Radiative Corrections to K_{e3} Decays

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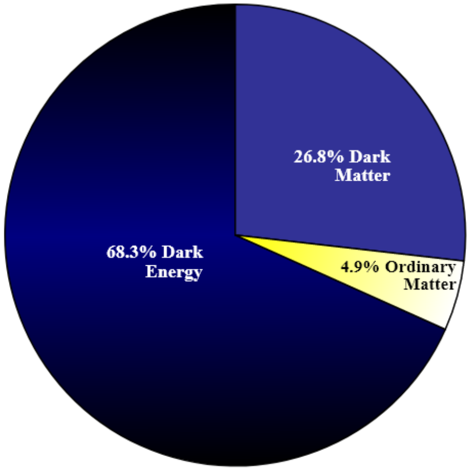
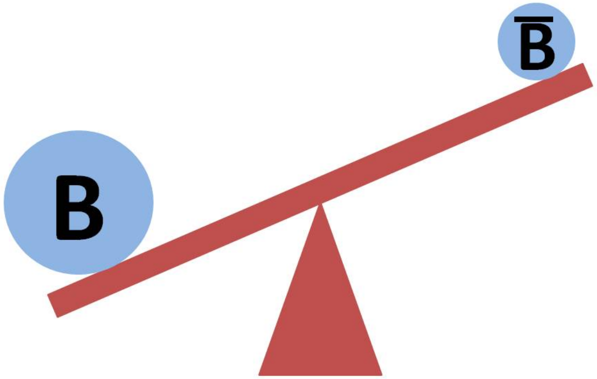
The V_{us} anomaly

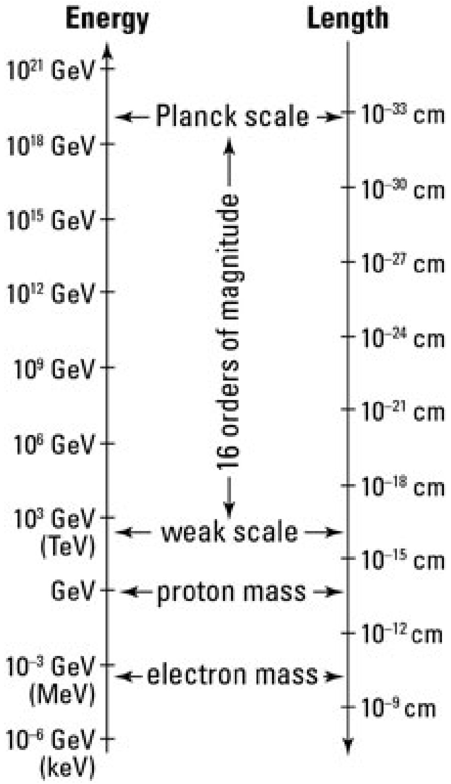
Many unresolved problems call for physics beyond the Standard Model (BSM)



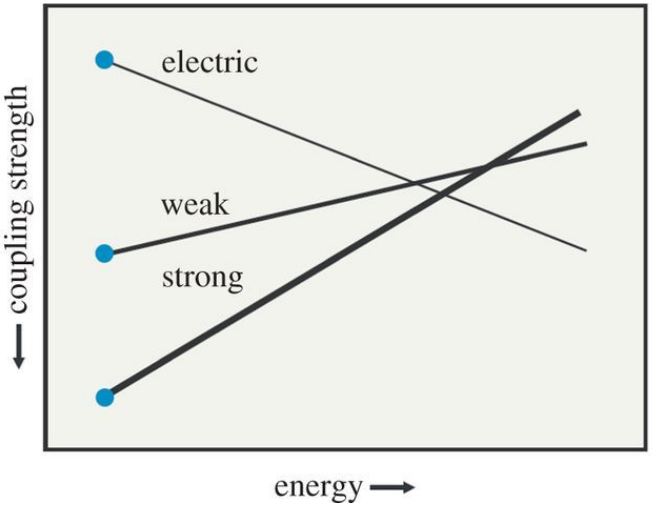
Dark energy, dark matter



Matter-antimatter asymmetry



Hierarchy problem



Unification of forces

The V_{us} anomaly

Beta decays place **one of the most stringent tests of SM** through precision measurements of the **first-row CKM matrix elements V_{ud} and V_{us}**

V_{ud}

	$\lvert V_{ud}\rvert$
Superallowed nuclear decays ($0^+ \to 0^+$)	0.97373(31)
Free n decay	0.97377(90)
Mirror nuclei decays	0.9739(10)
Pion semileptonic decay (π_{e3})	0.9740(28)

V_{us}

	$\lvert V_{us}\rvert$
Kaon semileptonic decays ($K_{\ell 3}$)	0.22309(56)
Tau decays	0.2221(13)
Hyperon decays	0.2250(27)

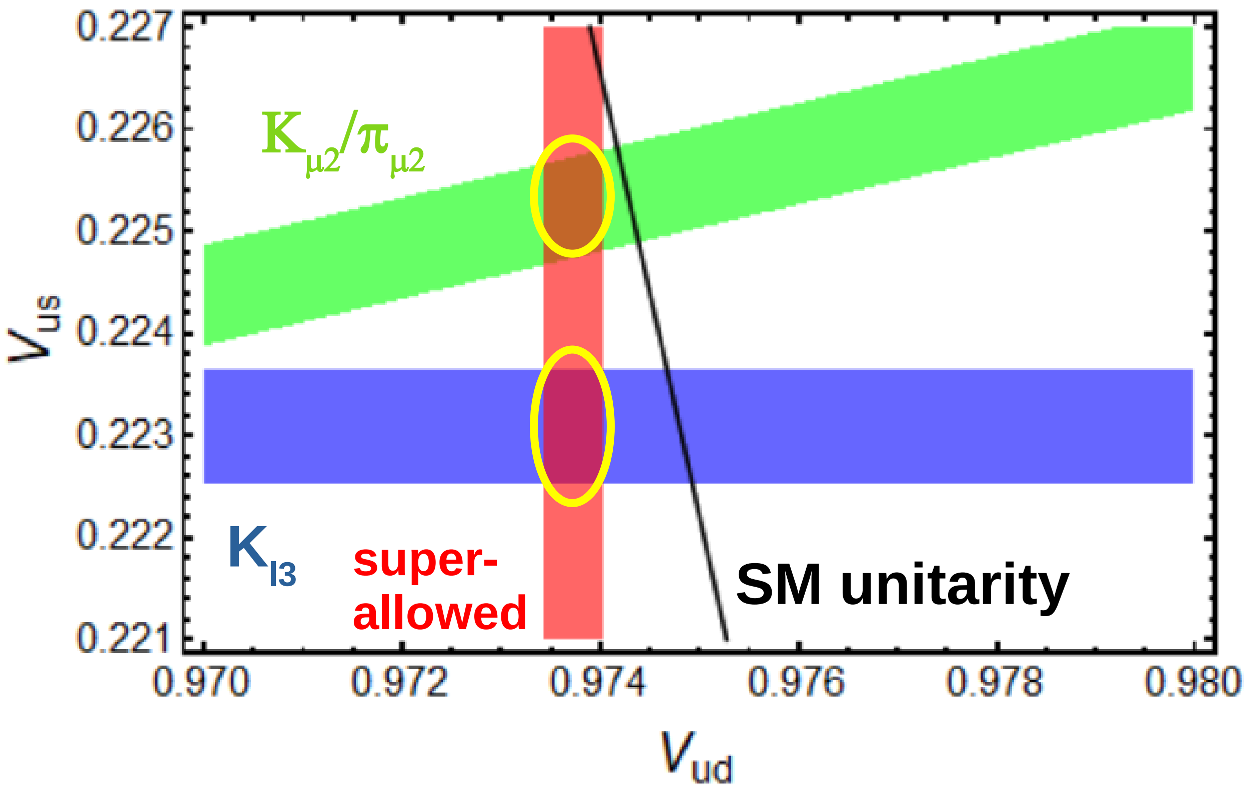
CYS, Galviz, Marciano and Meißner, 2107.14708

