Strong Phase Measurements at BESIII

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Outline

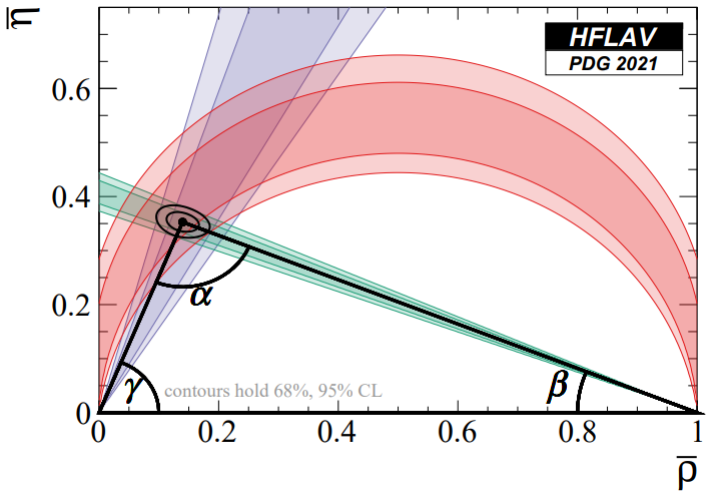
- **CKM Angle γ/φ_3**
  - Measurement of γ/φ_3
  - Measurement of strong phase parameters
- **BESIII Experiment**
  - BEPCII & BESIII
  - Quantum correlated $D^0\overline{D}^0$ sample at BESIII
- **Strong Phase Measurement**
  - $D^0 \to K_S^0\pi^+\pi^-$
  - $D^0 \to K_S^0K^+K^-$
  - $D^0 \to K^-\pi^+\pi^+\pi^-$ & $K^-\pi^+\pi^0$
- **Summary**

Measurement of γ/φ_3

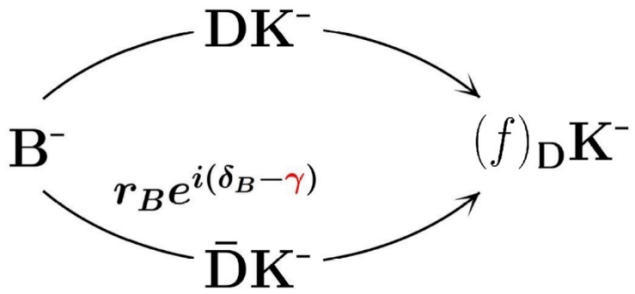
- **CKM Matrix**
  - Only source of CPV in SM
  - γ can be determined in tree level process with small NP effects and theoretical uncertainty

$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = (66.2^{+3.4}_{-3.6})^\circ$$

[HFLAV & PDG2021]



- **Measurement of γ/φ_3**
  - Measured in the tree level decay $B \to DK$
  - Inputs from $D^0 \to f$ and $\bar{D}^0 \to f$ decays are needed



$$\frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} = r_B e^{i(\delta_B - \gamma)}$$

$$\frac{A(D^0 \to f)}{A(\bar{D}^0 \to f)} = r_D^f e^{i\delta_D^f}$$

- **Extract γ in different D decays :**
  - ADS: CF and DCS decays (eg. $K\pi, K\pi\pi^0$) $\leftarrow R_f, \delta_D^f$
    [PRL 78 (1997) 3257; PRD 63 (2001) 036005]
  - GLW: (Quasi-)CP eigenstates (eg. $KK, \pi^+\pi^-\pi^0$) $\leftarrow F_+$
    [PLB 265 (1991) 172; PLB 253 (1991) 483]
  - GGSZ: Multi-body Self-conjugate decay (eg. $K_S^0\pi^+\pi^-$) $\leftarrow c_i, s_i$
    [PRD 68 (2003) 054018; PRD 67 (2003) 071301]

Measurement of Strong Phase Parameters

- **Quantum correlated (QC) $D^0\overline{D}^0$ decay at $\psi(3770)$**

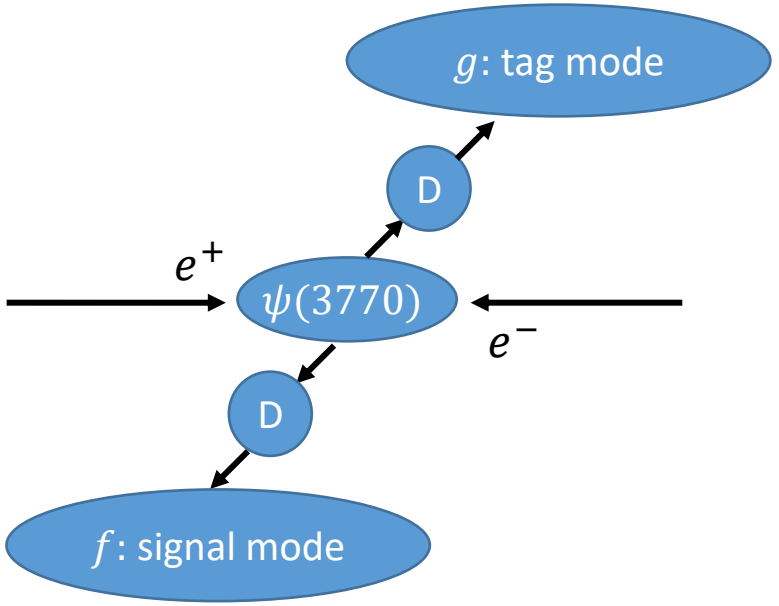
$$e^+e^- \to \psi(3770) \to D^0\overline{D}^0 \to fg$$

$$C_{\psi(3770)} = -1 \Downarrow$$

$$|\psi(3770)\rangle \to \frac{1}{\sqrt{2}}(|D^0\rangle|\overline{D}^0\rangle - |\overline{D}^0\rangle|D^0\rangle) \Downarrow \text{ ignore } D^0\text{-}\overline{D}^0 \text{ mixing}$$

$$\Gamma(f|g) \propto [(r_D^f)^2 + (r_D^g)^2 - 2r_D^f r_D^g R_f R_g \cos(\delta_D^f - \delta_D^g)]$$

$\propto$ Number of events; coherence factor; strong phase difference



- **Strong phase parameters of $D^0 \to f$ decay can be measured in quantum correlated $D^0\overline{D}^0$ data**

BEPCII & BESIII

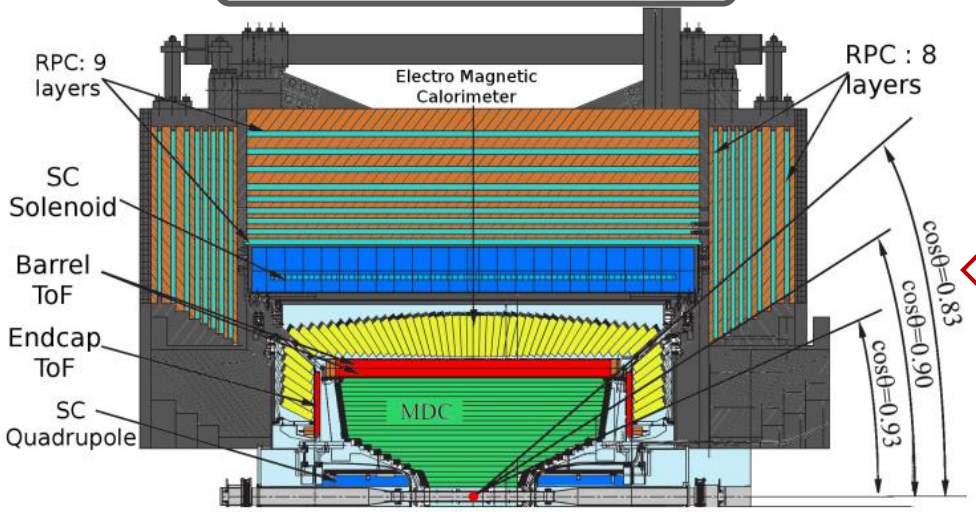
MDC: small cell & Gas:
$He/C_3H_8(60/40)$, 43 layers
$\sigma_p/p = 0.5\%@1GeV$,
$\sigma_{dE/dx} = 6\%$

TOF: Barrel: $\sigma_T = 100ps$
endcap: $\sigma_T = 110ps$
(60ps for endcap after upgraded to MRPC in 2015)

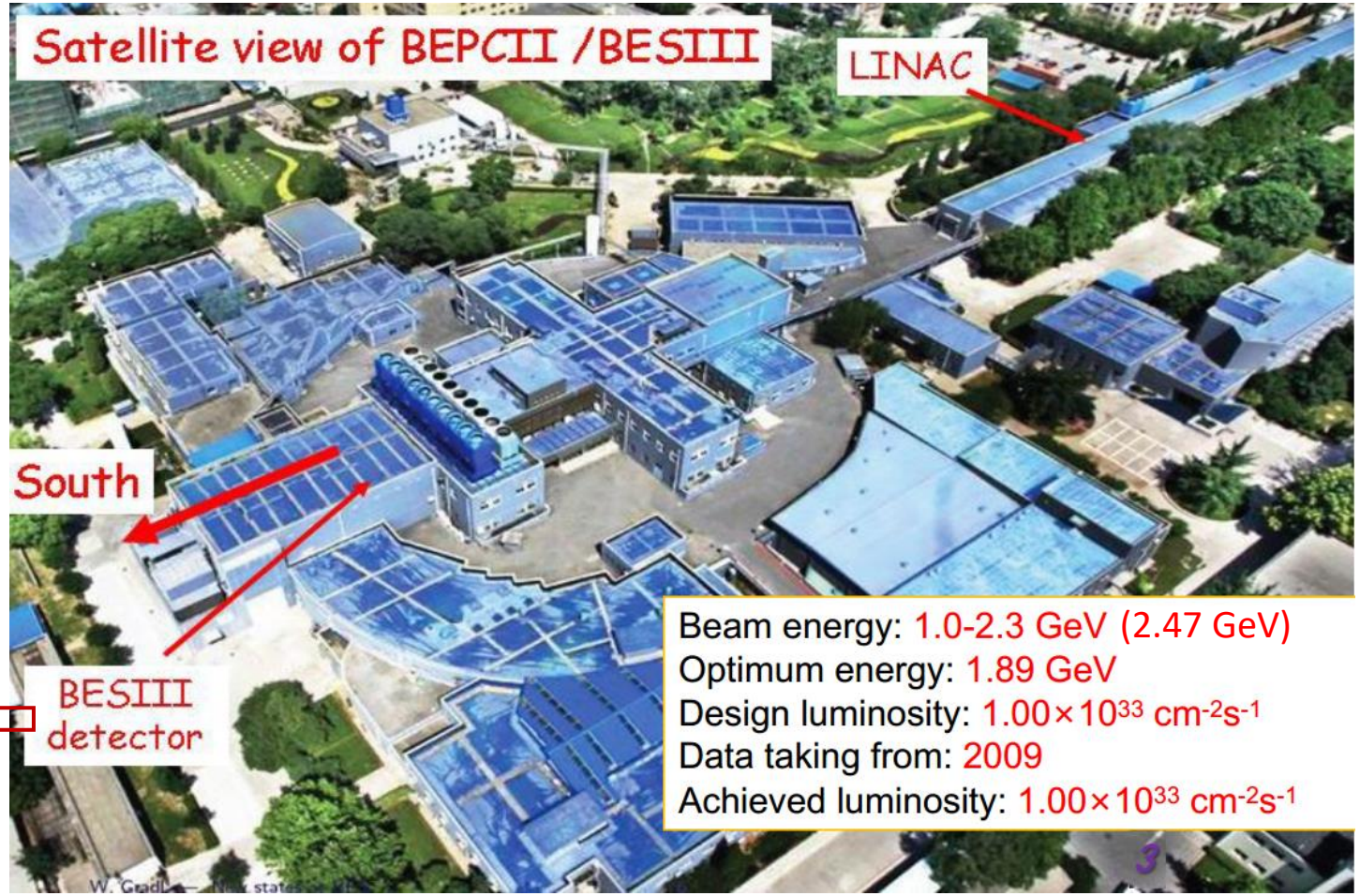
EMC: CsI crystal. 28cm
$\Delta E/E = 2.5\%@1GeV$,
$\sigma_z = 0.6cm/\sqrt{E}$

