Charm lifetimes and mixing

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The theoretical framework

The total decay width of a heavy hadron

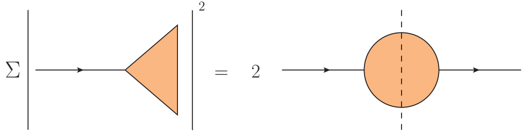
- $\Gamma(H_Q)$ can be computed as [Shifman, Voloshin '85], see also review [Lenz '14]

$$\Gamma(H_Q) = \frac{1}{2m_{H_Q}} \mathrm{Im} \langle H_Q | i \int d^4x \, \mathrm{T}\Big\{ \mathcal{H}_{eff}(x) \,, \mathcal{H}_{eff}(0) \Big\} | H_Q \rangle$$

- $\mathcal{H}_{eff}$ weak effective Hamiltonian describing Q decays

[Buchalla, Buras, Lautenbacher '96]

- Use optical theorem



to relate Γ with imaginary part of forward scattering amplitude

The HQE

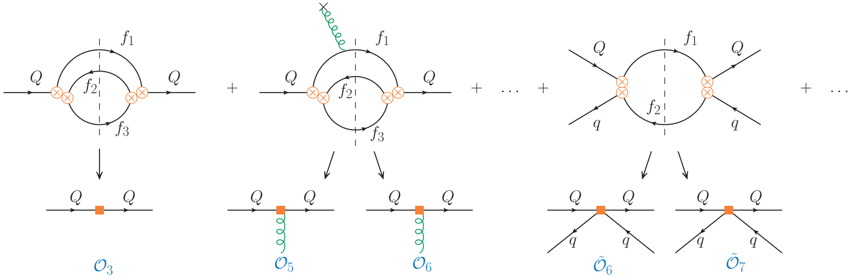
- Exploit $m_Q \gg \Lambda_{QCD}$ and use $p_Q^\mu = m_Q v^\mu + k^\mu$ with $k \sim \Lambda_{QCD}$
- Systematic expansion

$$\Gamma(H_Q) = \underbrace{\Gamma_3}_{\Gamma(Q)} + \underbrace{\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right]}_{\delta\Gamma(H_Q)}$$

  - $\Gamma_d, \tilde{\Gamma}_d$ - short distance coefficients
  - $\mathcal{O}_d, \tilde{\mathcal{O}}_d$ - local operators bilinear in the heavy quark field
  - $\Gamma(Q)$ - total decay width of free quark Q
  - $\delta\Gamma(H_Q)$ - effects due to interaction with soft gluons and quarks

The HQE

$$\Gamma(H_Q) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right]$$



Very advanced framework thanks to huge effort of big community

Status of HQE: perturbative side

$$\Gamma_d = \Gamma_d^{(0)} + \left(\frac{\alpha_s(m_Q)}{4\pi}\right)\Gamma_d^{(1)} + \left(\frac{\alpha_s(m_Q)}{4\pi}\right)^2 \Gamma_d^{(2)} + \dots$$

Semileptonic (SL) modes	
$\Gamma_3^{(3)}$	Fael, Schönwald, Steinhauser '20 * ; Czakon, Czarnecki, Dowling '21
$\Gamma_3^{(2)}$	Czarnecki, Melnikov, v. Ritbergen, Pak, Dowling, Bonciani, Ferroglia, Biswas, Brucherseifer, Caola '97-'13
$\Gamma_5^{(1)}$	Alberti, Gambino, Nandi, Mannel, Pivovarov, Rosenthal '13-'15
$\Gamma_6^{(1)}$	Mannel, Pivovarov '19
$\Gamma_7^{(0)}$	Dassinger, Mannel, Turczyk '06
$\Gamma_8^{(0)}$	Mannel, Turczyk, Uraltsev '10

* see also talks by K. Schönwald and M. Fael

** Partial result

Non-leptonic (NL) modes	
$\Gamma_3^{(2)}$	Czarnecki, Slusarcyk, Tkachov '05 **
$\Gamma_3^{(1)}$	Ho-Kim, Pham, Altarelli, Petrarca, Voloshin, Bagan, Ball, Braun, Gosdzinsky, Fiol, Lenz, Nierste, Ostermaier, Krinner, Rauh '84-'13
$\Gamma_5^{(0)}$	Bigi, Uraltsev, Vainshtein, Blok, Shifman '92
$\Gamma_6^{(0)}$	Lenz, MLP, Rusov, Mannel, Moreno, Pivovarov '20-'21
$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste, Franco, Lubicz, Mescia, Tarantino, Rauh '02-'13
$\tilde{\Gamma}_7^{(0)}$	Gabbiani, Onishchenko, Petrov '03-'04

