11th International Workshop on the CKM Unitarity Triangle
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V_{us} from kaon decays

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V_{us}, CKM unitarity, gauge universality

Standard-model coupling of quarks and leptons to W:

$$\frac{g}{\sqrt{2}} W_\alpha^+ \left(\overline{\mathbf{U}}_L \mathbf{V}_{\text{CKM}} \gamma^\alpha \mathbf{D}_L + \overline{e}_L \gamma^\alpha \nu_{e\,L} + \overline{\mu}_L \gamma^\alpha \nu_{\mu\,L} + \overline{\tau}_L \gamma^\alpha \nu_{\tau\,L} \right) \quad + \quad \text{h.c.}$$

Single gauge coupling

Unitary matrix

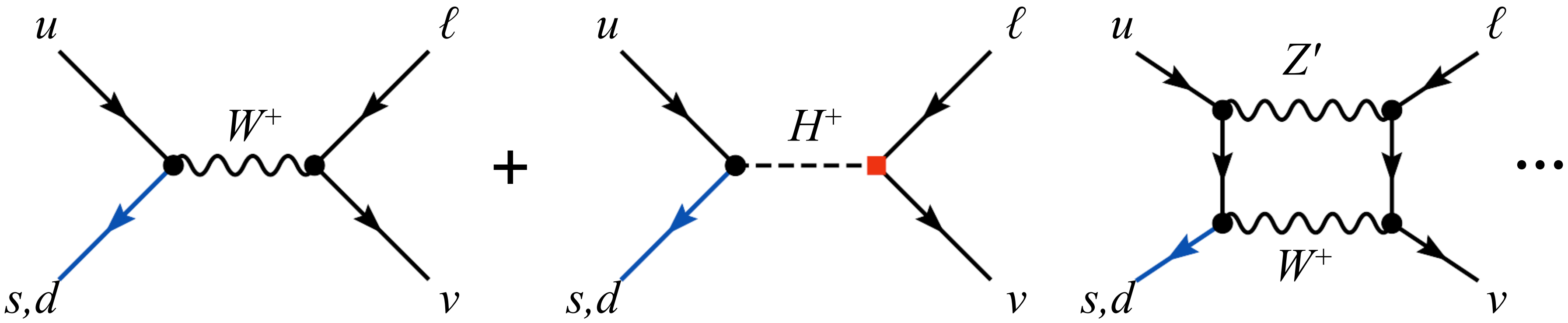
$$|V_{ud}|^2 + |V_{us}|^2 + \underset{\approx 2\times10^{-5}}{|\cancel{V_{ub}}|^2} = 1$$

Most precise test of CKM unitarity

Universality: Is G_F from μ decay equal to G_F from π, K, nuclear β decay?

$$G_\mu^2 = (g_\mu g_e)^2 / M_W^4 \quad \overset{?}{=} \quad G_{\text{CKM}}^2 = (g_q g_\ell)^2 \left(|V_{ud}|^2 + |V_{us}|^2\right) / M_W^4$$

Physics beyond the Standard Model can break gauge universality:



V_{us} from $K_{\ell 3}$: Déjà vu all over again?

→ 2002 (2004 PDG)	**Old $K_{\ell 3}$ data give 1 – $\vert V_{ud}\vert^2$ – $\vert V_{us}\vert^2$ = 0.0035(15)** A 2.3σ hint of unitarity violation?
2003	**BNL 865 measures BR($K^+ \rightarrow \pi^0 e^+ \nu$) = 5.13(10)%** Value for V_{us} consistent with unitarity
2004-2008 (mostly)	**Many new measurements from KTeV, ISTRA+, KLOE, NA48** • **BRs, lifetimes, form-factor slopes** • **Much higher statistics** than older measurements • Importance of **radiative corrections** • Proper reporting of **correlations** between measurements
2008- present	**Much progress on hadronic constants from lattice QCD** Value of V_{us} used in precision tests of the Standard Model
2018- present	**New evaluation of radiative corrections for V_{ud}** $> 3\sigma$ evidence of unitarity violation in first row?

Experiment, theory, and evaluation

V_{us} from $K_{\ell 3}$ & $K_{\ell 2}$

- ~100 measurements of ~10 experimental parameters
- 50+ (and counting!) lattice results for 2 hadronic matrix elements
- Radiative and SU(2)-breaking corrections, ChPT results, etc.

FlaviA net **Kaon WG**

2006-2010 (EU 6FP)

Experimental averages, fits, etc

Selection of results (experiments, corrections)

Evaluation, discussion and intepretation

Final report: EPJC 69 (2010) 399

Periodic, informal updates at conferences since 2010

Corresponding effort to synthesize results from **lattice QCD:**

FLAG

Flavour Lattice Averaging Group

Participation by all major lattice collaborations

Biannual review of lattice results for π, K, B, D physics

2019 review: EPJC 80 (2020) 113

2020 web update available at http://flag.unibe.ch

Determination of V_{us} from $K_{\ell 3}$ data

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2	1/2 for K^+, 1 for K^0
$S_{\rm EW}$	Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$	Rates with well-determined treatment of radiative decays: • Branching ratios • Kaon lifetimes
$I_{K\ell}(\{\lambda\}_{K\ell})$	Integral of form factor over phase space: λs parameterize evolution in t • K_{e3}: Only λ_+ (or λ_+', λ_+'') • $K_{\mu 3}$: Need λ_+ and λ_0

Inputs from theory:

$f_+^{K^0\pi^-}(0)$	Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)
$\Delta_K^{SU(2)}$	Form-factor correction for $SU(2)$ breaking
$\Delta_{K\ell}^{\rm EM}$	Form-factor correction for long-distance EM effects

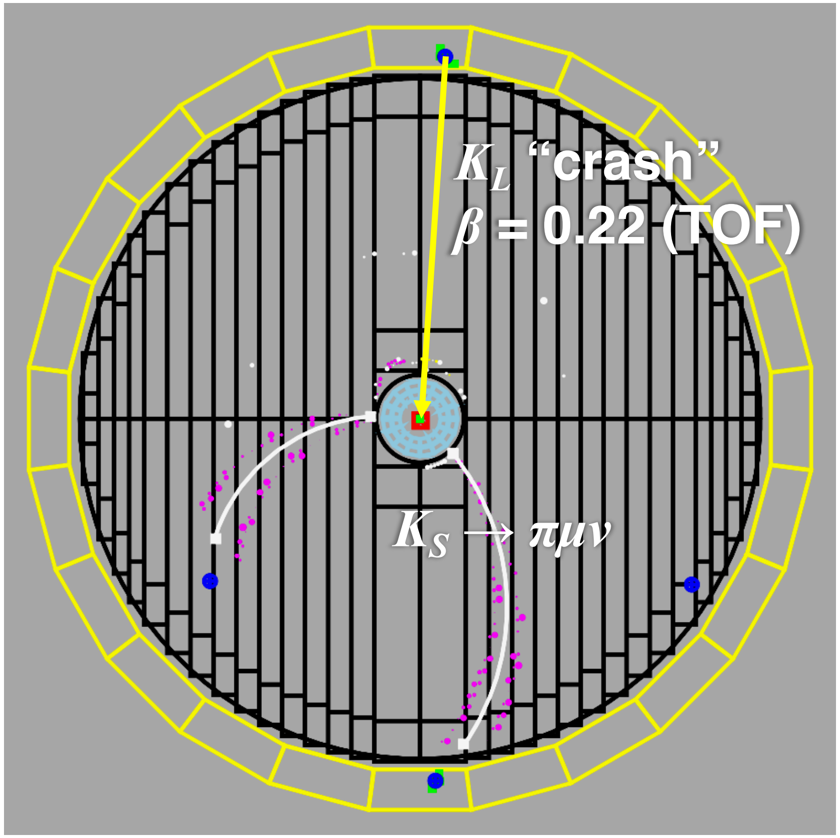
Modern experimental data for V_{us} from $K_{\ell 3}$

Experiment	Measurement	Year
BNL865	**BR($K^+ \rightarrow \pi^0{}_D e^+\nu$)/BR($K^+ \rightarrow \pi^0{}_D X^+$)**	**2003**
KTeV	**$\tau(K_S)$**	**2003**
	BR(K_{Le3}), BR($K_{L\mu3}$), $\lambda_+(K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	**2004**
ISTRA+	**$\lambda_+(K^-{}_{e3})$, $\lambda_{+,0}(K^-{}_{e3})$**	**2004**
KLOE	**$\tau(K_L)$**	**2005**
	BR(K_{Le3}), BR($K_{L\mu3}$), BR(K_{Se3}), $\lambda_+(K_{Le3})$	**2006**
	$\lambda_{+,0}(K_{L\mu3})$	**2007**
	$\tau(K^\pm)$, BR(K_{Le3}), BR($K_{L\mu3}$)	**2008**
NA48	**$\tau(K_S)$**	**2002**
	BR(K_{Le3}/2 tracks), $\lambda_+(K_{Le3})$	**2004**
	$\Gamma(K_{Se3}/K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	**2007**
NA48/2	**BR($K^+{}_{e3}/\pi^+\pi^0$), BR($K+_{\mu3}/\pi^+\pi^0$)**	**2007**

Above data set used for 2010 FlaviaNet review (fits, averages, etc.)

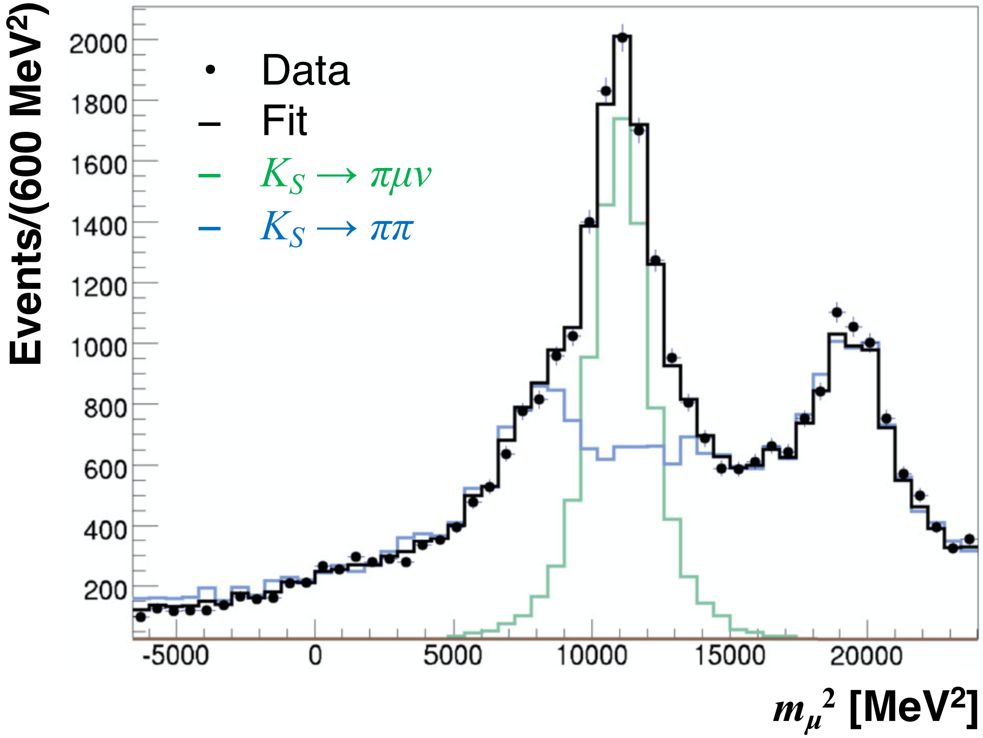
BR($K_S \to \pi\mu\nu$) from KLOE-2

K_S from $\phi \to K_L K_S$ tagged by K_L interaction in calorimeter barrel



Preselection with kinematic BDT and time-of-flight $\pi\mu$ assignment

Fit to $m_\mu^2 = (E_{KS} - E_\pi - p_{\text{miss}})^2 - \mathbf{p}_\mu^2$



KLOE-2
PLB 804 (2020)

