Lattice QCD prediction of ε'/ε
(and status of ε, $K \to \pi\bar{\nu}\nu$)

Mattia Bruno
for the RBC/UKQCD collaboration

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Motivations

CP violation in SM too small for observed matter/anti-matter asymm.

tantalizing hint for physics beyond SM

CP violation discovered in $K \to \pi\pi$ decays

physical states $K_{L,S}$ linear combo of CP eigenstates

$\Delta m_K = M_L - M_S$ mass difference

indirect CP violation: ε

direct CP violation: ε'

From experiments we get ratios of amplitudes, $\eta_{ij} = \frac{\mathcal{A}[K_L \to \pi^i \pi^j]}{\mathcal{A}[K_S \to \pi^i \pi^j]}$

we can relate $\eta_{00}, \eta_{+-} \leftrightarrow \varepsilon, \varepsilon'$ and get

$|\varepsilon| = 2.228(11) \cdot 10^{-3}$

$\mathrm{Re}\,(\varepsilon'/\varepsilon) = 1.66(0.23) \cdot 10^{-3}$

1. Hadronic ($\simeq$ low-energy) weak decays (=mediated by W bosons)
   Effective field theory → integrate heavy degrees of freedom
   W, Z, top, bottom, but also charm → $N_f = 2+1$ theory

$$\mathcal{H}_{\Delta S=1} = \frac{G_\mathrm{F}}{\sqrt{2}} V^\star_{us} V_{ud} \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu)$$

   z_i, y_i Wilson coefficients, known to 1-loop in $\overline{\mathrm{MS}}$
   $\tau = -V^\star_{ts} V_{td} / (V^\star_{us} V_{ud})$ complex → CP-violation
   $Q_i(\mu)$ four-quark ops, must be computed in $\overline{\mathrm{MS}}$

2. Using isospin symmetry, classify amplitudes $A_I e^{i\delta_I} = \langle (\pi\pi)_I | \mathcal{H}_W | K \rangle$
   e.g. $\varepsilon'/\varepsilon = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2} - \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}\right], \quad \omega = \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0}$

3. Given non-perturbative nature of A_I we use Lattice QCD
   our biggest contribution is $\langle (\pi\pi)_I | Q_i(\mu) | K \rangle$

Lattice field theories

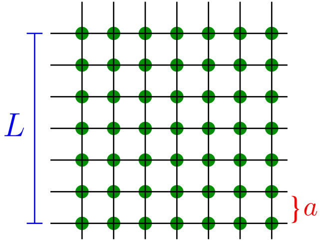
Mathematically sound non-perturbative formulation of QCD

lattice spacing $a \to$ regulate UV divergences

finite size $L \to$ infrared regulator

Continuum theory $a \to 0$, $L \to \infty$

Euclidean metric $\to$ Boltzman interpretation of path integral



$$\langle O \rangle = \mathcal{Z}^{-1} \int [DU] e^{-S[U]} O(U) \approx \frac{1}{N} \sum_{i=1}^{N} O[U_i]$$

Very high dimensional integral $\to$ Monte-Carlo methods

Hadronic weak decays

Theoretical Challenges - I

Formulation of LQCD w/ good chiral symmetry very important

often prevents power divergences $1/a^k$ [Capitani, Giusti '01, ...]

suppresses mixing w/ wrong chiralities (simpler renormalization)

Fermion doubling $\leftrightarrow$ chiral symmetry [Nielsen-Ninomiya '81]

domain-wall formulation (DWF) [Kaplan '92, Shamir '93, Brower et al. '12]

other formulations: staggered, Wilson-clover and twisted mass

Well-defined non-pertubative renormalization scheme

momentum schemes [Martinelli et al. '95][Sturm et al '09]

regularization independent $\rightarrow$ pert. conversion to $\overline{\mathrm{MS}}$

other schemes (Schrödinger functional, Wilson flow) under devel

Hadronic weak decays

Theoretical Challenges - II

[Lüscher '85, ...][Lellouch-Lüscher '00, ...]

