Precise Radiative Corrections to V_{ud}

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History of Radiative Corrections to β Decay

1950's: V - A Fermi theory;

$$\mathcal{L} = -\frac{G_\mu}{\sqrt{2}}[\bar{\psi}_{\nu_\mu}\gamma^\mu(1-\gamma_5)\psi_\mu][\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_{\nu_e}] + \text{h.c.}$$

Calculating radiative corrections to muon decay: important evidence for V-A theory

RC to muon decay - UV finite for V and A interactions but UV divergent for S, PS

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}F(x)[1+\delta_\mu]$$

Tree-level phase space: $F(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$ $x = m_e^2/m_\mu^2$

RC (2-loop): $\delta_\mu = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left[1 + \frac{2\alpha}{3\pi}\ln\left(\frac{m_\mu}{m_e}\right)\right] + 6.700\left(\frac{\alpha}{\pi}\right)^2 + \cdots = -4.19818 \times 10^{-3}$

Precise measurement of muon lifetime: $\tau_\mu = 2196980.3(2.2)ps$

Precise determination of Fermi constant: $G_F = G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

History of Radiative Corrections to β Decay

$$\mathcal{L}_{\beta-\mathrm{decay}} = -\frac{G_V}{\sqrt{2}}[\bar{\psi}_p\gamma^\mu(1-\gamma_5)\psi_n][\bar{\psi}_e\gamma_\mu(1-\gamma_5)\psi_{\nu_e}] + \mathrm{h.c.}$$

However, RC to neutron decay - UV divergent even in V-A theory

Uncorrected spectrum for Fermi transition: $P^0 d^3p = \frac{8G_V^2}{(2\pi)^4}(E_m - E)^2 d^3p$

RC to spectrum: $\Delta P d^3p = \frac{\alpha}{2\pi}P^0 d^3p \left\{6\ln\left(\frac{\Lambda}{m_p}\right) + g(E, E_m) + \frac{9}{4}\right\}$

UV cut-off

Sirlin's function $g(E, E_m)$: QED beyond Coulomb distortion

Current algebra: UV div. part $\frac{\alpha}{2\pi}P^0 d^3p\ 3[1+2\bar{Q}]\ln(\Lambda/M)$

$\bar{Q}$: average charge of fields involved: $1 + 2\bar{Q}_{\mu,\nu_\mu} = 0$ but $1 + 2\bar{Q}_{n,p} = 2$

Neutron and nuclear beta decay rates: $G_V < G_\mu$

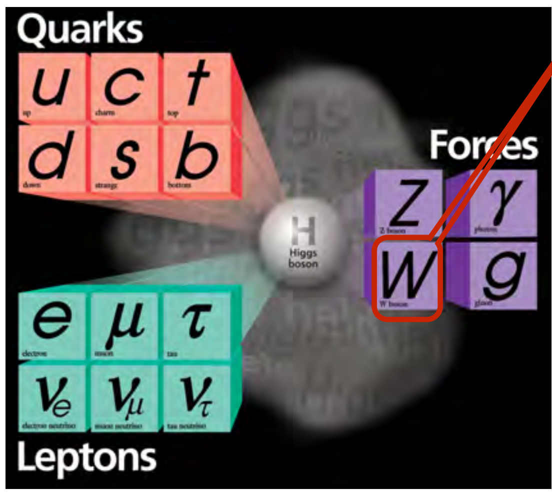
Kaon and hyperon decays? ($\Delta S = 1$)

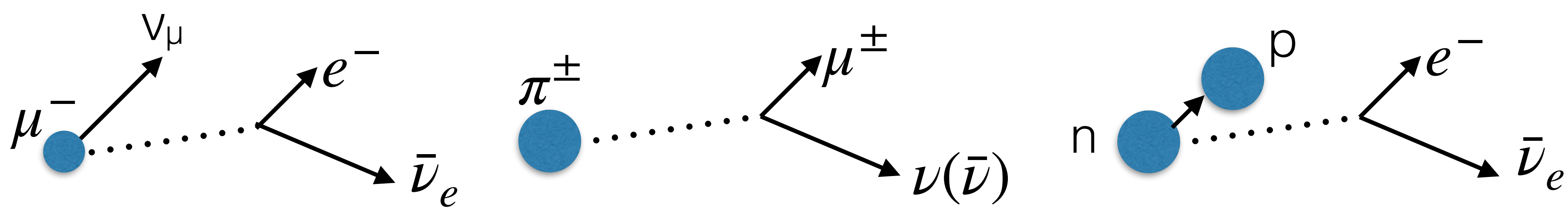
Is weak interaction universal? Strong interaction effects?

Quark Mixing & CKM Unitarity

1960's: electroweak theory - $SU(2)_L$ x $U(1)_Y$, massive W, Z bosons, EW mixing, …

Charged current interaction - β-decay (μ, π±, n)





Weak interaction of lepton and quarks is universal
But its strength is distributed among quark families

Cabbibo-Kabayashi-Maskawa: mass vs. flavor eigenstates

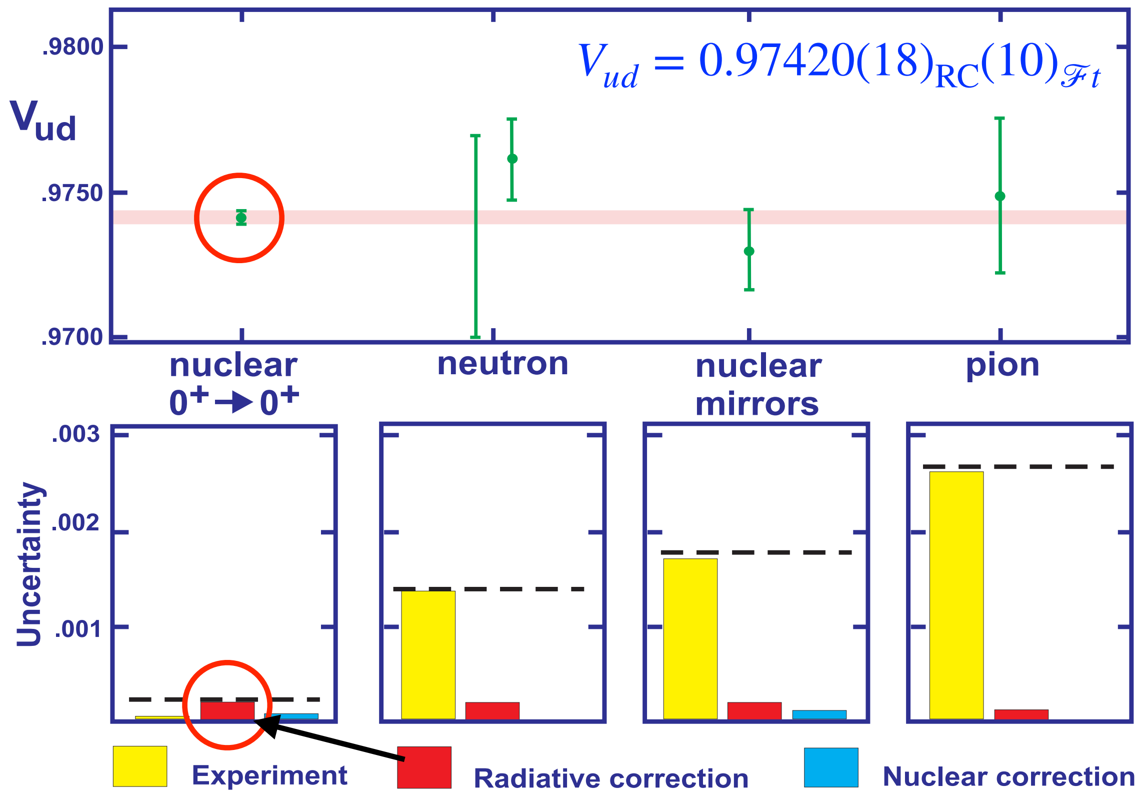
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM unitarity - measure of completeness of the SM: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Landscape Change in Top-Row CKM Unitarity

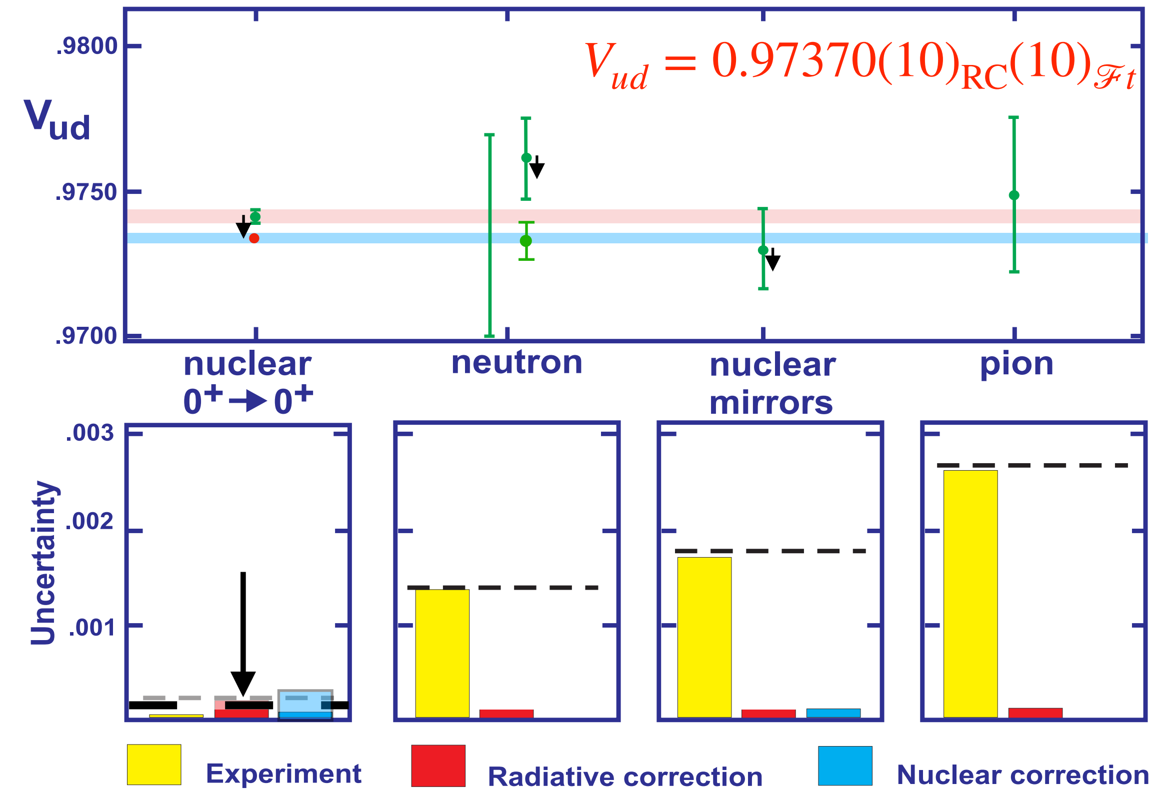
PDG 2018:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{V_{ud}}(2)_{V_{us}}$$



