Neutral meson mixing from Lattice QCD

Justus Tobias Tsang
tsang@imada.sdu.dk
The University of Southern Denmark

CKM 2021, Melbourne, AU

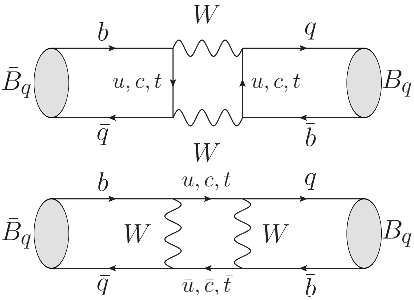
24 November 2021

The project leading to this application has received funding from the European Union's Horizon 2020 research and innovation programme under the **Marie Skłodowska-Curie** grant agreement No 894103.

Outline

Neutral $B_{(s)}$ meson mixing - background

Neutral mesons oscillate:



where $q = d, s$

mass eigenstate $\neq$ flavour eigenstate

$$|B_{L,H}\rangle = p\,|B_q^0\rangle \pm q\,|\bar{B}_q^0\rangle$$

$\Rightarrow$ **splittings** in mass eigenstates:

- mass splitting $\Delta m_q \equiv m_H - m_L$
- width splitting $\Delta\Gamma_q \equiv \Gamma_L - \Gamma_H$

Time dependence:

$$|B_q^0(t)\rangle = g_+(t)\,|B_q^0\rangle + \frac{q}{p} g_-(t)\,|\bar{B}_q^0\rangle$$

$$|\bar{B}_q^0(t)\rangle = \frac{p}{q} g_-(t)\,|B_q^0\rangle + g_+(t)\,|\bar{B}_q^0\rangle$$

Occurs at loop level in SM $\Rightarrow$ **sensitive probe of new physics**!

Neutral $B_{(s)}$ Meson Mixing - experiment

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos\left(\Delta m_q t\right)\right]$$

Δm experimentally accessible as a frequency!

B_d^0: Many results

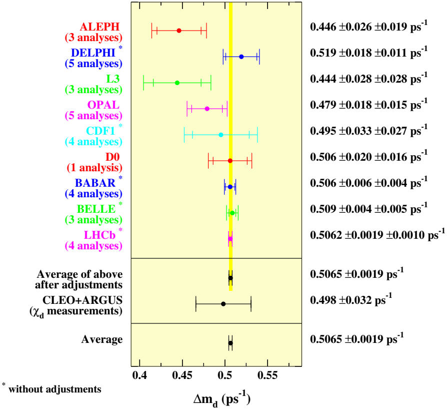
B_s^0: "Only" CDF and LHCb

$$\Delta m_d = 0.5065(19)\mathrm{ps}^{-1}$$

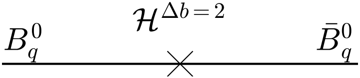
$$\Delta m_s = 17.757(21)\mathrm{ps}^{-1}$$

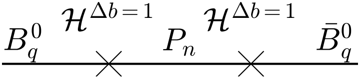
Well below per cent level!

[HFLAV]



Neutral $B_{(s)}$ Meson Mixing - theory





$$\left\langle B_q^0 \right| \mathcal{H}_{\text{eff}} \left| \bar{B}_q^0 \right\rangle \propto \underbrace{\left\langle B_q^0 \right| \mathcal{H}^{\Delta b=2} \left| \bar{B}_q^0 \right\rangle}_{\text{Short distance}} + \underbrace{\sum_n \frac{\left\langle B_q^0 \right| \mathcal{H}^{\Delta b=1} \left| n \right\rangle \left\langle n \right| \mathcal{H}^{\Delta b=1} \left| \bar{B}_q^0 \right\rangle}{E_n - M_{B_q}}}_{\text{Long distance}}$$

$$\text{short distance} \propto \left| \sum_{q'=u,c,t} \frac{m_{q'}^2}{M_W^2} V_{q'b} V_{q'q}^* \right|^2 \approx \frac{m_t^4}{M_W^4} \left| V_{tb} V_{tq}^* \right|^2$$

SD: Top enhanced: $m_t^2 V_{tb} V_{tq}{}^* \gg m_c^2 V_{cb} V_{cq}^* \gg m_u^2 V_{ub} V_{uq}^*$
LD: Only m_c, m_u in intermediate states: no top + CKM suppressed
$\Rightarrow$ **Short distance dominated**.

