ε'/ε, Standard Model and Beyond

Andrzej J. Buras
(TUM – Institute for Advanced Study)

CKM 2021
Melbourne

Overture

Homeoffice in Ottobrunn

March 2020 →

Effective Hamiltonian and OPE

$$H_{eff} = \sum_i C_i O_i^{SM} + \sum_j C_j^{NP} O_j^{NP}$$

$$C_i = C_i^{SM} + \Delta_i^{NP}$$

Absent in SM

Wilson Coefficients

$$A(K \to \pi\pi) = \sum_i \underbrace{C_i(\mu)}_{SD} \underbrace{\langle \pi\pi | O_i^{SM}(\mu) | K \rangle}_{LD} + \sum_j \underbrace{C_j^{NP}(\mu)}_{SD} \underbrace{\langle \pi\pi | O_j^{NP}(\mu) | K \rangle}_{LD}$$

<u>Examples:</u>

$$O^{SM} = \left(\bar{s}\gamma_\mu(1-\gamma_5)d\right)\left(\bar{d}\gamma^\mu(1-\gamma_5)d\right)$$

$$O^{NP} = \left(\bar{s}(1-\gamma_5)d\right)\left(\bar{d}(1-\gamma_5)d\right)$$

$\mu \approx 1-3$ GeV

Renormalization scale

Impact of QCD at SD and LD Scales

(K-physics)

SD

Fully under control: NLO + NNLO
AJB: „Climbing NLO and NNLO Summits of Weak Decays“
(1102.5650; last update 2014)
Including BSM

(Munich, Rome + Gorbahn, Brod, Haisch, Jäger, Nierste, Cèrda-Sevilla)
(early 1990s)

LD

Lattice QCD (ETM, SWME, RBC-UKQCD, ...)

(Numerical sophisticated and demanding calculations lasting many years) (from first principles)

BSM operators only for $K^0 - \bar{K}^0$ mixing

Analytic

Dual QCD (Bardeen, AJB, Gérard, 1986)
(Much faster than LQCD, very suitable for non-leptonic transitions)
($K^0 - \bar{K}^0$ mixing, $K \to \pi\pi$)

SM+ BSM (2018)

Chiral Pertubation Theory (Gasser, Leutwyler, 1980 → Ecker, Pich..)
(Much faster than LQCD, very suitable for leptonic, semi-leptonic decays)
($K_L \to \pi ll$, $K \to ll$, $K \to \pi\nu\bar{\nu}$)

(less useful for $K\to\pi\pi$, ε'/ε, K^0-$\bar{K}^0$ mixing)

ε'/ε Controversy

2015-2020

$$\left(\varepsilon'/\varepsilon\right)_{exp} = \left(16.6 \pm 2.3\right) \cdot 10^{-4}$$

(NA48, KTeV)

$$\left(\varepsilon'/\varepsilon\right)_{SM} = \left(14 \pm 5\right) \cdot 10^{-4}$$

Chiral Perturbation Theory
(Pich et al)

No Anomaly

★ $$\left(\varepsilon'/\varepsilon\right)_{SM} = \left(5 \pm 2\right) \cdot 10^{-4}$$

Hep-arxiv: 2101.00020

Insight from Dual QCD + NNLO QCD

(AJB + Gérard)

Anomaly

$$\left(\varepsilon'/\varepsilon\right)_{SM} = \left(21.7 \pm 8.4\right) \cdot 10^{-4}$$

RBC – UKQCD

No Anomaly

CKM-Sonatina Nr. 1
(Premiere – 25.11.2021)

1. Dual QCD

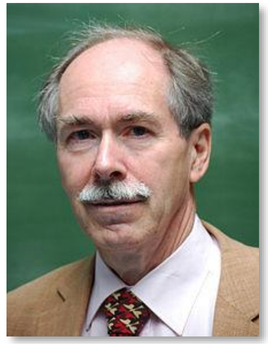
2. ε'/ε in the Standard Model

3. ε'/ε Beyond SM

4. Summary

1. Dual QCD

Large N
QCD



Gerard ´t Hooft
(1974)

Edward Witten
(1979, 1980)

At large N QCD becomes a theory of weakly interacting mesons

with coupling $\frac{1}{f_\pi^2} \sim \frac{1}{N}$

⇩

In the strict Large N limit QCD becomes a free theory of mesons.

⇩

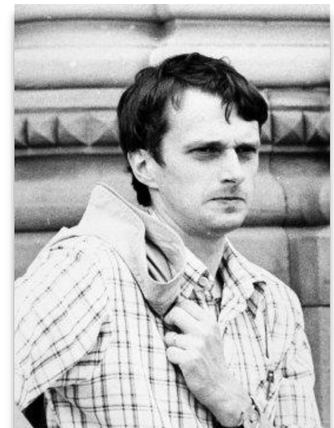
Factorization of hadronic matrix elements ★

AJB (1985)
Gérard
Rückl

Dual QCD Approach for Weak Decays

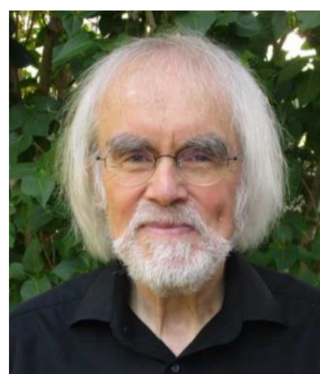
Successful low energy approximation of QCD

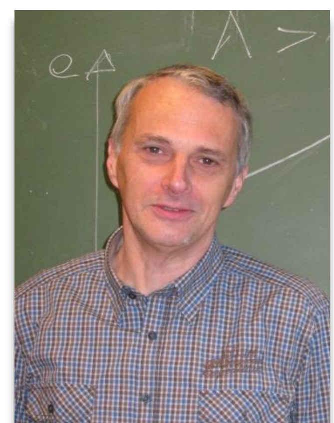
1986



W. Bardeen | **AJB** | **J.-M. Gérard**

2021





Basic Structure of DQCD for $K \to \pi\pi$, $K^0 - \bar{K}^0$ mixing

(ε'/ε, ε, $\Delta I = 1/2$ Rule, ΔM_K)

SM and BSM Operators

Reviews: 1401.1385, 1408.4820, 1809.02616, Cambridge Book

Λ_{NP}

EW

1 GeV

SMEFT with SM and BSM Operators

QCD + QED with SM and BSM Operators (Quark-Gluon Evolution)

RG Evolution

α_s, α_2, α_1

top Yukawa

α_s, α_{QED}

Non-Factorizable contributions

Λ_{DQCD} ≈ 0.8 GeV

m_π, m_K

DQCD: Meson Evolution (1/N)

$0(m_\pi, m_K) \to \Lambda_{DQCD} \approx 0.8$ GeV

SM and BSM operators

Factorization Scale

for hadronic matrix elements

($N \to \infty$)

★ Crucial strong dynamics
Responsible for $\Delta I = 1/2$ Rule, ε'/ε, ε, ΔM_K, $K \to \pi\pi$ in general.

★ Very different philosophy from Chiral PTh in which meson evolution not included.

As the existence of Meson Evolution has been questioned over last 30 years by some Chiral Experts by some Lattice Experts

Let me demonstrate its existence by considering BSM operators in $(K^0 - \bar{K}^0$ Mixing)

Very good test !

Important : The controversal issue of Final State interactions is absent here !!!

and four parameters to our disposal B_2, B_3, B_4, B_5

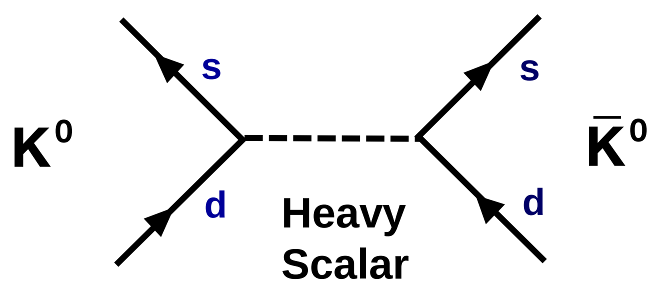
BSM $\Delta S = 2$ Operators $K^0 - \bar{K}^0$ Mixing

$$P_{L,R} = \frac{1}{2}\left(1 \pm \gamma_5\right)$$

BSM

$$\begin{cases} O_2 = \left(\bar{s}^\alpha P_L d^\alpha\right)\left(\bar{s}^\beta P_L d^\beta\right) \to B_2 \\ O_3 = \left(\bar{s}^\alpha P_L d^\beta\right)\left(\bar{s}^\beta P_L d^\alpha\right) \to B_3 \\ O_4 = \left(\bar{s}^\alpha P_L d^\alpha\right)\left(\bar{s}^\beta P_R d^\beta\right) \to B_4 \\ O_5 = \left(\bar{s}^\alpha P_L d^\beta\right)\left(\bar{s}^\beta P_R d^\alpha\right) \to B_5 \end{cases}$$

$$\langle O_i(\mu) \rangle \approx \frac{B_i(\mu)}{m_s^2(\mu)}$$



DQCD : Explaining Values for B_2, B_3, B_4, B_5 from Lattice QCD

(AJB + Gérard, 1804.02401) (ETM15, SWME, RBC-UKQCD)

μ	B_2	B_3	B_4	B_5	$K^0-\bar{K}^0$ mixing
3 GeV	0.49	0.77	0.90	0.65	AJB Lattice Average (± 5%)
Quark Gluon Evolution ⇩ 1 GeV					B_2=B_3=B_4=B_5=1 Vacuum insertion
					} gap : Vector mesons
(0.70) GeV Meson Evolution ⇧					Meson Evolution in the chiral limit
Factorization Scale ≈ 0					← $N \to \infty$

