Latest Measurements of the CKM angle γ/ϕ_3 at Belle

Seema Bahinipati, IIT Bhubaneswar
On behalf of the Belle Collaboration

CKM2021
November 22-26, 2021
University of Melbourne, Australia

CKM matrix

- Cabibbo-Kobayashi-Maskawa quark mixing matrix (CKM) matrix, V_{CKM} contains coupling constants of weak interaction in the quark sector.

- CP violation (CPV) enters Standard Model as an irreducible phase in the CKM matrix.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

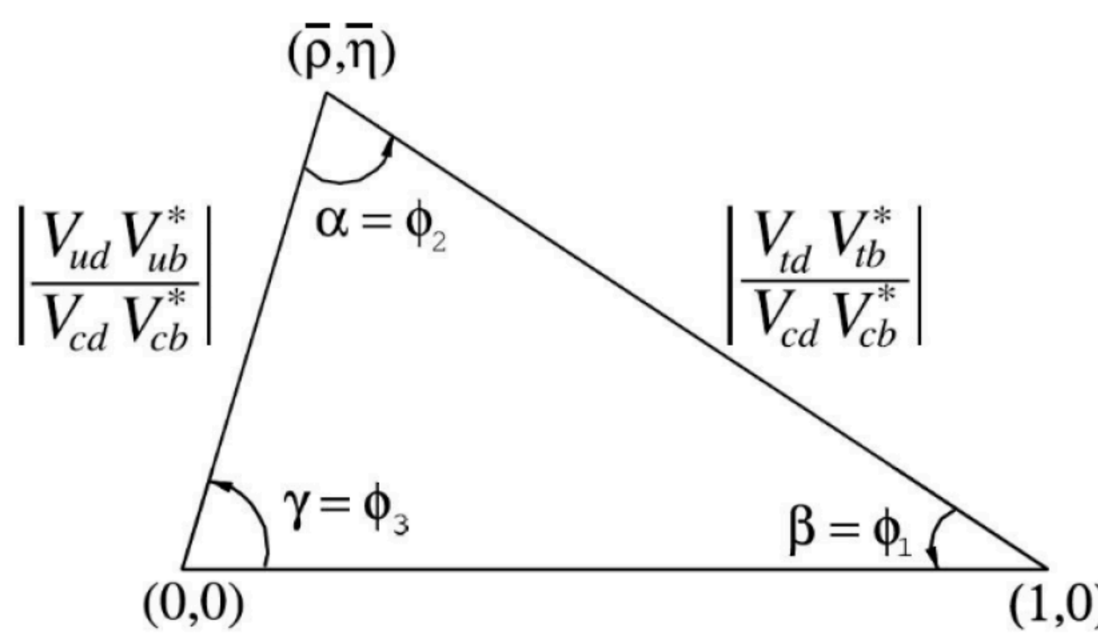
$\sim e^{i\varphi 3}$

- V_{CKM} unitarity leads to 6 relations that can be represented as triangles in the complex plane.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

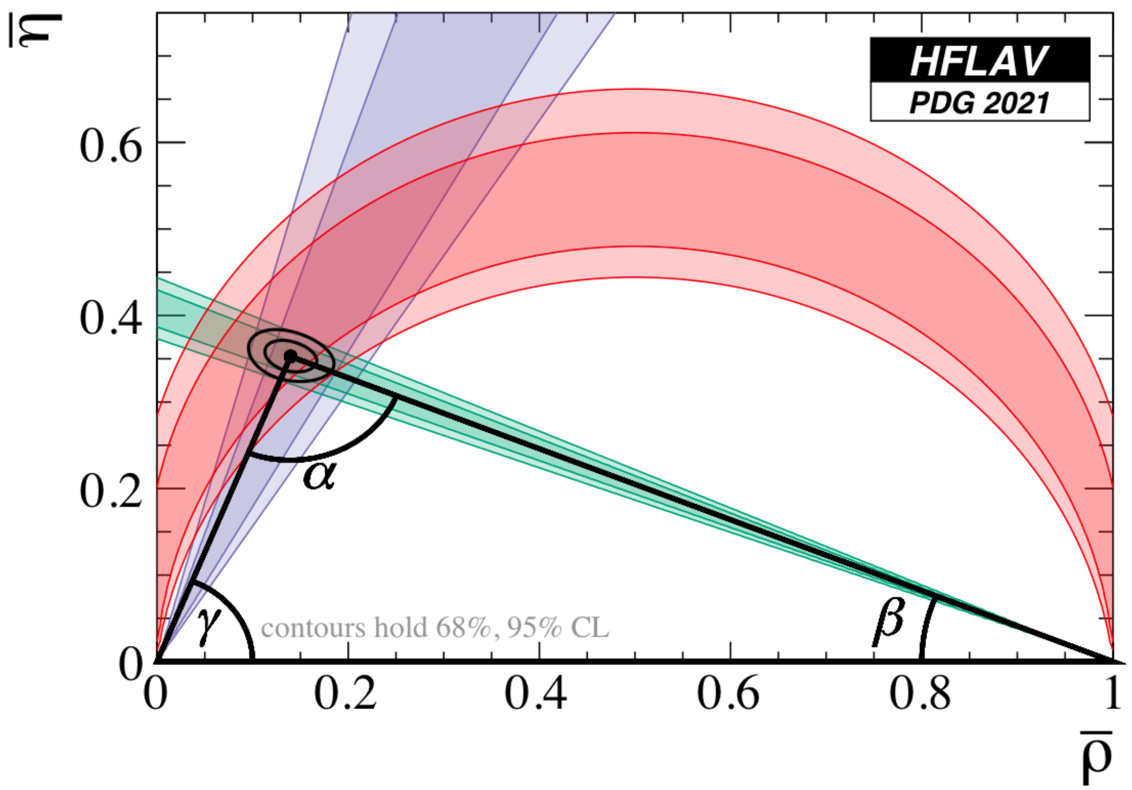
$$\Rightarrow \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

$$\phi_3/\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$



Current Status

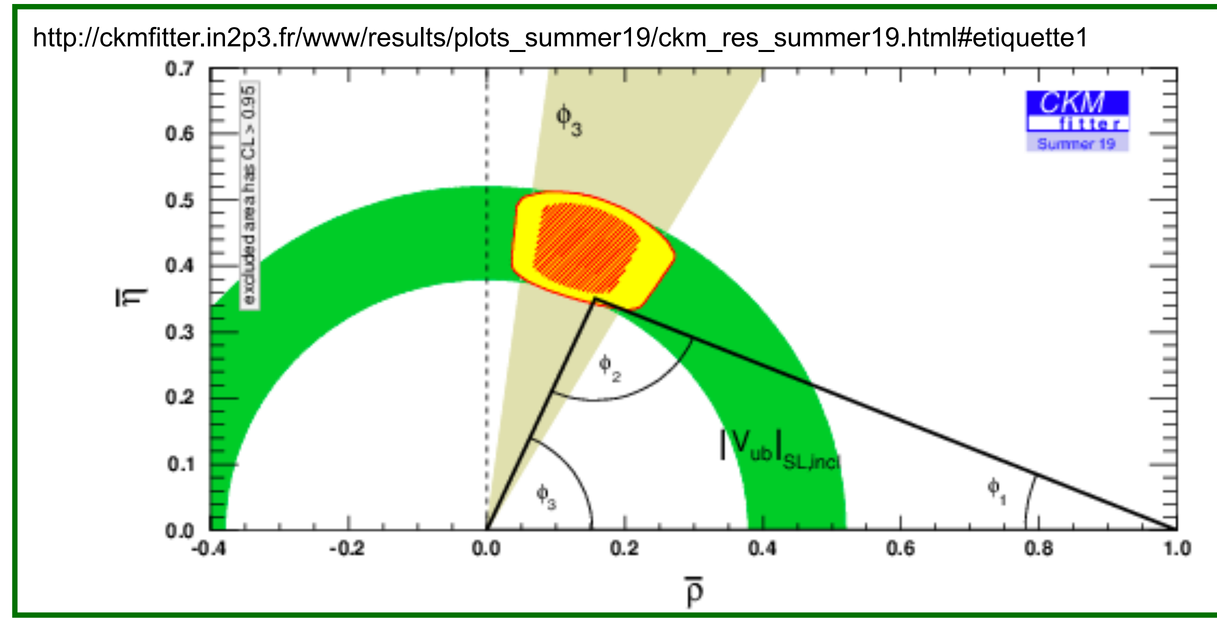
HFLAV2021



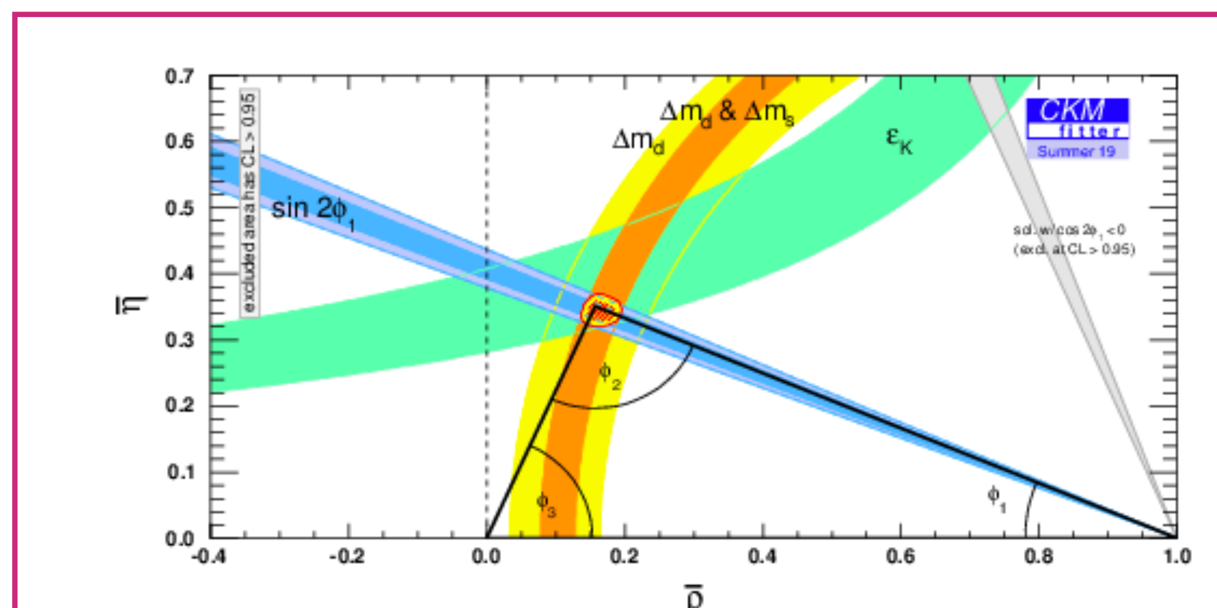
https://hflav-eos.web.cern.ch/hflav-eos/triangle/pdg2021/index.shtml

