QCD CORRECTIONS TO $\Delta\Gamma_s$

Vladyslav Shtabovenko
Karlsruhe Institute of Technology
Institute for Theoretical Particle Physics

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in collaboration with M. Gerlach, U. Nierste and M. Steinhauser

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Outline

- Neutral meson systems can oscillate between their flavor eigenstates: $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B_q^0 - \bar{B}_q^0$ with $q = s, d$
- Loop-induced FCNC processes
- B meson properties equally well accessible to theory and experiment
- Many new exciting experimental measurements, see talks by Alessandro Gaz, Niels Tuning, Ramon Angel Ruiz Fernandez, Anna Lupato, Lukas Novotny, Alibordi Muhammad, Thibaud Humair, . . .

- $B_s^0 - \bar{B}_s^0$ oscillations between flavor eigenstates $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$

$$i\frac{d}{dt}\begin{pmatrix}|B_s^0(t)\rangle\\|\bar{B}_s^0(t)\rangle\end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix}|B_s^0(t)\rangle\\|\bar{B}_s^0(t)\rangle\end{pmatrix},$$

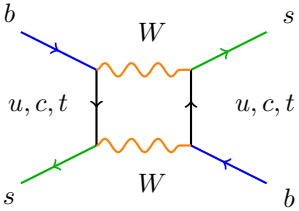
$$\hat{M} = \begin{pmatrix}M_{11} & M_{12}\\M_{12}^* & M_{22}\end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\\Gamma_{12}^* & \Gamma_{22}\end{pmatrix}$$

- Diagonalize the matrices

$$|B_{s,L}\rangle = p\,|B_s^0\rangle + q\,|\bar{B}_s^0\rangle$$
$$|B_{s,H}\rangle = p\,|B_s^0\rangle - q\,|\bar{B}_s^0\rangle$$

- Mass eigenstates: $|B_{s,L}\rangle$ (lighter) and $|B_{s,H}\rangle$ (heavier)

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto$$



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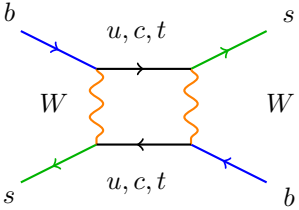


- Physical observables depend on: $|M_{12}|$, $|\Gamma_{12}|$, ϕ_s
- Δm_s: $B_s^0 - \bar{B}_s^0$ oscillation frequency

$$\Delta M_s = M_H - M_L \approx 2|M_{12}|$$

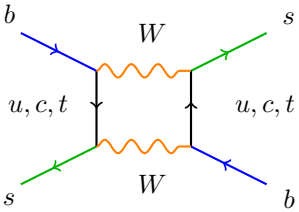
  t quark is dominant in SM, sensitivity to NP in the loops
- $\Delta\Gamma_s$: $B_s^0 - \bar{B}_s^0$ width difference

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi_s$$

  only u and c contribute, precision probe of SM, little room for NP
- ϕ_s: CP-asymmetry in the mixing

$$a_{\rm fs} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_s$$

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto$$



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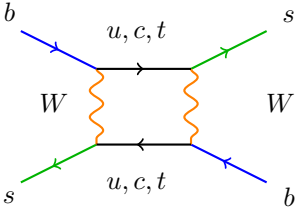


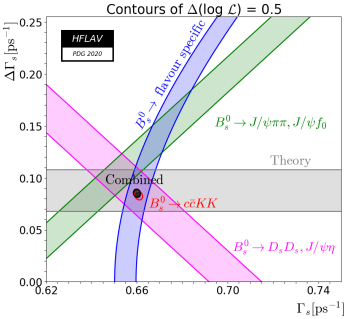
- Our interest: $\Delta\Gamma_s$ from $B_s^0 - \bar{B}_s^0$
- Experimental value (HFLAV 2020 average)

$$\Delta\Gamma^{\rm exp} = (0.085 \pm 0.004)\ {\rm ps}^{-1}$$

- Theory prediction (NLO + n_f-piece of NNLO QCD corrections) [Beneke et al., 1999; Ciuchini et al., 2002, 2003; Lenz & Nierste, 2007; Asatrian et al., 2020, 2017]