V_{us}/V_{ud}

	$\lvert V_{us}/V_{ud}\rvert$
K/π leptonic decays ($K_{\mu 2}/\pi_{\mu 2}$)	0.23131(51)
K/π semileptonic decays ($K_{\ell 3}/\pi_{e3}$)	0.22908(87)

The V_{us} anomaly

~2σ **discrepancy** exists between V_{us} extracted from **leptonic kaon decay ($K_{\mu 2}$)** and **semileptonic kaon decay (K_{l3})**

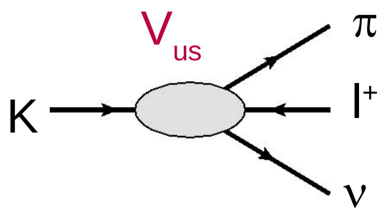


Signature of **BSM physics**, or **unidentified SM effects**?

Kaon semileptonic decays (K_{l3}) and the long-distance RC

Best measurement of **V_{us}**:

Kaon semileptonic decay (K_{l3})



$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\rm EW} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}\right)$$

Anlaysis based on 20++ years of accumulated data

2021 PDG (online)

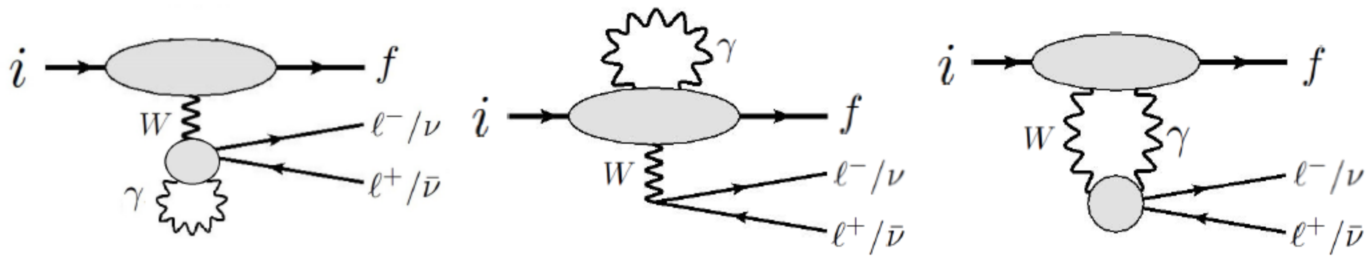
Non-trivial theory inputs:

- $K\pi$ form factor at zero momentum transfer
- Phase space factor
- Electromagnetic radiative corrections (RC)
- Isospin-breaking corrections (ISB)

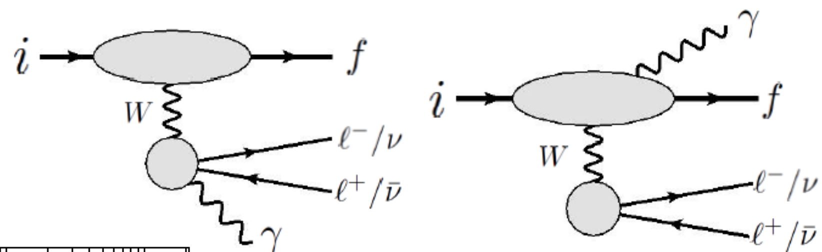
Kaon semileptonic decays (K_{l3}) and the long-distance RC

Long-distance electromagnetic radiative corrections (RC)

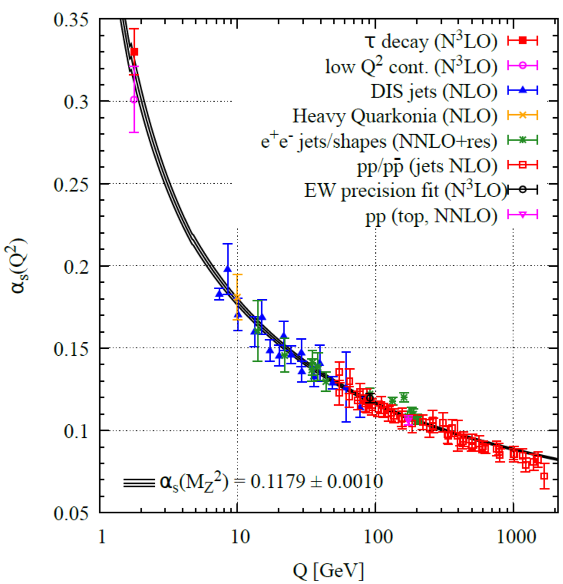
Virtual corrections:



Real corrections:



"**Long-distance**" : W-propagator shrinks to a point (Fermi interaction)



Main issue: Strong interactions governed by QCD become non-perturbative at $Q^2 \sim 1\ \text{GeV}^2$

Major theory challenge in the K_{l3} RC

for the past 5 decades

Chiral perturbation theory (ChPT):

- A (the?) **low-energy effective field theory (EFT)** of QCD
- Lagrangian constructed based on the **spontaneously-broken chiral symmetry** of QCD
- Pseudoscalar mesons, leptons and photons as **dynamical degrees of freedom (DOFs)**
- **Chiral power-counting** scheme ensures convergence
- Effects of non-perturbative QCD contained in the **low-energy constants (LECs)**

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\gamma} + \mathcal{L}_{\text{ChPT}}}$$

$$\begin{aligned} \mathcal{L}_{\text{lepton}} &= \sum_{\ell}\left[\bar{\ell}(i\not{\partial} + e\not{A} - m_{\ell})\ell + \bar{\nu}_{\ell L} i\not{\partial}\nu_{\ell L}\right] \\ \mathcal{L}_{\gamma} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial\cdot A)^2 + \frac{1}{2}M_{\gamma}^2 A_{\mu}A^{\mu} \end{aligned}$$

