# Lattice QCD inputs for $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$, and $|V_{cs}|$

**Elvira Gámiz**

UNIVERSIDAD DE GRANADA

Centro Andaluz de Física de Partículas Elementales

· 11th International Workshop on the CKM Unitarity Triangle (CKM 2021) ·
· 23 November 2021 ·

# Contents

$$V_{CKM} = \begin{pmatrix} \begin{matrix}|V_{ud}|\\ \pi \to \ell\nu \\ \text{nucleon charges} \\ \textbf{RC}\end{matrix} & \begin{matrix}|V_{us}|\\ K \to \ell\nu \\ K \to \pi\ell\nu \\ \textbf{RC}\end{matrix} & \begin{matrix}|V_{ub}|\\ \text{WG2}\end{matrix} \\ \begin{matrix}|V_{cd}|\\ D \to \ell\nu \\ D \to \pi\ell\nu \\ \\ B_c \to B^0\ell\nu \ \ldots\end{matrix} & \begin{matrix}|V_{cs}|\\ D_s \to \ell\nu \\ D \to K\ell\nu \\ \Lambda_c \to \Lambda\ell\nu \\ B_c \to B_s^0\ell\nu \ \ldots\end{matrix} & \begin{matrix}|V_{cb}|\\ \text{WG2}\end{matrix} \\ \begin{matrix}|V_{td}|\\ \text{WG6}\end{matrix} & \begin{matrix}|V_{ts}|\\ \text{WG6}\end{matrix} & \begin{matrix}|V_{tb}|\\ \text{WG6}\end{matrix} \end{pmatrix}$$

**Lattice inputs for neutron and nuclear $\beta$ decays:** Talk by A. Nicholson, $V_{ud}$ session on Tues.

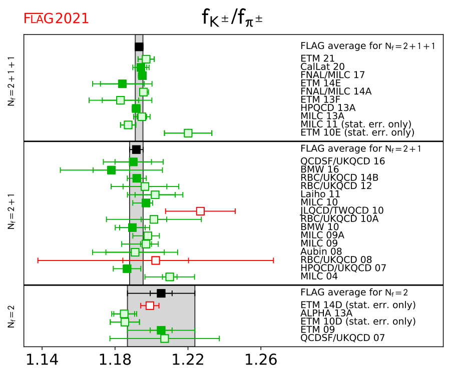
**RC for $|V_{ud}|$:** Talk by M. Gorchtein $V_{ud}$ session on Tuesday

**RC for $|V_{us}|$:** Talks by N. Tantalo and C.Y. Seng: $V_{us}$ session on Tuesday

**FLAG 2021 just appeared:** 2111.09849 and http://flag.unibe.ch/2021/

# Testing first-row CKM unitarity

# Leptonic decays of light mesons: $|V_{us}|/|V_{ud}|$




Pure QCD including $SU(2)$ IB corrections.

$N_f = 2+1+1$ **FLAG 21** $\boxed{\dfrac{f_{K^\pm}}{f_{\pi^\pm}} = 1.1932(21)}$

0.18% error

Reduction of errors in the last years thanks to physical light quark masses, improved actions, NPR or no renormalization.

$$\frac{\Gamma(K^+ \to l^+\nu_l(\gamma))}{\Gamma(\pi^+ \to l^+\nu_l(\gamma))} \propto \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f^2_{K^\pm}}{f^2_{\pi^\pm}}\frac{\left(1+\delta^l_{EM,K}\right)}{\left(1+\delta^l_{EM,\pi}\right)} \xrightarrow{\text{PDG20}} \frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(29)(24)$$

$\delta^l_{EM}$ includes structure dependent EM corrections, traditionally estimated phenomenologically within ChPT, **Cirigliano et al** 1107.6001

# Leptonic decays of light mesons: $|V_{us}|/|V_{ud}|$

- Calculate leptonic decay rates including QCD and QED on the lattice **See N. Tantalo talk**
  - For $K_{\mu 2}/\pi_{\mu 2}$, EM and SIB effects $\delta_{SU(2)} + \delta_{EM}$ found to be very compatible with $ChPT$ estimates: -1.26(14)% vs -1.12(21)%. But smaller errors.

    **Carrasco et al**, 1502.00257, **Giusti et al** 1711.06537, **Di Carlo et al** 1904.08731

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi}\Bigg|^{Moulson21}_{Di\,Carlo\,et\,al} = 0.27679(28)(20)$$

  - Together with the $N_f = 2+1+1$ isospin-symmetric average $\frac{f_K}{f_\pi} = 1.1967(18)$ gives

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23129(24)_{exp}(17)(35)_{latt} = 0.23129(46)$$

  - Preliminary work by **RBC/UKQCD** to calculate $\delta_{SU(2)} + \delta_{EM}$ for $K_{\mu 2}/\pi_{\mu 2}$, talk at Lattice 21 by **A.Z. Ning Yong**

# Leptonic decays of light mesons: $|V_{us}|$

Experimental average from PDG20 and $N_f = 2+1+1$ FLAG21 average for $\frac{f_{K^\pm}}{f_{\pi^\pm}}$:

(See update of exp. average in **M. Moulson talk**)

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23130(24)_{exp}(20)_{RC}(41)_{latt} = 0.23130(51)$$

- Extraction of $|V_{us}|$: Need external input for $|V_{ud}|$: superallowed $\beta$ decays, neutron decays, pion $\beta$ decays ... **See talks in next session**
  - Most precise determination from superallowed $\beta$ decays.
    - Recent updates of universal single-nucleon radiative corrections, $\Delta_R^V$: **Seng et al** 1812.03352, 1807.10197 (SGPRM), **Czarnecki, Marciano & Sirlin** 1907.06737 (CMS) $\rightarrow$ Shift central value
    - New nuclear structure-dependent corrections **Gorchtein** 1812.04229, **Seng et al** 1812.03352 $\rightarrow$ Increase the errors.