BR($K_S \to \pi\mu\nu$) = (4.56 ± 0.20) × 10^{-4}
First measurement of this BR

Dominant systematic from $\pi\mu$ PID by TOF

Fit to K_S rate data

7 input measurements:

- **KLOE** BR $\pi^0\pi^0/\pi^+\pi^-$
- **KLOE** BR $\pi e\nu/\pi^+\pi^-$
- **NA48** $\Gamma(K_S \to \pi e\nu)/\Gamma(K_L \to \pi e\nu)$, τ_S
- **KLOE '11** τ_S
- **KTeV '11** τ_S
- **KLOE-2 '20** BR $\pi\mu\nu$ **New!**

2 possible constraints:

- **Σ BR = 1**
- **BR(K_{e3})/BR($K_{\mu3}$) = 0.6640(17)**
  From ratio of phase-space integrals from current fit to dispersive $K_{\ell3}$ form factor parameters

Only sum constraint used for fit

Parameter	Value
BR($\pi^+\pi^-(\gamma)$)	**69.20(5)%**
BR($\pi^0\pi^0$)	**30.69(5)%**
BR(K_{e3})	**7.05(8) × 10^{-4}**
BR($K_{\mu3}$)	**4.56(20) × 10^{-4}**
τ_S	**89.58(4) ns**

χ^2/ndf = 0.55/3 (Prob = 91%)

No correlation from fit for K_{e3} $K_{\mu3}$

Input measurements essentially unchanged

Fit to K_L rate data

21 input measurements:

- **5 KTeV** ratios
- **NA48** BR(K_{e3}/2 track)
- **4 KLOE** BRs
  with dependence on τ_L
- **KLOE**, **NA48** BR($\pi^+\pi^-/K_{\ell 3}$)
- **KLOE**, **NA48** BR($\gamma\gamma/3\pi^0$)
- **BR($2\pi^0/\pi^+\pi^-$) from K_S fit, Re ε'/ε**
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh '72** τ_L
- **KTeV** BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)
- **E731, 2 KTeV** BR($\pi^+\pi^-\gamma_{\mathrm{DE}}/\pi^+\pi^-\gamma$)

1 constraint: Σ BR = 1

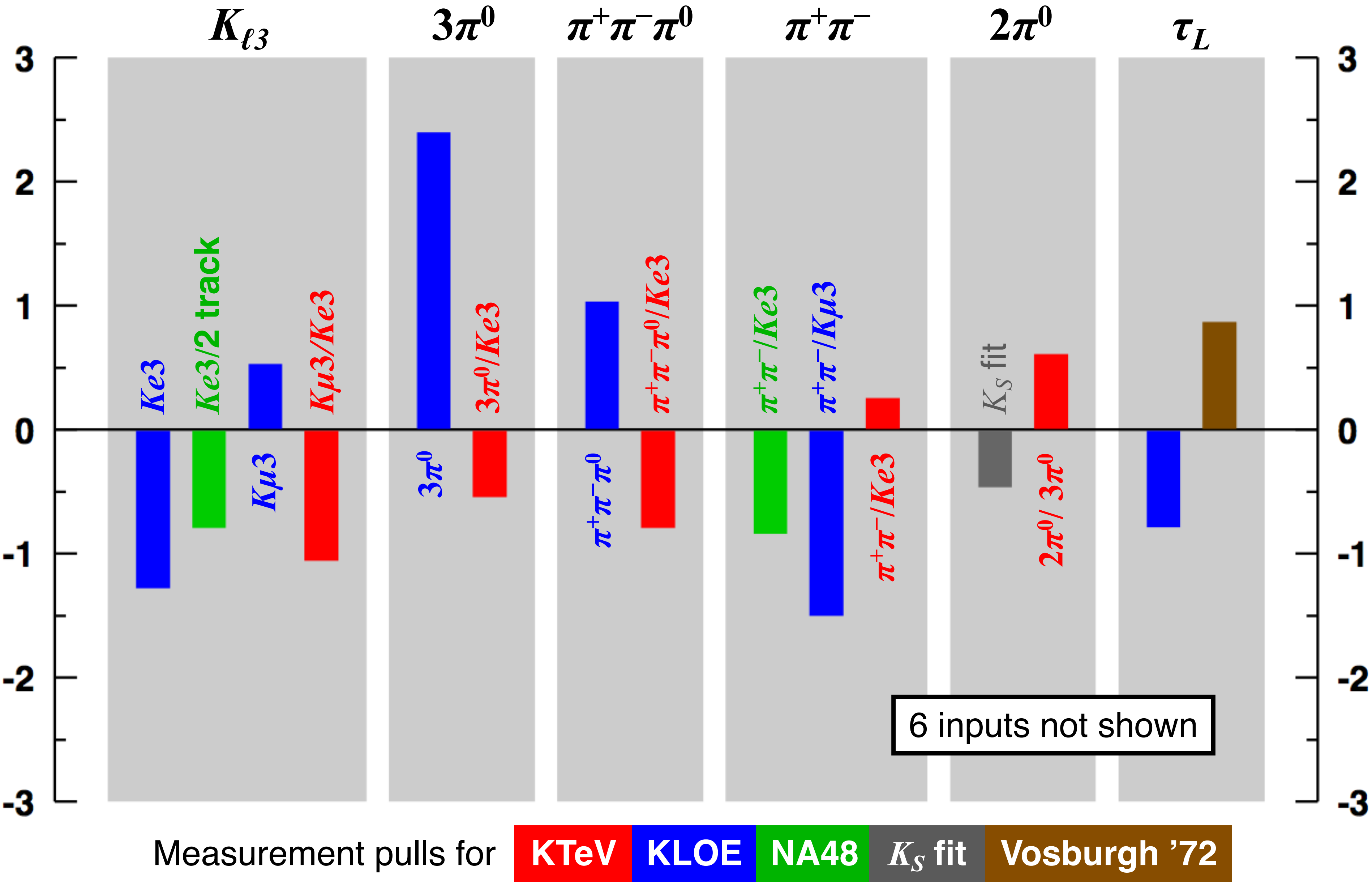
Parameter	**Value**	S
BR(K_{e3})	**0.4056(9)**	**1.3**
BR($K_{\mu 3}$)	**0.2704(10)**	**1.5**
BR($3\pi^0$)	**0.1952(9)**	**1.2**
BR($\pi^+\pi^-\pi^0$)	**0.1254(6)**	**1.3**
BR($\pi^+\pi^-(\gamma_{\mathrm{IB}})$)	**$1.967(7) \times 10^{-3}$**	**1.1**
BR($\pi^+\pi^-\gamma$)	**$4.15(9) \times 10^{-5}$**	**1.6**
BR($\pi^+\pi^-\gamma_{\mathrm{DE}}$)	**$2.84(8) \times 10^{-5}$**	**1.3**
BR($2\pi^0$)	**$8.65(4) \times 10^{-4}$**	**1.4**
BR($\gamma\gamma$)	**$5.47(4) \times 10^{-4}$**	**1.1**
τ_L	**51.16(21) ns**	**1.1**

χ^2/ndf = 19.8/12 (Prob = 7.0%)

Essentially same result as 2010 fit

Current PDG (since '09): **37.4/17** (0.30%)

Comparison: K_L fit result vs. input data



Updated fit to $K^{\pm}$ rate data

17 input measurements:

- **3 old τ values in PDG**
- **KLOE τ**
- **KLOE BR $\mu\nu$, $\pi\pi^0$**
- **KLOE BR K_{e3}, $K_{\mu3}$**
  with dependence on τ
- **NA48/2 BR $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$**
- **E865 BR K_{e3}/KDal**
- **3 old BR $\pi\pi^0/\mu\nu$**
- **KEK-246 $K_{\mu3}/K_{e3}$**
- **KLOE BR $\pi\pi\pi$, $\pi\pi^0\pi^0$**
- (Bisi ’65 BR $\pi\pi^0\pi^0/\pi\pi\pi$ removed)

1 constraint: Σ BR = 1

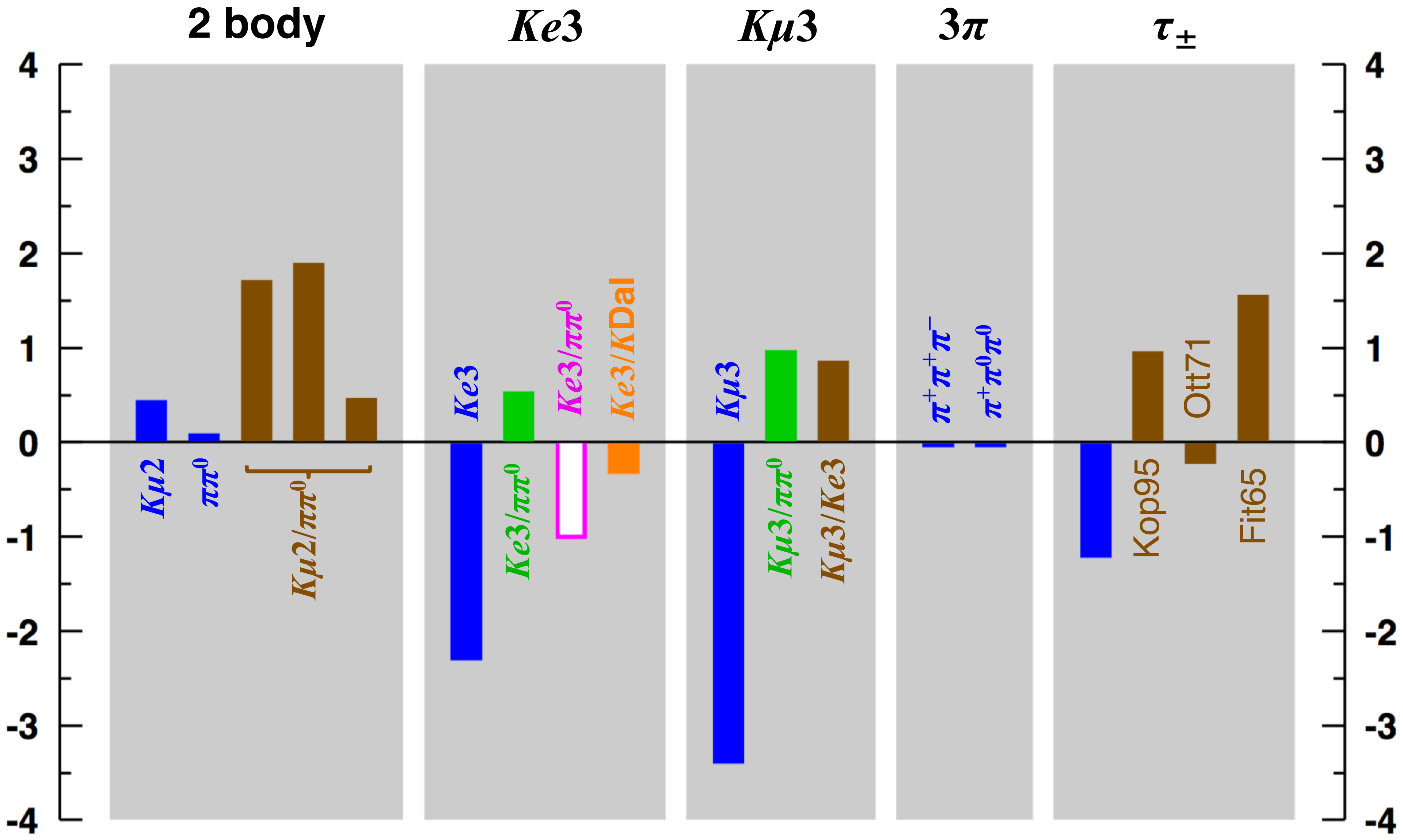
Much more selective than PDG fit
PDG ’16: 35 inputs, 8 parameters

