

# Transverse momentum fluctuation in ultra-central Pb+Pb collision

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in collaboration with Jean-Yves Ollitrault, Matthew Luzum, Jiangyong Jia,  
Somadutta Bhatta, Joao-Paulo Pichhetti

...based on arXiv:2303.15323 and arXiv:2306.09294

XVI Polish Workshop on Relativistic Heavy-Ion Collisions, Kielce, Dec 2, 2023

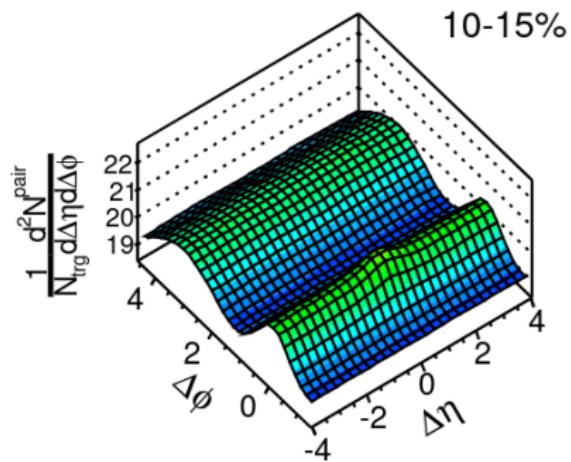


AGH UNIVERSITY OF SCIENCE  
AND TECHNOLOGY



## Motivation

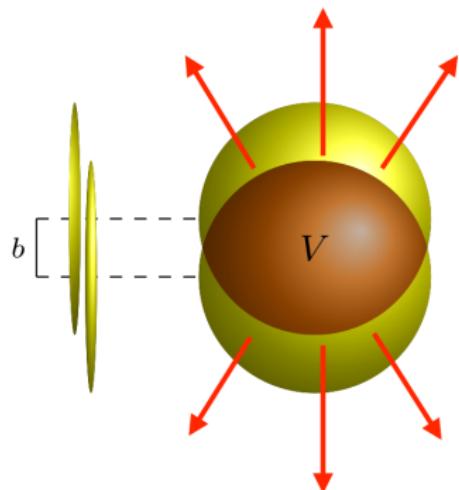
- Experimental evidence for the formation of a **little fluid** in Pb+Pb collision → **azimuthal correlations between particles** seen in detectors.



CMS:1201.3158

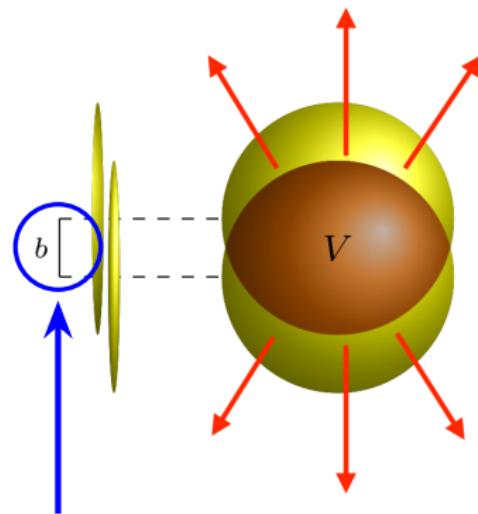
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- Evidence is **indirect** ! → azimuthal distribution of particles is **not isotropic** → **anisotropy driven by pressure gradients** within a fluid.



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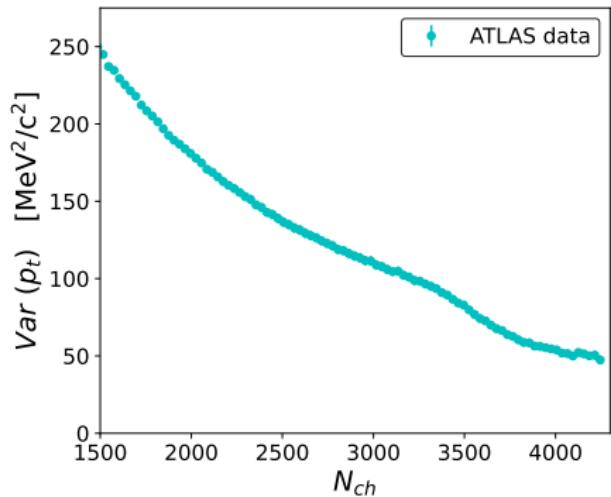
- Experimental evidence for the formation of a **little fluid** in Pb+Pb collision → **azimuthal correlations between particles** seen in detectors.
- Evidence is **indirect** ! → azimuthal distribution of particles is not isotropic → **anisotropy driven by pressure gradients** within a fluid.
- We report **more direct evidence** of **local thermalization** in Pb+Pb collisions → **does not involve directions** of outgoing particles, but solely their momenta.



**$b = \text{impact parameter}$**   
**(important in this talk ! )**

## ATLAS data for $[p_t]$ fluctuation

- Recent ATLAS data shows multiplicity ( $N_{ch}$ ) dependence of the variance of transverse momentum per particle,  $[p_t]$ .



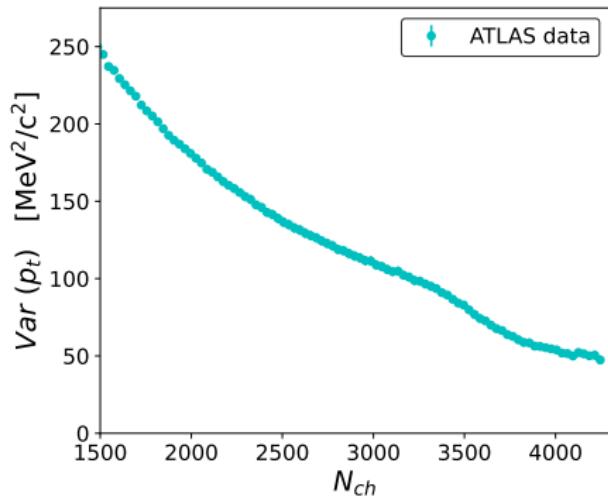
Variance of  $[p_t]$  for Pb+Pb @ 5.02 TeV

PhysRevC.107.054910

Table 374 in <https://www.hepdata.net/record/ins2075412>

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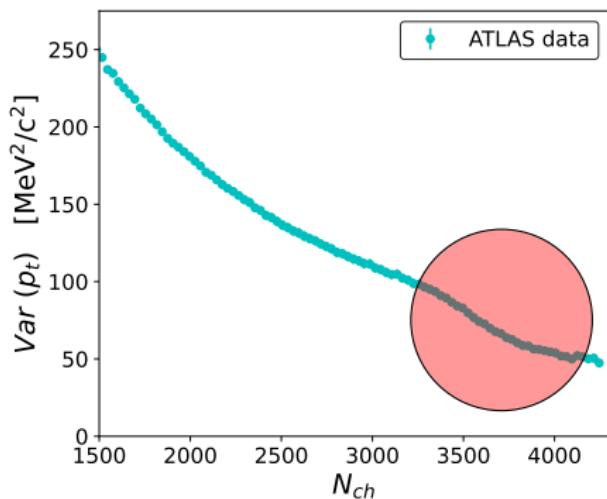
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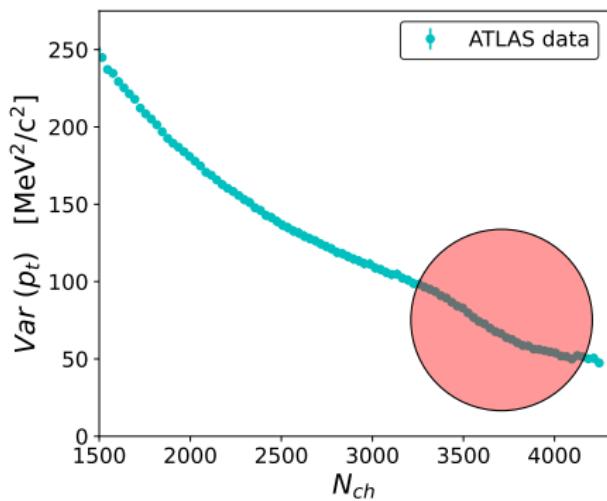


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- We will show that this could be a consequence of thermalization !