PDG 2020:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(3)_{V_{ud}}(4)_{V_{us}}$$



Improvement (factor 2) in universal RC

Seng, MG, Patel, Ramsey-Musolf, 1807.10197;
Seng, MG, Ramsey-Musolf, 1812.03352
MG, 1812.04229

Scrutiny of nuclear uncertainties $\mathscr{F}t = 3072.1(7) \to 3072.0(2.0)$

If taken at face value: $V_{ud}^{0^+} = 0.9737(1)_{\mathrm{RC}}(3)_{\mathscr{F}t}$

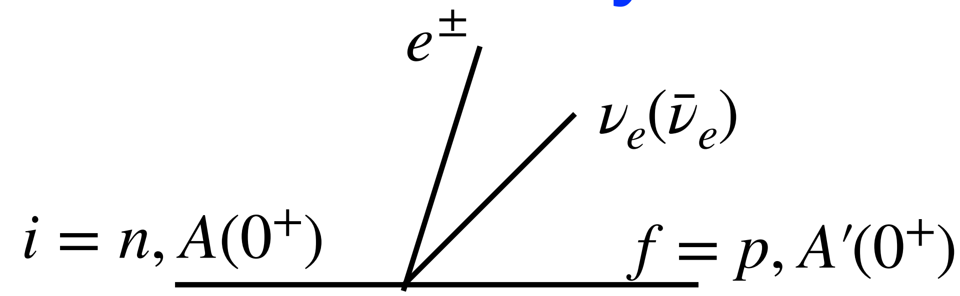
g_A from neutron decay asymmetry improved by factor 4

$$V_{ud} = 0.9763(5)_{\tau_n}(15)_{g_A}(2)_{\mathrm{RC}} \longrightarrow V_{ud} = 0.9733(3)_{\tau_n}(3)_{g_A}(1)_{\mathrm{RC}}$$

PERKEO-III
Märkisch et al., 1812.04666

RC to beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for Vud extraction 1×10^{-4}

Electron carries away energy $E \leq Q$
Energy scales:

Weak boson scale
$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

Nuclear scale
$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

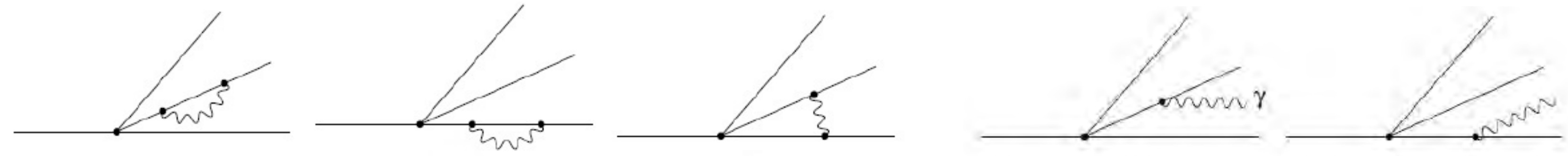
Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

E-dep RC: $\dfrac{\alpha}{2\pi}\left(\dfrac{E}{\Lambda}, \ln\dfrac{E}{\Lambda}, \ldots\right)$

RC to beta decay: overall setup

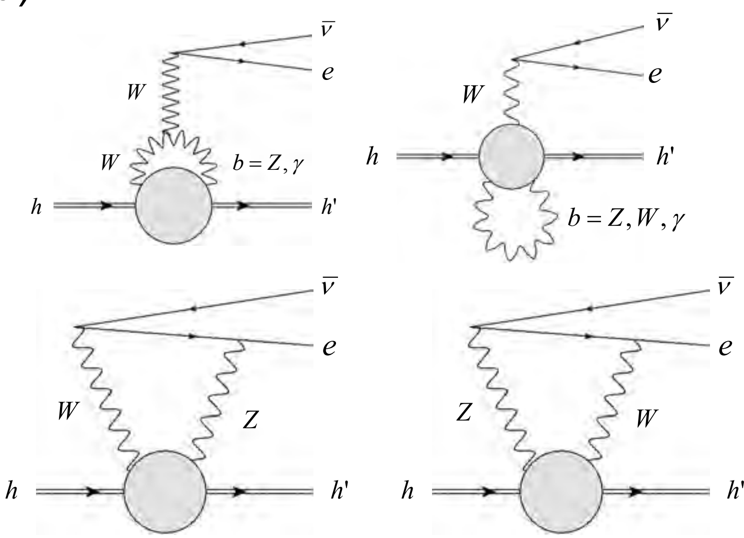
Outer (depend on e-energy):
retain only IR divergent pieces



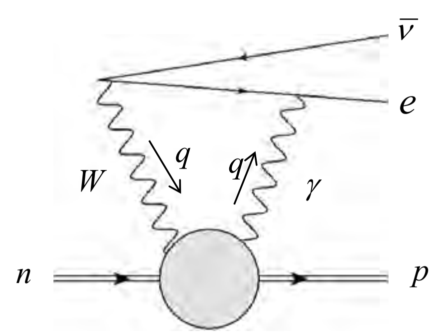
E/m_e not small, need to account for exactly.
Coulomb distortion: resummation of $(Z\alpha)^n$ —> Dirac equation in the Coulomb field
IR finite piece: can set m_e=0 —> if energy-dependent ~ $(\alpha/2\pi)$ x (E/Λ_{had})

Inner RC Δ_R^V — energy-independent

W,Z-exchange:
UV-sensitive, pQCD;
model-independent



When γ involved:
sensitive to long range physics;
model-dependent!



$$\Delta_R^V = 2\Box_{\gamma W}^{A\times V} + \text{ model independent}$$

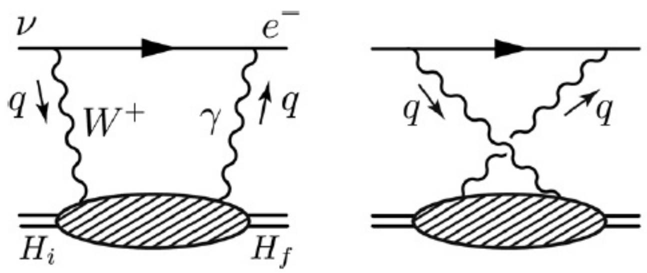
V x V correlator protected by CVC - no hadronic uncertainty
Axial vector not conserved —> A x V correlator from γW box sensitive to hadron structure

Superalloweds $|V_{ud}|^2 = \dfrac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$

Neutron $|V_{ud}|^2 = \dfrac{5024.7\text{ s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R)}$

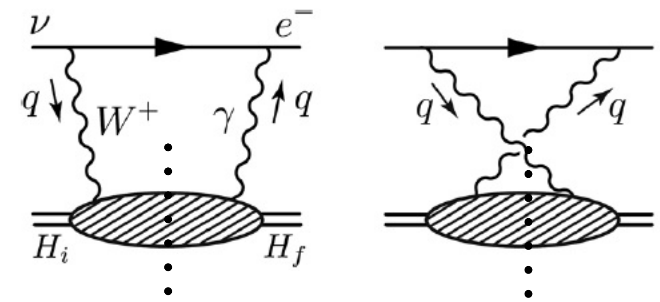
Universal RC from dispersion relations

Model-dependent part or RC: γW-box



Generalized Compton tensor
time-ordered product — complicated!

$$\int dx e^{iqx} \langle H_f(p) \,|\, T\{J^\mu_{em}(x) J^{\nu,\pm}_W(0)\} \,|\, H_i(p)\rangle$$



Commutator (Im part) - only on-shell
hadronic states — related to data

$$\int dx e^{iqx} \langle H_f(p) \,|\, [J^\mu_{em}(x), J^{\nu,\pm}_W(0)] \,|\, H_i(p)\rangle$$

Physics of model dependence: virtual photon polarizes the nucleus;
Long-range part of the box sensitive to hadronic polarizabilities;
Polarizabilities are related to the excitation spectrum via a dispersion relation (sum rule)

Interference γW structure function

$$\mathrm{Im} T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F_3^{\gamma W}(x, Q^2)$$

Box ~ 1st Nachtmann moment of $F_3^{\gamma W(0)}$
Symmetry: only isoscalar photons contribute

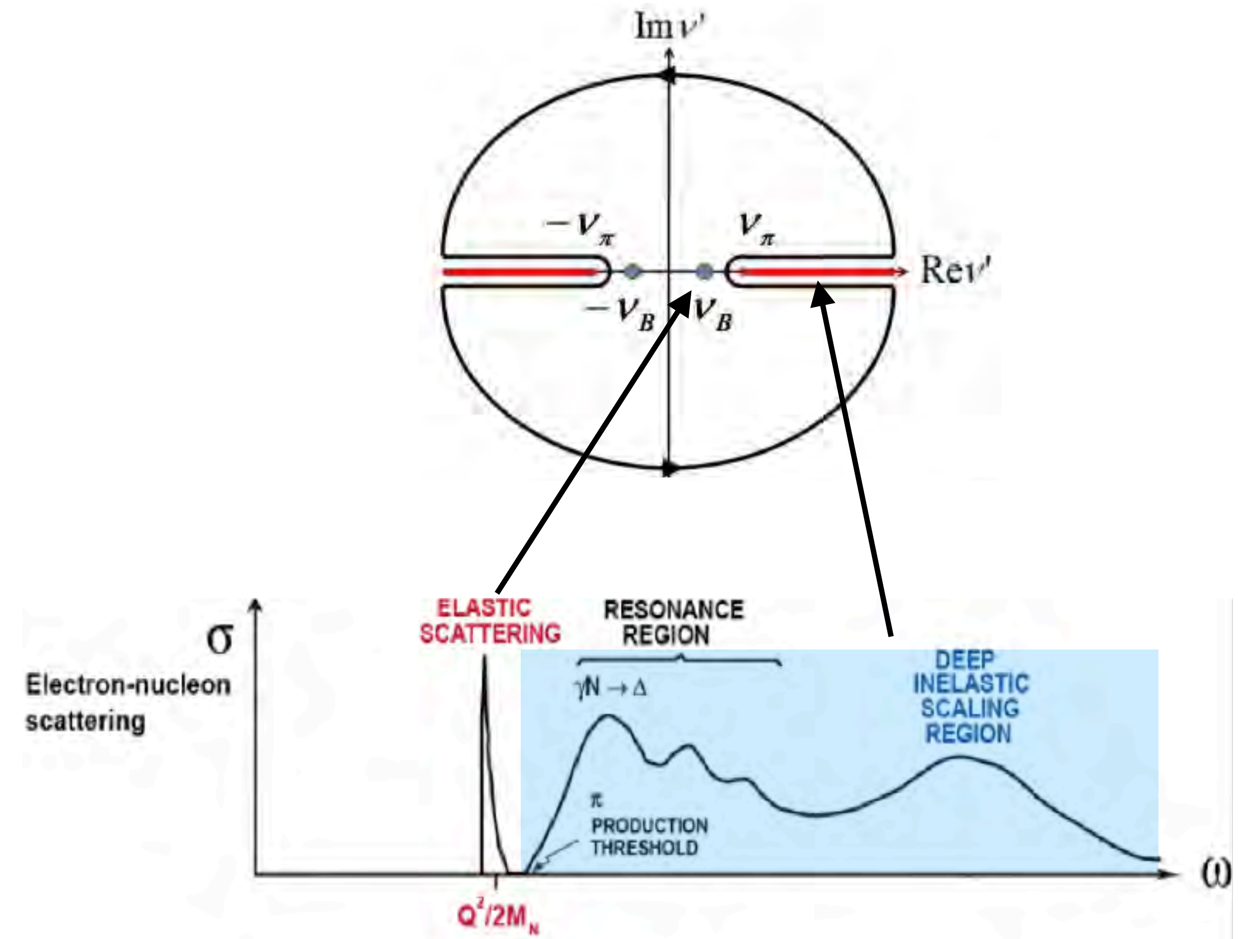
$$\Box^{VA}_{\gamma W} = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} M_3^{\gamma W(0)}(Q^2)$$