Lattice contribution to the calculation of RC **See M. Gorchtein talk**

# Leptonic decays of light mesons: $|V_{us}|$

Experimental average from PDG20 and $N_f = 2+1+1$ FLAG21 average for $\frac{f_{K^\pm}}{f_{\pi^\pm}}$:

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23130(24)_{exp}(20)_{RC}(41)_{latt} = 0.23130(51)$$

plus reevaluation by **Hardy, Towner** PRC102 (2020):

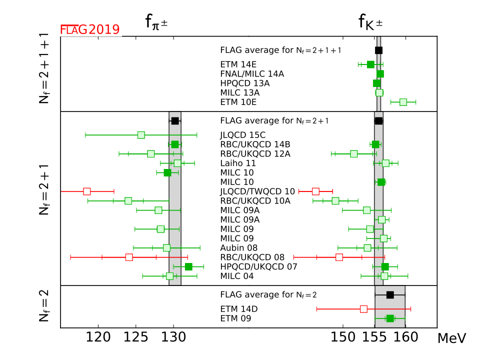
$$V_{ud}^{\text{sup. }\beta} = 0.97373(11)_{exp}(9)_{RC}(27)_{NS} = 0.97373(31)$$

(Incorporate new nuclear corrections and weighted average of $\Delta_R^V$ from SGPRM and CMS)

give

$$|V_{us}|^{K_{\ell 2}/\pi_{\ell 2}} = 0.22523(24)_{exp}(40)_{latt}(20)_{RC}(6)_{NS} = 0.22523(51)$$

# Leptonic decays of light mesons: $f_{\pi^\pm}$ and $f_{K^\pm}$




$f_{K^\pm}^{N_f=2+1+1}$ 0.19% error

**But** many existing LQCD calculations use $f_\pi$ to set the lattice scale (implicitly rely on $|V_{ud}|$ and the SM).

$\rightarrow$ use a different external input: For example, $M_\Omega$ used by BMW, CalLat, RBC/UKQCD, work in progress by FNAL/MILC ...

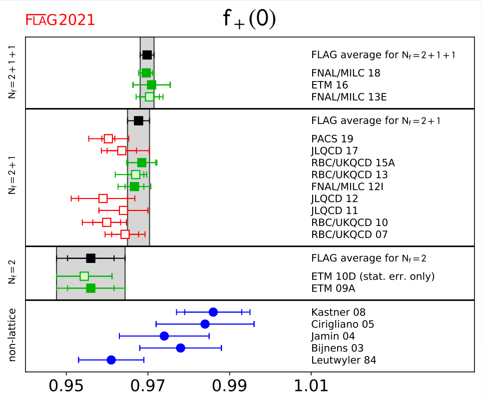
* With $f_{K^\pm}^{FLAG21,2+1+1} = 155.7(3)$ MeV and $|V_{us} f_{K^\pm}|^{PDG20} = 35.09(4)_{exp}(4)_{RC}$

$$|V_{us}| = 0.2254(4)_{latt}(3)_{exp}(3)_{RC}$$

* With $|V_{us} f_K| = 35.23(4)_{exp}(2)_{th}$ **Di Carlo et al** 1904.08731 and $f_K^{2+1+1} = 156.0(3)$ MeV

$$|V_{us}| = 0.2258(4)_{latt}(3)_{exp}(1)_{th}$$

# $K$ semileptonic decays: Direct $|V_{us}|$




$$f_+(0)^{FLAG21}_{N_f=2+1+1} = 0.9698(17)$$

0.18% error

* Preliminary $N_f = 2+1$ **PACS** result presented at Lattice 21 (**talk by T.Yamazaki**): consistent with **PACS19**.

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \left(1 + \delta^{Kl}_{\rm EM} + \delta^{K\pi}_{\rm SU(2)}\right)$$

$\delta^{Kl}_{EM}$ includes structure-dependent EM corrections and the SIB $\delta^{K\pi}_{SU(2)}$ is defined as a correction with respect to the $K^0$ decay. Currently estimated phenomenologically within ChPT, **Cirigliano et al** 1107.6001

# $K$ semileptonic decays: RC

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

$\delta_{SU(2)}$ looks solid at current level of precision, ChPT estimate for $\delta_{EM}$ into question (plus relevant source of error)

- Calculation of $\delta_{EM}^{Ke}$ based on Sirlin's representation of RC and new lattice QCD inputs for $\gamma W - box$ diagrams **Seng et al** 1910.13208, 2009.00459, 2103.00975, 2103.04843 See **C.Y. Seng** talk

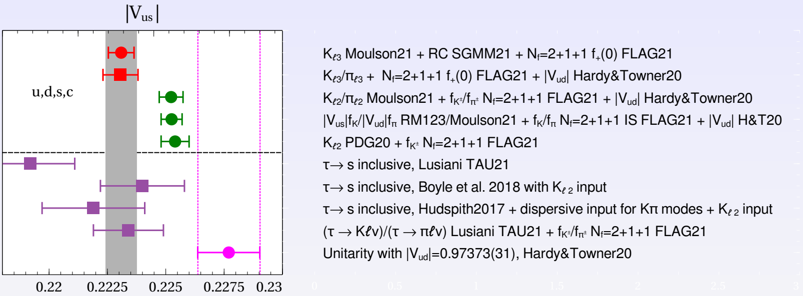
**Cirigliano, Moulson, Passemar,** talk at UMass Amherst19 $\longrightarrow$ See **M. Moulson** talk.