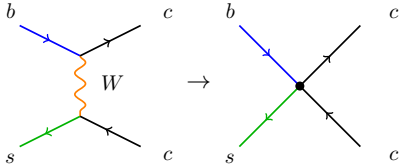
$$\Delta\Gamma_{\rm OS} = (0.077 \pm 0.015_{\rm pert.} \pm 0.002_{B,\tilde{B}_S} \pm 0.017_{\Lambda_{\rm QCD}/m_b})\ {\rm ps}^{-1}$$
$$\Delta\Gamma_{\overline{\rm MS}} = (0.088 \pm 0.011_{\rm pert.} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{\Lambda_{\rm QCD}/m_b})\ {\rm ps}^{-1}$$

- Large perturbative uncertainty from the uncalculated NNLO corrections (pert.)
- Can be reduced by including relevant 2- and 3-loop QCD corrections
- Theory under pressure, full NNLO corrections highly desirable

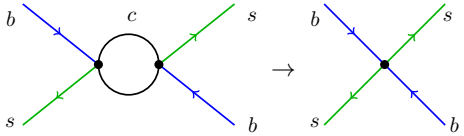


Overview of the matching calculation

- $|\Delta B| = 1$ EFT ($m_b \ll m_W, m_t$)

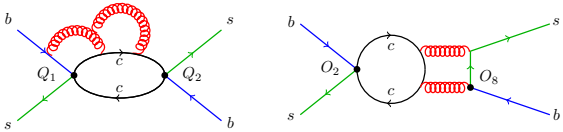


- $|\Delta B| = 2$ EFT (via HQE)

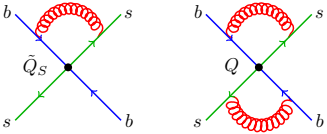


$$\Gamma_{12} \sim \frac{1}{m_b^3} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_3^{(i)} + \frac{1}{m_b^4} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_4^{(i)} + \dots$$

- Calculation done using $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$ in the CMM operator basis for $b \to sc\bar{c}$ [Chetyrkin et al., 1998]
- Representative diagrams in the $|\Delta B| = 1$ EFT needed for the NNLO accuracy



matched to the $|\Delta B| = 2$ EFT



$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb}^\dagger \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u)$$
$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

- Current operators

$$Q_1 = \bar{s}_L \gamma_\mu T^a c_L \; \bar{c}_L \gamma^\mu T^a b_L,$$
$$Q_2 = \bar{s}_L \gamma_\mu c_L \; \bar{c}_L \gamma^\mu b_L,$$
$$Q_1^u = \bar{s}_L \gamma_\mu T^a u_L \; \bar{u}_L \gamma^\mu T^a b_L,$$
$$Q_2^u = \bar{s}_L \gamma_\mu u_L \; \bar{u}_L \gamma^\mu b_L,$$
$$Q_1^{cu} = \bar{s}_L \gamma_\mu T^a u_L \; \bar{c}_L \gamma^\mu T^a b_L,$$
$$Q_2^{cu} = \bar{s}_L \gamma_\mu u_L \; \bar{c}_L \gamma^\mu b_L,$$
$$Q_1^{uc} = \bar{s}_L \gamma_\mu T^a c_L \; \bar{u}_L \gamma^\mu T^a b_L,$$
$$Q_2^{uc} = \bar{s}_L \gamma_\mu c_L \; \bar{u}_L \gamma^\mu b_L,$$

- Penguin operators

$$Q_3 = \bar{s}_L \gamma_\mu b_L \textstyle\sum_q \bar{q} \gamma^\mu q\,,$$
$$Q_4 = \bar{s}_L \gamma_\mu T^a b_L \textstyle\sum_q \bar{q} \gamma^\mu T^a q\,,$$
$$Q_5 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q\,,$$
$$Q_6 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q\,,$$
$$Q_8 = \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \sigma^{\mu\nu} T^a b_R \, G^a_{\mu\nu}\,,$$

$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}}\Bigg[-V_{ts}^* V_{tb}^\dagger \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8\Big) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u)$$
$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc}\Bigg] + \text{h.c.}\,,$$

- 4-fermion vertices generate Dirac structures with multiple insertions of γ matrices

$$(P_L)_{ij} \times (P_L)_{kl}, \quad (\gamma^\mu P_L)_{ij} \times (\gamma_\mu P_L)_{kl}, \quad (\gamma^\mu \gamma^\nu P_L)_{ij} \times (\gamma_\mu \gamma_\nu P_L)_{kl},$$
$$(\gamma^\mu \gamma^\nu \gamma^\rho P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho P_L)_{kl}, \quad (\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma P_L)_{kl},$$
$$(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_\tau P_L)_{kl}, \ldots$$