Non-hadronic piece

$$\boxed{\mathcal{L}_{\text{ChPT}} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \ldots\,.}$$

$$\mathcal{L}^{p^2} = \frac{F_0^2}{4}\left\langle D_{\mu}U(D^{\mu}U)^{\dagger} + \chi U^{\dagger} + U\chi^{\dagger}\right\rangle, \quad \mathcal{L}^{e^2} = ZF_0^4\left\langle q_L U^{\dagger} q_R U\right\rangle$$

Leading-order (LO) Chiral Lagrangian

Chiral Perturbation Theory

Next-to-leading-order (NLO) Chiral Lagrangian:

Pure mesonic

$$\begin{aligned}\mathcal{L}_{p^4} = {} & L_1\langle D_\mu U(D^\mu U)^\dagger\rangle^2 + L_2\langle D_\mu U(D_\nu U)^\dagger\rangle\langle D^\mu U(D^\nu U)^\dagger\rangle \\ & + L_3\langle D_\mu U(D^\mu U)^\dagger D_\nu U(D^\nu U)^\dagger\rangle + L_4\langle D_\mu U(D^\mu U)^\dagger\rangle\langle \chi U^\dagger + U\chi^\dagger\rangle \\ & + L_5\langle D_\mu U(D^\mu U)^\dagger(\chi U^\dagger + U\chi^\dagger)\rangle + L_6\langle \chi U^\dagger + U\chi^\dagger\rangle^2 \\ & + L_7\langle \chi U^\dagger - U\chi^\dagger\rangle^2 + L_8\langle U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger\rangle \\ & - iL_9\left\langle f^R_{\mu\nu} D^\mu U(D^\nu U)^\dagger + f^L_{\mu\nu}(D^\mu U)^\dagger D^\nu U\right\rangle + L_{10}\left\langle U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu}\right\rangle .\end{aligned}$$

Gasser and Leutwyler, NPB 1985

With dynamical photons

$$\begin{aligned}\mathcal{L}^{e^2p^2}_{\{K\}} = {} & F_0^2\Big\{\tfrac{1}{2}K_1\langle D^\mu U(D_\mu U)^\dagger\rangle\langle q_R q_R + q_L q_L\rangle + K_2\langle D^\mu U(D_\mu U)^\dagger\rangle\langle q_R U q_L U^\dagger\rangle \\ & + K_3(\langle (D^\mu U)^\dagger q_R U\rangle\langle (D_\mu U)^\dagger q_R U\rangle + \langle D^\mu U q_L U^\dagger\rangle\langle D_\mu U q_L U^\dagger\rangle) \\ & K_4\langle (D^\mu U)^\dagger q_R U\rangle\langle D_\mu U q_L U^\dagger\rangle + K_5\langle q_L q_L (D^\mu U)^\dagger D_\mu U + q_R q_R D^\mu U(D_\mu U)^\dagger\rangle \\ & + K_6\langle (D^\mu U)^\dagger D_\mu U q_L U^\dagger q_R U + D^\mu U(D_\mu U)^\dagger q_R U q_L U^\dagger\rangle \\ & + \tfrac{1}{2}K_7\langle \chi^\dagger U + U^\dagger\chi\rangle\langle q_R q_R + q_L q_L\rangle + K_8\langle \chi^\dagger U + U^\dagger \chi\rangle\langle q_R U q_L U^\dagger\rangle \\ & + K_9\langle (\chi^\dagger U + U^\dagger\chi) q_L q_L + (\chi U^\dagger + U\chi^\dagger) q_R q_R\rangle \\ & + K_{10}\langle (\chi^\dagger U + U^\dagger\chi) q_L U^\dagger q_R U + (\chi U^\dagger + U\chi^\dagger) q_R U q_L U^\dagger\rangle \\ & + K_{11}\langle (\chi^\dagger U - U^\dagger\chi) q_L U^\dagger q_R U + (\chi U^\dagger - U\chi^\dagger) q_R U q_L U^\dagger\rangle \\ & + K_{12}\langle (D^\mu U)^\dagger [\nabla_\mu q_R, q_R] U + D^\mu U[\nabla_\mu q_L, q_L] U^\dagger\rangle \\ & + K_{13}\langle \nabla^\mu q_R U \nabla_\mu q_L U^\dagger\rangle + K_{14}\langle \nabla^\mu q_R \nabla_\mu q_R + \nabla^\mu q_L \nabla_\mu q_L\rangle\Big\} ,\end{aligned}$$

Urech, NPB 1995

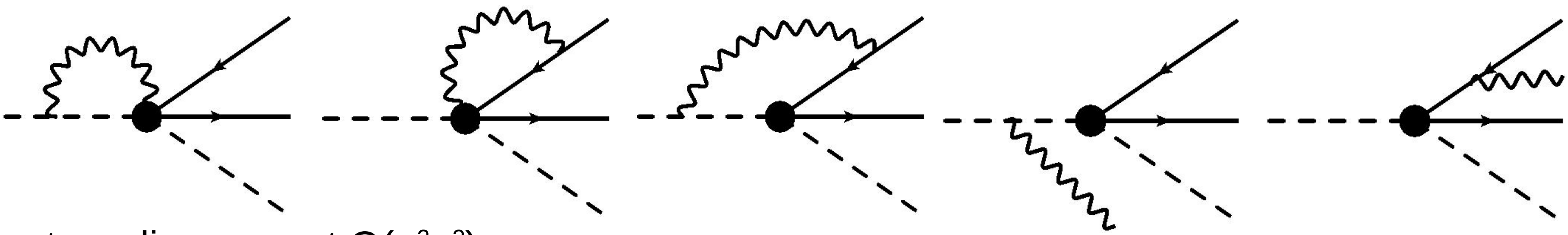
With dynamical leptons

$$\begin{aligned}\mathcal{L}^{e^2p^2}_{\{X\}} = {} & e^2 F_0^2 \textstyle\sum_\ell \{X_1 \bar{\ell}\gamma_\mu \nu_{\ell L}\langle u^\mu\{\mathcal{Q}^{\rm em}_{\rm R}, \mathcal{Q}^{\rm w}_{\rm L}\}\rangle + X_2 \bar{\ell}\gamma_\mu \nu_{\ell L}\langle u^\mu[\mathcal{Q}^{\rm em}_{\rm R}, \mathcal{Q}^{\rm w}_{\rm L}]\rangle \\ & + X_3 m_\ell \bar{\ell}\nu_{\ell L}\langle \mathcal{Q}^{\rm w}_{\rm L}\mathcal{Q}^{\rm em}_{\rm R}\rangle + h.c.\} + e^2\textstyle\sum_\ell X_6 \bar{\ell}(i\not{\partial} + e\not{A})\ell ,\end{aligned}$$

