Progress on hadronic uncertainties in $b \to s\ell\ell$

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CKM ~~2020~~ 2021

November 22nd 2021

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Introduction

Scope of this presentation

- treatment of hadronic uncertainties for theory predictions in and beyond the SM, which is crucial to understand the $b \to s\ell\ell$ anomalies
- for review of the anomalies and phenomenology implications, see the plenary talk by Wolfgang Altmannshofer earlier today
- here: focus on recent progress for non-local contributions in $b \to s\ell\ell$

in the SM $b \to s\ell\ell$ is described by the following set of $D = 6$ effective operators

$$\mathcal{L}_{\text{SM}}^{\text{eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}}\left[\lambda_t \sum_i \mathcal{C}_i \mathcal{O}_i + \lambda_c \sum_i \mathcal{C}_i^c \mathcal{O}_i^c + \lambda_u \sum_i \mathcal{C}_i^u \mathcal{O}_i^u\right]$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R T^A b) G^A_{\mu\nu}$$

$$\mathcal{O}_9 = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{10} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_1^q = (\bar{q}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L q) \qquad \mathcal{O}_2^q = (\bar{q}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a q)$$

$$\mathcal{O}_i = (\bar{s}\Gamma P_X b) \sum_q (\bar{q}\Gamma' q) \qquad \text{(QCD penguins)}$$

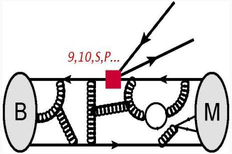
with $\lambda_q \equiv V_{qb} V_{qs}^*$

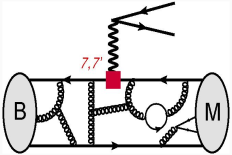
- ▶ SM contributions to $\mathcal{C}_i(\mu_b)$ known to NNLL

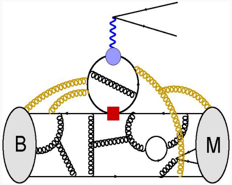
[Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04]

[Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]

Anatomy of exclusive $b \to s\ell^+\ell^-$ decay amplitudes







$$\mathcal{A}_\lambda^\chi = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

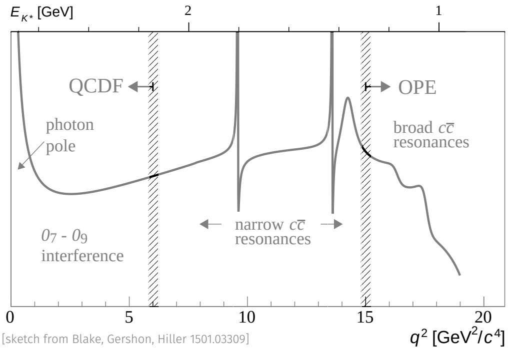
nomenclature of the essential hadronic matrix elements $\qquad q^2 = m_{\ell\ell}^2$

- $\mathcal{F}_\lambda$ local form factors of dimension-three $\bar{s}\gamma^\mu b$ & $\bar{s}\gamma^\mu\gamma_5 b$ currents
- $\mathcal{F}_\lambda^T$ local dipole form factors of dimension-three $\bar{s}\sigma^{\mu\nu} b$ currents
- $\mathcal{H}_\lambda$ nonlocal form factors of dimension-five nonlocal operators

all three needed for consistent description to leading-order in α_e

Spectrum

$$\mathcal{H}_\lambda(q^2) = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{\rm em}(x), [C_1 O_1^c + C_2 O_2^c](0) \} |H_b\rangle$$



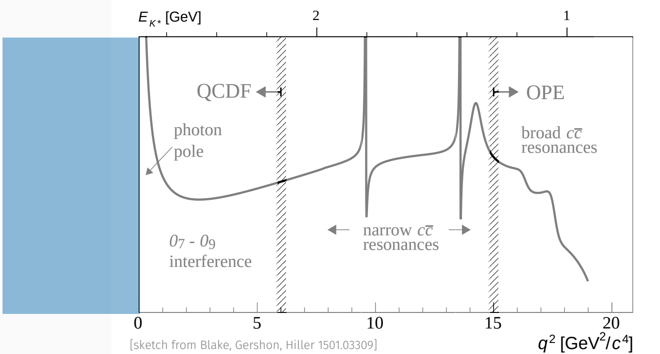
[sketch from Blake, Gershon, Hiller 1501.03309]

- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

source of dominant systematic uncertainties in theoretical predictions!