Operator Product Expansion

OPE factorises this into

- **Perturbative model-dependent Wilson coefficients** $C_i(\mu)$
- **Non-perturbative model-independent matrix elements**

$$\left\langle B^0_{(s)} \right| \mathcal{H}^{\Delta b=2} \left| \bar{B}^0_{(s)} \right\rangle = \sum_i C_i(\mu) \left\langle B^0_{(s)} \right| \mathcal{O}_i^{\Delta b=2}(\mu) \left| \bar{B}^0_{(s)} \right\rangle$$

- 5 independent (parity even) operators $\mathcal{O}_i$.
- Only $\mathcal{O}_1$ is relevant for Δm:

$$\mathcal{O}_1 = \left(\bar{b}_a \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_a\right) \left(\bar{b}_b \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_b\right) = \mathcal{O}_{VV+AA}$$

- Define bag parameters: $B_i = \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle / \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle_{VSA}$

$$\Delta m_q = \left| V^*_{tb} V_{tq} \right|^2 \times f^2_{B_q} \hat{B}^{(1)}_{B_q} \times m_{B_q} \mathcal{K}$$

$\Rightarrow$ **Non-perturbative matrix elements calculable on the lattice**

Lattice QCD in a nutshell

Based on the **Path Integral** formulation.

$$\langle \mathcal{O} \rangle_M = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \overline{\psi}, U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{iS[\psi,\overline{\psi},U]}$$

Minkowski: Highly oscillatory, infinite dimensional integral. ✗

⇒ Wick rotate to Euclidean (i.e. imaginary) time ($t \to i\tau$).

$$\langle \mathcal{O} \rangle_E = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \overline{\psi}, U]\, \mathcal{O}_E[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

Euclidean: Exponentially decaying, infinite dimensional integral. ✗

⇒ Discretise space-time and **interpret as a probability distribution**.

- Lattice spacing a (UV regulator)
- Box of length L (IR regulator)
- $\int \to \sum$, $\partial \to$ finite differences
- Evaluate **stochastically**

Lattice: Exponentially decaying and finite dimensional ✓

Recovering continuum physics

Lattice vs Continuum

We simulate:

- at finite lattice spacing a
- in finite volume L^3
- Euclidean space
- lattice regularised
- some bare input quark masses
  am_l, am_s, am_c, am_b
  In general: $m_\pi \neq m_\pi^{\rm phys}$

We want:

- $a = 0$
- $L = \infty$
- Minkowski space
- some continuum scheme
- $m_l = m_l^{\rm phys}$
- $m_s = m_s^{\rm phys}$
- $m_h = m_c^{\rm phys}, m_b^{\rm phys}$

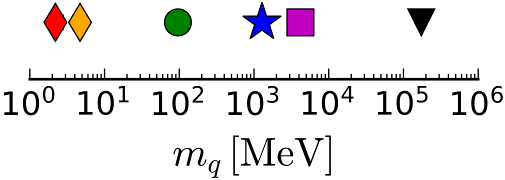
$\Rightarrow$ **Need to control all limits!**
$\rightarrow$ particularly simultaneously control FV and discretisation.

$\Rightarrow$ **Universality**: Different discretisations **must** give same results.

Multiple scale problem on the lattice: back of the envelope

Control effects of IR (finite volume) and UV (discretisation) regulators:

$$m_\pi L \gtrsim 4 \qquad\qquad a^{-1} \gg \text{Mass scale of interest}$$



For $m_\pi = m_\pi^{\rm phys} \sim 140\,\mathrm{MeV}$ and $\overline{m_b}(m_b) \approx 4.2\,\mathrm{GeV}$:

$$L \gtrsim 5.6\,\mathrm{fm} \qquad\qquad a^{-1} \gg 4.2\,\mathrm{GeV} \approx (0.05\,\mathrm{fm})^{-1}$$

Requires $N \equiv L/a \gg 120 \Rightarrow N^3 \times (2N) \gg 4 \times 10^8$ lattice sites.

VERY EXPENSIVE to satisfy both constraints simultaneously...

... needs to be repeated for different values of a.

How to simulate the b-quark?