MUC: 9layers RPC
(8 layers in Endcap)
$\sigma_{R\phi} = 1.4\sim1.7cm$

Magnet: 1T Super conducting



[Nucl. Instr. Meth. A614, 345(2010)]

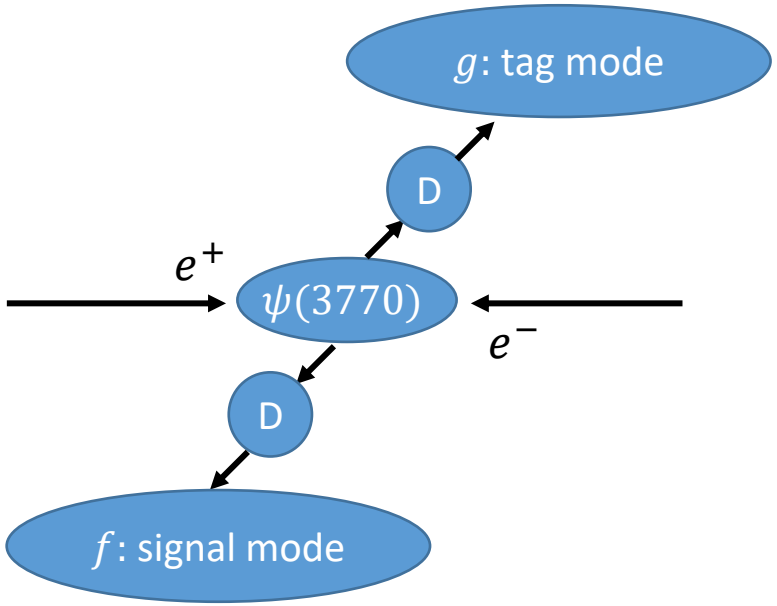


Quantum Correlated $D^0\overline{D}^0$ Data at BESIII

- **Quantum correlated $D^0\overline{D}^0$ produced at BESIII**

$$e^+e^- \to \psi(3770) \to D^0\overline{D}^0$$

  - 2.93fb^{-1}@ $\text{E}_{\text{cm}} = 3.773$ GeV (~3.6x CLEO's)
  - ~10.5M $D^0\overline{D}^0$ pairs produced



- **Analysis method (pair production):**
  - Single Tag(ST): reconstruct one of $D\overline{D}$
  - Double Tag(DT): reconstruct both of $D\overline{D}$
- **Advantages:**
  - Absolute branching fraction
  - Quantum correlated $D^0\overline{D}^0$
  - Clean background
  - Full kinematic constraint reconstruct missing particle (ν, K_L^0)
- **Typical Tag modes:**

Flavor	$K^\pm\pi^\mp, K^\pm\pi^\mp\pi^0, K^\pm\pi^\mp\pi^\pm\pi^\mp, K^\pm e^\mp\nu_e$
CP even	$K^+K^-, \pi^+\pi^-, K_S^0\pi^0\pi^0, K_L^0\pi^0, \pi^+\pi^-\pi^0$
CP odd	$K_S^0\pi^0, K_S^0\eta^{(\prime)}, K_S^0\omega, K_L^0\pi^0\pi^0$
Mixed CP	$K_S^0\pi^+\pi^-, K_L^0\pi^+\pi^-$

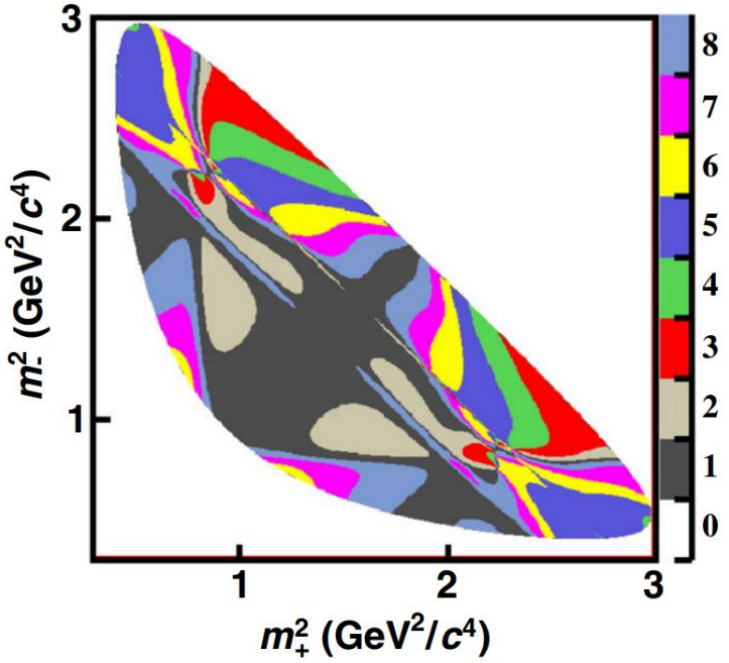
$D^0 \to K^0_S \pi^+ \pi^-$

[PRL 124, 241802 (2020); PRD 101, 112002 (2020)]

- **Measurement of γ (GGSZ)** ← binned parameters c_i, s_i

$$K_i = \int_i |A_f|^2 \mathrm{d}m_+^2 \mathrm{d}m_-^2 \qquad c_i = \frac{1}{\sqrt{K_i K_{-i}}} \int_i |A_f||\bar{A}_f| \cos[\Delta\delta_D] \, \mathrm{d}m_+^2 \mathrm{d}m_-^2 \xrightarrow{\cos \to \sin} s_i$$

- **Divide phase space (DP) into symmetrical bins** *Binning scheme (N=8) [Phys. Rev. D 82, 112006 (2010)]



- **Observables (DT yields)**
  - Flavor tag: $\propto K_i/f_i$ (f_i: correct factor for hadronic flavor tag modes)
  - CP tag: $\propto [K_i + K_{-i} - (2F_+ - 1)2c_i\sqrt{K_i K_{-i}}]$
  - $K^0_S\pi^+\pi^-$ tag (binned): $\propto [K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-i} K_j K_{-j}}(c_i c_j + s_i s_j)]$
  - $K^0_L\pi^+\pi^-$ tag (binned): $\propto [K_i K'_{-j} + K_{-i} K'_j + 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)]$

- **Extraction of DT yields:**
  - Full reconstruction: Fit to M^{sig}_{bc}
  - Partial reconstruction: Fit to U_{miss} for ν and M^2_{miss} for other

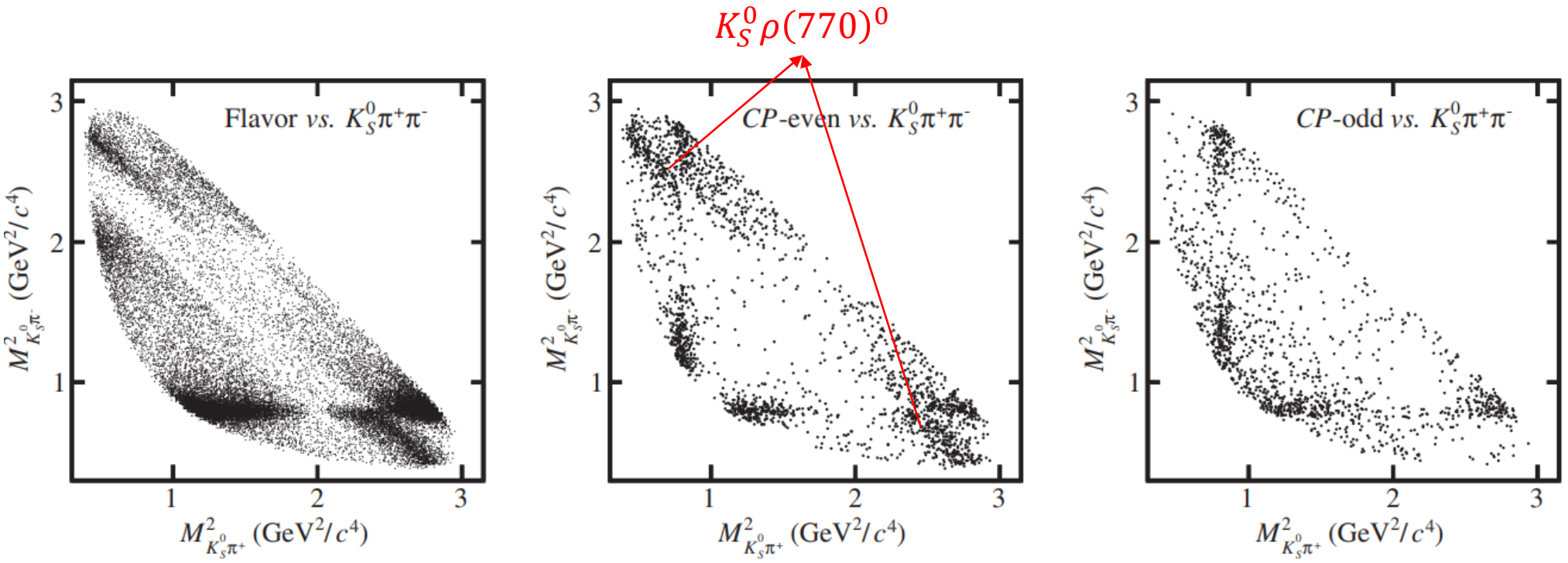
$$M_{bc} = \sqrt{(\sqrt{s}/2)^2 - |\vec{p}_D|^2}$$

$$U_{miss} = E_{miss} - |\vec{p}_{miss}|$$

$$M^2_{miss} = E^2_{miss} - |\vec{p}_{miss}|^2$$

$D^0 \to K^0_S\pi^+\pi^-$

[PRL 124, 241802 (2020); PRD 101, 112002 (2020)]

