Theoretical aspects of charm mixing

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Introduction

- **Charm quark mass is a unique scale.**

$$m_c \approx 1.3 - 1.7 \text{ GeV}$$

— too heavy for ChPT

— too light for $\Lambda_{\rm QCD}/m_c$ expansion?

Theoretically challenging

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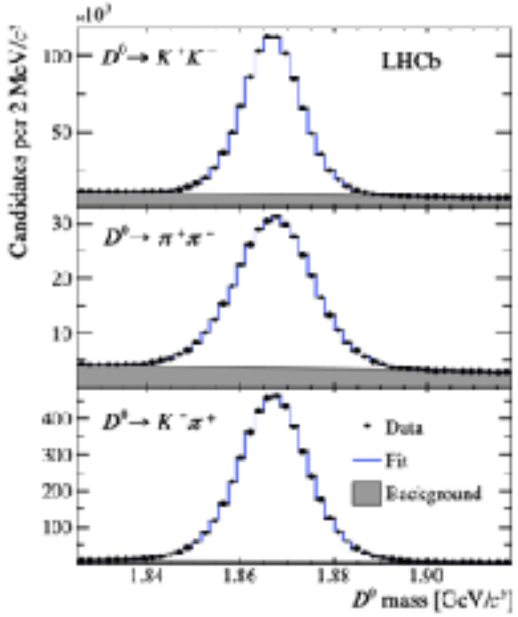
Theoretically challenging

- **High statistics data are provided.**

— in a good stage to test theories



LHCb [1810.06874]

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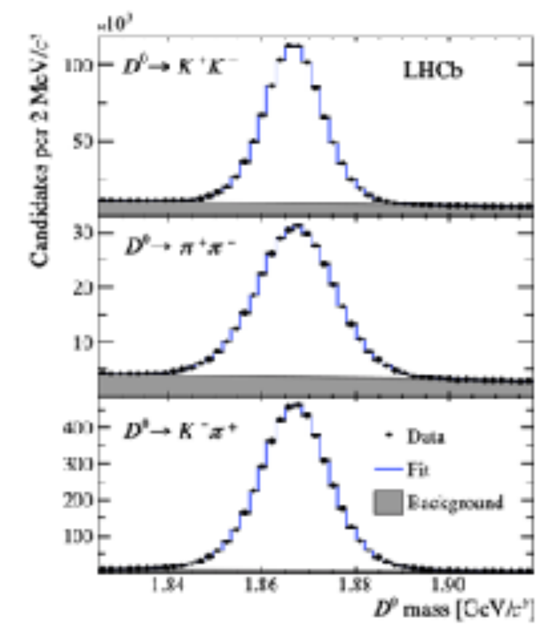


LHCb [1810.06874]

- **<u>$D^0 - \bar{D}^0$ mixing</u>.**

— Λ_{QCD}/m_c expansion is not successful

— experimental data are not quantitatively reproduced yet

Outline

(A) $D^0 - \bar{D}^0$ mixing

— Exclusive approach: Hadronic processes

— Inclusive approach: Heavy quark expansion (HQE)

(B) Quark-hadron duality in the ’t Hooft model

— Duality violation in the $D^0 - \bar{D}^0$ mixing

JHEP 09 (2021) 066 [arXiv:2106.06215 [hep-ph]]

(C) Summary

$D^0 - \bar{D}^0$ mixing

(notation in PDG)

Time evolution

$D^0 \longleftrightarrow \bar{D}^0$

Time evolution Eq. $$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

Mass eigenstate $$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

$q/p \neq$ unity

$\Updownarrow$

CP violation

◉ D_1 (D_2) coincides with a CP-odd (even) state in the CP conserving limit.

(CP conserving limit)

Observables
$$\begin{cases} x = (M_2 - M_1)/\Gamma \overset{\downarrow}{=} 2M_{12}/\Gamma & \text{mass difference} \\ y = (\Gamma_2 - \Gamma_1)/2\Gamma = \Gamma_{12}/\Gamma & \text{width difference} \end{cases}$$

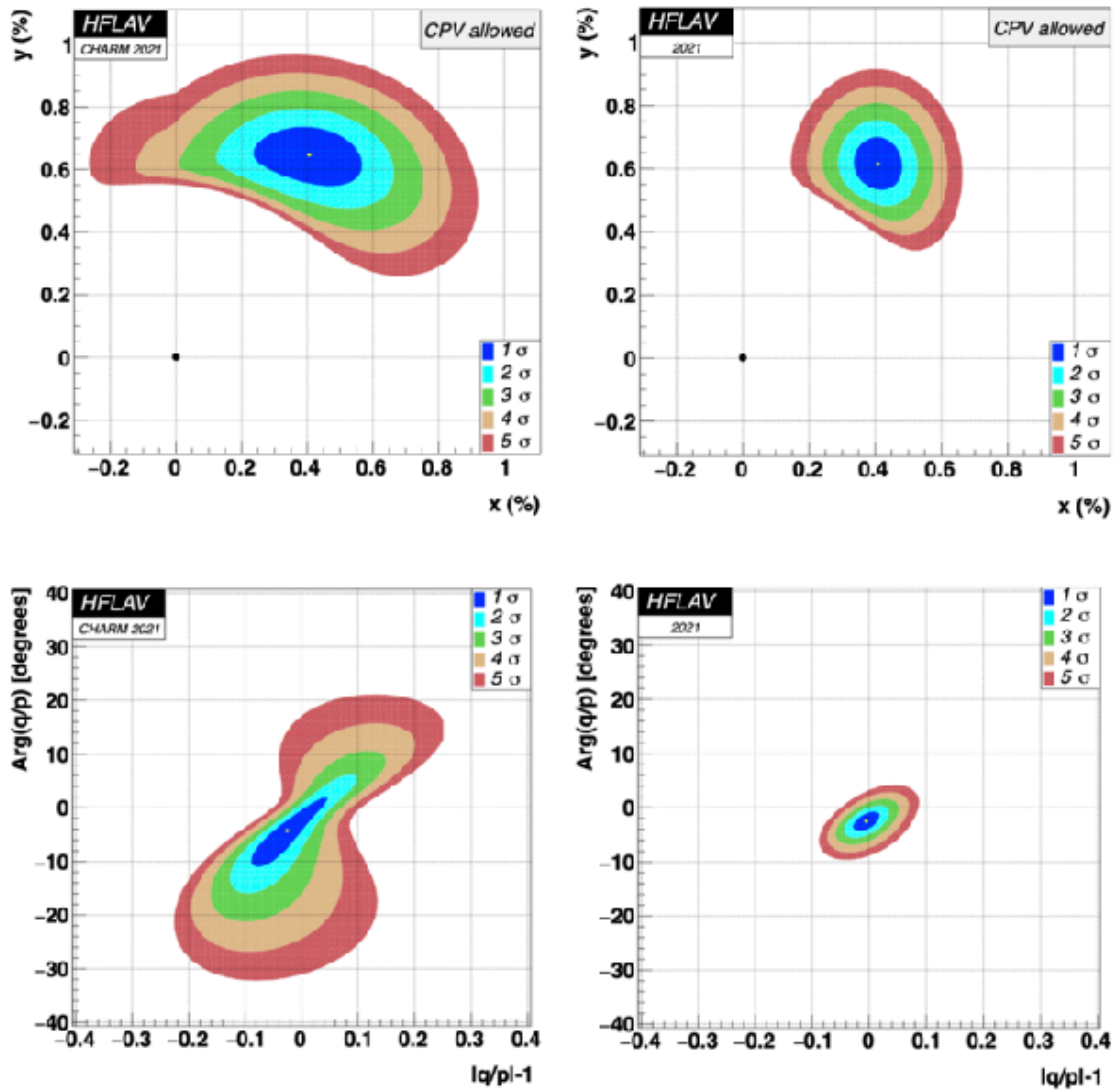
Γ : total width

$D^0 - \bar{D}^0$ mixing: experiment

before after LHCb 5.4fb^{-1} $D^0 \to K_S\pi^+\pi^-$

x, y

CP violation



No mixing $(x, y) = (0, 0)$ is excluded by >>11.5σ

$D^0 - \bar{D}^0$ mixing: theory

Two methods

◉Exclusive

Hadronic-level analysis

Hard to calculate $\begin{cases} \Gamma[D \to \pi\pi] \\ \Gamma[D \to KK] \end{cases}$

↓

Data are used

◉Inclusive

Quark-level analysis

without data

purely theoretical method

(quark-hadron duality is assumed)

Exclusive approaches

$D_{\pm}$: CP eigenstate

(CP limit)

exclusive sum

$$y \approx \frac{\Gamma_+ - \Gamma_-}{2\Gamma} = \frac{1}{2}\sum_n (\mathcal{B}(D_+ \to n) - \mathcal{B}(D_- \to n))$$

$$= \frac{1}{2\Gamma}\sum_n \rho_n \left(|\langle D_+|H_w|n\rangle|^2 - |\langle D_-|H_w|n\rangle|^2\right),$$