For now choose between:

Effective action for b

- Can tune to m_b
- comes with systematic errors which are hard to estimate/reduce

Relativistic action for b

- Theoretically cleaner and systematically improvable
- Need to control extrapolation in heavy quark mass

Different properties:

- computational cost
- chirality
- tuning errors
- systematic errors
- cut off effects
- renormalisation

BUT SOON:

Huge efforts in the community to produce **very fine lattice spacings**:

$\Rightarrow$ **Direct simulation of $\approx m_b^{\mathrm{phys}}$ will become possible!**

Extracting CKM matrix elements

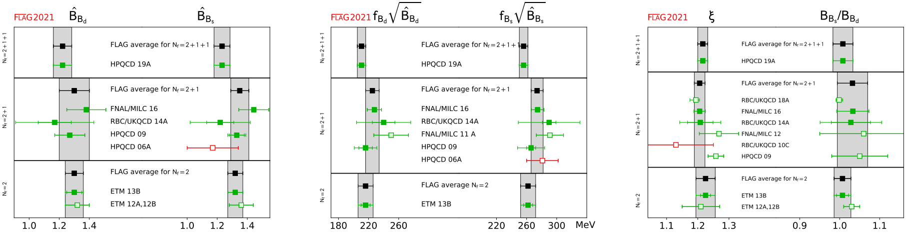
We write Δm_q in terms of the Renormalisation Group Independent (RGI) bag parameter $\hat{B}_{B_q}$:

$$\Delta m_q = |V_{tb}^* V_{tq}|^2 \frac{G_F^2 m_W^2 m_{B_q}^2}{6\pi^2} S_0(x_t)\eta_{2B} f_{B_q}^2 \hat{B}_{B_q}^{(1)}$$

- Δm_d and Δm_s are known experimentally to 0.4% and 0.1% accuracy
- Combined other inputs are known to 0.4%
- Typical precision for $f_{B_q}\sqrt{\hat{B}_{B_q}^{(1)}}$ is a few percent.
- Part of statistic and systematic errors cancel in $SU(3)$ breaking ratios:

$$\xi^2 \equiv \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_d}^2 \hat{B}_{B_d}} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}} \qquad \Rightarrow \text{access to } |V_{td}/V_{ts}|$$

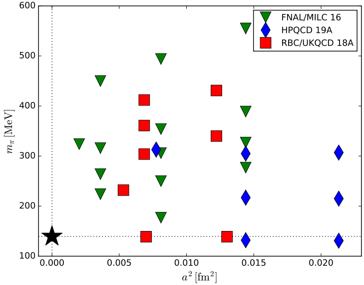
$B - \bar{B}$ mixing results: FLAG 2021 [2111.09849]



Fewer results than in the light sector, but very complementary results from

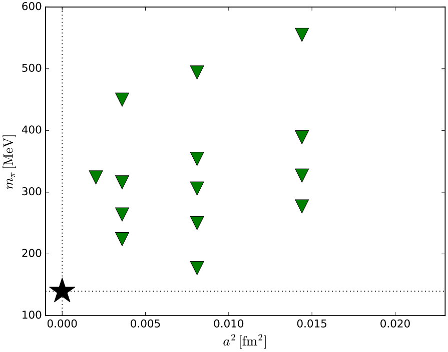
- different gauge field configurations
- different valence light actions
- different valence heavy actions
- different methodologies

Includes computations with m_π^{phys}!



FNAL/MILC: set-up [1602.03560]

- $N_f = 2 + 1$ flavours of staggered quarks (asqtad) in the sea
- 4 lattice spacings, pion masses from $177 - 555\,\mathrm{MeV}$
- valence light & strange: asqtad
- Fermilab method for the b-quark
- *mostly non-perturbative* 1-loop lattice perturbation theory
- Computation of $f_{B_q}\sqrt{\hat{B}_{B_q}}$ and ξ
- f_{B_q} taken from the PDG average to access to $\hat{B}_{B_q}$
- all 5 operators for B and B_s



$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 227.7(9.5)\,\mathrm{MeV}$$

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 274.6(8.4)\,\mathrm{MeV}$$

$$\xi = 1.206(18)$$

FNAL/MILC: error budget [%] [1602.03560]