Nachtmann moments:

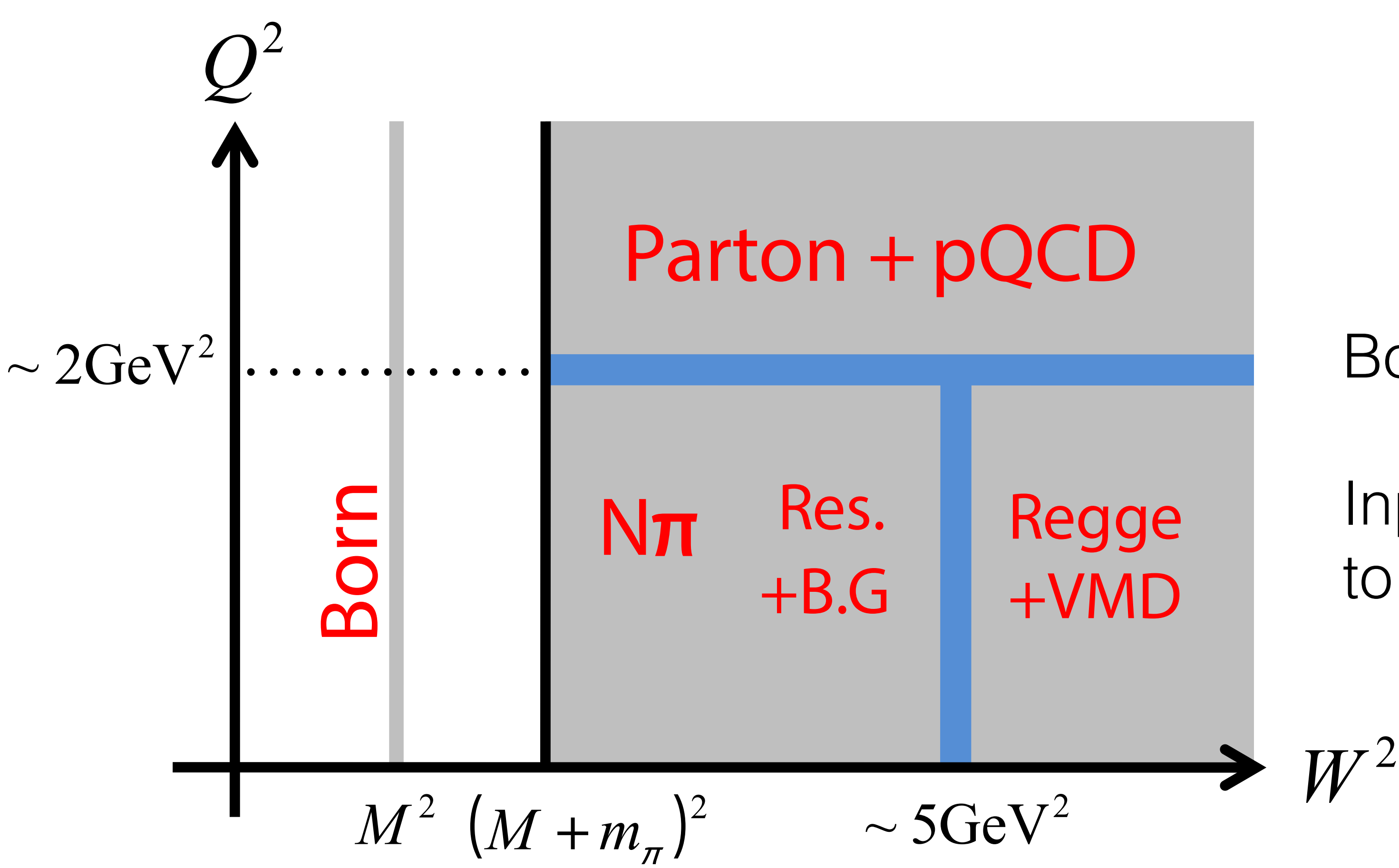
$$M_3(n, Q^2) = \frac{n+1}{n+2} \int_0^1 \frac{dx \xi^n}{x^2} \frac{2x(n+1) - n\xi}{n+1} F_3(x, Q^2), \qquad \xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q²:
scanning dominant physics pictures



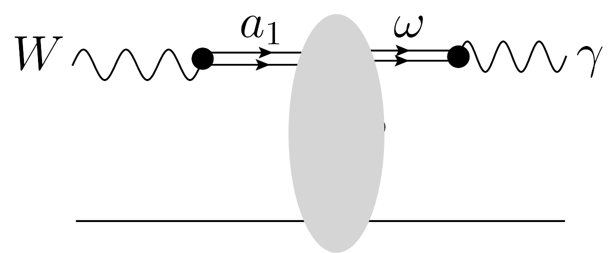
Boundaries between regions - approximate

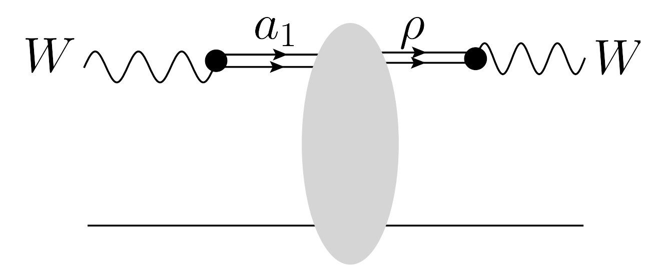
Input in DR related (directly or indirectly) to experimentally accessible data

Input into dispersion integral - $\nu/\bar{\nu}$ data

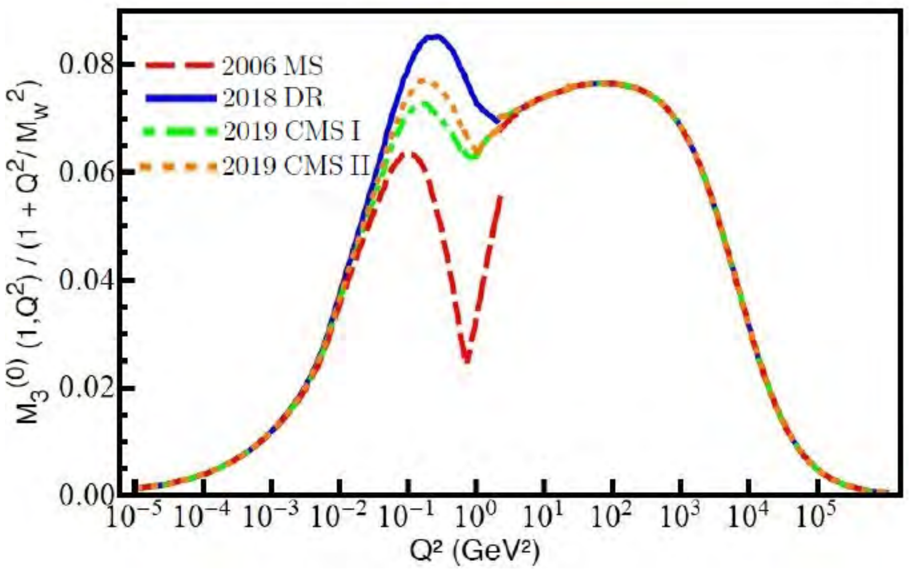
Isospin symmetry: vector-isoscalar current related to vector-isovector current

Mixed CC-NC γW SF (no data) <—> Purely CC SF (inclusive neutrino data)

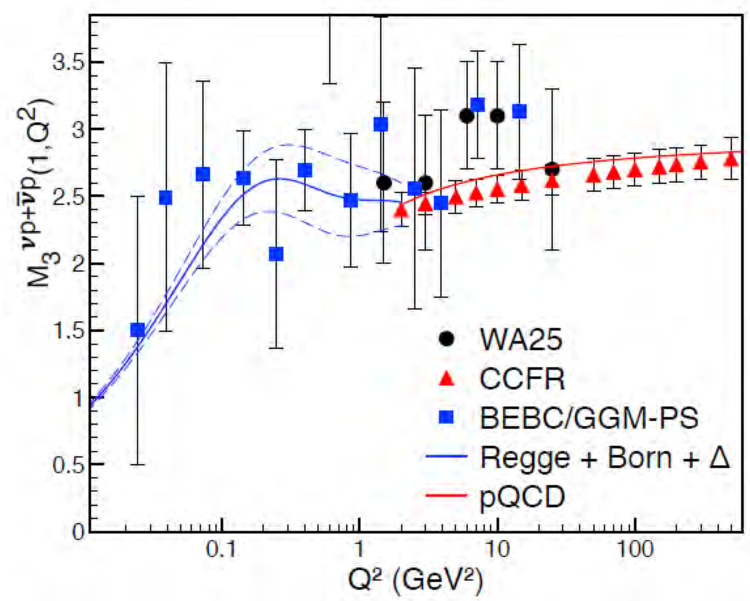




Free neutron γW box



Neutrino scattering data



Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $|V_{ud}| = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $|V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

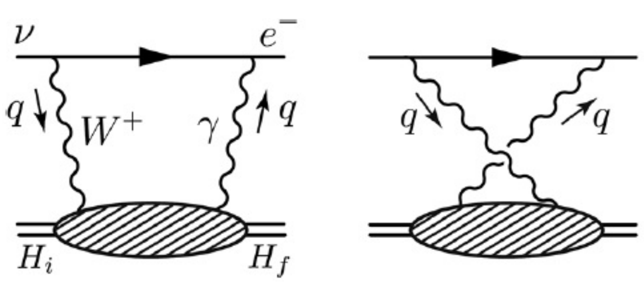
Main limitation: low quality of neutrino data (old bubble-chamber experiments)
Better neutrino data from DUNE (Snowmass 2022 LOI in preparation)
Next breakthrough: first principle calculation on the lattice

First lattice QCD calculation of γW-box

Neutron γW-box - complicated
Address (very rare! BR ~ 10^{-8}) pion decay $\pi^+ \to \pi^0 + e^+ + \nu_e$