Knecht, Neufeld, Rupertsberger and Talavera, EPJC 2000

Chiral Perturbation Theory

K_{l3} RC in ChPT:



+ tree diagrams at $O(e^2p^2)$

Cirigliano, Knecht, Neufeld, Rupertsberger and Talavera, 2002 EPJC
Cirigliano, Neufeld and Pichl, 2004 EPJC
Cirigliano, Giannotti and Neufeld, 2008 JHEP

Connection to the **full EW theory** was done through **a specific combination of LECs**:

$$\begin{aligned} X_6^{\rm phys} &\equiv X_6^r - 4K_{12}^r \\ &= \left(X_6^{\rm phys}\right)_{\rm SD} + \left(X_6^{\rm phys}\right)_{\rm LD} \\ S_{\rm EW} &= 1 - e^2\left(X_6^{\rm phys}\right)_{\rm SD} \end{aligned}$$

Chiral Perturbation Theory

Quantification of **theory uncertainties** in ChPT:

- Uncertainties due to **neglected terms at $O(e^2p^4)$**: Estimated using standard chiral power counting, i.e. multiplying central values by $(M_K/\Lambda_\chi)^2 \sim 1/4$
- Uncertainties due to **unknown LECs at $O(e^2p^2)$**: LECs calculated using resonance models, and assign 100% uncertainty

Ananthanarayan and Moussallam, JHEP 2004
Descotes-Genon and Moussallam, EPJC 2005

	$\delta_{\mathrm{EM}}^{K\ell}(\%)$
K_{e3}^{0}	$0.99 \pm 0.19_{e^2p^4} \pm 0.11_{\mathrm{LEC}}$
$K_{e3}^{\pm}$	$0.10 \pm 0.19_{e^2p^4} \pm 0.16_{\mathrm{LEC}}$
$K_{\mu3}^{0}$	$1.40 \pm 0.19_{e^2p^4} \pm 0.11_{\mathrm{LEC}}$
$K_{\mu3}^{\pm}$	$0.016 \pm 0.19_{e^2p^4} \pm 0.16_{\mathrm{LEC}}$

Uncertainty ~$\mathbf{10^{-3}}$

"**Natural limitations**" of the ChPT precision

Cirigliano, Giannotti and Neufeld, JHEP 2008

Plans for direct lattice calculations of the full RC: ~10 years to reach 10^{-3} precision

Boyle et al., SnowMass 2021 LoI

Sirlin's representation

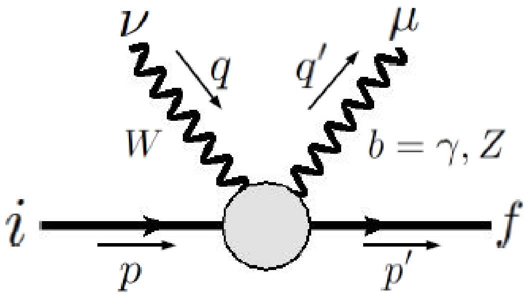
"Sirlin's representation" of the $O(G_F\alpha)$ electroweak RC (EWRC):

- First constructed by Sirlin to deal with EWRC in superallowed beta decays
  Sirlin, 1978 Rev.Mod.Phys
- Resurrected and generalized recently to study EWRC in general semileptonic decays
  CYS, Galviz and Meißner, 2020 JHEP
  CYS, 2021 Particles

Basic ingredients: **"Generalized Compton tensors"**

$$T^{\mu\nu}_{(b)}(q';p',p) = \int d^4x e^{iq'\cdot x}\langle\phi_f(p')|T\left\{J^\mu_b(x)J^\nu(0)\right\}|\phi_i(p)\rangle$$

$$\Gamma^{\mu}_{(b)}(q';p',p) = \int d^4x e^{iq'\cdot x}\langle\phi_f(p')|T\left\{J^\mu_b(x)\partial\cdot J(0)\right\}|\phi_i(p)\rangle$$



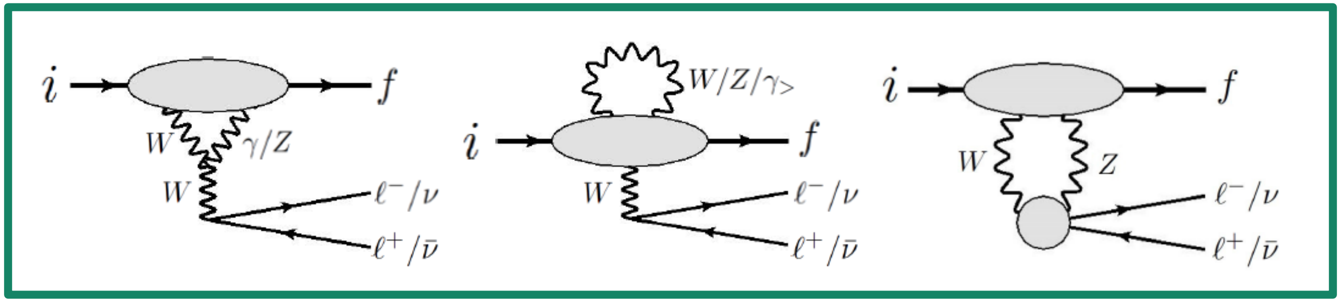
Theory Foundations:

Current Algebra (CA) and **Ward identities (WI)**, both are exact relations in QCD

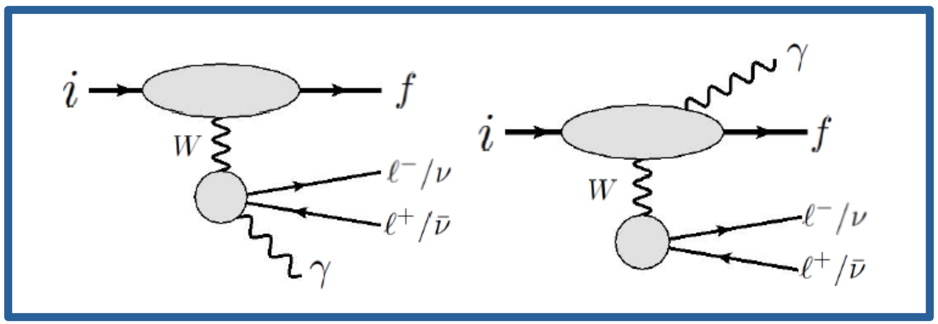
Sirlin's representation

$$\frac{1}{q'^2} = \frac{1}{q'^2 - M_W^2} + \frac{M_W^2}{M_W^2 - q'^2}\frac{1}{q'^2}$$