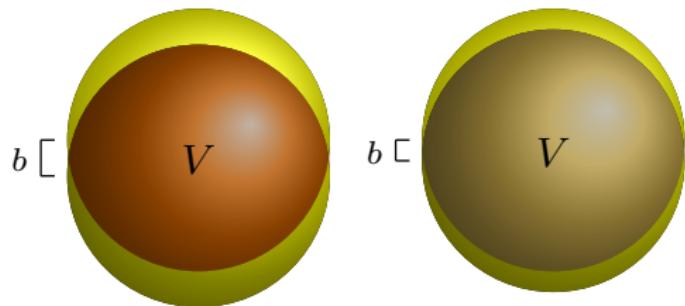


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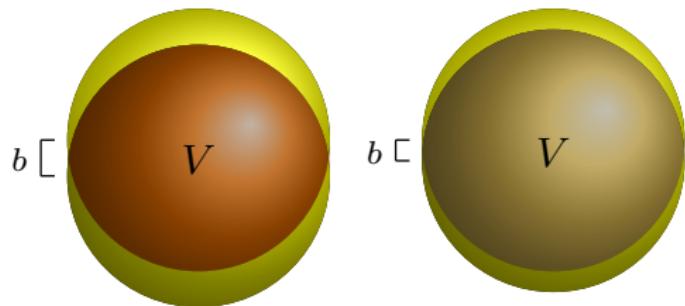
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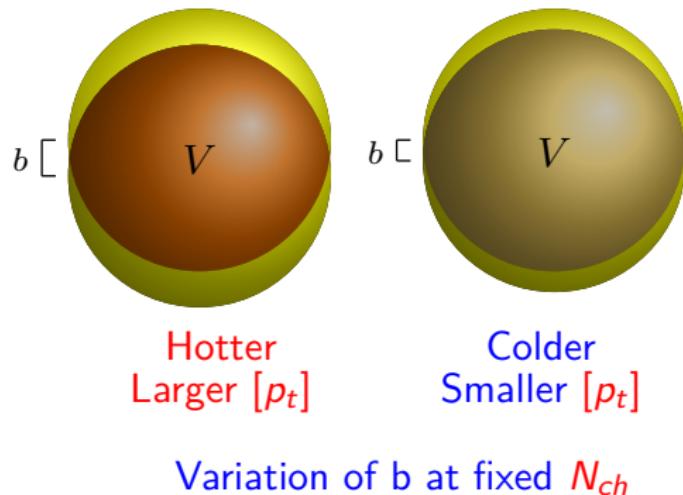


Hotter  
Larger  $[p_t]$       Colder  
Smaller  $[p_t]$

Variation of  $b$  at fixed  $N_{ch}$

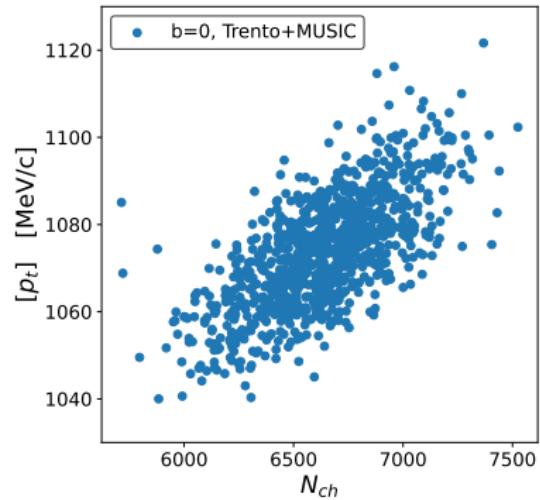
## Impact parameter ( $b$ ) is important !

- In experiment  $b$  is not known !  $\Rightarrow [p_t]$  fluctuation is measured for fixed  $N_{ch}$
- Fixed  $N_{ch} \Rightarrow$  finite range of  $b$  !
- Variation of  $b$  gives a contribution to the variation of  $[p_t] \Rightarrow$  goes to 0 in ultracentral collisions !



## Hydrodynamic simulation: $b$ is known !

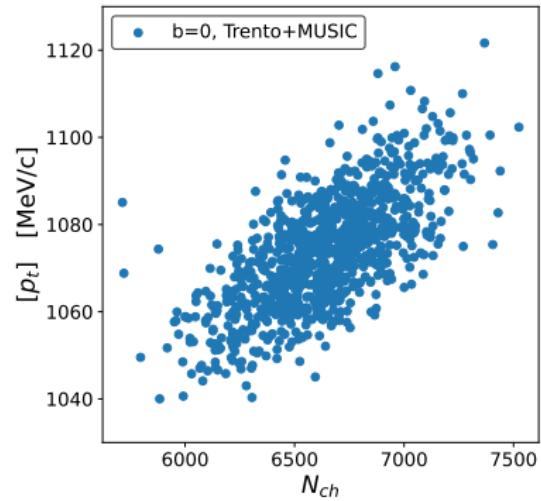
- Hydro : assumes thermalization !  
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Pb+Pb @ 5.02 TeV

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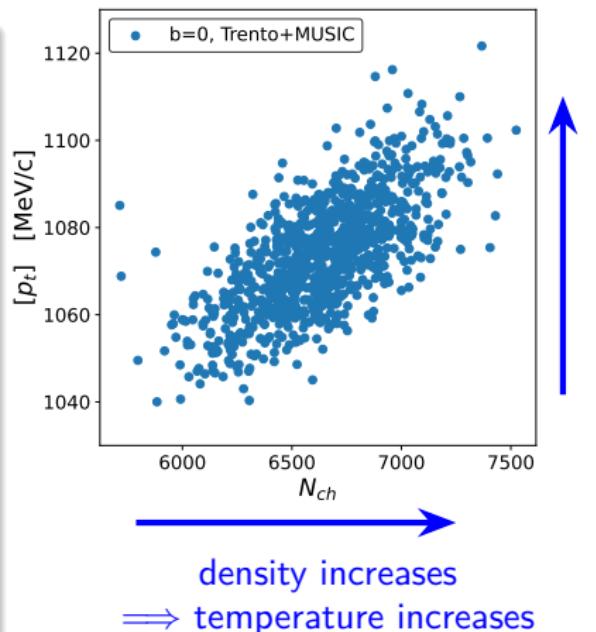
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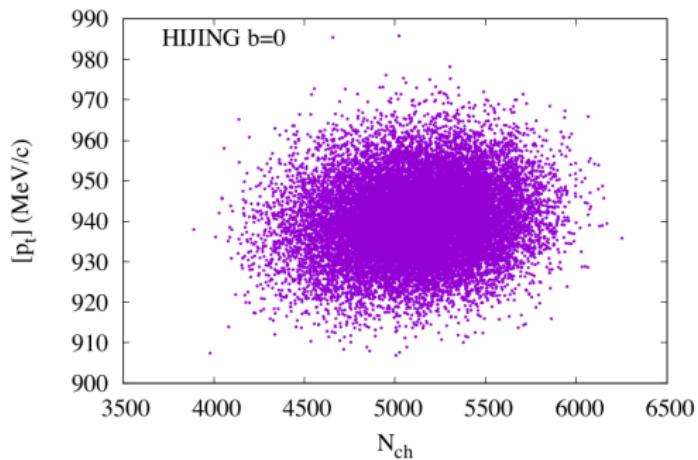
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- ▶ Fixed  $b \Rightarrow$  fixed collision volume  
Larger  $N_{ch} \Rightarrow$  larger density  
⇒ larger temperature  
⇒ larger energy per particle  
⇒ larger  $[p_t]$



## Comparing other models : HIJING simulation

Wang, Gyulassy, arXiv:nucl-th/9502021

- HIJING: microscopic model of HI collision  $\implies$  the system doesn't thermalize !

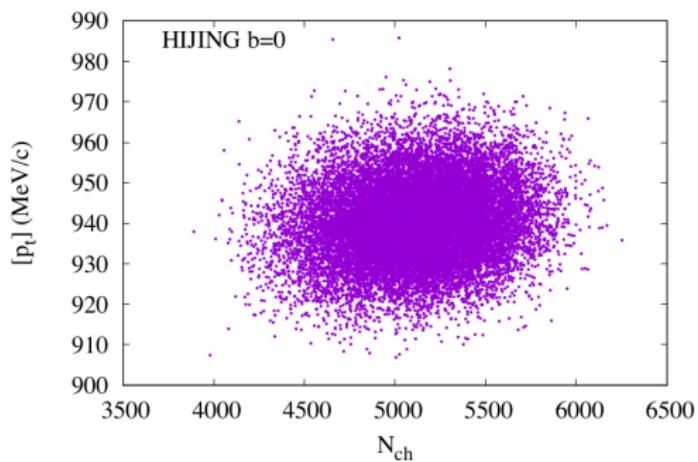


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- Very small correlation between  $N_{ch}$  and  $[p_t] \sim 10 \times$  smaller !!



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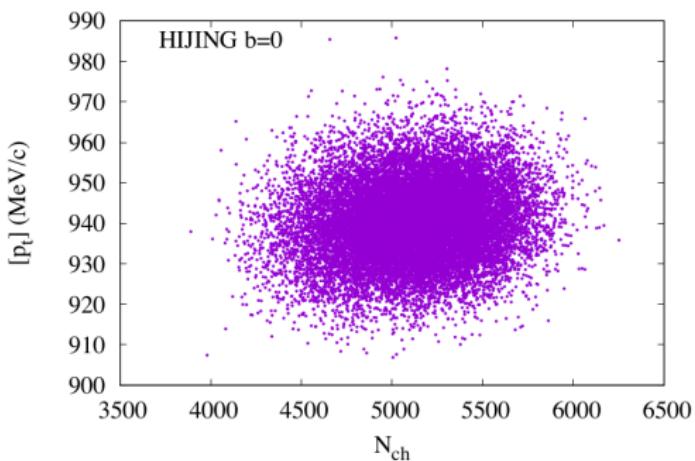
No thermalization

$\Rightarrow$  Very little correlation !