Parameter	**Value**	**S**
BR($\mu\nu$)	**63.58(11)%**	**1.1**
BR($\pi\pi^0$)	**20.64(7)%**	**1.1**
BR($\pi\pi\pi$)	**5.56(4)%**	**1.0**
BR(K_{e3})	**5.088(27)%**	**1.2**
BR($K_{\mu3}$)	**3.366(30)%**	**1.9**
BR($\pi\pi^0\pi^0$)	**1.764(25)%**	**1.0**
$\tau_{\pm}$	**12.384(15) ns**	**1.2**

χ^2/ndf = 25.5/11 (Prob = 0.78%)
compare PDG ’16: 53/28 (0.26%)

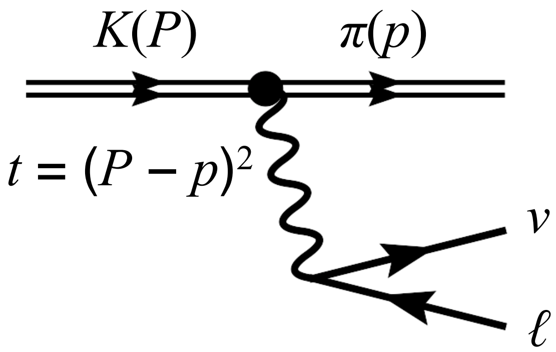
With **ISTRA+ ’14 BR($K^{-}_{e3}/\pi^{-}\pi^0$)**
- **BR(K_{e3}) = 5.083(27)%**
- Negligible changes in other parameters, fit quality

Comparison: $K^{\pm}$ fit result vs. input data



Measurement pulls for **KLOE** **E865** **NA48/2** **ISTRA+** **others**

$K_{\ell 3}$ form factors



Hadronic matrix element:

$$\langle\pi|J_\alpha|K\rangle = f(0) \times \left[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha\right]$$

K_{e3} decays: Only **vector form factor:** $\tilde{f}_+(t)$

$K_{\mu 3}$ decays: Also need **scalar form factor:** $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \frac{t}{m_K^2 - m_\pi^2}$

For V_{us}, need integral over phase space of squared matrix element:

Parameterize form factors and fit distributions in t (or related variables)

Parameterizations based on systematic expansions

Taylor expansion:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0}\left(\frac{t}{m_{\pi^+}^2}\right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0}\left(\frac{t}{m_{\pi^+}^2}\right) + \lambda''_{+,0}\left(\frac{t}{m_{\pi^+}^2}\right)^2$$

Notes:

Many parameters: λ_+', λ_+'', λ_0', λ_0''

Large correlations, unstable fits

Higher-order terms ignored

$K_{\ell 3}$ form-factor parameterizations

Parameterizations incorporating physical constraints

Pole dominance:

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t}$$

Notes:

What does M_S correspond to?

Dispersion relations:

$$\tilde{f}_{+}(t) = \exp\left[\frac{t}{m_\pi^2}(\Lambda_+ - H(t))\right]$$

$$\tilde{f}_{0}(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2}(\ln C - G(t))\right]$$

Notes:

Allows tests of ChPT & low-energy dynamics

$H(t)$, $G(t)$ evaluated from $K\pi$ scattering data and given as polynomials

Bernard et al., PRD 80 (2009)

Uncertainties from representations $H(t)$, $G(t)$ of $K\pi$ phase-shift data contribute to fit results for Λ_+, $\ln C$

– Small compared to other uncertainties for single measurements (so far)

2010 FlaviaNet analysis used average of FF parameters from dispersive fits

– Parameterization uncertainties beginning to dominate averages for Λ_+, $\ln C$

$K_{\ell 3}$ form factor data

Form-factor parameter measurements in FlaviaNet 2010 fit:

K_L: **KTeV, KLOE, NA48** (K_{e3} only)

K^-: **ISTRA+**

Even if not in the original publications, all experiments have:

- Obtained results for Taylor, pole, and dispersive parameterizations
- Supplied parameter correlation coefficients

Recent measurements

NA48/2
JHEP 1810 (2018)

2.3×10^6 $K^\pm_{\mu 3}$
4.4×10^6 $K^\pm_{e3}$

Updates 2012 preliminary

K^+ and K^- simultaneously acquired in dedicated minimum-bias run

Taylor, pole, and dispersive fits with complete investigation of systematics

OKA
JETPL 107 (2018)

5.25×10^6 K^+_{e3}

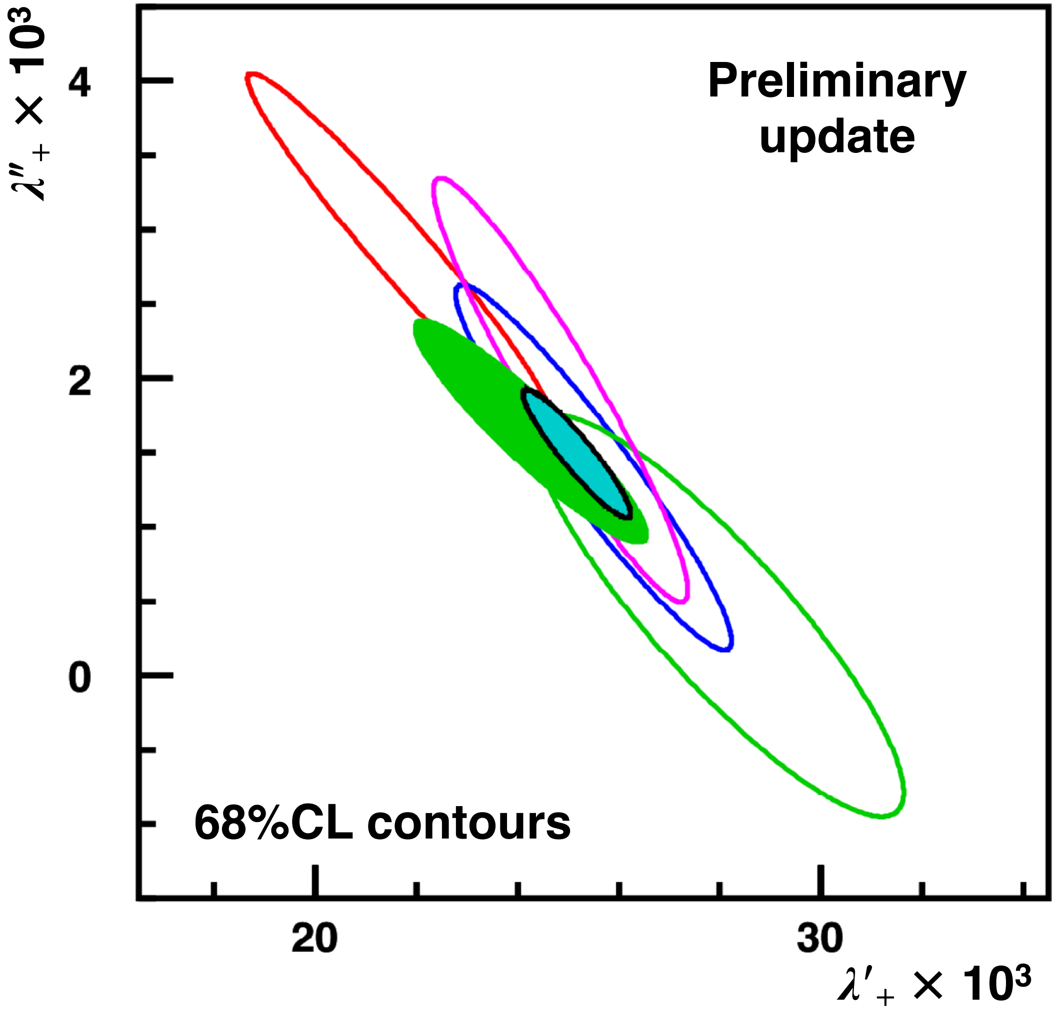
Extraordinarily high precision claimed, esp. for λ_+', λ_+''

Rudimentary discussion of systematics

Not yet included in updated K_{e3} fit

Fit to K_{e3} form-factor slopes

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Current**



Slope parameters × 10^3

λ'_+ = **25.17 ± 0.70**

λ''_+ = **1.49 ± 0.29**

$\rho(\lambda'_+, \lambda''_+)$ = −0.929

χ^2/ndf = 6.4/10 (61%)

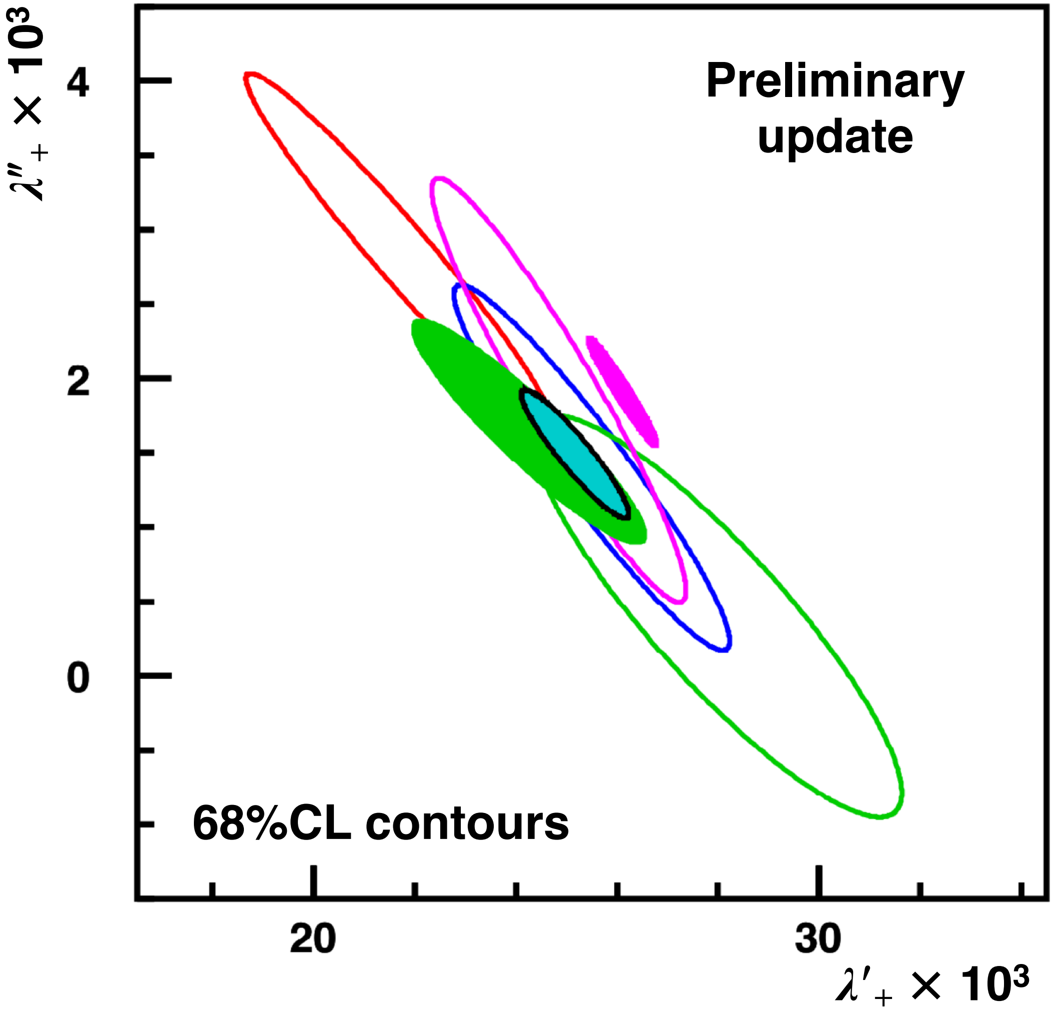
Excellent compatibility

$I(K^0_{e3})$ = 0.15458(19)

$I(K^+_{e3})$ = 0.15895(20)

Fit to K_{e3} form-factor slopes

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Current**



Slope parameters × 10^3

$$\lambda'_+ = \mathbf{25.17 \pm 0.70}$$

$$\lambda''_+ = \mathbf{1.49 \pm 0.29}$$

$$\rho(\lambda'_+, \lambda''_+) = -0.929$$

$$\chi^2/\text{ndf} = 6.4/10\ (61\%)$$

OKA
JETPL 107 (2018)