DQCD : Explaining Values for B_2, B_3, B_4, B_5 from Lattice QCD

(AJB + Gérard, 1804.02401) (ETM15, SWME, RBC-UKQCD)

μ	B_2	B_3	B_4	B_5	$K^0-\bar{K}^0$ mixing
3 GeV	0.49	0.77	0.90	0.65	AJB Lattice Average (± 5%)
Quark Gluon Evolution ⇩					$B_2=B_3=B_4=B_5=1$ Vacuum insertion
1 GeV					
					gap : Vector mesons
(0.70) GeV					
Meson Evolution ⇧					Meson Evolution in the chiral limit
Factorization Scale ≈ 0	1.2	3.0	1.0	0.23	← $N \to \infty$

DQCD : Explaining Values for B_2, B_3, B_4, B_5 from Lattice QCD

(AJB + Gérard, 1804.02401) (ETM15, SWME, RBC-UKQCD)

μ	B_2	B_3	B_4	B_5	$K^0 - \bar{K}^0$ mixing
3 GeV	0.49	0.77	0.90	0.65	AJB Lattice Average ($\pm$ 5%)
Quark Gluon Evolution 1 GeV	0.62	1.10	0.90	0.45	$B_2=B_3=B_4=B_5=1$ Vacuum insertion
					gap : Vector mesons
(0.70) GeV Meson Evolution					Meson Evolution in the chiral limit
Factorization Scale ≈ 0	1.2	3.0	1.0	0.23	← $N \to \infty$

DQCD : Explaining Values for B_2, B_3, B_4, B_5 from Lattice QCD

(AJB + Gérard, 1804.02401) (ETM15, SWME, RBC-UKQCD)

μ	B_2	B_3	B_4	B_5	$K^0-\bar{K}^0$ mixing
3 GeV	0.49	0.77	0.90	0.65	AJB Lattice Average (± 5%)
Quark Gluon Evolution ⇩					$B_2=B_3=B_4=B_5=1$ Vacuum insertion
1 GeV	0.62	1.10	0.90	0.45	
					} gap : Vector mesons
(0.70) GeV	0.79	0.96	0.83	0.30	
Meson Evolution ⇧					Meson Evolution in the chiral limit
Factorization Scale ≈ 0	1.2	3.0	1.0	0.23	← $N \to \infty$

Support for ε'/ε Anomaly from DQCD

Main Messages from our Papers ★

The inclusion of meson evolution in the phenomenology of any non-leptonic transition like $K^0 - \bar{K}^0$ mixing, $K \to \pi\pi$ decays ($\Delta I = 1/2$ Rule, ε'/ε) is mandatory !

Meson Evolution is hidden in LQCD results but among analytic approaches only DQCD takes this important QCD dynamics into account.

MESON EVOLUTION

The pattern of operator LD mixing found to agree with SD mixing both for SM and BSM operators. ★

2.

ε´/ε in the Standard Model

Review: AJB, 2021.00020

The ε´/ε - Story: 1976 - 2021

Main Actors in ε'/ε in SM

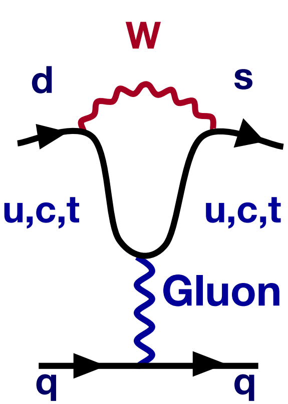
Q_6 – QCD Penguin operator

Q_8 – Electroweak Penguin operator

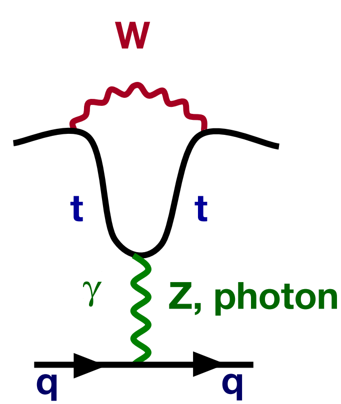
$$Q_6 = \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_q \left(\bar{q}_\beta q_\alpha\right)_{V+A}$$

$$Q_8 = \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_q e_q \left(\bar{q}_\beta q_\alpha\right)_{V+A}$$

$Q_6 =$



$Q_8 =$



Importance of Q_8: (1989)

Flynn + Randall
Buchalla, AJB, Harlander

32nd anniversary of the suppression of ε'/ε by Q_8 at large m_t

QCD and Electroweak Penguin Matrix Elements

1986	BBG strict Large N limit		$B_6^{(1/2)} = B_8^{(3/2)} = 1$	$(\mu \approx 0(m_\pi))$
2015	AJB + Gérard 1507.06326	Including 1/N (meson evolution for B_6, B_8)	$B_6^{(1/2)} < B_8^{(3/2)} < 1$	at $\mu \geq 1\text{GeV}$

$$\langle Q_6(\mu)\rangle_0 = -4\left[\frac{m_K^2}{m_s(\mu)}\right]^2 (F_K - F_\pi) B_6^{(1/2)}(\mu)$$

$$\langle Q_8(\mu)\rangle_2 = \sqrt{2}\left[\frac{m_K^2}{m_s(\mu)}\right]^2 F_\pi B_8^{(3/2)}(\mu)$$

$B_6^{(1/2)}(\mu)$, $B_8^{(3/2)}(\mu)$: very weak μ dependence for $\mu > 1\text{GeV}$
significantly stronger for $\mu < 1\text{GeV}$
through meson evolution

Four dominant contributions to ε′/ε in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)
Aebischer, Bobeth, AJB (2020)

$$\left(\varepsilon'/\varepsilon\right)_{SM} = \mathrm{Im}\left(V_{td}V_{ts}^{*}\right)\left[\mathrm{QCDP} - \mathrm{EWP}\right]$$

$$\mathrm{QCDP} = \left(1-\hat{\Omega}_{eff}\right)\left[-2.9 + 15.4 \cdot B_6^{(1/2)}\right]$$

(V-A) ⊗ (V-A)
QCD Penguins

(V-A) ⊗ (V+A)
QCD Penguins

$$\mathrm{EWP} = \left[-2.0 + 8.0 \cdot B_8^{(1/2)}\right]$$

(V-A) ⊗ (V-A)
EW Penguins

(V-A) ⊗ (V+A)
EW Penguins

$\hat{\Omega}_{eff} = 0$
in present
RBC-UKQCD

(isospin breaking)

(η-η′ mixing)
in L_7

DQCD

(Nonet) (η-η′ mixing)

$$\hat{\Omega}_{eff}^{(9)} = \left(29 \pm 7\right) \cdot 10^{-2}$$

AJB, Gérard (2020)

(Explicit Octet)

$$\hat{\Omega}_{eff}^{(8)} = \left(17 \pm 9\right) \cdot 10^{-2}$$

ChPT

Cirigliano et al. (2019)

Estimates of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ (2021)

At μ = 1GeV

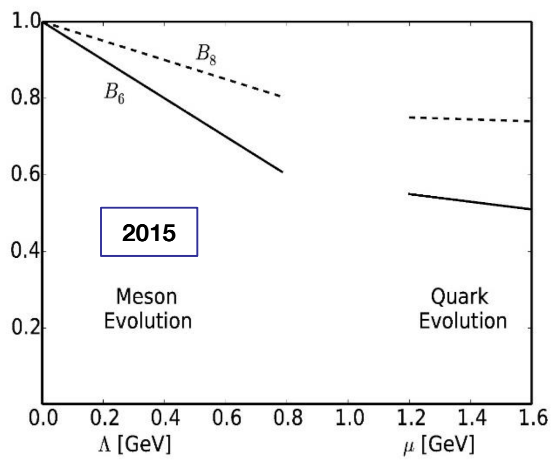
$B_6^{(1/2)} \leq 0.6$ $B_8^{(3/2)} = 0.80 \pm 0.10$ (DCQD-2015)

$B_6^{(1/2)} = 1.49 \pm 0.25$ $B_8^{(3/2)} = 0.85 \pm 0.05$ (RBC-UKQCD-2020)

$B_6^{(1/2)} = 1.35 \pm 0.20$ $B_8^{(3/2)} = 0.55 \pm 0.20$ (ChPT-2019)

Scale Dependence of B_6 and B_8

AjB+ Gerard (1507.06326)



Additional Messages

1. NNLO Corrections to **electroweak penguins**
AJB, Gambino, Haisch (1998)

$$\Delta(\varepsilon'/\varepsilon) \approx -1.3 \cdot 10^{-4}$$

Remove large renormalization scheme dependence from NLO + scale dependence in $m_t(\mu)$

2. NNLO Corrections to **QCD penguins**
Maria Cerda-Sevilla, Martin Gorbahn, Sebastian Jäger, Ahmet Kokulu

$$\Delta(\varepsilon'/\varepsilon) \approx -(1-2) \cdot 10^{-4}$$

(2021?)

3. Meson Evolution suppressing ε'/ε more important than enhancement through FSI (claimed by Pallante + Pich)
Problem in separating Q_2-Q_1 (current-current) from Q_6 in CHPT. The same problem in Gisbert + Pich (1712.06147)

AJB + Gérard (1603.05686)

Good News on ε'/ε

ε'/ε = QCD Penguins – Electroweak Penguin

$$\left(\frac{\varepsilon'}{\varepsilon}\right)^{EWP}_{SM} = -(7 \pm 1)\cdot 10^{-4} \quad \text{(RBC – UKQCD and DQCD)}$$

Perfect Agreement!