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- HIJING: microscopic model of HI collision  $\Rightarrow$  the system doesn't thermalize !
- Very small correlation between  $N_{ch}$  and  $[p_t] \sim 10 \times$  smaller !!
- Hence the correlation could be a signature of thermalization !

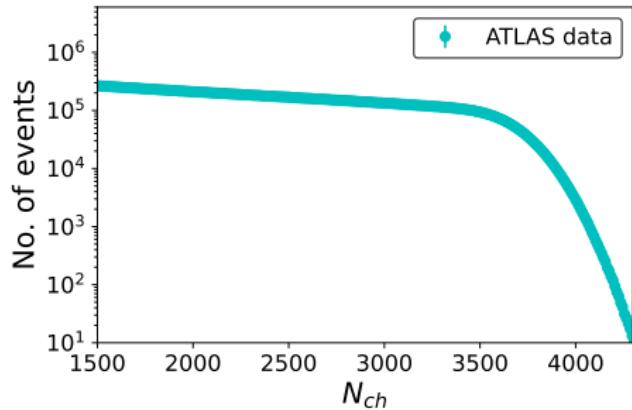


Pb+Pb @ 5.02 TeV

No thermalization  
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## Bayesian reconstruction of $P(\mathbf{b} | N_{ch})$

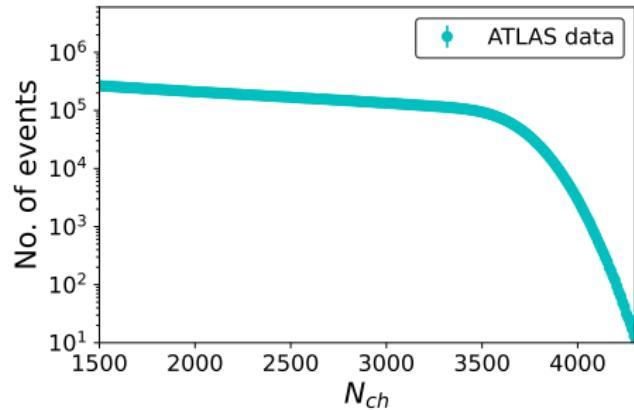
- First we solve the inverse problem:  
what is the distribution of  $N_{ch}$  at fixed  $\mathbf{b}$  i.e.  $P(N_{ch}|\mathbf{b})$  ?



$N_{ch}$  distribution  
for centrality classification !

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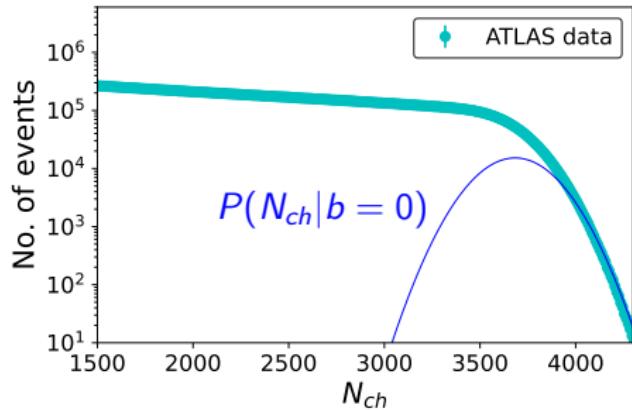
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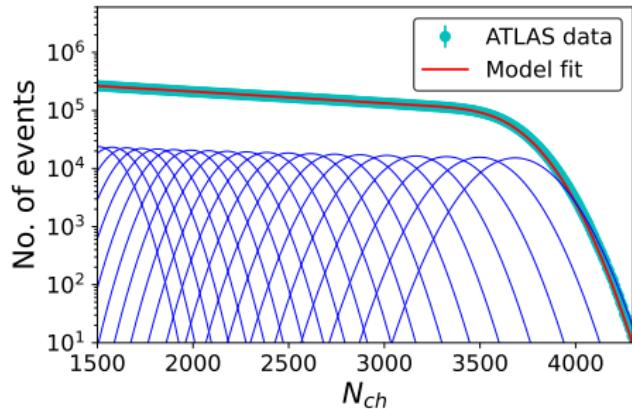
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- We assume  $P(N_{ch}|\mathbf{b})$  to be Gaussian !



$N_{ch}$  distribution at fixed b  
Gaussian assumption !

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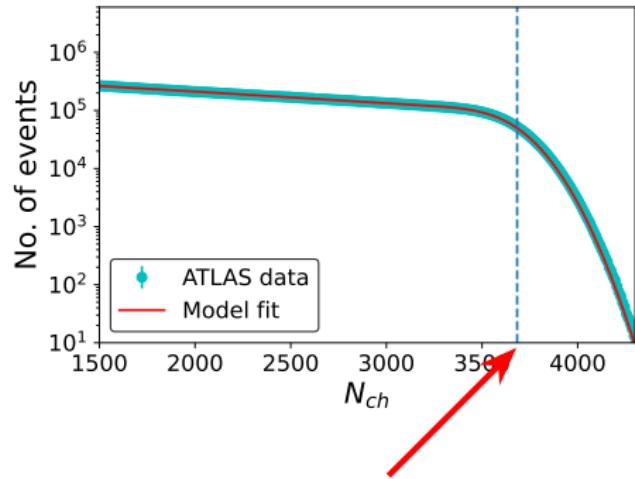


Sum of Gaussians at fixed  $\mathbf{b}$

Das, Giacalone, Monard, Ollitrault  
arXiv:1708.00081

## Bayesian reconstruction of $P(\mathbf{b} | N_{ch})$

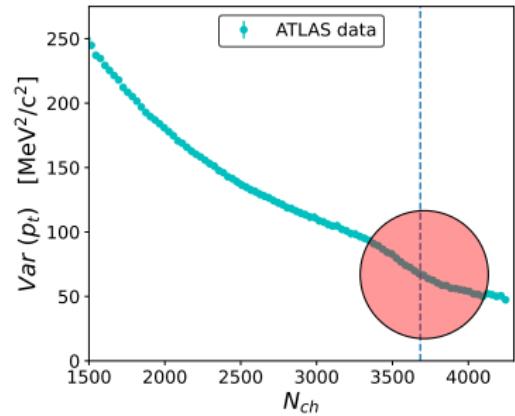
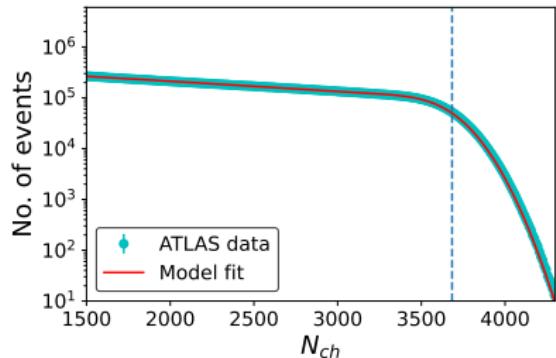
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- We precisely reconstruct the knee (mean  $N_{ch}$  at  $\mathbf{b}=0$ )



Precise construction of knee  
 $\langle N_{ch} | b = 0 \rangle$

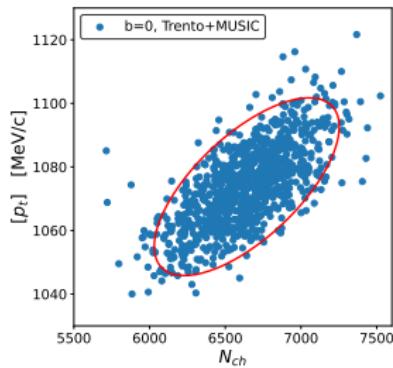
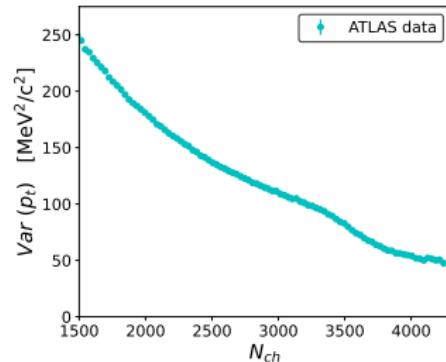
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- The steep fall of the variance precisely occur at the knee !



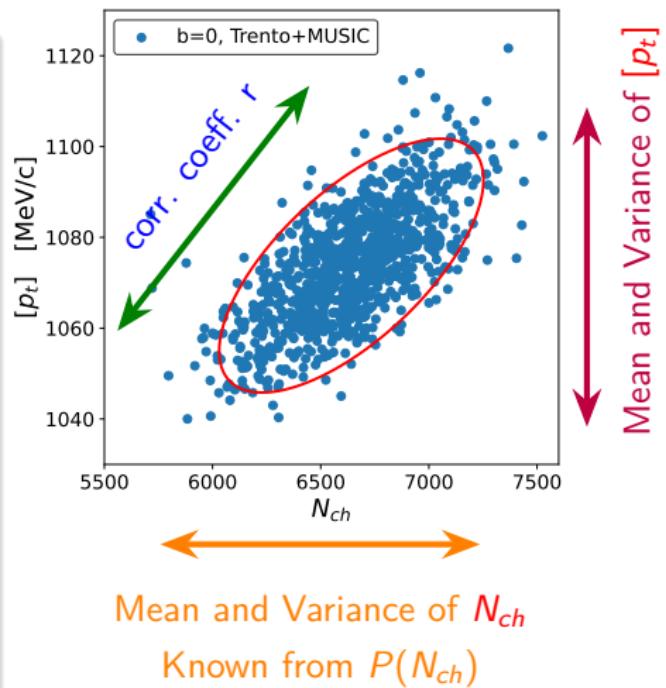
## Understanding $[p_t]$ fluctuation data : Parametrizing $P(N_{ch}, [p_t] | b)$

- We assume a simple 2D correlated Gaussian between  $[p_t]$  and  $N_{ch}$  at fixed impact parameter  $b$  :  $P([p_t], N_{ch} | b)$ .