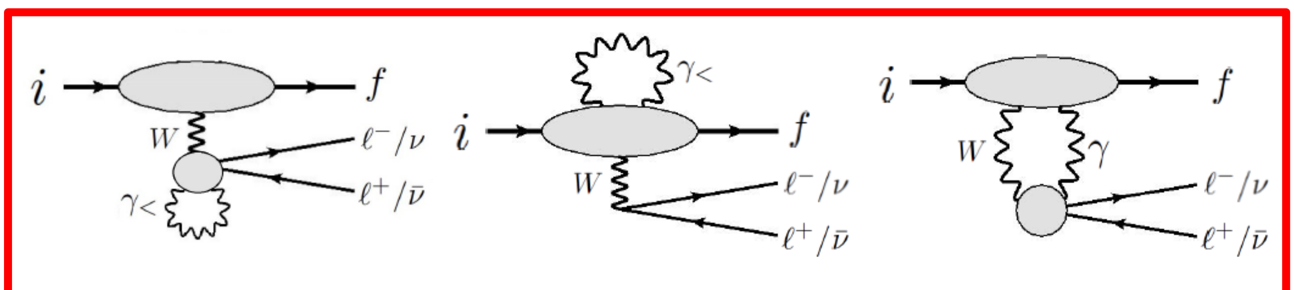
"$\gamma_>$" "$\gamma_<$"



"Weak" RC:
Calculable perturbatively to satisfactory precision



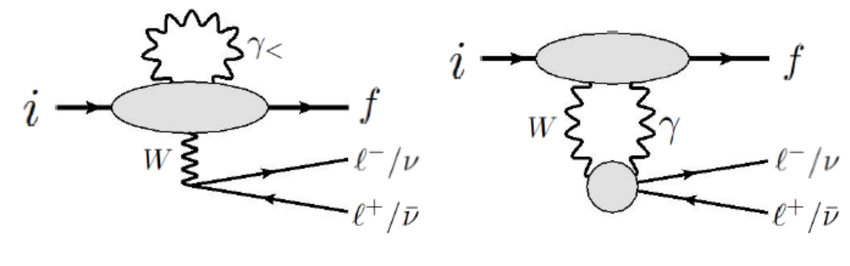
Bremsstrahlung



(Virtual) electromagnetic RC

Sirlin's representation

Further separation of the non-trivial virtual electromagnetic RC:



$$\left(\delta\mathfrak{M}_2 + \delta\mathfrak{M}^a_{\gamma W}\right)_{\text{int}} - \frac{G_F}{\sqrt{2}}\delta F_3^\lambda L_\lambda$$

Depends on physics at
$Q^2 << 1\text{GeV}^2$.

$$\delta\mathfrak{M}^b_{\gamma W}$$

Depends on physics at
$Q^2 \sim 1\text{GeV}^2$.

Sirlin's representation

Important concept: the **"convection term"**

$$T^{\mu\nu}_{\text{conv}}(q';p',p) = \frac{iZ_f(2p'+q')^\mu F^\nu(p',p)}{(p'+q')^2 - M_f^2} + \frac{iZ_i(2p-q')^\mu F^\nu(p',p)}{(p-q')^2 - M_i^2}$$

Meister and Yennie, PR 1963

is the simplest structure that satisfies the **exact EM Ward identity**, and thus **gives the full IR-divergent contribution** in both the loop and phase-space integrals.

Simplifications in the **near-degenerate limit:**

$\delta F_3^\lambda(p',p)$: vanishes

$\left(\delta\mathfrak{M}_2 + \delta\mathfrak{M}^a_{\gamma W}\right)_{\text{int}}$, $\mathfrak{M}_{\text{brems}}$: saturated by the convection term ("outer corrections") *Wilkinson and Macefield, 1970 Nucl.Phys.A*

$\delta\mathfrak{M}^b_{\gamma W}$: IR-finite and probes non-perturbative QCD ("inner corrections"). Can take the forward limit

The only non-trivial piece

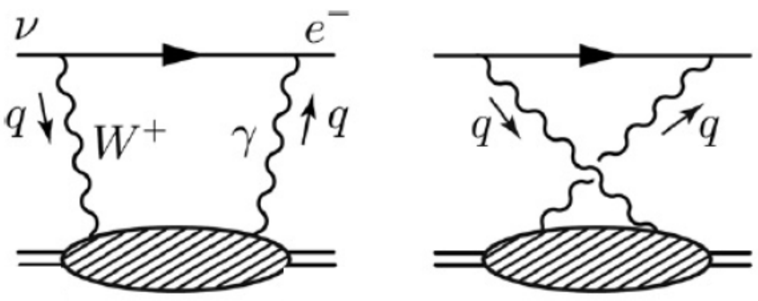
Inputs from Lattice QCD

In the degenerate limit, the main source of uncertainty: **the forward γW-box diagram**

$$\delta\mathfrak{M}^b_{\gamma W} = \Box_{\gamma W}(\phi_i,\phi_f,M)\mathfrak{M}_0$$

$$\Box_{\gamma W}(\phi_i,\phi_f,M) \equiv \frac{ie^2}{2M^2}\int\frac{d^4q}{(2\pi)^4}\frac{M_W^2}{M_W^2-q^2}\frac{1}{q^2}\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta\frac{T^{if}_{\mu\nu}(q;p,p)}{F^{if}_+(0)}$$

For spinless hadrons (e.g. pion), it probes the **AXIAL** charge weak current

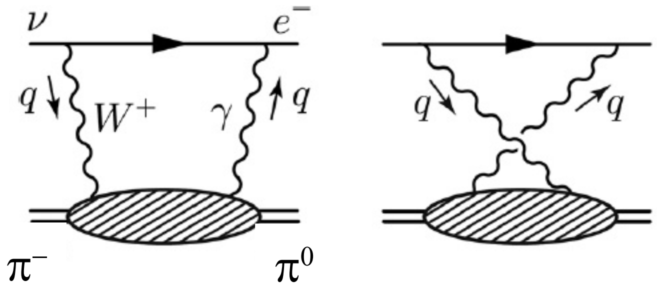


Sensitive to **non-perturbative QCD** at q~1GeV, which can be studied on **lattice**

Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL (pion box diagram)
Ma, Feng, Gorchtein, Jin and CYS, 2021 PRD (Kπ box diagram)

Inputs from Lattice QCD

Charged pion γW-box diagrams in pion semileptonic decay (π_{e3})



$$\Box_{\gamma W}(\pi^+, \pi^0, M_\pi) = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\mathbb{M}_\pi(Q^2)$$