Chiral Pert Th: $\approx (-3.5 \pm 2.0) \cdot 10^{-4}$

Disagreements on QCD Penguin contribution.

RBC-UKQCD $\approx 28 \cdot 10^{-4}$

ChPT $\approx 18 \cdot 10^{-4}$

DQCD $\approx 12 \cdot 10^{-4}$

Hopefully clarified in this decade!

Important Message for Non-Experts to take Home

RBC-UKQCD collaboration and ChPT Experts do not claim that there is no New Physics in ε'/ε. But as of 2021 their methods are not sufficiently powerful to see an anomaly in ε'/ε.

Dual QCD approach, even if approximate, can much faster see the underlying dynamics, even analytically. Both in ε'/ε and the $\Delta I = 1/2$ rule !!

Main Dynamics responsible for ε'/ε Anomaly

	DQCD	RBC-UKQCD	ChPT
Large m_t	✓	✓	✓
Meson Evolution	✓	✓	-
Enhancement of EWP at NNLO	✓	-	-
Suppression of QCDP at NNLO	✓	-	-
Suppression of QCDP by IB (Octet)	✓	-	✓
Suppression of QCDP by η-η'mixing (Nonet)	✓	-	L_7 ?

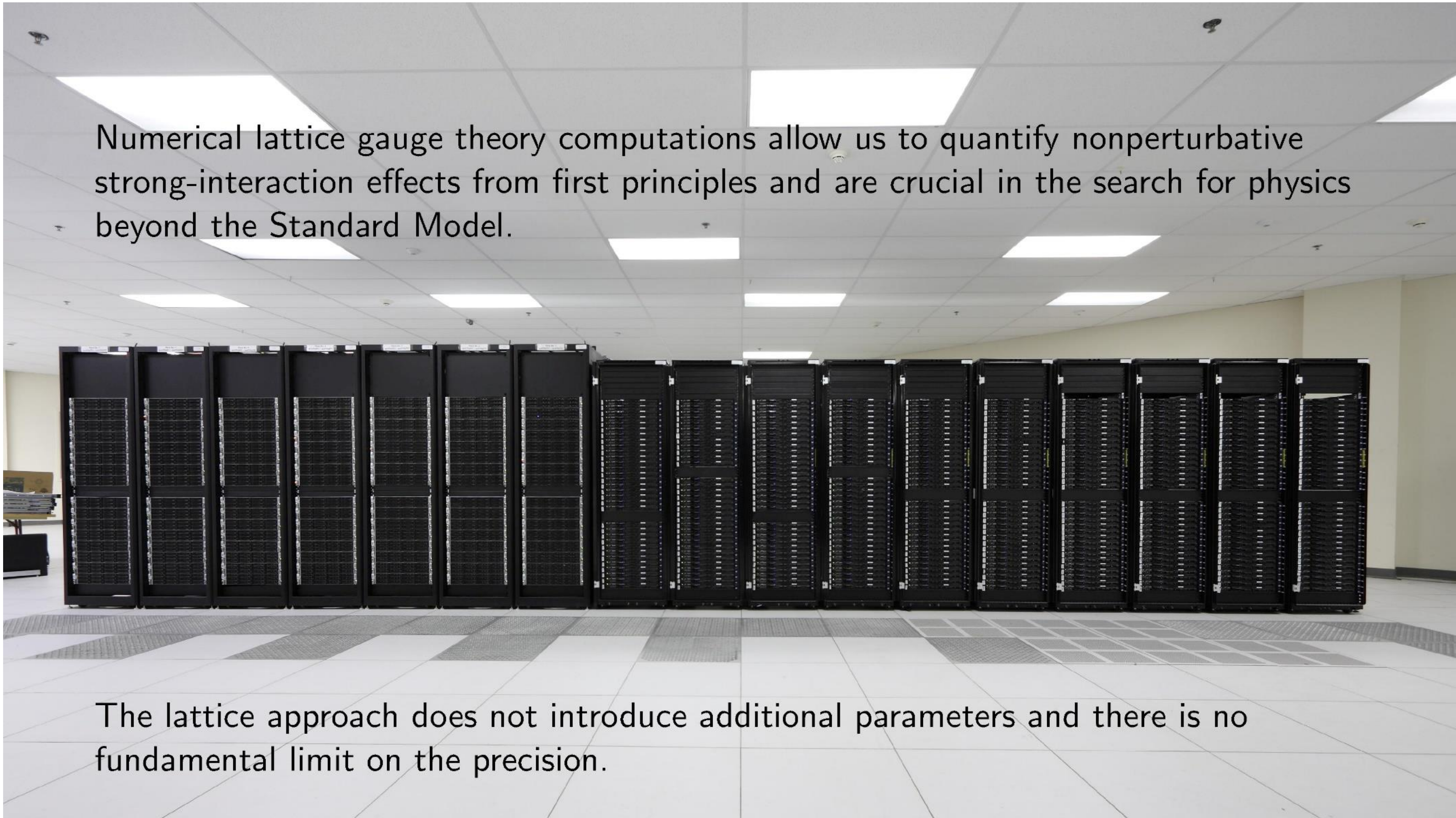
Important: **The enhancement by FSI:**

DQCD $$\left(\frac{\varepsilon'}{\varepsilon}\right)_{SM} = (5 \pm 2) \cdot 10^{-4}$$

Included for FSI ≤ Meson Evolution suppression

AJB + Gérard: 1603.05686

Lattice QCD (from Stefan Meinel)



Numerical lattice gauge theory computations allow us to quantify nonperturbative strong-interaction effects from first principles and are crucial in the search for physics beyond the Standard Model.

The lattice approach does not introduce additional parameters and there is no fundamental limit on the precision.

(2021) Age Sum Rule

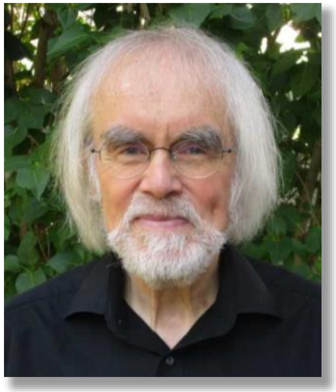
Accurate to 1‰ !

N. Christ

+

C. Sachrajda

$$\frac{\text{N. Christ} + \text{C. Sachrajda}}{2} = \text{AJB} = \boxed{75}$$



AJB

A. Pich

(2021) **Age Sum Rule**

Accurate to 1‰ !

N. Christ

+

C. Sachrajda

$$\frac{\text{N. Christ} + \text{C. Sachrajda}}{2} = \text{AJB} = \boxed{75}$$

AJB

A. Pich

★ **Good health for us 4 the full decade**

3.

ε'/ε Beyond SM

(NLO)

Status of ε′/ε in the SM before February 2020

RBC-UKQCD (1505.07863)	$(\varepsilon'/\varepsilon)_{SM} = (1.4 \pm 6.9)\cdot 10^{-4}$	No isospin breaking correction (IB)
AJB, Gorbahn, Jäger Jamin (1507.06345)	$(\varepsilon'/\varepsilon)_{SM} = (1.9 \pm 4.5)\cdot 10^{-4}$	Lattice results + IB
AJB + Gérard (1507.06326)	$(\varepsilon'/\varepsilon)_{SM} < (6.0 \pm 2.4)\cdot 10^{-4}$	Dual QCD bound
Kitahara, Nierste, Tremper (1607.06727)	$(\varepsilon'/\varepsilon)_{SM} = (1.1 \pm 5.1)\cdot 10^{-4}$	Lattice results + IB
Gisbert, Pich (1712.06147) (1912.04736)	$(\varepsilon'/\varepsilon)_{SM} = (14 \pm 5)\cdot 10^{-4}$	Chiral Pert. Th. (No meson evolution!!) but FSI
Experiment (NA48, KTeV)	$(\varepsilon'/\varepsilon)^{exp} = (16.6 \pm 2.3)\cdot 10^{-4}$	

NP Models and ε´/ ε Anomaly

(Only SM operators)

Dominated by Q_8

Littlest Higgs (T parity)	Blanke, AJB, Recksiegel (1507.06316)
Z-FCNC	AJB (1601.00005), Bobeth, AJB, Celis, Jung (1703.04753) Endo, Kitahara, Mishima, Yamamoto (1612.08839)
Z´-Models	AJB (1601.00005), AJB, Buttazzo, Knegjens (1507.08672)
331- Models	AJB, De Fazio (1512.02869, 1604.02344)
Vector-Like Quarks	Bobeth, AJB, Celis, Jung (1609.04783)
SUSY	Tanimoto, Yamamoto (1603.07960) Kitahara, Nierste, Tremper (1604.07400) Endo, Mishima, Ueda, Yamamoto (1608.01444) Crivellin, D'Ambrosio, Kitahara, Nierste (1703.05786) Endo, Goto, Kitahara, Mishima, Ueda, Yamamoto (1712.04959)
Right-handed Currents	Cirigliano, Dekens, De Vries, Meraghetti (1703.04751)
$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$	Haba, Umeeda, Yamada (1802.09903, 1806.03424)
Leptoquark Models	Bobeth, AJB (1712.01295)

NP Models and ε'/ε Anomaly continued

2HDM Chen, Nomura (1804.06017, 1805.07522)

SU(8) Matsuzaki, Nishiwaki, Yamamoto (1806.02312)

Diquarks Chen, Nomura (1808.04097) (1811.02315)

Most papers address correlations with $K \to \pi\nu\bar{\nu}$ and EDMS.