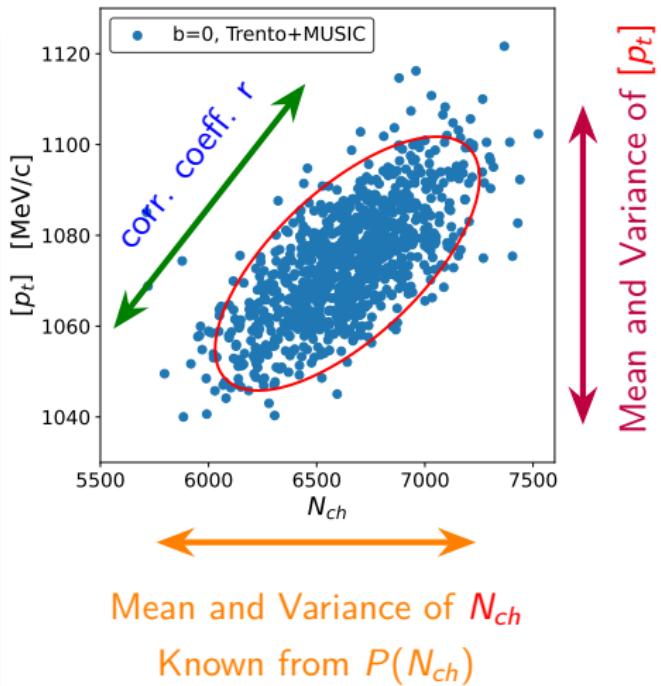
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- The distribution has 5 parameters : Mean and variance of  $N_{ch}$ , Mean and variance of  $[p_t]$  and correlation coefficient  $r$  between  $N_{ch}$  and  $[p_t]$ .

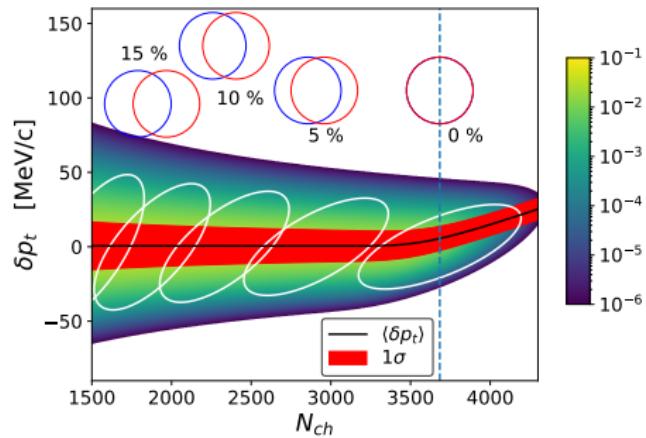


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- ▶ The distribution has 5 parameters : Mean and variance of  $N_{ch}$ , Mean and variance of  $[p_t]$  and correlation coefficient  $r$  between  $N_{ch}$  and  $[p_t]$ .
- ▶ Mean value of  $[p_t]$  is constant at fixed  $b$  and assuming it is independent of  $b \implies$  we fit  $P(\delta p_t, N_{ch} | b)$   
 $\delta p_t = [p_t] - \langle [p_t] \rangle$



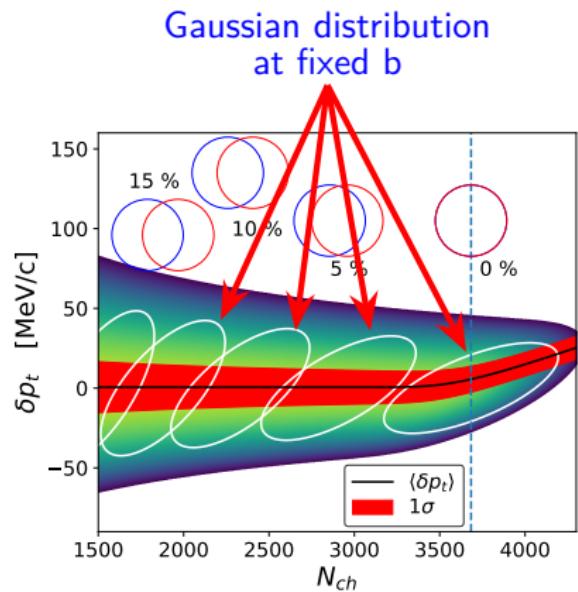
Fit result :  $P(N_{ch}, \delta p_t)$



2D correlated gaussian  
distribution of  $\delta p_t$  and  $N_{ch}$

**Fit result :  $P(N_{ch}, \delta p_t)$**

- We get,  $P(N_{ch}, \delta p_t) = \int P(N_{ch}, \delta p_t | b) P(b) db$

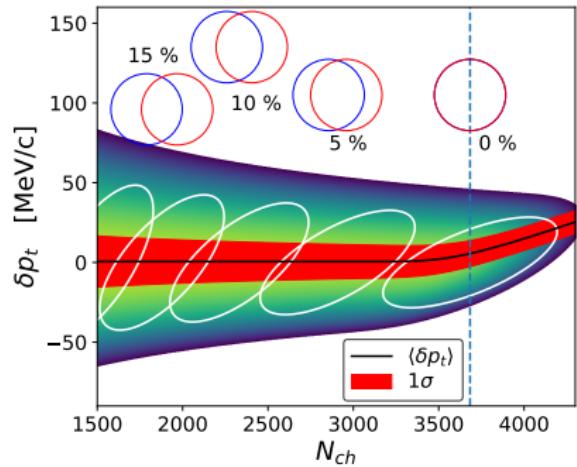


**Fit result :**  $P(N_{ch}, \delta p_t)$

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  - By conditional probability  

$$P(\delta p_t | N_{ch}) = \frac{P(N_{ch}, \delta p_t)}{P(N_{ch})}$$

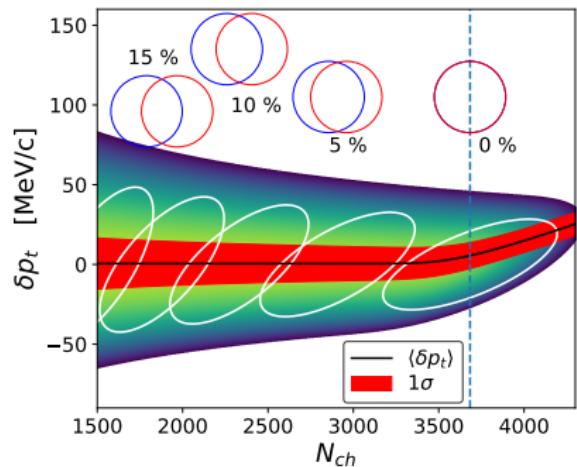
$$\Rightarrow \text{Var}([\delta p_t] | N_{ch}) \text{ is the squared width of } P(\delta p_t | N_{ch})$$



## 2D correlated gaussian distribution of $\delta p_t$ and $N_{ch}$

## Fit result : $P(N_{ch}, \delta p_t)$

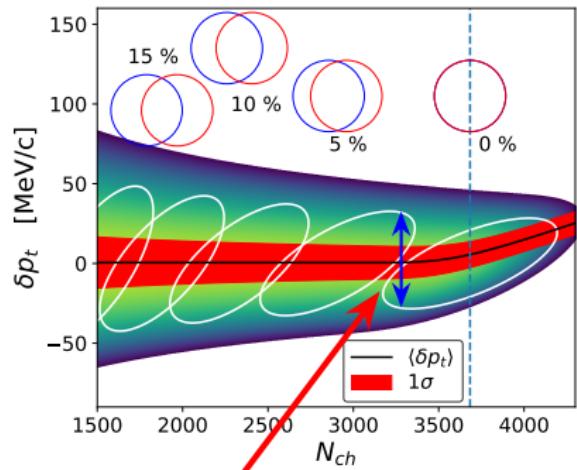
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- The width of  $[\delta p_t]$  fluctuation has two contributions :