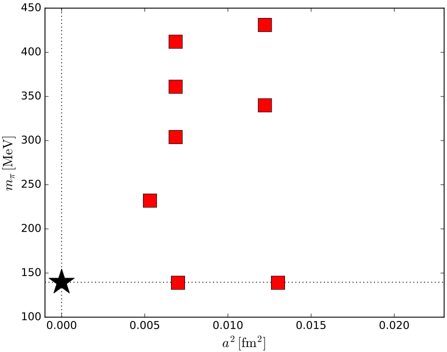
	statistics	inputs	κ tuning	matching	chiral	LQ disc	HQ disc	fit total
$\langle\mathcal{O}_1^d\rangle$	4.2	0.4	2.1	3.2	2.3	0.6	4.6	7.7
$\langle\mathcal{O}_2^d\rangle$	4.6	0.3	1.1	3.7	2.6	0.6	4.6	8.0
$\langle\mathcal{O}_3^d\rangle$	8.7	0.2	2.1	12.6	4.8	1.2	9.9	19.0
$\langle\mathcal{O}_4^d\rangle$	3.7	0.4	1.7	2.2	1.9	0.5	3.9	6.4
$\langle\mathcal{O}_5^d\rangle$	4.7	0.5	2.5	4.7	2.7	0.8	4.9	9.1
$\langle\mathcal{O}_1^s\rangle$	2.9	0.4	1.5	2.1	1.6	0.4	3.2	5.4
$\langle\mathcal{O}_2^s\rangle$	3.1	0.3	0.8	2.5	1.6	0.4	3.1	5.5
$\langle\mathcal{O}_3^s\rangle$	5.9	0.3	1.4	8.6	3.0	0.7	6.9	13.0
$\langle\mathcal{O}_4^s\rangle$	2.7	0.4	1.2	1.6	1.3	0.3	2.9	4.8
$\langle\mathcal{O}_5^s\rangle$	3.4	0.4	1.8	3.4	1.9	0.5	3.6	6.7
ξ	0.8	0.4	0.3	0.5	0.4	0.1	0.7	1.4

Uncertainty dominated by chiral-continuum limit fit, in particular

- statistical
- heavy quark discretisation errors
- matching

RBC/UKQCD: set-up [1812.08791]

- $N_f = 2+1$ flavours of chirally symmetric domain wall fermions in the sea and the valence
- 3 lattice spacings
- 2 physical pion mass ensembles with m_π up to $430\,\mathrm{MeV}$
- Domain wall heavy quarks with $m_c^{\mathrm{phys}} \lesssim m_h^{\mathrm{sim}} \lesssim \frac{m_b^{\mathrm{phys}}}{2}$
- Slightly different DWF parameters between heavy and light sector
- Computation of $SU(3)$ breaking ratios f_{B_s}/f_{B_d}, B_{B_s}/B_{B_d} and ξ
- Renormalisation constants cancel

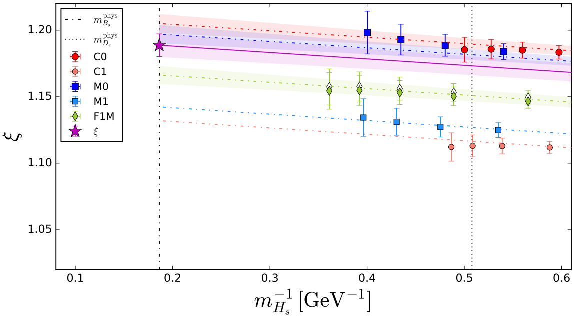


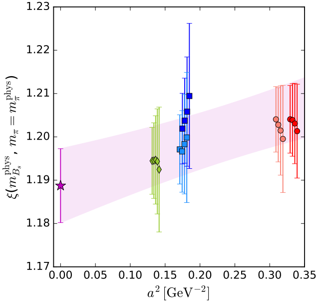
$$f_{B_s}/f_{B_d} = 1.1949(60)_{\mathrm{stat}} \left({}^{+95}_{-175}\right)_{\mathrm{sys}}$$

$$B_{B_s}/B_{B_d} = 0.9984(45)_{\mathrm{stat}} \left({}^{+80}_{-63}\right)_{\mathrm{sys}}$$

$$\xi = 1.1939(67)_{\mathrm{stat}} \left({}^{+95}_{-177}\right)_{\mathrm{sys}}$$

RBC/UKQCD: error budget [%] [1812.08791]