Finite volume L: no asymptotic states, scattering? decays?

single-particle states are e^{-mL} close to $L = \infty$

multi-particle states generate $1/L^k$ effects

removable below 4 particle threshold, such that $O(e^{-mL})$

$\rightarrow m_K < 4m_\pi$, but $m_D \gg 4m_\pi$ (new ideas under devel)

Euclidean metric: correlator $\langle 0|O^\dagger(t)O(0)|0\rangle = \langle 0|O^\dagger(0)e^{-\hat{H}t}O(0)|0\rangle$

Eucl. metric filters low energies at $t \gg 0$ [Maiani, Testa '90]

$\rightarrow$ higher states, e.g. $\hat{H}|\pi\pi\rangle = m_K|\pi\pi\rangle$, exponentially suppressed

boundary conditions to constrain $\pi\pi$ ground state at m_K

[Blum et al. '12][Christ et al. '19]

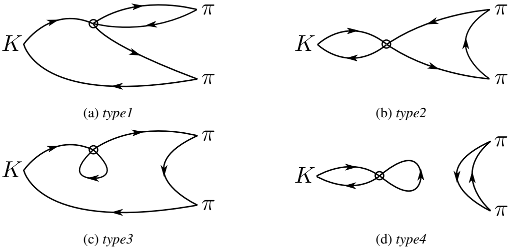


FIG. 2: The four classes of $K \to \pi\pi$ Wick contractions.

Signal-to-noise problem for lattice correlators at large separations

signal $\propto e^{-M_s|x_0-y_0|}$, error $\propto e^{-M_e|x_0-y_0|}$ [Parisi '84, Lepage '89]

e.g. type1 $M_s \approx m_K$, $M_e \approx 2m_\pi$

type4 noisiest: $M_e = 0$ at large t

Status $\Delta I = 1/2$ rule

$K \to (\pi\pi)_{I=2}$ complete calculation [RBC/UKQCD '15]

no disconnected diagrams, numerically simpler

continuum limit from 2 lattice spacings; phys. quark masses

lattice $\quad \mathrm{Re}\, A_0 / \mathrm{Re}\, A_2 = 19.9(5.0)$

$\mathrm{Re}\, A_0$ [RBC/UKQCD '20] and $\mathrm{Re}\, A_2$ [RBC/UKQCD '15]

experiment $\quad \mathrm{Re}\, A_0 / \mathrm{Re}\, A_2 = 22.46(6)$

QCD induces remarkable cancellation at phys. quark masses

matrix element $\Delta I = 3/2$, $Q_2 \simeq -0.7 Q_1$

understanding of $\Delta I = 1/2$ from first principles [RBC/UKQCD '15]

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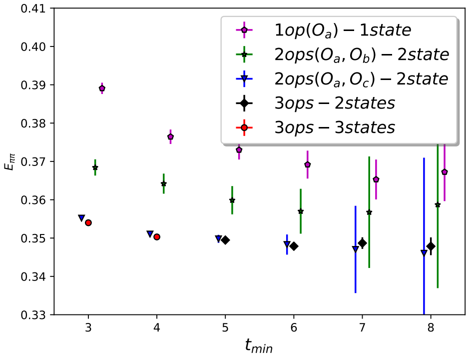
Status $\pi\pi$ scattering

[RBC/UKQCD '21]

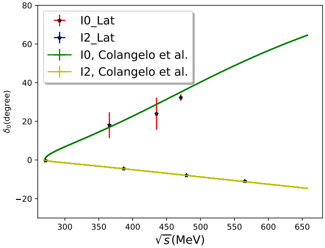
multi operators w/ same quantum numbers

better constrain spectrum and amplitudes

significant improvement syst. errors



solved $I = 0$ $\pi\pi$ phase shift discrepancy w/ dispersive approach



same multi-ops technique also for $K \to \pi\pi$

STATUS ε'/ε - I

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{\omega}{\sqrt{2}|\varepsilon|}\mathrm{Re}\left[ie^{i(\delta_2-\delta_0-\phi_\varepsilon)}\right]\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]$$