Spectrum

$$\mathcal{H}_\lambda(q^2) = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{\text{em}}(x), [C_1 O_1^c + C_2 O_2^c](0) \} |H_b\rangle$$



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- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

- for $q^2 - 4m_c^2 \ll \Lambda_{\text{had}} m_b$, expand T-product in light-cone operators [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- leading contributions expressed through local form factors $\mathcal{F}_\lambda$
- correction suppressed by $1/(q^2 - 4m_c^2)$ can by systematically obtained

Spectrum

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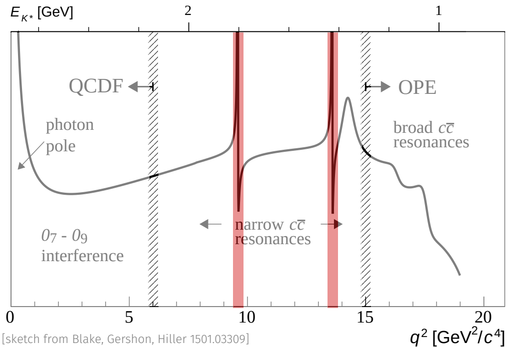

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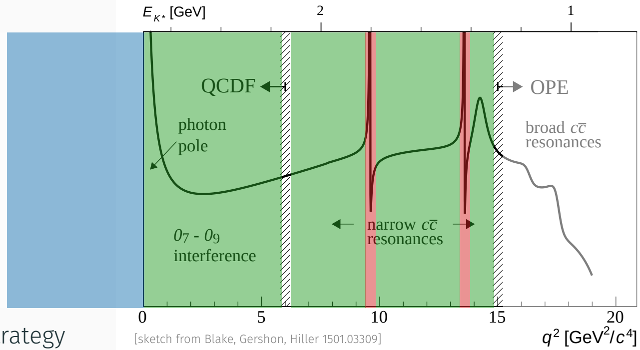
- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

- for $q^2 = M^2_{J/\psi}$ and $q^2 = M^2_{\psi(2S)}$, spectrum dominated by non-leptonic decays
- experimental measurements provide additional information about $\mathcal{H}_\lambda$

$$\mathcal{H}_\lambda(q^2) = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{\mathrm{em}}(x), [C_1 O_1^c + C_2 O_2^c](0) \} |H_b\rangle$$



[sketch from Blake, Gershon, Hiller 1501.03309]

- $O_{1,2}^c \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

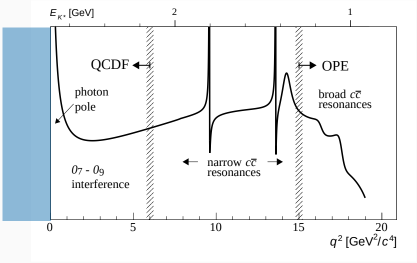
[Bobeth,Chrzaszcz,DvD,Virto '17]

new strategy

- compute $\mathcal{H}_\lambda$ at spacelike q^2
- extrapolate to timelike $q^2 \leq 4M_D^2$ using suitable parametrization
- include information from non-leptonic decays to narrow charmonia J/ψ and $\psi(2S)$

Compute Light-Cone OPE

$$4m_c^2 - q^2 \gg \Lambda^2_{\text{hadr.}}$$

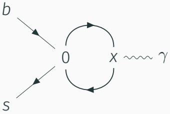


- expansion in operators w/ light-like sep. $x^2 \simeq 0$

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

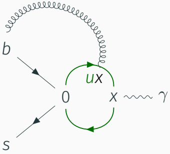
- employing light-cone expansion of charm propagator