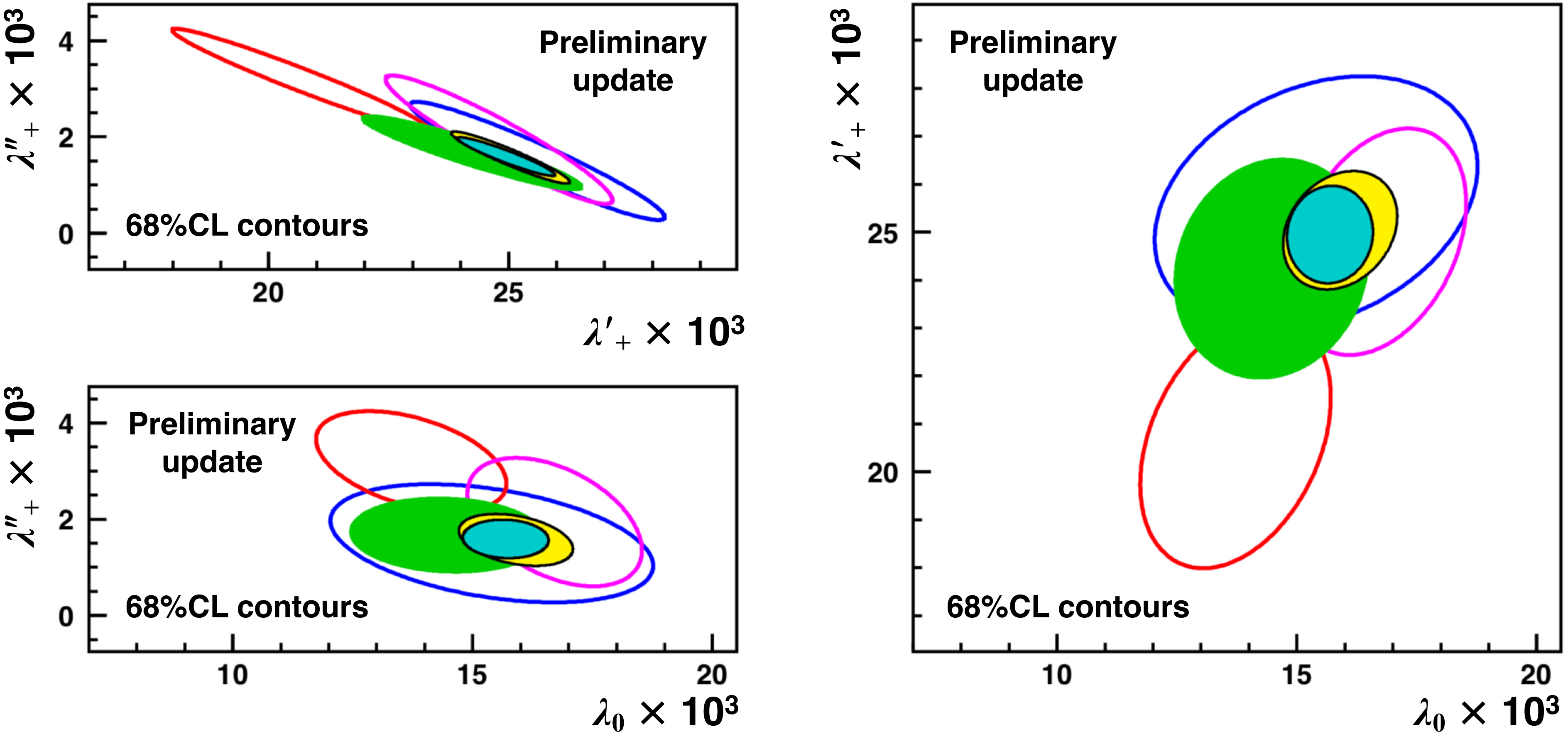
Not included in fit

- Stated as preliminary
- If included: χ^2/ndf → 45/10 ($P \sim 10^{-6}$)

Fits to $K_{e3} + K_{\mu 3}$ form-factor slopes

KTeV **KLOE** **ISTRA+** **NA48/2**   **2010 fit** **Current**

NA48 K_{e3} data included in fits but not shown



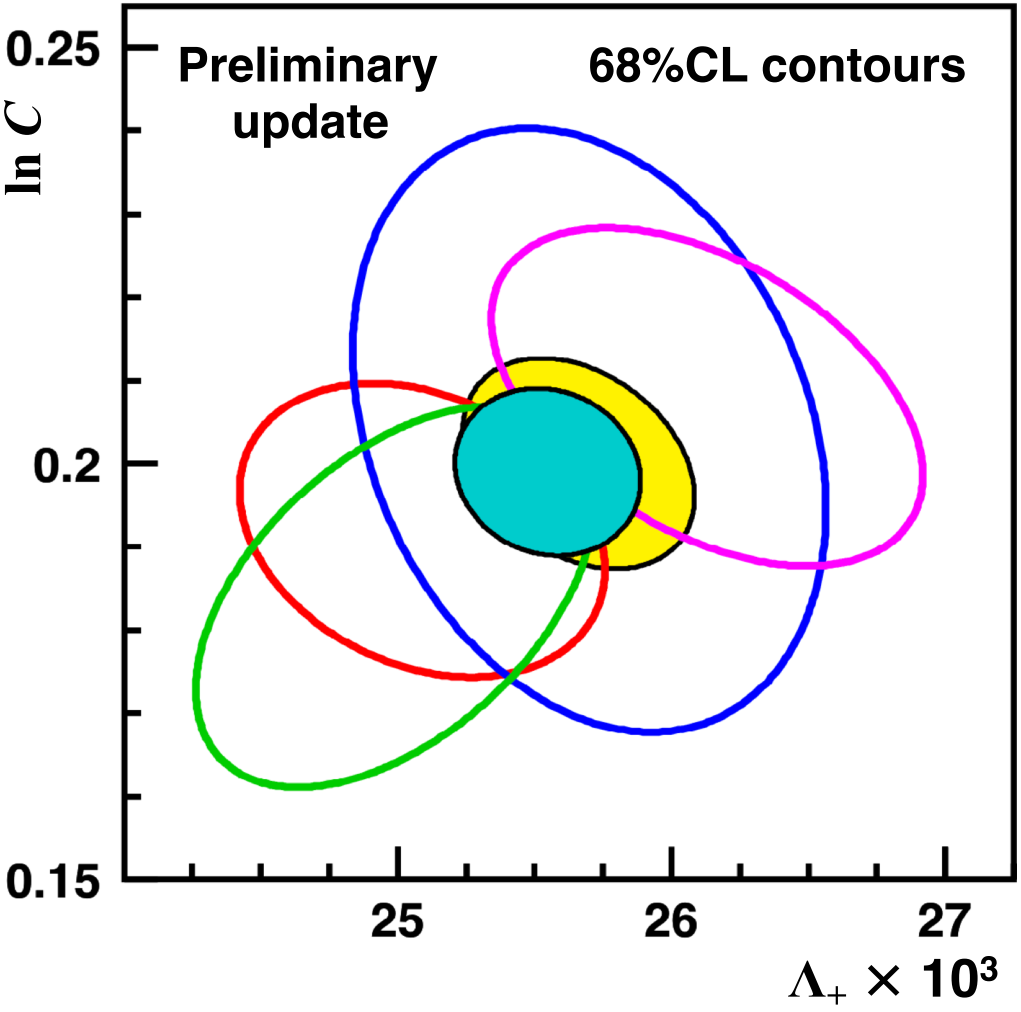
2010: χ^2 = 12.1/8 (P = 14.5%) | Update: χ^2 = 13.4/11 (P = 26.8%)

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Current**



$$\Lambda_+ \times 10^3 = \mathbf{25.55 \pm 0.38}$$

$$\ln C = \mathbf{0.1992(78)}$$

$$\rho(\Lambda_+, \ln C) = -0.110$$

$$\chi^2/\text{ndf} = 7.5/7\ (38\%)$$

Integrals

Mode	Update	2010
$K^0{}_{e3}$	**0.15470(15)**	0.15476(18)
$K^+{}_{e3}$	**0.15915(15)**	0.15922(18)
$K^0{}_{\mu 3}$	**0.10247(15)**	0.10253(16)
$K^+{}_{\mu 3}$	**0.10553(16)**	0.10559(17)

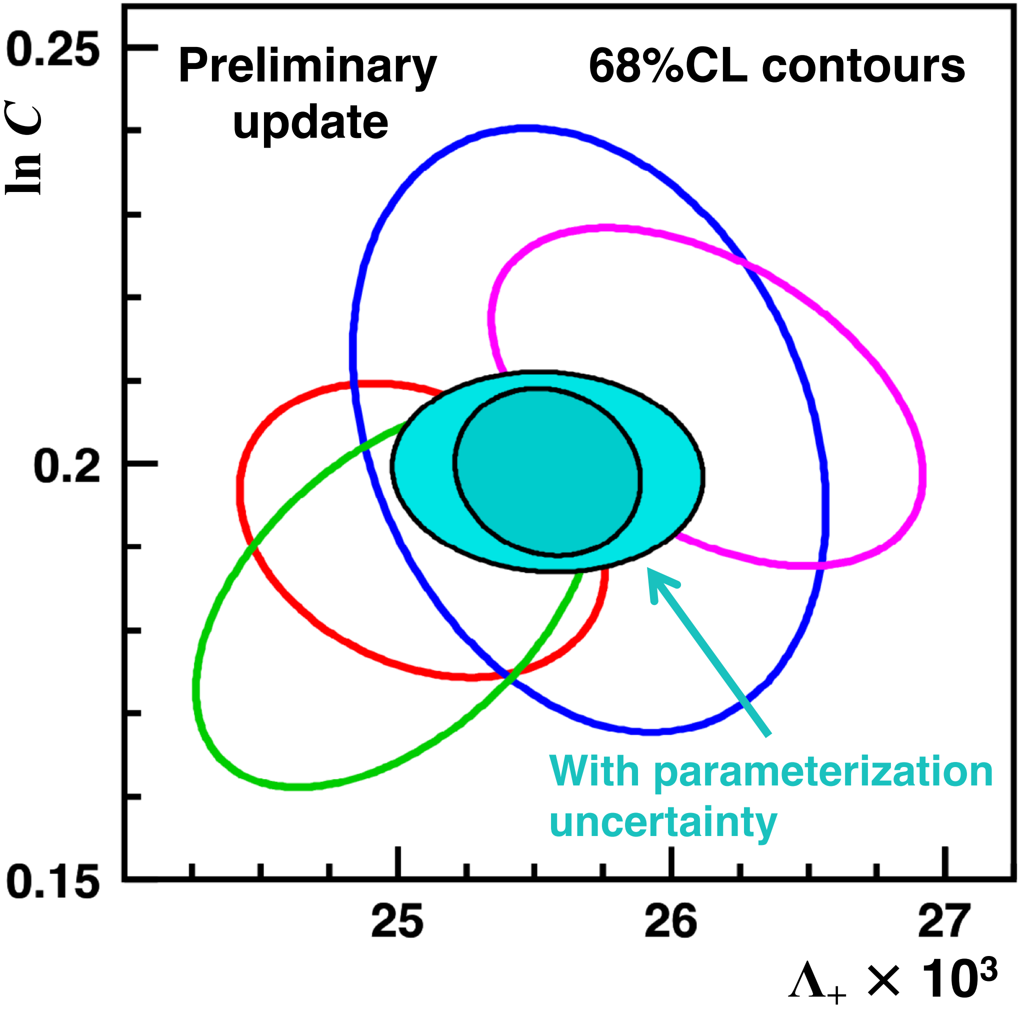
Only tiny changes in central values

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Current**



$$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$$

$$\ln C = 0.1992(78)$$

$$\rho(\Lambda_+, \ln C) = -0.110$$

$$\chi^2/\mathrm{ndf} = 7.5/7\ (38\%)$$

Fit results include common uncertainty from $H(t)$, $G(t)$:

$\sigma_{\mathrm{param}}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{\mathrm{param}}(\ln C) = 0.0040$

KTeV, Bernard et al. ’09

Confidence ellipses shown **without** common uncertainty

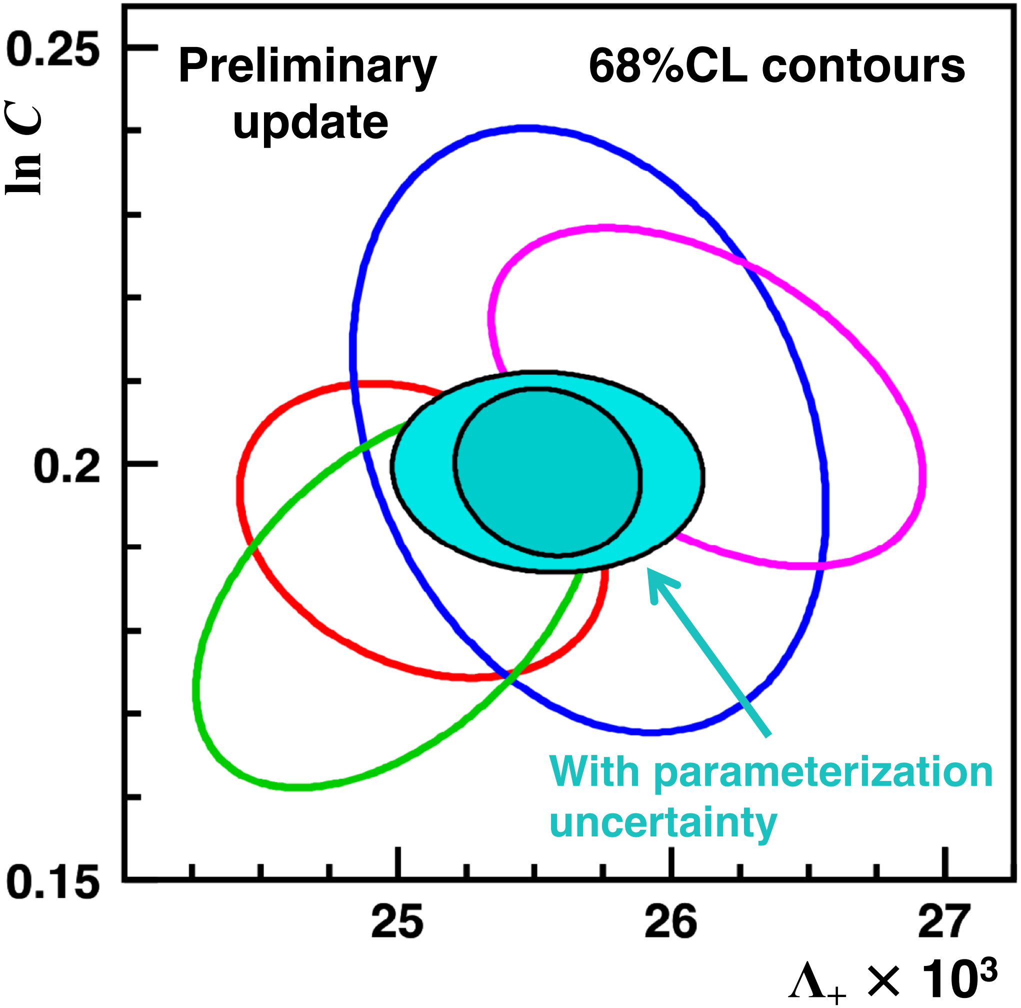
(except as indicated)

Dispersive parameters for $K_{\ell3}$ form factors

$K_{\ell3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Current**



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Fit results include common uncertainty from $H(t)$, $G(t)$

Without common uncertainty:

$\sigma(\Lambda_+)$ (0.38 → 0.22) × 10^{-3}

$\sigma(\ln C)$ 0.0078 → 0.0067

$\sigma(K_{e3}$ int) 0.10% → 0.09%

$\sigma(K_{\mu3}$ int) 0.15% → 0.11%

Long-distance EM corrections

Mode-dependent corrections $\Delta^{\mathrm{EM}}{}_{K\ell}$ to phase-space integrals $I_{K\ell}$ from EM-induced Dalitz plot modifications

- Values depend on acceptance for events with additional real photon(s)
- All recent measurements assumed fully inclusive

FlaviaNet analysis and updates used Cirigliano et al. ’08

- Comprehensive analysis at fixed order e^2p^2

Seng et al.
2107.14708

Calculation of complete EW RC using hybrid current algebra and ChPT with resummation of largest terms to all chiral orders

- Reduced uncertainties at O(e^2p^4)
- Lattice evaluation of QCD contributions to γW box diagrams
- Conventional value of S_{EW} subtracted from results for use with standard formula for V_{us}

Only K_{e3} at present
For $K_{\mu3}$ modes continue to use **Cirigliano et al. ’08**

	Cirigliano et al. ’08	**Seng et al. ’21**
$\Delta^{\mathrm{EM}}(K^0{}_{e3})$ [%]	0.50 ± 0.11	**0.580 ± 0.016**
$\Delta^{\mathrm{EM}}(K^+{}_{e3})$ [%]	0.05 ± 0.13	**0.105 ± 0.024**
ρ	+0.081	**–0.039**

$K_{\ell 3}$ data and lepton universality

For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\text{obs}}}{(R_{\mu e})_{\text{SM}}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\text{ obs}}}{[|V_{us}|\,f_+(0)]^2_{e3,\text{ obs}}} = \frac{g_\mu^2}{g_e^2}$$

Modes	2004 BRs*,†	Current†
K_L	1.054(14)	**1.004(5)**
$K^\pm$	1.014(12)	**1.000(9)**
Avg	1.030(9)	**1.003(5)**

*Assuming current values for form-factor parameters and Δ^{EM} †K_S not included

As statement on lepton universality

Compare to other precise tests:

$\pi \to \ell\nu$ $(r_{\mu e})$ = **1.0020(19)**

PDG with PIENU ’15 result

$\tau \to \ell\nu\nu$ $(r_{\mu e})$ = **1.0036(28)**

HFLAV May ’19 unofficial prelim.

As statement on calculation of Δ^{EM}

Confirmed at per-mil level

New calcuations for $K_{\mu 3}$ modes?

$SU(2)$-breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2-\hat{m}^2}{m_d^2-m_u^2}$$

$\chi_p{}^4 = 0.252$

NLO in strong interaction

O(e^2p^2) term $\varepsilon_{\mathrm{EM}}{}^{(4)} \sim 10^{-6}$

Cirigliano et al., ’02; Gasser & Leutwyler, ’85

$=$ **+2.61(17)%** **Calculated using:**

$Q = 22.1(7)$ Colangelo et al. ’18, avg. from $\eta \to 3\pi$

$m_s/\hat{m} = 27.23(10)$ FLAG ’20, $N_f = 2+1+1$ avg.

$M_K = 494.2(3)$

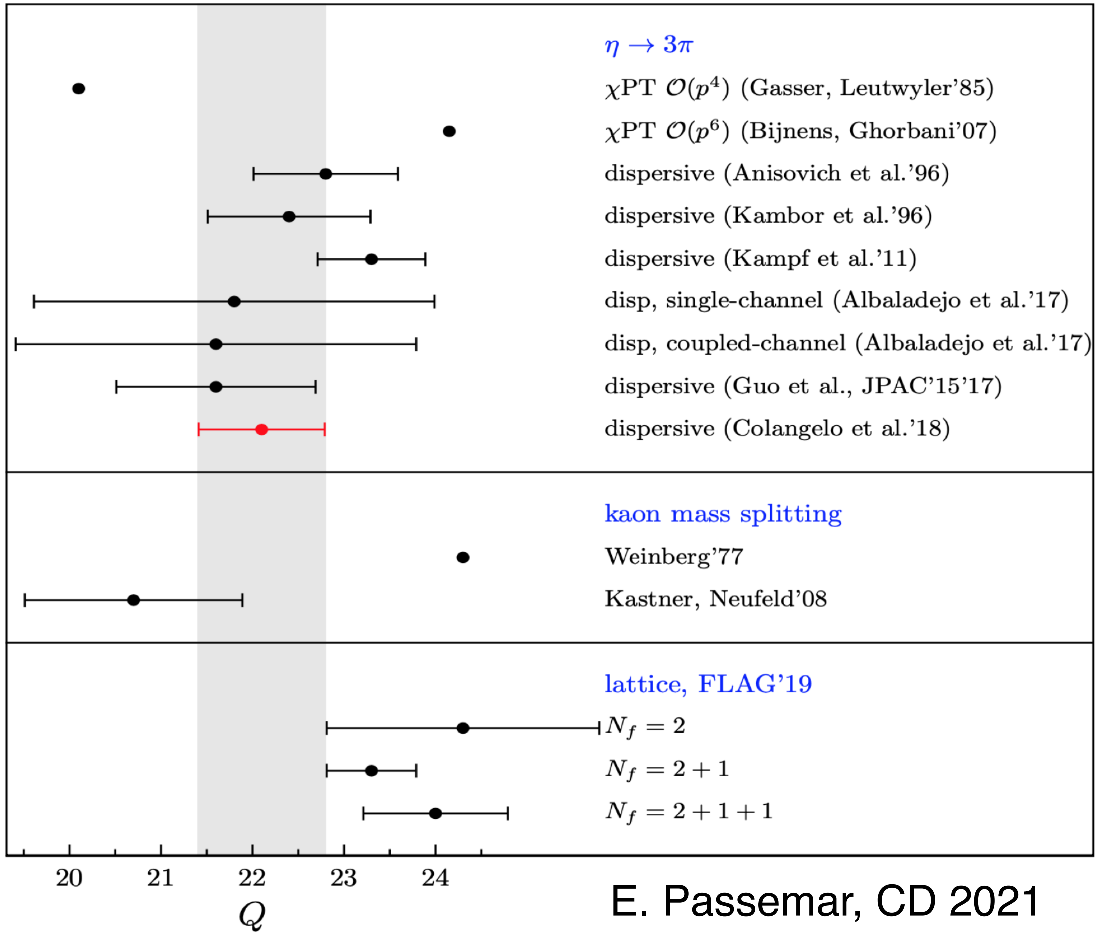
$M_\pi = 134.8(3)$

Isospin-limit meson masses from FLAG ’17

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require **$\Delta^{SU(2)} = 2.86(34)\%$**

Accuracy of $SU(2)$-breaking correction

Previous to recent results for Q, uncertainty on $\Delta^{SU(2)}$ was leading contributor to uncertainty on V_{us} from $K^{\pm}$ decays



E. Passemar, CD 2021

Reference value of Q from dispersion relation analyses of $\eta \to 3\pi$ Dalitz plots

Colangelo et al., '18

Q = 22.1 ± 0.7

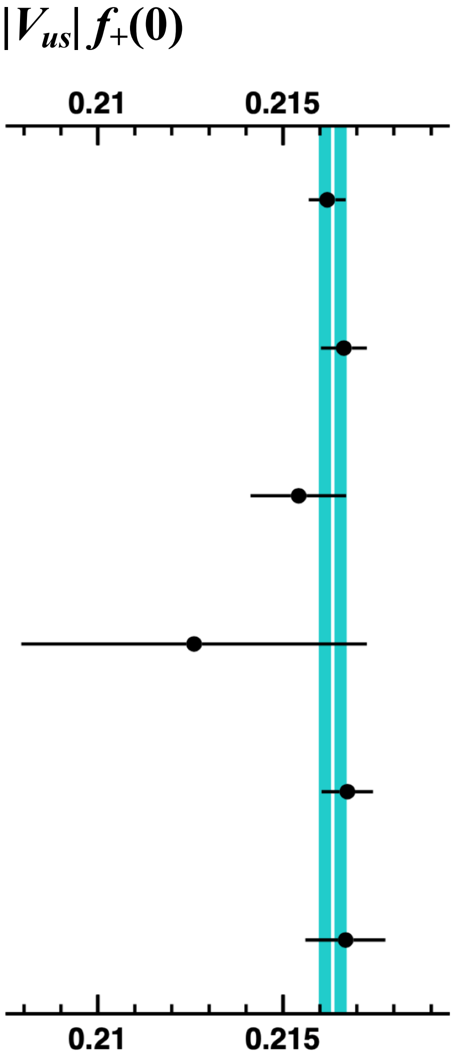
Lattice results for Q somewhat higher than analytical results