Correlation with tensions in $B \to \pi K$ (LHCb) through $U(2)^3$ flavour symmetry

Crivellin, Gross, Pokorski, Vernazza (1909.02101)

2018 Results in DQCD

: BSM hadronic Matrix elements

1. Matrix elements of chromomagnetic penguins
AJB + Gérard 1803.08052 (First on-shell $K\rightarrow\pi\pi$ calculation to date)

Confirmation of $K\rightarrow\pi$ matrix element by ETM collaboration 1712.09824

$B_{CMO} \approx 1/3$

Much smaller than early estimates in chiral quark model

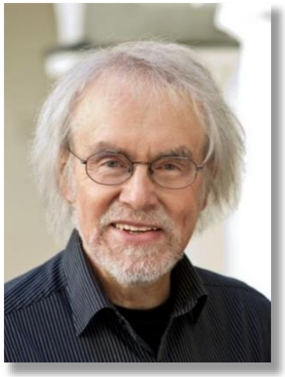
2. Insight into BSM B_i parameters ($K^0 - \bar{K}^0$ Mixing) obtained by Lattice QCD 1804.02401 (AJB + Gérard)

Meson Evolution

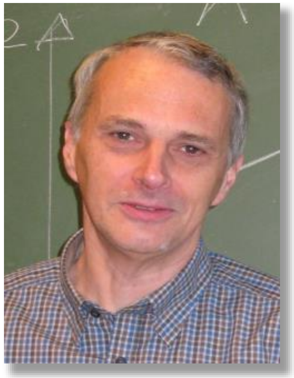
(see 1. Movement)

3. $K \rightarrow \pi\pi$ matrix elements of all BSM 4-quark operators

Jason Aebischer



AJB



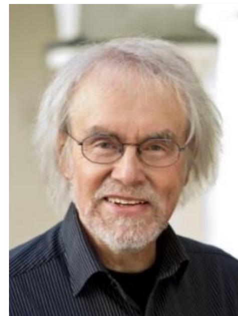
J.-M. Gérard

(1807.01709)

4. Master Formula for ε'/ε Beyond SM (1807.02520)

Jason Aebischer

Christoph Bobeth



AJB

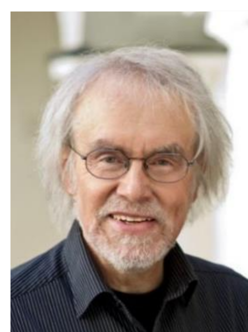
Jean-Marc Gérard

David Straub

5. Anatomy of ε'/ε Beyond SM (1808.00466) First SMEFT analysis

Jason Aebischer

Christoph Bobeth



AJB

David Straub

All Dimension 6 BSM Four-Quark Operators (linearly independent)

ABG (1807.01709) ABBS (1808.00466)

QCD x QED invariant

SM : 7

BSM : 33 = 7´ + 13 + 13´

Obtained from SM : L ↔ R Matrix Elements reverse sign (→ 7´)

Operators unrelated to SM operators (→ 13 + 13´)

Matrix Elements only calculated in DQCD

SMEFT invariant under SM gauge Group

BSM : 21 = 7´ + 7 + 7´

See Backup for the list of operators

Master Formula for ε′/ε Beyond SM

$$\left(\frac{\varepsilon'}{\varepsilon}\right) = \left(\frac{\varepsilon'}{\varepsilon}\right)_{SM} + \left(\frac{\varepsilon'}{\varepsilon}\right)_{NP}$$

Valid in ANY extension of SM

★
$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{NP} = \sum_i P_i(\mu_{ew}) \,\mathrm{Im}\left[C_i(\mu_{ew}) - C_i'(\mu_{ew})\right]$$ (LO)

i = 1,…40 4-quark operators + 1 dipole operator $(P_L \leftrightarrow P_R)$

Model independent: $P_i(\mu_{ew})$ - Include hadronic elements + renormalization group effects from $\mu \approx 0$ (1 GeV) to $\mu_{ew} \approx 0$ (m_t)

All listed in

J. Aebischer, C. Bobeth, AJB, J.-M. Gérard, D. Straub
1807.02520

Islands of K $\rightarrow\pi\pi$ Matrix Elements

ABBS (1808.00466) ABBGS (1808.02520) ABG (1807.01709)

SM Operators

Lattice
DQCD

BSM
Four-Quark Operators

DQCD

Chromomagentic
Penguins

DQCD
Lattice

Conquered in 2018

Useful paper:
de Blas, Criado, Perez-Victoria, Santiago (1711.10391)

Most important for ε'/ε Anomaly

Left-Right Operators

Z´, W´, G´

Vector-like Quarks

Scalar and Tensor Operators

Heavy Scalars

Being presently explored

BSM anatomy of ε'/ε: Aebischer, Bobeth, AJB, Straub (1808.00466) SMEFT analysis

Leptoquarks cannot explain this anomaly because of bounds from rare Kaon decays (Bobeth, AJB, 1712.01295)

Basically only U_1 model survives

but only if LH and RH couplings present

(see details in back up)

General non-leptonic $\Delta F = 1$ WET at NLO in QCD
(2107.10262)

J. Aebischer, C. Bobeth, AJB, J. Kumar, M. Misiak

$$\vec{C}_{BMU}(\mu_{had}) = \underbrace{\hat{U}_{BMU}(\mu_{had},\mu_{ew})\hat{M}_{JMS}}_{\text{WET Operators}}(\mu_{ew})\vec{C}_{JMS}(\mu_{ew})$$

BMU basis: useful for QCD RG evolution
(AJB, Misiak, Urban (2000))

JMS basis: useful for matching to SMEFT
(Jenkins, Manohar, Stoffer)
(Dekens, Stoffer)

$\hat{M}_{JMS}$ = Matching of JMS on BMU
Careful treatment of Evanescent Operator required

Tables of

$$\left[\hat{U}_{BMU}(\mu_{had},\mu_{ew})\hat{M}_{JMS}\right]_{ab}$$

$a = \text{BMU} \qquad b = \text{JMS}$


J. Kumar


M. Misiak

BSM Master Formula for ε'/ε in the WET Basis at NLO in QCD
(2107.12391)

J. Aebischer, C. Bobeth, AJB, J. Kumar

*)

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{BSM}} = \sum_b P_b(\mu_{ew})\,\text{Im}\left[C_b(\mu_{ew}) - C'_b(\mu_{ew})\right]\cdot(1\,\text{TeV})^2$$

$P_b(\mu_{ew})$ calculated in NLO QCD in JMS basis (important step towards NLO QCD analysis of ε'/ε in SMEFT)

SM Hadronic matrix element: LQCD
BSM Hadronic matrix element: DQCD

*) Generalization of 2018 LO master formula [1807.01709, 1807.02520]

Links between $(\varepsilon'/\varepsilon)_{BSM}$ and $\left(K \to \pi\nu\bar{\nu}\right)_{BSM}$

AJB: 1601.00005; 1805.11096

Aebischer, Bobeth, AJB, Straub: 1808.00466

See Backup

Strategy (Z´)

AJB (1601.00005)

$(\varepsilon'/\varepsilon)^{NP} = \kappa_{\varepsilon'} \cdot 10^{-3}$

$0.5 \leq \kappa_{\varepsilon'} \leq 1.5$

(Im)

$\varepsilon_K^{NP} = \kappa_\varepsilon \cdot 10^{-3}$

$0.1 \leq \kappa_\varepsilon \leq 0.4$

(Im, Re)

ε_K more important than $K_L \to \mu^+\mu^-$ in Z´ models

Re and Im Parts: Z´ Couplings

$\Delta_L^{sd}(Z'), \quad \Delta_R^{sd}(Z')$

$K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, ΔM_K

(Re, Im) (Im) (Re) (Im, Re)

Functions of $\kappa_{\varepsilon'}, \kappa_\varepsilon$

4.

Summary

Main Homework for Coming Years

RBC-UKQCD :

a) Isospin breaking and QED Corrections (including η-η' mixing)
b) Inclusion of charm

ChPT :

a) Matching to short distance (L_5)
b) Better inclusion of η-η' mixing (L_7)

DQCD :

Final state interactions

Inclusion of NNLO QCD to QCD Penguins

BSM hadronic matrix elements from LQCD

known only from DQCD

Contributions from other LQCD Groups

Ishizuka et al 1809.03893; Hernandez et al. 2003.10293

$\Delta I = 1/2$ Rule

$$R_{exp} = \frac{A\left(K \to (\pi\pi)_{I=0}\right)}{A\left(K \to (\pi\pi)_{I=2}\right)} = 22.4$$

Puzzle since
1954 (Gell-Mann + Pais)

$R_{th} = \sqrt{2}$ (without QCD)