Integral sensitive to **all values of Q^2**

Lattice not applicable at **large Q^2** (> 2 GeV^2) due to large lattice artifacts. But **perturbative QCD** works well:

$$\mathbb{M}_\pi(Q^2) = \frac{1}{12}\left[1 - \tilde{C}_1\left(\frac{\alpha_S}{\pi}\right) - \tilde{C}_2\left(\frac{\alpha_S}{\pi}\right)^2 - \tilde{C}_3\left(\frac{\alpha_S}{\pi}\right)^3 - \tilde{C}_4\left(\frac{\alpha_S}{\pi}\right)^4 - \ldots\right]$$

$$\begin{aligned}\tilde{C}_1 &= 1\\ \tilde{C}_2 &= 4.583 - 0.333n_f\\ \tilde{C}_3 &= 41.44 - 7.607n_f + 0.177n_f^2\\ \tilde{C}_4 &= 479.4 - 123.4n_f + 7.697n_f^2 - 0.1037n_f^3\end{aligned}$$

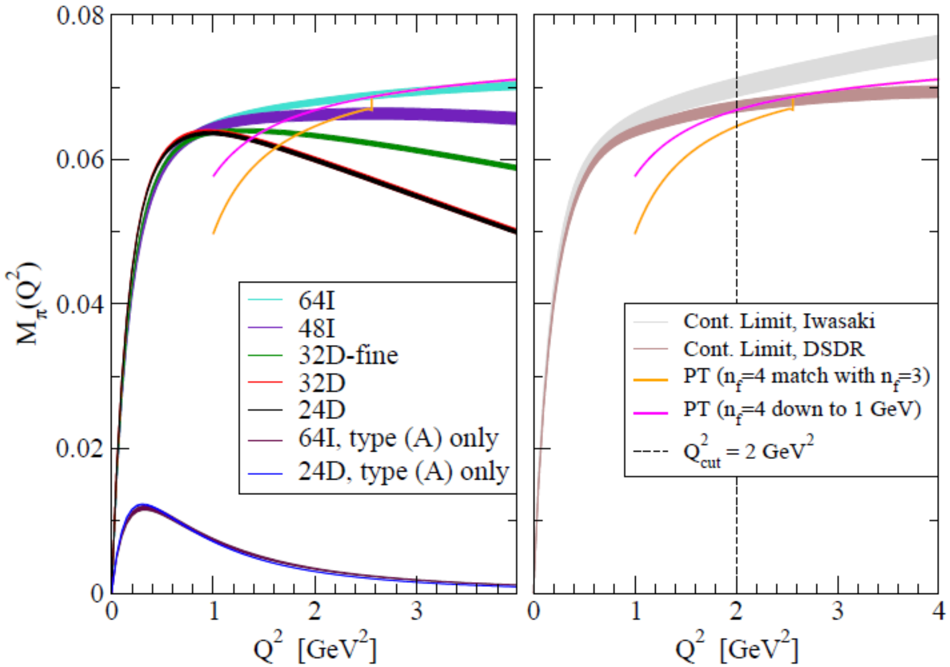
Baikov, Chetyrkin and Kuhn, 2010 PRL

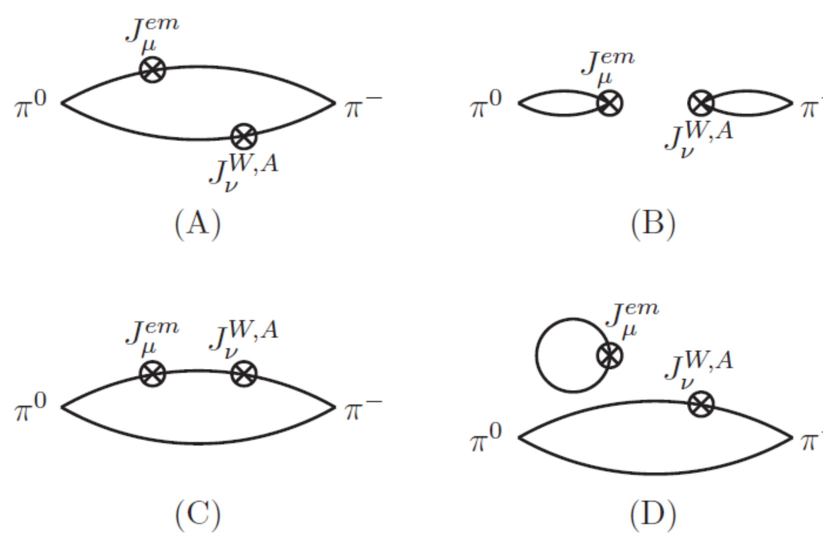
Inputs from Lattice QCD

At **low Q²** (< 2 GeV²): **direct lattice computation** of the generalized Compton tensor

$$\mathcal{H}^{VA}_{\mu\nu}(x) = \langle \pi^0(p) | T[J^{em}_\mu(x) J^{W,A}_\nu(0)] | \pi^-(p) \rangle$$

$$\mathrm{M}_\pi(Q^2) = -\frac{1}{6\sqrt{2}} \frac{\sqrt{Q^2}}{m_\pi} \int d^4x \omega(Q,x) \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}^{VA}_{\mu\nu}(x)$$