But, lattice results have finite correction to LO expectation:

$$Q_M^2 \equiv \frac{\hat{M}_K^2}{\hat{M}_\pi^2}\frac{\hat{M}_K^2 - \hat{M}_\pi^2}{\hat{M}_{K^0}^2 - \hat{M}_{K^+}^2}$$

Low-energy theorem: Q has no correction at NLO

$|V_{us}|\,f_+(0)$ from world data: Update

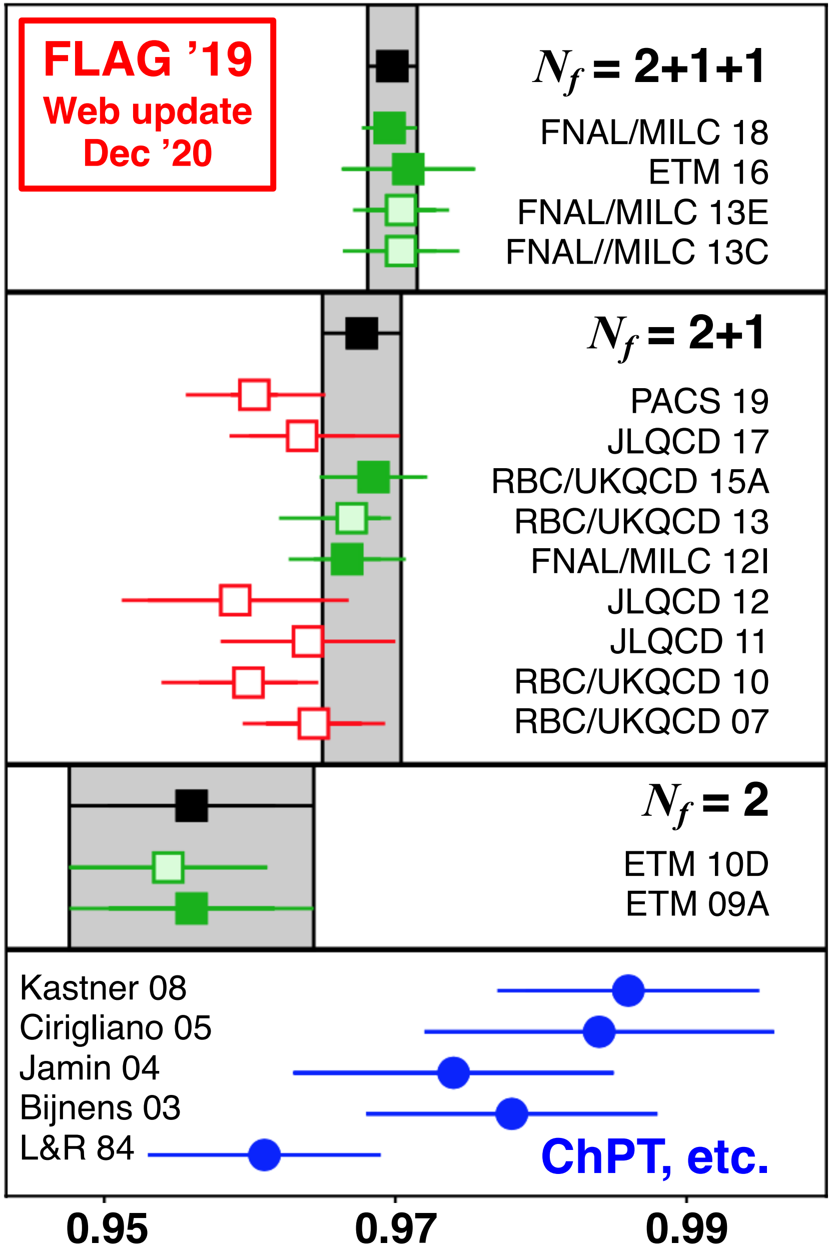


Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
K_Le3	**0.2162(5)**	**0.23**	0.09	0.20	0.02	0.05
$K_L\mu3$	**0.2167(6)**	**0.29**	0.15	0.18	0.11	0.07
K_Se3	**0.2154(13)**	**0.60**	0.60	0.02	0.02	0.05
$K_S\mu3$	**0.2126(47)**	**2.2**	2.2	0.02	0.11	0.07
$K^{\pm}e3$	**0.2167(7)**	**0.32**	0.27	0.06	0.17	0.05
$K^{\pm}\mu3$	**0.2167(11)**	**0.50**	0.45	0.06	0.21	0.07

Average: $|V_{us}|\,f_+(0)$ = 0.21635(38) χ^2**/ndf = 2.14/5 (83%)**

Evaluations of $f_+(0)$



FLAG '20 web update averages:

$N_f = 2+1+1$ $f_+(0) = 0.9698(17)$

Uncorrelated average of:

FNAL/MILC 18: HISQ, 5sp, $m_\pi \to 135$ MeV, new ensembles added to FNAL/MILC 13E

ETM 16: TwMW, 3sp, $m_\pi \to 210$ MeV, full q^2 dependence of f_+, f_0

$N_f = 2+1$ $f_+(0) = 0.9677(27)$

Uncorrelated average of:

FNAL/MILC 12I: HISQ, $m_\pi \sim 300$ MeV

RBC/UKQCD 15A: DWF, $m_\pi \to 139$ MeV

JLQCD 17 not included because only single lattice spacing used

ChPT $f_+(0) = 0.970(8)$

Ecker 15, Chiral Dynamics 15:
Calculation from Bijnens 03,
with new LECs from Bijnens, Ecker 14

Evaluations of $f_+(0)$

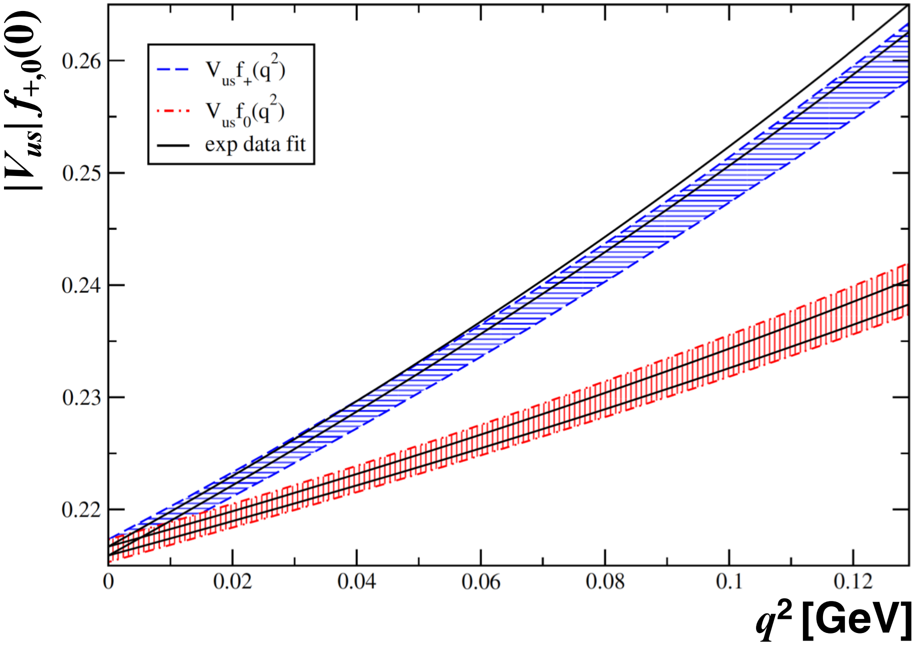
ETM
PRD 93 (2016)

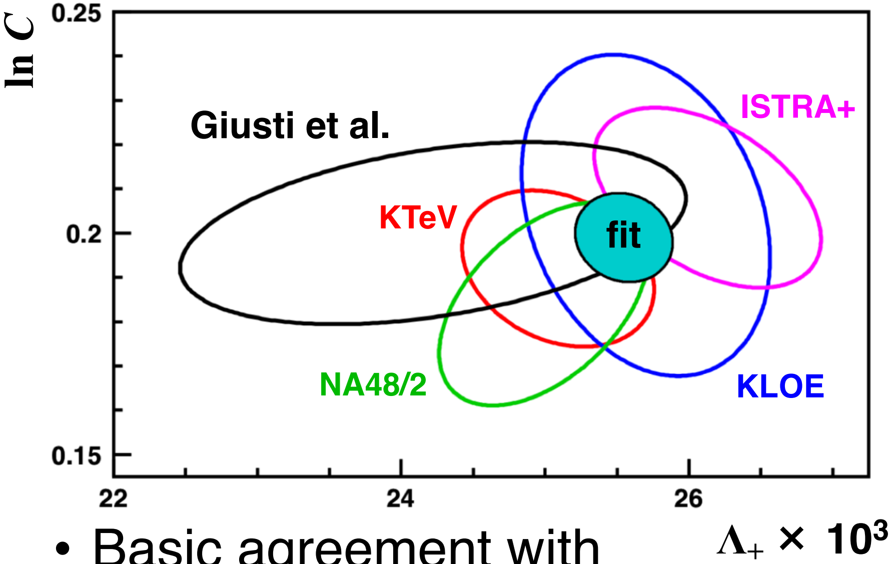
N_f **= 2+1+1**

$f_+(0) = 0.9709(44)_{st}(9)_{sy}(11)_{ext}$

Full q^2 dependence of f_+, f_0

See also:
PACS PRD101 (2020)
ETM 2111.08135





Fit synthetic data points with dispersive parameterization

Λ_+ **= 24.22(1.16) × 10^{-3}**

ln C = 0.1998(138)

$\rho(\Lambda_+, f_+(0)) = -0.228$

$\rho(\ln C, f_+(0)) = -0.719$

$\rho(\Lambda_+, \ln C) = +0.376$

- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

$|V_{ud}|$ from $0^+ \rightarrow 0^+$ nuclear β decays

For a given decay mode:

$$ft\,(1+\mathrm{RC}) = \frac{K}{2(V_{ud}G_F)^2(1+\Delta_R^V)}$$

f	statistical rate function
t	measured half life
K	constant
RC	mode-dependent radiative corr.
$\Delta_R{}^V$	inner (universal) radiative corr.