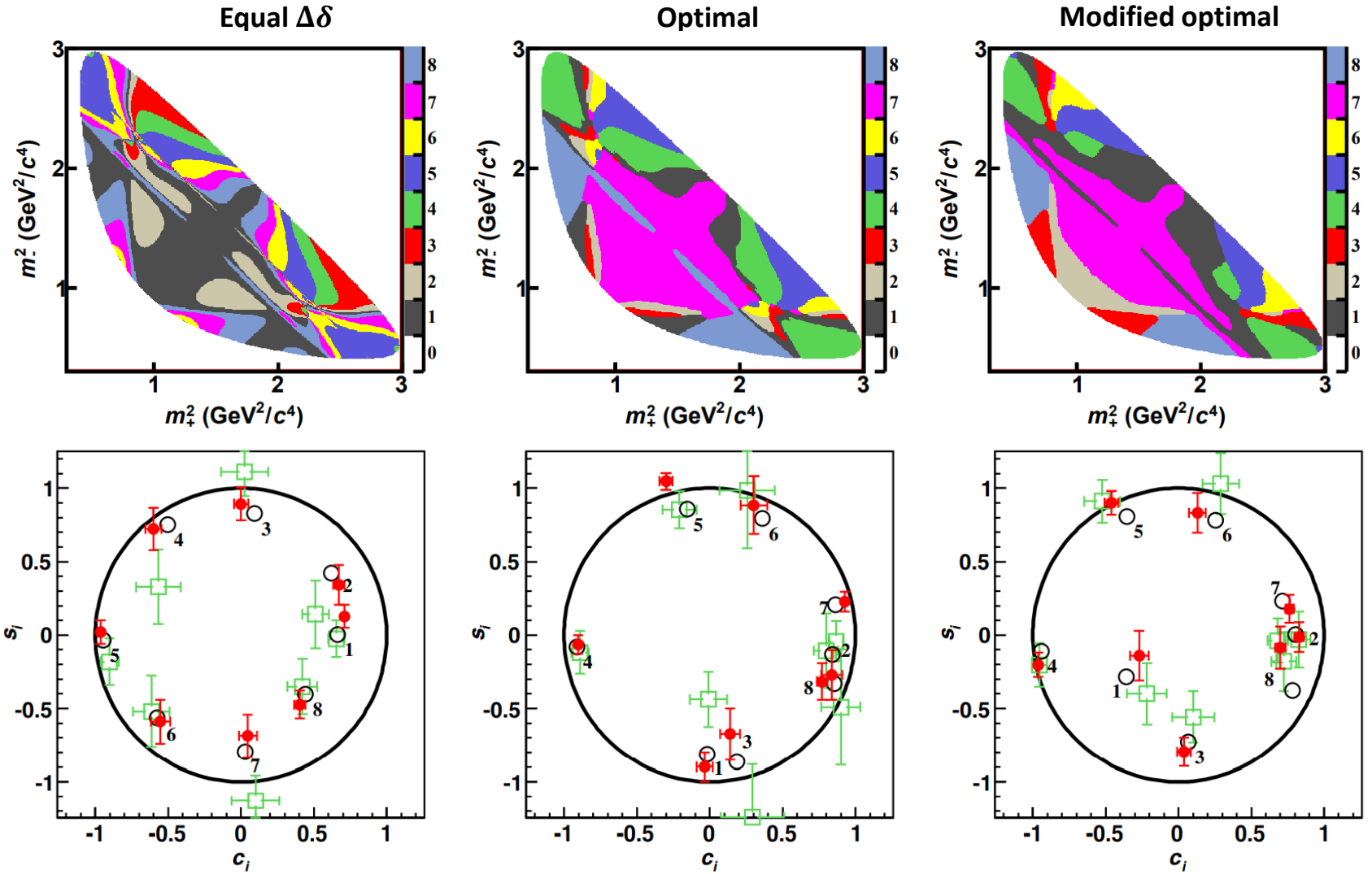


- Quantum correlated effects in Dalitz Plot (DP)
  $K^0_S\rho(770)^0$ exist symmetrically in DP with CP-even tag mode and disappear with CP-odd tag mode

$D^0 \to K^0_S \pi^+ \pi^-$

[PRL 124, 241802 (2020); PRD 101, 112002 (2020)]

- Strong phase parameters are obtained by MLH fit with expected and observed DT yields
- The strong phase parameters are limited by statistical errors
- On average a factor of ~2.5 (2.0) more precise for $c_i(s_i)$ than CLEO-c measurements
- The associated uncertainties on γ are expected to be 0.7°, 1.2° and 0.8° for equal $\Delta\delta$, optimal and modified optimal binning schemes.



BESIII CLEO-c [PRD 82, 112006 (2010)] Model [PRD 98, no.11, 112012 (2018)]

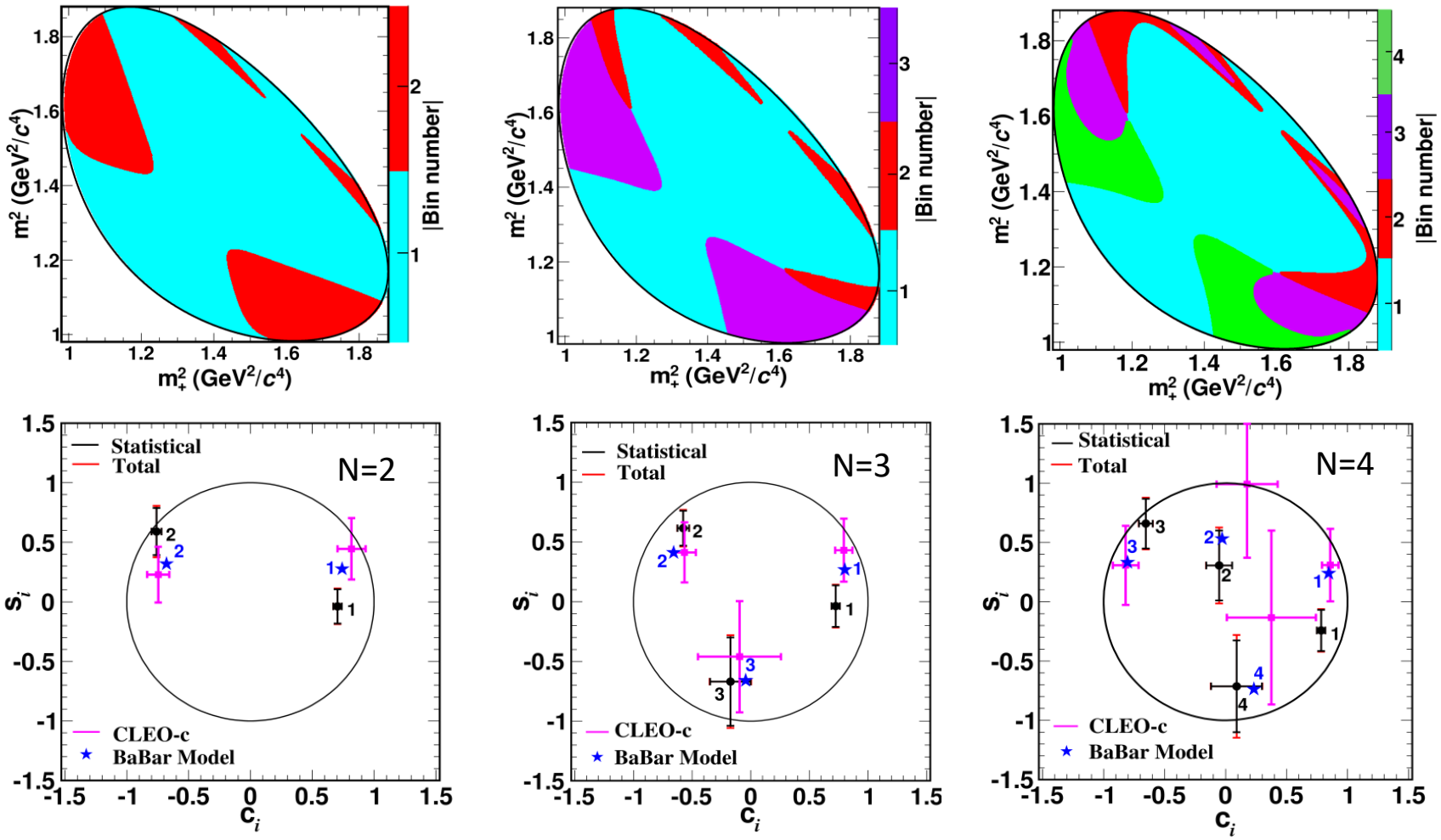
$D^0 \to K^0_S K^+ K^-$

[PRD 102, 052008 (2020)]

- Measurement of γ (GGSZ) ← c_i, s_i
- The strong phase parameters are limited by statistical errors
- Compatible with CLEO-c measurement with improved precision
- The associated uncertainty on γ is expected to be ~1.3° (N=3,4)
- The results of $K^0_S h^+ h^-$ have been used on γ measurement by LHCb and BelleII. The uncertainty from charm inputs is 1°.

LHCb: [JHEP 02 (2021) 169]
BelleII: [arXiv:2110.12125]

Equal $\Delta\delta_D$ binning scheme **[PRD 78, 034023 (2008)]**



BESIII CLEO-c **[PRD 82, 112006 (2010)]** Model **[PRD 78, 034023 (2008)]**

$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

- **Measurement of γ (ADS)** $\leftarrow \boldsymbol{R_f, \delta_D^f}$ $\quad (r_D^f)^2 = \int |\bar{A}_f|^2 d\Phi / \int |A_f|^2 d\Phi \quad \boldsymbol{R_f e^{-i\delta_D^f}} = \dfrac{\int A_f^* \bar{A}_f d\Phi}{\sqrt{\int |A_f|^2 d\Phi \int |\bar{A}_f|^2 d\Phi}}$
- **Global analysis and binned analysis for $\boldsymbol{D^0 \to K^-\pi^+\pi^+\pi^-}$**

***Binning scheme (N=4)**
[PLB 802, 135188 (2020)]

- **Observables**
  - $\rho = \dfrac{DT\ Yield\ with\ QC}{DT\ Yield\ without\ QC}$ (*D^0- $\bar{D}^0$ mixing is ignored for simplicity, but considered in the analysis)
    - ✓ CP tag: $\rho^{\rm f}_{\rm CP\pm} = 1 \mp \dfrac{2r_D^f \boldsymbol{R_f}}{1+(r_D^f)^2}\cos\left(\boldsymbol{\delta_D^f}\right)$, $\Delta^f_{CP} = \pm(\rho^{\rm f}_{\rm CP\pm} - 1)$ (e.g. f vs CP + $\bar{f}$ vs CP)
    - ✓ Like-sign tag (same charge of Kaon in tag side and signal side):

$$\rho^{\rm f}_{\rm LS} = 1 - \boldsymbol{R_f^2} \qquad \rho^{\rm f}_{\rm T,LS} = 1 - \frac{2r_D^f r_D^T}{(r^{\rm f}_{\rm D})^2+(r_D^T)^2}\mathbf{R_f}\mathrm{R_T}\cos(\delta_D^T - \boldsymbol{\delta_D^f})$$

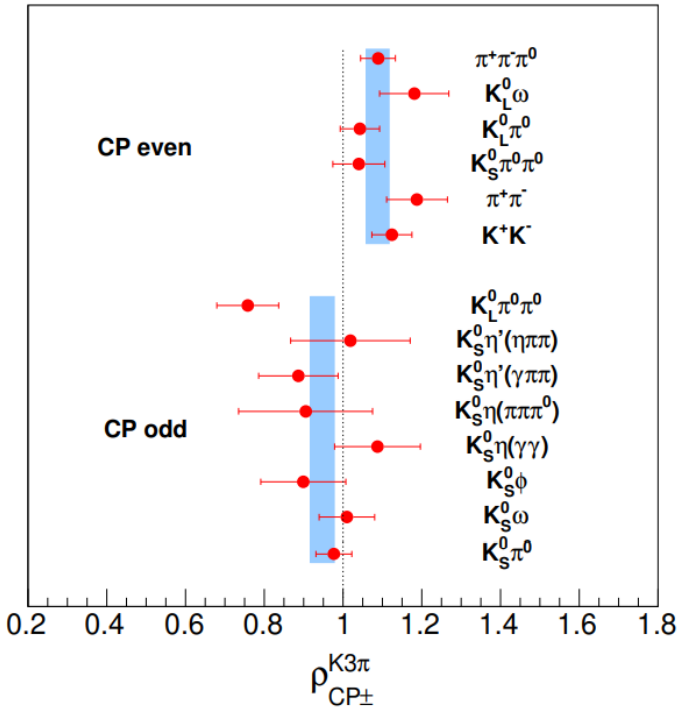
(e.g. f vs f + $\bar{f}$ vs $\bar{f}$) (e.g. $K^-\pi^+$ vs f + $K^+\pi^-$ vs $\bar{f}$)

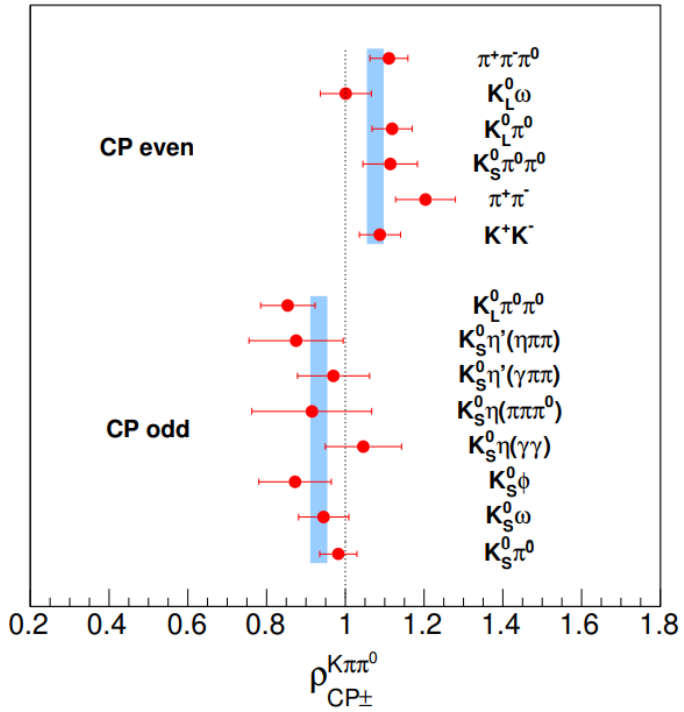
  - Y_i (DT yield for $K_S^0\pi^+\pi^-$ tag): $\propto [K_i + (r_D^f)^2 K_{-i} - 2r_D^f \boldsymbol{R_f}\sqrt{K_i K_{-i}}(c_i \cos\left(\boldsymbol{\delta_D^f}\right) - s_i \sin(\boldsymbol{\delta_D^f}))]$
- **Extraction of DT yields**
  - Full reconstruction: Fit to M_{bc}^{sig}
  - Partial reconstruction: Fit to M_{miss}^2