Partial decay width: $$\Gamma_{\pi\ell 3} = \frac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3}(1+\delta)I_\pi = 0.3988(23)\,\mathrm{s}^{-1}$$

Form factor: well under control
RC: estimate in χPT: $\delta = 0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}}$ *Cirigliano et al., 2003*



$$\Box_{\gamma W}^{VA} = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_{3\pi}^{\gamma W(0)}(Q^2)$$

All values of Q contribute to the integral
Use perturbative QCD 4-loop result for $Q^2 \geq 2\,\mathrm{GeV}^2$

$$M_\pi(Q^2) = \frac{1}{12}\left[1 - \tilde{C}_1\left(\frac{\alpha_S}{\pi}\right) - \tilde{C}_2\left(\frac{\alpha_S}{\pi}\right)^2 - \tilde{C}_3\left(\frac{\alpha_S}{\pi}\right)^3 - \tilde{C}_4\left(\frac{\alpha_S}{\pi}\right)^4 - \ldots\right]$$

For low $Q^2 \leq 2\,\mathrm{GeV}^2$: direct lattice calculation of the generalized Compton tensor
Feng, MG, Jin, Ma, Seng 2003.09798

First lattice QCD calculation of γW-box

Main executors: Xu Feng (Peking U.), Lu-Chang Jin (UConn/RIKEN BNL)
Supercomputers: Blue Gene/Q Mira computer (Argonne, USA),
Tianhe 3 prototype (Tianjin, China)

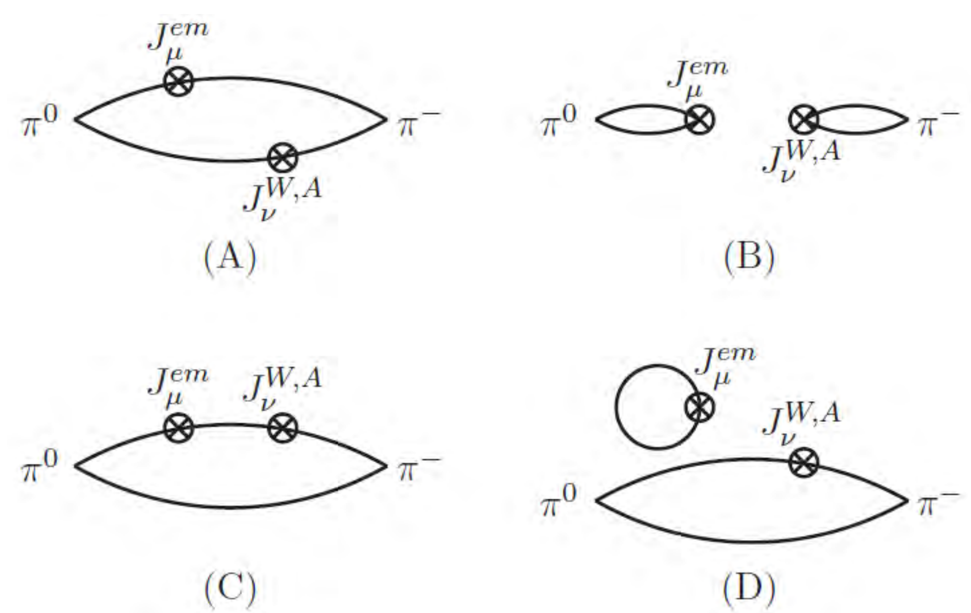
$$\mathcal{H}^{VA}_{\mu\nu}(t,\vec{x}) \equiv \langle H_f(P)|T\left[J^{em}_{\mu}(t,\vec{x})J^{W,A}_{\nu}(0)\right]|H_i(P)\rangle$$

$$M^{\gamma W(0)}_{3\pi}(Q^2) = -\frac{1}{6\sqrt{2}}\frac{Q}{m_\pi}\int d^4x\,\omega(Q,x)\epsilon_{\mu\nu\alpha 0}x_\alpha \mathscr{H}^{VA}_{\mu\nu}(x)$$

Lattice setup:
5 LQCD gauge ensembles at physical pion mass
Generated by RBC and UKQCD collaborations
w. 2+1 flavor domain wall fermion

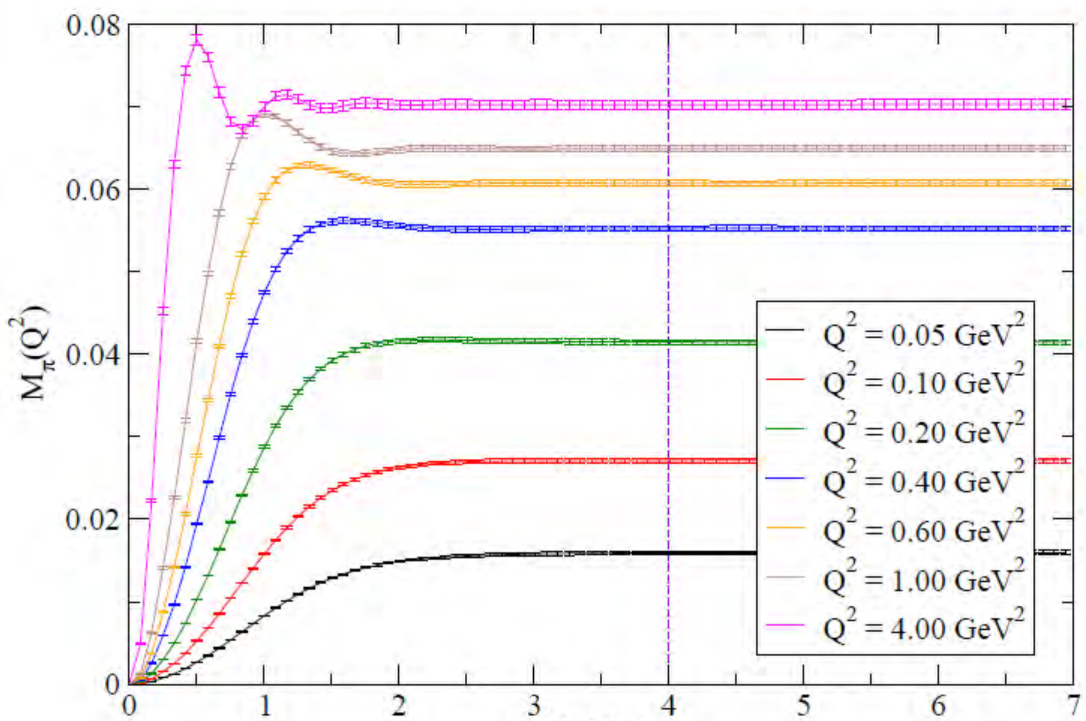
Ensemble	m_π [MeV]	L	T	a^{-1} [GeV]	N_{conf}	N_r	$\Delta t/a$
24D	141.2(4)	24	64	1.015	46	1024	8
32D	141.4(3)	32	64	1.015	32	2048	8
32D-fine	143.0(3)	32	64	1.378	71	1024	10
48I	135.5(4)	48	96	1.730	28	1024	12
64I	135.3(2)	64	128	2.359	62	1024	18

Blue: DSDR
Red : Iwasaki

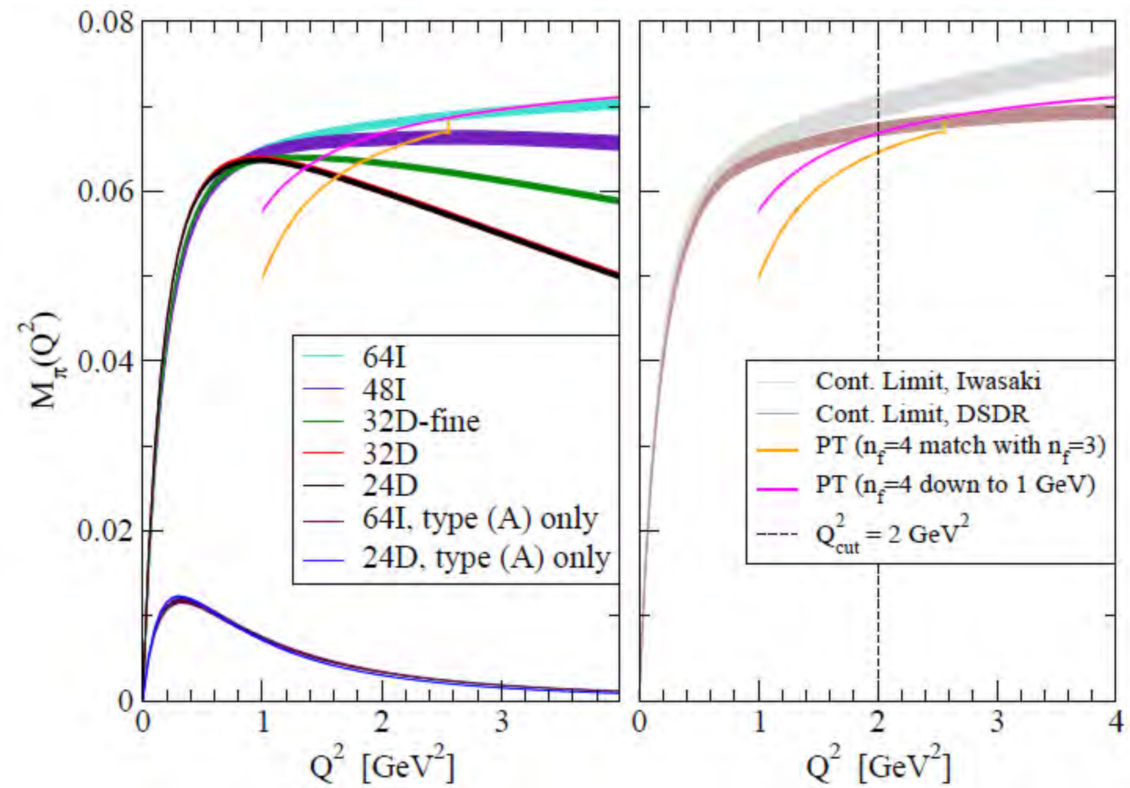


Quark contraction diagrams

First lattice QCD calculation of γW-box



(integral range, 64I)



(before cont. extrapolation) (after cont. extrapolation)

Estimate of major systematic effects:

- **Lattice discretization effect**: Estimated using the discrepancy between DSDR and Iwasaki
- **pQCD calculation**: Estimated from the difference between 3-loop and 4-loop results
- **Higher-twist effects at large Q^2**: Estimated from lattice calculation of type (A) diagrams

Final result: $\square^{VA}_{\gamma W}\big|_\pi = 2.830(11)_{\mathrm{stat}}(26)_{\mathrm{syst}} \times 10^{-3}$

Significant reduction of the uncertainty! $\delta: \quad 0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}} \to 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}$

Cleanest way to access V_{ud} theoretically: $|V_{ud}| = 0.9740(28)_{\mathrm{exp}}(1)_{\mathrm{th}}$