Hardy & Towner
1807.01146

$|V_{ud}|$ = 0.97420(21) updating 2015 published value
World $0^+ \rightarrow 0^+$ data set very robust: 14 transitions with compatible measurements at 0.1% precision or better
$\Delta_R{}^V$ = 2.361(38)% from Marciano & Sirlin ’06

Seng et al.
PRL 125 (2018)

$|V_{ud}|$ = 0.97370(14) **$\Delta_R{}^V$ = 2.467(22)%**
New calculation of γW-box contribution to $\Delta_R{}^V$ using dispersion relations and DIS structure functions

Czarnecki et al.
PRL 125 (2018)

$|V_{ud}|$ = 0.97389(18) **$\Delta_R{}^V$ = 2.426(32)%**
Improved use of Bjorken sum rule to constrain strong-interaction corrections to axial-vector component of the γW-box

Hardy & Towner
PRC 102 (2020)

$|V_{ud}|$ = 0.97373(31) **$\Delta_R{}^V$ = 2.454(19)%**
Use weighted average of above values for $\Delta_R{}^V$
23 new publications, some older measurements eliminated

$|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \to 0^+)$ unitarity test

Hardy & Towner
PRC 102 (2020)

$|V_{ud}|$ **= 0.97373(31)**
$0^+ \rightarrow 0^+$ with Δ_R^V = 2.454(19)%

From FlaviaNet 2010 $K_{\ell 3}$ analysis

$|V_{us}|\, f_+(0)$ **= 0.2163(5)** $|V_{us}|$ **= 0.2254(13)**

with $f_+(0) =$ **0.959(5)** **with** $|V_{ud}| =$ **0.97425(22)**

Δ_{CKM} **= +0.0000(8)**

Update with $|V_{us}|\, f_+(0)$ = 0.21635(38)

N_f = 2+1
$f_+(0)$ = 0.9677(27)

V_{us} **= $0.22358(39)_{\mathrm{exp}}(62)_{\mathrm{lat}}$**

Δ_{CKM} **= $-0.00185(19)_{\mathrm{exp}}(28)_{\mathrm{lat}}(60)_{ud}$** $= -2.7\sigma$

N_f = 2+1+1
$f_+(0)$ = 0.9698(17)

V_{us} **= $0.22309(39)_{\mathrm{exp}}(39)_{\mathrm{lat}}$**

Δ_{CKM} **= $-0.00206(17)_{\mathrm{exp}}(17)_{\mathrm{lat}}(60)_{ud}$** $= -3.2\sigma$

It ain't over till it's over: ~3σ tension with first-row unitarity

NB: 1.5-2σ inconsistency with unitarity first seen with 2014-era lattice results

V_{us}/V_{ud} and $K_{\ell 2}$ decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{1}{2}\delta_{\text{EM}} - \frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from theory:

Cirigliano, Neufeld '11

$\delta_{\text{EM}} = -0.0069(17)$

Long-distance EM corrections

$\delta_{SU(2)} = -0.0043(5)(11)$

Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

Lattice: f_K/f_π

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

Inputs from experiment:

Updated $K^\pm$ BR fit:

$\text{BR}(K^\pm_{\mu2(\gamma)}) = 0.6358(11)$

$\tau_{K\pm} = 12.384(15)$ ns

PDG:

$\text{BR}(\pi^\pm_{\mu2(\gamma)}) = 0.9999$

$\tau_{\pi\pm} = 26.033(5)$ ns

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm} =$ 0.27599(37)

No $SU(2)$-breaking correction

V_{us}/V_{ud} and $K_{\ell 2}$ decays

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- $N_f = 2+1+1$ Twisted-mass Wilson fermions

$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)$$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld ’11:

$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)$$

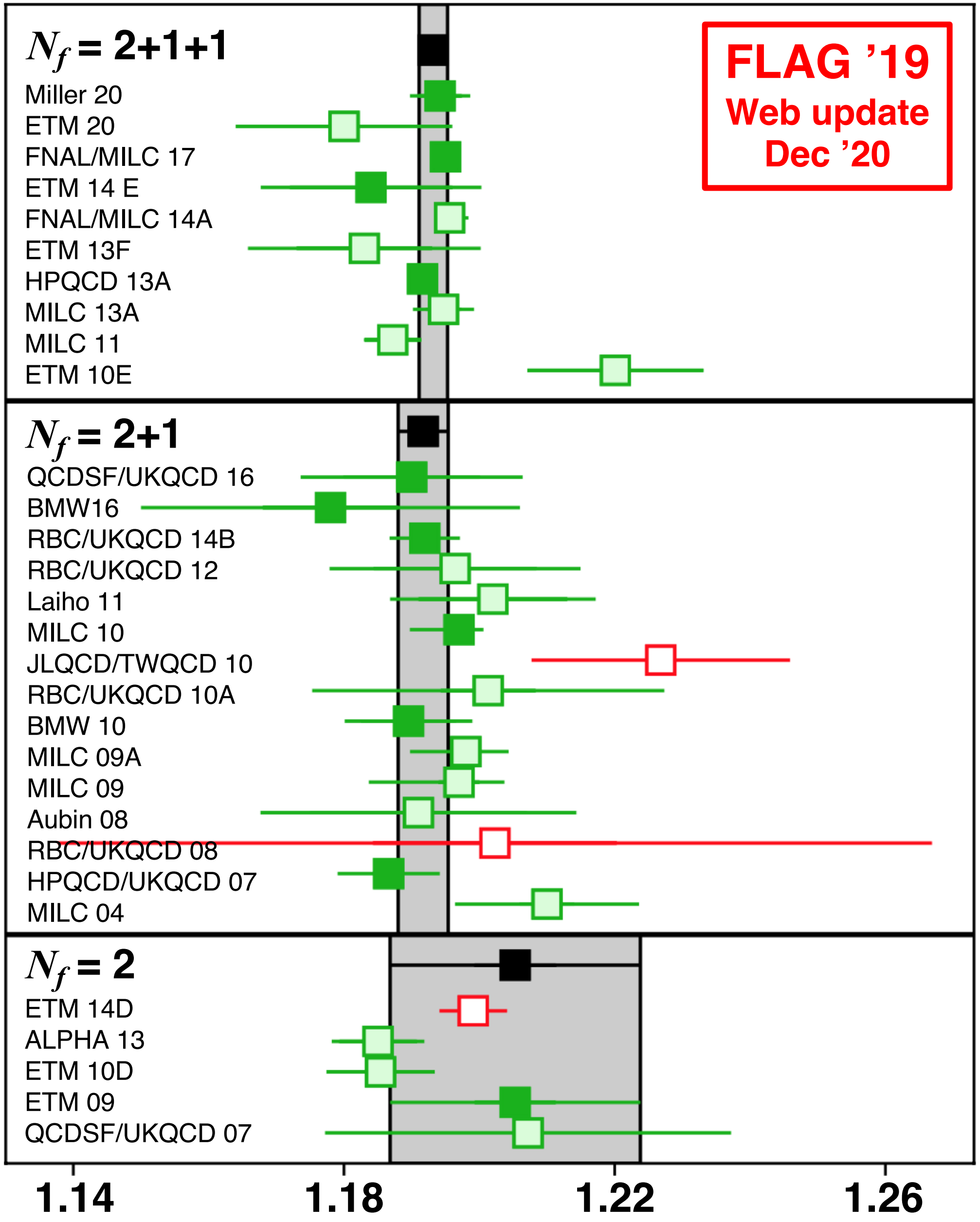
Di Carlo et al.
PRD 100 (2019)

Update, extended description, and systematics of Giusti et al.

$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)$$

$$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{\mathrm{BR}}(20)_{\mathrm{corr}}$$

Lattice evaluations of f_K/f_π



N_f = 2+1+1 $f_{K\pm}/f_{\pi\pm}$ = 1.1932(19)

Miller 20

HISQ + DWF, 4sp, $m_\pi \to$ 130 MeV
Uses FNAL/MILC ensembles

FNAL/MILC 17

HISQ, 4sp, m_π phys
Updates MILC 13A, FNAL/MILC 14A

ETM 14E

TwM, 3sp, m_π = 210-450 MeV

HPQCD 13A

HISQ, 3sp, m_π phys
Uses FNAL/MILC ensembles

N_f = 2+1 $f_{K\pm}/f_{\pi\pm}$ = 1.1917(37)

QCDSF/UKQCD 16

Clover, 4sp, $m_\pi \to$ 220 MeV

BMW 16

Clover, 5sp, $m_\pi \to$ 139 MeV

RBC/UKQCD 14B

DWF, m_π = 139 MeV
f_K and f_π separately (isospin limit)

Lattice results for f_K/f_π

Recalculate FLAG averages for results without $SU(2)$-breaking

Isospin-limit results as reported in original papers

N_f = 2+1+1

f_K/f_π = 1.1978(22) S = 1.1

Average is problematic with correlations assumed by FLAG, dominated by FNAL/MILC17 (symmetrized)

ETM 21 New!	**1.1995(44)(7)**

TM quarks, 3sp, $m_\pi \rightarrow$ physical
Not yet in FLAG 2020 web average
Replaces ETM 14E in our average

Miller 20	1.1964(44)
FNAL/MILC17	$1.1980(^{+13}{}_{-19})$
HPQCD13A	1.1948(15)(18)

Share ensembles
Partially correlated uncertainties using FLAG prescription

N_f = 2+1

f_K/f_π = 1.1946(34)*

QCDSF/UKQCD17	1.192(10)(13)
BMW16	1.182(10)(26)
RBC/UKQCD14B	1.1945(45)
BMW10	1.192(7)(6)
HPQCD/UKQCD07	1.198(2)(7)

* MILC10 omitted from average because unpublished

$|V_{us}|(K_{\ell 2})$ and $|V_{ud}|(0^+ \to 0^+)$: Update

$|V_{us}/V_{ud}| \times f_K/f_\pi$ = 0.27679(34) and $|V_{ud}|$ = 0.97370(31)

$\delta_{SU(2)} + \delta_{\text{EM}} = -0.0126(14)$ from Di Carlo et al. ’19

N_f = 2+1

f_K/f_π = 1.1946(34)

$V_{us}/V_{ud} = 0.23170(29)_{\text{exp}}(66)_{\text{lat}}$

$V_{us} = 0.22562(28)_{\text{exp}}(64)_{\text{lat}}(07)_{ud}$

$\Delta_{\text{CKM}} = -0.00093(13)_{\text{exp}}(29)_{\text{lat}}(64)_{ud}$ $= -1.3\sigma$

N_f = 2+1+1

f_K/f_π = 1.1967(18)

$V_{us}/V_{ud} = 0.23108(29)_{\text{exp}}(42)_{\text{lat}}$

$V_{us} = 0.22501(28)_{\text{exp}}(41)_{\text{lat}}(07)_{ud}$

$\Delta_{\text{CKM}} = -0.00120(12)_{\text{exp}}(18)_{\text{lat}}(63)_{ud}$ $= -1.8\sigma$

$K_{\ell 2}$ results give somewhat better agreement with unitarity via V_{ud} than $K_{\ell 3}$ results

V_{us} and CKM unitarity: All data

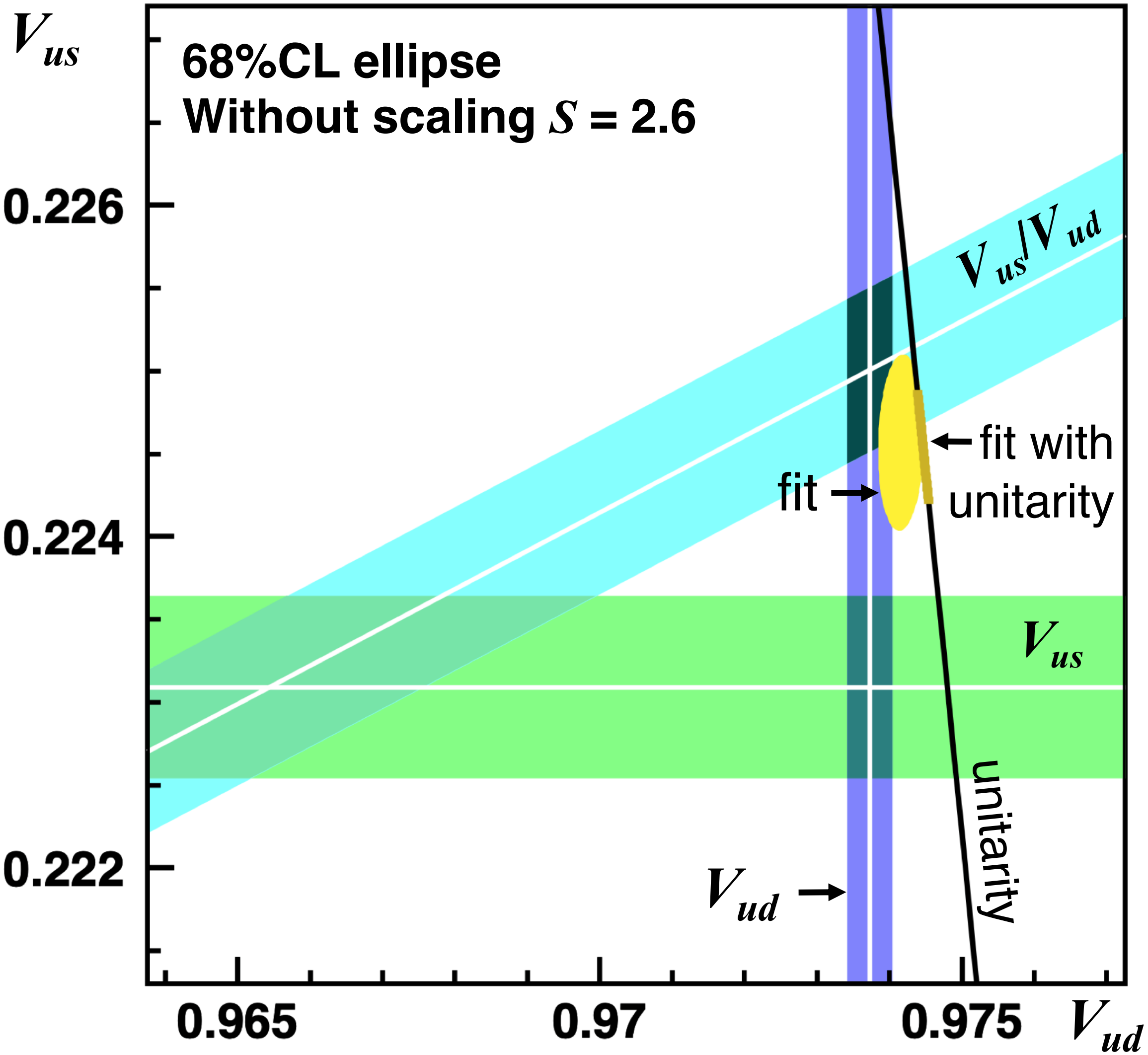
$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$