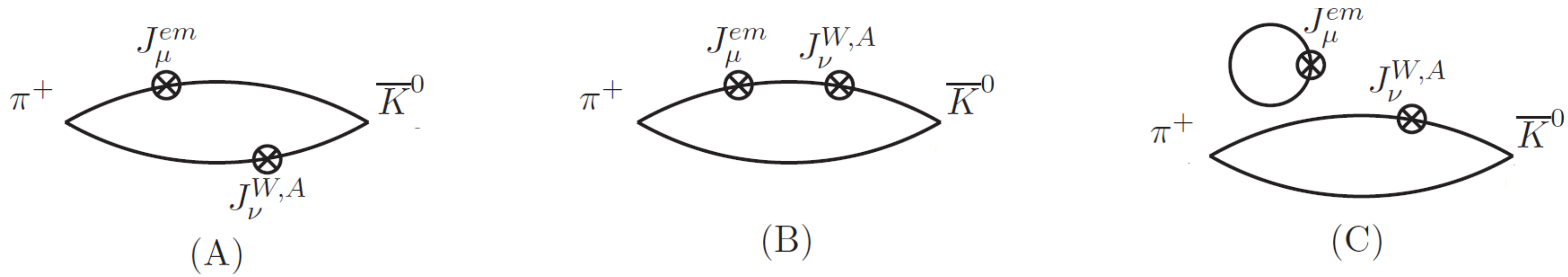
Quark contraction diagrams

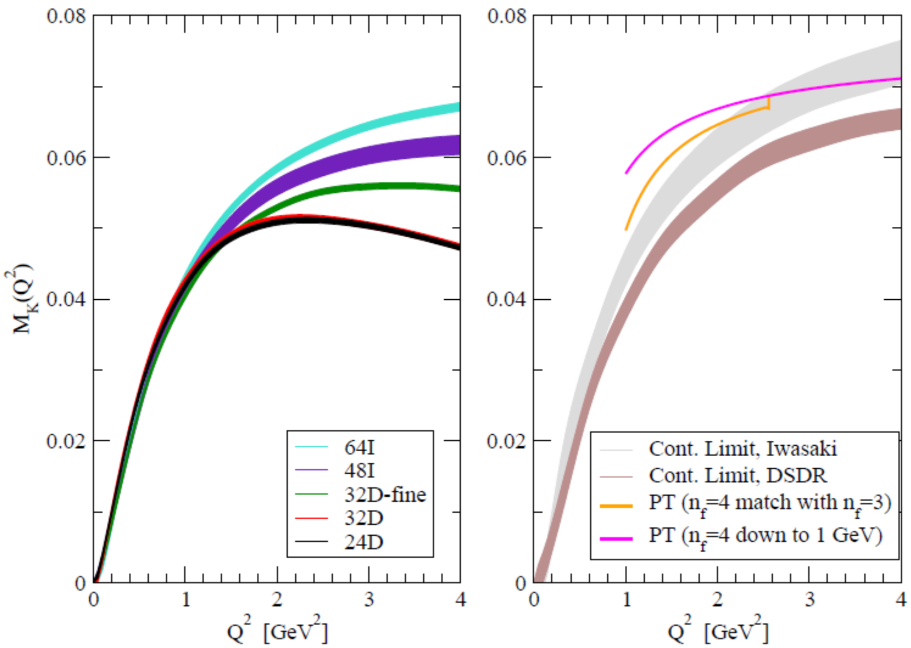
Final result: $\Box_{\gamma W}(\pi^+, \pi^0, M_\pi) = 2.830(11)_{\mathrm{stat}}(26)_{\mathrm{syst}} \times 10^{-3}$

Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL

Inputs from Lattice QCD

Similar calculation of the **Kπ box diagram** in the **degenerate limit**:





Final result:

$$\Box_{\gamma W}(K^0, \pi^-, M_\pi) = 2.437(44) \times 10^{-3}$$

Ma, Feng, Gorchtein, Jin and CYS, 2021 PRD

Inputs from Lattice QCD

First application of lattice QCD inputs in kaon decays:

Comparing the **ChPT** and **Sirlin's representation** in the **forward limit** gives the **matching conditions between LECs and box diagrams**:

$$\frac{4}{3}X_1 + \bar{X}_6^{\mathrm{phys}}(M_\rho) = -\frac{1}{2\pi\alpha}\left(\Box_{\gamma W}^{VA}(\pi_0,\pi_+) - \frac{\alpha}{8\pi}\ln\frac{M_W^2}{M_\rho^2}\right) + \frac{1}{8\pi^2}\left(\frac{5}{4} - \tilde{a}_g\right)$$

$$-\frac{8}{3}X_1 + \bar{X}_6^{\mathrm{phys}}(M_\rho) = -\frac{1}{2\pi\alpha}\left(\left(\Box_{\gamma W}^{VA}(\pi_-,K_0)\right)_{\mathrm{SU(3)}} - \frac{\alpha}{8\pi}\ln\frac{M_W^2}{M_\rho^2}\right) + \frac{1}{8\pi^2}\left(\frac{5}{4} - \tilde{a}_g\right)$$

CYS, Feng, Gorchtein, Jin and Meißner, 2020 JHEP

Improved determination of **LECs**:

	Resonance model	Lattice
X_1	$(-3.7 \pm 3.7) \times 10^{-3}$	$(-2.2 \pm 0.4) \times 10^{-3}$
$\bar{X}_6^{\mathrm{phys}}$	$(10.4 + 3.0 \pm 10.4) \times 10^{-3}$	$(16.9 \pm 0.7) \times 10^{-3}$

Improved determination of the **K_{l3} EMRC (%)**:

$$\delta_{K^0}^{e} = 0.99(19)_{e^2p^4}(11)_{\mathrm{LEC}} \quad \to \quad 1.00(19)$$

$$\delta_{K^0}^{\mu} = 1.40(19)_{e^2p^4}(11)_{\mathrm{LEC}} \quad \to \quad 1.41(19)$$

$$\delta_{K^\pm}^{e} = 0.10(19)_{e^2p^4}(16)_{\mathrm{LEC}} \quad \to \quad -0.01(19)$$

$$\delta_{K^\pm}^{\mu} = 0.02(19)_{e^2p^4}(16)_{\mathrm{LEC}} \quad \to \quad -0.09(19).$$

Ma, Feng, Gorchtein, Jin and CYS, 2021 PRD

Electroweak RC in K_{e3}

Unlike π_{e3}, the K_{l3} RC is unsettled even with the aforementioned lattice inputs!
Reason:

- **In ChPT language:** Fixing LECs are not enough because the major uncertainties from $O(e^2p^4)$ corrections remain
- **In Sirlin's representation**: The non-forward $K\to\pi$ decay causes several extra complications:

$\delta F_3^{\lambda}(p',p)$: **does not vanish**

$\left(\delta\mathfrak{M}_2+\delta\mathfrak{M}_{\gamma W}^a\right)_{\text{int}}$, $\mathfrak{M}_{\text{brems}}$: **not saturated by the convection term**

$\delta\mathfrak{M}_{\gamma W}^b$: **cannot simply take forward limit**

Further analysis of the these terms is needed

Electroweak RC in K_{e3}

Learning from the tree-level squared amplitude formula:

$$|\mathfrak{M}_0^{(0)}|^2 = 2G_F^2|V_{us}|^2M_K^4\Big\{A_1^{(0)}(y,z)|f_+^{K\pi}(t)|^2 + A_2^{(0)}(y,z)f_+^{K\pi}(t)f_-^{K\pi}(t) + A_3^{(0)}(y,z)|f_-^{K\pi}(t)|^2\Big\}$$

$$A_1^{(0)}(y,z) = 4(1-y)(y+z-1) + r_\ell(4y+3z-3) - 4r_\pi + r_\ell(r_\pi - r_\ell)$$

$$A_2^{(0)}(y,z) = 2r_\ell(3-2y-z+r_\ell-r_\pi)$$

$$A_3^{(0)}(y,z) = r_\ell(1-z+r_\pi-r_\ell)\,.$$

$$r_\ell \equiv m_\ell^2/M_K^2$$

with $K\pi$ **charged weak form factors**:

$$F_\mu^{K\pi}(p',p) \equiv \langle \pi(p')|\,(J_\mu^W)^\dagger\,|K(p)\rangle = V_{us}^*\left[\underline{f_+^{K\pi}(t)}(p+p')_\mu + \underline{f_-^{K\pi}(t)}(p-p')_\mu\right]$$

We restrict ourselves to **K_{e3}**, where the **tree-level** and **virtual EWRC** contributions from **f_-** to the squared amplitude are **suppressed by r_e~10^{-6}**, bringing great simplifications