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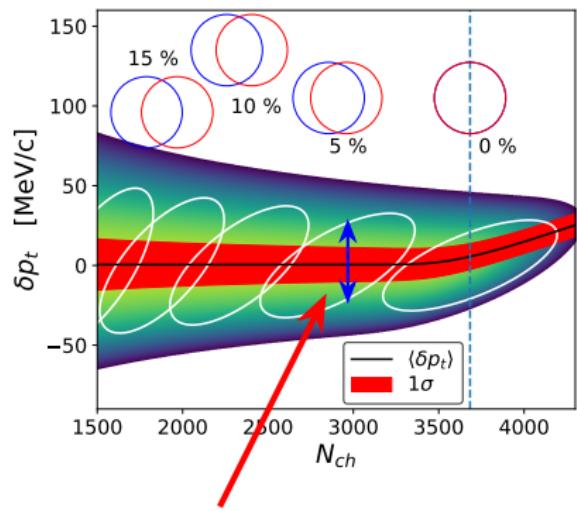
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fluctuation from the variation of b  
(several ellipses contribute)

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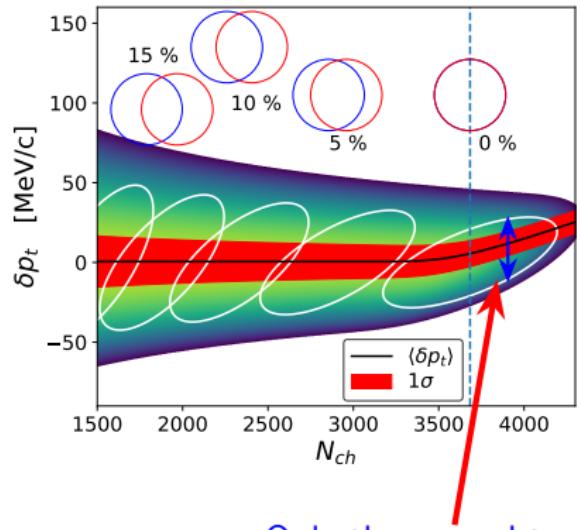
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    - ② the true intrinsic fluctuation



fluctuation of  $[p_t]$  at  
fixed b and fixed  $N_{ch}$   
(height of a single ellipse)

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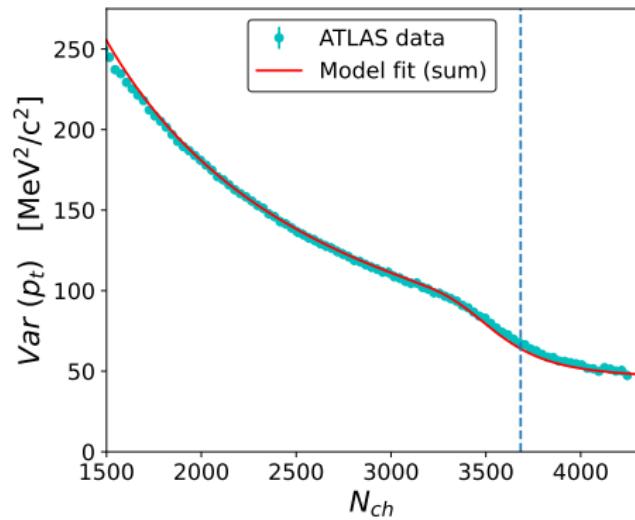
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- The width of  $[\delta p_t]$  fluctuation has two contributions :
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  - ② the true intrinsic fluctuation
- Only the second term contributes above knee in the ultracentral regime.



Only the second term remains in ultracentral collisions

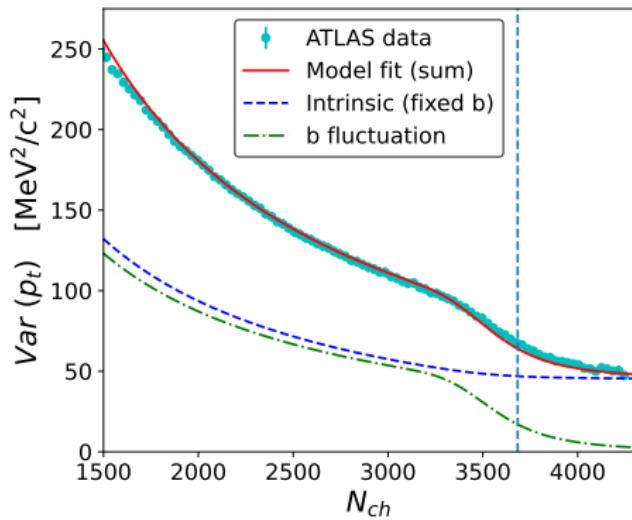
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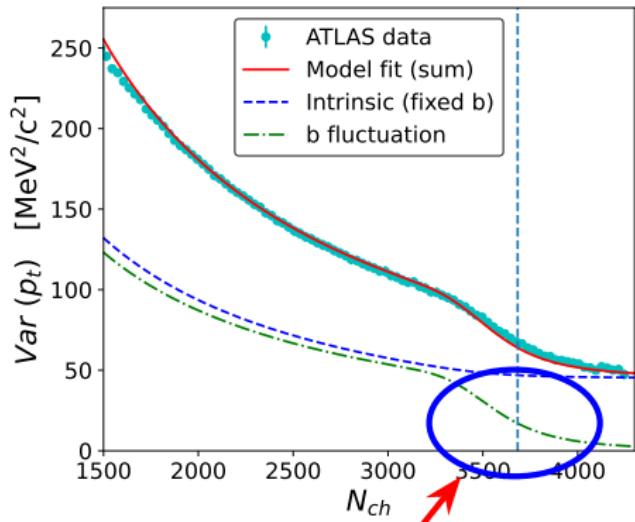
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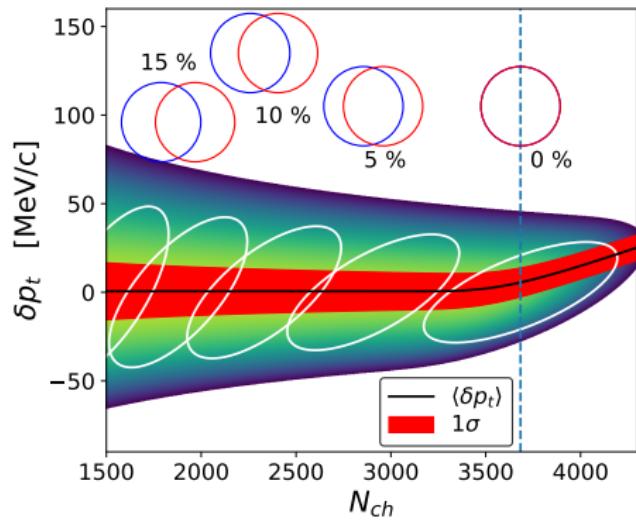
- Our simple model naturally reproduces the steep fall in the ATLAS data very well !
- Below the knee, half of the contribution is from impact parameter fluctuation
- The contribution gradually disappears around the knee !



Contribution of b-fluctuation disappears above knee

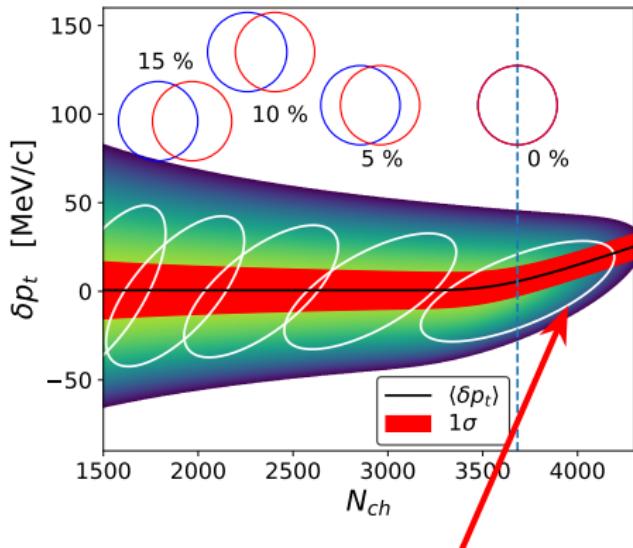
## Hint of Thermalization !

- Our model fit returns  $r = 0.676$  !



## Hint of Thermalization !

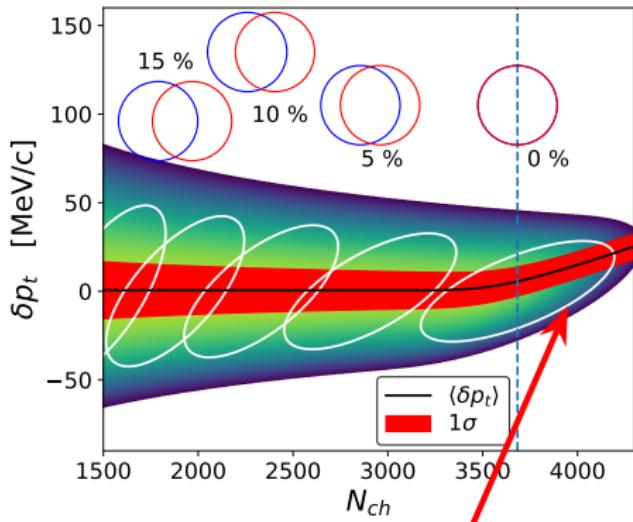
- Our model fit returns  $r = 0.676$  !
- It suggests **strong correlation** between  $[p_t]$  and  $N_{ch}$  at fixed  $b$



Strong correlation between  
 $[p_t]$  and  $N_{ch}$  at fixed  $b$   
from our model fit

## Hint of Thermalization !

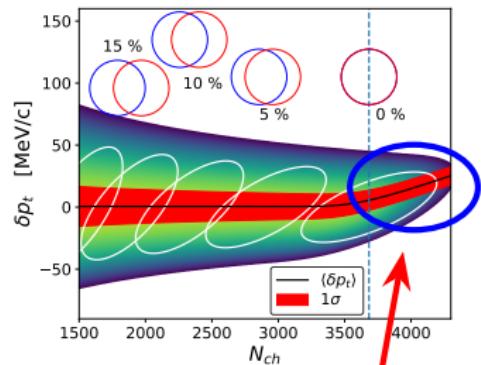
- Our model fit returns  $r = 0.676$  !
- It suggests strong correlation between  $[p_t]$  and  $N_{ch}$  at fixed  $b$
- thermalization is at work ?