1. use $\omega, \mathrm{Re}\,A_0, \mathrm{Re}\,A_2$ from experiment
2. phases either from dispersive or lattice, no difference
3. take $\mathrm{Im}\,A_2$ from previous LQCD [RBC/UKQCD '15]
4. take $\mathrm{Im}\,A_0$ from new work [RBC/UKQCD '20]

$\mathrm{Re}\,(\varepsilon'/\varepsilon) = 21.7(2.6)(6.2)(5.0) \cdot 10^{-4}$ from lattice [RBC/UKQCD '20]
$\mathrm{Re}\,(\varepsilon'/\varepsilon) = 16.6(2.3) \cdot 10^{-4}$ from experiment

Errors:

(2.6) statistical, (6.2) systematic, (5.0) isospin-breaking

Status ε'/ε - II

Error budget $\mathrm{Im}A_0 = -6.98(0.62)(1.44) \times 10^{-11}\mathrm{GeV}$

9%: statistical, remarkable achievement

21%: systematic error $[(16\%)^2 + (12\%)^2 + (6\%)^2]^{1/2}$

$\rightarrow 16\%$: lattice syst. errors $[(12\%)^2 + (7\%)^2 + \cdots]^{1/2}$

$\rightarrow 12\%$: cont. limit

$\mathrm{Im}A_0$ from single lattice spacing

estimate taken from $\Delta I = 3/2$ matrix elements

$\rightarrow 7\%$: finite volume effects

$\rightarrow \ldots$

$\rightarrow 12\%$: Wils. Coefficients

due to perturbative truncation

lack of charm in calculation leads to large effects

$\rightarrow 6\%$: parametric errors, e.g. τ, α_s

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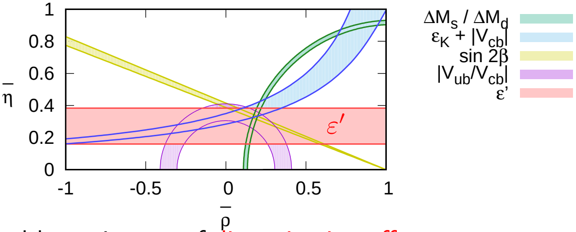
Future of ε'/ε

unitarity triangle
[Lehner et al. '15]
[RBC/UKQCD '20]



Lattice errors dominated by estimates of discretization effects

- next-gen computers unlock opportunity for cont. limit
- current plan: add two additional ensembles [Kelly Lattice '21]

Independent calculation w/ available ensembles periodic BC

- need to extract states exponentially suppressed [Tomii Lattice '21]

On-going work for $3 \to 4$ flavor matching using LQCD

- crucial to bypass PT at charm scale [PoS LATTICE2018 (2019) 216]

New devel include EM effects in two-particle quantization condition
[Karpie Lattice '21][Christ et al. '21]

ε_K

In usual two-state picture w/ $|K_0\rangle, |\bar{K}_0\rangle$ and $H_{ij} = M_{ij} - i\Gamma_{ij}$

$\Delta m_K = M_L - M_S$ and ε_K generated from $K_0 - \bar{K}_0$ mixing

$$\Delta m_K = 2\mathrm{Re}\, M_{12}\,, \quad |\varepsilon_K| \propto \left[\frac{\mathrm{Im}\, M_{12}}{\Delta m_K} + \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}\right]$$

$$\mathrm{Im}\, M_{12} = \mathrm{Im}\, M_{12}^{\mathrm{SD}} + \mathrm{Im}\, M_{12}^{\mathrm{LD}}$$ [Buras et al '10]

Short-distance from $\langle \bar{K}_0 | \mathcal{H}_{\Delta S=2} | K_0 \rangle \to B_K$ parameter

single operator + external kaons $\to$ high-precision from LQCD

but renormalization: break chiral symm. mixing w/ wrong chiralities

$$B_K^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 0.5570(71)\,[1.2\%]\,, \quad N_f = 2+1$$ [FLAG '21]