ρ_n : phase space factor

For PP mode

$$\begin{aligned} y_{PP} = {} & \mathcal{B}(\pi^+\pi^-) + \mathcal{B}(\pi^0\pi^0) + \mathcal{B}(\pi^0\eta) + \mathcal{B}(\pi^0\eta') + \mathcal{B}(\eta\eta) + \mathcal{B}(\eta\eta') + \mathcal{B}(K^+K^-) + \mathcal{B}(K^0\bar{K}^0) \\ & -2\cos\delta_{K^+\pi^-}\sqrt{\mathcal{B}(K^-\pi^+)\mathcal{B}(K^+\pi^-)} - 2\cos\delta_{K^0\pi^0}\sqrt{\mathcal{B}(\bar{K}^0\pi^0)\mathcal{B}(K^0\pi^0)} \\ & -2\cos\delta_{K^0\eta}\sqrt{\mathcal{B}(\bar{K}^0\eta)\mathcal{B}(K^0\eta)} - 2\cos\delta_{K^0\eta'}\sqrt{\mathcal{B}(\bar{K}^0\eta')\mathcal{B}(K^0\eta')} \, . \end{aligned}$$

data is used to determine y

Exclusive approaches

Exp: $y = (0.615^{+0.056}_{-0.055})\ \%$ (HFLAV2021, CPV allowed)

Topological approach

H-Y Cheng, C-W Chiang, 2010

$y_{PP} = (0.086 \pm 0.041)\%.$

two solutions

$$y_{PV} = (0.269 \pm 0.253)\%\ (A, A1)$$
$$y_{PV} = (0.152 \pm 0.220)\%\ (S, S1)$$

H-Y Jiang, F-S Yu, Q. Qin, H-n. Li and C-D Lu, 2017

Factorization assisted topological (FAT) approach

$$D \to PP \qquad D \to PV \qquad D \to VV$$

VV :negligible

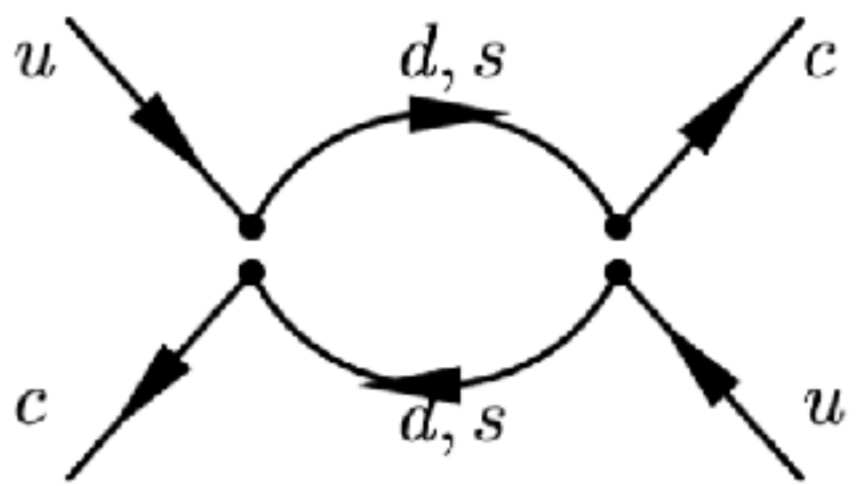
$$y_{PP+PV} = (0.21 \pm 0.07)\%,$$ below data

➡ imply other two/multi-body final states' contribution

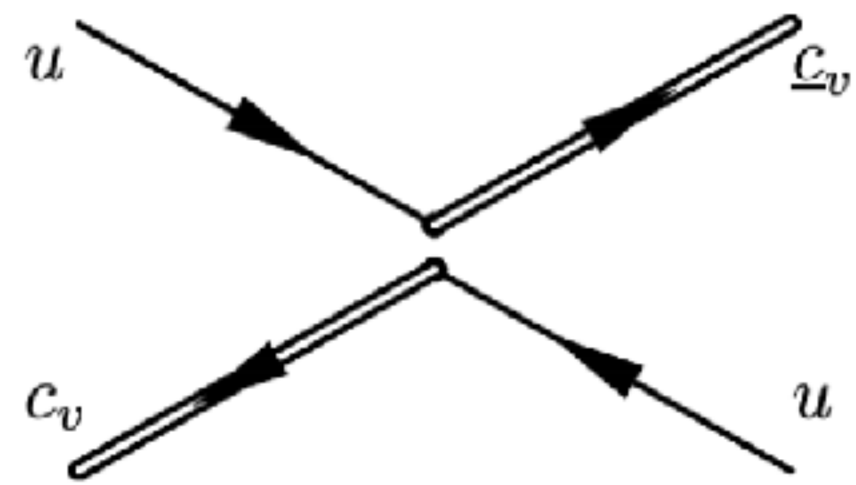
Exclusive ➡ <u>half-value explainable</u>

Inclusive approach

Quark-level



box diagrams (W integrated out)



4-quark operator

Heavy quark expansion (HQE)

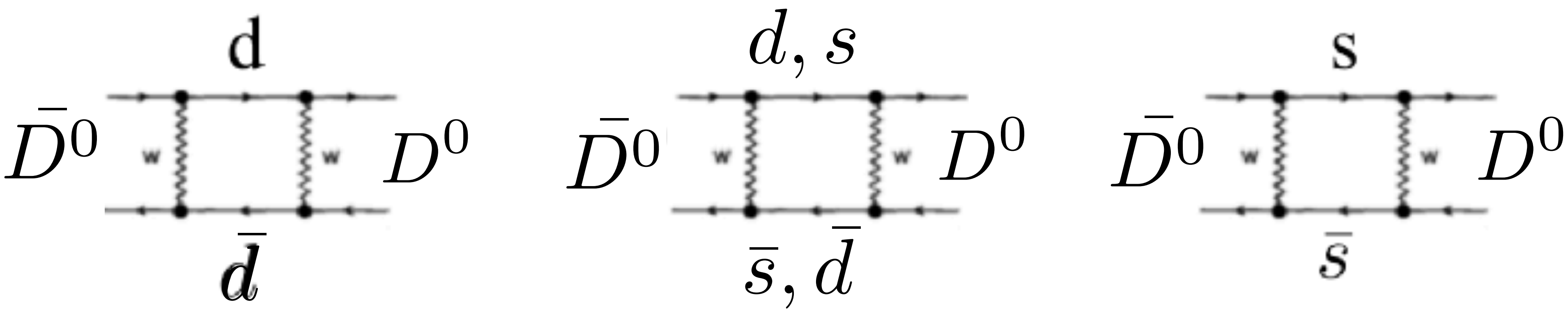
$$A_{12} = M_{12} - \frac{i}{2}\Gamma_{12} \quad \longrightarrow \quad \Gamma_{12} = \sum_n \frac{C_n}{m_c^n} \langle D^0 | \mathcal{O}_n^{\Delta C=2} | \bar{D}^0 \rangle$$

C_n : Wilson coefficient ($\propto m_c^8$)

n : dimension of operator

Contributions

$\lambda_i = V_{ci} V_{ui}^*$



CKM suppressed

~~+ bottom loop~~

neglect

For $m_s = m_d$

CKM unitarity

$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$

neglect

summation $\propto \lambda_d^2 + 2\lambda_d \lambda_s + \lambda_s^2$

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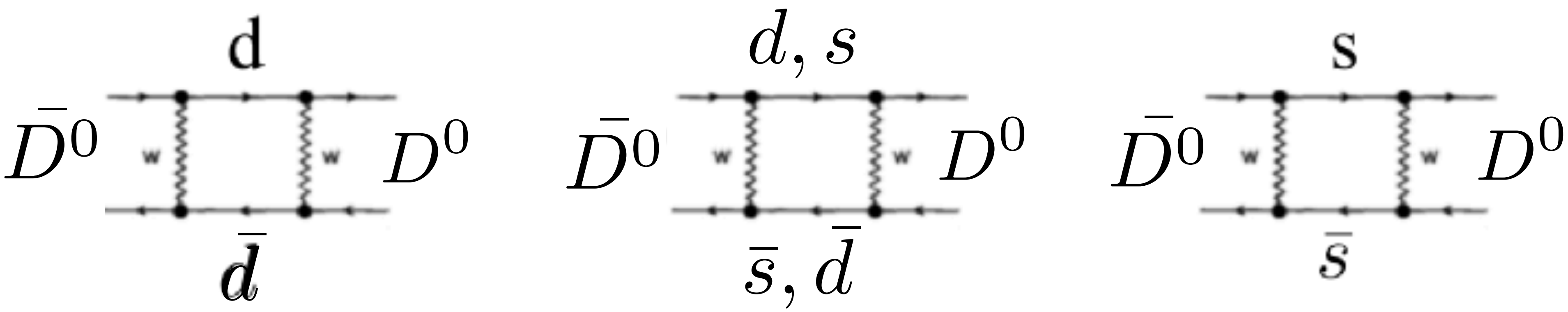

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For $m_s = m_d$

CKM unitarity

$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$

neglect

summation $\propto \lambda_d^2 + 2\lambda_d \lambda_s + \lambda_s^2 = 0$

➡ Suppressed by the GIM mechanism.