$$|f_+(0)V_{us}| = 0.21652(41) \rightarrow |f_+(0)V_{us}| = 0.21635(38)$$

* And using $N_f = 2+1+1$ FLAG21 average for $f_+(0) = 0.9698(17)$

$$|V_{us}|^{\rm semil} = 0.22309(39)_{exp+\delta_{EM}+\delta_{SU(2)}}(39)_{latt} = 0.22309(56)$$

# Testing first-row CKM unitarity

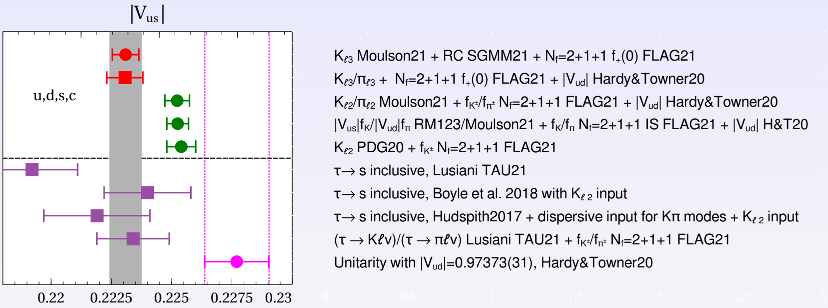



* $K_{\ell 3}/\pi_{\ell 3}$ from **Seng et al** 2107.14708 includes improved EW RC for $K_{e3}$ and $\pi_{e3}$.

* **Inclusive hadronic $\tau$ decays:** $D > 4$ condensates from the lattice **Hudspith et al** 1702.01767, replacing OPE expansion by lattice HVP functions and optimizing weight functions **RBC/UKQCD Boyle et al** 1803.07228. Updates **T. Izubuchi** talk at CKM18 and **K. Maltman** talk at TAU18

* $|V_{us}|$ from **hyperon decays**: Preliminary work by **RBC/UKQCD** on the calculation of form factors for $\Sigma^- \to n\ell^-\bar{\nu}$ presented at Lattice21, talk by **R. Hodgson.**

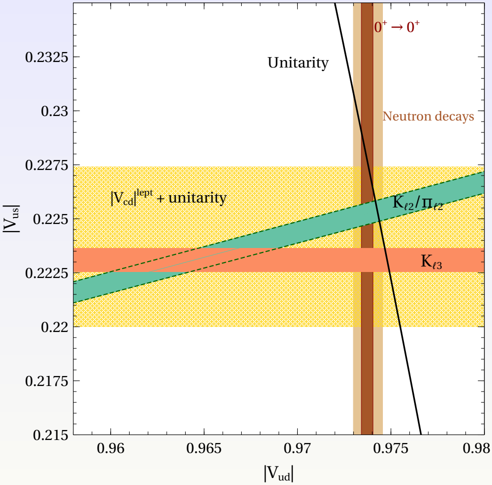
# Testing first-row CKM unitarity




**Tensions with first-row unitarity at $\sim 2-3\sigma$ level**

- Internal tensions between leptonic and semileptonic determinations of $|V_{us}|$ (with $|V_{ud}|$ as external input): $\sim 3\sigma$

# Testing first-row CKM unitarity




$\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$

- $V_{us}^{semil}$ and $|V_{ud}^{sup\,\beta}|$

  $\Delta_u = -0.0021(2)_{V_{us}}(6)_{V_{ud}}$ $\quad \sim \mathbf{3\sigma}$

- $V_{us}^{semil}$ and $|V_{us}|/|V_{ud}|^{K_{\ell 2}\,/\,\pi_{\ell 2}}$

  $\Delta_u = -0.020(5)_{V_{us}}(4)_{V_{us}/V_{ud}}$ $\quad \sim \mathbf{3\sigma}$

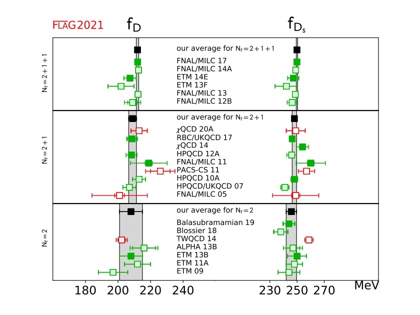
$$|V_{us}|/|V_{ud}|^{K_{\ell 2}/\pi_{\ell 2}} = 0.23129(24)_{exp}(17)_{RC}(35)_{latt}$$

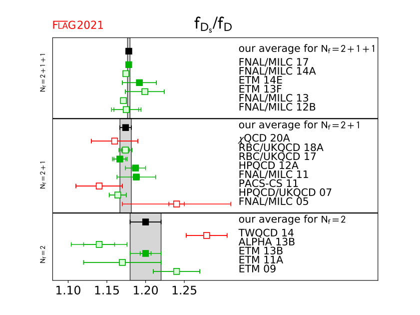
(with $\delta_{EM} + \delta_{SU(2)}$ from Di Carlo et al 1904.08731)

# Testing second-row CKM unitarity

# Leptonic decays of $D$ and $D_s$ mesons

**Errors at 0.33-0.20%, 0.14% for the ratio**: physical light quark masses, improved actions, NPR or no renormalization + small lattice spacings + same action for all flavors




* Also, final results soon on $N_f = 2+1$ CLS ensembles, see **F. Joswig** talk at Lattice 21