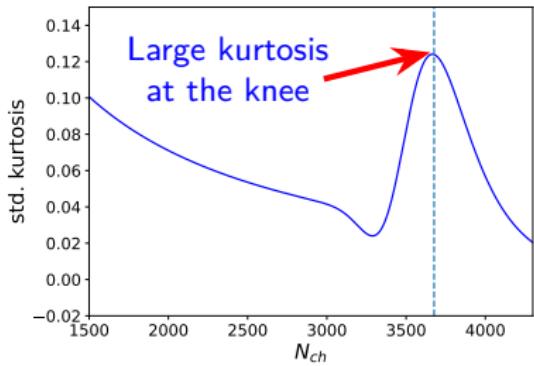
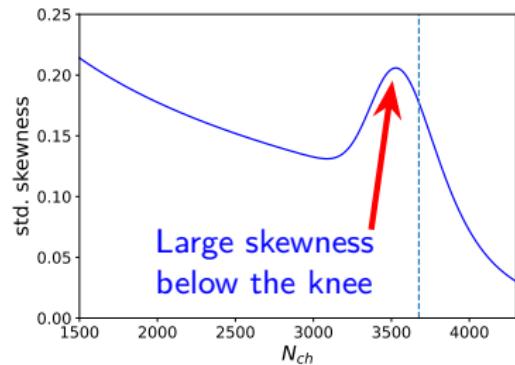
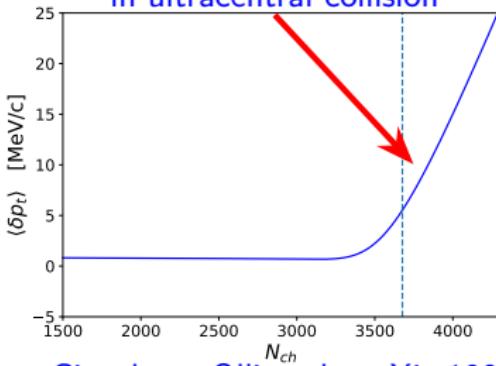
Strong correlation between  
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from our model fit

## Further predictions : Mean, Skewness and kurtosis

RS, Picchetti, Luzum, Ollitrault, arXiv:2306.09294

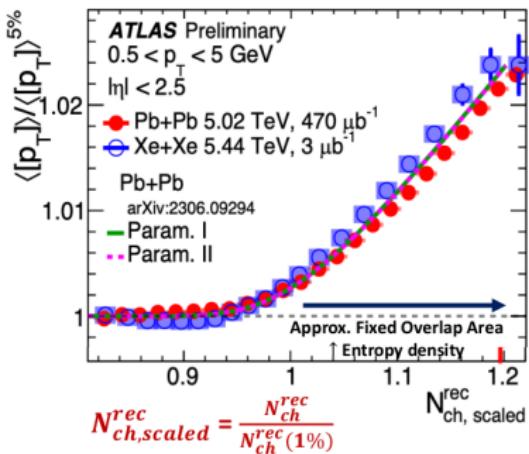


Slight increase of mean  $[p_t]$   
in ultracentral collision

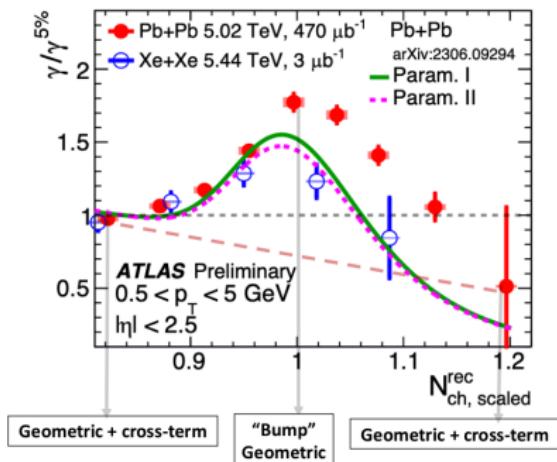


Gardim, Giacalone, Ollitrault, arXiv:1909.11609

ATLAS Preliminary, presented in QM2023

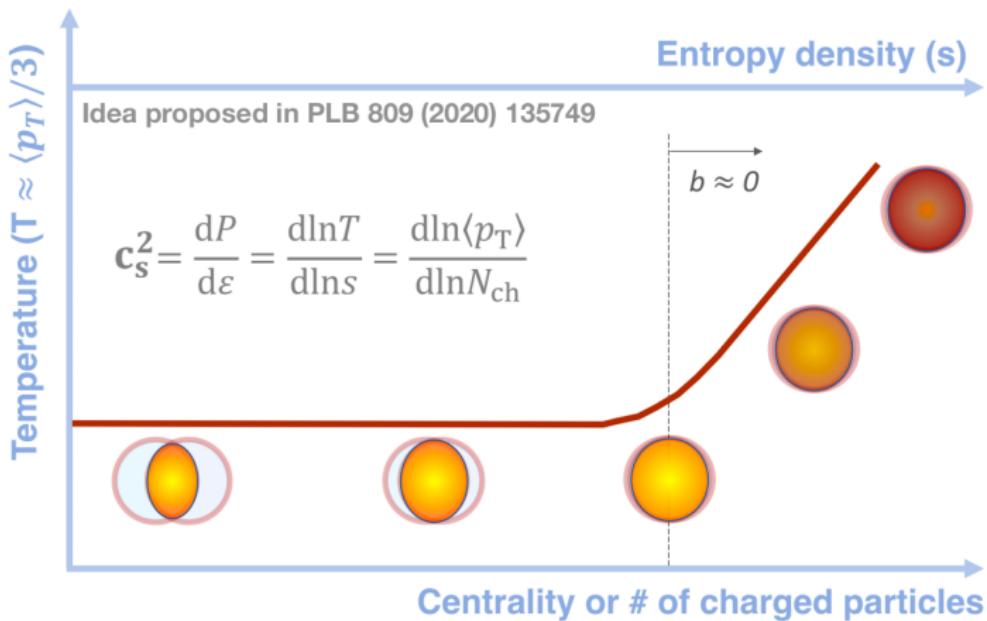


mean  $[p_t]$



## [ $p_t$ ]- skewness

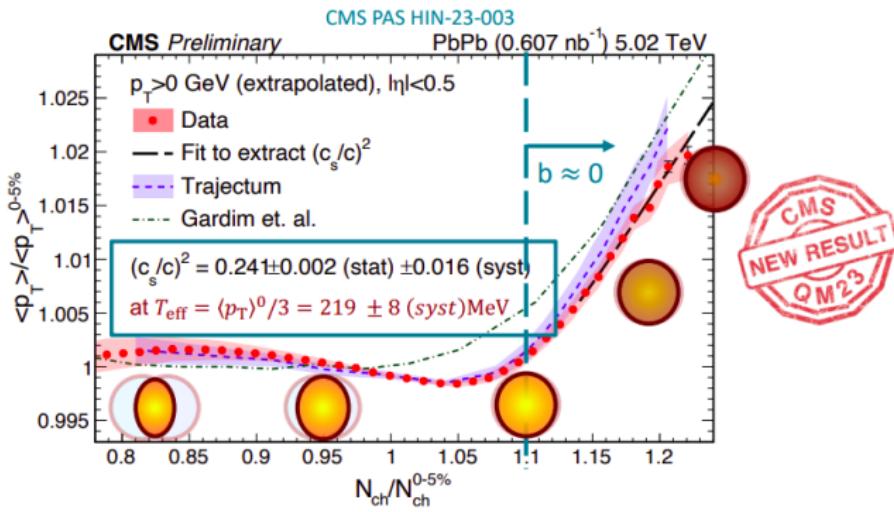
## Application : Extraction of speed of sound in QGP from mean [ $p_t$ ]



