H}^{li}_{j} (M)^{j}_k (M)^{k}_l + T_{ES} B_i \bar{H}^{ij}_{l} (M)^{l}_j (M)^{j}_k + T_{AS} B_i \bar{H}^{ij}_{l} (M)^{l}_j (M)^{j}_k$$

$$+ T_{S} B_i (M)^i_j \bar{H}^{lj}_{k} (M)^{l}_k + T_{PA} B_i \bar{H}^{li}_{l} (M)^{k}_j (M)^{j}_k + T_{P} B_i (M)^i_j (M)^{j}_k \bar{H}^{lk}_l$$

$$+ T_{SS} B_i \bar{H}^{li}_{l} (M)^{j}_j (M)^{j}_k$$

- The $\bar{H}^{ij}_{k}$ are contain the CKM elements, e.g.

$$\bar{H}^{12}_1 = \lambda^{(d)}_u, \quad \bar{H}^{13}_1 = \lambda^{(s)}_u.$$
QCD-factorization and flavour symmetries

**SU(3)-irreducible decomposition of the sub-amplitudes**

\[ T^{IRA} = A_3^T B_i(\bar{H}_3)^i(M)_k(M)^j_j + C_3^T B_i(M)_j^j(M)_k^k(M)^i_j + B_3^T B_i(\bar{H}_3)^i(M)_k^k(M)^j_j \\
+ D_3^T B_i(M)^i_j(\bar{H}_3)^j(M)_k^k + A_6^T B_i(\bar{H}_6)^i_j(M)_k^k(M)^l_l + C_6^T B_i(M)^i_j(\bar{H}_6)^j_l(M)_k^k \\
+ B_6^T B_i(\bar{H}_6)^i_j(M)_k^k(M)^l_l + A_{15}^T B_i(\bar{H}_{15})^i_j(M)_k^k(M)^l_l + C_{15}^T B_i(M)^i_j(\bar{H}_{15})^j_k^k(M)_k^k \\
+ B_{15}^T B_i(\bar{H}_{15})^i_j(M)_k^k(M)^l_l. \]

**SU(3) decomposition:**

\[ H_k^{ij} = \frac{1}{8} (H_{15})^{ij}_k + \frac{1}{4} (H_6)^{ij}_k - \frac{1}{8} (H_3)^{i}_j \delta^j_k + \frac{3}{8} (H_3')^j_k \delta^i_k \]

**Gives linear relations** between topological and \(SU(3)\)-invariant amplitudes, e.g. [He,Wang'18]

\[ A_3^T = -\frac{A}{8} + \frac{3E}{8} + T_{PA}, \quad B_3^T = T_{SS} + \frac{3T_{AS} - T_{ES}}{8}, \]

\[ A_6^T = \frac{1}{4}(A - E), \quad B_6^T = \frac{1}{4}(T_{ES} - T_{AS}) \]
QCD-factorization and flavour symmetries

- Determine the SU(3)-invariant amplitudes through a $\chi^2$-fit.
  - 20 complex amplitudes (10 for trees, 10 for penguins)
  - One overall phase and the complex amplitudes $A_T^6$ and $A_P^6$ can be absorbed
    \[ \implies \text{ 35 real parameters.} \]

- Use the following experimental input for branching fractions and CP asymmetries
  - Branching fractions: 23 measurements plus 6-upper bounds
  - CP Asymmetries: 17 measurements plus 1-upper bound

- Implement $\eta-\eta'$ mixing in the FKS scheme (a single mixing angle)

- Determine uncertainties through likelihood ratio test, determine $p$ value from Wilk's theorem with 2 degrees of freedom.
**SU(3) fit: Results**

- **Good overall fit quality:** \( \chi^2/d.o.f. = 0.851 \)

---

**Figures:**

- **Left Top:** Imaginary part of \( A_T^3 \) vs. real part of \( A_T^3 \) in GeV.
- **Right Top:** Imaginary part of \( A_T^{15} \) vs. real part of \( A_T^{15} \) in GeV.
- **Left Bottom:** Imaginary part of \( B_P^3 \) vs. real part of \( B_P^3 \) in GeV.
- **Right Bottom:** Imaginary part of \( B_T^{3} \) vs. real part of \( B_T^{3} \) in GeV.
SU(3) fit: Results

[Tetlai matzi-Xolocotzi, TH'21]