Lattice QCD calculations at $O(1-2\%)$ are standard

careful if evaluate B_K with dynamical charm

Long-distance effects from double insertions of $\mathcal{H}_{\Delta S=1}$

new frontier for Lattice QCD

$$\int d^4x \langle f| \mathrm{T}\big[\mathcal{O}_1(x)\mathcal{O}_2(0)\big]|i\rangle$$

[Isidori et al. '05][Chirst et al. '12]

1. new divergences $x \to 0$? [Christ et al. '16][Christ et al. '18]

Δm_K (and $K \to \pi \ell^+ \ell^-$) fully protected $N_f = 4$

ε_K and $K \to \pi \bar{\nu} \nu$ no power divergeces $1/a^k$ w/ χ symm.

additional renormalization (log divergence)

$$\sum_{t=0}^{T} \langle f|\mathcal{O}_1(t)\mathcal{O}_2(0)|i\rangle \simeq \sum_n \frac{\langle f|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|i\rangle}{m_f - E_n}\big[1 - e^{-(E_n - m_f)T}\big]$$

2. growing exponentials $E_n < m_f$, problem of analytic continuation

if $f, i = m_\pi, m_K$ and $m_\pi L \simeq 4$ they can be handled

for f, i heavy mesons still a challenge (new methods under study)

3. finite volume effects [Christ, Feng, Martinelli, Sachrajda '15]

worked out from extension Lellouch-Lüscher correction

Status - LD effects

Numerical challenges

Δm_K

$\Delta m_K^{\text{exp}} = 3.483(6) \cdot 10^{-12}$ MeV

first numerical results only in PoS [Wang Lattice '18 '19]

preliminary results at $m_\pi^{\text{phys}} \to \Delta m_K = 6.7(1.7) \cdot 10^{-12}$ MeV

- dominated by discr. errors from charm
- $m_\pi^{\text{phys}} \to$ large $L \oplus$ charm $\to$ fine a: challenging

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ε_K

$|\varepsilon_K^{\text{exp}}| = 2.228(11) \cdot 10^{-3}$

conference papers, latest $\varepsilon_K^{\text{LD}} = 0.17(1) \cdot 10^{-3}$ [Bai Lattice '16]

- significant amount of Wick contractions and topologies

preliminary results at $m_\pi \simeq 390$ MeV, unphys. charm

- approx 5% consistent w/ expectation but requires improvements

Status - LD effects

$K \to \pi\bar{\nu}\nu$

- FCNC ideal probes for new physics effects
- mostly dominated by short-distance effects, QCD input from $K_{\ell 3}$
  - current theory predictions around 10 % [Buras et al '15]
  - LD effects potentially up to 6% in $K^+ \to \pi^+\bar{\nu}\nu$
- exploratory calculation at unphys. kinematics [RBC/UKQCD '17 '18]
  - cancellation of WW vs Z-exchange → will survive at m_π^{phys}?
- 2nd calculation $m_\pi \simeq 170\ \mathrm{MeV}$, unphys. charm [RBC/UKQCD '19]
  - small mom. dependence, clarified role of intermediate $(\pi\pi)$

Exciting results to be expected over next years for LD effects

- advent of (pre-)exascale era in computing crucial
  - fine lattice spacings required for including charm (safely)

Thanks for your attention!

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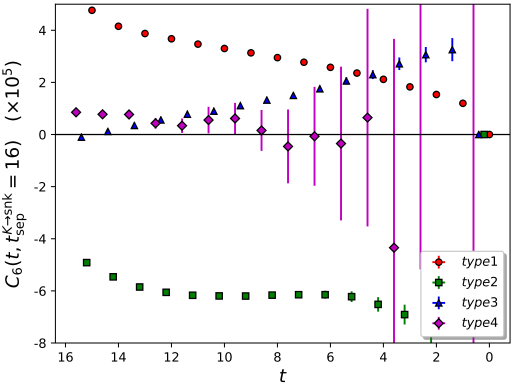
Q_6 IN $\text{Im}\, A_0$



Example of signal-to-noise depending on diagr. topology

type1 best case $M_e = 2m_\pi$

type2 and 3 $M_e = m_\pi$

type4 worst case $M_e = 0$