$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

- Observed values of $\rho_{CP\pm}$

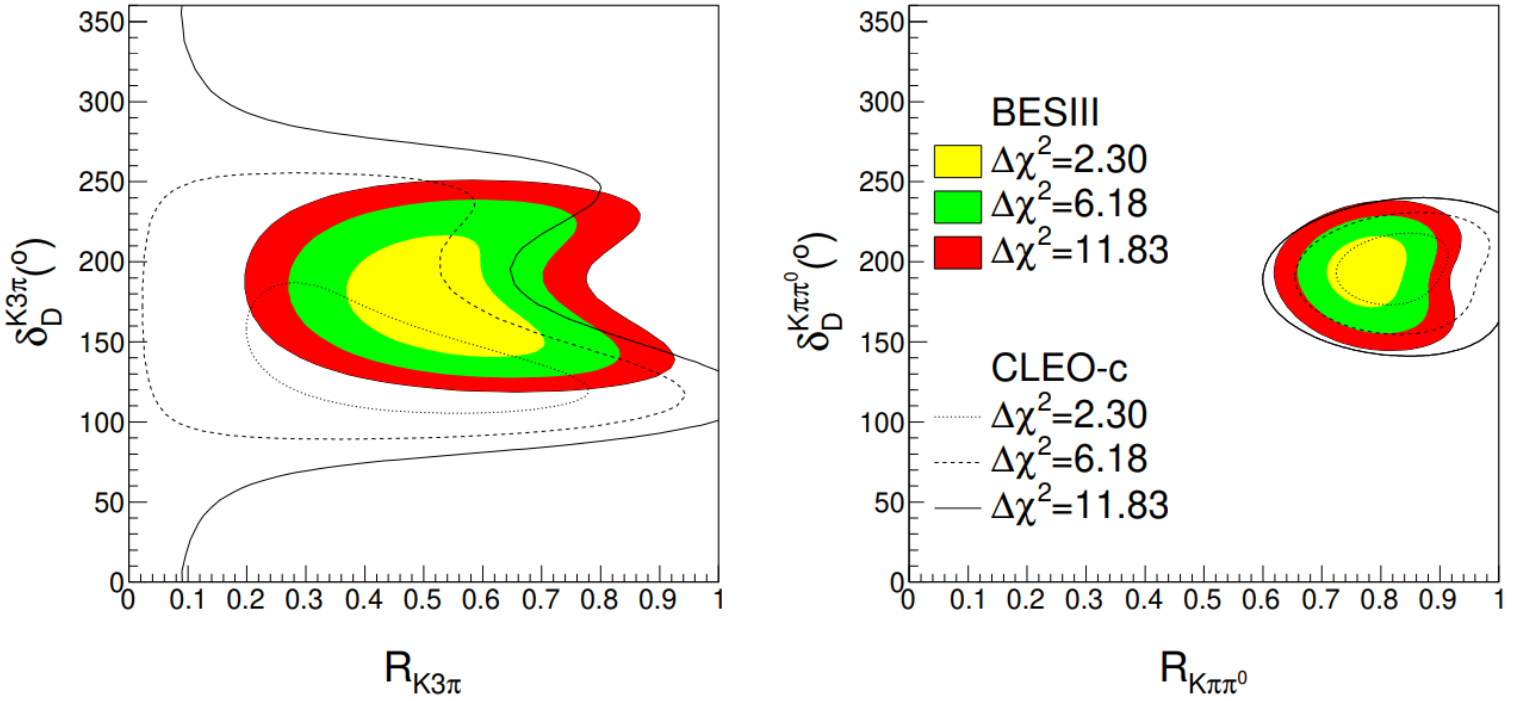




- Deviation from uncorrelated prediction 1 indicates that significant quantum correlated effects are observed in data

$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$ (Global)

[JHEP 05, 164 (2021)]



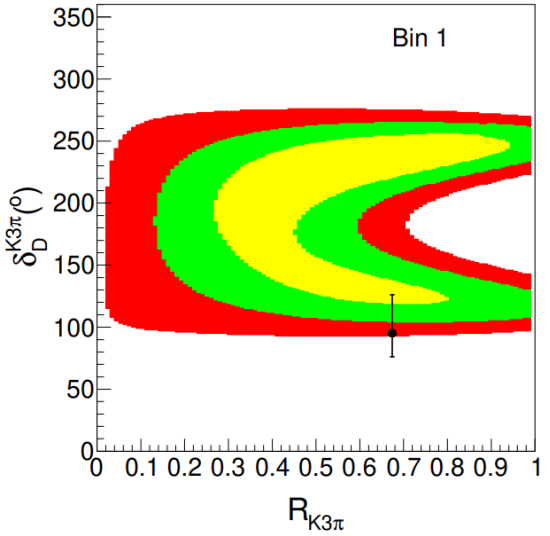
$$R_{K3\pi} = 0.52^{+0.12}_{-0.10}, \qquad \delta_D^{K3\pi} = (167^{+31}_{-19})^\circ,$$
$$R_{K\pi\pi^0} = 0.78 \pm 0.04, \qquad \delta_D^{K\pi\pi^0} = (196^{+14}_{-15})^\circ.$$

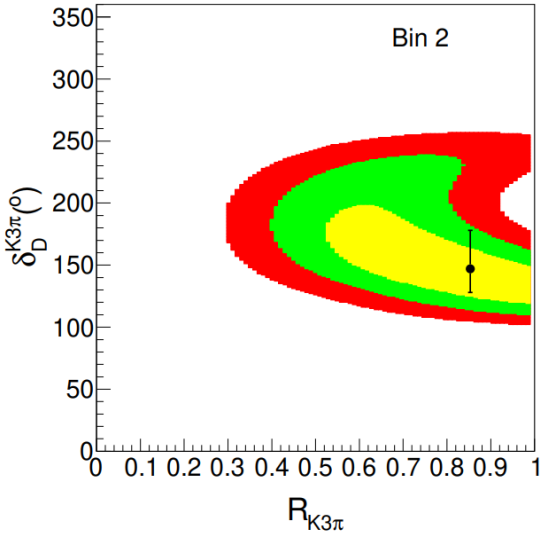
- Global strong phase parameters are obtained by min χ^2 fit with expected and observed ρ & Y_i
- The strong phase parameters are limited by statistical errors
- The results have significant improvement compared to CLEO-c results **[PLB 757, 520 (2016)]**

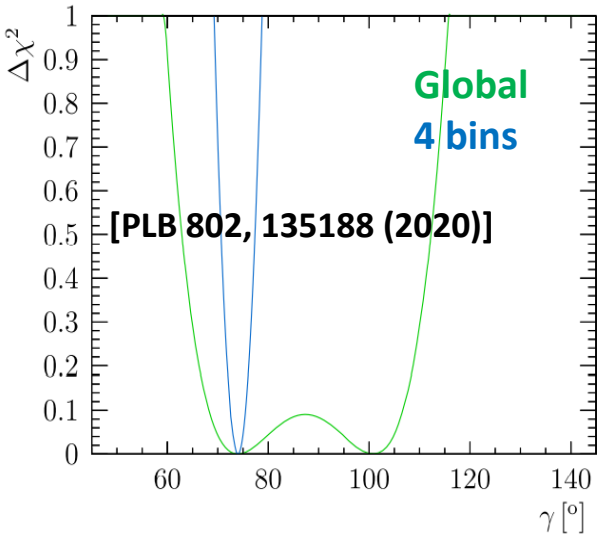
$D^0 \to K^-\pi^+\pi^+\pi^-$ (Binned)

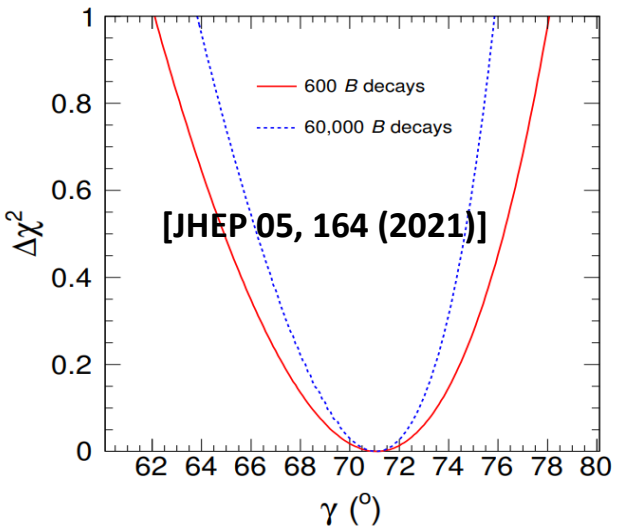
[JHEP 05, 164 (2021)]

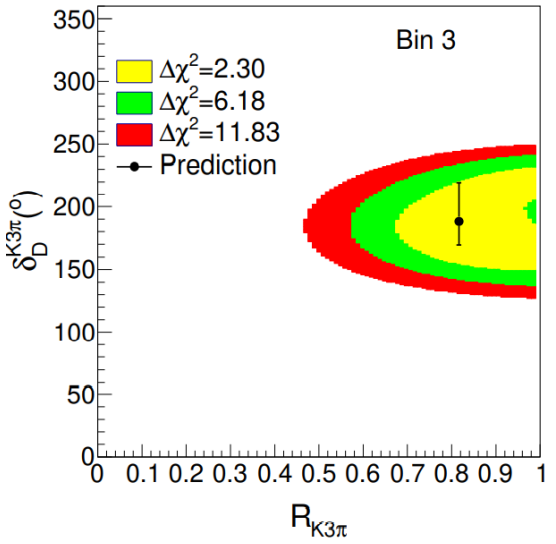
- Sensitivity on γ can be improved with binned measurement in $D^0 \to K^-\pi^+\pi^+\pi^-$ **[PLB 802, 135188 (2020)]**
- Binned strong phase parameters has been measured with improved precision compared to CLEO-c's results **[PLB 802, 135188 (2020)]**
- The associated uncertainty on γ measurement in $K3\pi$ for 600 B decays (~run1 & run2 @ LHCb) is expected to be ~6° (Total uncertainty ~8°)

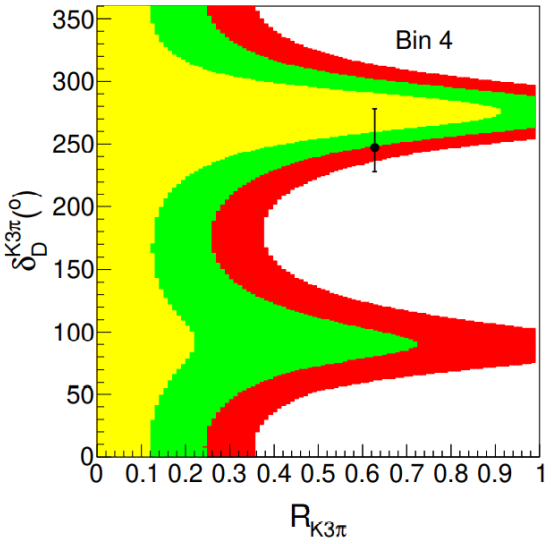












Summary

- BESIII provides unique quantum correlated $D^0\overline{D}^0$ data to measure the strong-phase parameters in D decays as inputs to LHCb and Belle II for CKM angle γ measurement in the b sector