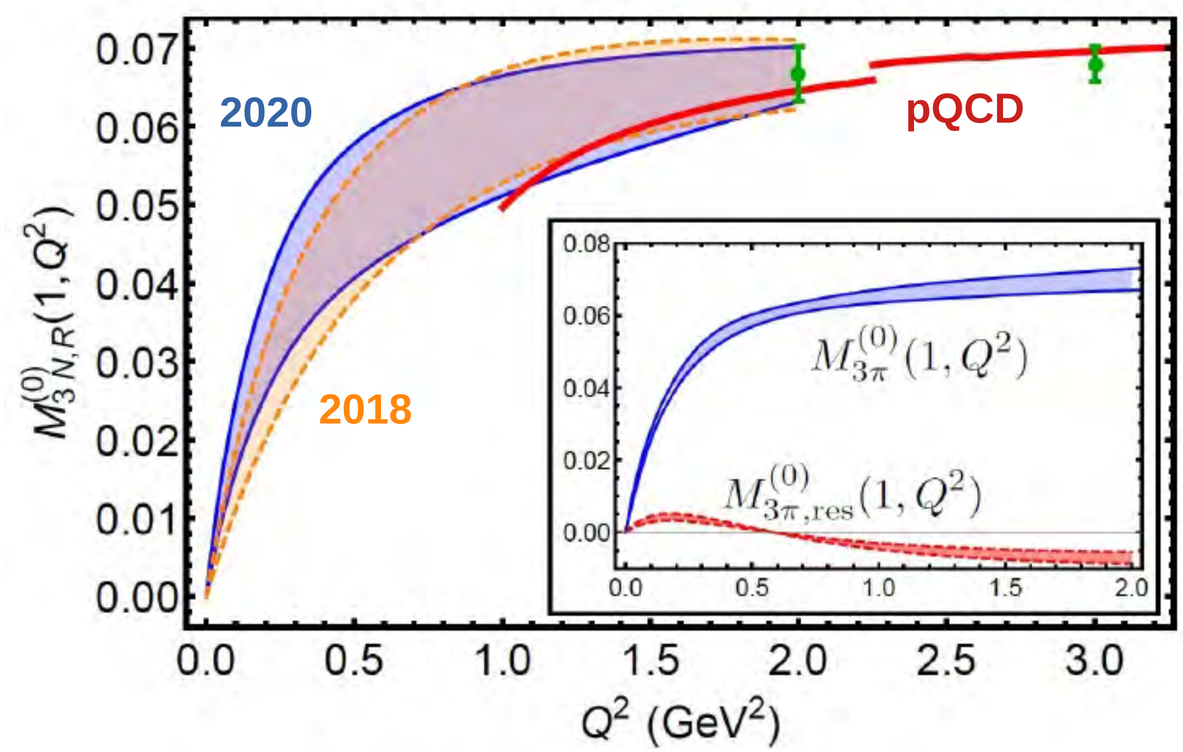
Next-gen experiments: aim at 1 o.o.m. exp. uncertainty improvement

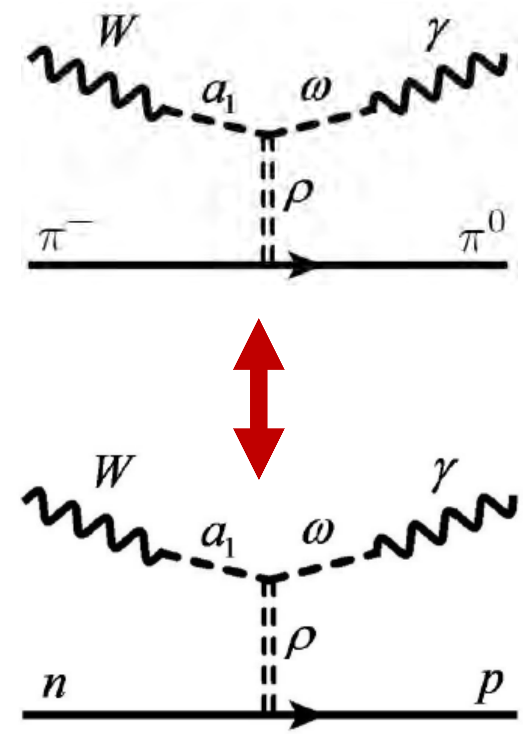
Implications for the free nucleon γW-box

Main uncertainty of the DR calculation of the free neutron γW-box:
Poorly constrained parameters of the Regge contribution which dominates the Nachtmann moment at $Q^2 \sim 1-2\,\mathrm{GeV}^2$
Use the Regge universality and a body of $\pi\pi$, πN, NN scattering data.

$$\frac{T^{\rho}_{W^+ + \pi^- \to \gamma + \pi^0}}{T^{\rho}_{W^+ + n \to \gamma + p}} = \frac{T^{\rho}_{\pi\pi \to \pi\pi}}{T^{\rho}_{\pi N \to \pi N}} = \frac{T^{\rho}_{\pi N \to \pi N}}{T^{\rho}_{NN \to NN}}$$

Seng, MG, Feng, Jin, 2003.11264





Independent confirmation of the empirical DR result AND uncertainty

$$\Delta_R^V = 0.02467(22)_{\mathrm{DR}} \to 0.02477(24)_{\mathrm{LQCD+DR}}$$

DR result confirmed by other groups

Czarnecki, Marciano, Sirlin, Phys.Rev. D 100 (2019) 7, 073008
Shiells, Blunden, Melnitchouk, Phys.Rev.D 104 (2021) 3, 033003
Hayen, Phys.Rev.D 103 (2021) 11, 113001

Summary of universal RC

0⁺-0⁺ nuclear decays $|V_{ud}|^2 = \frac{2984.43 s}{\mathscr{F}t(1+\Delta_R^V)}$ $|V_{ud}^{0^+-0^+}| = 0.9737\,(1-3)_{\text{exp+nucl}}\,(1)_{RCn}$

Free neutron decay $|V_{ud}|^2 = \frac{5024.7\ \text{s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R)}$ $|V_{ud}^{\text{free n}}| = 0.9733\,(3)_{\tau_n}\,(3)_{g_A}\,(1)_{RCn}$

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$ $|V_{ud}|^2 = \frac{0.9799}{(1+\delta)}\frac{\Gamma_{\pi\ell 3}}{0.3988(23)\ \text{s}^{-1}}$ $|V_{ud}^{\pi\ell 3}| = 0.9739\,(27)_{exp}\,(1)_{RC\pi}$

RC uncertainty halved, model dependence (of the uncertainty!) thoroughly tested

Reason for improvement:
a new method (dispersion relations) allowed to combine independent inputs

Experimental neutrino data + lattice QCD + ChPT + Regge phenomenology

Fully up to date for a 0.01% determination of V_{ud}

With improved experimental precision for (τ_n, g_A) neutron decay becomes competitive

Status of δ_{NS}

Splitting the γW-box into Universal and Nuclear Parts

RC for nuclear decay $ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_\mathrm{R}^\mathrm{V}) = \mathrm{ft}(1 + \delta_\mathrm{R}')(1 - \delta_\mathrm{C} + \delta_\mathrm{NS})(1 + \Delta_\mathrm{R}^\mathrm{V})$

RC on a free neutron $$\Delta_R^V \propto F_3^{\mathrm{free\,n}} \propto \int dx e^{iqx} \sum_X \langle p | J_{em}^{\mu,(0)}(x) | X \rangle \langle X | J_W^{\nu,+}(0) | n \rangle$$

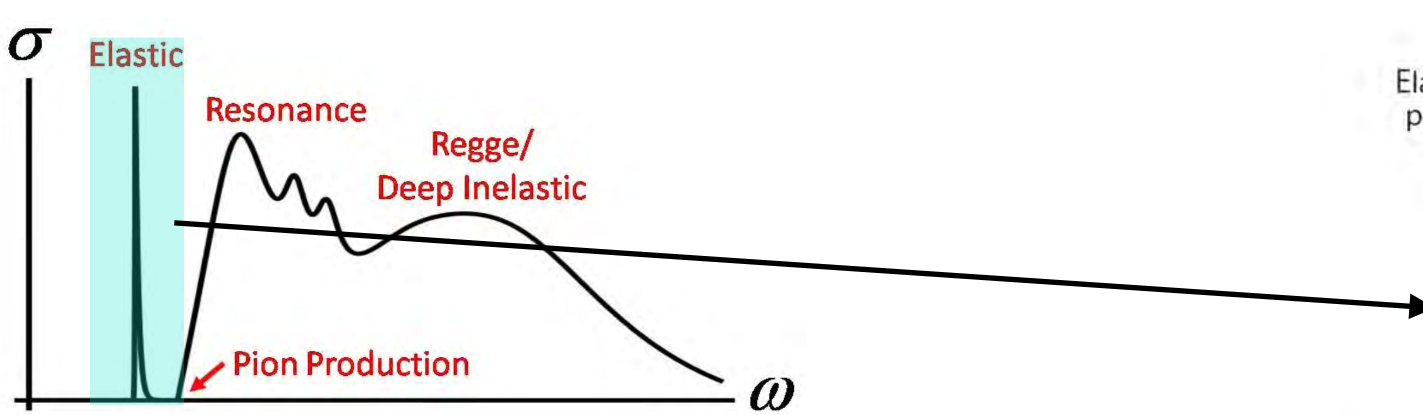
RC on a nucleus $$\Delta_R^V + \delta_{NS} \propto F_3^{\mathrm{Nucl.}} \propto \int dx e^{iqx} \sum_{X'} \langle A' | J_{em}^{\mu,(0)}(x) | X' \rangle \langle X' | J_W^{\nu,+}(0) | A \rangle$$

NS correction reflects this extraction of the free box

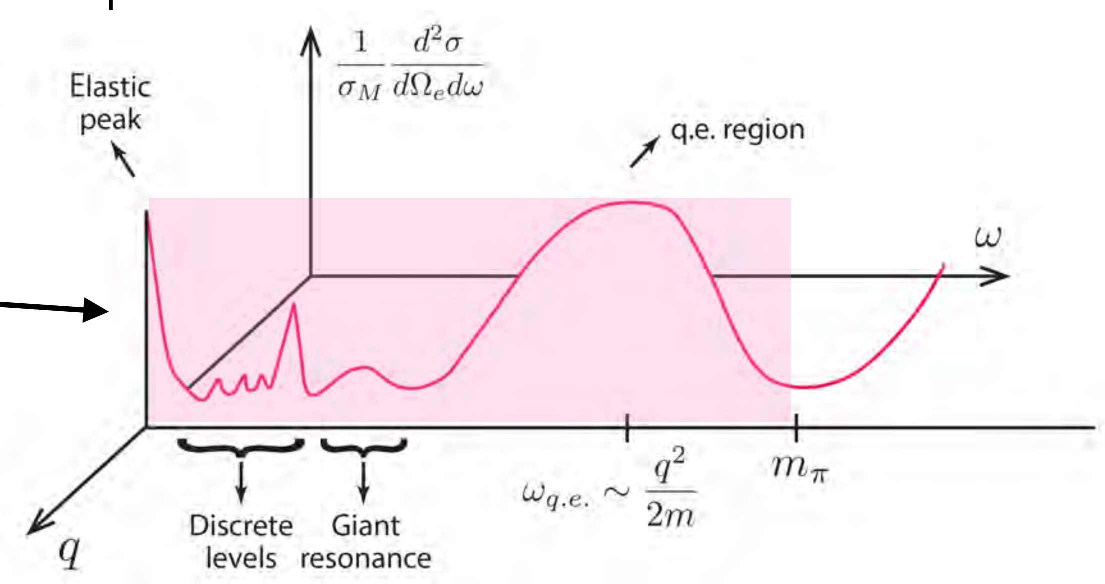
$$\Box_{\gamma W}^{\mathrm{VA,\ Nucl.}} = \Box_{\gamma W}^{\mathrm{VA,\ free\ n}} + \left[\Box_{\gamma W}^{\mathrm{VA,\ Nucl.}} - \Box_{\gamma W}^{\mathrm{VA,\ free\ n}}\right] \qquad \delta_\mathrm{NS} = 2[\,\Box_{\gamma W}^{\mathrm{VA,\,nucl}} - \Box_{\gamma W}^{\mathrm{VA,\,free\,n}}\,]$$