- 4-dimensions: Products of γ matrices reducible using Fierz and Chisholm identities

$$\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu}\gamma^\rho + g^{\nu\rho}\gamma^\mu - g^{\mu\rho}\gamma^\nu + i\varepsilon^{\mu\nu\rho\sigma}\gamma_\sigma \gamma^5$$

- d-dimensions: Fierz and Chisholm identities become ambiguous
- Proper treatment using evanescent operators [Dugan & Grinstein, 1991; Herrlich & Nierste, 1995]

$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb}^\dagger \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u)$$
$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

- $|\Delta B| = 1$ LO evanescent operators

$$E_1^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a c_L \; \bar{c}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L - 16 Q_1\,,$$
$$E_2^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} c_L \; \bar{c}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L - 16 Q_2\,,$$
$$E_3^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} b_L \textstyle\sum_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} q \; - \; 20 Q_5 \; + \; 64 Q_3\,,$$
$$E_4^{(1)} = \bar{s}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} T^a b_L \textstyle\sum_q \bar{q} \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_{\mu_5} T^a q \; - \; 20 Q_6 \; + \; 64 Q_4$$

$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin et al., 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb}^\dagger \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - V_{us}^* V_{ub}^\dagger \sum_{i=1}^{2} C_i (Q_i - Q_i^u)$$
$$+ V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

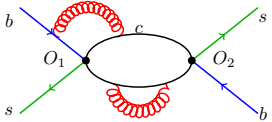
- $|\Delta B| = 1$ NLO evanescent operators

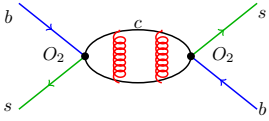
$$E_1^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} T^a c_L \; \bar{c}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T^a b_L - 20 E_1^{(1)} - 256 Q_1\,,$$
$$E_2^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} c_L \; \bar{c}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} b_L - 20 E_2^{(1)} - 256 Q_2\,,$$
$$E_3^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}\gamma^{\mu_6}\gamma^{\mu_7} b_L \textstyle\sum_q \bar{q}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\gamma_{\mu_6}\gamma_{\mu_7} q \; - \; 336 Q_5 \; + \; 1280 Q_3\,,$$
$$E_4^{(2)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} T^a b_L \textstyle\sum_q \bar{q}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T^a q \; - \; 336 Q_6 \; + \; 1280 Q_4$$

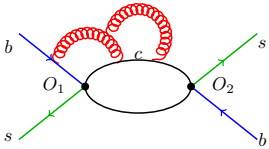
- Evanescent operators are of $\mathcal{O}(\varepsilon)$, formally vanishing in the $d \to 4$ limit
- However: A pole multiplying tree-level matrix element of an ev. operator $\langle E_i^{(j)} \rangle / \varepsilon$ is $\mathcal{O}(\varepsilon^0)$

$|\Delta B| = 1$ side of the matching: representative diagrams

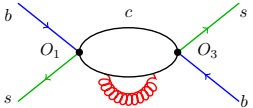
3-loop $O_{1,2} \times O_{1,2}$ correlators

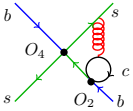


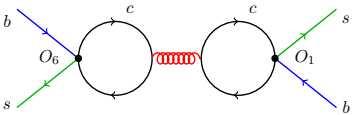




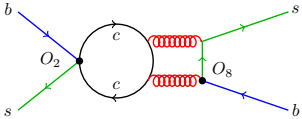
2-loop $O_{1,2} \times O_{3-6}$ correlators

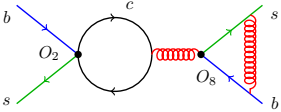






2-loop $O_{1,2} \times O_8$ correlators





$|\Delta B| = 2$ side of the matching: operator basis

- $\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke et al., 1999; Lenz & Nierste, 2007; Asatrian et al., 2017]
- Using Heavy Quark Expansion [Khoze & Shifman, 1983; Shifman & Voloshin, 1985; Khoze et al., 1987; Chay et al., 1990; Bigi & Uraltsev, 1992; Bigi et al., 1992, 1993; Blok et al., 1994; Manohar & Wise, 1994] (expansion in $\Lambda_{\rm QCD}/m_b$) one arrives at

$$\Gamma_{12} = -(\lambda_c^q)^2\Gamma_{12}^{cc} - 2\lambda_c^q\lambda_u^q\Gamma_{12}^{uc} - (\lambda_u^q)^2\Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{ab}(z)\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle\right] + \mathcal{O}\left(\Lambda_{\rm QCD}/m_b\right)$$