**FLAG2021** $N_f = 2+1+1$ averages:

$$f_D = 212.0(0.7)\ \text{MeV} \quad f_{D_s} = 249.9(0.5)\ \text{MeV} \quad f_{D_s}/f_D = 1.1783(0.0016)$$

With dominant SIB from **FNAL/MILC** 1712.09262:

# Leptonic $D$ decays: Extraction of $|V_{cd(cs)}|$

$$\Gamma(D^+_{(s)} \to \ell^+ \nu(\gamma)) \propto S_{EW}\,(1+\delta_{\rm EM})\,|V_{cd(cs)}|^2 f^2_{D^+_{(s)}}$$

- **Experimental data:** CLEO-c, Belle, Babar, and BESIII: precision better $\sim$2-3% for $e, \mu$

  **WG1 talks on Thursday** and plenary talk by **L. Dong** on Monday

- **EW and EM corrections:** Accounted for in the exp. rates. **PDG19** adds 2.8% uncertainty to purely leptonic decay rate. Adding that to **HFLAV2021** averages:

  $|V_{cd}| f_{D^+} = 46.2(1.0)(0.6)$ MeV $\qquad |V_{cs}| f_{D_s^+} = 245.9(2.9)(3.4)$ MeV

  * Short-distance EW corrections $\sim$1.8% **Sirlin** NP82
  * Structure-dependent EM: Reduce $\sim$1% $\mu$ channels **Dobrescu, Kronfeld** 0803.0512
  * Long-distance EM: removed with PHOTOS.

Together with $N_f = 2+1+1$ **FLAG21** averages (with dominant SIB corrections):

$$|V_{cd}| = 0.2173(47)_{exp}(28)(7)_{latt} \qquad |V_{cs}| = 0.984(12)_{exp}(14)(2)_{latt}$$

* EW+EM corrections important source of error. Preliminary work to calculate radiative decay rates on the lattice: See **S. Meinel** plenary talk on Monday

  **Meinel, Kane et al** 1907.00279, 2110.13196, **Desiderio et al** 2006.05358

# Semileptonic $D$ decays: Extraction of $|V_{cd(cs)}|$

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} \propto S_{EW}\,(1+\delta_{\rm EM})\,|V_{cd(cs)}|^2|f_+^{DP}(q^2)|^2$$

(neglecting contribution proportional to $m_\ell^2$)

- **Experimental data:** CLEO-c, Belle, Babar, and BESIII. **WG1 talks on Thursday** and plenary talk by **L. Dong** on Monday
- **EW and EM corrections:** Should be accounted for in exp. rates.
  - * Short-distance EW corrections ∼1.8% **Sirlin** NP82
  - * Structure-dependent EM: use ChPT $K_{\ell 3}$ calculations to estimate $\sim 1\%$
  - * Long-distance EM: removed with PHOTOS.

# Semileptonic $D$ decays: Extraction of $|V_{cd(cs)}|$

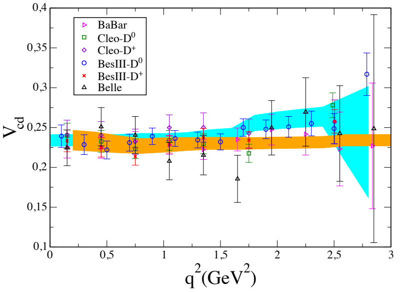
Twisted mass $N_f = 2+1+1$, $m_\pi^{\min} \approx 220$ MeV and 3 lattice spacings down to $0.06$ fm **ETM** 1706.03017, 1706.03657 $f_T^{DK(\pi)}(q^2)$ in 1803.04807

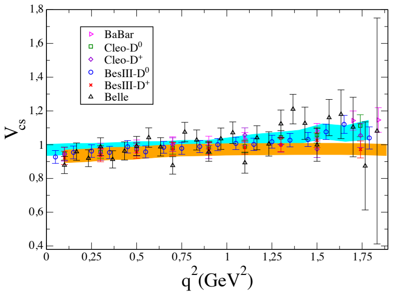
* Double ratios of correlation functions to avoid non-trivial renormalization.
* Many kinematical conditions using twisted boundary conditions.
* Modified z-expansion based on hard-pion $SU(2)$ $\chi$PT.

# Semileptonic $D$ decays: Extraction of $|V_{cd(cs)}|$

Twisted mass $N_f = 2+1+1$, $m_\pi^{\min} \approx 220$ MeV and 3 lattice spacings down to 0.06 fm

**Lubicz, Riggio et al** 1706.03017, 1706.03657 $f_T^{DK(\pi)}(q^2)$ in 1803.04807




$|V_{cd}| = 0.2341(74)$ and $|V_{cs}| = 0.970(33)$ with $\sim 80\%$ theory error.

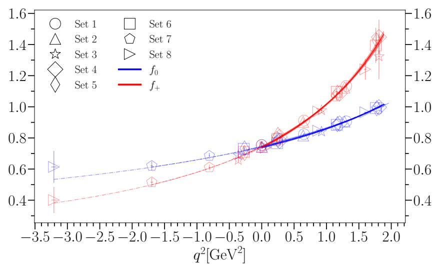
* If $\eta_{EW} = S_{EW}^{1/2} = 1.009(2)$ is applied: $|V_{cd}| = 0.2320(73)$ and $|V_{cs}| = 0.961(32)$

- Using instead $f_+^{D\to\pi}(0) = 0.612(35)$ and $f_+^{D\to K}(0) = 0.765(31)$ **ETM** 1706.03017 and $\eta_{EW}|V_{cs}|f_+(0) = 0.7180(33)$, $\eta_{EW}|V_{cd}|f_+(0) = 0.1426(18)$ **HFLAV19**

$|V_{cd}| = 0.231(14)$ and $|V_{cs}| = 0.930(38)$

# Semileptonic $D$ decays: $D \to K$

**New:** $D \to K$ form factors covering whole physical $q^2$ range HPQCD 2104.09883 on $N_f = 2+1+1$ MILC HISQ ensembles.