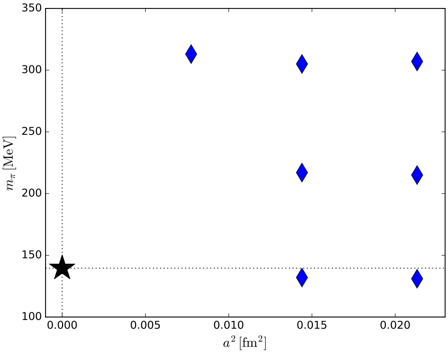
	f_{D_s}/f_D	f_{B_s}/f_B	ξ	B_{B_s}/B_{B_d}
central	1.1740	1.1949	1.1939	0.9984
stat	0.43%	0.50%	0.56%	0.45%
fit chiral-CL	$^{+0.31}_{-0.32}$%	$^{+0.34}_{-0.54}$%	$^{+0.38}_{-0.45}$%	$^{+0.42}_{-0.01}$%
fit heavy mass	$^{+0.07}_{-0.05}$%	$^{+0.00}_{-0.82}$%	$^{+0.00}_{-0.87}$%	$^{+0.27}_{-0.22}$%
H.O. heavy	0.00%	0.47%	0.35%	0.21%
H.O. disc.	0.01%	0.01%	0.12%	0.17%
$m_u \neq m_d$	0.08%	0.07%	0.08%	0.01%
finite size	0.18%	0.18%	0.18%	0.18%
total systematic	$^{+0.38}_{-0.38}$%	$^{+0.61}_{-1.38}$%	$^{+0.56}_{-1.37}$%	$^{+0.66}_{-0.45}$%
total sys+stat	$^{+0.58}_{-0.58}$%	$^{+0.79}_{-1.47}$%	$^{+0.80}_{-1.48}$%	$^{+0.80}_{-0.63}$%

Combined fit in m_π, m_H and a^2
Uncertainty dominated by

- chiral-continuum fit
- heavy quark extrapolation
- estimates of higher order $1/m_H$ terms

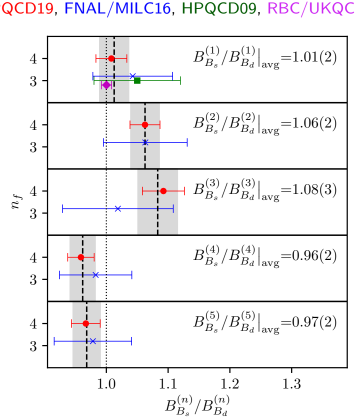
HPQCD: set-up [1907.01025]

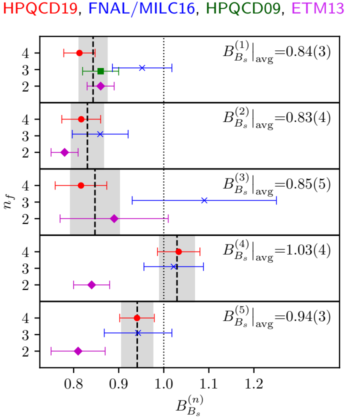
- $N_f = 2 + 1 + 1$ flavours of staggered quarks (HISQ) in sea
- light quarks using HISQ in the valence
- 3 lattice spacings, 2 physical pion mass ensembles
- improved nonrelativistic QCD action for the b
- all 5 operators for B_d and B_s
- blinded analysis
- Computation of the $\hat{B}^{(i)}_{B_q}$.
- ξ and $f_{B_q}\sqrt{\hat{B}_{B_q}}$ accessed by using decay constants taken from a different computation



$$\hat{B}^{(1)}_{B_d} = 1.222(61)$$
$$\hat{B}^{(1)}_{B_s} = 1.232(53)$$
$$\hat{B}^{(1)}_{B_s}/\hat{B}^{(1)}_{B_d} = 1.008(25)$$

HPQCD: error budget and results [1907.01025]