1986–2014: $R = 16 \pm 2$ — Dual QCD — Bardeen, AJB, Gérard

Current-Current, not QCDP

2020: $R = 19.19 \pm 4.8$ — RBC-UKQCD Lattice Collaboration

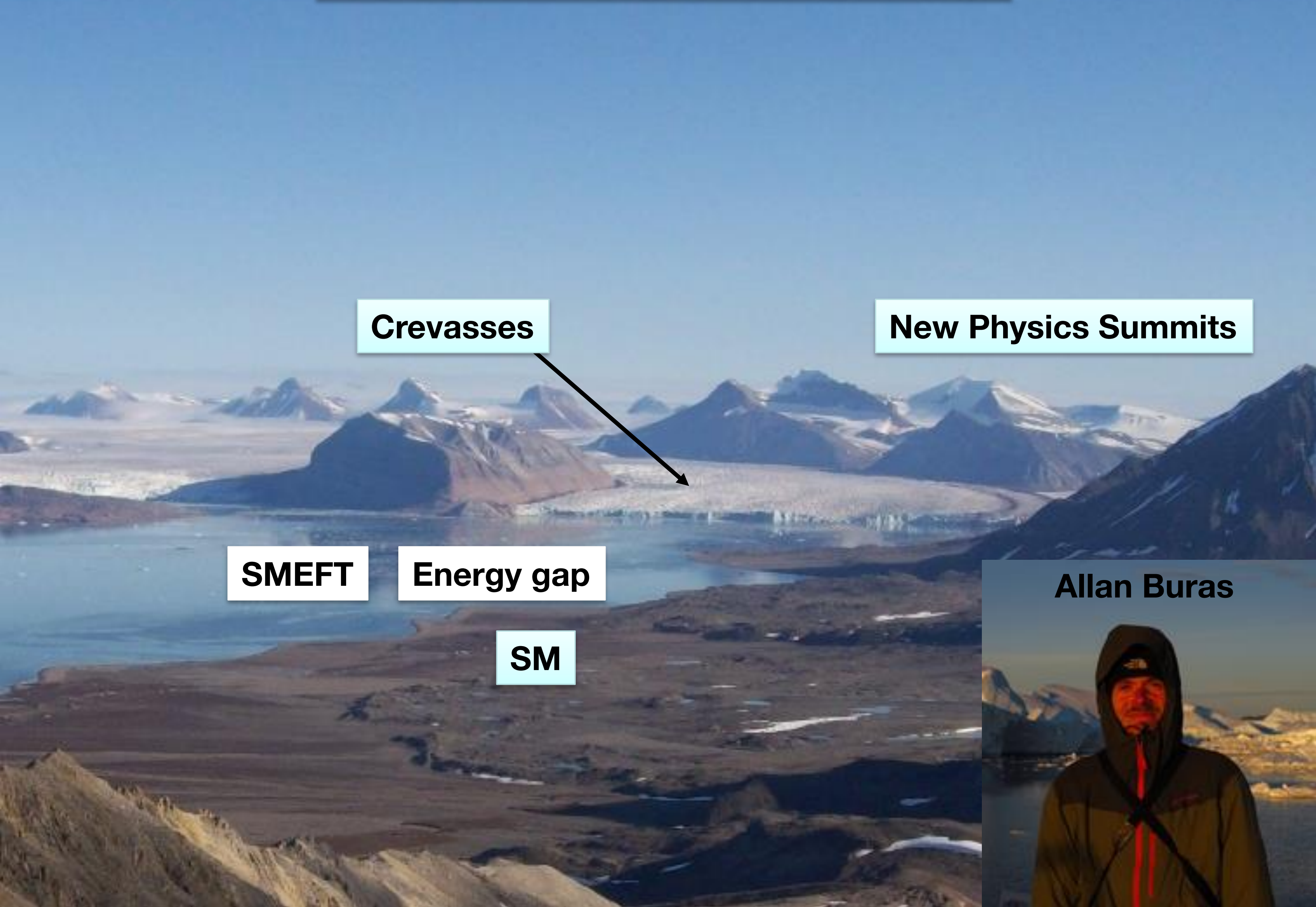
QCD dynamics dominate this rule
but New Physics could still contribute

AJB
F. de Fazio
J. Girrbach-Noe
(1404.3824)

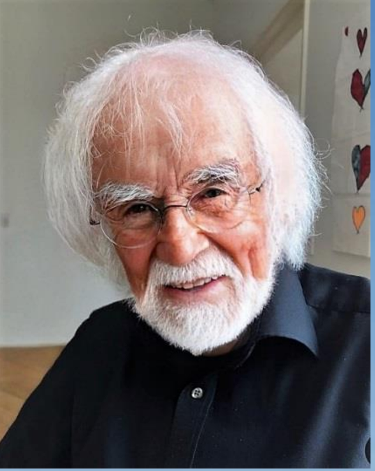
Note: Relative to no QCD case must
enhance A_0 by 7.5
suppress A_2 by 2.1

Hep-arxiv: 2101.00020

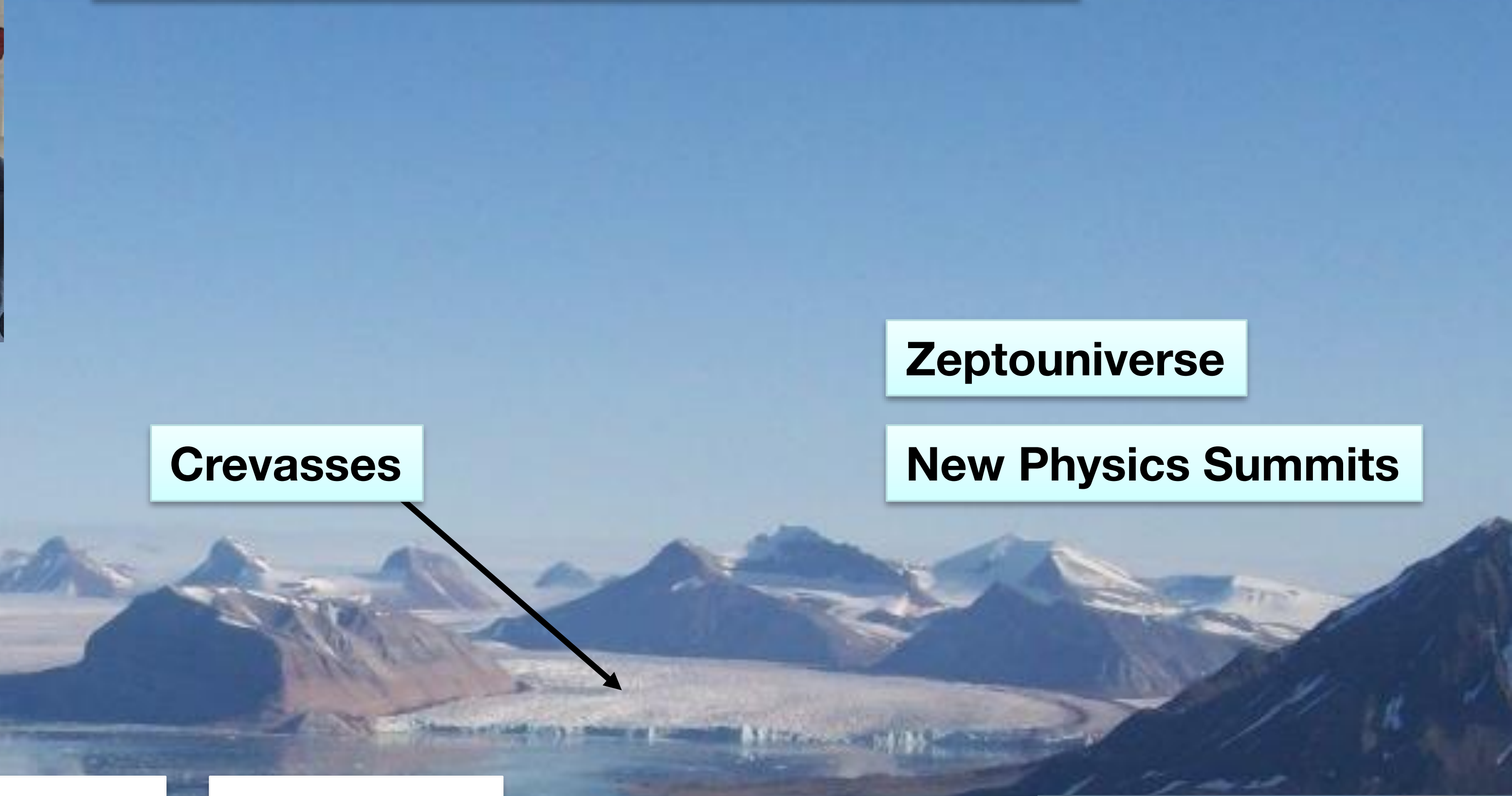
Flavour Physics (2020 -)



Flavour Physics (2020 -)



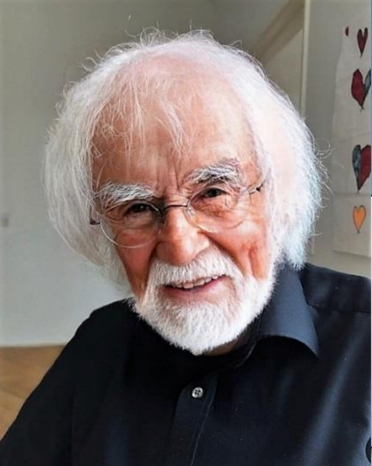
(KAON 2031)