non-zero contributions to $D^0 - \bar{D}^0$ mixing

$$\begin{cases} \text{SU(3) breaking: } \left(\dfrac{m_s^2 - m_d^2}{m_c^2}\right)^n \\ \text{small CKM } : \lambda_b = \mathcal{O}(\lambda^5) \end{cases}$$

Theory/Experiment comparison (for inclusive)

D meson

Box diagram: Hagelin 1981, Cheng 1982
Buras, Slominski and Steger 1984

NLO QCD: ✔Golowich and Petrov 2005
Bobrowski *et al.* 2010

SM
$$\begin{cases} x \simeq 6 \cdot 10^{-7} \\ y \simeq 6 \cdot 10^{-7} \end{cases}$$

suppressed by GIM cancellation

Ref. [1]

Exp.
$$\begin{cases} x = (0.409^{+0.048}_{-0.049})\ \% \\ y = (0.615^{+0.056}_{-0.055})\ \% \end{cases}$$

B_s meson

Lenz and Tetlalmatzi-Xolocotzi 2019

SM $\Delta\Gamma_s = (0.091 \pm 0.013)\ \mathrm{ps}^{-1}$

Ref. [2]

Exp. $\Delta\Gamma_s = (0.082 \pm 0.005)\ \mathrm{ps}^{-1}$

B_d meson

Lenz and Tetlalmatzi-Xolocotzi 2019

SM $\Delta\Gamma_d = (2.6 \pm 0.4) \times 10^{-3}\ \mathrm{ps}^{-1}$

Ref. [3]

Exp. $\Delta\Gamma_d/\Gamma_d = (0.001 \pm 0.010)$

- For B_s, B_d mesons, the experimental data are reproduced.
  ($\Delta\Gamma_d$ is consistent within the experimental uncertainty.)
- For D meson, the order of magnitude is not reproduced within 4-quark op.

[1] HFLAV2021 https://hflav-eos.web.cern.ch/hflav-eos/charm/CHARM21/results_mix_cpv.html

[2] HFLAV2020 https://hflav-eos.web.cern.ch/hflav-eos/osc/PDG_2020/

[3] PDG2020 https://pdg.lbl.gov/2021/reviews/rpp2020-rev-b-bar-mixing.pdf

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SM

1

−1

Ref. [1]

Exp.
$$\begin{cases} x = (0.409^{+0.048}_{-0.049})\ \% \\ y = (0.615^{+0.056}_{-0.055})\ \% \end{cases}$$

Exp.

- For B_s, B_d mesons, the exp
  ($\Delta\Gamma_d$ is consistent within the
- For *D* meson, the order of m

Possibilities discussed in the literature

✔ Violation of quark-hadron duality?

— 20% violation explains the data (based on a simple model).

Jubb, Kirk, Lenz and Tetlalmatzi-Xolocotzi, 2017

Contribution of higher dim. operators?

Suggested by Georgi, 1992, prior to the experimental measurement

— It gives a source of SU(3) breaking linear in m_s,
avoiding severe GIM cancellation?

$x, y \sim \mathcal{O}(10^{-3})$ Bigi and Uraltsev, 2001

— With some assumption about hadronic matrix elements,

$x \sim y \lesssim 10^{-3}$ Falk, Grossman, Ligeti and Petrov, 2001

Beyond the standard model?

e.g., Golowich, Pakvasa and Petrov, 2007
Golowich, Hewett, Pakvasa and Petrov, 2009

[1] HFLAV2021 https://hflav-eos.wel

[2] HFLAV2020 https://hflav-eos.web.cern.ch/hflav-eos/osc/PDG_2020/

[3] PDG2020 https://pdg.lbl.gov/2021/reviews/rpp2020-rev-b-bar-mixing.pdf

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JHEP 09 (2021) 066 [arXiv:2106.06215 [hep-ph]]

(C) Summary

Inclusive and exclusive processes

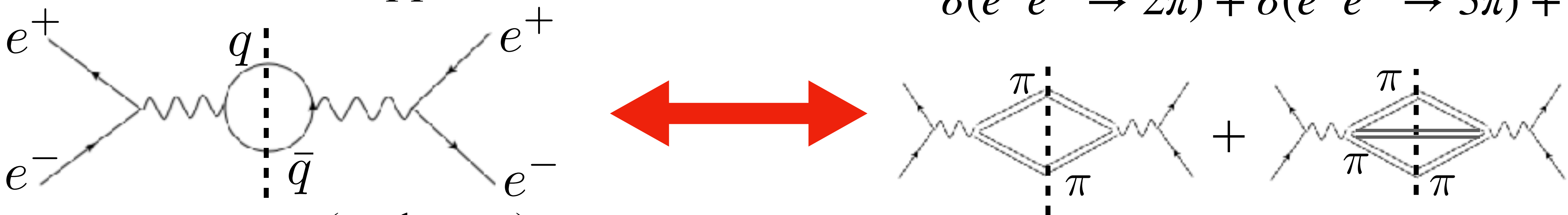
Quark

hadron

$\sigma^{(\mathrm{inc})}(e^+e^- \to q\bar{q})$

$e^+e^- \to$ hadrons

$\sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$



+ (condensates)

Poggio, Quinn and Weinberg, 1976 for the case with smearing

Quark-hadron duality: inclusive = sum of exclusive

(one definition of)
Duality violation: inclusive $\neq$ sum of exclusive

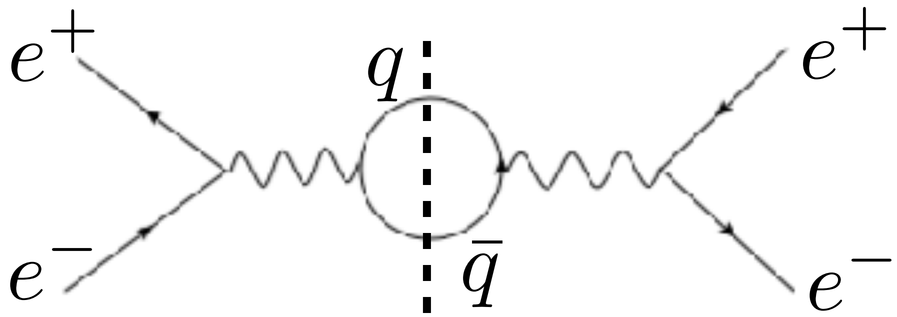
Inclusive and exclusive processes

Quark

hadron

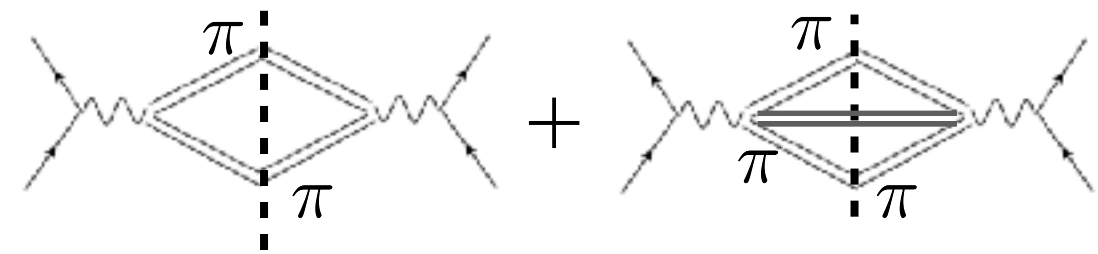
$$e^+e^- \to \text{hadrons}$$

$$\sigma^{(\text{inc})}(e^+e^- \to q\bar{q})$$



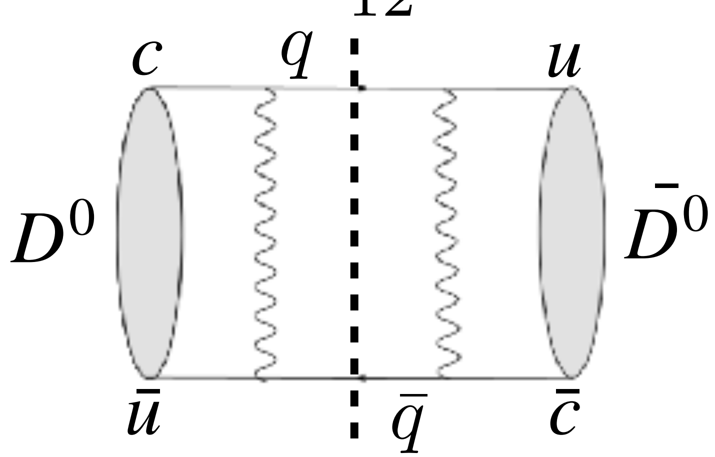
+ (condensates)