4 momenta for each ensemble.

Phys. $m_l$ ensembles: Sets 1, 2, 3.

* Relativistic (HISQ) description of all flavors: very small discretization errors
* 5 lattice spacings $a \approx 0.15 - 0.042$ fm.

  $m_s$ and $m_c$ close to physical, 3 ensembles with physical $m_l$.
* NPR imposing Ward identities at $q^2_{max}$
* Modified z-expansion: chiral interpolation, mass mistunings, continuum extrapol. and $q^2$ dependence.

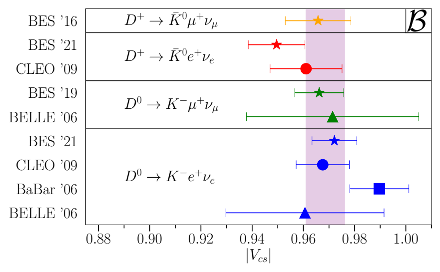
Also results for $f_T(q^2)$.

# Semileptonic $D$ decays: $D \to K$

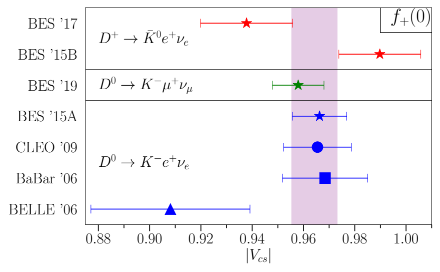
**New:** HPQCD 2104.09883 on $N_f = 2+1+1$ MILC HISQ ensembles.

- Three methods to extract $V_{cs}$: differ in experimental data included.

Using total branching fraction




With $\eta_{EW}\sqrt{1+\delta_{EM}}|V_{cs}|f_+(0) = 0.7180(33)$ HFLAV 2019



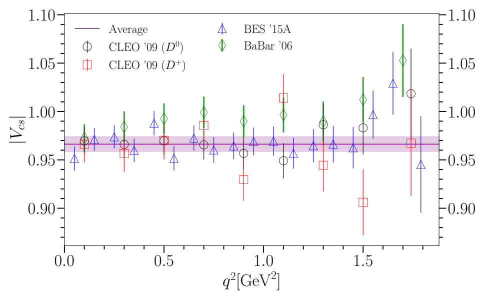

$f_+(0) = 0.7830(44)$ agrees with ETMC but $\sim 2\sigma$ tension for $f_+(q^2_{max})$

$$|V_{cs}|^{\mathcal{B}} = 0.9686(54)_{\rm latt}(39)_{\rm exp}(19)_{\eta_{EW}}(30)_{\delta_{EM}}$$
$$|V_{cs}|^{f+(0)} = 0.9643(57)_{\rm latt}(44)_{\rm exp}(19)_{\eta_{EW}}(48)_{\delta_{EM}}$$

# Semileptonic $D$ decays: $D \to K$

**New:** HPQCD 2104.09883 on $N_f = 2+1+1$ MILC HISQ ensembles.

HPQCD preferred method: Using $q^2$ binned differential rates.




Need binned experimental data.

Include only exp. results with a covariance matrix for the partial rates between $q^2$ bins.

$$|V_{cs}|^{\mathcal{B}} = 0.9663(53)_{\rm latt}(39)_{\rm exp}(19)_{\eta_{EW}}(40)_{\delta_{EM}}$$

- For the first time, semileptonic determination of $|V_{cs}|$ with smaller errors than leptonic determination.

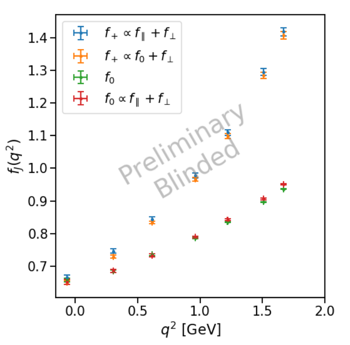
# Semileptonic decays of $D$ mesons: in progress

**FNAL/MILC** on $N_f = 2+1+1$ MILC HISQ ensembles: $D \to K$, $D \to \pi$, $D_s \to K$ form factors over whole physical $q^2$ range. $f_+$, $f_0$ and $f_T$.

**Preliminary results** at Lattice 21: **W. Jay, A. Lytle et al** 2111.05184

- 8 ensembles at 5 lattice spacings down to $\sim 0.042$ fm, 4 of them with phys. light quark masses.
- Several valence heavy-quark masses: interpolate to correct $m_c$ and study $B$ semileptonic decays. **A. Lytle** talk on Thursday WG2