$$\phi_1 = (22.2 \pm 0.7)^\circ, \phi_3 = \left(66.2^{+3.4}_{-3.6}\right)^\circ$$

CKMFitter2019



Constraints from "Tree" quantities in the (ρ-bar,η-bar) plane with only input on $|V_{ub}|$ from inclusive semileptonic B decays (only γ(DK) is used).



Constraints from "Loop" quantities

$$\phi_3 = \left(72.1^{+5.4}_{-5.7}\right)^\circ$$ Direct measurement

$$\phi_3 = \left(65.66^{+0.90}_{-2.65}\right)^\circ$$ Indirect prediction

ϕ_3 Measurement

- ϕ_3 is a Standard Model benchmark since it is the only CKM angle that can be accessed at tree level. Together with $|V_{ub}|$, it provides an unique testbed for new physics searches as well.

- Three methods to extract ϕ_3 using $B \to DK$ decays:

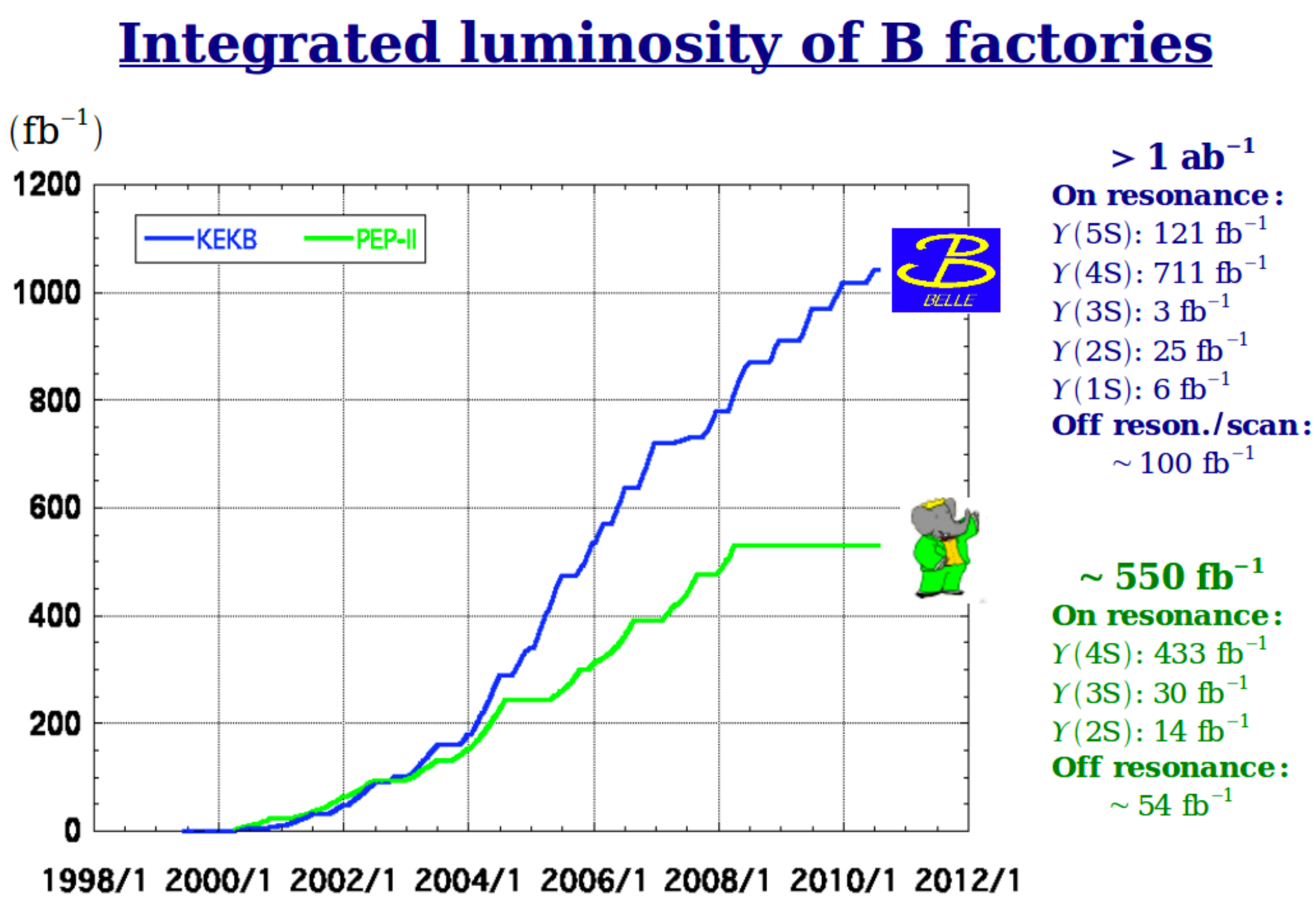
1. GLW method (interference with CP eigenstates) [PLB 253, 483 (1991)] Typical final states of D^0: $K^+K^-, \pi^+\pi^-, K_S\pi^0$

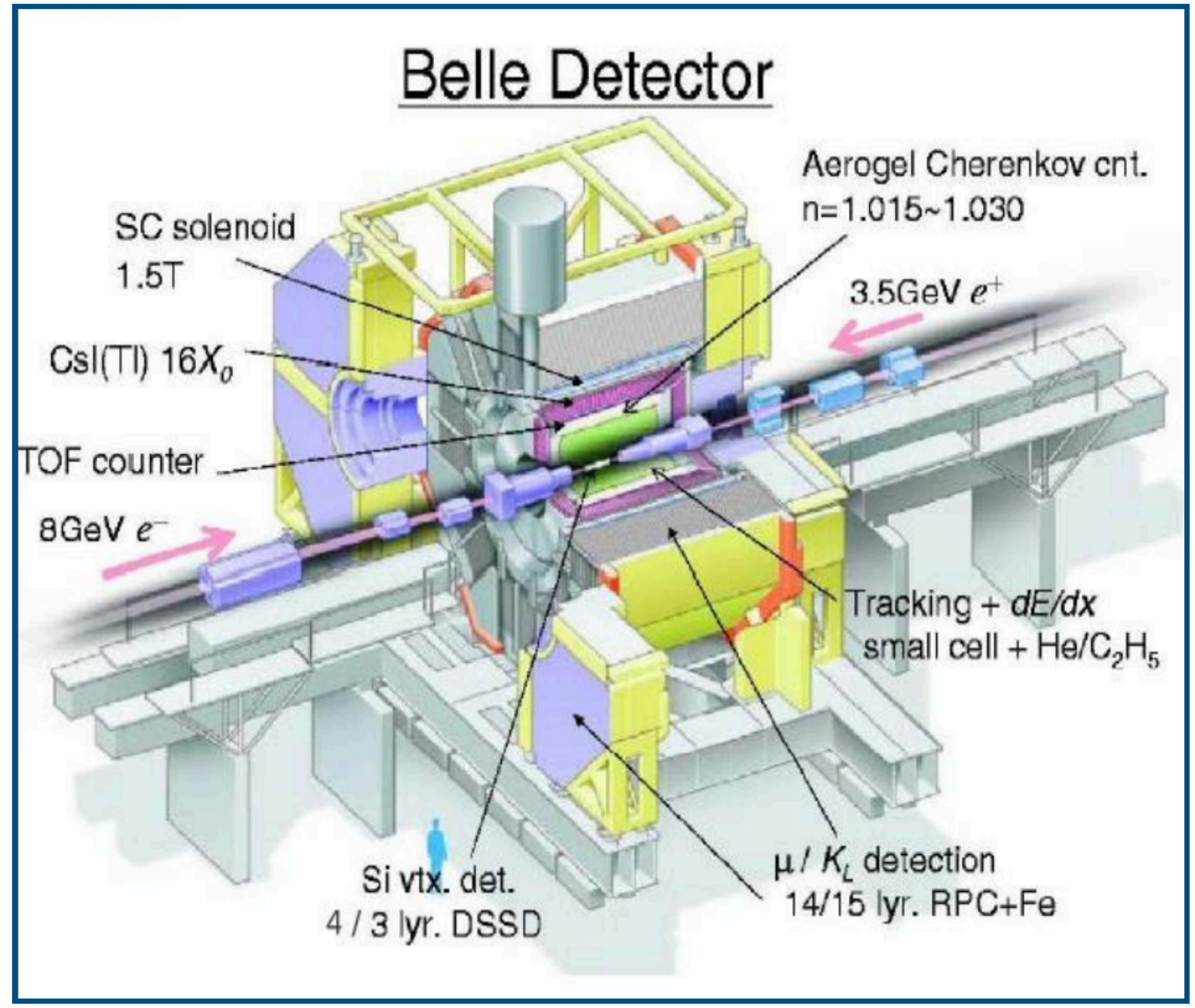
2. ADS method (interference with flavour specific states) [PRL78, 3257 (1997)] Typical final states of D^0: CF and DCS D decays such as $K\pi, K\pi\pi^0, K\pi\pi\pi$

3. BPGGSZ method (self-conjugate of D decays using Dalitz plot) [PRD 68, 054018 (2003), PRD 70, 072003] Typical final states of D^0: three-body D decays such as $K_S\pi^+\pi^-, K_SK^+K^-$

ADS, BPGGSZ methods need inputs from charm

Belle detector





On-resonance:
- 4S : 711 fb^{-1} largest
- 5S : 121 fb^{-1} largest
- 3S : 3 fb^{-1}
- 2S : 25 fb^{-1} largest
- 1S : 6 fb^{-1} largest

Off-resonance/scan
~100 fb^{-1}

Successful data-taking for over a decade with ~600 physics publications!!