$$\sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 3\pi) + \cdots$$



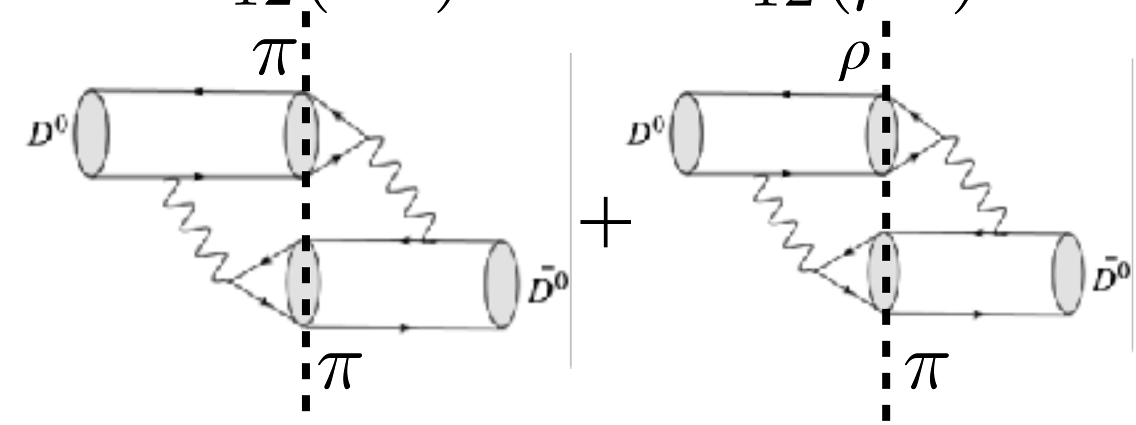
$$D^0 - \bar{D}^0$$

$$\Gamma_{12}^{(\text{inc})}$$



+ (higher dimensional operators)

$$\Gamma_{12}(\pi\pi) \quad + \quad \Gamma_{12}(\rho\pi) + \cdots$$



+ (all the other exclusive states)

One non-trivial point

✔Does duality violation possibly give a large correction to the box diagram? This talk

Considered sources of duality violation

◉ Accuracy of the OPE is limited up to non-perturbative effects in perturbative series.

divergences
- (1) Proliferation of Feynman diagrams
- (2) Renormalons
- (3) OPE series

◉ Duality violation is modeled by

- (a) Instanton-based approach
  — takes accout of the effect of (fixed sized) background instanton, discussed more or less as an orientation.
- (b) Resonance-based approach ✔
  — large-N_c + linear Regge tragectory.
  factorial divergence (sign-flipping) for (3) is captured

◉ The previous works showed that duality violating terms are suppressed exponentially in Euclidean domain while having an oscillatory shape in Minkowski domain.
For both cases, it is indicated that duality violation is suppressed for large energy or mass.

Method to investigate duality violation

$$\Gamma_{12}^{(\mathrm{inc})} \stackrel{?}{=} \sum \Gamma_{12}^{(\mathrm{exc})}$$

Comparison between $d = 4$ and $d = 2$

	inclusive (HQE)	exclusive (Exp.)	exclusive (Theo.)
$d = 4$	✔	✔	✘
$d = 2$	✔	✘	✔

QCD is solvable in the large-N_c limit

't Hooft, 1974

The 't Hooft model (QCD$_2$ in the large-N_c limit)

◎ Asymptotic free theory. ◎ HQE is in common with d=4.

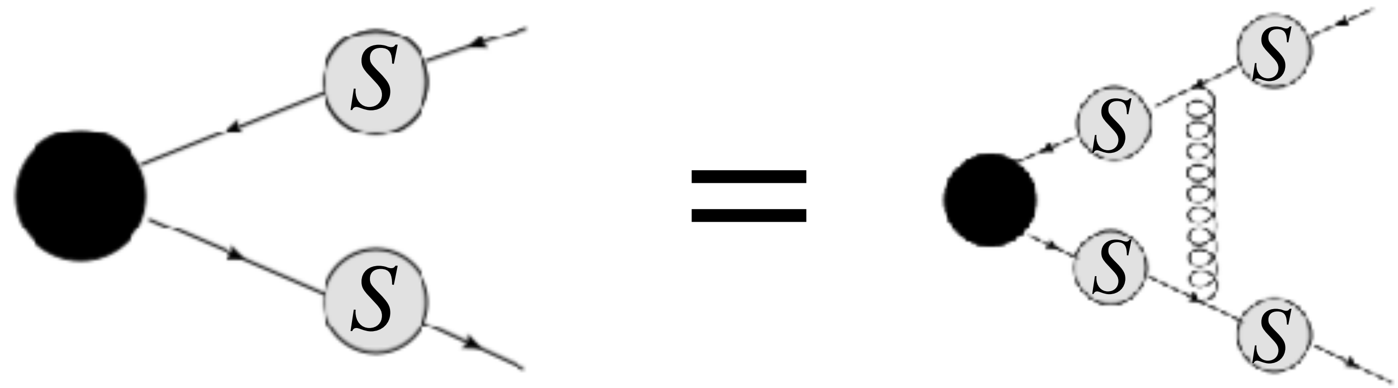
◎ Confinement is built-in. ◎ Gluon is not dynamical.

◎ Phase space offen has singularity in $d = 2$.

➡ Duality violation is qualitatively testable.

Bound state equation in the 't Hooft model

Bethe-Salpeter equation (in the light-cone gauge):



The 't Hooft equation: $M_k^2\phi_k(x) = \left(\frac{m_1^2-\beta^2}{x} + \frac{m_2^2-\beta^2}{1-x}\right)\phi_k(x) - \beta^2 \mathrm{Pr}\int_0^1 \frac{\phi_k(y)\mathrm{d}y}{(x-y)^2}$

QCD coupling: $\beta^2 = \frac{g^2}{2\pi}\left(N_c - \frac{1}{N_c}\right), \quad \lim_{N_c\to\infty}\beta = \text{const.}$

Masses and wavefunctions for mesons can be determined within the formalism.

Numerical method to solve the 't Hooft equation

$M_n^2\phi_n^{q_1\bar{q}_2}(x) = \left(\frac{m_1^2-\beta^2}{x} + \frac{m_2^2-\beta^2}{1-x}\right)\phi_n^{q_1\bar{q}_2} - \beta^2 \mathrm{Pr}\int_0^1 \mathrm{d}y \frac{\phi_n^{q_1\bar{q}_2}(y)}{(x-y)^2}$ for $q_1\overline{q_2}$ bound state

The BSW-improved Multhopp technique — Brower, Spence and Weis, 1979

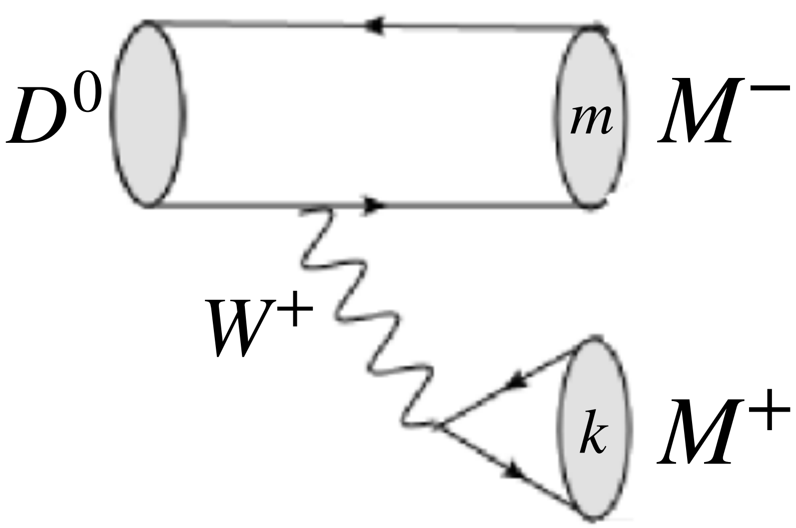
eigenvalue problem: $M^2 a_i = (H_0 + V)_{ij} a_j \qquad \phi = \sum_{k=1}^{N} a_k \sin(k\theta), \quad x = \frac{1+\cos\theta}{2}$

Exclusive processes for $D^0 - \bar{D}^0$ mixing

Width difference in CP conserving limit:

$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|} \longleftarrow \text{phase space in 2D}$$

$T^{(k,m)}$: color-allowed tree diagram



$M = \pi, K, +$ (excited states)

k, m: labels of excited states

$k = 0,\ m = 0$: ground states

Grinstein, Lebed 1997

$$T^{(k,m)}_{(Q\bar{q})(i,j)} = 2\sqrt{2} G_F (c_V^2 - c_A^2) \sqrt{\frac{N_c}{\pi}} c_k^{(qi)} \left[\sum_{n=0} \frac{[(-1)^k q^2 + (-1)^n M_n^2] c_n^{(Qj)}}{q^2 - M_n^2} F_{nm} + (-1)^{k+1} q^2 \mathcal{C}_m + m_Q m_j \mathcal{D}_m \right],$$

$F_{nm},\ \mathcal{C}_m,\ \mathcal{D}_m$: overlap integrals of meson wave functions

Inclusive result for $D^0 - \bar{D}^0$ mixing

$$\Gamma_{12} = C_A < \overline{D^0} \,|\, (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) \,|\, D^0 > \; + C_P < \overline{D^0} \,|\, (\bar{u}^\alpha i\gamma_5 c^\alpha)(\bar{u}^\beta i\gamma_5 c^\beta) \,|\, D^0 >$$

$$\begin{cases} C_A = +\dfrac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{ci}^* V_{uj} [(c_V^2 - c_A^2)(F_{ij}^{(\mathrm{th})} + 2G_{ij}^{(\mathrm{th})}) - (c_V^2 + c_A^2)(H_{ij}^{(\mathrm{th})} + H_{ji}^{(\mathrm{th})})] \\ C_P = -\dfrac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{ci}^* V_{uj} [(c_V^2 - c_A^2)(G_{ij}^{(\mathrm{th})} + 2H_{ij}^{(\mathrm{th})}) + (c_V^2 + c_A^2)(H_{ij}^{(\mathrm{th})} + H_{ji}^{(\mathrm{th})})] \end{cases}$$

generalized weak vertex: $\dfrac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^\mu (c_V + c_A \gamma_5)$

the standard model

$c_V = \dfrac{1}{2}, \quad c_A = -\dfrac{1}{2}$

Matrix elements are calculated in the large-N_c factorization.