- Using 2.93 fb^{-1} e^+e^- collision data taken @ 3.773 GeV with BESIII detector, strong phase parameters of four D^0 decays are reported
  - $K_S^0\pi^+\pi^-, K_S^0K^+K^-, K^-\pi^+\pi^0, K^-\pi^+\pi^+\pi^-$

[CPC 44, 040001 (2020)]

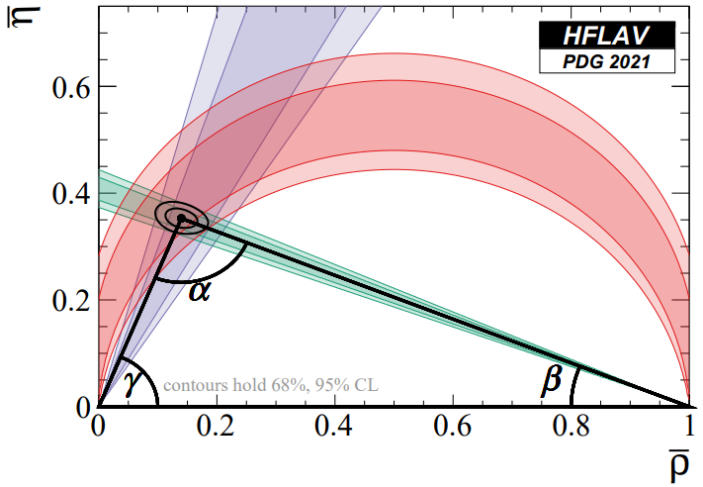
- $20\ \mathrm{fb}^{-1}$ $\psi(3770)$ data will be collected in the near future @ BESIII
  - More decays (e.g. $K_S^0\pi^+\pi^-\pi^0, \pi^+\pi^-\pi^+\pi^-, K^+K^-\pi^+\pi^-$ …)
  - Higher precision (e.g. uncertainty on γ $\sim 1^\circ \rightarrow \sim 0.4^\circ$ for $K_S^0h^+h^-$)

Thank you!

BACK-UP

Measurement of γ/φ_3

- **CKM Matrix**

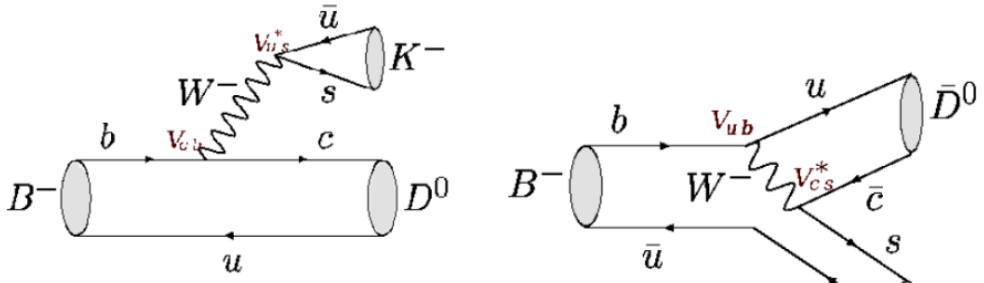


$$\alpha \equiv \varphi_1 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = (85.2^{+4.8}_{-4.3})°$$

$$\beta \equiv \varphi_2 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = (22.7 \pm 0.7)°$$

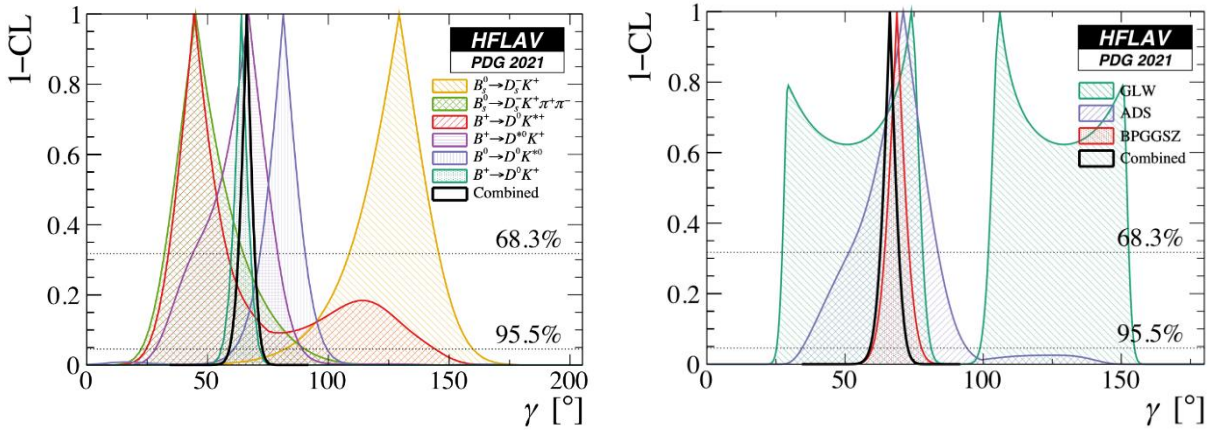
$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = (66.2^{+3.4}_{-3.6})°$$

- **Measurement of γ/φ_3**



$$A(B^- \to D^0 K^-) = A_B A_D$$

$$A(B^- \to \bar{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)} A_{\bar{D}}$$



Measurement of γ/φ_3

- Expected γ/φ_3 precision of LHCb[1] and Belle II[2] experiment

[1]. arXiv: 1808.08865
[2]. E. Kou et al. (Belle II Collaboration), PTEP 2019, 123C01 (2019)

Runs	Collected / Expected integrated luminosity	Year attained	γ/ϕ_3 sensitivity
LHCb Run-1 [7, 8 TeV]	3 fb^{-1}	2012	8°
LHCb Run-2 [13 TeV]	6 fb^{-1}	2018	4°
Belle II Run	50 ab^{-1}	2025	1.5°
LHCb upgrade I [14 TeV]	50 fb^{-1}	2030	< 1°
LHCb upgrade II [14 TeV]	300 fb^{-1}	(>)2035	< 0.4°

- γ/φ_3 uncertainty from strong phase inputs

$2.93 fb^{-1}$@ $E_{cm} = 3.773$ GeV @ BESIII
$D^0 \to K_S^0\pi^+\pi^- \sim 1°$ [3]

[JHEP 02 (2021) 169]

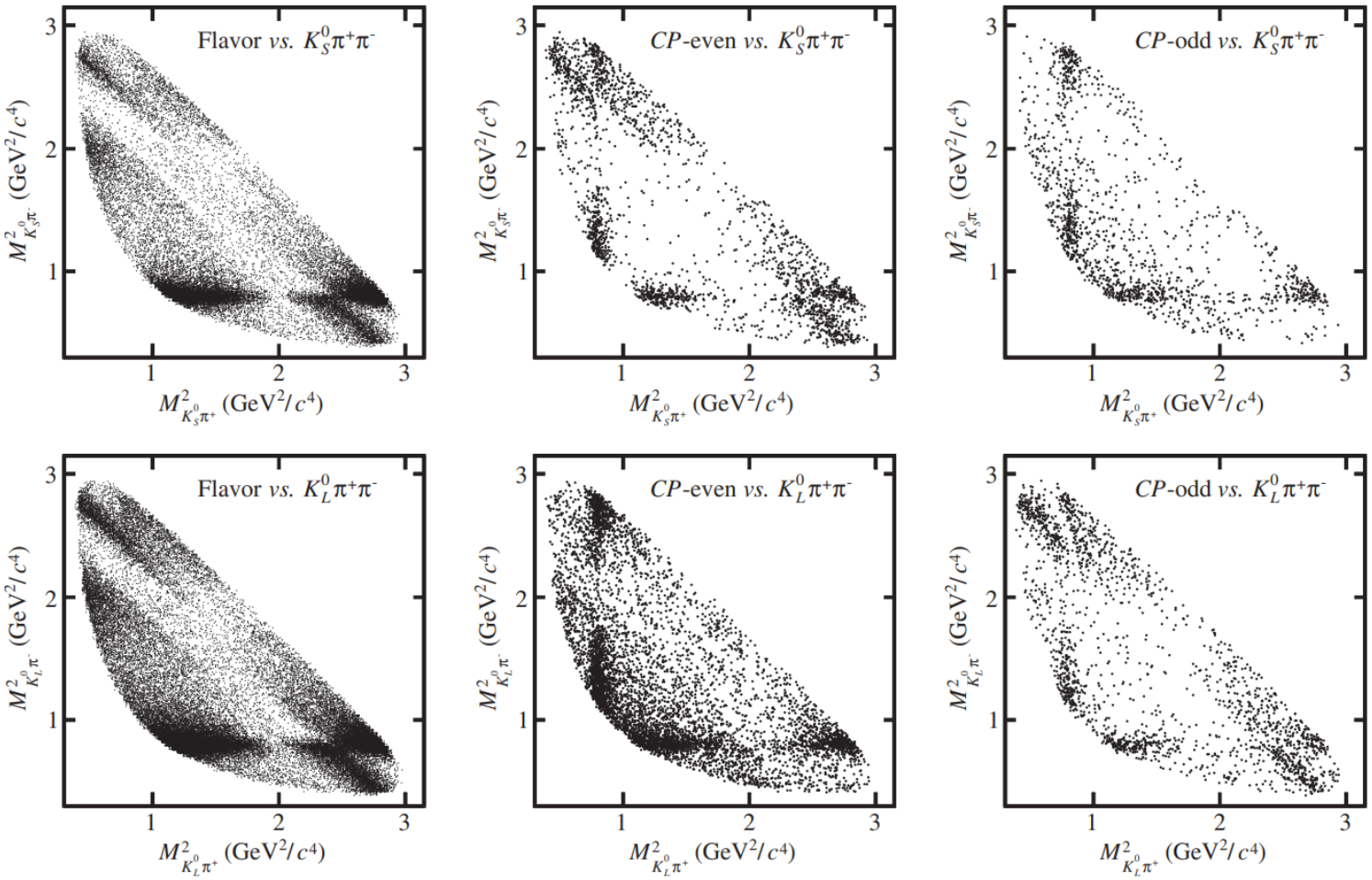
$20 fb^{-1}$@ $E_{cm} = 3.773$ GeV @ BESIII
$D^0 \to K_S^0\pi^+\pi^- \sim 0.4°$ [4]

[BESIII White Paper, Chinese Phys. C 44, 040001 (2020)]

$D^0 \to K^0_{S/L}\pi^+\pi^-$

[PRL 124, 241802 (2020)] [PRD 101, 112002 (2020)]