Nuclear modification in the lower part of the spectrum

Input in the DR for the universal RC



Input in the DR for the RC on a nucleus



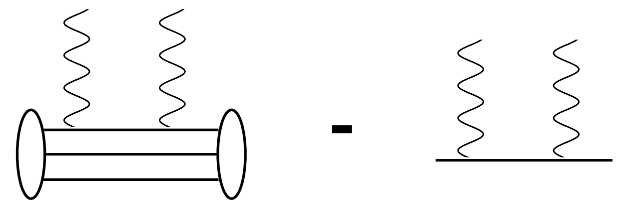
Splitting the γW-box into Universal and Nuclear Parts

Need to know the full nuclear Green's function
indices k, l count the nucleon d.o.f. in a nucleus

$$T^{\gamma W\,\mathrm{nuc}}_{\mu\nu} \sim \sum_{k,\ell} \langle f | J^W_\mu(k)\, G_{\mathrm{nuc}}\, J^{\mathrm{EM}}_\nu(\ell) | i \rangle$$

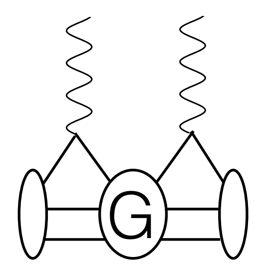
$\delta_{\mathrm{NS}} =$

(A) same active nucleon



Modified Born

(B) two nucleons correlated by G



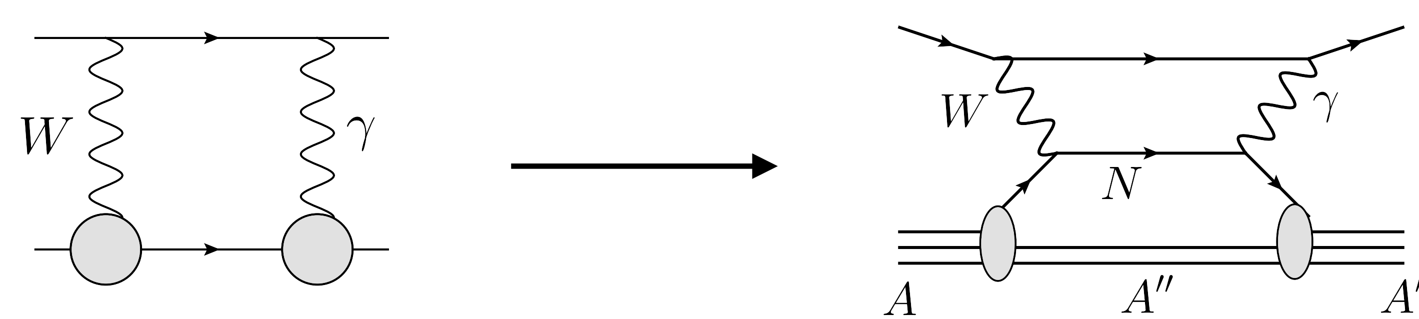
Specifically nuclear effect

Case (A): non-interacting (=on-shell) neutron propagating between interaction vertices
Case (B): all two-nucleon contributions (QE 2p2h and nuclear excitations)

Insert on-shell intermediate states:

$$T^A_{\mu\nu} \to \sum_k \langle f | J^W_\mu(k) [S^N_F \otimes G^{A''}_{nuc}] J^{EM}_\nu(k) | i \rangle$$

The elastic nucleon box
is replaced by a single N QE knockout



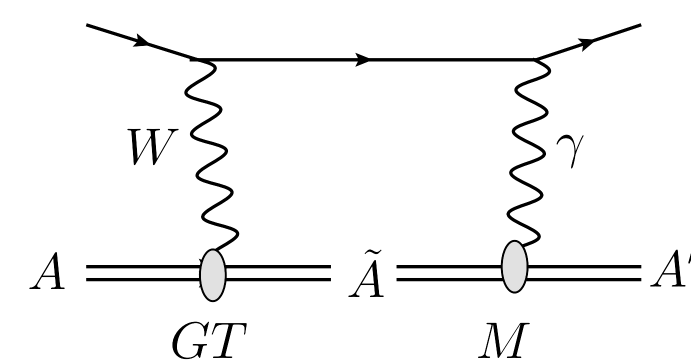
Universal vs. Nuclear Corrections

Towner 1994 and ever since: $\Box_{\gamma W}^{\text{quenched Born}} - \Box_{\gamma W}^{\text{Born}} = [q_S^{(0)} q_A - 1]\,\Box_{\gamma W}^{\text{Born}}$

Idea: calculate Gamow-Teller and magnetic nuclear transitions in the shell model;
Insert the single nucleon spin operators —> predict the strength of nuclear transitions
"Quenching of spin operators in nuclei": shell model overestimates those strengths!
Each vertex is suppressed by 10-15%

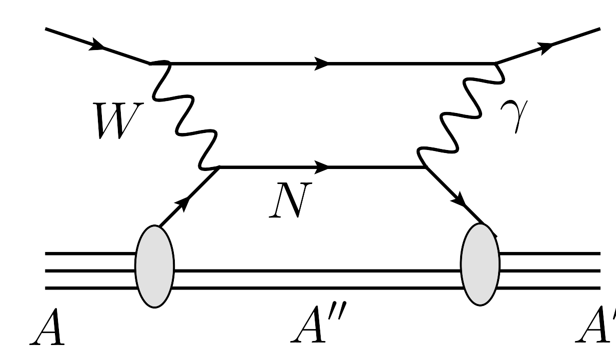
Numerically: on average between the 14 superallowed decays

$$\delta_{NS}^{quenched\,Born} = [q_S^{(0)} q_A - 1] 2\,\Box_{\gamma W}^{\text{free n, Born}} \approx -0.058(14)\,\%$$



But from dispersion relation perspective it corresponds
to a contribution of an excited nuclear state,
not to the modified box on a free nucleon!
The correct estimate should base on quasielastic knockout
with an on-shell N + spectator in the intermediate state

Note that continuum is outside shell model Hilbert space!



Splitting the γW-box into Universal and Nuclear Parts

$$\delta_{NS} = \frac{\alpha}{\pi}\left[C_{NS} + C_B^{\text{quenched}}\right] \approx 0.22\,\%\,\left[C_{NS} + C_B^{\text{quenched}}\right]$$

Hardy, Towner 2002 review

Parent	Unquenched	Quenched C_{NS}					$(q-1)\times$	$\delta_{NS}(\%)$	
nucleus	C_{NS}	os	ss	ov	sv	total	$C_{\text{Born}}(\text{free})$	Quenched	Adopted
$T_z=-1$:									
^{10}C	−1.669	0.002	−0.283	−0.002	−1.065	−1.348	−0.188	−0.357	−0.360(35)
^{14}O	−1.360	−0.008	−0.341	0.082	−0.782	−1.049	−0.221	−0.295	−0.250(50)
^{18}Ne	−1.531	−0.011	−0.249	−0.119	−0.812	−1.191	−0.210	−0.325	−0.290(35)
^{22}Mg	−1.046	−0.009	−0.222	−0.067	−0.497	−0.796	−0.226	−0.237	−0.240(20)
^{26}Si	−0.986	−0.007	−0.224	−0.086	−0.424	−0.741	−0.242	−0.228	−0.230(20)
^{30}S	−0.800	0.002	−0.287	0.020	−0.300	−0.566	−0.257	−0.191	−0.190(15)
^{34}Ar	−0.770	0.014	−0.322	0.061	−0.272	−0.519	−0.273	−0.184	−0.185(15)
^{38}Ca	−0.693	0.041	−0.358	0.091	−0.214	−0.440	−0.288	−0.169	−0.180(15)
^{42}Ti	−1.011	−0.016	−0.181	−0.225	−0.354	−0.776	−0.256	−0.240	−0.240(20)
$T_z=0$:									
^{26m}Al	0.352	−0.007	−0.224	0.086	0.424	0.279	−0.242	0.009	0.009(20)
^{34}Cl	−0.135	0.015	−0.333	−0.064	0.280	−0.101	−0.273	−0.087	−0.085(15)
^{38m}K	−0.276	0.042	−0.363	−0.093	0.216	−0.198	−0.288	−0.113	−0.100(15)
^{42}Sc	0.472	−0.016	−0.182	0.228	0.358	0.389	−0.256	0.031	0.030(20)
^{46}V	0.101	−0.004	−0.197	0.099	0.198	0.096	−0.263	−0.039	−0.040(7)
^{50}Mn	0.054	−0.009	−0.184	0.104	0.152	0.063	−0.270	−0.048	−0.042(7)
^{54}Co	0.161	−0.013	−0.180	0.133	0.203	0.144	−0.277	−0.031	−0.029(7)
^{62}Ga	0.172	0.005	−0.289	−0.058	0.445	0.103	−0.289	−0.043	−0.040(20)
^{66}As	0.124	0.006	−0.291	−0.070	0.421	0.066	−0.295	−0.053	−0.050(20)
^{70}Br	0.077	0.009	−0.295	−0.083	0.401	0.032	−0.301	−0.063	−0.060(20)
^{74}Rb	0.155	0.009	−0.261	0.006	0.353	0.106	−0.306	−0.046	−0.065(20)

Nuclear Structure Modification

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352
MG, arXiv: 1812.04229

δ_{NS} from DR with energy dependence averaged over the spectrum

$$\delta_{NS} = \frac{2\alpha}{\pi NM}\int_0^{1\,\mathrm{GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu}\left[\frac{\nu+2q}{(\nu+q)^2}\left(F_3^{(0)\,Nucl.} - F_3^{(0),\,B}\right) + \frac{2\langle E\rangle}{3}\frac{\nu+3q}{(\nu+q)^3}F_3^{(-)\,Nucl.}\right]$$

$\Lambda_{\rm nuc} \sim Q \sim 10\,\mathrm{MeV}$: nuclear structure leaks from inner into outer RC ("γW-box inside-out")

Compare the effect on the average Ft value:

HT value 2018:	Old estimate:	$\delta\mathscr{F}t = -\,(1.8 \pm 0.4)s \quad + \quad (0 \pm 0)s$
$\mathscr{F}t = 3072.1(7)s$	New estimate:	$\delta\mathscr{F}t = -\,(3.5 \pm 1.0)s \quad + \quad (1.6 \pm 0.5)s$

Two 2σ corrections that cancel each other;
The cancellation is delicate: the two terms are highly correlated
Larger E-dep. term will correspond to a smaller negative E-indep. term and vv.