- Physical $|\Delta B| = 2$ operators

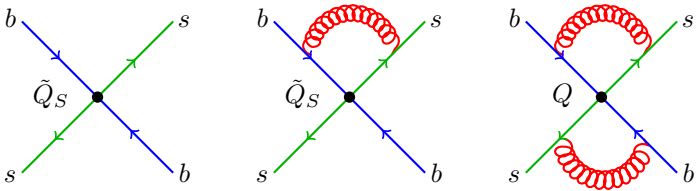
$$Q = \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_i\ \bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_j \qquad \widetilde{Q}_S = \bar{s}_i\left(1-\gamma^5\right)b_j\ \bar{s}_j\left(1-\gamma^5\right)b_i$$

- Additional operators needed at intermediate stages (e. g. basis changes, def. of evanescent operators)

$$\widetilde{Q} = \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_j\ \bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_i\,, \qquad Q_S = \bar{s}_i\left(1-\gamma^5\right)b_i\ \bar{s}_j\left(1-\gamma^5\right)b_j\,,$$

- Not shown here: evanescent $|\Delta B| = 2$ operators and the $1/m_b$ suppressed operator R_0
- $H(z)$ and $\widetilde{H}_S(z)$: Wilson coefficients from the perturbative matching of $|\Delta B| = 1$ to $|\Delta B| = 2$, $z \equiv m_c^2/m_b^2$
- Nonperturbative ME $\langle B_s|Q|\bar{B}_s\rangle$ and $\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle$ (also for B_d mesons) from QCD/HQET sum rules [Ovchinnikov & Pivovarov, 1988; Reinders & Yazaki, 1988; Korner et al., 2003; Mannel et al., 2011; Grozin et al., 2016; Kirk et al., 2017; King et al., 2019], lattice QCD [Bazavov et al., 2016; Dowdall et al., 2019] or combined [Di Luzio et al., 2019]

$|\Delta B| = 2$ side of the matching: representative diagrams



Wilson coefficients of the $|\Delta B| = 2$ theory determined in the matching to $|\Delta B| = 1$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}\left(\Lambda_{\rm QCD}/m_b\right)$$

$|\Delta B| = 1$ contributions needed for NNLO (always 2 insertions from $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$)

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1, 2 \\ \alpha_s & \text{for } i = 3, 4, 5, 6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

Important scale: $z \equiv m_c^2/m_b^2$

- LO contributions to $\Delta\Gamma_s$
  - 1-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Hagelin, 1981; Franco et al., 1982; Chau, 1983; Buras et al., 1984; Khoze et al., 1987; Datta et al., 1987, 1988]
- NLO contributions to $\Delta\Gamma_s$ (z-exact)
  - 2-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1999]
  - 1-loop $O_{1-2} \times O_8$ correlators (z-exact) [Beneke et al., 1999]

$|\Delta B| = 1$ contributions needed for NNLO (always 2 insertions from $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$)

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1, 2 \\ \alpha_s & \text{for } i = 3, 4, 5, 6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

Important scale: $z \equiv m_c^2/m_b^2$

- NNLO contributions to $\Delta\Gamma_s$
  - 3-loop $O_{1-2} \times O_{1-2}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, $\mathcal{O}(z^3)$)
  - 2-loop $O_{1-2} \times O_{3-6}$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 2-loop $O_{1-2} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_{3-6} \times O_{3-6}$ correlators (z-exact) [Beneke et al., 1996]
  - 1-loop $O_{3-6} \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
  - 1-loop $O_8 \times O_8$ correlators [Asatrian et al., 2017, 2020] (n_f piece only, z-exact)
- This work
  - Full (n_f + non-n_f) results for all 2-loop correlators at $\mathcal{O}(z)$ (including $O_8 \times O_8 \Rightarrow$ N^{3}LO)
  - Full (n_f + non-n_f) results for the 3-loop $O_{1-2} \times O_{1-2}$ at $\mathcal{O}(z^0)$
  - WIP: Final checks for the 3-loop result, higher order expansions in z, possibly z-exact results for selected correlators