Latest Belle results

$\bar{B}^0 \to D^+ h^- (h = \pi, K)$

NEW RESULT

$B^\pm \to D^0(K_S \pi^+ \pi^-) K^\pm$

$B^- \to D(K^+ \pi^- \pi^0) K^- (D = D^0/\bar{D}^0)$

$B^0 \to D^0(K\pi) K^{*0}$

$B^0 \to D^0(K_S \pi^+ \pi^-) K^{*0}$

$B^\pm \to D^0(K_S \pi^+ \pi^- \pi^0) K^\pm$

$\bar{B}^0 \to D^+ h^- (h = \pi, K)$

- Improved measurements of color-favored hadronic two-body decays of B mesons will lead to a better understanding of quantum chromodynamics (QCD) effects

- The colour-allowed tree-level decays $\bar{B}^0 \to D^{(*)+} h^- (h = \pi, K)$ triggered quite some interest recently since they differ significantly from the QCD factorisation expectation. [PRL102, 021801(2009), PRD 85, 032008 (2012)]

- Discrepancy has been observed between theory and data in two-body non-leptonic tree-level decays into heavy-light final states. [arXiV:1606.02888 (2016), arXiV:2007.10338 (2020)]

- Implications of New Physics in these modes will be quite interesting [PRD 102 (2020) 071701]

- These modes are high statistics control samples for hadronic B-decay measurements related to time dependent CPV and ϕ_3 measurement

$\bar{B}^0 \to D^+h^-(h = \pi, K)$

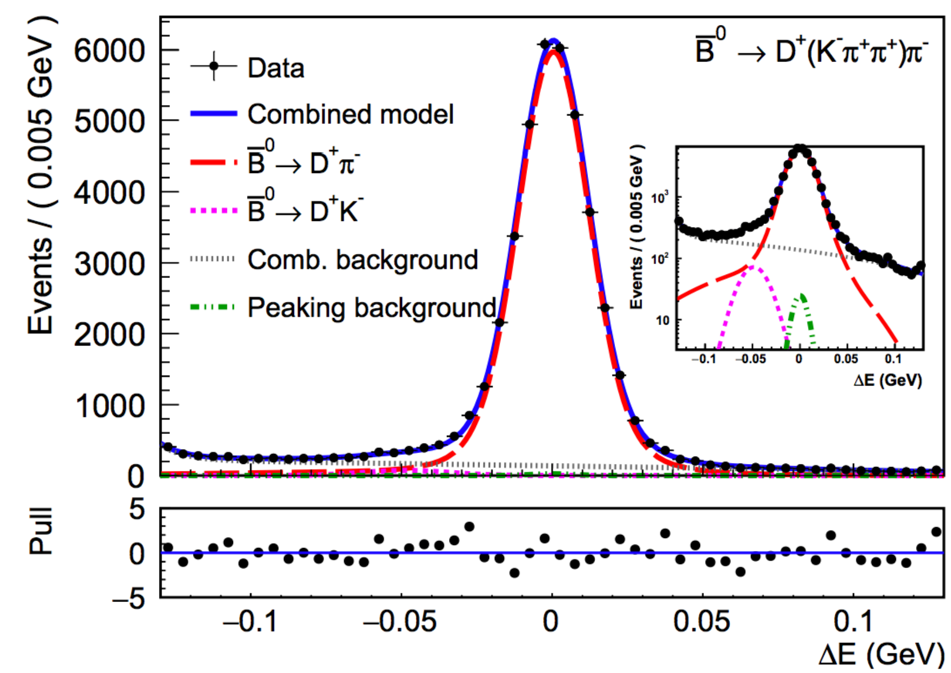
- Analysis using full Belle dataset of 711 fb^{-1} [arXiV: 2111.04978 (2021)]

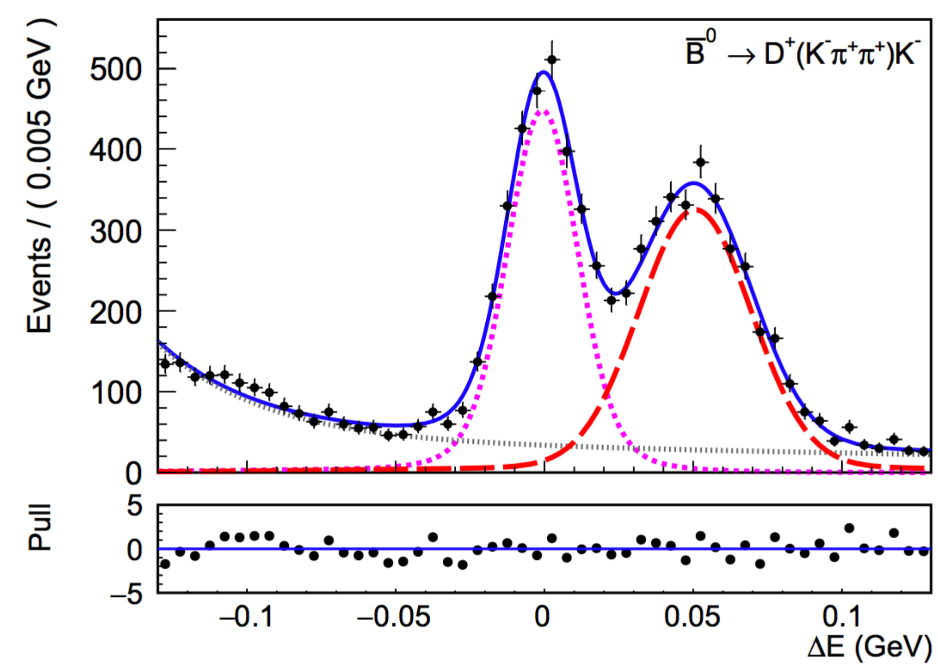
- Individual Branching fractions of the Cabibbo favored and the Cabibbo suppressed measured $BF(\bar{B}^0 \to D^+\pi^-) = (2.48 \pm 0.01 \pm 0.09 \pm 0.04) \times 10^{-3}$ and the Cabibbo suppressed $BF(\bar{B}^0 \to D^+K^-) = = (2.03 \pm 0.05 \pm 0.07 \pm 0.03) \times 10^{-4}$

- Ratio of branching fractions of CS and CF is measured as $R^D = (8.19 \pm 0.20 \pm 0.23) \times 10^{-2}$

- This ratio facilitates tests of theoretical predictions, particularly those of factorization and SU(3) symmetry breaking in QCD.

- Individual branching fractions are lower than the theory predictions, however, the ratio agrees within uncertainties [arXiV:1606.02888 (2016)].





$\bar{B}^0 \to D^{*+}h^-(h = \pi, K)$

- Large branching fraction modes - high statistics sample available

- Allow for high precision tests of the theoretical framework to calculate hadronic B decays -reduction of theoretical uncertainties on CPV phases and CKM angles

- Using semi-leptonic decay rates $d\Gamma(\bar{B}^0 \to D^{*+}l\bar{\nu})/dq^2$ at fixed lepton momentum transfer, $q^2 = m_h^2$, prediction for a fundamental parameter of hadronic B-decays, $|a_1(q^2)|$ can be made

- QCD Factorization test (QCD factorisation prediction [arXiv:2111.04478 (2021)]) to measure this factor $|a_1(q^2)|$ and tests for SU(3) symmetry performed at Belle - **Results will be published soon with full Belle dataset**

- Individual branching fractions and ratios of branching fraction will also be presented