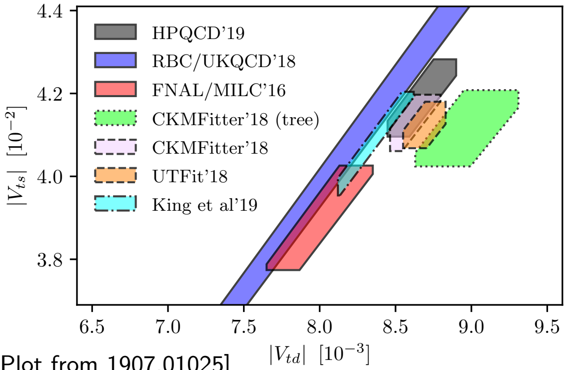


	$B^{(1)}_{B_s}$	$B^{(2)}_{B_s}$	$B^{(3)}_{B_s}$	$B^{(4)}_{B_s}$	$B^{(5)}_{B_s}$	$B^{(1)}_{B_s}/B^{(1)}_{B_d}$
lattice data	1.4	1.4	1.5	1.6	1.5	1.5
η_i^q	0.0	2.3	2.3	2.1	1.2	0.0
α_s^2 terms	2.1	2.9	5.2	1.9	1.5	0.1
$\alpha_s\Lambda_{\rm QCD}/m_b$ terms	2.9	2.8	2.9	2.8	2.7	0.0
$(a\Lambda_{\rm QCD})^{2n}$ terms	1.8	1.9	2.3	1.5	1.8	0.1
m_l extrapolation	0.4	0.4	0.7	0.5	0.4	1.9
Total	4.3	5.3	7.0	4.6	4.1	2.5

Uncertainty dominated by matching terms α_s^2 and $\alpha_s \Lambda_{QCD}/m_b$.

CKM matrix elements

	$f_{B_d}\sqrt{\hat{B}_{B_d}}$ [GeV]	$f_{B_s}\sqrt{\hat{B}_{B_s}}$ [GeV]	ξ	$\lvert V_{td}\rvert$	$\lvert V_{ts}\rvert$	$\lvert V_{ts}/V_{td}\rvert$
HPQCD19	0.2106(55)	0.2561(57)	1.216(16)	0.00867(23)	0.04189(93)	0.2071(27)
FNAL/MILC16	0.2277(98)	0.2746(88)	1.206(18)	0.00800(35)	0.0390(13)	0.2052(33)
RBC/UKQCD18			$1.194\left(^{+12}_{-19}\right)$			$0.2033\left(^{+16}_{-30}\right)$



[Plot from 1907.01025]

- Reasonable agreement between lattice results, but some spread
- Tree-only fit somewhat differs
- Uncertainty is still dominated by theory

$\Rightarrow$ **There is some work yet to be done!**

Aside: Non-perturbative renormalisation on the lattice

We want

$$\text{Amplitude} = C^{\overline{\text{MS}}}(\mu)\,\langle\mathcal{O}\rangle^{\overline{\text{MS}}}(\mu)$$

- Wilson coefficients (typically) computed in $\overline{\text{MS}}$ at some scale μ.
- Operators $\langle\mathcal{O}\rangle^{\text{bare}}(a)$ computed with lattice regulator a^{-1}.
- Renormalise $\langle\mathcal{O}\rangle^{\text{bare}}(a)$ at scale μ in regularisation independent (RI) scheme, by computing a non-pert. renormalisation factor $Z^{RI}(\mu, a)$.

$$\langle\mathcal{O}\rangle^{RI}(\mu) = \lim_{a^2\to 0} Z^{RI}(\mu, a)\,\langle\mathcal{O}\rangle^{\text{bare}}(a)$$

- Match to preferred scheme (e.g. $\overline{MS}$) using P.T. at μ: $R^{\overline{MS}\leftarrow RI}(\mu)$
- If the operators mix: C and $\langle\mathcal{O}\rangle$ become vectors, R and Z matrices.

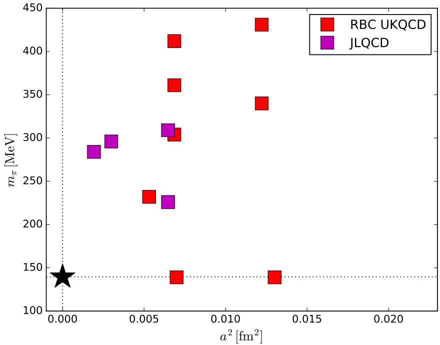
$$\text{Amplitude} = C_i^{\overline{\text{MS}}}(\mu) R_{ij}^{\overline{MS}\leftarrow RI}(\mu) \lim_{a\to 0} Z_{jk}^{RI}(\mu, a)\,\langle\mathcal{O}_k\rangle^{\text{bare}}(a)$$

- Chirally symmetric fermions $\Rightarrow$ R and Z are block diagonal.