$f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$

$|V_{ud}| = 0.97373(31)$

$|V_{us}| = 0.2231(6)$

$|V_{us}|/|V_{ud}| = 0.2311(5)$



Fit results, no constraint

$V_{ud} = 0.97365(30)$

$V_{us} = 0.22414(37)$

$\chi^2/\text{ndf} = 6.6/1\ (1.0\%)$

$\Delta_{\text{CKM}} = -0.0018(6)$

-2.7σ

With scale factor $S = 2.6$

$V_{ud} = 0.9737(8)$

$V_{us} = 0.2241(10)$

What can we learn from $\pi^+ \to \pi^0 e\nu$?

Czarnecki, Marciano, Sirlin, PRD101 (2020)

$$\frac{\Gamma(K_L \to \pi e \nu(\gamma))}{\Gamma(\pi^+ \to \pi^0 e \nu(\gamma))} = \frac{1}{3}\left(\frac{m_{K^0}}{m_{\pi^+}}\right)^5 \left(\frac{V_{us} f_+^K(0)}{V_{ud} f_+^\pi(0)}\right)^2 \left(\frac{I_K}{I_\pi}\right)\left(\frac{1+\mathrm{RC}_K}{1+\mathrm{RC}_\pi}\right)$$

Ratio not sensitive to short-distance EW radiative corrections

- $(1 + \mathrm{RC}_K - \mathrm{RC}_\pi) = 1.000(2)_K(1)_\pi$
- Includes S_{EW} and short-distance radiative corrections for π
- $\Delta_{\mathrm{EM}}{}^K$ (long-distance correction) fortuitously cancels for K_{Le3} mode

Consider K_{Le3} mode as an example:

Most precise value of V_{us}; LD corrections known better than for other modes

$$\frac{V_{us} f_+^K(0)}{V_{ud} f_+^\pi(0)} = 0.2221(53)_{\Gamma(K)}(64)_{\Gamma(\pi)}(22)_{\mathrm{RC}}(12)_{\mathrm{int}} \quad \mathbf{= 0.2221(87)}$$

	K_{Le3}/π_{e3}*	**$K_{\mu 2}/K_{\pi 2}$†**	**$K_{\ell 3}$* & $0^+ \to 0^+$**
V_{us}/V_{ud}	**$0.2291(9)_{\mathrm{exp}}(4)_{\mathrm{lat}}$**	0.2311(5)	$0.2291(56)_{us}(7)_{ud}$
diff with K_{Le3}/π_{e3}	–	**+1.7σ**	**0σ**

*with $f_+^K(0) = 0.9698(17)$ and $f_+^\pi(0) = 1$ in $SU(2)$ limit †with $f_K/f_\pi = 1.1967(18)$

V_{us} from kaons: Not over till it's over

Experimental results from kaons

With $|V_{ud}|(0^+ \rightarrow 0^+)$ and $N_f = 2+1+1$ lattice

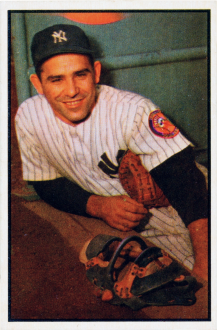
$|V_{us}|\, f_+(0)$ = **0.21635(38)** → Δ_{CKM} = **–0.00206(65) = –2.7σ**

$|V_{us}/V_{ud}| \times f_K/f_\pi$ = **0.27679(34)** → Δ_{CKM} = **–0.00120(67) = –1.8σ**

$K_{\ell 2}$ result shows better agreement with unitarity than $K_{\ell 3}$ result when $|V_{ud}|$ obtained from $0^+ \rightarrow 0^+$ decays

Normalization of $\Gamma(K_{Le3})$ to $\Gamma(\pi_{e3})$ result agrees with $K_{\ell 3}$ results obtained with V_{ud} from $0^+ \rightarrow 0^+$ decays

- Discrepancy not in RC for $0^+ \rightarrow 0^+$?
- K_{Le3}/π_{e3} not sensitive to universal radiative corrections: can this be a clue?



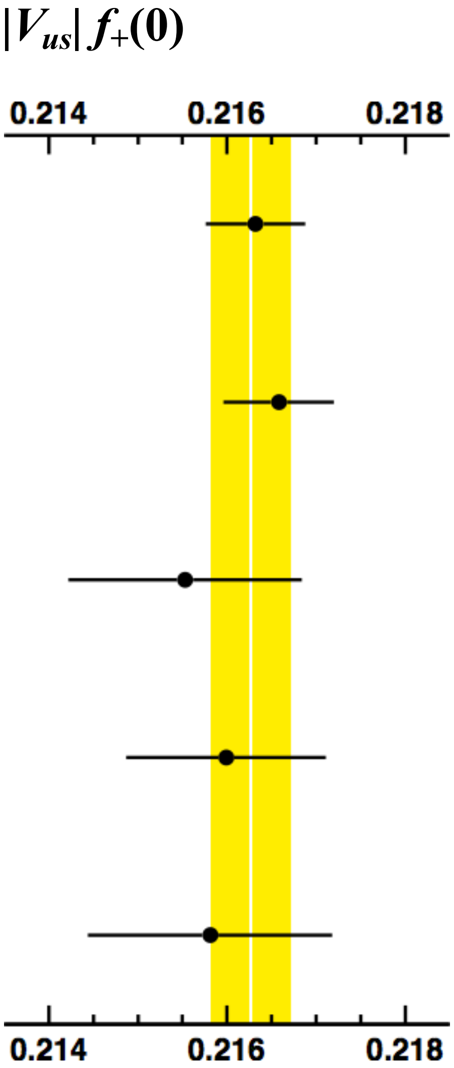
"It's hard to make predictions, especially about the future"

Attributed to Yogi Berra
Catcher, NY Yankees, 1946-1963

V_{us} from kaon decays

Additional information

$|V_{us}|\,f_+(0)$ from world data: 2010



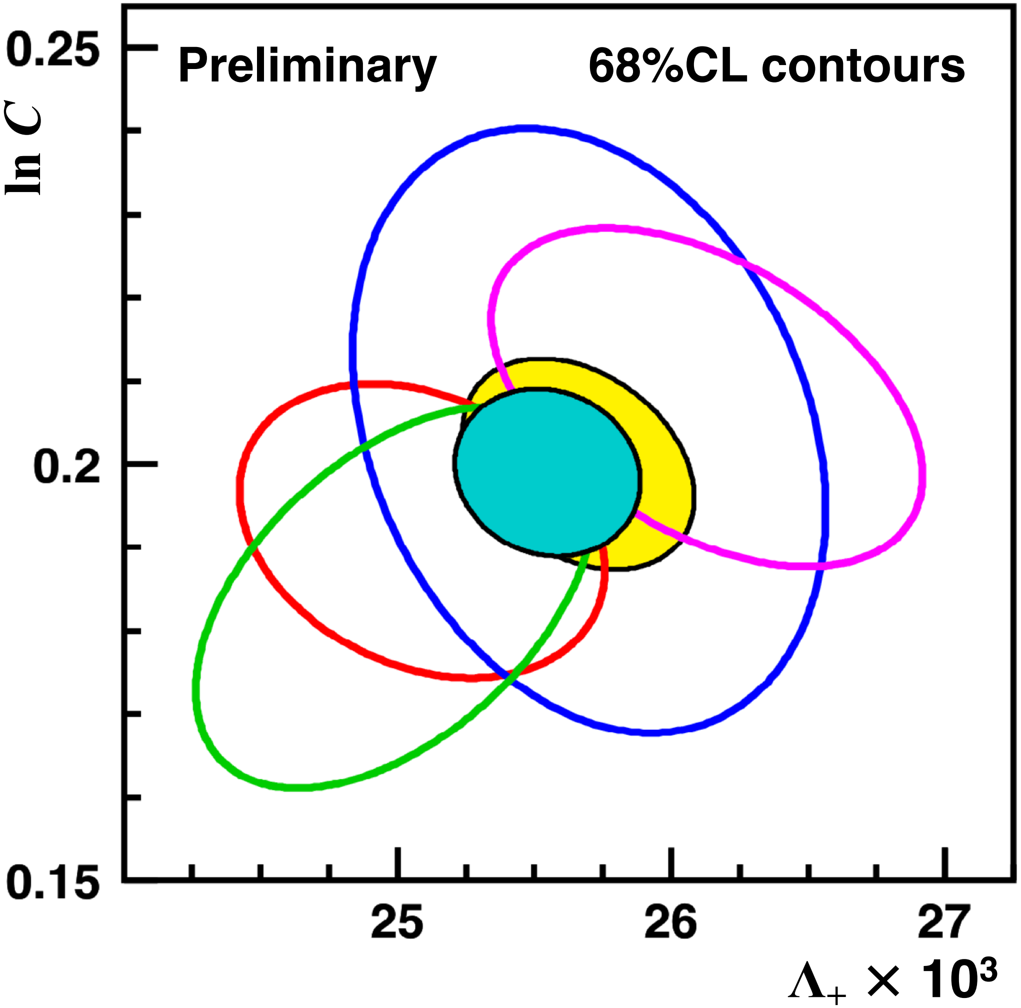
			Approx. contrib. to % err from:			
		% err	**BR**	τ	Δ	**Int**
K_Le3	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.06
$K_L\mu3$	**0.2166(6)**	**0.29**	0.15	0.18	0.11	0.08
K_Se3	**0.2155(13)**	**0.61**	0.60	0.03	0.11	0.06
$K^\pm e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.40	0.06
$K^\pm\mu3$	**0.2158(14)**	**0.63**	0.47	0.08	0.39	0.08

Average: $|V_{us}|\,f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.77/4 (94%)**

Phase-space integrals 2021

Averages of form-factor parameters for dispersive parameterization Λ_+ and ln C

Integrals calculated from average values



$\mathbf{\Lambda_+ \times 10^3}$ = **25.55 ± 0.38**

ln C = **0.1992(78)**

$\rho(\Lambda_+, \ln C)$ = −0.110

χ^2/ndf = 7.5/7 (38%)

Integrals	
$K^0{}_{e3}$	**0.15470(15)**
$K^+{}_{e3}$	**0.15915(15)**
$K^0{}_{\mu3}$	**0.10247(15)**
$K^+{}_{\mu3}$	**0.10553(16)**

Correlation matrix for integrals

$K^0{}_{e3}$	1	1	0.530	0.521
$K^+{}_{e3}$		1	0.530	0.521
$K^0{}_{\mu3}$			1	1
$K^+{}_{\mu3}$				1