Conservative uncertainty estimate: 100% $\quad \mathscr{F}t = (3072 \pm 2)s$

Emphasize: until a complete dispersive $\delta_{\rm NS}$ calculation exists this is only a hint!

Status of δ_C

Isospin symmetry breaking in superallowed β-decay

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

Fermi matrix element: $$M_F = \sum_{\alpha,\beta} \langle f | a^\dagger_\alpha a_\beta | i \rangle \langle \alpha | \tau_+ | \beta \rangle$$

$a_\alpha{}^\dagger$ creates a neutron in the state α
a_β annihilates a proton in the state β

Single-particle m. e.

$$\langle \alpha | \tau_+ | \beta \rangle = \delta_{\alpha,\beta} \int_0^\infty R^n_\alpha(r) R^p_\beta(r) r^2 dr \equiv \delta_{\alpha,\beta} r_\alpha$$

R^n_α, R^p_β - neutron and proton radial WF

Insert a complete set of states (in practice - dominant shells)

$$M_F = \sum_{\alpha,\pi} \langle f | a^\dagger_\alpha | \pi \rangle \langle \pi | a_\alpha | i \rangle r^\pi_\alpha$$

Exact isospin symmetry: $\langle \pi | a_\alpha | i \rangle = \langle f | a^\dagger_\alpha | \pi \rangle^*$ and $r^\pi_\alpha = 1$

$$M_0 = \sum_{\alpha,\pi} |\langle f | a^\dagger_\alpha | \pi \rangle|^2 \xrightarrow{\text{ISB}} |M_F|^2 = |M_0|^2 (1 - \delta_C)$$

$$\delta_C = \delta_{C1} + \delta_{C2}$$

TABLE X. Corrections δ'_R, δ_{NS}, and δ_C that are applied to experimental ft values to obtain $\mathcal{F}t$ values.

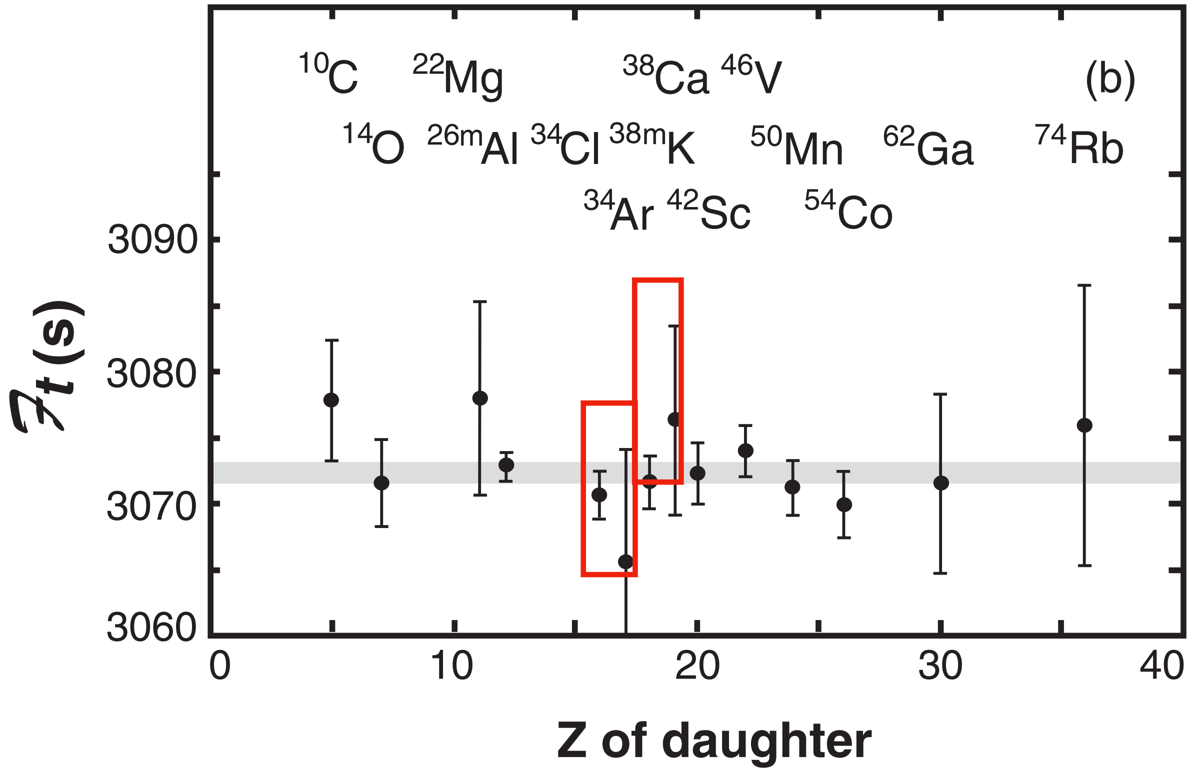
Parent nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	δ_C (%)
$T_z = -1$					
^{10}C	1.679	−0.345(35)	0.010(10)	0.165(15)	0.175(18)
^{14}O	1.543	−0.245(50)	0.055(20)	0.275(15)	0.330(25)
^{18}Ne	1.506	−0.290(35)	0.155(30)	0.405(25)	0.560(39)
^{22}Mg	1.466	−0.225(20)	0.010(10)	0.370(20)	0.380(22)
^{26}Si	1.439	−0.215(20)	0.030(10)	0.405(25)	0.435(27)
^{30}S	1.423	−0.185(15)	0.155(20)	0.700(20)	0.855(28)
^{34}Ar	1.412	−0.180(15)	0.030(10)	0.665(55)	0.695(56)
^{38}Ca	1.414	−0.175(15)	0.020(10)	0.745(70)	0.765(71)
^{42}Ti	1.427	−0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0$					
^{26m}Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
^{34}Cl	1.443	−0.085(15)	0.100(10)	0.550(45)	0.650(46)
^{38m}K	1.440	−0.100(15)	0.105(20)	0.565(50)	0.670(54)
^{42}Sc	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
^{46}V	1.445	−0.035(10)	0.075(30)	0.545(55)	0.620(63)
^{50}Mn	1.444	−0.040(10)	0.035(20)	0.610(50)	0.645(54)
^{54}Co	1.443	−0.035(10)	0.050(30)	0.720(60)	0.770(67)
^{62}Ga	1.459	−0.045(20)	0.275(55)	1.20(20)	1.48(21)
^{66}As	1.468	−0.060(20)	0.195(45)	1.35(40)	1.55(40)
^{70}Br	1.486	−0.085(25)	0.445(40)	1.25(25)	1.70(25)
^{74}Rb	1.499	−0.075(30)	0.115(60)	1.50(26)	1.62(27)

HT: calculate $\delta_{C1,C2}$ in shell model with *phenomenological* Woods-Saxon potential locally adjusted:

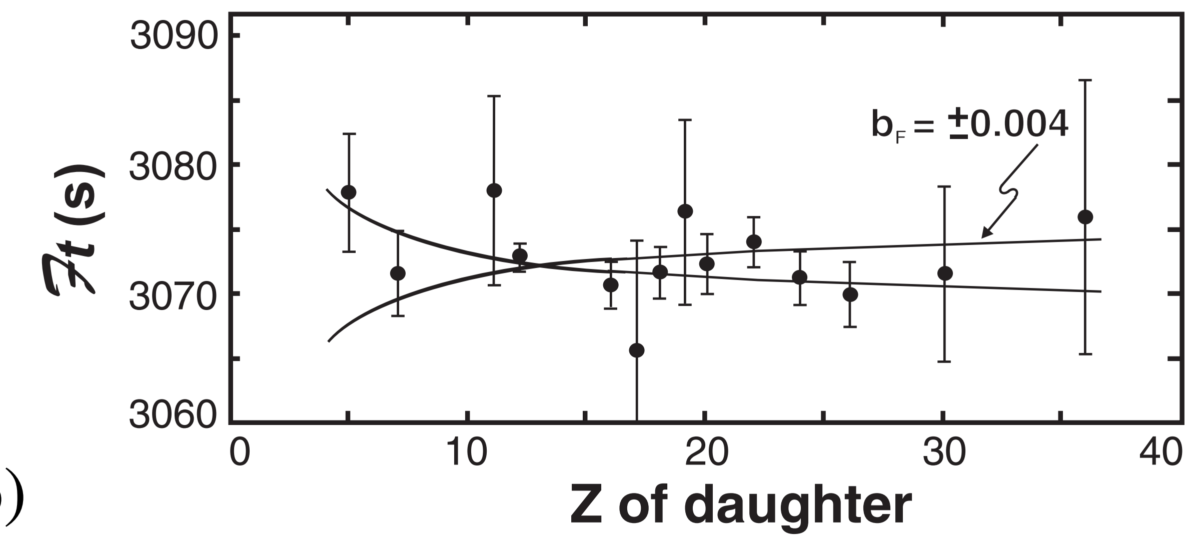
- Masses of the isobaric multiplet T=1, 0+ (e.g. $^{34}Ar - {}^{34}Cl - {}^{34}S$)
- Neutron and proton separation energies
- Known proton (charge) radii of stable isotopes

ISB in superallowed β-decay and test of CVC

Conserved vector current hypothesis —> Ft constant



If Ft were not constant:
Presence of scalar currents - BSM
Fierz interference term $\sim b_F m_e / E_e$



Fit to 14 transitions:
Ft constant within 2×10^{-4} and $b_F = -0.0028(26)$

However: to achieve this precision the model was adjusted locally in each iso-multiplet