$B^{\pm} \rightarrow D^0(K_S^0\pi^+\pi^-)K^{\pm}$

GGSZ

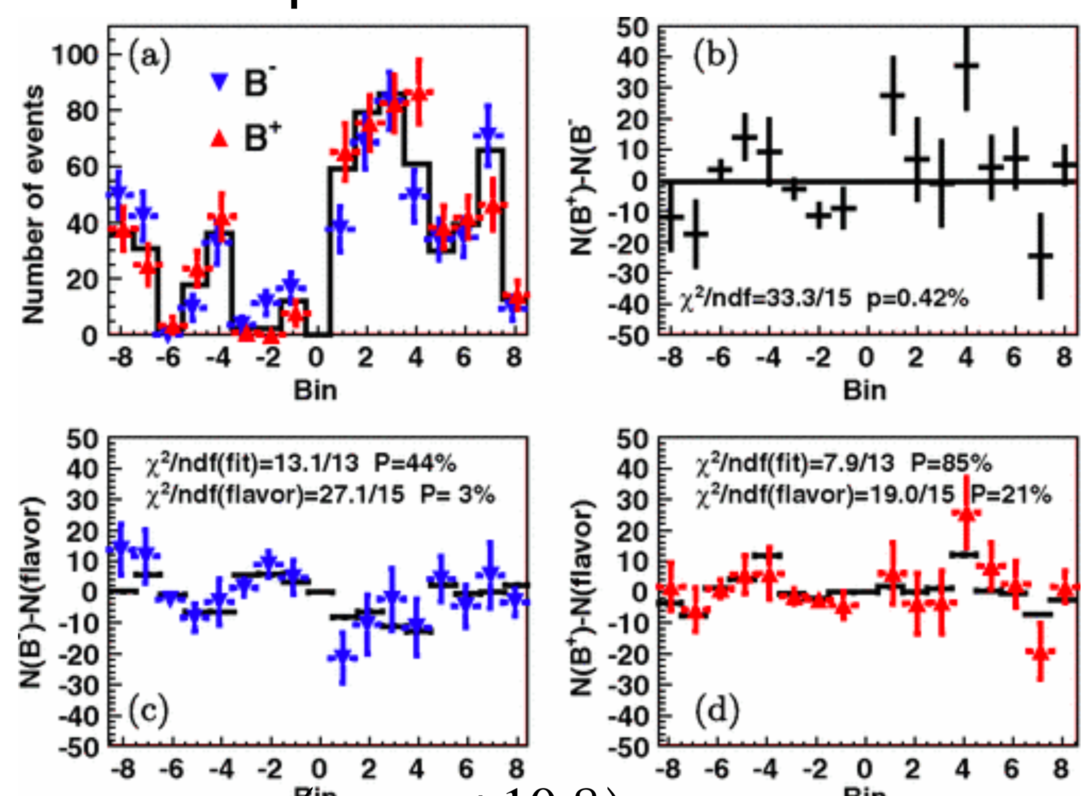
- First measurement of ϕ_3 using a model-independent Dalitz plot analysis of $B \rightarrow D(K_S^0\pi^+\pi^-)K$ [PRD85, 112014 (2012)]. Combined Belle and Belle II analysis result will be presented by Niharika (next talk)!!

- Uses measurements of the strong phase of the amplitude from the CLEO Collaboration as input

- Full Belle dataset of 711 fb^{-1}

- $\phi_3 = (77.3^{+15.1}_{-14.9}) \pm 4.1 \pm 4.3)^{\circ}$

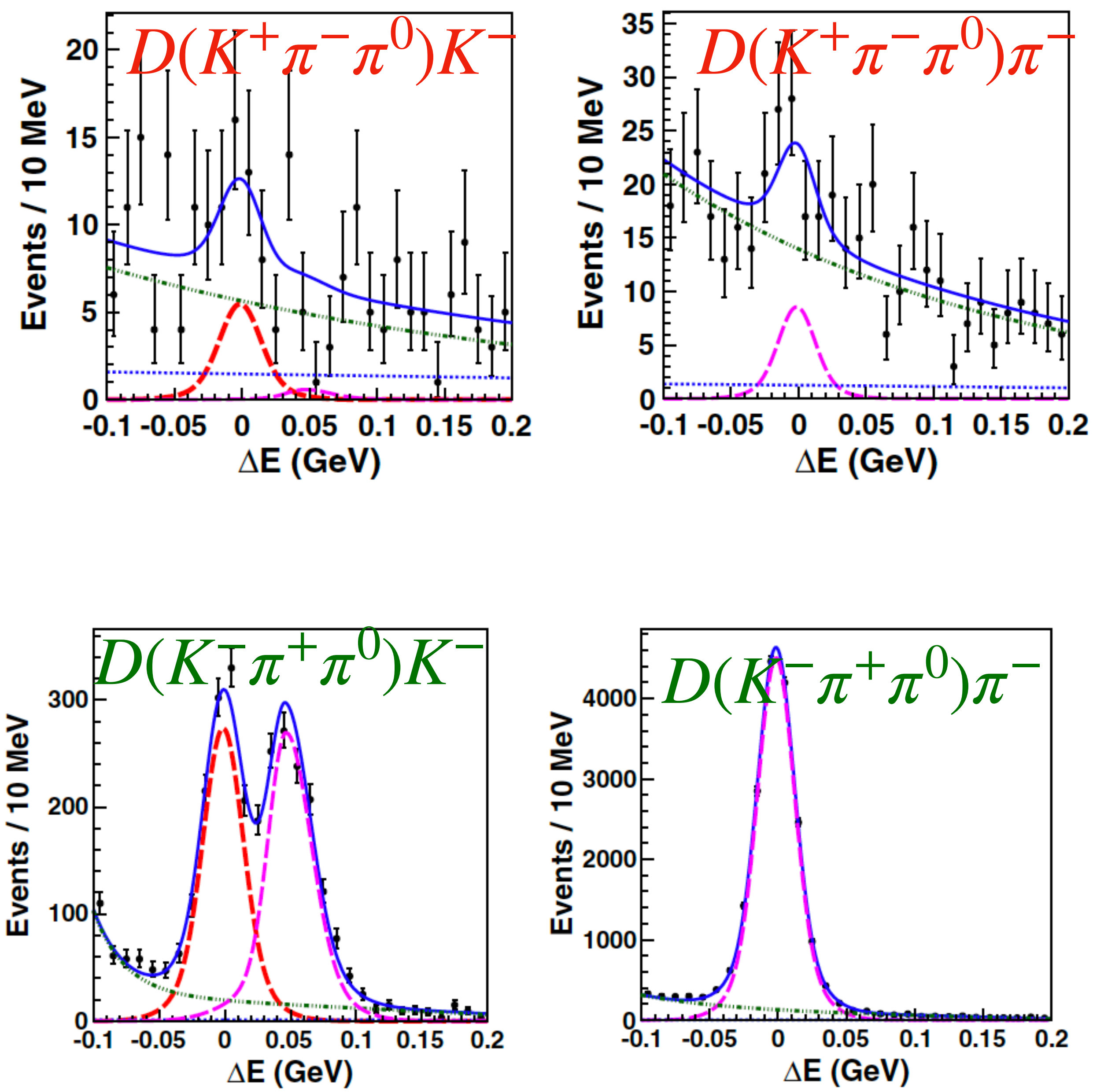
- $r_B = 0.145 \pm 0.030 \pm 0.010 \pm 0.011$



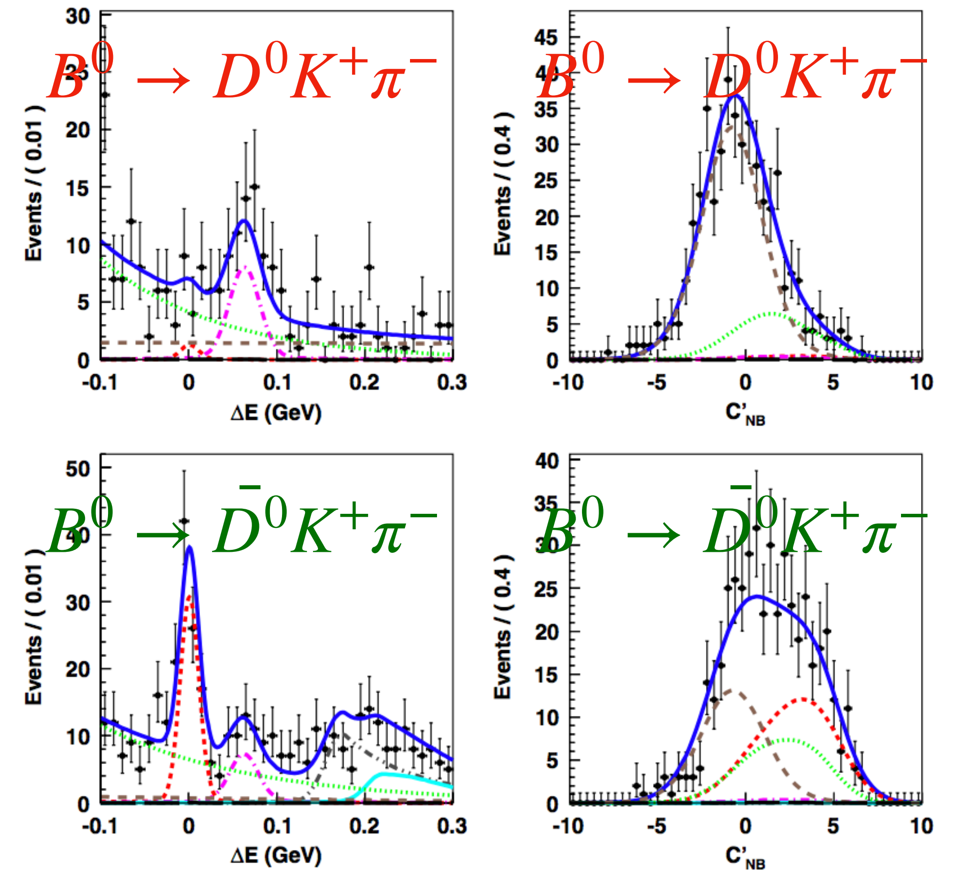
- Latest Belle result using D*K (605 fb^{-1} data): $\phi_3 = (78.4^{+10.8}_{-11.6}) \pm 3.6 \pm 8.9^{\circ}$ [PRD81, 112002 (2010)]

$B^- \to D(K^+\pi^-\pi^0)K^-$ ADS

- $B^- \to D(K^+\pi^-\pi^0)K^-$ analysis using full Belle dataset of 711 fb^{-1} [PRD88, 091104 (R) (2013)]
- First evidence of the signal for this suppressed decay with a significance of 3.2 σ
- Direct CP asymmetry between the suppressed B$^-$ and B$^+$ decays: $A_{DK} = (0.41 \pm 0.30 \pm 0.05)$
- $R_{DK} = (1.98 \pm 0.62 \pm 0.24) \times 10^{-2}$
- These results can be used to constrain ϕ_3 using the ADS method



$B^0 \to D^0(K\pi)K^{*0}(K\pi)$ ADS



- Usually ϕ_3 measurement has been advanced mainly by exploiting charged B meson decays into $D^{(*)}K^{\pm}$, this analysis exploits neutral B meson decays

- $B^0 \to D(K^-\pi^+)K^{*0}(K^+\pi^-)$ analysis done using full Belle dataset of 711 fb-1 [PRD 86, 011101(R) (2012)]

- $R_{DK^{*0}} = \Gamma(B^0 \to [K^-\pi^+]_D K^+\pi^-)/\Gamma(B^0 \to [K^+\pi^-]_D K^+\pi^-) = \left(4.5^{+5.6+2.8}_{-5.0-1.8}\right) \times 10^{-2}$

- Since the value is not significant, credible upper limit set: $R_{DK^{*0}} < 0.16(95\,\%\,CL)$

- $R_{DK^{*0}}$ can be used to extract ϕ_3 by combining with other observables related to the same dynamical parameters r_s, δ_s, k.