RBC/UKQCD/JLQCD: follow-up project [2111.11287]

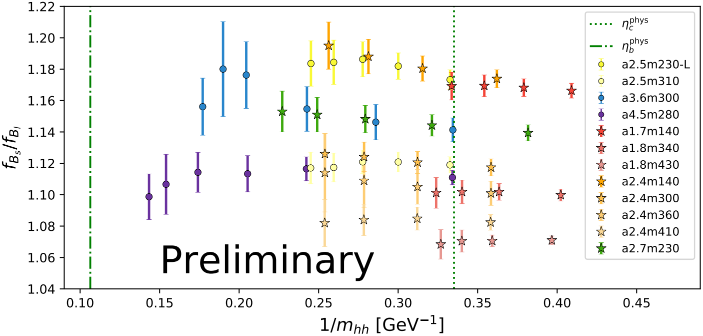
RBC/UKQCD18 budget dominated

- by chiral-continuum fit
- by heavy quark extrapolation
- by estimates of higher order $1/m_H$ terms



- Supplement RBC/UKQCD dataset with very fine ensembles from JLQCD
  $\Rightarrow$ reduce extrapolation in m_H significantly
- all domain wall fermion set-up
  $\Rightarrow$ mixed action NPR in RI-SMOM scheme in progress
  $\Rightarrow$ all 5 operators $\hat{B}^{(i)}_{B_d}$ and $\hat{B}^{(i)}_{B_d}$
- 6 lattice spacings, 2 ensembles with physical pion mass
  $\Rightarrow$ good control over all required limits
- new correlator fitting strategy

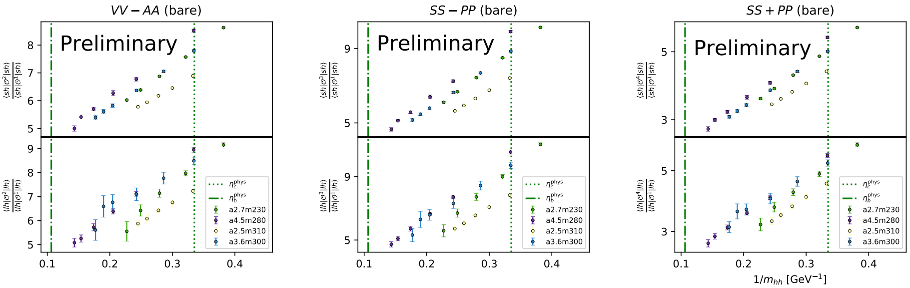
RBC/UKQCD/JLQCD: 1st look – decay constants [2111.11287]

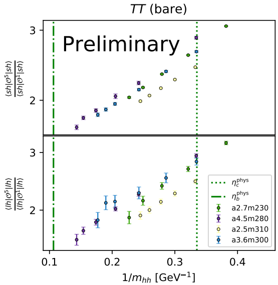


unfitted data, i.e. a function of (a, m_π, m_K, m_H). But promising since

- required extrapolation to $m_b^{\rm phys}$ small
- benign behaviour with $1/m_H$

RBC/UKQCD/JLQCD: 1st look – full operator basis [2111.11287]





Un-renormalised ratios $\langle B_q | \mathcal{O}_i | \bar{B}_q \rangle / \langle B_q | \mathcal{O}_1 | \bar{B}_q \rangle$

- More ensembles to be fitted soon
- Required extrapolation to $m_b^{\rm phys}$ small
- Benign behaviour with $1/m_H$
- NPR in progress
- Developing fit strategy to take limits

Summary and Outlook

Disclaimer

- Focussed on the three most recent results (two new since CKM 18)
- Many technical lattice details omitted
- Only covered lattice results of dim 6 operators. Omitted
  $\Rightarrow$ new sum rules result [1904.00940]
  $\Rightarrow$ new lattice result for width difference [HPQCD 1910.00970]

Summary

- Comparably few but very complementary lattice results:
  - ensembles
  - light quark action
  - heavy quark action
  - renormalisation
- Physical pion mass results
- Results for full operator basis
- First results without need for effective action for the b-quark

Status and Future

- $|V_{td}|$ and $|V_{ts}|$ known at $\approx 2.5\%$ level, $|V_{td}/V_{ts}|$ at $\approx 1.5\%$ level
- Uncertainty theory dominated - work is ongoing