$F_{ij}^{(\mathrm{th})} = \sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}$, : 4D-like phase space

$G_{ij}^{(\mathrm{th})} = \dfrac{z_i + z_j - (z_i - z_j)^2}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}$: 2D-specific phase space

$H_{ij}^{(\mathrm{th})} = \dfrac{\sqrt{z_i z_j}}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}$: 2D-specific phase space

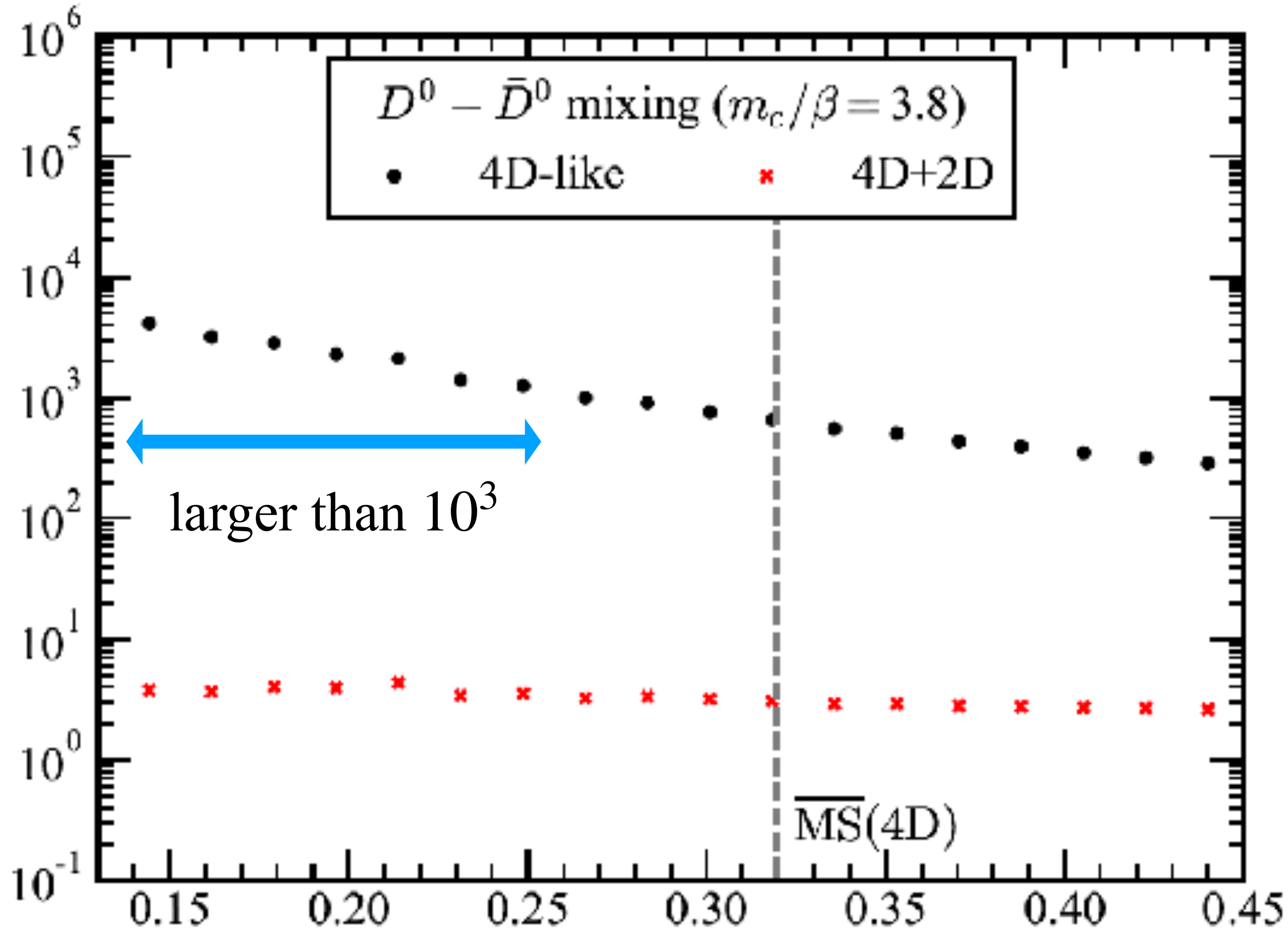
$$\begin{cases} z_d = m_d^2 / m_c^2 \\ z_s = m_s^2 / m_c^2 \end{cases}$$

expansion parameter

We present the two cases: (1) only 4D-like term
(2) pure 2D result (all of the 4D-like and 2D-specific terms are included)

Numerical result: $D^0 - \bar{D}^0$ mixing

$\left|\dfrac{\Delta\Gamma_D^{(\mathrm{exc})}}{\Delta\Gamma_D^{(\mathrm{inc})}}\right|$



m_s/β

$m_c = m_c^{\overline{\mathrm{MS}},4\mathrm{D}}$

points: based only on the 4D-like phase space
crosses: based on the 4D-like phase space + 2D-specific one

◎ The exclusive rate is enhanced by more than 10^3, confirmed for $0.14 < m_s/\beta < 0.25$, when only the 4D-like phase space function is used.

Summary

◎ For the $D^0 - \bar{D}^0$ mixing, it is known that the predicition of the HQE, based on the four-quark operator, is smaller than the experimental value by $\mathcal{O}(10^{-4})$ while the order of magnitude is reproduced in the exclusive approach.

◎ Possible solutions include
- (1) violation of quark-hadron duality
- (2) GIM-favored higher dimensional contributions
- (3) beyond the standard model

◎ We have studied quark-duality for $D^0 - \bar{D}^0$ mixing on the basis of one certain dynamical mechanism.

— For the $D^0 - \bar{D}^0$ mixing, the order of magnitude for exclusive $\Delta\Gamma_D$ is enhanced by more than 10^3, confirmed for $0.14 < m_s/\beta < 0.25$, if the phase space function is given by 4D-like one.

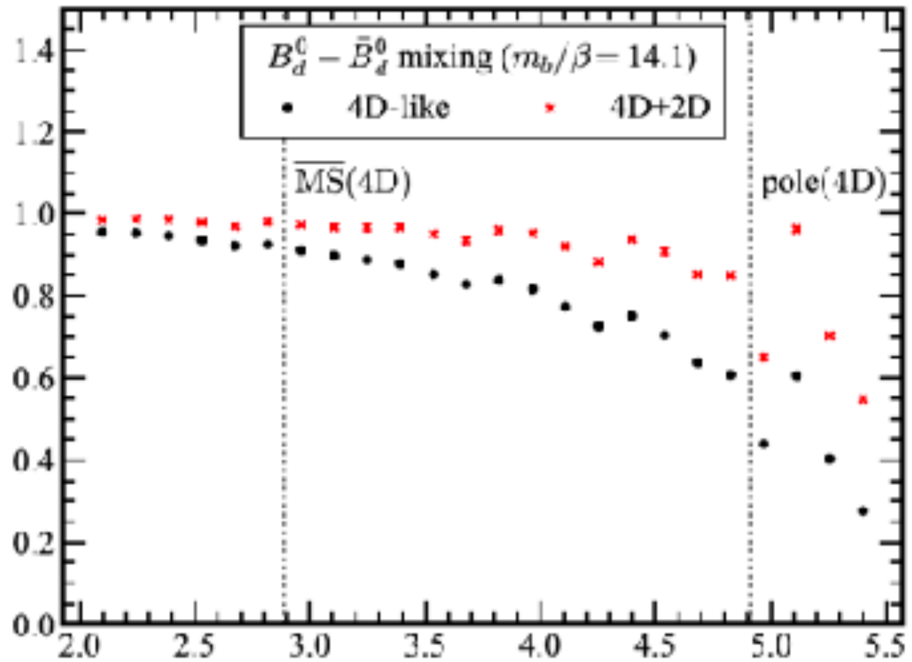
→ The result does not exclude the possibility that large duality violation in $D^0 - \bar{D}^0$ mixing exists.