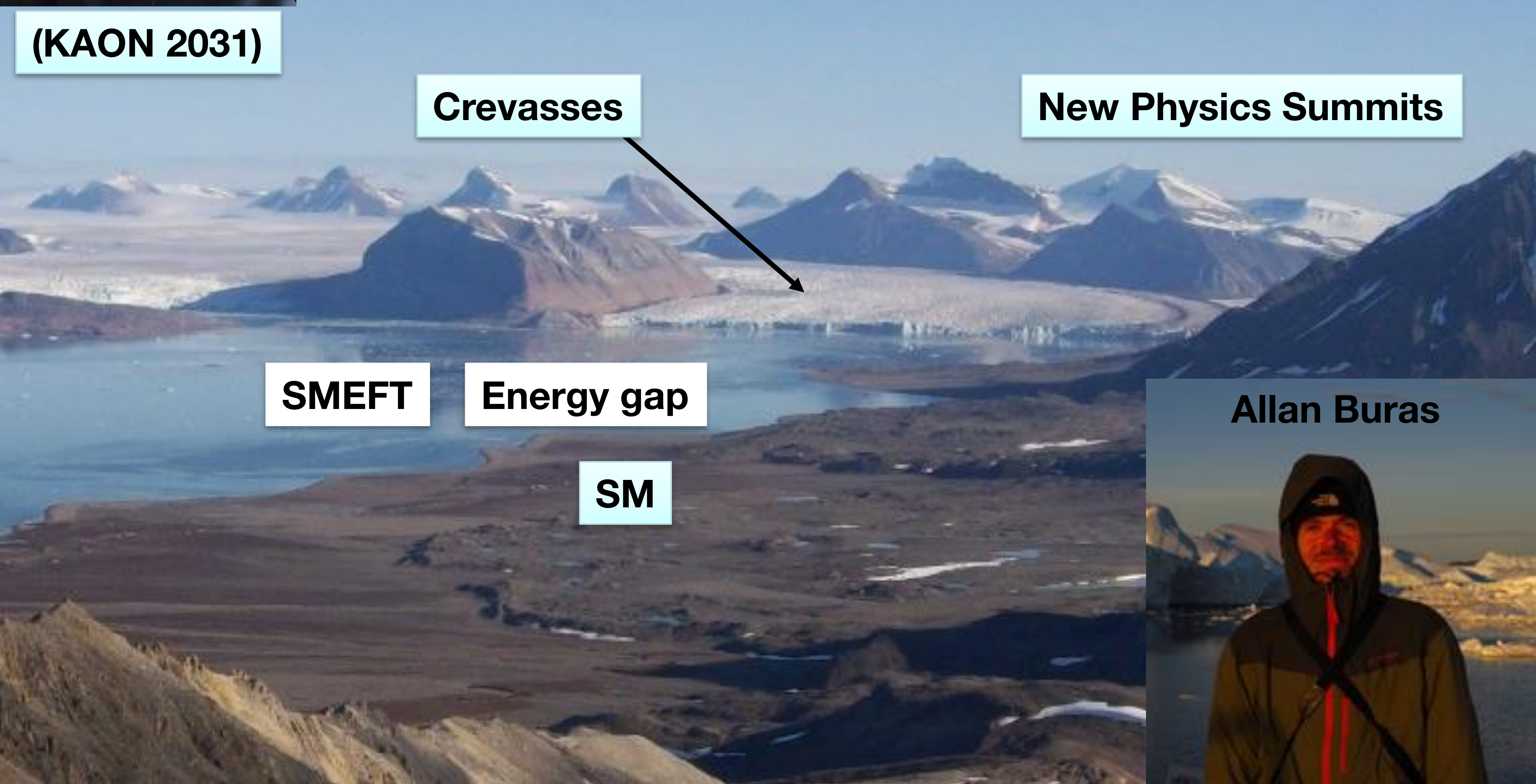
Allan Buras

Flavour Physics (2020 -)

Thank You !



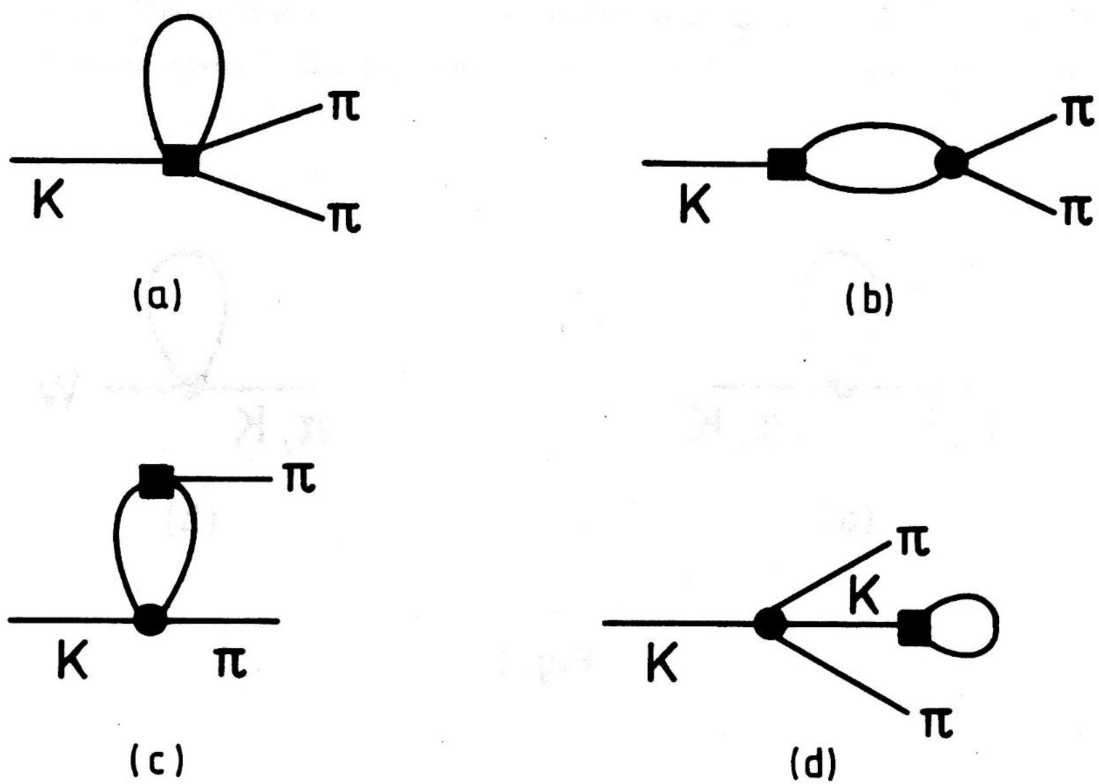
(KAON 2031)



Backup

Meson Evolution

Loops with a physical cutt-off Λ : 1/N non-factorizable contributions



■ weak

● strong

Very different philosophy from Chiral PTh

No dimensional regularisation !!!

QCD and Electroweak Penguin Matrix Elements (DQCD)

1986 BBG strict Large N limit $B_6^{1/2} = B_8^{3/2} = 1$ $\left(\mu \approx 0(m_\pi)\right)$

2015 AJB + Gérard 1507.06326 Including 1/N (meson evolution for B_6, B_8) $B_6^{1/2} < B_8^{3/2} < 1$ at $\mu \geq 1\,\text{GeV}$

$$B_6^{(1/2)} = 1 - 0.66\ \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_6^2}\right) \Rightarrow B_6^{(1/2)} < 0.54$$

$$B_8^{(3/2)} = 1 - 0.17\ \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right) \Rightarrow B_8^{(3/2)} \approx 0.8 \pm 0.1$$

RBC-UKQCD

$$B_6^{(1/2)} = 0.57 \pm 0.19$$

$$B_8^{(3/2)} = 0.76 \pm 0.05$$

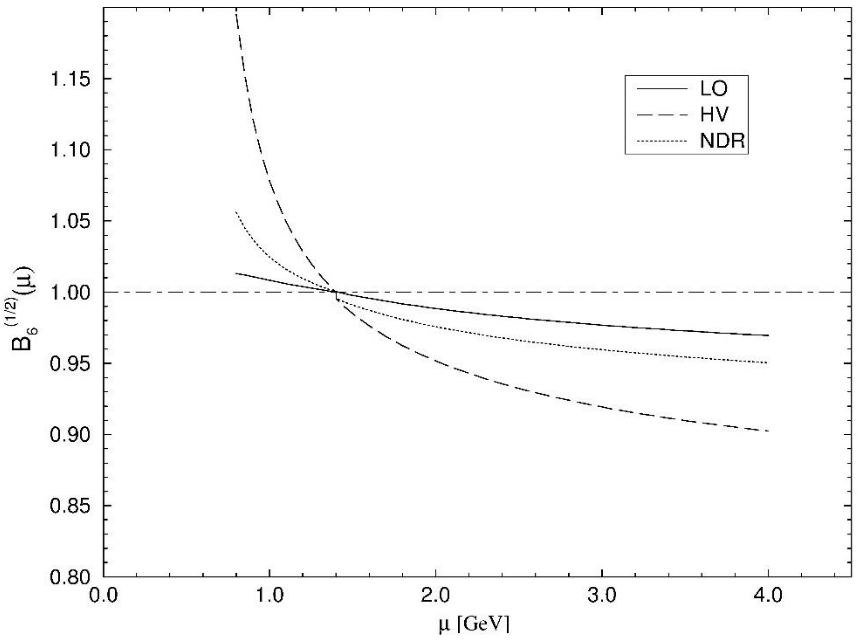
$\tilde{m}_{6,8} < \Lambda$

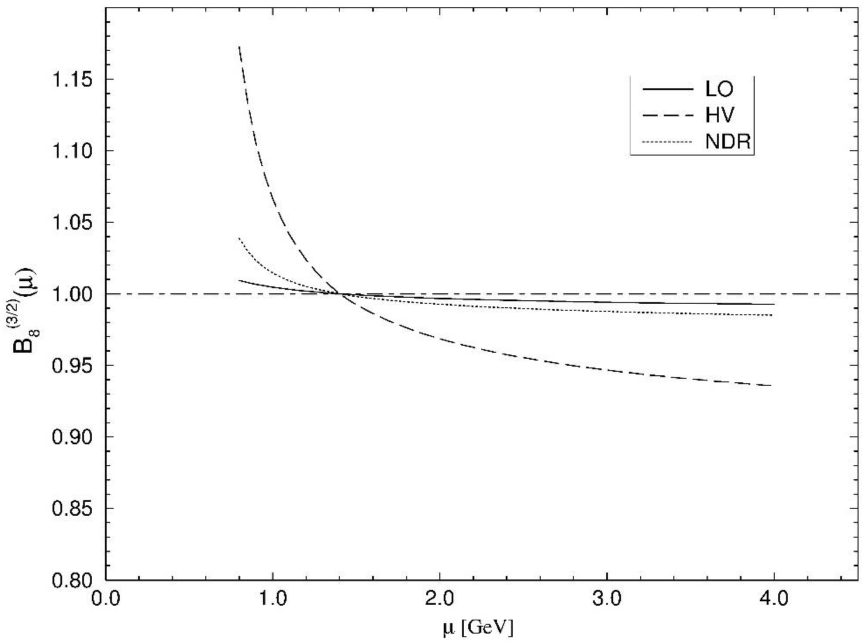
Λ = physical cut-off

$$(\varepsilon'/\varepsilon)_{SM} < (6.0 \pm 2.4) \cdot 10^{-4}$$ (2015)

B_6 and B_8 in the Perturbative Regime (1993!)