$B^0 \to D(K_S^0\pi^+\pi^-)K^{*0}$

GGSZ

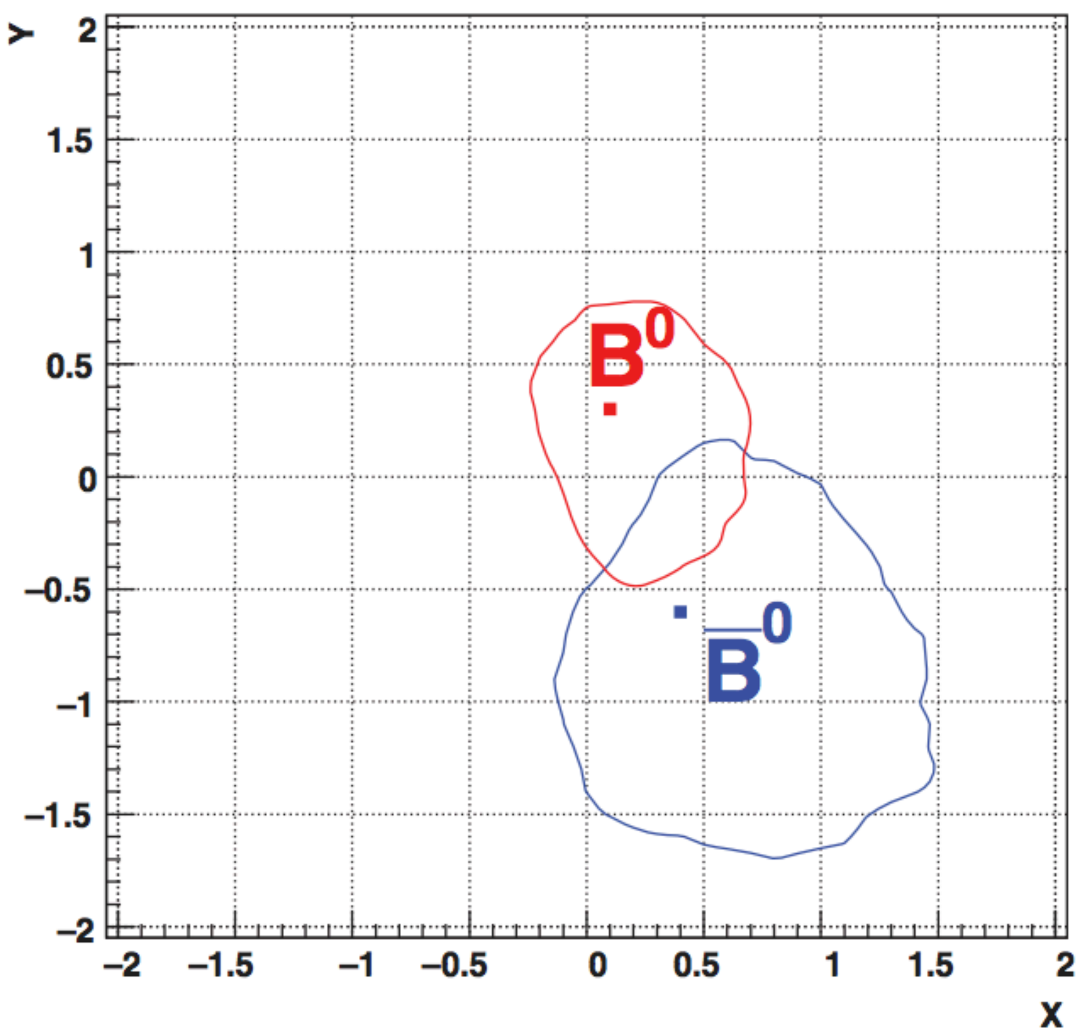
- First model-independent Dalitz analysis of $B^0 \to D(K_S^0\pi^+\pi^-)K^{*0}$ using full Belle dataset of 711 fb^{-1} [PTEP 043C01 (2016)]

$$x_\pm = r_s \cos(\delta_s \pm \phi_3)$$
$$y_\pm = r_s \sin(\delta_s \pm \phi_3)$$

$$r_S^2 \equiv \frac{\Gamma(B^0 \to D^0 K^+\pi^-)}{\Gamma(B^0 \to \bar{D}^0 K^+\pi^-)}$$

$$x_- = +0.4^{+1.0}_{-0.6}\,{}^{+0.0}_{-0.1} \pm 0.0$$
$$y_- = -0.6^{+0.8}_{-1.0}\,{}^{+0.1}_{-0.0} \pm 0.1$$
$$x_+ = +0.1^{+0.7}_{-0.4}\,{}^{+0.0}_{-0.1} \pm 0.1$$
$$y_+ = +0.3^{+0.5}_{-0.8}\,{}^{+0.0}_{-0.1} \pm 0.1$$

$$r_s < 0.87 (68\,\%\ CL)$$



$B^{\pm} \to D(K_S^0\pi^+\pi^-\pi^0)K^{\pm}$

GGSZ

- First measurement of ϕ_3 using full Belle dataset using $B^{\pm} \to D(K_S^0\pi^+\pi^-\pi^0)K^{\pm}$ [JHEP10(2019)178]
- Measurements of the strong-phase difference of the of $D \to K_S^0\pi^+\pi^-\pi^0$ decays using 0.82 fb^{-1} data collected at the ψ(3770) resonance by the CLEO-c [JHEP01, 82 (2018)] used as input.
- cos (c_i) and sine(s_i) of the strong phase difference between D^0 and $\bar{D}^0$ averaged over the region of phase space, obtained using CLEO data used as input in the Belle analysis
- Several resonance substructures seen
- Binning the phase space around these resonances in the absence of an amplitude model done

Bin	Resonance
1	ω
2	$K^{*-}\rho^+$
3	$K^{*+}\rho^-$
4	K^{*-}
5	K^{*+}
6	K^{*0}
7	ρ^+
8	ρ^-
9	remainder

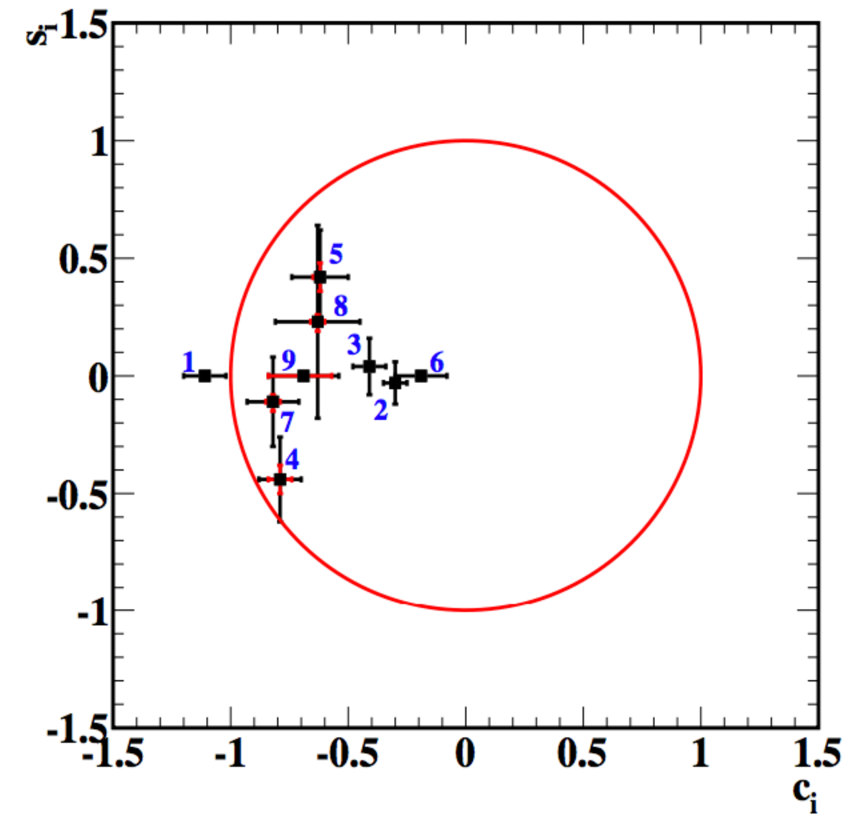
$B^{\pm} \to D(K_S^0\pi^+\pi^-\pi^0)K^{\pm}$