Status of HQE: non-perturbative side

★ Fit to experimental data on semileptonic B decays (see also talks by P. Gambino and K. Vos) ★ HQET sum rules ★ Lattice QCD

	B_d, B^+	B_s
$\langle \mathcal{O}_5 \rangle$	Gambino, Schwanda, Alberti Healey, Nandi '13-'14 ★ Ball, Braun, Neubert '93-'95 ★ Kronfeld, Simone, Gambino, Melis, Simula '00-'17 ★	Spectroscopy relations for μ_G^2
$\langle \mathcal{O}_6 \rangle$	Gambino, Schwanda, Alberti Healey, Nandi '13-'14 ★ EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	Kirk, Lenz, Rauh '17 ★	King, Lenz, Rauh (to appear) ★
$\langle \tilde{\mathcal{O}}_7 \rangle$	VIA	VIA

- No independent computation for D: rely only heavy-quark symmetry

  leads to large uncertainties

- For baryons: even less results available

Study of lifetimes of charmed mesons

Based on arXiv:2109.13219

In collaboration with A. Lenz, A. Rusov, Th. Rauh, D. King, Ch. Vlahos

HQE: bottom vs. charm

- Two expansion parameters: $\alpha_s(m_Q)$ and $\frac{\Lambda_{QCD}}{m_Q}$
- If $m_Q \gg \Lambda_{QCD}$, HQE well defined series
  - Expect $\Gamma(H_Q) = \Gamma(Q) + \delta\Gamma(H_Q)$ with $\delta\Gamma(H_Q) \ll \Gamma(Q)$
- For $Q = b$: $\alpha_s \sim 0.22$ and $\frac{\Lambda_{QCD}}{m_b} \sim 0.10$
  - Lifetime ratios of bottom mesons differ from 1 up to 10 percent

    [Kirk, Lenz, Rauh '17]
  - Good agreement with experimental data
- For $Q = c$: $\alpha_s \sim 0.33$ and $\frac{\Lambda_{QCD}}{m_c} \sim 0.30$
  - Is charm heavy enough to apply HQE?

Charmed mesons - Experimental status

- Charmed meson lifetimes are measured precisely

	D^0	D^+	D_s^+
τ [ps]	0.4101(15)	1.040(7)	0.504(4)
Γ [ps^{-1}]	2.44(1)	0.96(1)	1.98(2)
$\tau(D_X)/\tau(D^0)$	1	2.54(2)	1.20(1)

[PDG 2021] *

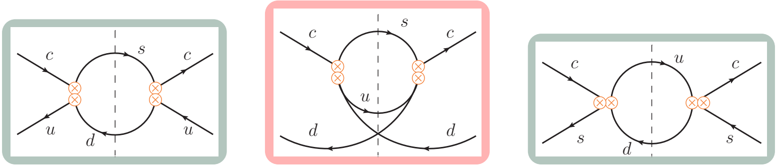
- Spectator quark effects must give large contribution

$$\Gamma(D) = \Gamma(c) + \delta\Gamma(D) \quad \text{with} \quad \delta\Gamma(D) \sim \Gamma(c)$$

- Can HQE explain this pattern?

* New measurement of $\tau(D^{0,+})$ by Belle II arXiv:2108.03216 not included
(see talks by T. Humair and S. Prell)

Spectator quark effects



- Very different size of four-quark operator diagrams
  - Dominant contribution of Pauli interference to D^+ [Guberina et al. '79]]
  - Small contribution due to helicity suppression for D^0 and D_s^+
- Large NLO-QCD corrections for D^+ at dimension-six
- Dimension-seven contributions important

Our set-up

- Most up to date analysis

$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \ldots \right]$$

$$\tau(D_{(s)}^+)/\tau(D^0) = 1 + \left[\Gamma(D^0)^{\rm HQE} - \Gamma(D_{(s)}^+)^{\rm HQE} \right] \tau(D_{(s)}^+)^{\rm exp}$$