AJB, Jamin, Lautenbacher, (9303284)





B_6 and B_8 decrease with increasing μ!

Note $B_6 = B_8 = 1$ at $\mu = m_c$ wrong!!

Dual QCD Approach

Bardeen, AJB, Gérard

High Energy Scale

Wilson Coefficients

Standard Renormalization Group Evolution within Quark + Gluon Phase

Short-Distances

0 (1 GeV)

Hadronic Matrix Elements

Evolution within Meson Phase of QCD (Meson Evolution)

Long-Distance Scales

Large N Limit

Factorization Scale

The only analytic approach allowing matching of short distance and long-distance contributions

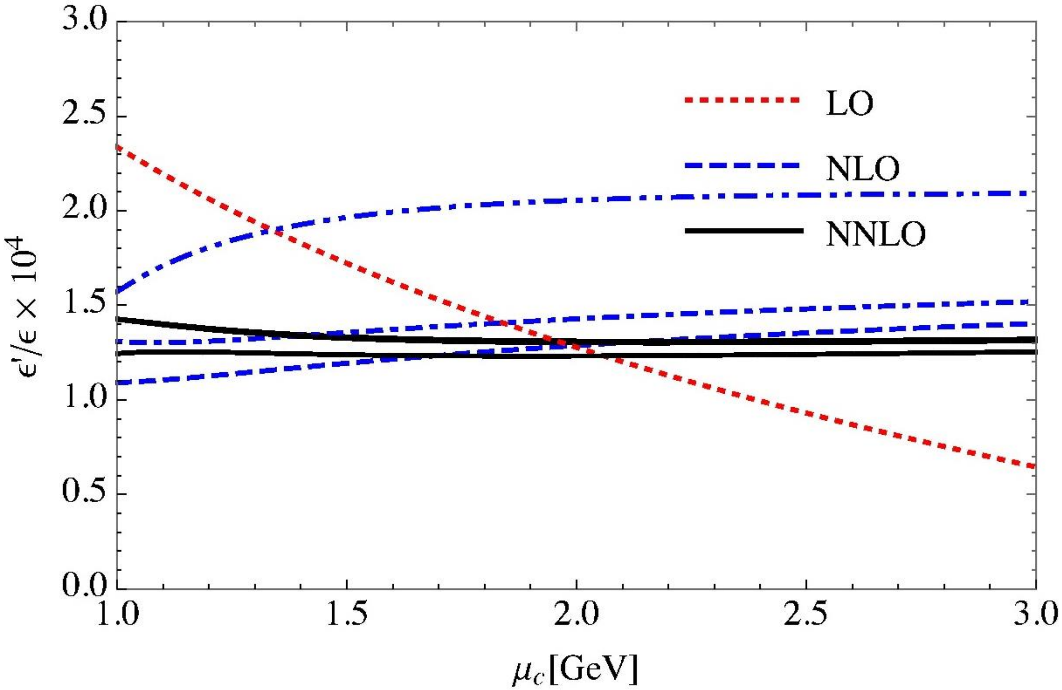
AJB, Gérard 2016

Meson evolution (hidden in lattice QCD) is crucial strong dynamics responsible for $\Delta I=1/2$ rule, ε'/ε, ε, $K\to\pi\pi$ in general

First QCDP NNLO Result for $(\varepsilon'/\varepsilon)_{SM}$

2018

(preliminary) **1611.08276 + 2 talks of Maria**



Lattice values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$

Maria Cerda-Sevilla

Martin Gorbahn

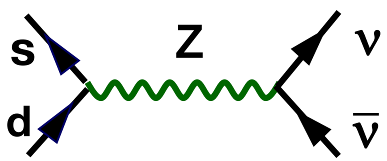
Sebastian Jäger

Ahmet Kokulu

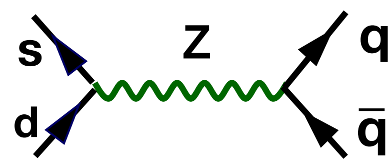
Links between $(\varepsilon'/\varepsilon)_{BSM}$ and $\left(K \to \pi\nu\bar{\nu}\right)_{BSM}$

With few exceptions these links are not direct unless one makes specific assumptions about flavour diagonal couplings:

Examples

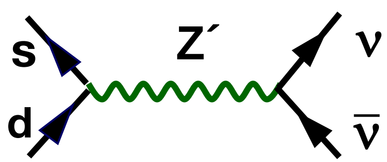


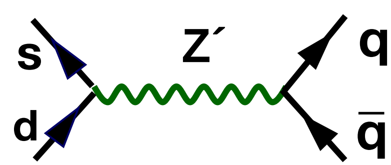
$K \to \pi\nu\bar{\nu}$



ε'/ε

$Z\nu\bar{\nu}$ $Zq\bar{q}$ known





$Z'\nu\bar{\nu}$ $Z'q\bar{q}$ arbitrary and unrelated to each other

In loop induced decays concrete models are required, but often in view of many parameters no strict relation.

(MSSM, L-R symmetric models)

Induced Z-mediated FCNCs

LH FCNCs

Enhancement of ε'/ε implies suppression of $K_L \to \pi^0 \nu\bar{\nu}$

$$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \leq 2\ \mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)_{\mathrm{SM}}$$

($K_L \to \mu\bar{\mu}$ bound)

RH FCNCs

Suppression of $K_L \to \pi^0 \nu\bar{\nu}$

$$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \leq 1.5\ \mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)_{\mathrm{SM}}$$

(ε_K bound SMEFT)

LH+RH FCNCs

$K_L \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$ can be both enhanced if necessary (no definite prediction)

Literature:

AJB (1601.00005), Bobeth, AJB, Celis, Jung (1703.04753)
Endo, Kitahara, Mishima, Yamamoto (1612.08839)

ε'/ε and rare K Processes in Leptoquark Models

Bobeth, AJB (1712.01295)

1. Assuming that the upper bound on $(\varepsilon'/\varepsilon)_{SM}$ from Dual QCD is correct: Largest anomaly!

2. But in contrast to R_D, R_{D^*} (LQs contribute there at tree level) in ε'/ε leptoquarks contribute at one-loop (RG running and box contributions)

   ⇨ **Large Im(Y) couplings required**

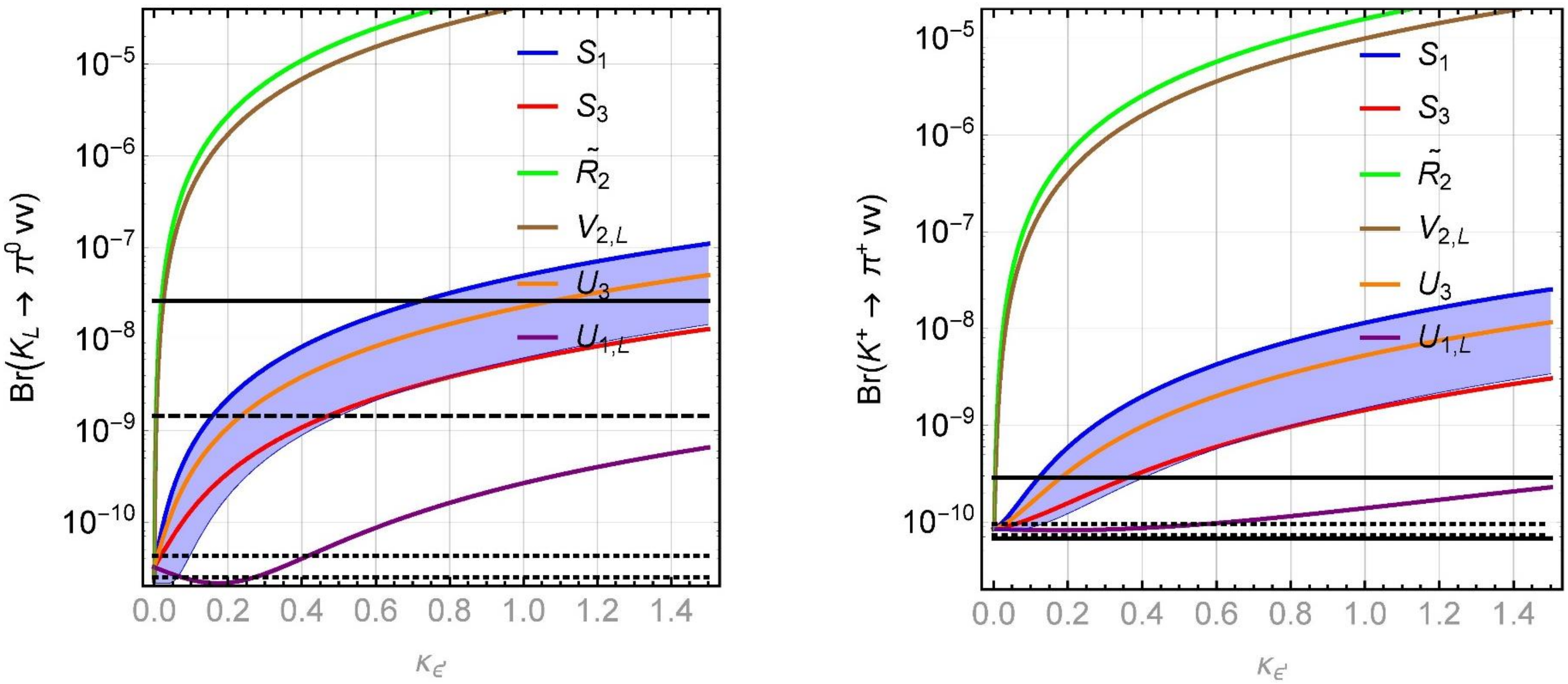
3. Problems with rare decays

   $K \to \pi\nu\bar{\nu}$, $K_L \to \pi^0 l^+ l^-$, $K_S \to \mu^+\mu^-$ (tree – level)