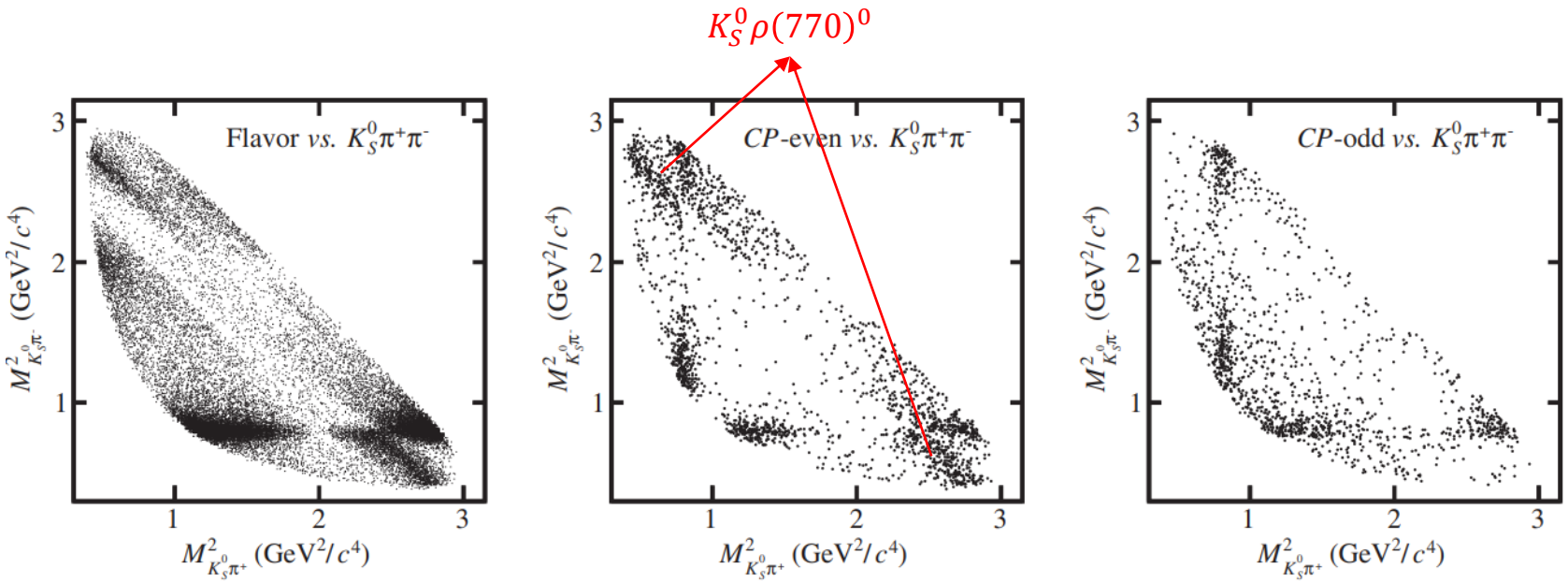
Mode	$N_{\rm ST}$	$N^{\rm DT}_{K^0_S\pi^+\pi^-}$	$N^{\rm DT}_{K^0_L\pi^+\pi^-}$
Flavor tags			
$K^+\pi^-$	549373 ± 756	4740 ± 71	9511 ± 115
$K^+\pi^-\pi^0$	1076436 ± 1406	5695 ± 78	11906 ± 132
$K^+\pi^-\pi^-\pi^+$	712034 ± 1705	8899 ± 95	19225 ± 176
$K^+e^-\bar{\nu}_e$	458989 ± 5724	4123 ± 75	
CP-even tags			
K^+K^-	57050 ± 231	443 ± 22	1289 ± 41
$\pi^+\pi^-$	20498 ± 263	184 ± 14	531 ± 28
$K^0_S\pi^0\pi^0$	22865 ± 438	198 ± 16	612 ± 35
$\pi^+\pi^-\pi^0$	107293 ± 716	790 ± 31	2571 ± 74
$K^0_L\pi^0$	103787 ± 7337	913 ± 41	
CP-odd tags			
$K^0_S\pi^0$	66116 ± 324	643 ± 26	861 ± 46
$K^0_S\eta_{\gamma\gamma}$	9260 ± 119	89 ± 10	105 ± 15
$K^0_S\eta_{\pi^+\pi^-\pi^0}$	2878 ± 81	23 ± 5	40 ± 9
$K^0_S\omega$	24978 ± 448	245 ± 17	321 ± 25
$K^0_S\eta'_{\pi^+\pi^-\eta}$	3208 ± 88	24 ± 6	38 ± 8
$K^0_S\eta'_{\gamma\pi^+\pi^-}$	9301 ± 139	81 ± 10	120 ± 14
$K^0_L\pi^0\pi^0$	50531 ± 6128	620 ± 32	
Mixed CP tags			
$K^0_S\pi^+\pi^-$	188912 ± 756	899 ± 31	3438 ± 72
$K^0_S\pi^+\pi^-_{\rm miss}$		224 ± 17	
$K^0_S(\pi^0\pi^0_{\rm miss})\pi^+\pi^-$		710 ± 34	



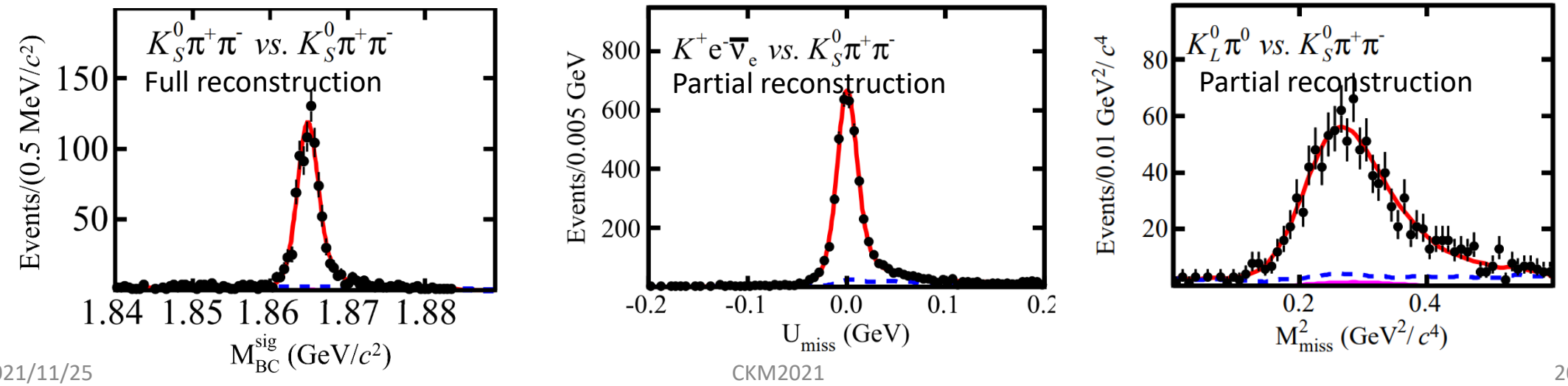
$D^0 \to K^0_S \pi^+ \pi^-$

[PRL 124, 241802 (2020); PRD 101, 112002 (2020)]

- Dalitz Plot



- Fit Plot



$D^0 \to K^0_{S/L}\pi^+\pi^-$

[PRL 124, 241802 (2020)] [PRD 101, 112002 (2020)]

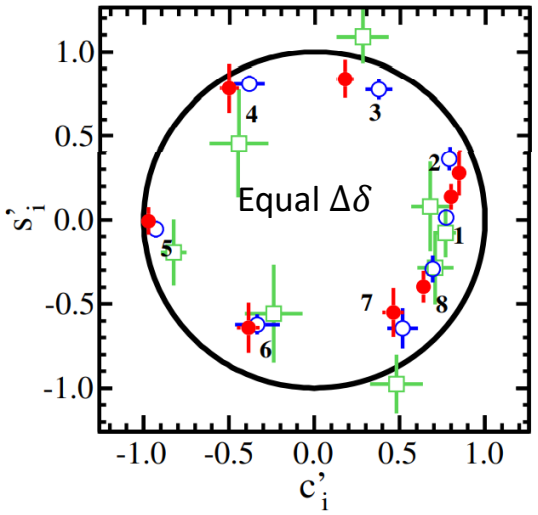
Likelihood function in fit:

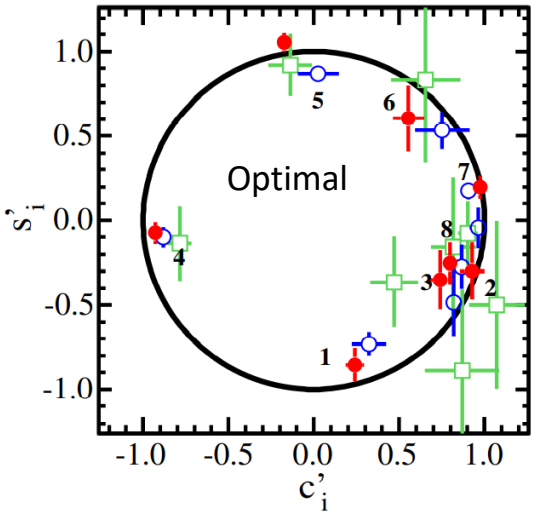
$$-2\log\mathcal{L} = -2\sum_{i=1}^{8}\ln P(N_i^{\rm obs}, \langle N_i^{\rm exp}\rangle)_{CP,K^0_S\pi^+\pi^-}$$
$$-2\sum_{i=1}^{8}\ln P(N_i^{\rm obs}, \langle N_i^{\rm exp}\rangle)_{CP,K^0_L\pi^+\pi^-}$$
$$-2\sum_{n=1}^{72}\ln P(N_n^{\rm obs}, \langle N_n^{\rm exp}\rangle)_{K^0_S\pi^+\pi^-,K^0_S\pi^+\pi^-}$$
$$-2\sum_{n=1}^{128}\ln P(N_n^{\rm obs\prime}, \langle N_n^{\rm exp}\rangle)_{K^0_L\pi^+\pi^-,K^0_S\pi^+\pi^-} + \chi^2$$

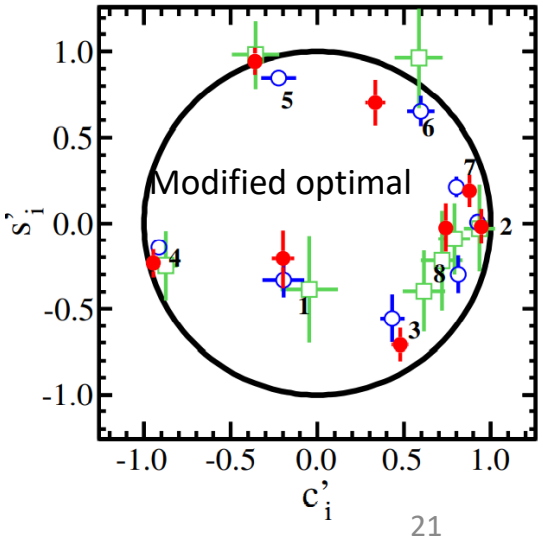
$$\chi^2 = \sum_i\left(\frac{c_i' - c_i - \Delta c_i}{\delta\Delta c_i}\right)^2 + \sum_i\left(\frac{s_i' - s_i - \Delta s_i}{\delta\Delta s_i}\right)^2$$

[Phys. Rev. D 81, 112002 (2010); Phys. Rev. Lett. 95, 121802 (2005)]

- c_i' and s_i' for $D^0 \to K^0_L\pi^+\pi^-$
- A factor of 2.8 (2.2) more precise for c_i' and s_i' than previous measurements







$D^0 \to K^0_{S/L}\pi^+\pi^-$

[PRL 124, 241802 (2020)] [PRD 101, 112002 (2020)]

- Systematic uncertainty for equal $\Delta\delta$ binning scheme

Uncertainty	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
K_i and K_i'	0.004	0.013	0.005	0.007	0.005	0.014	0.006	0.007
ST yields	0.007	0.007	0.013	0.008	0.004	0.014	0.019	0.011
MC statistics	0.001	0.003	0.003	0.003	0.001	0.004	0.004	0.003
DT peaking-background subtraction	0.002	0.003	0.002	0.007	0.005	0.007	0.003	0.002
DT yields	0.001	0.002	0.002	0.001	0.001	0.002	0.003	0.002
Momentum resolution	0.002	0.003	0.012	0.011	0.010	0.010	0.011	0.009
$D^0\bar{D}^0$ mixing	0.001	0.000	0.002	0.001	0.000	0.002	0.002	0.001
Total systematic	0.009	0.016	0.019	0.017	0.013	0.024	0.023	0.017
Statistical plus $K^0_L\pi^+\pi^-$ model	0.020	0.035	0.047	0.053	0.019	0.062	0.057	0.036
$K^0_L\pi^+\pi^-$ model alone	0.011	0.009	0.027	0.030	0.007	0.034	0.033	0.017
Total	0.022	0.039	0.051	0.055	0.023	0.066	0.061	0.039