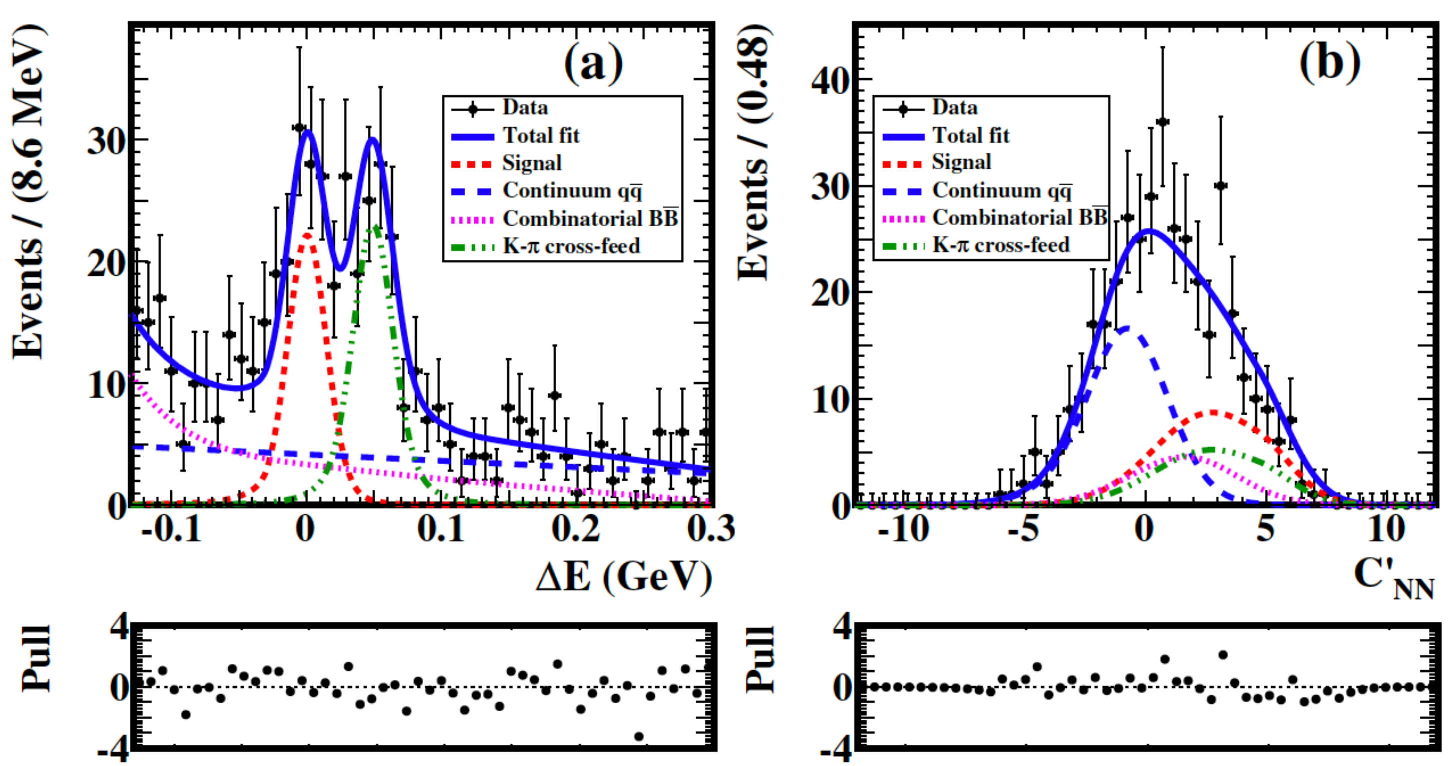
GGSZ

[JHEP01, 82 (2018)]

Bin	Resonance
1	ω
2	$K^{*-}\rho^+$
3	$K^{*+}\rho^-$
4	K^{*-}
5	K^{*+}
6	K^{*0}
7	ρ^+
8	ρ^-
9	remainder

Bin	c_i	s_i
1	$-1.11 \pm 0.09^{+0.02}_{-0.01}$	0.00
2	$-0.30 \pm 0.05 \pm 0.01$	$-0.03 \pm 0.09^{+0.01}_{-0.02}$
3	$-0.41 \pm 0.07^{+0.02}_{-0.01}$	$0.04 \pm 0.12^{+0.01}_{-0.02}$ *
4	$-0.79 \pm 0.09 \pm 0.05$	$-0.44 \pm 0.18 \pm 0.06$
5	$-0.62 \pm 0.12^{+0.03}_{-0.02}$	$0.42 \pm 0.20 \pm 0.06$ *
6	$-0.19 \pm 0.11 \pm 0.02$	0.00
7	$-0.82 \pm 0.11 \pm 0.03$	$-0.11 \pm 0.19^{+0.04}_{-0.03}$
8	$-0.63 \pm 0.18 \pm 0.03$	$0.23 \pm 0.41^{+0.04}_{-0.03}$ *
9	$-0.69 \pm 0.15^{+0.15}_{-0.12}$	0.00





Signal-enhanced fit projections

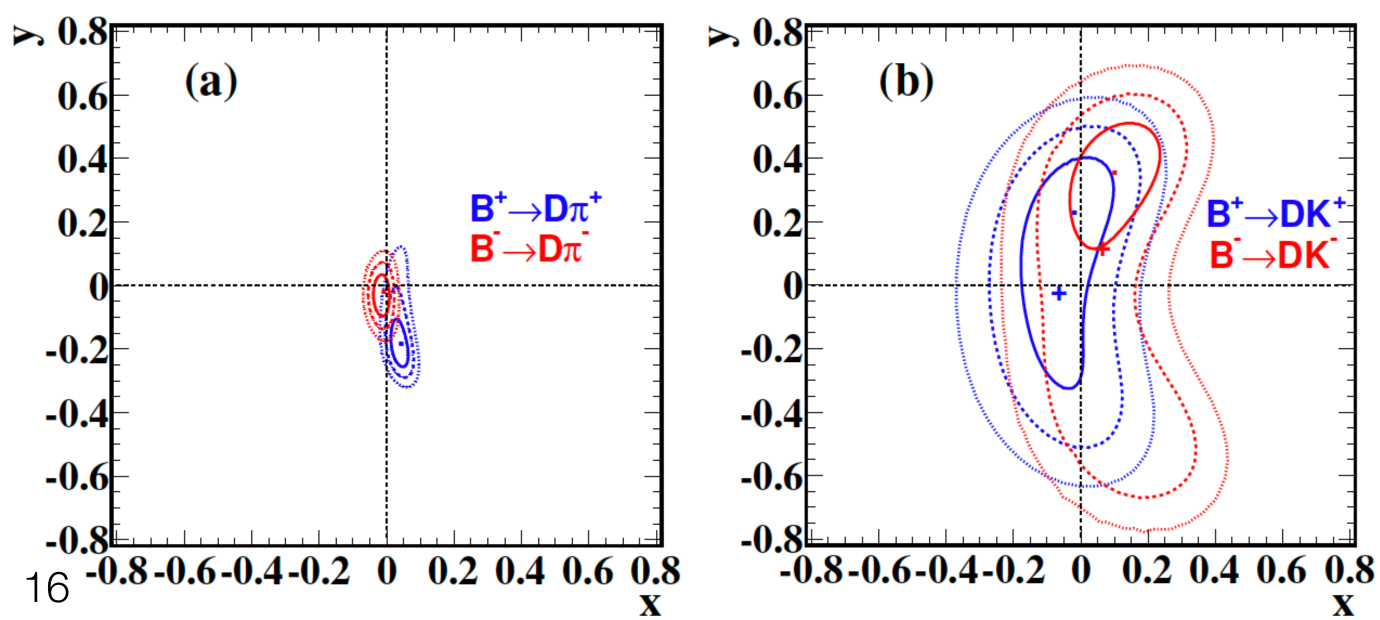
[JHEP10(2019)178]

$B^{\pm} \to D(K_S^0\pi^+\pi^-\pi^0)K^{\pm}$

[JHEP10(2019)178]

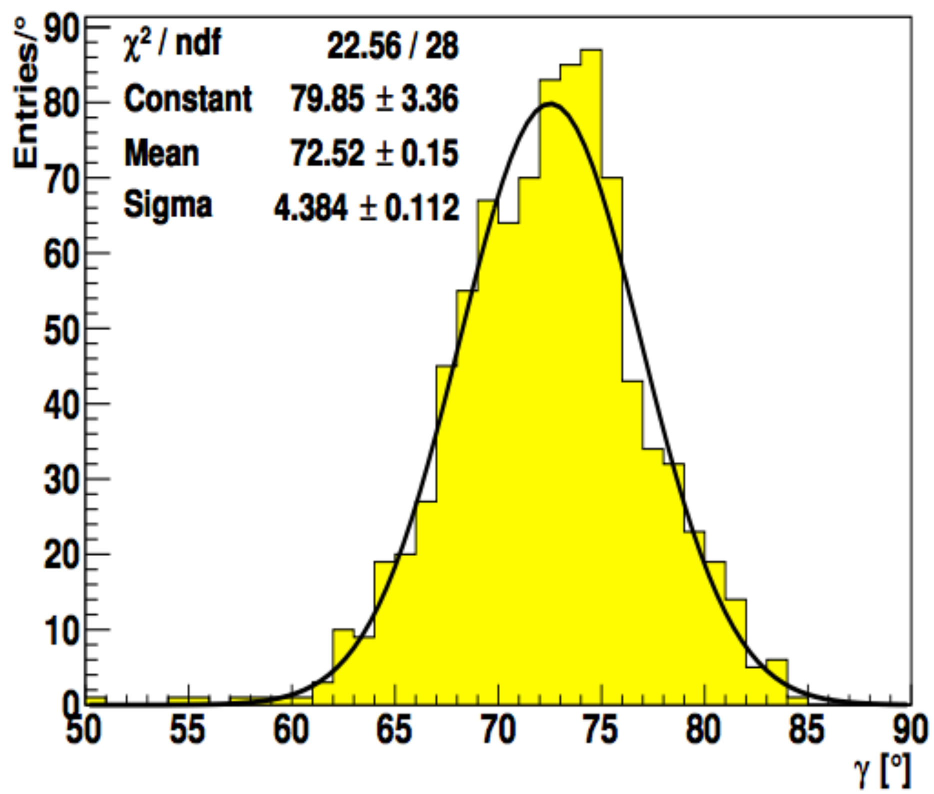
- $\phi_3 = \left(5.7^{+10.2}_{-8.8}\right) \pm 3.5 \pm 5.7^\circ$
- 95% CL interval on ϕ_3 $(-29.7, 109.5)^\circ$ consistent with the current world average
- $r_B = 0.323 \pm 0.147 \pm 0.023 \pm 0.051$
- $\delta_B = \left(83.4^{+18.3}_{-16.6}\right) \pm 3.1 \pm 4.0^\circ$