- Calculational strategy
  - Matching done on-shell: $p_b^2 = m_b^2$
  - The s-quark mass is neglected $\Rightarrow p_s = 0$
  - Asymptotic expansion in $z \equiv m_c^2/m_b^2$ (at first up to $\mathcal{O}(z)$ for 2-loop and $\mathcal{O}(z^0)$ for 3-loop)
  - Only the imaginary part of the $|\Delta B| = 1$ diagrams enters the matching
- Regularization
  - Dimensional regularization used both for UV- and IR-divergences
  - Cross-check: massive gluons in IR-divergent diagrams at 2-loops
  - $\varepsilon_{\rm UV} + m_g$: renormalized amplitudes manifestly finite $\Rightarrow$ the limit $d \to 4$ is safe
  - $\varepsilon = \varepsilon_{\rm UV} = \varepsilon_{\rm IR}$: products of $1/\varepsilon_{\rm IR}$ and evanescent ME are of $\mathcal{O}(\varepsilon^0)$

NLO matching with $\varepsilon = \varepsilon_{\rm IR} = \varepsilon_{\rm UV}$ (no gluon mass) [Ciuchini et al., 2002]

- Normally, only the matching coefficients of physical $|\Delta B| = 2$ operators are relevant
- Here matching coefficients of evanescent operators are also needed (at intermediate stages)
- $|\Delta B| = 2$ matching coefficients obtain $\mathcal{O}(\varepsilon)$ pieces

$$C = f_0^{(0)} + \varepsilon\, f_1^{(0)} + \frac{\alpha_s}{4\pi} f_0^{(1)}, \qquad C_E = f_{E,0}^{(0)} + \varepsilon\, f_{E,1}^{(0)} + \frac{\alpha_s}{4\pi} f_{E,0}^{(1)}$$

- LO matching must be carried out up to $\mathcal{O}(\varepsilon)$: fixes $f_0^{(0)}, f_1^{(0)}, f_{E,0}^{(0)}, f_{E,1}^{(0)}$
- At NLO we only need $\mathcal{O}(\varepsilon^0)$
- Upon inserting $f_0^{(0)}, f_1^{(0)}, f_{E,0}^{(0)}, f_{E,1}^{(0)}$ at NLO all $1/\varepsilon_{\rm IR}$ poles must cancel.
- Finally, the difference

$$A_{\rm ren}^{|\Delta B|=1} - A_{\rm ren}^{|\Delta B|=2}$$

  is manifestly finite $\Rightarrow$ determine $f_0^{(1)}$
- Only $f_0^{(0)}$ and $f_0^{(1)}$ enter the physical matching coefficient
- What about $f_{E,1}^{(0)}$? Not needed at NLO, must be determined for the NNLO calculation!
- At NNLO, the LO matching must be done up $\mathcal{O}(\varepsilon^2)$, the NLO matching up to $\mathcal{O}(\varepsilon)$
- The explicit cancellation of IR poles (and of ξ) is a highly nontrivial cross-check of the whole calculation

- All computations done using our well-tested automatic setup
  - Diagram generation with **QGRAF** [Nogueira, 1993]
  - Insertion of Feynman rules and topology identification using **Q2E/EXP** [Seidensticker, 1999; Harlander et al., 1998] or **TAPIR** [Gerlach, Herren, 2022]
  - Feynman amplitude evaluation: in-house **CALC** setup written in **FORM** [Ruijl et al., 2017]
  - IBP-reduction: **FIRE** 6 [Smirnov & Chuharev, 2020]
  - Analytic computation of master integrals: **HyperInt** [Panzer, 2015], **HyperlogProcedures** [Schnetz], **PolyLogTools** [Duhr & Dulat, 2019]
  - All master integrals checked numerically using **FIESTA** [Smirnov, 2016] and **pySecDec** [Borowka et al., 2018]
- Cross-checks of selected intermediate results using **FeynArts** [Hahn, 2001], **FeynRules** [Christensen & Duhr, 2009; Alloul et al., 2014] and **FeynCalc** [VS et al., 2020]
- Two complementary approaches to the treatment of tensor integrals in **FORM**
  - Explicit decomposition formulas (1 ext. momentum, max. rank 10), calculated using **FeynCalc** and **FERMAT** [Lewis]
  - Projections to the occurring 4-fermion Dirac structures

$$\{(P_L)_{ij}, (\gamma^\mu P_L)_{ij}, (\gamma^\mu\gamma^\nu P_L)_{ij}, \ldots\} \otimes \{(P_L)_{kl}, (\gamma_\mu P_L)_{kl}, (\gamma_\mu\gamma_\nu P_L)_{kl}, \ldots\}$$

  - Both methods lead to the same results!