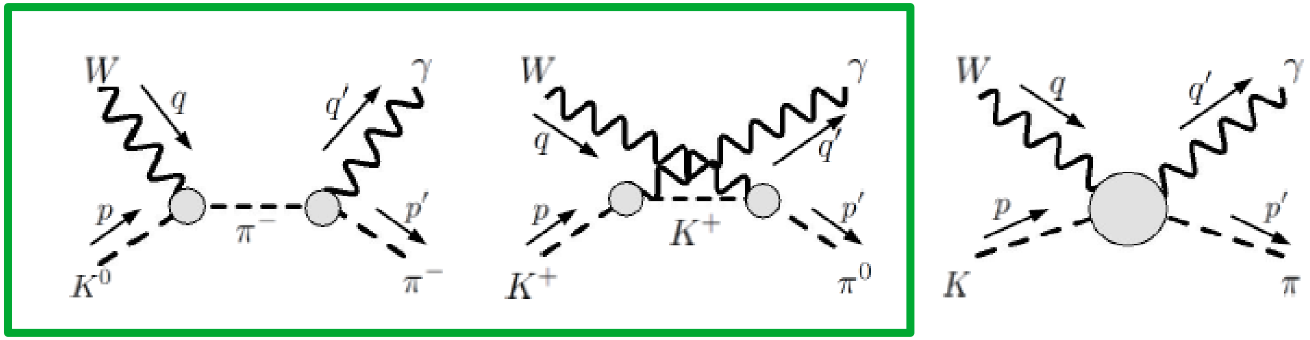
Electroweak RC in K_{e3}

Non-trivial piece (A) :

"Residual integral" + the vector current contribution to $\delta\mathfrak{M}^b_{\gamma W}$

$$I^\lambda_{\mathfrak{A}} = -e^2\int\frac{d^4q'}{(2\pi)^4}\frac{1}{[(p_e-q')^2-m_e^2][q'^2-M_\gamma^2]}\left\{\frac{2p_e\cdot q' q'^\lambda}{q'^2-M_\gamma^2}T^\mu_\mu + 2p_{e\mu}T^{\mu\lambda} -(p-p')_\mu T^{\lambda\mu} + i\Gamma^\lambda - i\epsilon^{\mu\nu\alpha\lambda}q'_\alpha (T_{\mu\nu})_V\right\}.$$

Its contribution to f_+ is saturated by the "**pole" terms**" in $T_{\mu\nu}$ and Γ_μ :



- Required inputs: **K/π EM form factors** and **charged weak form factors**, well-measured in experiments
- Equivalent to **re-summing the most important $O(e^2p^n)$ corrections** in ChPT

Electroweak RC in K_{e3}

Non-trivial piece (B) :

The axial current contribution to $\delta \mathfrak{M}^b_{\gamma W}$

$$I^\lambda_{\mathfrak{B}} = ie^2 \int \frac{d^4q'}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q'^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q'_\alpha (T_{\mu\nu})_A}{[(p_e - q')^2 - m_e^2] q'^2}$$

Its contribution to f_+ is directly related to the lattice QCD calculation of the **forward axial Kπ box diagram**:

$$(\delta f_+)_{\mathfrak{B}} = \underbrace{\Box^{>}_{\gamma W} f_+(t)}_{\text{pQCD}} + \{\overbrace{\Box^{<}_{\gamma W}(K,\pi,M_\pi)}^{\text{lattice input}} + \mathcal{O}(M_K^2/\Lambda_\chi^2)\} f_+(t)$$

A **non-forward uncertainty** is assigned using standard chiral power counting.

Electroweak RC in K_{e3}

Non-trivial piece (C) : f_+ contributed by the 3-pt function

$$\delta f_{+,3} = (\delta f_+)_{\text{III}} + \left\{ (\delta f_{+,3})^{\text{fin}}_{e^2 p^2} + \mathcal{O}(e^2 p^4) \right\}$$

Resummed term to ensure IR-finiteness

Regular terms calculated at fixed-order ChPT

Non-trivial piece (D) : Bremsstrahlung contribution

$$\mathfrak{M}_{\text{brems}} = \mathfrak{M}_A + \mathfrak{M}_B$$

$$T^{\mu\nu} = T^{\mu\nu}_{\text{conv}} + \left\{ (T^{\mu\nu} - T^{\mu\nu}_{\text{conv}})_{p^2} + \mathcal{O}(p^4) \right\}$$

Full convection term contribution, contains full IR-divergence

Regular terms calculated at fixed-order ChPT

Electroweak RC in K_{e3}

Final Result:

	ChPT	New result
$\delta_{\mathrm{EM}}^{K^+e}(\%)$	$0.10(19)_{e^2p^4}(16)_{\mathrm{LEC}}$	$0.21(2)_{\mathrm{inel}}(1)_{r_K}(1)_{\mathrm{lat}}(4)_{\mathrm{NF}}(1)_{e^2p^4}$
$\delta_{\mathrm{EM}}^{K^0e}(\%)$	$0.99(19)_{e^2p^4}(11)_{\mathrm{LEC}}$	$1.16(2)_{\mathrm{inel}}(1)_{\mathrm{lat}}(1)_{\mathrm{NF}}(2)_{e^2p^4}$

CYS, Galviz, Gorchtein and Meißner, 2021 PLB
CYS, Galviz, Gorchtein and Meißner, 2103.04843 (accepted by JHEP)

Sources of uncertainty:

- **inel**: Contributions from inelastic states to the residual integral
- **rk**: Experimental uncertainty of the kaon charge radius
- **lat**: Lattice uncertainty in the γW-box diagram
- **NF**: Uncertainty due to non-forward effects in the γW-box diagram
- **e^2p^4**: Higher-order ChPT corrections

- Consistent with the pure ChPT result within error bars
- Significant improvement of precision: **$10^{-3} \rightarrow 10^{-4}$**

Summary

- ~2σ discrepancy exists between V_{us} extracted from $K_{\mu 2}$ and K_{l3}. SM theory inputs must be further scrutinized to ensure that it does not originate from unexpected SM corrections
- Electromagnetic RC in K_{l3} carries a "natural limitation" in precision ~10^{-3}, which is irreducible using ChPT and other model-dependent frameworks
- Adopting Sirlin's representation of RC in K_{e3} successfully overcomes the "natural limitation" (precision improves from 10^{-3} to 10^{-4}) by:
  - Effectively re-summing the most important $O(e^2p^n)$ contributions in ChPT to reduce the higher-order uncertainties
  - Using appropriate lattice QCD inputs to effectively reduce the uncertainties from non-perturbative QCD (LECs in ChPT language)
- The outcome is consistent with pure ChPT results. The V_{us} anomaly is unlikely to originate from SM RC
- Future step is to generalize the analysis above to $K_{\mu 3}$. More complicated error analysis is expected