	$B^{\pm} \to D\pi^{\pm}$	$B^{\pm} \to DK^{\pm}$
x_+	$0.039 \pm 0.024\ ^{+0.018}_{-0.013}\ ^{+0.014}_{-0.012}$	$-0.030 \pm 0.121\ ^{+0.017}_{-0.018}\ ^{+0.019}_{-0.018}$
y_+	$-0.196\ ^{+0.080}_{-0.059}\ ^{+0.038}_{-0.034}\ ^{+0.032}_{-0.030}$	$0.220\ ^{+0.182}_{-0.541} \pm 0.032\ ^{+0.072}_{-0.071}$
x_-	$-0.014 \pm 0.021\ ^{+0.018}_{-0.010}\ ^{+0.019}_{-0.010}$	$0.095 \pm 0.121\ ^{+0.017}_{-0.016}\ ^{+0.023}_{-0.025}$
y_-	$-0.033 \pm 0.059\ ^{+0.018}_{-0.019}\ ^{+0.019}_{-0.010}$	$0.354\ ^{+0.144}_{-0.197}\ ^{+0.015}_{-0.021}\ ^{+0.032}_{-0.049}$



$B^{\pm} \to D(K_S^0\pi^+\pi^-\pi^0)K^{\pm}$

- This measurement can be improved upon once a suitable amplitude model for is available to provide guidance in choosing a more sensitive binning.
- Precise inputs for c_i, s_i from BESIII will help reduce the systematic uncertainty.
- Single-mode uncertainty on ϕ_3 of $4.4°$ is achievable with a 50 ab^{-1} sample of data at Belle II experiment.



Summary

- Precision measurement of ϕ_3 is important to establish CPV in Standard model.
- Latest measurements by Belle have strong impact in improving ϕ_3 precision. Several results are already available with full Belle dataset.
- Belle II and LHCb will be major players for further improving ϕ_3 precision in future.
- ϕ_3 measurement precision will improve the precision to about $1-2°$ with 50 ab^{-1} to be collected at Belle II.

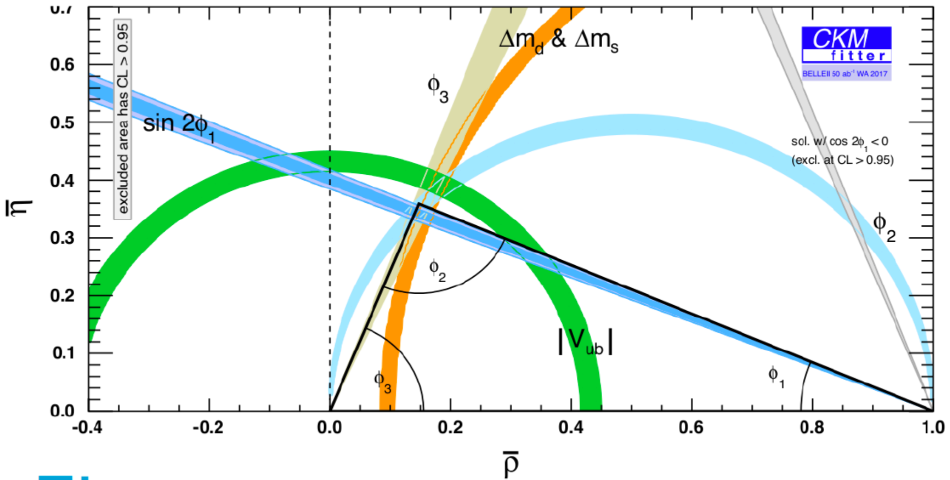


Figure : Future (50 ab^{-1} Belle II data)

BACK-UP

arXiV:2007.10338 (2020)

measurement	value	source
$\mathcal{B}(B_s^0 \to D_s^- \pi^+)$	$(3.6 \pm 0.5 \pm 0.5)\ 10^{-3}$	Belle
$\frac{f_s}{f_d} \frac{\mathcal{B}(B_s^0 \to D_s^-(\to \phi(\to K^+K^-)\pi^-)\pi^+)}{\mathcal{B}(B^0 \to D^-(\to K^+\pi^-\pi^-)\pi^+)}$	$(6.7 \pm 0.5)\%$	CDF
$\frac{f_s}{f_d} \frac{\mathcal{B}(B_s^0 \to D_s^-(\to K^+K^-\pi^-)\pi^+)}{\mathcal{B}(B^0 \to D^-(\to K^+\pi^-\pi^-)\pi^+)}$	0.174 ± 0.007	LHCb
$\frac{f_s}{f_d} \frac{\mathcal{B}(B_s^0 \to D_s^-(\to K^+K^-\pi^-)\pi^+)}{\mathcal{B}(B^0 \to D^-(\to K^+\pi^-\pi^-)K^+)}$	2.08 ± 0.08	LHCb
$\frac{\mathcal{B}(B^0 \to D^-K^+)}{\mathcal{B}(B^0 \to D^-\pi^+)}$	$(8.22 \pm 0.28)\%$	LHCb
$\frac{\mathcal{B}(B^0 \to D^-K^+)}{\mathcal{B}(B^0 \to D^-\pi^+)}$	$(6.8 \pm 1.7)\%$	Belle
$f_{00}\mathcal{B}(B^0 \to D^-(\to K^+\pi^-\pi^-)\pi^+)$	$(1.21 \pm 0.05)\ 10^{-4}$	BaBar/CLEO
$\mathcal{B}(B^0 \to D^-(\to K^+\pi^-\pi^-)\pi^+)$	$(2.88 \pm 0.29)\ 10^{-4}$	BaBar
$\frac{\mathcal{B}(B_s^0 \to D_s^{*-}\pi^+)}{\mathcal{B}(B_s^0 \to D_s^-\pi^+)}$	0.66 ± 0.16	Belle
$\frac{\mathcal{B}(B^0 \to D^{*-}K^+)}{\mathcal{B}(B^0 \to D^{*-}\pi^+)}$	$(7.75 \pm 0.30)\%$	LHCb/BaBar/Belle
$f_{00}\mathcal{B}(B^0 \to D^{*-}\pi^+)$	$(2.72 \pm 0.14)\ 10^{-3}$	BaBar/CLEO
$\frac{\mathcal{B}(B^0 \to D^{*-}\pi^+)}{\mathcal{B}(B^0 \to D^-\pi^+)}$	0.99 ± 0.14	BaBar

$\bar{B}^0 \to D^{(*)+}h^-(h = \pi, K)$
Theory predictions

Table 2: CP-averaged branching ratios (in units of 10^{-3} for $b \to c\bar{u}d$ and 10^{-4} for $b \to c\bar{u}s$ transitions) of $\bar{B}_{(s)} \to D_{(s)}^{(*)+} L^-$ decays. The vector- and axial-vector final states refer to the longitudinal polarization amplitudes only. The theoretical errors shown correspond to the uncertainties due to renormalization scales μ and μ_0, the CKM as well as the hadronic parameters, added in quadrature. The experimental data is taken from refs. [62,66,81,82].

Decay mode	LO	NLO	NNLO	Exp.
$\bar{B}_d \to D^+\pi^-$	3.58	$3.79^{+0.44}_{-0.42}$	$3.93^{+0.43}_{-0.42}$	2.68 ± 0.13
$\bar{B}_d \to D^{*+}\pi^-$	3.15	$3.32^{+0.52}_{-0.49}$	$3.45^{+0.53}_{-0.50}$	2.76 ± 0.13
$\bar{B}_d \to D^+K^-$	2.74	$2.90^{+0.33}_{-0.31}$	$3.01^{+0.32}_{-0.31}$	1.97 ± 0.21
$\bar{B}_d \to D^{*+}K^-$	2.37	$2.50^{+0.39}_{-0.36}$	$2.59^{+0.39}_{-0.37}$	2.14 ± 0.16

$\frac{\mathrm{Br}(\bar{B}_d \to D^+K^-)}{\mathrm{Br}(\bar{B}_d \to D^+\pi^-)}$	0.077	$0.077^{+0.002}_{-0.002}$	$0.077^{+0.002}_{-0.002}$	0.074 ± 0.009
$\frac{\mathrm{Br}(\bar{B}_d \to D^{*+}K^-)}{\mathrm{Br}(\bar{B}_d \to D^{*+}\pi^-)}$	0.075	$0.075^{+0.002}_{-0.002}$	$0.075^{+0.002}_{-0.002}$	0.078 ± 0.007

[arXiV:1606.02888]