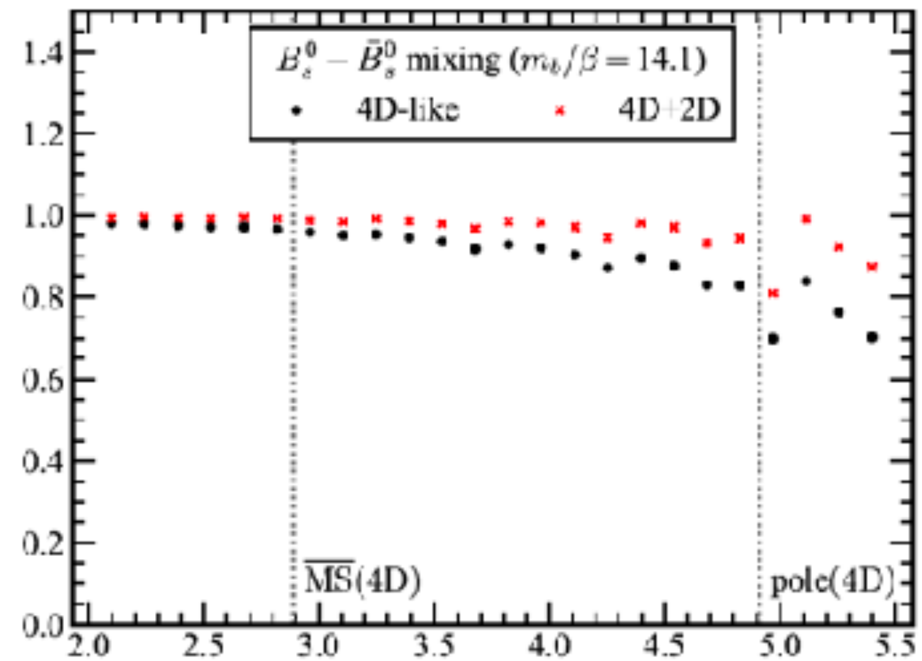
Backup

Numerical result: $B_q^0 - \bar{B}_q^0 \, (q = d, s)$

$B_d^0 - \bar{B}_d^0$ $\qquad\qquad$ $B_s^0 - \bar{B}_s^0$

$\left| \frac{\Delta\Gamma_q^{(\mathrm{exc})}}{\Delta\Gamma_q^{(\mathrm{inc})}} \right|$





m_c/β $\qquad\qquad$ m_c/β

$m_b = m_b^{\mathrm{pole,4D}}$

For $m_c < m_c^{\mathrm{pole,4D}}$

◎ For the $B_d^0 - \bar{B}_d^0$ mixing, the correction to the inclusive rate up to 40% is observed.

⟶ The correction of this size is not excluded yet.

(The experimental data for $\Delta\Gamma_{B_d}/\Gamma_{B_d}$ is still consistent with zero.)

◎ For the $B_s^0 - \bar{B}_s^0$ mixing, the correction to the inclusive rate up to 18% is observed.

⟶ The result is consistent with what is currently indicated in 4D.

(The ratio of the HFLAV data to the HQE gives $\Delta\Gamma_{B_s}^{(\mathrm{ex})}/\Delta\Gamma_{B_s}^{(\mathrm{th})} = 0.99 \pm 0.15$.)

A. Lenz and G. Tetlalmatzi-Xolocotzi, 2020

Previous works for heavy meson ***decays***

◉ non-leptonic decays (color-allowed tree)

[1] B. Grinstein and R. F. Lebed, Phys. Rev. D**57**, 1366-1378 (1998) [arXiv:hep-ph/9708396 [hep-ph]].

◉ semi-leptonic decay and also non-leptonic decay

[2] I. I. Y. Bigi, M. A. Shifman, N. Uraltsev and A. I. Vainshtein, Phys. Rev. D**59**, 054011 (1999) [arXiv:hep-ph/9805241 [hep-ph]].

◉ non-leptonic decay (weak annhilation)

[3] B. Grinstein and R. F. Lebed, Phys. Rev. D**59**, 054022 (1999) [arXiv:hep-ph/9805404 [hep-ph]].

◉ non-leptonic decays (weak annhilation, Pauli interference)

[4] I. I. Y. Bigi and N. Uraltsev, Phys. Rev. D60, 114034 (1999) [arXiv:hep-ph/9902315 [hep-ph]]; Phys. Lett. B**457**, 163-169 (1999) [arXiv:hep-ph/9903258 [hep-ph]].

◉ semi-leptonic decay

[5] R.~F. Lebed and N. G. Uraltsev, Phys. Rev. D**62**, 094011 (2000) [arXiv:hep-ph/0006346 [hep-ph]].

This work → heavy meson **mixings**

・Mixing is suppressed by the GIM cancellation.

➡ Tiny duality violation is possibly enlarged after cancellation.

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^\mu (c_V + c_A \gamma_5)$

the standard model

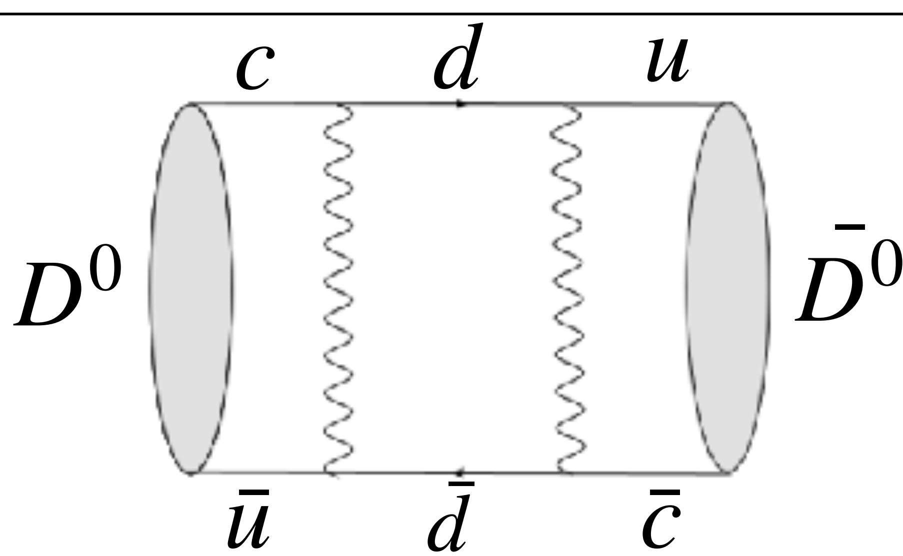
$c_{\mathrm{V}} = \frac{1}{2}, \quad c_{\mathrm{A}} = -\frac{1}{2}$

◉ Inclusive width difference

$$\Gamma_{12} = C_A < \overline{D^0} \,|\, (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) \,|\, D^0 > \; + C_P < \overline{D^0} \,|\, (\bar{u}^\alpha i\gamma_5 c^\alpha)(\bar{u}^\beta i\gamma_5 c^\beta) \,|\, D^0 >$$

$$\begin{cases} C_A = +\dfrac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_A^2)(F_{dd}^{(\mathrm{th})} + 2G_{dd}^{(\mathrm{th})}) - (c_V^2 + c_A^2)(I_{dd}^{(\mathrm{th})} + I_{dd}^{(\mathrm{th})})] \\ C_P = -\dfrac{2G_F^2}{M_{D^0}}(c_V^2 - c_A^2) V_{cd}^* V_{ud} [(c_V^2 - c_A^2)(G_{dd}^{(\mathrm{th})} + 2H_{dd}^{(\mathrm{th})}) + (c_V^2 + c_A^2)(I_{dd}^{(\mathrm{th})} + I_{dd}^{(\mathrm{th})})] \end{cases}$$

$F_{dd}^{(\mathrm{th})}, G_{dd}^{(\mathrm{th})}, H_{dd}^{(\mathrm{th})}, I_{dd}^{(\mathrm{th})}$: phase space functions $\qquad F_{dd}^{(\mathrm{th})} = \sqrt{1 - 4m_d^2/m_c^2}$



large-N_c factorization: $\qquad R = [M_H/(m_Q + m_q)]^2$

$$\begin{cases} \dfrac{\langle \bar{H} |\, (\bar{q}^\alpha \gamma^\mu \gamma_5 Q^\alpha)(\bar{q}^\beta \gamma_\mu \gamma_5 Q^\beta) \,| H \rangle}{2M_H} = f_H^2 M_H \\ \dfrac{\langle \bar{H} |\, (\bar{q}^\alpha i\gamma_5 Q^\alpha)(\bar{q}^\beta i\gamma_5 Q^\beta) \,| H \rangle}{2M_H} = f_H^2 M_H R \end{cases}$$

down quark massless limit: $\qquad \Gamma_{12} \to 4(c_{\mathrm{V}}^2 - c_{\mathrm{A}}^2)^2 V_{cd}^* V_{ud} G_F^2 f_{D^0}^2 M_{D^0}$

Analytical check of local duality

generalized weak vertex: $\frac{-ig_2}{\sqrt{2}} V_{\mathrm{CKM}} \gamma^\mu (c_V + c_A \gamma_5)$

the standard model
$c_\mathrm{V} = \frac{1}{2}, \quad c_\mathrm{A} = -\frac{1}{2}$