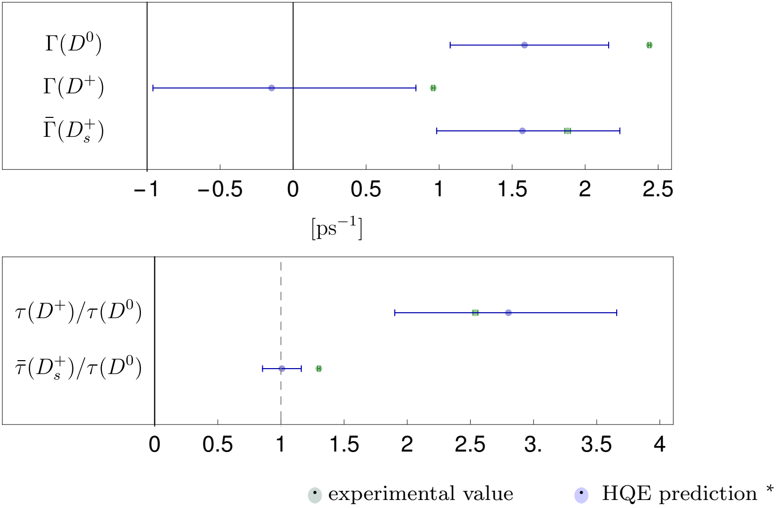
- What we have included

	SL	NL
Γ_3	NLO	NLO
Γ_5	LO	LO
Γ_6	LO	LO New *
$\tilde{\Gamma}_6$	NLO	NLO
$\tilde{\Gamma}_7$	LO	LO

	Source
$\langle \mathcal{O}_5 \rangle$	Heavy quark symmetry; Spectroscopy relations
$\langle \mathcal{O}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$ New
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET sum rules New **
$\langle \tilde{\mathcal{O}}_7 \rangle$	VIA

* Based on [Lenz, MLP, Rusov '20] ** [King, Lenz, Rauh (to appear)]

Result



* Obtained using charm quark mass in kinetic scheme with cutoff of $\mu = 0.5$ GeV

Conclusion and outlook for charm lifetimes

- HQE consistent with experimental data albeit huge uncertainties

- Large room for theoretical improvement

  - For $\langle \mathcal{O}_5 \rangle$, $\langle \mathcal{O}_6 \rangle$ exp. data highly desirable → control of $SU(3)_F$

    BESIII, Belle II, super tau charm factory?

  - Lattice determination for $\langle \tilde{\mathcal{O}}_6 \rangle$

    Planned by M. Black, O. Witzel (RBC-UKQCD, Siegen)

  - Complete dimension-seven contribution at LO-QCD: $\Gamma^{(0)}_{7,\mathrm{NL}}$

  - Higher order QCD corrections, particularly $\Gamma^{(2)}_{3,\mathrm{NL}}$, $\tilde{\Gamma}^{(2)}_{6}$, and $\tilde{\Gamma}^{(1)}_{7}$

    Planned by U. Nierste, M. Steinhauser (Karlsruhe)

  - Study dependence of charm quark mass definition

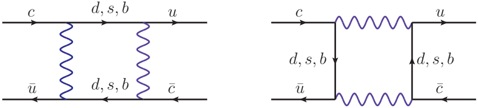
Analysis of GIM cancellations for D-mixing

Based on arXiv:2007.03022

In collaboration with A. Lenz, Ch. Vlahos

Charm mixing

- Neutral mesons mix with their antiparticles via box diagrams



- Evolution described by 2×2 Hamiltonian matrix $\hat{\mathcal{H}}$

$$i \frac{d}{dt} \begin{pmatrix} D(t) \\ \overline{D}(t) \end{pmatrix} = \left(\hat{M} - i \frac{\hat{\Gamma}}{2} \right) \begin{pmatrix} D(t) \\ \overline{D}(t) \end{pmatrix}$$

- Flavour eigenstates D, $\overline{D}$ have same mass and decay width

- Find $\hat{\mathcal{H}}$ eigenstates D_H, D_L with proper mass and decay width

$$\Delta M_D = M_H - M_L \qquad \Delta \Gamma_D = \Gamma_L - \Gamma_H$$

Charm mixing

- Determine mixing observables as see e.g. review [Lenz, Wilkinson '20]

$$\Delta M_D = 2|M_{12}| + \mathcal{O}(\phi_{12}^2) \qquad \Delta\Gamma_D = 2|\Gamma_{12}| + \mathcal{O}(\phi_{12}^2) \qquad \phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right)$$

- Exp. : $x = \dfrac{\Delta M_D}{\Gamma_D} = \left(0.409^{+0.048}_{-0.049}\right)\%$ $\quad y = \dfrac{\Delta\Gamma_D}{2\Gamma_D} = \left(0.615^{+0.056}_{-0.055}\right)\%$

HFLAV 2021, LHCb arXiv:2106.03744 see also talks by L. Dong, J. Libby and D. Cervenkov
see talk of A. Schwartz