$D_S \to K$ renormalized FF on the phys.-mass $a \approx 0.09$ fm ensemble




Several ways of extracting $f_{+(0)}$

$$f_\parallel = Z_{V^0} \frac{\langle P|V^0|D\rangle}{\sqrt{2M_D}}$$

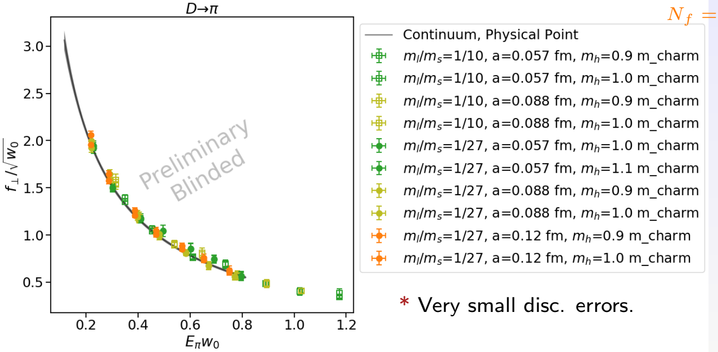
$$f_\perp = Z_{V^i} \frac{\langle P|V^i|D\rangle}{\sqrt{2M_D}} \frac{1}{p_L^i}$$

$$f_0 = \frac{m_h - m_\ell}{M_D^2 - M_P^2} \langle P|\, S\,|D\rangle$$

NP renormalization factors from Ward identity at all $q^2$.

# Semileptonic decays of $D$ mesons: in progress

- Extrapolate to the continuum + interpolate to the physical quark masses: Example




$N_f = 2+1+1$ **FNAL/MILC**

hard-pion SU(2) HMRS$\chi$PT + NNLO analy. terms

* Very small disc. errors.

* Checking different fit functions (hard K/$\pi$) $SU(2)$ vs $SU(3)$ HMRS$\chi$PT + higher order analytical terms: very small variations

**First results soon:** Reduction of errors in $|V_{cd}^{sem}|$, correlated ratios of FF and branching fractions from different channels ...

* More on-going work: **JLQCD** 1711.11235, **RBC/UKQCD, ALPHA** ...

# Alternate ways of extracting $|V_{cd(cs)}|$

- $\Lambda_c \to \Lambda \ell^+ \nu_\ell$: $N_f = 2+1$ form factors calculated by **Meinel et al,** 1611.09696. Combined with **BESIII** 1510.02610, 1611.04382:

$$|V_{cs}| = 0.949(24)_{latt}(14)_{\tau_{\Lambda_c}}(49)_{\mathcal{B}}$$

- $B_c \to B^0_{(s)}$ **HPQCD** 2003.00914 $N_f = 2+1+1$ calculation of form factors over complete phys. $q^2$ range. Measurable at LHCb.

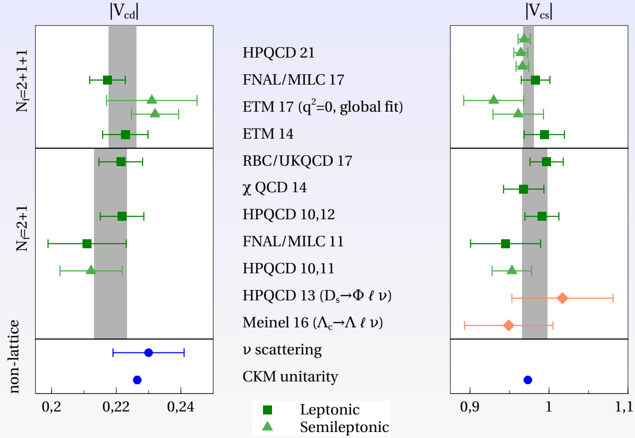
$$\Gamma(B_c^+ \to B_s^0 \bar{\ell} \nu_\ell) = 26.3(9)_{CKM}(8)_{latt} \cdot 10^9 s^{-1}$$

$$\Gamma(B_c^+ \to B^0 \bar{\ell} \nu_\ell) = 1.65(6)_{CKM}(8)_{latt} \cdot 10^9 s^{-1}$$

- $\Xi_c \to \Xi \ell^+ \nu_\ell$: First calculation of form factors with LQCD **Q.A. Zhang et at** 2103.07064 gives
  $|V_{cs}| = 0.834(74)_{latt}(127)_{exp}$ together with **Belle** 2103.06496 data

  $|V_{cs}| = 0.883(88)_{latt}(167)_{exp}$ together with **ALICE, J. Zhu,** PoS ICHEP2020 (2021) 524

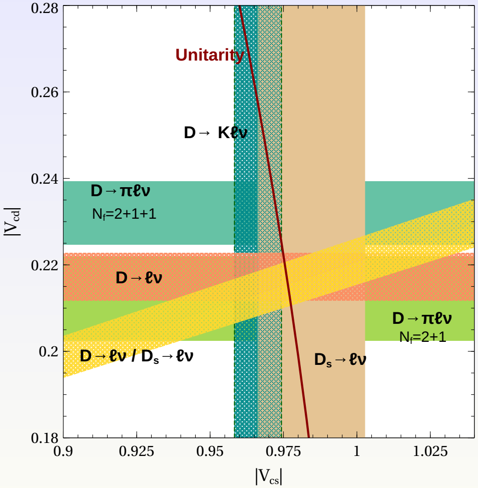
# Second row unitarity




- **Grey bands**: FLAG21 averages (without EW/EM semil. corrections or errors, besides those included by exp.)
- Exper. leptonic and CKM unit. inputs: PDG20 (+HFLAV update for $|V_{cs} f_{D_s}|$)
- Exp. $D/D_s$ Semileptonic: HFLAV19 + $\eta_{EW} = 1.009(2)$, except in HPQCD21 and ETM17 (global fit)

- Large reduction of errors in semileptonic $|V_{cs}|$.
- Leptonic and semileptonic determinations, as well as $N_f = 2+1$ vs $N_f = 2+1+1$, in good agreement.
  - Exception: Semileptonic $|V_{cd}|$, need further investigation/LQCD results.