◉ Sum of exclusive width difference

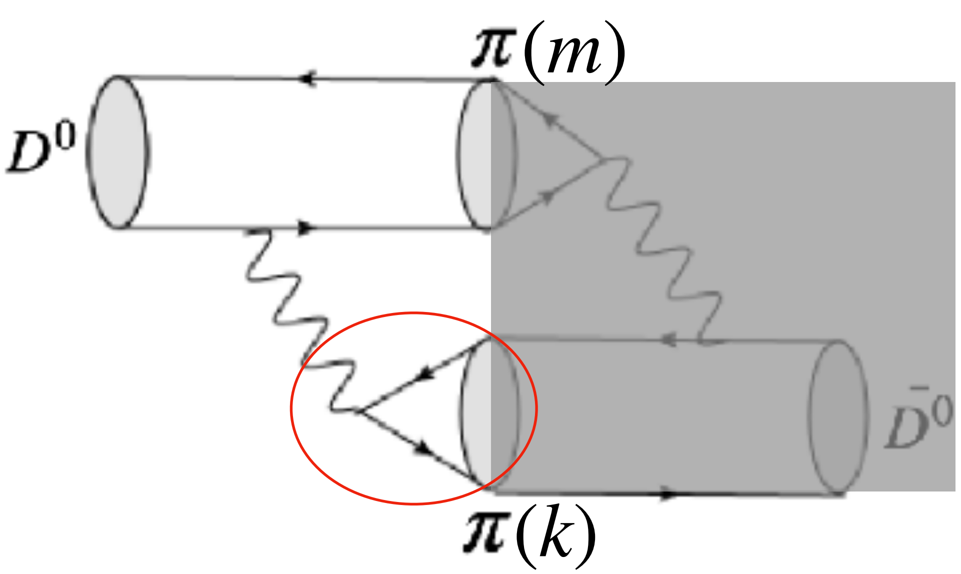
massless limits for u and d quark for $\pi(u\bar{d})$

analog of the Pauli interference (Bigi and Uraltsev, 1999)

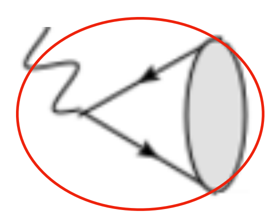
$$\Gamma_{12}^{(D^0)} = \sum_{k,m} (-1)^{k+m} \frac{T^{(k,m)} T^{(m,k)*}}{4M_{D^0}^2 |p_{km}|} \rightarrow \frac{T^{(0,0)} T^{(0,0)*}}{4M_{D^0}^2 |p_{00}|} = 4(c_\mathrm{V}^2 - c_\mathrm{A}^2)^2 G_F^2 V_{cd}^* V_{ud} M_{D^0} \frac{N_c}{\pi} \left(\int_0^1 \phi_{D^0}(x) \phi_\pi(x) \right)^2$$

$$= 4(c_\mathrm{V}^2 - c_\mathrm{A}^2)^2 G_F^2 V_{cd}^* V_{ud} f_{D^0}^2 M_{D^0}$$

agrees with the inclusive result



$k = 0,\ m = 0$: ground states



$\propto f_\pi^{(k)} p_\mu \qquad f_\pi^{(k)} = \sqrt{\frac{N_c}{\pi}} \int_0^1 \phi_k(x) dx$

(a) exact solution, $\phi_0(x) = 1$.

(b) completeness: $\sum_{k=0}^{\infty} \phi_k(x) \phi_k^*(y) = \delta(x-y)$

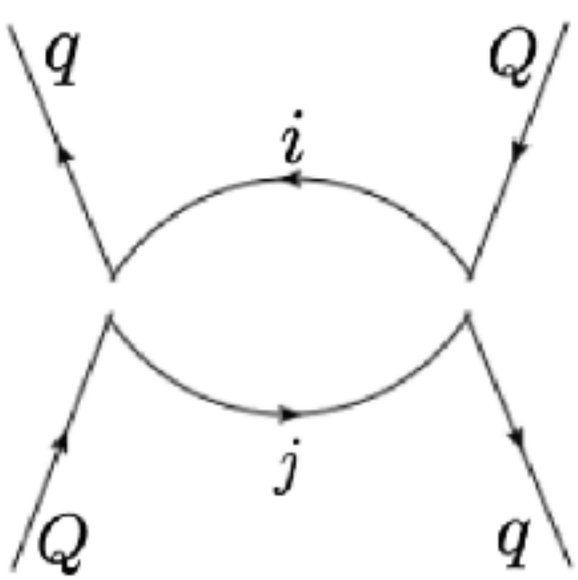
$$\Rightarrow f_\pi^{(k)} = \begin{cases} \sqrt{N_c/\pi} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

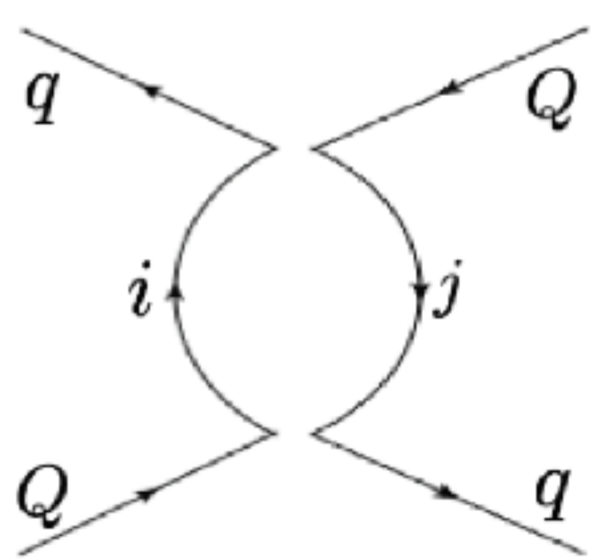
Box diagrams in two-dimensions

$$\Gamma_{12} = C_A < \overline{D^0} \,|\, (\bar{u}^\alpha \gamma^\mu \gamma_5 c^\alpha)(\bar{u}^\beta \gamma_\mu \gamma_5 c^\beta) \,|\, D^0 > \; + C_P < \overline{D^0} \,|\, (\bar{u}^\alpha i \gamma_5 c^\alpha)(\bar{u}^\beta i \gamma_5 c^\beta) \,|\, D^0 >$$

$$\begin{cases} C_A = + \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{ci}^* V_{uj} [(c_V^2 - c_A^2)(F_{ij}^{(\mathrm{th})} + 2G_{ij}^{(\mathrm{th})}) - (c_V^2 + c_A^2)(I_{ij}^{(\mathrm{th})} + I_{ji}^{(\mathrm{th})})] \\ C_P = - \dfrac{2G_F^2}{M_{D^0}} (c_V^2 - c_A^2) V_{ci}^* V_{uj} [(c_V^2 - c_A^2)(G_{ij}^{(\mathrm{th})} + 2H_{ij}^{(\mathrm{th})}) + (c_V^2 + c_A^2)(I_{ij}^{(\mathrm{th})} + I_{ji}^{(\mathrm{th})})] \end{cases}$$

$$i, j = d, s \qquad\qquad z_i = m_i^2 / m_c^2$$





$$F_{ij}^{(\mathrm{th})} = \sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}$$

$$G_{ij}^{(\mathrm{th})} = \frac{z_i + z_j - (z_i - z_j)^2}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}$$

$$H_{ij}^{(\mathrm{th})} = \frac{\sqrt{z_i z_j}}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}$$

$$I_{ij}^{(\mathrm{th})} = \frac{\sqrt{z_i}(1 + z_i - z_j)}{\sqrt{1 - 2(z_i + z_j) + (z_i - z_j)^2}}$$

Definition of amplitude and overlap integrals

Grinstein, Lebed 1997, Bigi, Uraltsev 1999

Amplitude
$$T^{(k,m)}_{(Q\bar{q})(i,j)} = 2\sqrt{2}G_F(c_V^2 - c_A^2)\sqrt{\frac{N_c}{\pi}}c_k^{(q\bar{i})}\left[\sum_{n=0}\frac{[(-1)^k q^2 + (-1)^n M_n^2]c_n^{(Q\bar{j})}}{q^2 - M_n^2}F_{nm} + (-1)^{k+1}q^2\mathcal{C}_m + m_Q m_j \mathcal{D}_m\right],$$

Overlap int.
$$\begin{cases} F_{nm} = \omega(1-\omega)\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y \frac{\phi_n^{(Q\bar{j})}(x)\phi_m^{(j\bar{q})}(y)}{[\omega(1-x)+(1-\omega)y]^2} \\ \qquad\times\{\phi_0^{(Q\bar{q})}(\omega x) - \phi_0^{(Q\bar{q})}[1-(1-\omega)(1-y)]\}, \\ \mathcal{C}_m = -\frac{1-\omega}{\omega}\int_0^1 \mathrm{d}x\phi_0^{(Q\bar{q})}[1-(1-\omega)(1-x)]\phi_m^{(j\bar{q})}(x), \\ \mathcal{D}_m = -\omega\int_0^1 \mathrm{d}x\frac{\phi_0^{(Q\bar{q})}[1-(1-\omega)(1-x)]}{1-(1-\omega)(1-x)}\frac{\phi_m^{(j\bar{q})}(x)}{x}, \end{cases}$$