[Balitsky, Braun 1989]

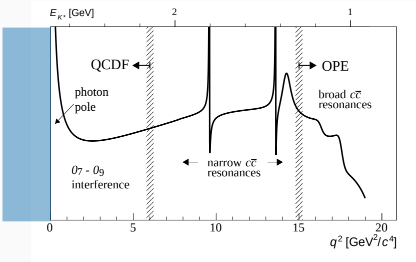


$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{coeff \#1}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$+ (\text{coeff \#2}) \times [\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L]$$



$$4m_c^2 - q^2 \gg \Lambda_{\text{hadr.}}^2$$



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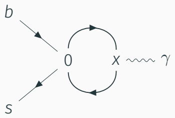
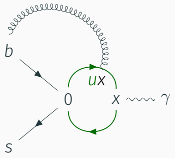


$$\Rightarrow \mathcal{H}_\lambda = \text{coeff \#1} \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}} + \text{coeff \#2} \times \tilde{\mathcal{V}}_\lambda$$



- ▶ leading part identical to QCD fact. results [Beneke, Feldmann, Seidel '01&'04]
- ▶ subleading matrix element $\tilde{\mathcal{V}}_\lambda$ can be inferred from B-LCSRs

[Khodjamirian, Mannel, Pivovarov, Wang '10; Gubernari, DvD, Virto '21] matrix elements of a single operator appearing at subleading power in the LCOPE

$$\tilde{\mathcal{V}}_\lambda \sim \langle M|\, \bar{s}(0)\gamma^\rho P_L G^{\alpha\beta}(-un^\mu)b(0)\, |\bar{B}\rangle$$

for $B \to K^{(*)}$ and $B_s \to \phi$ transitions

- matrix element has been prev. calculated in light-cone sum rules

[Khodjamirian, Mannel, Pivovarov, Wang '10]

- physical picture provides that the soft gluon field originates from the $\bar{B}$ meson
  - analytical results independent of two-particle $b\bar{q}$ Fock state inside the $\bar{B}$
  - expressions start with three-particle $b\bar{q}G$ Fock state, and their light-cone distribution amplitudes (LCDAs)

$$\Phi(t,u) \sim \langle 0|\, \bar{q}(x) G^{\mu\nu}(ux)\Gamma h_v^b(0)\, |\bar{B}(vM_B)\rangle \qquad x^\mu = tn^\mu$$

  - original results lacking four out of eight three-particle LCDAs

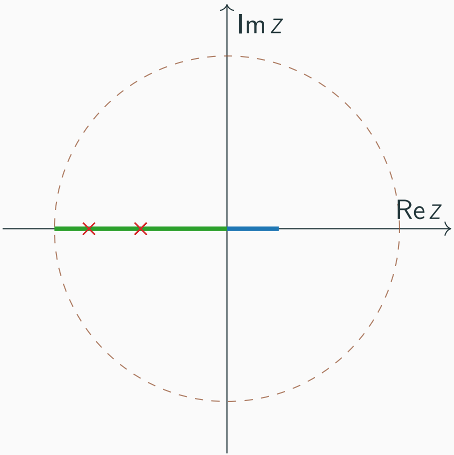
[Gubernari,DvD,Virto '20]

Soft gluon matrix elements

- we calculate the soft-gluon contributions $\tilde{V}_\lambda$ to the full set of $B \to V$ and $B \to P$ nonlocal form factors using light-cone sum rules [Gubernari,DvD,Virto '20]
  - analytic results for restricted set of LCDAs in full agreement with KMPW2010 [Khodjamirian, Mannel, Pivovarov, Wang 2010]
  - result of restricted set fails to reproduce duality thresholds obtained from local form factor sum rules [Gubernari, Kokulu, DvD '18]
  - cross check: our results reproduce the (local) duality thresholds!
  - our numerical results differ significantly from KMPW2010
    - reduction by factor ~ 100, differences well understood!
    - reduction by ~ 10 from update inputs, and ~ 10 from cancellations due to new terms
  - conclusion: soft-gluon contributions are not numerically relevant for $q^2 < 0$