# CMS Result on mean $\langle p_T \rangle$ !

See CMS preliminary in QM 2023

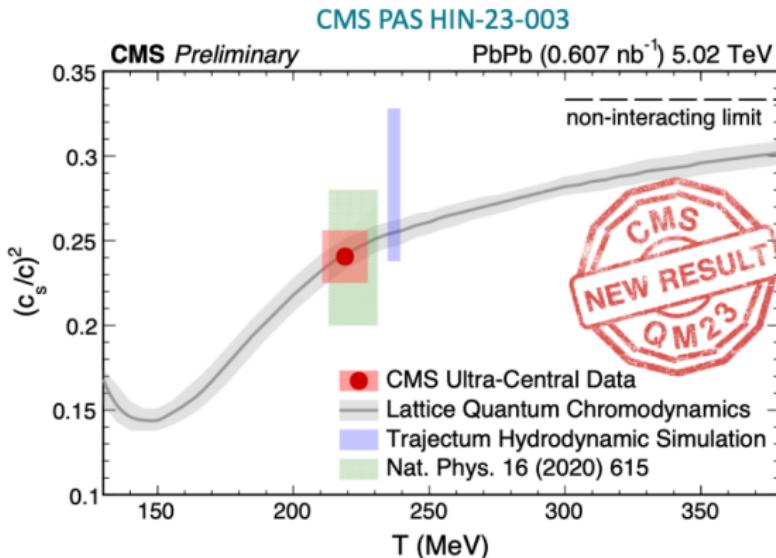
Significant increase of  $\langle p_T \rangle$  toward UCC events as predicted by the simulations



Speed of sound extracted from the fit and  $T_{\text{eff}}$  from  $\langle p_T \rangle^0$

## How precise is the measurement ?

See CMS preliminary in QM 2023



Speed of sound in QGP is predicted and measured with great precision !!

## Summary and Outlook

- Two separate contributions to  $[p_t]$  fluctuation :
  - ➊ intrinsic fluctuation → originates from quantum fluctuation in the initial state
  - ➋ impact parameter fluctuation at fixed  $N_{ch}$  → disappears in ultracentral region → causes the steep fall at the knee

Our methodology paves a way to separate the geometrical and quantum fluctuations

- We present predictions for mean  $[p_t]$ , skewness and kurtosis of  $[p_t]$ -fluctuation → the unique patterns of the cumulants of  $[p_t]$  fluctuation at the ultracentral regime originates mostly due to b-fluctuation !
- Prediction of increase of mean  $[p_t]$  with  $N_{ch}$  leads to the precise extraction of speed of sound ( $c_s^2$ ) in QGP
- Transverse momentum fluctuation in ultra central collision could be a probe of the thermalization !

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... Thank you for your attention !



$P(\delta p_t | N_{ch}, c_b)$  in terms of  $k1$  and  $k2$

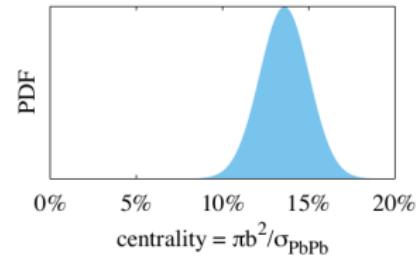
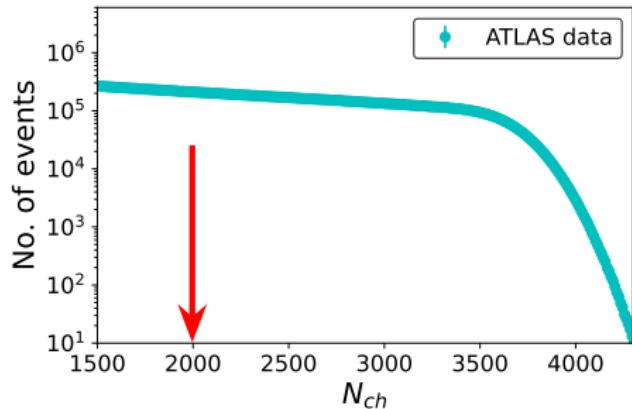
$$P(\delta p_t | N_{ch}, c_b) = \frac{1}{\sqrt{2\pi\kappa_2(c_b)}} \exp\left(-\frac{(\delta p_t - \kappa_1(c_b))^2}{2\kappa_2(c_b)}\right)$$

$$\kappa_1(c_b) = r \frac{\sigma_{p_t}(c_b)}{\sigma_{N_{ch}}(c_b)} (N_{ch} - \overline{N_{ch}}(c_b)),$$

$$\kappa_2(c_b) = (1 - r^2) \sigma_{p_t}^2(c_b).$$

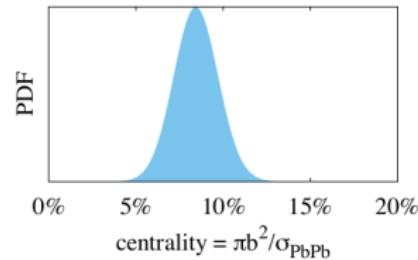
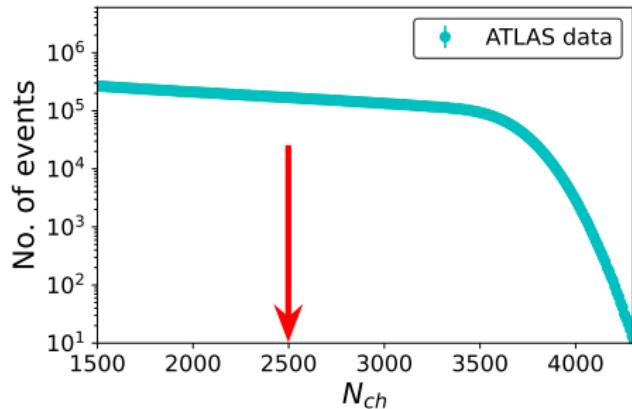
## $P(b | N_{ch})$ from Bayesian reconstruction

- At smaller  $N_{ch}$  the distribution  $P(b|N_{ch})$  is a full Gaussian



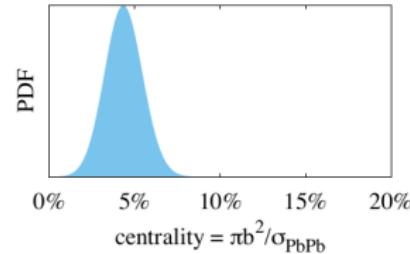
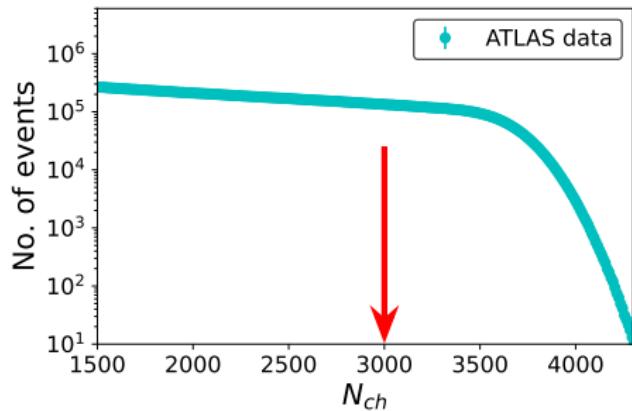
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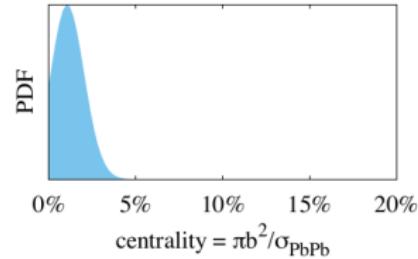
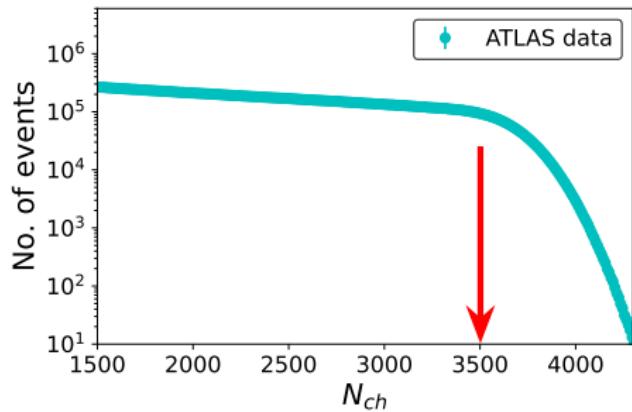
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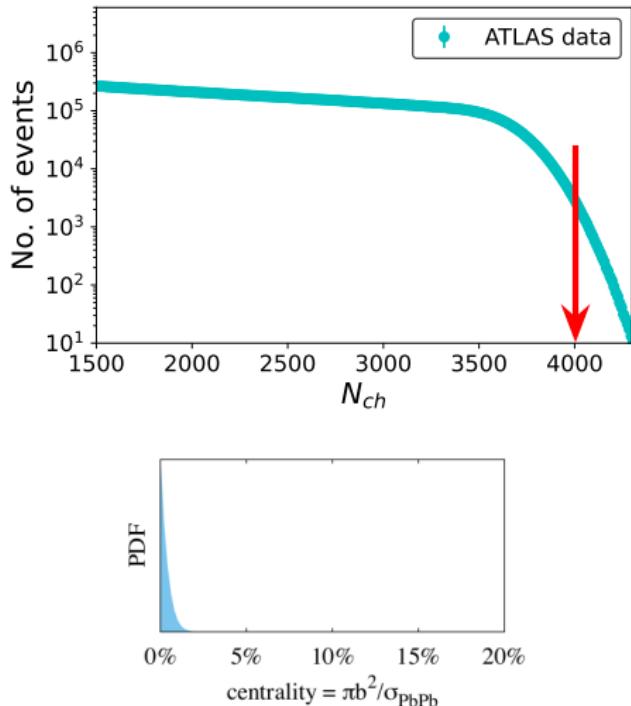
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- But as we move closer and closer to the knee,  $P(b|N_{ch})$  becomes truncated due to the limit  $b \geq 0$
- Above the knee it gets extremely truncated  $\Rightarrow$  the impact parameter fluctuation gradually disappears !