Kinematical val.
$$\omega = \frac{1}{2}\left[1 + \left(\frac{q^2 - M_m^2}{M_0^2}\right) - \sqrt{1 - 2\left(\frac{q^2+M_m^2}{M_0^2}\right) + \left(\frac{q^2-M_m^2}{M_0^2}\right)^2}\right]$$

$q^2 = M_k^2$ for on-shell amplitudes

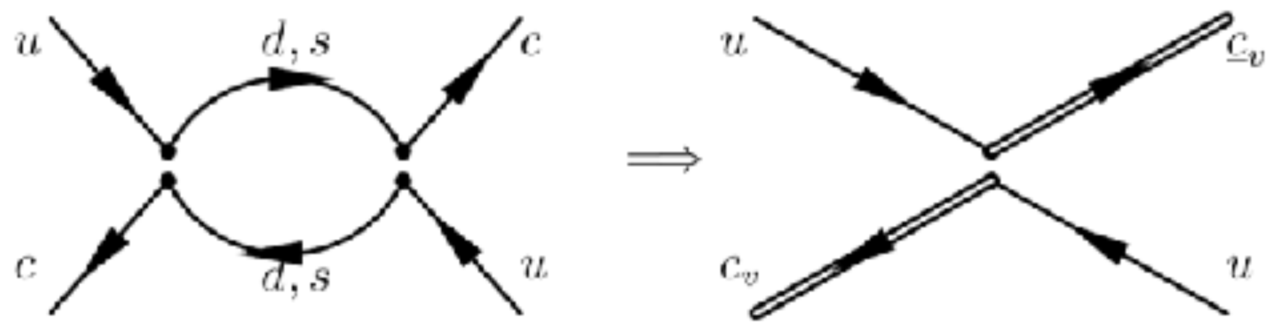
Structure of higher-dimensional operators

Ohl, Ricciardi and Simmons [9301212]

D=6

Leading in $1/m_q$

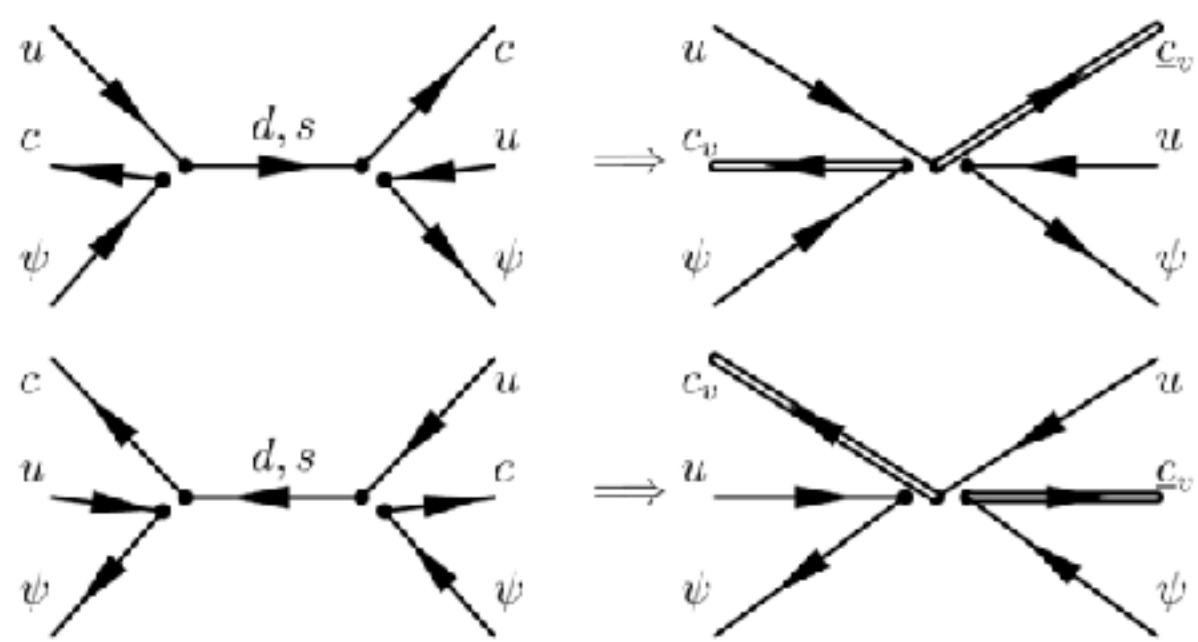
$(\bar{c}_v \Gamma_1 u)\,(\bar{\underline{c}}_v \Gamma_2 u)$



D=9

Subleading in $1/m_q$

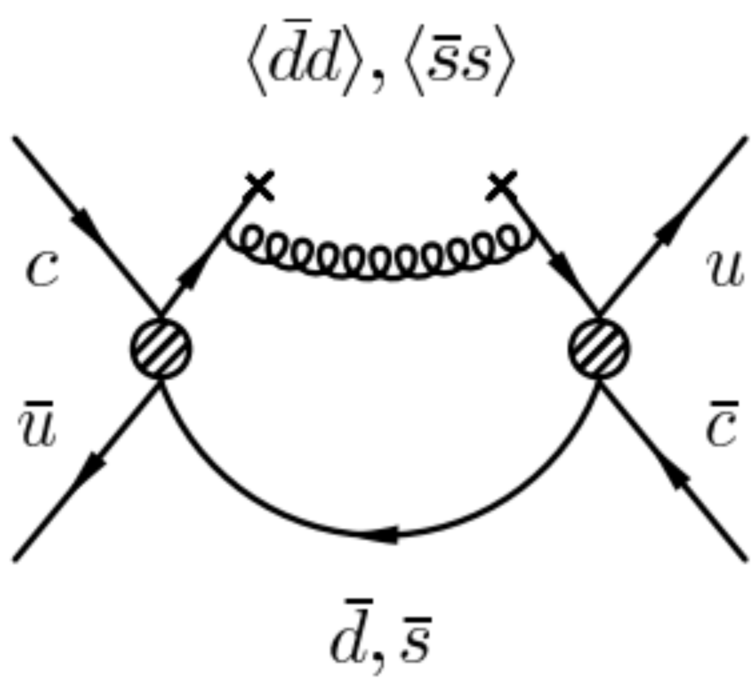
$(\bar{\psi}\Gamma_1 u)\,(\bar{c}_v \Gamma_2 \psi)\,(\bar{\underline{c}}_v \Gamma_3 u)$



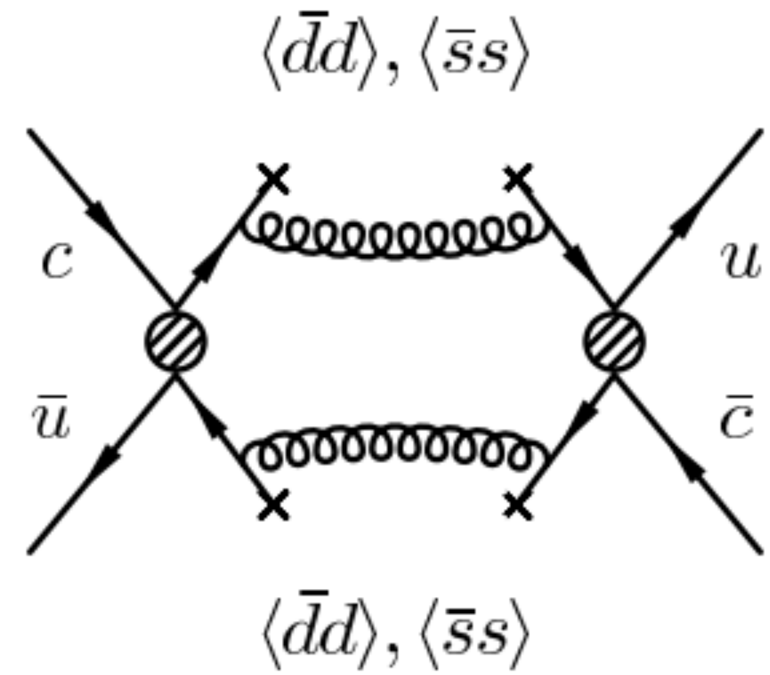
Γ_i : color/Dirac structure

Higer-dimensional operators with condensates

Bobrowski, Lenz and Riedl [1002.4794]



D=9



D=12

$\mathcal{O}(\alpha_s(4\pi)\langle\bar{q}q\rangle/m_c^3)$

$\mathcal{O}(\alpha_s^2(4\pi)^2\langle\bar{q}q\rangle^2/m_c^6)$

y	no GIM	with GIM
$D=6,7$	$2\cdot10^{-2}$	$5\cdot10^{-7}$
$D=9$	$5\cdot10^{-4}$	?
$D=12$	$2\cdot10^{-5}$	?