Parametrisation of the nonlocal form factors

- map q^2 to new variable z that develops branch cut at $q^2 = 4M_D^2$ [Bobeth, Chrzaszcz, DvD, Virto '17]
  - branch cut is mapped onto unit circle in z
- data and theory live inside the unit circle
  - real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued z
- expand in z
  - \+ resonances J/ψ, $\psi(2S)$ can be included (poles/Blaschke factors)
  - \+ easy to use in a fit to theory and data
  - \+ compatible with analyticity
  - \- expansion coefficients unbounded!



matrix elements $\mathcal{H}$ arise from nonlocal operator [Gubernari,DvD,Virto '20]

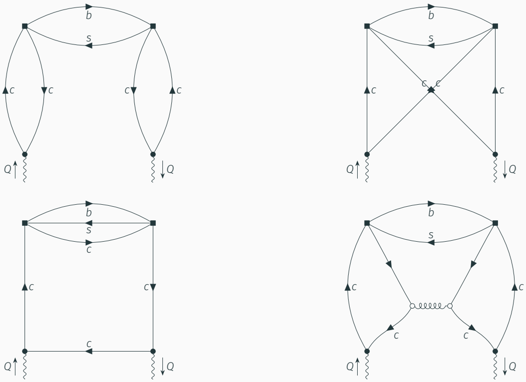
$$\mathcal{O}^\mu(Q;x) \sim \int d^4y\, e^{iQ\cdot y}\, T\{J^\mu_{\text{em}}(x+y), [C_1 O_1 + C_2 O_2](x)\}$$

construct four-point operator to derive a dispersive bound

- define matrix element of "square" operator

$$\left[\frac{Q^\mu Q^\nu}{Q^2} - g^{\mu\nu}\right] \Pi(Q^2) \equiv \int d^4x\, e^{iQ\cdot x} \langle 0|\, T\{\mathcal{O}^\mu(Q;x)\mathcal{O}^{\dagger,\nu}(Q;0)\}\, |0\rangle$$

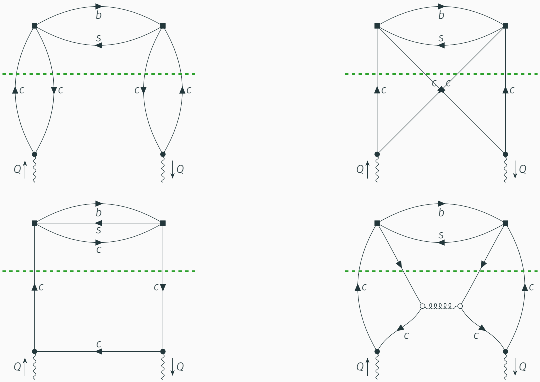
- $\Pi(Q^2)$ has two types of discontinuities
  - from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
  - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)



diagrams start at three loops

- diagrams to LO in α_s: top, and bottom left
- one diagram to NLO in α_s (bottom right), for illustration only

Extrapolate Cuts of Π



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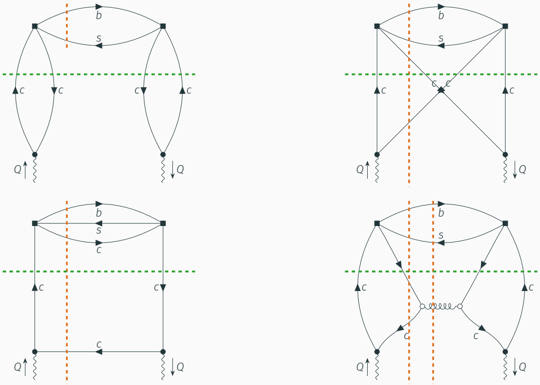