   but also ΔM_K, ε_K **Basically only U_1 model survives**

but only if LH and RH couplings present ★

Leptoquarks facing ε'/ε and $K \to \pi\nu\bar{\nu}$



Exp. Bound

Grossman-Nir Bound

U_1

$$\left(\varepsilon'/\varepsilon\right)^{NP} = \kappa_{\varepsilon'} \cdot 10^{-3}$$

$$0.5 \leq \kappa_{\varepsilon'} \leq 1.5$$

U_1 Model meets ε'/ε and rare K Decays

1. Generation of Q_8 through RG group! +

2. No tree-level contributions to $K \to \pi\nu\bar{\nu}$, generated through RG but still consistent with bounds even for $\kappa_{\varepsilon'} \approx 1.0$ +

3. If only left-handed or right-handed couplings present ruled out through

$$\left(\begin{array}{l} K_L \to \pi^0 e^+ e^-,\ K_L \to \pi\mu^+\mu^+, \\ K_S \to \mu^+\mu^- \end{array} \right)$$

(the only hope: couplings between τ and d, s) -

4. Box contributions with left- and right-handed couplings could help but UV completion needed to do the calculation. Would also generate LR contributions to ΔM_K, ε_K : very dangerous! ?

Main Messages on LQs in ε'/ε and rare K Decays

If improved lattice calculations will confirm the ε'/ε anomaly at the level $(\varepsilon'/\varepsilon)_{NP} \geq 5 \cdot 10^{-4}$ LQs are likely not responsible for it.

But if ε'/ε anomaly disappears large NP effects from LQs in rare K decays still possible.

(Need non-zero couplings to first generation!!)
(Need imaginary couplings!)
(Need both left-handed and right-handed couplings!)

In contrast to most explanations of B-anomalies

RG Effects: $\varepsilon'/\varepsilon \leftrightarrow K \rightarrow \pi\nu\bar{\nu}$

(1808.00466)

In both directions governed by QED and EW effects

(Model independent)

1. **Large enhancement of ε'/ε does not imply large effects in $K \rightarrow \pi\nu\bar{\nu}$**

2. **Large enhancements of $K \rightarrow \pi\nu\bar{\nu}$ do not imply large enhancement of ε'/ε (Problem for leptoquark models)**

ε'/ε vs ΔM_K

Dream

Lattice 2019

RBC - UKQCD

ABB (2019) $\left(\frac{\varepsilon'}{\varepsilon}\right)_{SM} = (5.5 \pm 2.4)\cdot 10^{-4}$

$(\Delta M_K)_{SM} = (7.7 \pm 2.1)\cdot 10^{-15}\,\text{GeV}$

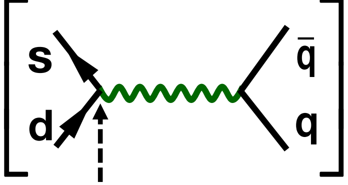
NP has to enhance ε'/ε

NP has to suppress ΔM_K

$(\Delta M_K)_{exp} = 3.5\cdot 10^{-15}\,\text{GeV}$

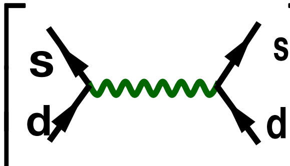
AJB 1601.00005

$\left(\frac{\varepsilon'}{\varepsilon}\right)_{NP} = \text{Im}\,[\ldots]$

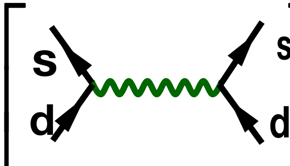


$g = \text{Re}\,g + i\,\text{Im}\,g$

$(\Delta M_K)^{NP} = \text{Re}\,[\ldots] \sim [\text{Re}\,g]^2 - [\text{Im}\,g]^2$



$\varepsilon_K = \text{Im}\,[\ldots] \sim \text{Re}\,g \cdot \text{Im}\,g$



Need sufficiently large Im g

To keep ε_K under control in the presence of large Im g:

$$[\text{Im}\,g \gg \text{Re}\,g] \Rightarrow (\Delta M_K)^{NP} < 0$$

Operators with Largest P_i

ABBGS (1807.02520)

(Most efficient in explaining ε′/ε anomaly)

(+ $P_L \leftrightarrow P_R$)

$$O^u_{VLR} = (\bar{s}^i \gamma_\mu P_L d^i)(\bar{u}^j \gamma^\mu P_R u^j)$$

$$\tilde{O}^u_{VLR} = (\bar{s}^i \gamma_\mu P_L d^j)(\bar{u}^j \gamma^\mu P_R u^i)$$

$$O^d_{VLR} = (\bar{s}^i \gamma_\mu P_L d^i)(\bar{d}^j \gamma^\mu P_R d^j)$$

$$\tilde{O}^d_{VLR} = (\bar{s}^i \gamma_\mu P_L d^j)(\bar{d}^j \gamma^\mu P_R d^i)$$

Present already in SM

(i, j = 1,2,3, colour) (generate Q_6, Q_8)

$$O^u_{TLL} = (\bar{s}^i \sigma_{\mu\nu} P_L d^i)(\bar{u}^j \sigma^{\mu\nu} P_L u^j)$$

$$\tilde{O}^u_{TLL} = (\bar{s}^i \sigma_{\mu\nu} P_L d^j)(\bar{u}^j \sigma^{\mu\nu} P_L u^i)$$

$$O^d_{TLL} = (\bar{s}^i \sigma_{\mu\nu} P_L d^i)(\bar{d}^j \sigma^{\mu\nu} P_L d^j)$$

$$O^u_{SLR} = (\bar{s}^i P_L d^i)(\bar{u}^j P_R u^j)$$

New Operators
(related to scalar-scalar
operators by Fierz identities)

Forbidden in
SMEFT = $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
Allowed by $SU(3)_C \otimes U(1)_Q$

Basic Formula for ε'/ε

$$\varepsilon'/\varepsilon = -\frac{\omega}{\sqrt{2}|\varepsilon|}\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\left[1-\frac{1}{\omega}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0}\right] \qquad \omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \approx \frac{1}{22}$$

$$= -\frac{\omega}{\sqrt{2}|\varepsilon|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\left(1-\hat{\Omega}_{\rm eff}\right) - \frac{\left(\mathrm{Im}A_2\right)^{\rm EWP}}{\mathrm{Re}A_2}\right] \qquad \left(\Delta I = 1/2 \text{ rule}\right)$$

$$\hat{\Omega}_{\rm eff} = \frac{1}{\omega}\frac{\left(\mathrm{Im}A_2\right)^{\rm IB}}{\mathrm{Re}A_0} + \text{ subleading QED corrections}$$

$$\mathrm{Im}A_0 = \left(\mathrm{Im}A_0\right)^{\rm QCDP} + \text{ subleading EWP contributions}$$

QCDP	EWP
Suppressed through IB $\sim 1/\omega$	Enhanced through $1/\omega$
Enhanced through FSI	Suppressed through FSI
Suppressed through Meson Evolution	Enhanced through large m_t

Importance of EW Penguins at large m_t

Flynn, Randall; Buchalla, AJB, Harlander (1989)

NLO Corrections to QCD and EW Penguins

AJB, Jamin, Lautenbacher, Weisz (1993)
Ciuchini, Franco, Martinelli, Reina (1993)

Dominant NNLO QCD Corrections to EW Penguins

AJB, Gambino, Haisch (1999)

Isospin Breaking corrections

Cirigliano et al (2019)   AJB + Gérard (1987)

RBC-UKQCD hadronic matrix elements (2019) ★

Supported by DQCD (2015, 2016) AJB + Gérard

⇩

3.3σ anomaly!

Dream

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{SM} = (5.5 \pm 2.4) \cdot 10^{-4}$$

Aebischer, Bobeth, AJB (2019)

Inclusion of NNLO QCD corrections to QCD penguins will lower this value.

Cerda-Sevilla, Gorbahn, Jäger, Kokulu

Lattice QCD	Dual QCD	Chiral Perturbation Theory	
New Physics	New Physics	New Physics	
Short Distance RG Evolution	Short Distance RG Evolution	Short Distance RG Evolution	
			0(1GeV)
Numerical sophisticated and demanding calculations lasting many years. (from first principles)	Meson Evolution: The only analytic approach allowing matching with short distance	Problems with matching with short distance, L_i (No meson evolution)	
		Based on global symmetries of QCD	$0(m_K)$

Meson evolution (hidden in lattice QCD) is crucial strong dynamics responsible for $\Delta I=1/2$ rule , ε'/ε , ε , $K\to\pi\pi$ in general