- New on-shell 3-loop integrals with massive (solid) lines
- Only imaginary parts are relevant and turn out to be very simple
- Appearing constants

$$\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Cl}_2(\pi/3), \sqrt{3},$$
$$\mathrm{Li}_4(1/2), \ln\left((1+\sqrt{5})/2\right)$$

- Real parts (obtained as a byproduct) more complicated but irrelevant for $\Delta\Gamma_s$

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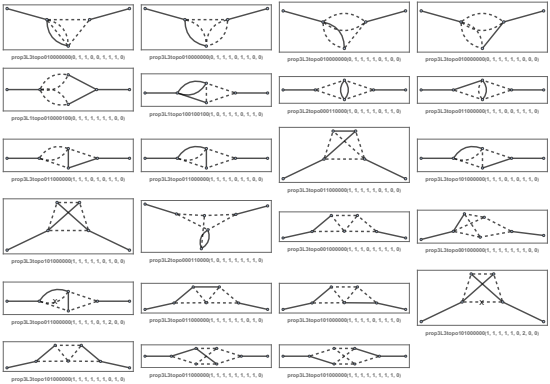

- Handling of master integrals facilitated using new **FeynCalc** functions added in the course of this project (see my talk at ACAT 2021 next Wednesday)
- Graph representation from propagator representation: `FCLoopIntegralToGraph`, `FCLoopGraphPlot`
- Derivation of the Feynman parametrization: `FCFeynmanParametrize`
- Mappings between master integrals: `FCLoopFindIntegralMappings`
- Official in the upcoming **FeynCalc** 10, however already publicly available and documented

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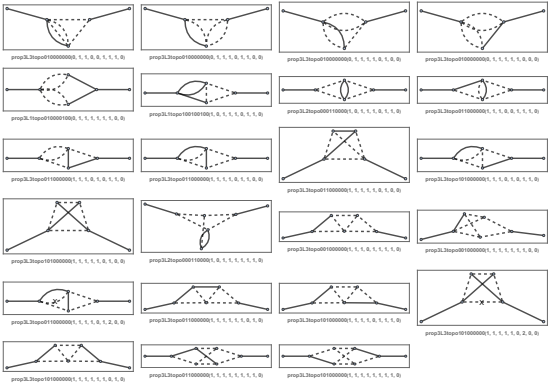

- New contributions to Γ_{12}^s computed in the course of this project ($z = m_c^2/m_b^2$)

Contribution	Literature result	This work
$Q_{1,2} \times Q_{3-6}$	2 loops, z-exact, n_f-part only [Asatrian et al., 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{1,2} \times Q_8$	2 loops, z-exact, n_f-part only [Asatrian et al., 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{3-6} \times Q_{3-6}$	1 loop, z-exact, full [Beneke et al., 1996]	2 loops, $\mathcal{O}(z)$, full
$Q_{3-6} \times Q_8$	1 loop, z-exact, n_f-part only [Asatrian et al., 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_8 \times Q_8$	1 loop, z-exact, n_f-part only [Asatrian et al., 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{1,2} \times Q_{1,2}$	3 loops, $\mathcal{O}(\sqrt{z})$, n_f-part only [Asatrian et al., 2017]	3 loops, $\mathcal{O}(z^0)$, full

- All 2-loop contributions to the NNLO correction already computed and cross-checked
- New theory predictions for the width difference $\Delta\Gamma_s$ and the CP asymmetry a_{fs}^s under way

$$\frac{\Delta\Gamma_s}{\Delta M_s} = -\text{Re}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right), \quad a_{\text{fs}}^s = \text{Im}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right)$$