**Prediction** for observables which have not been measured so far

<table>
<thead>
<tr>
<th>Channel</th>
<th>Branching ratio in units of 10^{-6}</th>
<th>Channel</th>
<th>Branching ratio in units of 10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Theoretical</td>
<td>Experimental</td>
</tr>
<tr>
<td>$B^- \rightarrow \pi^0\pi^-$</td>
<td>5.5 ± 0.4</td>
<td>6.04^{+2.42}_{-2.51}</td>
<td>$B^- \rightarrow \eta\pi^-$</td>
</tr>
<tr>
<td>$B^- \rightarrow K^0 K^-$</td>
<td>1.31 ± 0.17</td>
<td>1.36^{+0.17}_{-0.16}</td>
<td>$B^- \rightarrow \eta'\pi^-$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow \pi^+\pi^-$</td>
<td>5.12 ± 0.19</td>
<td>6.31^{+0.61}_{-0.50}</td>
<td>$\bar{B}^0 \rightarrow \eta\pi^0$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow \pi^0\pi^0$</td>
<td>1.59 ± 0.26</td>
<td>1.01^{+1.30}_{-0.51}</td>
<td>$\bar{B}^0 \rightarrow \eta'\pi^0$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow K^+ K^-$</td>
<td>0.078 ± 0.015</td>
<td>0.13^{+0.08}_{-0.07}</td>
<td>$\bar{B}_s \rightarrow \eta K^0$</td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow K^0 K^0$</td>
<td>1.21 ± 0.16</td>
<td>1.13^{+0.83}_{-0.91}</td>
<td>$\bar{B}_s \rightarrow \eta' K^0$</td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow \pi^- K^+$</td>
<td>5.8 ± 0.7</td>
<td>7.75^{+0.63}_{-0.09}</td>
<td>$B^- \rightarrow \eta K^-$</td>
</tr>
<tr>
<td>$B^- \rightarrow \pi^0 K^-$</td>
<td>12.9 ± 0.5</td>
<td>12.78^{+1.75}_{-1.94}</td>
<td>$B^- \rightarrow \eta' K^-$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>CP asymmetries in percent</th>
<th>Channel</th>
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<tbody>
<tr>
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<td>Theoretical</td>
<td>Experimental</td>
</tr>
<tr>
<td>$B^- \rightarrow \pi^0\pi^-$</td>
<td>3 ± 4</td>
<td>5.45^{+22.02}_{-20.60}</td>
<td>$B^- \rightarrow \eta\pi^-$</td>
</tr>
<tr>
<td>$B^- \rightarrow K^0 K^-$</td>
<td>4 ± 14</td>
<td>18.82^{+36.93}_{-30.83}</td>
<td>$B^- \rightarrow \eta'\pi^-$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow \pi^+\pi^-$</td>
<td>32 ± 4</td>
<td>35.01^{+32.19}_{-22.29}</td>
<td>$\bar{B}_s \rightarrow \eta K^0$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow \pi^0\pi^0$</td>
<td>33 ± 22</td>
<td>$-10.58^{+40.69}_{-89.40}$</td>
<td>$\bar{B}_s \rightarrow \eta' K^0$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow K^0 K^0$</td>
<td>$-60 \pm 70$</td>
<td>$-6.88^{+85.39}_{-81.37}$</td>
<td>$B^- \rightarrow \eta K^-$</td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow \pi^- K^+$</td>
<td>22.1 ± 1.5</td>
<td>20.84^{+2.39}_{-2.57}</td>
<td>$B^- \rightarrow \eta' K^-$</td>
</tr>
</tbody>
</table>
QCD-factorization and flavour symmetries

- Investigate connection to QCD factorization (QCDF)
- Amplitudes for two body non-leptonic $B$-meson decays in QCDF [Beneke,Neubert'03]

$$A_{\text{QCDF}} = \sum_{p=u,c} A_{M_1 M_2} \left\{ \begin{array}{l} BM_1 \left( \alpha_1 \delta_{p u} \hat{U} + \alpha_4^p \hat{I} + \alpha_{4, \text{EW}}^p \hat{Q} \right) M_2 \Lambda_p \\ + BM_1 \Lambda_p \cdot \text{Tr} \left[ (\alpha_2 \delta_{p u} \hat{U} + \alpha_3^p \hat{I} + \alpha_{3, \text{EW}}^p \hat{Q} ) M_2 \right] \\ + B \left( \beta_2 \delta_{p u} \hat{U} + \beta_3^p \hat{I} + \beta_{3, \text{EW}}^p \hat{Q} \right) M_1 M_2 \Lambda_p \\ + B \Lambda_p \cdot \text{Tr} \left[ (\beta_1 \delta_{p u} \hat{U} + \beta_4^p \hat{I} + \beta_{4, \text{EW}}^p \hat{Q} ) M_1 M_2 \right] \\ + B \Lambda_p \cdot \text{Tr} \left[ (\beta_5 \delta_{p u} \hat{U} + \beta_6^p \hat{I} + \beta_{6, \text{EW}}^p \hat{Q} ) M_1 \right] \cdot \text{Tr} M_2 \end{array} \right\}$$

- Establish transformation rules between the different approaches [Tetlalmatzi-Xolocotzi,TH'21]

$$T = A_{M_1 M_2} \left[ \alpha_1 + \frac{3}{2} \alpha_{4, \text{EW}}^u - \frac{3}{2} \alpha_{4, \text{EW}}^c \right], \quad C = A_{M_1 M_2} \left[ \alpha_2 + \frac{3}{2} \alpha_{3, \text{EW}}^u - \frac{3}{2} \alpha_{3, \text{EW}}^c \right],$$

$$E = A_{M_1 M_2} \left[ \beta_1 + \frac{3}{2} \beta_{4, \text{EW}}^u - \frac{3}{2} \beta_{4, \text{EW}}^c \right], \quad A = A_{M_1 M_2} \left[ \beta_2 + \frac{3}{2} \beta_{3, \text{EW}}^u - \frac{3}{2} \beta_{3, \text{EW}}^c \right]$$

- Translate fit results into constraints on QCDF amplitudes
  - Quantify the size of the annihilation amplitudes $\beta_i$ and $b_i$ as dictated by data
Annihilation amplitudes get constrained between the $\mathcal{O}(0.04)$ and $\mathcal{O}(0.3)$ level.
Interesting patterns in data vs. theory for color-allowed, tree-level nonleptonic decays

- Data consistently below theory, discrepancy up to $\sim 5\sigma$
- We will learn something from this situation:
  - power corrections in QCDF, input parameters, new physics, . . .

Precision in charmless nonleptonic decays must be further increased

- Get power corrections / flavour symmetry breaking better under control
- More data will help, in particular on direct CP asymmetries

$A_{CP}(B^0 \rightarrow \pi^0 \bar{K}^0)$ [2104.14871 + talk by Hazra], $A_{CP}(B^0_{(s)} \rightarrow \eta P)$, . . .