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- from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)

dispersive representation of the $b\bar{s}$ contribution to derivative of Π

$$\chi(Q^2) \equiv \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \Pi(Q^2) = \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \frac{1}{2i\pi}\int_{(m_b+m_s)^2}^{\infty} ds\, \frac{\mathrm{Disc}_{b\bar{s}}\,\Pi(s)}{s-Q^2} > 0$$

- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be computed in the local OPE
  $\rightarrow \chi^{\mathrm{OPE}}(Q^2)$
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be expressed in terms of the nonlocal form factors $|\mathcal{H}_\lambda|^2$
  $\rightarrow \chi^{\mathrm{had}}(Q^2)$

- global quark hadron duality suggests that $\chi^{\mathrm{OPE}}(Q^2) = \chi^{\mathrm{had}}(Q^2)$
- parametrize $\mathcal{H}_\lambda \propto \sum_n \alpha_{\lambda,n} f_n$ with orthonormal functions f_n

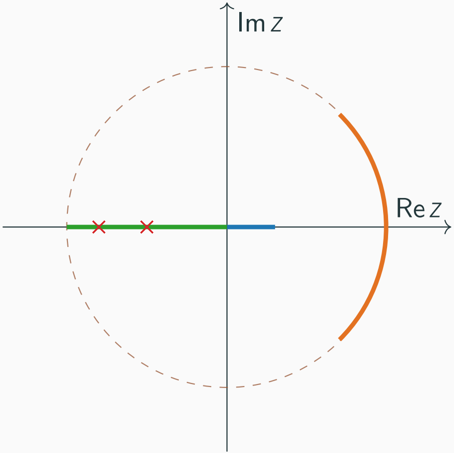
$$\Rightarrow \text{dispersive bound:} \quad \chi^{\mathrm{OPE}} \geq \sum_n |\alpha_{\lambda,n}|^2$$

- *first application* of such a bound to nonlocal form factors
- technically more challenging than for local form factors

- expand in z
  - $f_n(z)$ orthogonal on arc
  - \+ accounting for behaviour on arc produces dispersive bound on each parameter ✓

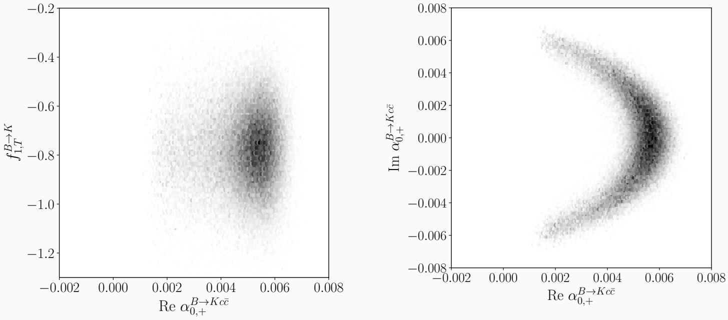
[Gubernari, DvD, Virto '20]

- turns so far hardly quantifiable systematic theory uncertainties into parametric uncertainties
- currently being implemented in EOS
  - open source software at `github.com/eos/eos`
  - available from PyPI for easy dissemination to both theory + experimental colleagues



- “first stage“ simultaneous fit of parameters of local and non-local form factors to theory inputs + $B_{(s)} \to \{K, K^*, \phi\} J/\psi$

[Gubernari, Reboud, DvD, Virto (to appear)]



- N.B.: non-local parameters are complex numbers
- cartesian parametrisation leads to non-gaussian posterior

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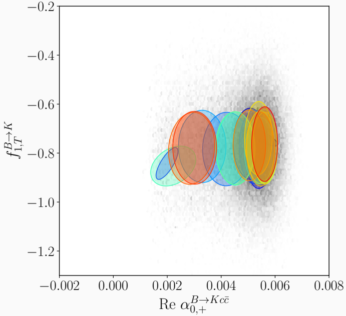