- Ingredients

$$\Gamma_{12}^s = -(\lambda_t^s)^2\left[\Gamma_{12}^{s,cc} + 2\frac{\lambda_u^s}{\lambda_t^s}\left(\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}\right) + \left(\frac{\lambda_u^s}{\lambda_t^s}\right)^2\left(\Gamma_{12}^{s,uu} + \Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc}\right)\right]$$

$$\Gamma_{s,12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\underbrace{\langle B_s|Q|\bar{B}_s\rangle}_{\frac{8}{3}M_{B_s}^2 f_{B_s}^2 B_{B_s}} + \widetilde{H}_S^{ab}(z)\underbrace{\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle}_{\frac{1}{3}M_{B_s}^2 f_{B_s}^2 \widetilde{B}'_{S,B_s}}\right] + \mathcal{O}\left(\Lambda_{\text{QCD}}/m_b\right)$$

$$M_{12} = (\lambda_t^s)^2\frac{G_F^2 M_{B_s}}{12\pi^2}M_W^2\hat{\eta}_B S_0\left(\frac{m_t^2}{M_W^2}\right)f_{B_s}^2 B_{B_s}$$

- Cancellation of $(\lambda_t^s)^2 = (V_{ts}^* V_{tb}^\dagger)^2$, f_{B_s}, M_{B_s} and to large extent bag parameters in the ratio Γ_{12}^s/M_{12}^s
- Following [Asatrian et al., 2020] we can can calculate

$$\Delta\Gamma_s = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)\Delta M_s^{\text{exp}}$$

- $|V_{cb}|$ controversy irrelevant!

- Theoretical predictions for the $\overline{\text{MS}}$ and pole schemes
- m_b^2 in the prefactor of Γ_{12} treated as $\left(m_b^{\text{OS}}\right)^2$ in the pole scheme and $\left(m_b^{\overline{\text{MS}}}(m_b)\right)^2$ in the $\overline{\text{MS}}$ scheme
- In both schemes we use $\bar{z} = (m_c^{\overline{\text{MS}}}(m_b)/m_b^{\overline{\text{MS}}}(m_b))^2$
- Numerical input [Tanabashi et al., 2018; Dowdall et al., 2019; Bazavov et al., 2018; Amhis et al., 2021]

$$
\begin{aligned}
M_{B_s} &= 5366.88 \text{ MeV} \quad f_{B_s} = (0.2307 \pm 0.0013) \text{ GeV}\,, \\
B_{B_s} &= 0.813 \pm 0.034\,, \quad \widetilde{B}'_{S,B_s} = 1.31 \pm 0.09\,, \\
\frac{\lambda_u^s}{\lambda_t^s} &= -(0.00865 \pm 0.00042) + (0.01832 \pm 0.00039)i \\
\Delta M_s^{\text{exp}} &= (17.749 \pm 0.020) \text{ ps}^{-1}
\end{aligned}
$$

- Numerical estimate on the impact of the new 2-loop $O_{1,2} \times O_{3-6}$ contribution [Gerlach, Nierste, VS, Steinhauser, 2021]

$$
\begin{aligned}
\text{1-loop (already known):} \quad & \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s^0}}{\Delta\Gamma_s} = 7.0\% \quad (\text{pole}) \qquad \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s^0}}{\Delta\Gamma_s} = 6.1\% \quad (\overline{\text{MS}})\,, \\
\text{full 2-loops (new):} \quad & \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s}}{\Delta\Gamma_s} = 0.2\% \quad (\text{pole})\,, \qquad \frac{\Delta\Gamma_s^{p,12\times 36,\alpha_s}}{\Delta\Gamma_s} = 1.4\% \quad (\overline{\text{MS}})\,,
\end{aligned}
$$

Summary

- Experimental precision of $\Delta\Gamma_s$ calls for the NNLO calculation!
- We calculated all building blocks needed to obtain the NNLO correction to $B_s^0 - \bar{B}_s^0$ mixing
- All the occurring 3-loop MI from the current-current contribution calculated analytically (for $m_c = 0$)
- The result for the 2-loop current-penguin contribution already published [Gerlach, Nierste, VS, Steinhauser, 2021]

Outlook

- Results for all the remaining 2-loop contributions and the 3-loop current-current piece under way
- New theory predictions for $\Delta\Gamma_s$ and the CP asymmetry $a_{\rm fs}^s$
- Higher order expansions in $z \equiv m_c^2/m_b^2$, ideally z-exact results at least for the 2-loop contributions