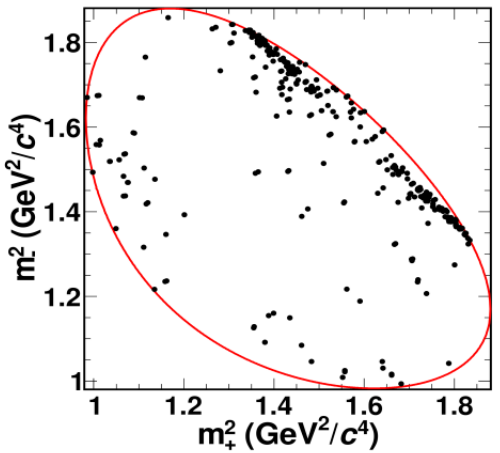
Uncertainty	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
K_i and K_i'	0.004	0.006	0.012	0.005	0.003	0.018	0.022	0.008
ST yields	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001
MC statistics	0.007	0.011	0.009	0.010	0.005	0.009	0.011	0.006
DT peaking-background subtraction	0.007	0.005	0.007	0.018	0.005	0.009	0.011	0.004
DT yields	0.005	0.005	0.003	0.004	0.003	0.004	0.005	0.003
Momentum resolution	0.012	0.005	0.011	0.001	0.003	0.022	0.006	0.025
$D^0\bar{D}^0$ mixing	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
Total systematic	0.017	0.015	0.020	0.022	0.009	0.031	0.028	0.027
Statistical plus $K^0_L\pi^+\pi^-$ model	0.076	0.134	0.112	0.143	0.081	0.147	0.143	0.091
$K^0_L\pi^+\pi^-$ model alone	0.017	0.029	0.022	0.018	0.012	0.017	0.036	0.028
Total	0.078	0.135	0.114	0.144	0.081	0.150	0.146	0.095

$D^0 \to K^0_{S/L} K^+ K^-$

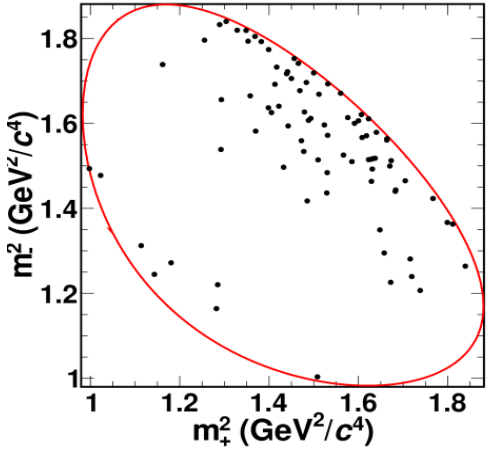
[PRD 102, 052008 (2020)]

Mode	ST		DT			
	$N_{\rm ST}$	$\epsilon_{\rm ST}(\%)$	$N_{\rm DT}^{K^0_S K^+K^-}$	$N_{\rm DT}^{K^0_L K^+K^-}$	$\epsilon_{\rm DT}^{K^0_S K^+K^-}(\%)$	$\epsilon_{\rm DT}^{K^0_L K^+K^-}(\%)$
Flavor-tags						
$K^-\pi^+$	524307 ± 742	63.31 ± 0.06	323	743	12.43 ± 0.07	15.85 ± 0.08
$K^-\pi^+\pi^0$	995683 ± 1117	31.70 ± 0.03	596	1769	5.86 ± 0.05	7.94 ± 0.06
$K^-e^+\nu_e$	752387 ± 12795		263		3.23 ± 0.04	
CP-even tags						
K^+K^-	53481 ± 247	61.02 ± 0.11	42	112	12.07 ± 0.07	15.52 ± 0.08
$\pi^+\pi^-$	19339 ± 163	64.52 ± 0.11	10	31	12.16 ± 0.07	15.70 ± 0.08
$K^0_S\pi^0\pi^0$	19882 ± 233	14.86 ± 0.08	7	45	2.49 ± 0.04	3.79 ± 0.04
$\pi^+\pi^-\pi^0$	99981 ± 618	37.65 ± 0.11	51	254	6.79 ± 0.06	9.54 ± 0.07
$K^0_L\pi^0$	209445 ± 14796		90		8.88 ± 0.06	
$K^0_L\eta(\gamma\gamma)$	40009 ± 2543		19		6.60 ± 0.06	
$K^0_L\omega$	207376 ± 11498		44		3.42 ± 0.04	
$K^0_L\eta'(\pi^+\pi^-\eta)$	33683 ± 1909		7		3.23 ± 0.04	
CP-odd tags						
$K^0_S\pi^0$	65072 ± 281	36.92 ± 0.11	39	89	6.75 ± 0.06	9.33 ± 0.07
$K^0_S\eta(\gamma\gamma)$	9524 ± 134	32.94 ± 0.11	9	10	6.05 ± 0.05	9.05 ± 0.06
$K^0_S\omega$	19262 ± 157	12.14 ± 0.07	16	27	2.20 ± 0.03	3.42 ± 0.04
$K^0_S\eta'(\pi^+\pi^-\eta)$	3301 ± 62	12.46 ± 0.07	2	5	2.20 ± 0.03	3.46 ± 0.04
Mixed CP tags						
$K^0_S\pi^+\pi^-$			78	265	6.35 ± 0.05	8.32 ± 0.06
$K^0_L\pi^+\pi^-$			282		9.56 ± 0.07	
$K^0_S K^+K^-$	12949 ± 119	18.35 ± 0.09	4	19	2.99 ± 0.04	3.40 ± 0.04

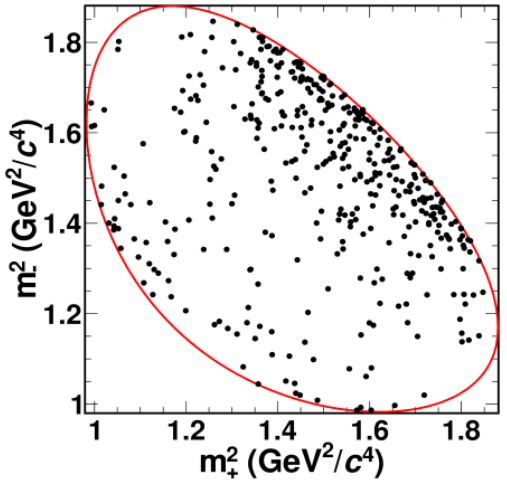
CP-even vs. $K^0_S K^+K^-$



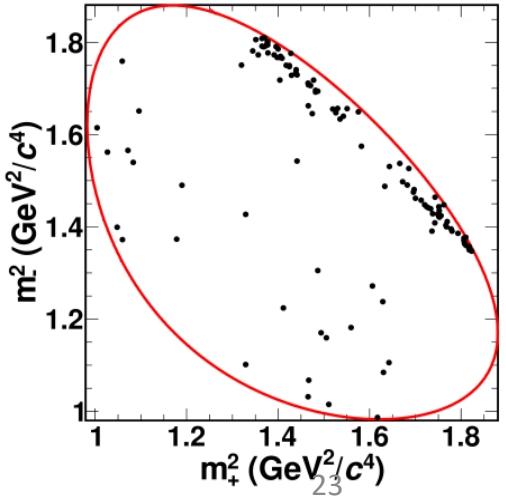
CP-odd vs. $K^0_S K^+K^-$



CP-even vs. $K^0_L K^+K^-$



CP-odd vs. $K^0_L K^+K^-$



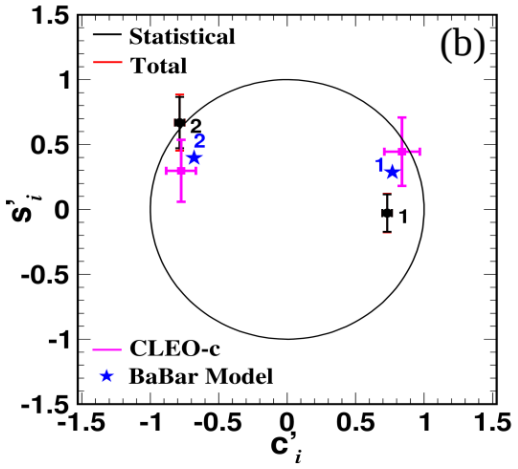
$D^0 \to K^0_{S/L} K^+ K^-$

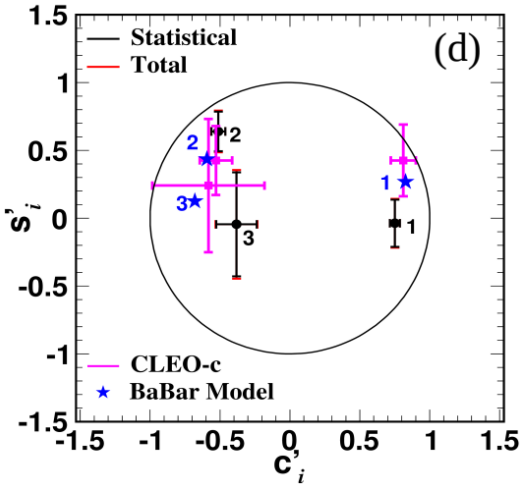
[PRD 102, 052008 (2020)]

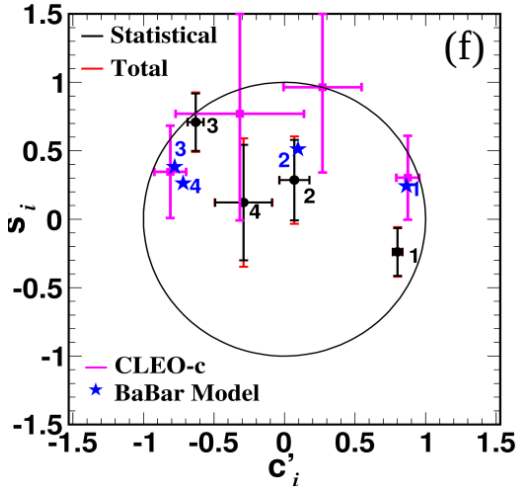
Likelihood function in fit:

$$
\begin{aligned}
-2\ln\mathcal{L} = & -2\sum_i \ln P\left(N_i^{\pm}, \langle N_i^{\pm}\rangle\right)_{K^0_S K^+K^-,\ CP} \\
& -2\sum_i \ln P\left(N_i'^{\pm}, \langle N_i'^{\pm}\rangle\right)_{K^0_L K^+K^-,\ CP} \\
& -2\sum_{i,j} \ln P\left(N_{ij}, \langle N_{ij}\rangle\right)_{K^0_S K^+K^-,\ K^0_S K^+K^-} \\
& -2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K^0_S K^+K^-,\ K^0_L K^+K^-} \\
& -2\sum_{i,j} \ln P\left(N_{ij}, \langle N_{ij}\rangle\right)_{K^0_S K^+K^-, K^0_S \pi^+\pi^-} \\
& -2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K^0_S K^+K^-,\ K^0_L \pi^+\pi^-} \\
& -2\sum_{i,j} \ln P\left(N'_{ij}, \langle N'_{ij}\rangle\right)_{K^0_L K^+K^-,\ K^0_S \pi^+\pi^-} \\
& +\chi^2\ . \qquad (28)
\end{aligned}
$$