# Second row unitarity




$N_f = 2+1+1$ LQCD inputs used in the plot unless otherwise stated.

$$\Delta_c = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1$$

Second row unitarity is fulfilled within $\sim 1\sigma$.

* For example, using

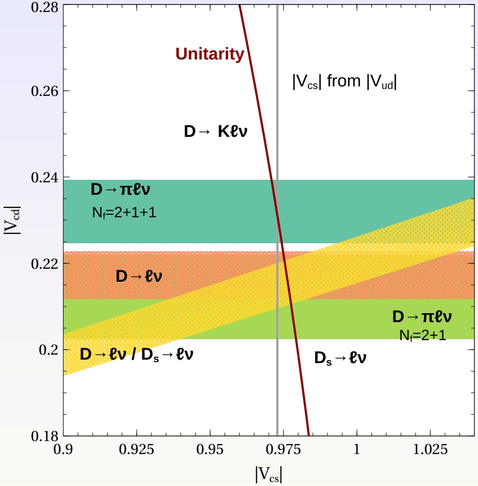
$$|V_{cd}|^{\rm lep}_{N_f=2+1+1} = 0.217(5)_{exp}(3)_{EM+EW}(1)_{latt}$$

$$|V_{cs}|^{\rm semil\ HPQCD21} = 0.9663(53)_{\rm latt}(39)_{exp}(19)_{\eta_{EW}}(40)_{\delta_{EM}}$$

and $|V_{cb}|^{PDG20} = (4.1\pm1.4)\cdot10^{-3}$

$$\Delta_c = -0.017(15)_{V_{cs}}(2)_{V_{cd}} = -0.017(16)$$

# Second row unitarity




$N_f = 2+1+1$ LQCD inputs used in the plot unless otherwise stated.

$$\Delta_c \equiv |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1$$

* Using Hardy&Towner 21

$V_{ud}^{\text{sup. }\beta} = 0.97373(31)$ corrected at $\lambda^4$:

$$|V_{cs}|_{V_{ud}} = 0.97291(31)$$

and $|V_{cd}|$ from leptonic decays

$$\Delta_c = -0.0045(6)_{V_{cs}}(25)_{V_{cd}}(1)_{V_{cb}} = -0.0045(26)$$

Equivalent to testing second-column CKM unitarity.

# Conclusions and outlook

- First-row CKM unitarity violated at the $\sim 3\sigma$ level
  - Need to settle (an improve) value of $|V_{ud}|$
    - Superallowed $\beta$ decays: structure-dependent nuclear corrections.
    - Determination from other sources with similar precision.
  - From the lattice:
    - Correlated analysis of $f_{K^\pm}/f_{\pi^\pm}$ and $f_+(0) \to$ Test without $|V_{ud}|$
      **FNAL/MILC** in progress
    - Not using $f_\pi$ to determine lattice scale $\to$ Independent (and precise) calculations of $f_\pi^\pm$ and $f_K^\pm$.
    - More calculations of $f_+(q^2)$ with $q^2 \neq 0$ and $f_T(q^2)$.
- Second row CKM unitarity fullfilled at current level of precision.
  - Experimental improvements: More urgent for leptonic decays.
  - From the lattice
    - Improved and correlated calculations of form factors, in particular for $D \to \pi$. Work in progress **FNAL/MILC, JLQCD, RBC/UKQCD, ALPHA...**

# Conclusions and outlook

- EM corrections significant source of uncertainty
  - Calculate QED corrections (including radiative corrections) directly in lattice QCD+QED.
    - First results for radiative $K/\pi$ leptonic decay rates: **Carrasco et al**, 1502.00257, **Giusti et al** 1711.06537, **Di Carlo et al** 1904.08731
    - Preliminary work for radiative heavy-light leptonic decay rates: **Meinel, Kane et al** 1907.00279, 2110.13196, **Desiderio et al** 2006.05358
  - Radiative corrections relevant to nuclear extractions of $|V_{ud}|$ **See M. Gorchtein talk**
  - Radiative corrections for Kaon semileptonic decays **See C.Y. Seng talk**

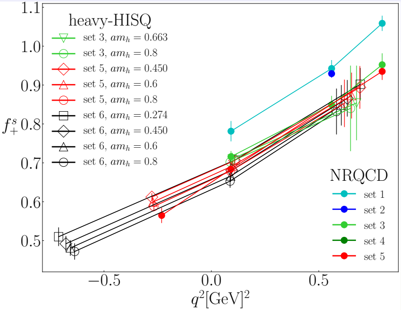
# Backup slides

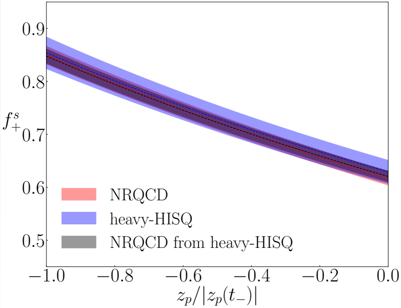
# $B_c \to B^0_{(s)}\bar{\ell}\nu$: (Potential) extraction of $|V_{cs,cd}|$

HPQCD 2003.00914: $N_f = 2+1+1$ using **two different descriptions for $b$**

- NRQCD: $am_h = am_b$ and Relativistic heavy-HISQ: $am_c \leq am_h \leq 0.8 am_b$ (both using HISQ for $u = d$ and $c$)