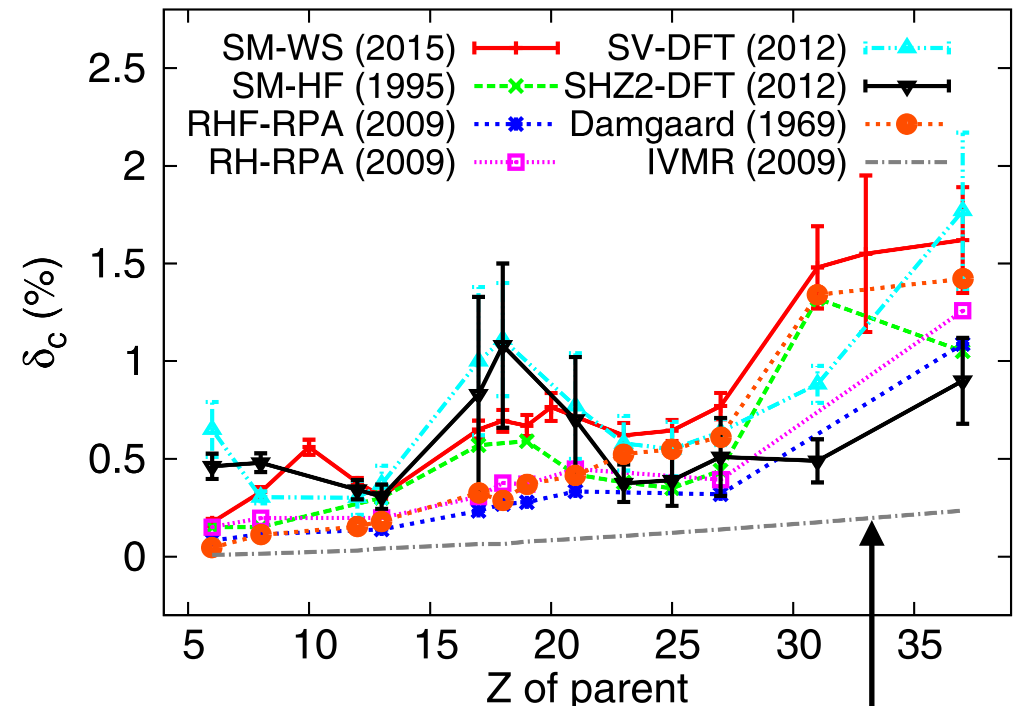
- Is this formalism the right tool to assess consistency amongst all the measurements?
- Red squares: even within one iso-multiplet ($^{34}Ar - {}^{34}Cl - {}^{34}S$, $^{38}Ca - {}^{38m}K - {}^{38}Ar$) discrepancies between central values may be larger than the total uncertainty
- Shell model does not cover all the model space (e.g. continuum)
- HT method criticized for using incorrect isospin formalism (G. Miller, A. Schwenk)
- Ab initio methods do not warrant such high precision

ISB in superallowed β-decay: nuclear model comparison

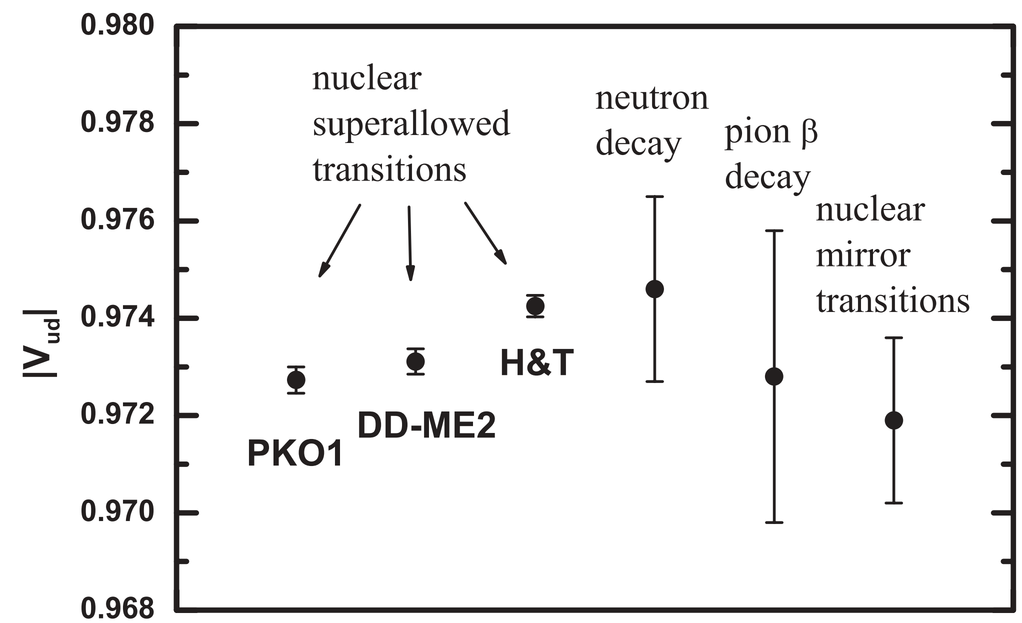
TABLE XI. Recent δ_C calculations (in percent units) based on models labeled SM-WS (shell-model, Woods-Saxon), SM-HF (shell-model, Hartree-Fock), RPA (random phase approximation), IVMR (isovector monopole resonance), and DFT (density functional theory). Also given is the χ^2/ν, χ^2 per degree of freedom, from the confidence test discussed in the text. *J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501*

			RPA				
	SM-WS	SM-HF	PKO1	DD-ME2	PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



IVMR (Iso-Vector Monopole Resonance) method: developed to relate δ_C to IV-sensitive observable



HT: χ^2 as criterium to prefer SM-WS;
V_{ud} and limits on BSM strongly depend on nuclear model

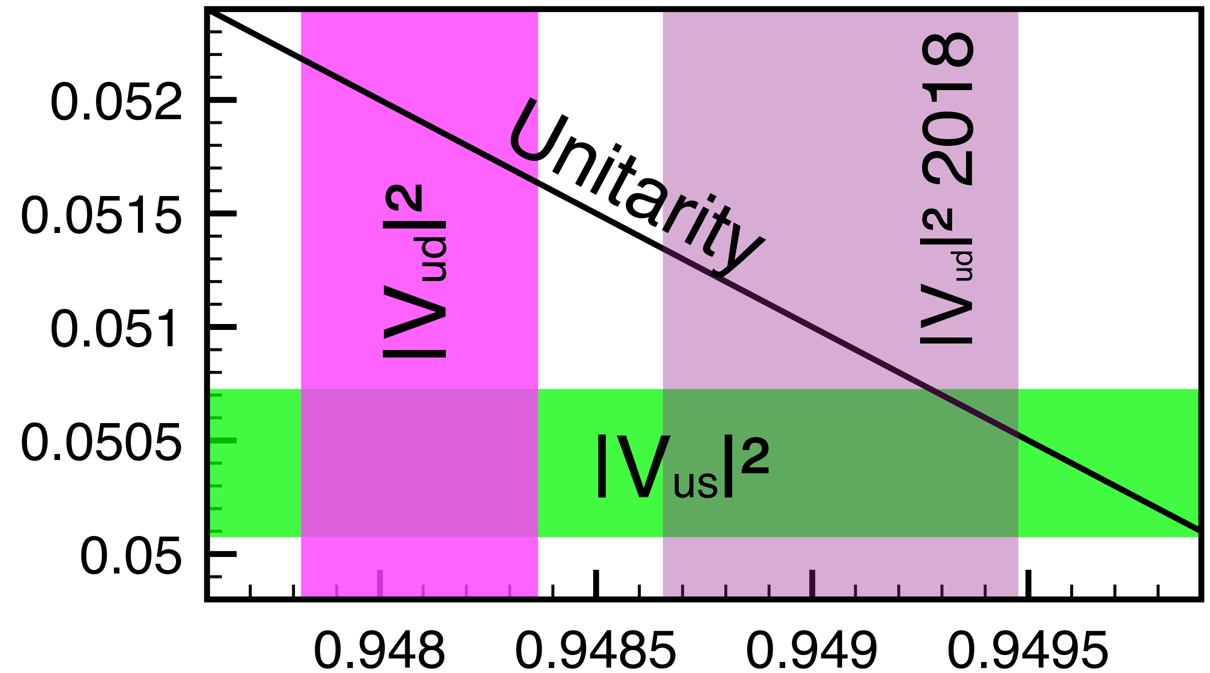
Nuclear community (Hagen, Forssen, Stroberg & friends):
exploratory ISB corrections with modern computational methods (not easy)
Better theory does not guarantee smaller uncertainties!

V_{ud} and top-row CKM unitarity summary

3-sigma CKM unitarity deficit established

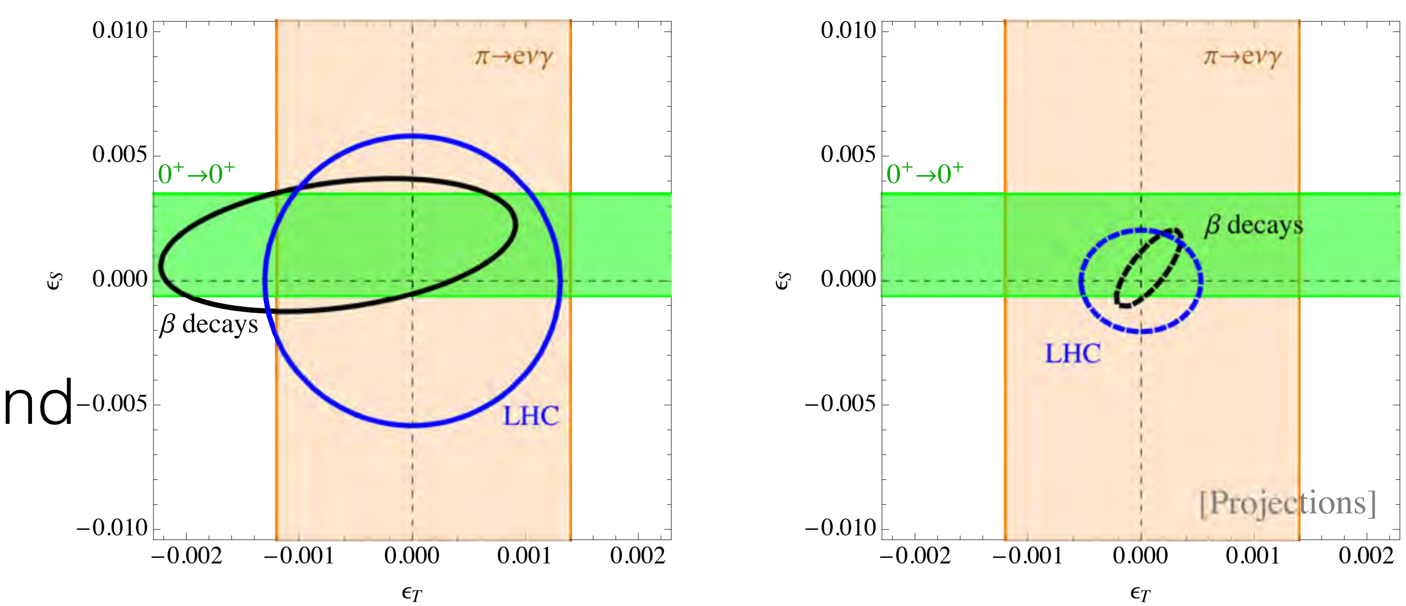
Significant shift in Vud due to RC

New dispersion relation method:
Combined Exp + LQCD + ChPT +…
Unified nuclear and universal RC
Further work necessary



Improvement in understanding theory issues does not guarantee smaller uncertainties

Beta decays remain a BSM testing ground even in the high-lumi LHC era



Gonzalez Alonso, Naviliat-Cuncic, Severijns 2019