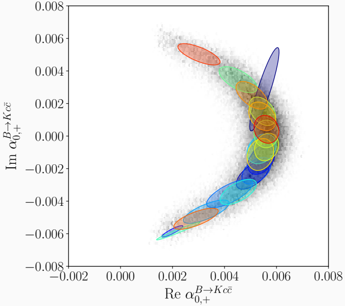

- N.B.: non-local parameters are complex numbers
- cartesian parametrisation leads to non-gaussian posterior
- successfully described by gaussian mixture density
- investigating polar parametrisation

- ▶ we plan to publish the mixture density in digital form, including a test statistic to determine a goodness of fit in BSM studies

Conclusion

Summary and Outlook

- exploitation of $b \to s\ell^+\ell^-$ data hinges on accurate and precise information of a number of hadronic form factors
  - nonlocal form factors contribute the single-largest systematic uncertainty in exclusive $b \to s\ell\ell$ decays
- clear road toward controlling these objects, but much work still needs to be done
- unitarity constraints provide a new parametrization with bounded parameters
- key is determination of parameter from a combined theory + data driven approach

Backup Slides

Extrapolate Parametrisations

- Taylor expand $\mathcal{H}_\lambda$ in q^2/M_B^2 around 0 [Ciuichini et al. '15]
  - \+ simple to use in a fit
  - \- incompatible with analyticity properties, does not reproduce resonances
  - \- expansion coefficients unbounded! $\Rightarrow$ impossible to estimate truncation error

- use information from hadronic intermediate states in a dispersion relation [Khodjamirian et al. '10]

$$\mathcal{H}_\lambda(q^2) - \mathcal{H}_\lambda(q_0^2) = \frac{q^2 - q_0^2}{2\pi} \int ds \frac{\text{Im}\,\mathcal{H}_\lambda(s)}{(s-s_0)(s-q^2)} + \dots$$

  - \+ reproduces resonances
  - \- hadronic information above the threshold must be modelled
  - \- complicated to use in a fit, relies on theory input in single point s_0

- expand the matrix elements in variable $z(q^2)$ with branch cut at $q^2 = 4M_D^2$ [Bobeth/Chrzaszcz/DvD/Virto '17]

Extrapolate Dispersion relation for Π

the hadronic representation reads schematically:

$$1 \geq \frac{1}{\chi^{\mathrm{OPE}}(Q^2)\,2!}\left[\frac{d}{dQ^2}\right]^2 \int_{(m_b+m_s)^2}^{\infty} ds \sum_\lambda \frac{\omega_\lambda(s)\,|\mathcal{H}_\lambda(s)|^2}{s - Q^2}$$

- aim: diagonalize this expression

Ansatz:

$$\hat{\mathcal{H}}_\lambda(q^2) \equiv P(q^2) \times \phi_\lambda(q^2) \times \mathcal{H}_\lambda(q^2) \equiv \sum_n \alpha_{\lambda,n}\, f_n(q^2)$$

- Blaschke factor $P(q^2)$ removes poles of narrow charmonia
- outer function ϕ_λ accounts for weight function ω_λ and Cauchy integration kernel
- orthonormal polynomials $f_n(q^2)$ diagonalize remainder of the expression

normalisation to χ^{OPE} leads to a diagonal bound

$$1 \geq \sum_\lambda \sum_n |\alpha_{\lambda,n}|^2$$

Compute Soft gluon matrix elements

Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\text{GeV}^2)$	GvDV2020	KMPW2010
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}$ GeV	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}$ GeV
$B \to K^*$	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}$ GeV	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}$ GeV
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}$ GeV	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}$ GeV
	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}$ GeV	—
$B_s \to \phi$	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}$ GeV	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}$ GeV	—

reduction by a factor of ~ 200

- new structures in three-particle LCDAs account for factor 10 (due to cancellations!)
- updated inputs that enter the sum rules (mostly) linearly account for further factor 10
- similar relative uncertainties, but absolute uncertainties reduced by $\mathcal{O}(100)$