- M_{12}, Γ_{12}: dispersive and absorptive part of $D^0 \to \overline{D}^0$ amplitude

$$\Gamma_{12} = \frac{1}{2m_D}\mathrm{Im}\,\langle\overline{D^0}|i\int d^4x\,\mathrm{T}\Big\{\mathcal{H}_{eff}(x)\,,\mathcal{H}_{eff}(0)\Big\}|D^0\rangle$$

- Compute Γ_{12} within the HQE

The HQE for mixing

$$\Gamma_{12} = 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{Q}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{Q}_7 \rangle}{m_Q^4} + \ldots \right]$$

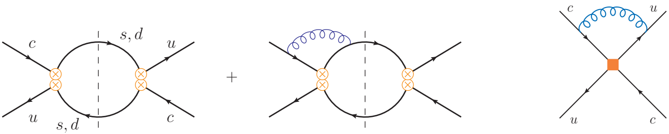
$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste '98 Beneke, Buchalla, Lenz, Nierste '03 Ciuchini, Franco, Lubicz, Mescia, Tarantino '03 Lenz, Nierste '06
$\tilde{\Gamma}_7^{(0)}$	Beneke, Buchalla, Dunietz '96 Dighe, Hürth, Kim, Yoshikawa '01

$\langle \tilde{Q}_6 \rangle$	arXiv:1706.04622 ★ Kirk, Lenz, Rauh '17 ★
$\langle \tilde{Q}_7 \rangle$	VIA, see also arXiv:1910.00970 ★

★ HQET sum rules ★ Lattice QCD

- Theoretical prediction four orders of magnitude smaller than data
- Complete failure of the HQE for charm mixing? see talk by H. Umeeda
  - HQE successfully predicts bottom mixing and lifetime ratios
  - No signal of breakdown of HQE for charm lifetimes

Peculiarities of charm mixing



◇ Three contributions to Γ_{12} with internal dd, ss, sd quark pairs

◇ Apply HQE and use $\lambda_d + \lambda_s + \lambda_b = 0$ with $\lambda_q = V_{cq} V^*_{uq}$

$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} \right) + 2\,\lambda_s \lambda_b \left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd} \right) - \lambda_b^2\, \Gamma_{12}^{dd}$$

◇ Interplay of CKM and GIM suppression [Glashow, Iliopoulos, Maiani '70]

$$\lambda_b^2 = -1.560 \times 10^{-8} + 1.757 \times 10^{-8} i \qquad \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} \right) = 0.07 z^2$$

$z = m_s^2 / m_c^2 \approx 0.006$

Alternative scale setting

- Consider the dependence of Γ_{12} on renormalisation scale $\mu_1^{q_1 q_2}$

$$\Gamma_{12} = \sum_{q_1 q_2 = ss, sd, dd} \tilde{\Gamma}_6^{q_1 q_2}(\mu_1^{q_1 q_2}, \mu_2^{q_1 q_2}) \langle \tilde{Q}_6 \rangle (\mu_2^{q_1 q_2}) \frac{1}{m_c^3} + \ldots$$

- $\Gamma_{12}^{ss,dd,sd}$ contribute to different decay channels
  - Reduction of dependence on $\mu_1^{q_1 q_2}$ within $\Gamma_{12}^{q_1 q_2}$ for each $q_1 q_2$ pair
  - Rescattering effects can only relate Γ_{12}^{dd} and Γ_{12}^{ss}
- Treat dependence on $\mu_1^{q_1 q_2}$ separately
  - Compute Γ_{12} varying $\mu_1^{ss}, \mu_1^{sd}, \mu_1^{dd}$ independently

$$\Omega \in [4.6 \times 10^{-5}, 1.3]$$

$$\Omega = 2|\Gamma_{12}|^{\mathrm{SM}} / \Delta\Gamma_D^{\mathrm{Exp}}$$

Conclusion and outlook for charm mixing

- Alternative scale setting
  - Uncertainty of HQE prediction enhanced, also exp. data covered
  - No sizeable effect for observables with no GIM cancellations
    - Bottom and charm lifetimes, $\Delta\Gamma_{(s)}$ and $\Delta M_{(s)}$, not affected
  - Can affect semileptonic CP asymmetries in $B_{(s)}$-mixing

Governed by weakly GIM suppressed contributions

- Future improvements
  - Include NNLO-QCD corrections at dimension-six when completed

[Gerlach et al. '21; Asatrian et al. '20; Asatrian et al. '17]

See talk by V. Shtabovenko

  - Include higher power corrections [Bobrowski, Lenz, Rauh '12 (partial dim. 9)]

Thanks for the attention