- c_i' and s_i' for $D^0 \to K^0_L K^+ K^-$



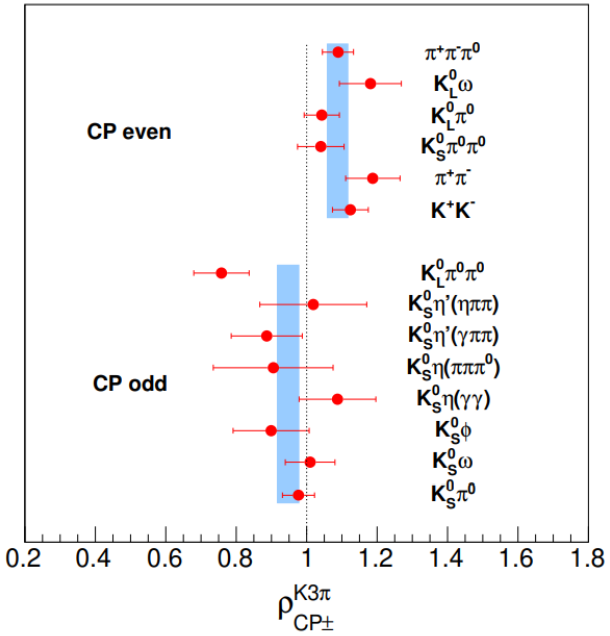


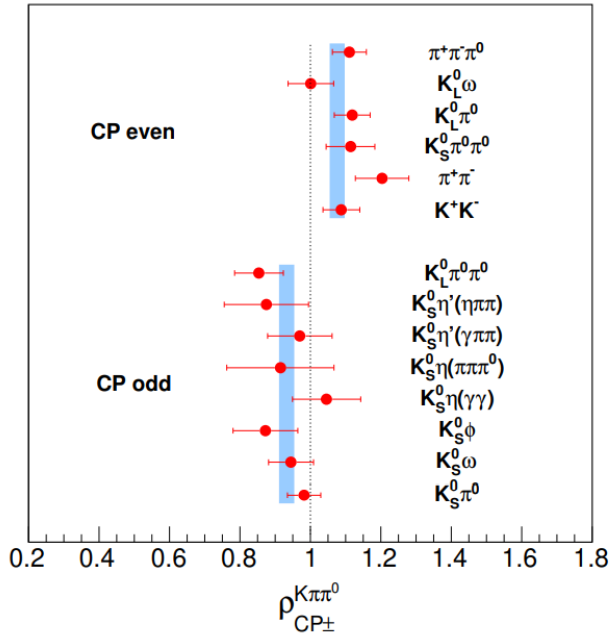


$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

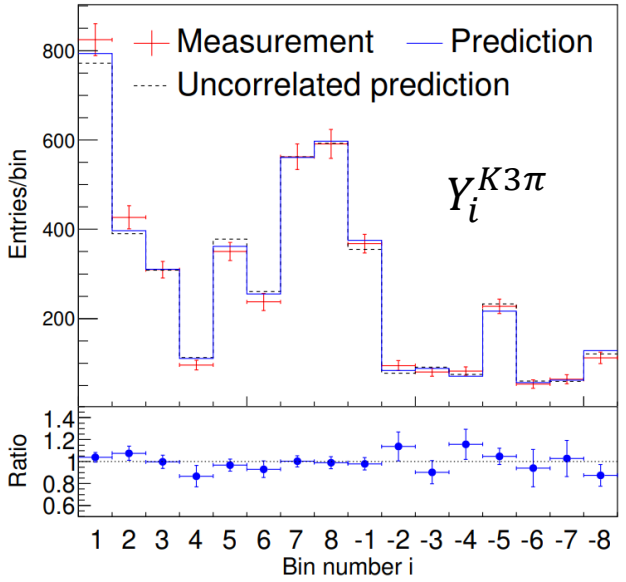
[JHEP 05, 164 (2021)]

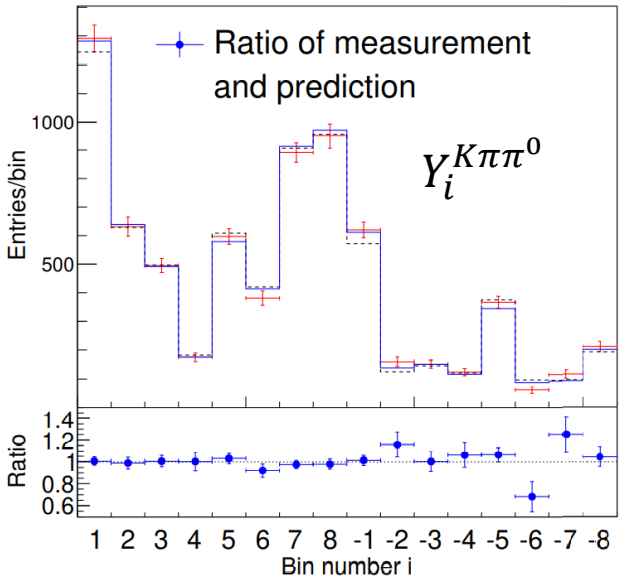
- Observed values of ρ & Y_i





Observable	Value	Observable	Value
$\Delta^{K3\pi}_{CP}$	$0.070 \pm 0.011 \pm 0.012$	$\Delta^{K\pi\pi^0}_{CP}$	$0.078 \pm 0.007 \pm 0.012$
$\rho^{K3\pi}_{LS}$	$0.740 \pm 0.157 \pm 0.161$	$\rho^{K\pi\pi^0}_{LS}$	$0.440 \pm 0.095 \pm 0.014$
$\rho^{K3\pi}_{K\pi,LS}$	$0.570 \pm 0.109 \pm 0.069$	$\rho^{K\pi\pi^0}_{K\pi,LS}$	$0.213 \pm 0.062 \pm 0.004$
$\rho^{K3\pi}_{K\pi\pi^0,LS}$	$0.715 \pm 0.094 \pm 0.089$		





$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

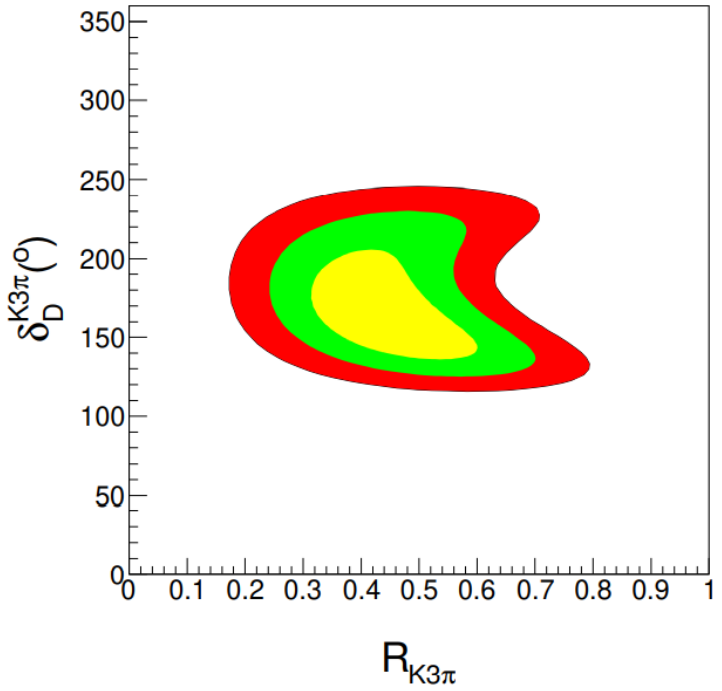
Systematics	$R_{K3\pi}$	$\delta_D^{K3\pi}$	$R_{K\pi\pi^0}$	$\delta_D^{K\pi\pi^0}$
Size of *CP*-tagged $D \to K^-\pi^+$ samples	0.04	7.0°	0.02	6.9°
K/π tracking and identification	0.02	3.8°	< 0.01	2.3°
π^0 reconstruction	< 0.01	$< 0.1°$	< 0.01	$< 0.1°$
Impact of resonance modelling on efficiency	< 0.01	2.5°	< 0.01	0.4°
Size of Monte Carlo samples	0.01	1.5°	< 0.01	1.3°
$D \to K^0_S K^-\pi^+$ background	0.05	1.0°	0.01	4.6°
Fit method for signal yields	0.02	3.4°	< 0.01	1.1°
c_i, s_i	$^{+0.01}_{-0.00}$	3.0°	< 0.01	$(^{+0.6}_{-0.7})°$
K_i	0.01	$(^{+6.7}_{-6.1})°$	0.01	$(^{+3.1}_{-4.4})°$
$\mathcal{B}(D^0 \to S)$, with $S = K^-\pi^+\pi^+\pi^-$ and $K^-\pi^+\pi^0$	0.01	$(^{+1.7}_{-1.5})°$	0.01	$(^{+3.4}_{-2.2})°$
$\mathcal{B}(D^0 \to \bar{S})/\mathcal{B}(D^0 \to S)$	$^{+0.02}_{-0.01}$	2.7°	< 0.01	0.2°
$\mathcal{B}(D^0 \to K^-\pi)$	0.01	$(^{+0.8}_{-1.2})°$	< 0.01	$(^{+0.9}_{-0.7})°$
$r_D^{K\pi}$	< 0.01	$(^{+0.2}_{-0.1})°$	< 0.01	0.2°
$\delta_D^{K\pi}$	< 0.01	$< 0.1°$	< 0.01	$< 0.1°$
x, y	< 0.01	$(^{+1.0}_{-1.1})°$	< 0.01	0.5°
$F^+_{\pi\pi\pi^0}$	< 0.01	$(^{+0.3}_{-0.4})°$	< 0.01	0.1°
Statistical	$^{+0.08}_{-0.09}$	$(^{+29.3}_{-18.7})°$	0.04	$(^{+10.6}_{-12.6})°$

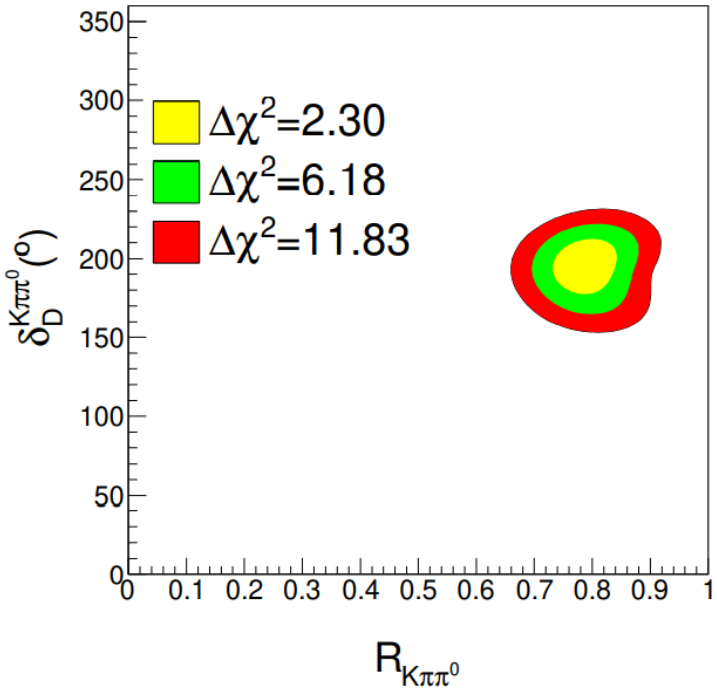
$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

Combination of global R & δ_D of BESIII, CLEO-c and LHCb

Parameter	BESIII and CLEO-c	BESIII, CLEO-c and LHCb
$R_{K3\pi}$	$0.49^{+0.11}_{-0.10}$	$0.44^{+0.10}_{-0.09}$
$\delta_D^{K3\pi}$	$\left(154^{+22}_{-14}\right)^\circ$	$\left(161^{+28}_{-18}\right)^\circ$
$r_D^{K3\pi}$ $(\times 10^{-2})$	5.46±0.08	5.50±0.07
$R_{K\pi\pi^0}$	0.79±0.04	0.79±0.04
$\delta_D^{K\pi\pi^0}$	$(196 \pm 11)^\circ$	$(196 \pm 11)^\circ$
$r_D^{K\pi\pi^0}$ $(\times 10^{-2})$	4.41±0.11	4.41±0.11





$D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^+\pi^-$

[JHEP 05, 164 (2021)]

Combination of binned $R_{K3\pi}$ & $\delta_D^{K3\pi}$ of BESIII and CLEO-c

Parameter	Bin 1	Bin 2	Bin 3	Bin 4
$R_{K3\pi}$	$0.66^{+0.18}_{-0.21}$	$0.85^{+0.14}_{-0.21}$	$0.78^{+0.12}_{-0.12}$	$0.25^{+0.16}_{-0.25}$
$\delta_D^{K3\pi}$	$(117^{+14}_{-9})^\circ$	$(145^{+23}_{-14})^\circ$	$(160^{+19}_{-20})^\circ$	$(288^{+15}_{-29})^\circ$
$r_D^{K3\pi}$ $(\times 10^{-2})$	5.43±0.10	5.78±0.11	5.76±0.10	5.06±0.12
$R_{K\pi\pi^0}$	0.80±0.04			
$\delta_D^{K\pi\pi^0}$	$(203 \pm 11)^\circ$			
$r_D^{K\pi\pi^0}$ $(\times 10^{-2})$	4.49±0.11