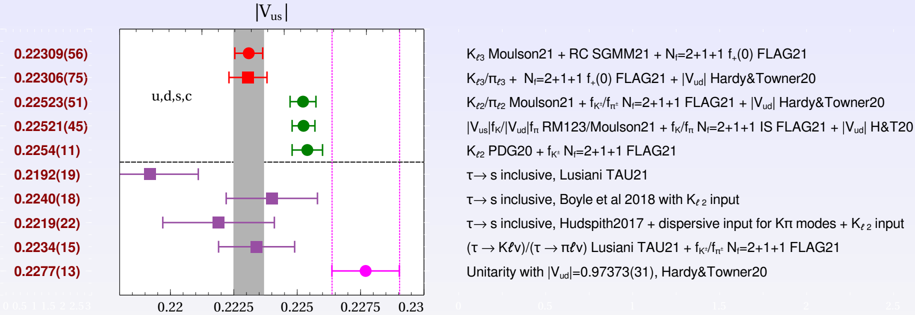
Measurable at LHCb

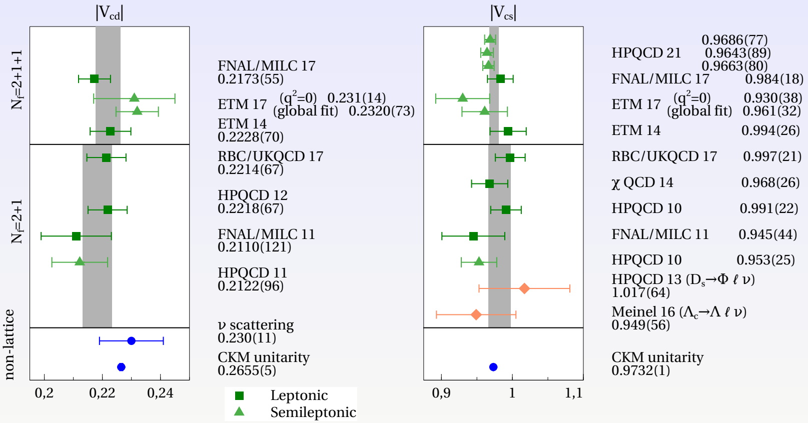



- Cover complete physical $q^2$ range
- Good agreement across entire physical $z$ range (for $B^0$ and $B^0_s$).

$$\Gamma(B_c^+ \to B_s^0 \bar{\ell}\nu_\ell) = 26.3(9)_{CKM}(8)_{latt} \cdot 10^9 s^{-1}, \quad \Gamma(B_c^+ \to B^0 \bar{\ell}\nu_\ell) = 1.65(6)_{CKM}(8)_{latt} \cdot 10^9 s^{-1}$$


$|V_{us}|$
0.22309(56)
0.22306(75)
0.22523(51)
0.22521(45)
0.2254(11)
0.2192(19)
0.2240(18)
0.2219(22)
0.2234(15)
0.2277(13)
u,d,s,c
0.22
0.2225
0.225
0.2275
0.23
$K_{\ell 3}$ Moulson21 + RC SGMM21 + $N_f$=2+1+1 $f_+(0)$ FLAG21
$K_{\ell 3}/\pi_{\ell 3}$ + $N_f$=2+1+1 $f_+(0)$ FLAG21 + $|V_{ud}|$ Hardy&Towner20
$K_{\ell 2}/\pi_{\ell 2}$ Moulson21 + $f_{K^\pm}/f_{\pi^\pm}$ $N_f$=2+1+1 FLAG21 + $|V_{ud}|$ Hardy&Towner20
$|V_{us}|f_K/|V_{ud}|f_\pi$ RM123/Moulson21 + $f_K/f_\pi$ $N_f$=2+1+1 IS FLAG21 + $|V_{ud}|$ H&T20
$K_{\ell 2}$ PDG20 + $f_{K^\pm}$ $N_f$=2+1+1 FLAG21
$\tau \to$ s inclusive, Lusiani TAU21
$\tau \to$ s inclusive, Boyle et al 2018 with $K_{\ell 2}$ input
$\tau \to$ s inclusive, Hudspith2017 + dispersive input for $K\pi$ modes + $K_{\ell 2}$ input
$(\tau \to K\ell\nu)/(\tau \to \pi\ell\nu)$ Lusiani TAU21 + $f_{K^\pm}/f_{\pi^\pm}$ $N_f$=2+1+1 FLAG21
Unitarity with $|V_{ud}|$=0.97373(31), Hardy&Towner20


|Vcd|
|Vcs|
Nf=2+1+1
Nf=2+1
non-lattice
FNAL/MILC 17
0.2173(55)
ETM 17 (q²=0) 0.231(14)
(global fit) 0.2320(73)
ETM 14
0.2228(70)
RBC/UKQCD 17
0.2214(67)
HPQCD 12
0.2218(67)
FNAL/MILC 11
0.2110(121)
HPQCD 11
0.2122(96)
ν scattering
0.230(11)
CKM unitarity
0.2655(5)
0,2
0,22
0,24
Leptonic
Semileptonic
0,9
1
1,1
HPQCD 21 0.9686(77)
0.9643(89)
0.9663(80)
FNAL/MILC 17 0.984(18)
ETM 17 (q²=0) 0.930(38)
(global fit) 0.961(32)
ETM 14 0.994(26)
RBC/UKQCD 17 0.997(21)
χ QCD 14 0.968(26)
HPQCD 10 0.991(22)
FNAL/MILC 11 0.945(44)
HPQCD 10 0.953(25)
HPQCD 13 (Ds→Φ ℓ ν)
1.017(64)
Meinel 16 (Λc→Λ ℓ ν)
0.949(56)
CKM unitarity
0.9732(1)