## Moments and cumulants of $[p_t]$ -fluctuation

$$\langle \delta p_t \rangle = \langle \kappa_1 \rangle,$$

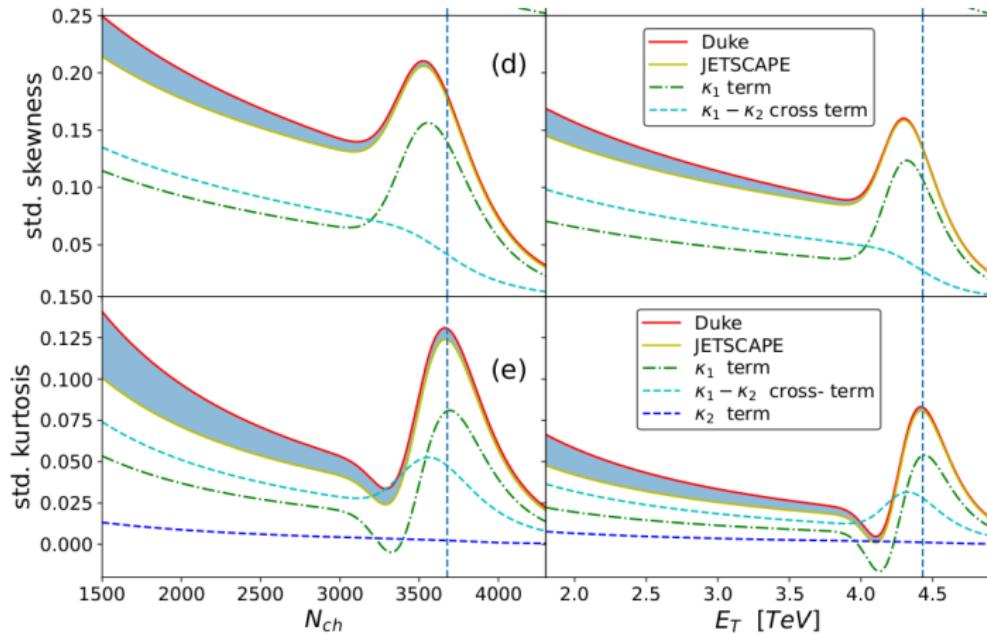
$$\langle \delta p_t | c_b \rangle = \kappa_1, \quad \text{Var}(p_t) = (\langle \kappa_1^2 \rangle - \langle \kappa_1 \rangle^2) + \langle \kappa_2 \rangle,$$

$$\langle \delta p_t^2 | c_b \rangle = \kappa_1^2 + \kappa_2, \quad \text{Skew}(p_t) = \langle \kappa_1^3 \rangle - 3\langle \kappa_1^2 \rangle \langle \kappa_1 \rangle + 2\langle \kappa_1 \rangle^3$$

$$\langle \delta p_t^3 | c_b \rangle = \kappa_1^3 + 3\kappa_2\kappa_1, \quad + 3(\langle \kappa_2\kappa_1 \rangle - \langle \kappa_2 \rangle \langle \kappa_1 \rangle),$$

$$\langle \delta p_t^4 | c_b \rangle = \kappa_1^4 + 6\kappa_2\kappa_1^2 + 3\kappa_2^2, \quad \text{Kurt}(p_t) = \langle \kappa_1^4 \rangle - 4\langle \kappa_1^3 \rangle \langle \kappa_1 \rangle + 6\langle \kappa_1^2 \rangle \langle \kappa_1 \rangle^2 - 3\langle \kappa_1 \rangle^4$$
$$+ 6(\langle \kappa_2\kappa_1^2 \rangle - \langle \kappa_2 \rangle \langle \kappa_1^2 \rangle - 2\langle \kappa_2\kappa_1 \rangle \langle \kappa_1 \rangle$$
$$+ 2\langle \kappa_2 \rangle \langle \kappa_1 \rangle^2) + 3(\langle \kappa_2^2 \rangle - \langle \kappa_2 \rangle^2),$$

## Detailed structure of skewness and kurtosis



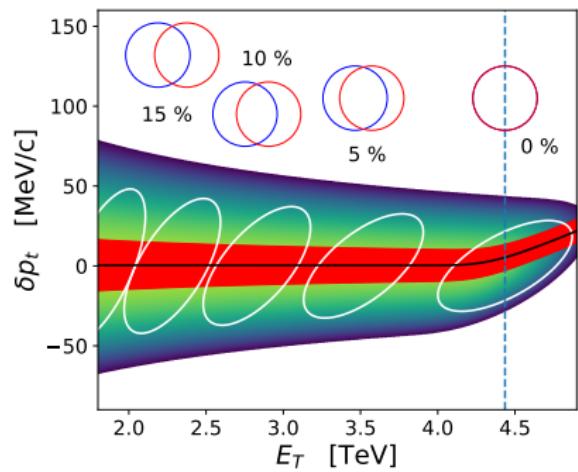
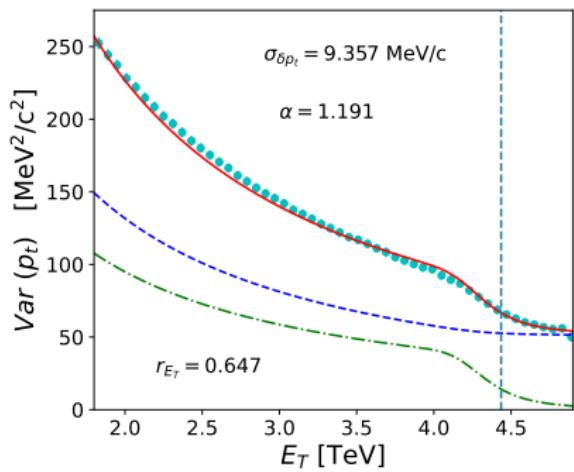
## b-dependence of the fit parameters

- We assume mean  $[p_t]$  to be independent of  $b$
- We assume  $\text{Var}([p_t])$  is a smooth function of mean multiplicity :

$$\sigma p_t^2 \left( \frac{\langle N_{ch}(0) \rangle}{\langle N_{ch}(b) \rangle} \right)$$

- We also assume  $r$  to be independent of  $b$  for simplicity

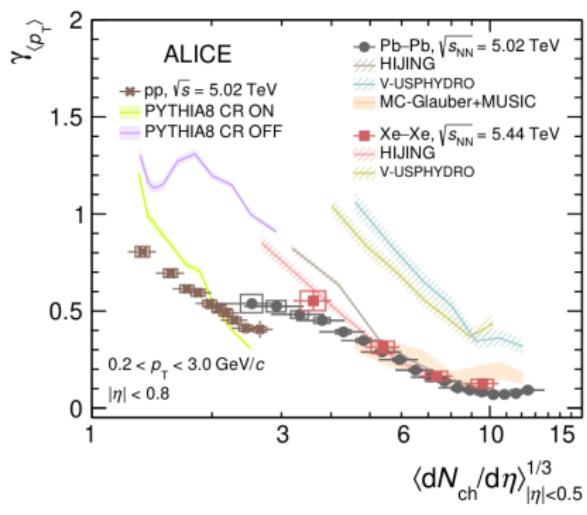
## $E_T$ -dependent [ $p_t$ ]-fluctuation



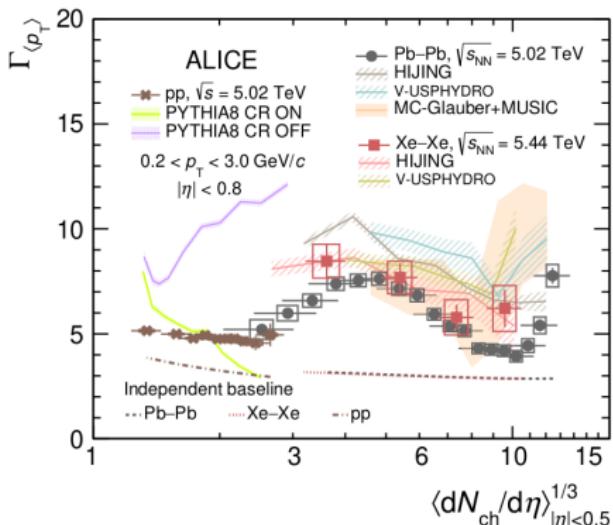
Impact parameter fluctuation is small !

# ALICE Measurements of $[p_t]$ -skewness !

arXiv: 2308.16217



standardized  $[p_t]$ -skewness



## intensive $[p_t]$ -skewness