$\bar{B}^0 \to D^{(*)+}h^-(h = \pi, K)$
Theory predictions

[arXiV:1606.02888]

$\lvert a_1(D^{(*)+}L^-)\rvert$	LO	NLO	NNLO	Exp.
$\lvert a_1(D^+\pi^-)\rvert$	1.025	$1.054^{+0.022}_{-0.020}$	$1.073^{+0.012}_{-0.014}$	0.89 ± 0.05
$\lvert a_1(D^{*+}\pi^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.071^{+0.013}_{-0.014}$	0.96 ± 0.03
$\lvert a_1(D^+\rho^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.072^{+0.012}_{-0.014}$	0.91 ± 0.08
$\lvert a_1(D^{*+}\rho^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.071^{+0.013}_{-0.014}$	0.86 ± 0.06
$\lvert a_1(D^+K^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.070^{+0.010}_{-0.013}$	0.87 ± 0.06
$\lvert a_1(D^{*+}K^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.069^{+0.010}_{-0.013}$	0.97 ± 0.04
$\lvert a_1(D^+K^{*-})\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.070^{+0.010}_{-0.013}$	0.99 ± 0.09
$\lvert a_1(D^+a_1^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.072^{+0.012}_{-0.014}$	0.76 ± 0.19

$\bar{B}^0 \to D^{(*)+}h^-(h = \pi, K)$

Theory predictions

arXiV:2007.10338

BFs in units of 10^{-3}

source	PDG	our fits (w/o QCDF)		our fit (w/ QCDF, no f_s/f_d)		QCDF prediction
scenario	—	no f_s/f_d	$(f_s/f_d)^{7\ \mathrm{TeV}}_{\mathrm{LHCb,sl}}$	ratios only	$\not{SU(3)}$	—
χ^2/dof	—	2.5/4	3.1/5	4.6/6	3.7/4	—
$\mathcal{B}(\bar{B}^0_s \to D^+_s\pi^-)$	3.00 ± 0.23	3.6 ± 0.7	3.11 ± 0.25	$3.11^{+0.21}_{-0.19}$	$3.20^{+0.20}_{-0.26}$ *	4.42 ± 0.21
$\mathcal{B}(\bar{B}^0 \to D^+K^-)$	0.186 ± 0.020	0.222 ± 0.012	0.224 ± 0.012	0.227 ± 0.012	0.226 ± 0.012	0.326 ± 0.015
$\mathcal{B}(\bar{B}^0 \to D^+\pi^-)$	2.52 ± 0.13	2.71 ± 0.12	2.73 ± 0.12	2.74 ± 0.12	$2.73^{+0.12}_{-0.11}$	—
$\mathcal{B}(\bar{B}^0_s \to D^{*+}_s\pi^-)$	2.0 ± 0.5	2.4 ± 0.7	2.1 ± 0.5	$2.46^{+0.37}_{-0.32}$	$2.43^{+0.39}_{-0.32}$	$4.3^{+0.9}_{-0.8}$
$\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$	0.212 ± 0.015	0.216 ± 0.014	0.216 ± 0.014	$0.213^{+0.014}_{-0.013}$	$0.213^{+0.014}_{-0.013}$	$0.327^{+0.039}_{-0.034}$
$\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-)$	2.74 ± 0.13	2.78 ± 0.15	2.79 ± 0.15	$2.76^{+0.15}_{-0.14}$	$2.76^{+0.15}_{-0.14}$	—

[arXiV:2111.04978 (2021)]

TABLE II. Systematic uncertainties in the measured R^D value and branching fractions for $\overline{B}^0 \to D^+\pi^-$ and $\overline{B}^0 \to D^+K^-$. The total systematic uncertainty is the quadratic sum of the uncorrelated uncertainties.

Source	R^D	$\mathcal{B}(\overline{B}^0 \to D^+\pi^-)$	$\mathcal{B}(\overline{B}^0 \to D^+K^-)$
$\mathcal{B}(D^+ \to K^-\pi^+\pi^+)$	–	1.71%	1.71%
Tracking	–	1.40%	1.40%
$N_{B\overline{B}}$	–	1.37%	1.37%
f^{00}/f^{+-}	–	1.92%	1.92%
$D^+ \to K^-\pi^+\pi^+$ model	–	0.69%	0.69%
PDF parameterization	2.71%	1.63%	1.79%
PID efficiency of K/π	0.88%	0.68%	0.73%
D^+ mass selection window	0.05%	0.56%	0.64%
J/ψ veto selection	0.12%	0.004%	0.15%
Peaking background yield	0.07%	0.04%	0.00%
MC statistics	< 0.01	0.04%	0.04%
Fit bias	–	0.58%	0.61%
Total	2.85%	3.43%	3.54%

Source	$B^\pm \to D\pi^\pm$				$B^\pm \to DK^\pm$			
	x_+	y_+	x_-	y_-	x_+	y_+	x_-	y_-
Efficiency uncertainty	$^{+0.013}_{-0.009}$	$^{+0.030}_{-0.027}$	$^{+0.012}_{-0.008}$	$^{+0.012}_{-0.013}$	$^{+0.012}_{-0.013}$	$^{+0.022}_{-0.023}$	$^{+0.012}_{-0.012}$	$^{+0.013}_{-0.016}$
Migration matrix uncertainty	$^{+0.011}_{-0.004}$	$^{+0.021}_{-0.019}$	$^{+0.011}_{-0.003}$	$^{+0.013}_{-0.014}$	$^{+0.007}_{-0.008}$	$^{+0.015}_{-0.016}$	$^{+0.007}_{-0.007}$	$^{+0.006}_{-0.012}$
$m_{\pi\pi\pi^0}$ resolution	0.003	0.001	0.004	0.001	0.001	0.001	0.001	0.003
K_i, $\overline{K}_i$ uncertainty	$^{+0.004}_{-0.001}$	$^{+0.007}_{-0.006}$	$^{+0.004}_{-0.001}$	$^{+0.002}_{-0.002}$	$^{+0.001}_{-0.002}$	$^{+0.001}_{-0.001}$	$^{+0.002}_{-0.002}$	$^{+0.001}_{-0.001}$
PDF shape	$^{+0.004}_{-0.008}$	$^{+0.004}_{-0.003}$	$^{+0.004}_{-0.004}$	$^{+0.001}_{-0.001}$	$^{+0.009}_{-0.008}$	$^{+0.017}_{-0.016}$	$^{+0.009}_{-0.007}$	$^{+0.001}_{-0.005}$
Fit bias	0.000	0.001	0.000	0.000	0.001	0.001	0.001	0.003
PID	0.001	0.001	0.001	0.000	0.002	0.001	0.002	0.001
Total systematic uncertainty	$^{+0.018}_{-0.013}$	$^{+0.038}_{-0.034}$	$^{+0.018}_{-0.010}$	$^{+0.018}_{-0.019}$	$^{+0.017}_{-0.018}$	$^{+0.032}_{-0.032}$	$^{+0.017}_{-0.016}$	$^{+0.015}_{-0.021}$
c_i, s_i uncertainty	$^{+0.014}_{-0.012}$	$^{+0.032}_{-0.030}$	$^{+0.010}_{-0.006}$	$^{+0.019}_{-0.010}$	$^{+0.019}_{-0.018}$	$^{+0.072}_{-0.071}$	$^{+0.023}_{-0.025}$	$^{+0.032}_{-0.049}$
Total statistical uncertainty	$^{+0.024}_{-0.024}$	$^{+0.080}_{-0.059}$	$^{+0.021}_{-0.021}$	$^{+0.059}_{-0.059}$	$^{+0.121}_{-0.121}$	$^{+0.182}_{-0.541}$	$^{+0.121}_{-0.121}$	$^{+0.144}_{-0.197}$

Table 9. Systematic uncertainties from various sources in $B^\pm \to D\pi^\pm$ and $B^\pm \to DK^\pm$ data samples.

[PRD88, 091104 (R) (2013)]

TABLE II. Summary of the systematic uncertainties for R_{Dh} and A_{Dh}. Negligible contributions are denoted by "· · · ."

Source	R_{DK} (%)	$R_{D\pi}$ (%)	A_{DK}	$A_{D\pi}$
ΔE and $\mathcal{C}'_{\rm NB}$ PDFs	$^{+6.5}_{-7.1}$	$^{+8.3}_{-10.3}$	$^{+0.03}_{-0.02}$	$^{+0.02}_{-0.03}$
Fit bias	+0.1	+0.4	· · ·	· · ·
Due to $B\bar{B}$ and $q\bar{q}$ bias	±3.0	· · ·	· · ·	· · ·
Peaking background	±9.5	±8.2	±0.04	±0.01
Efficiency	±0.1	±0.1	· · ·	· · ·
Detector asymmetry	· · ·	· · ·	±0.02	±0.02
Total	$^{+11.9}_{-12.2}$	$^{+11.7}_{-13.2}$	±0.05	$^{+0.03}_{-0.04}$

[PRD 86, 011101(R) (2012)]

TABLE II. Summary of the systematic uncertainties for $\mathcal{R}_{DK^{*0}}$.

Source	Uncertainty $[10^{-2}]$
Signal PDFs	+0.1–0.2
$\bar{D}^0\rho^0$ PDFs	+0.0–0.1
Combinatorial $B\bar{B}$ PDFs	+1.8–1.2
Peaking background PDFs	+0.1–0.1
$q\bar{q}$ PDFs	+2.2–1.4
$\bar{D}^0K^+$ and $\bar{D}^0\pi^+$ PDFs	+0.0–0.1
Fit bias	+0.1–0.1
Efficiency	+0.1–0.1
Charmless decay	+0.0–0.3
Total	+2.8–1.8