



Uniwersytet  
Wrocławski



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Rostock



# Thermodynamics of QCD with quarks and multi-quark clusters

Oleksii Ivanytskyi, David Blaschke and Gerd Röpke

arxiv:2308.07950 [nucl-th]

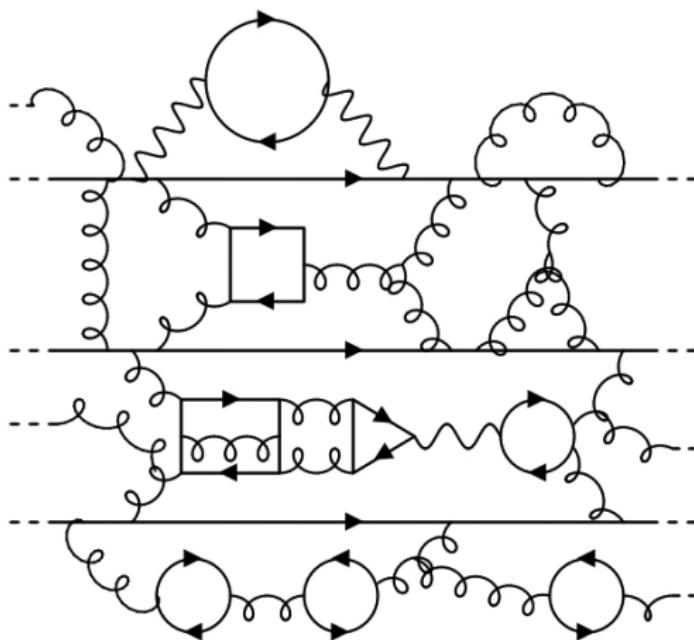
XVI Polish Workshop on Relativistic Heavy-Ion Collision

Kielce, 2-3 December 2023

# QCD: elegant Lagrangian, ...

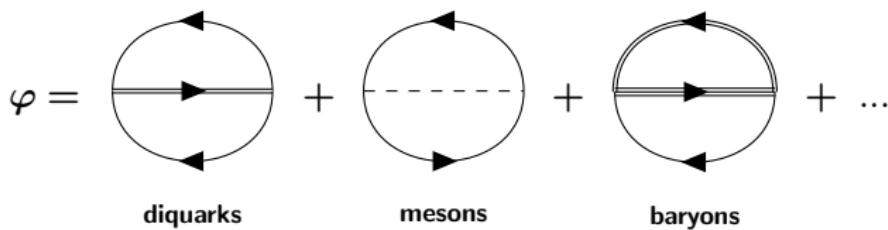
$$\mathcal{L}_{QCD} = \bar{q} \left( i \hat{D} - \hat{m} \right) q - \frac{1}{4} G^2$$

# QCD: ..., sophisticated nonperturbative dynamics



This is just a three quark bound state, e.g. nucleon

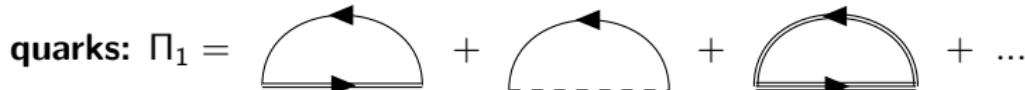
# Cluster decomposition in two-loop approximation



G. Baym, L.P. Kadanoff, Phys. Rev. 124, 287 (1961); G. Baym, Phys. Rev. 127, 1391 (1962)

- Two-loop self energies & Dyson-Schwinger propagators

$$\Pi_n = \frac{\delta \varphi}{\delta S_n}, \quad (S_n)^{-1} = (S_n^{\text{free}})^{-1} - \Pi_n, \quad n = 1, 2, 3, \dots$$



Dyson-Schwinger problem requires solving all  $S_n$  simultaneously

- Mean-field approximation for quark propagators

Dyson-Schwinger problem is reduced to subsequent solving  $S_n$  using  $S_{m \leq n}$

# Relativistic density functional with Polyakov loop

$$\mathcal{L} = \bar{q}(i\cancel{\partial} + g\cancel{A} - \hat{m})q - \mathcal{U}_\chi - \mathcal{U}_\Phi, \quad \hat{m} = \text{diag}(m_u, m_d, m_s)$$

$A_\mu$  - homogeneous static gluon field in the Polyakov loop

- **Density functional** (proper  $\chi$ -dynamics)

$$\mathcal{U} = D_0 \left[ (1 + \alpha) \langle \hat{\mathcal{O}} \rangle_0 - \hat{\mathcal{O}} \right]^{1/3}, \quad \hat{\mathcal{O}} = \frac{1}{2} \sum_{a=0,8} [(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2]$$

$D_0$  - coupling,  $\alpha \geq 0$  controls quark effective mass in the vacuum

D. Blaschke, O. Ivanytskyi, M. Shahrba, 2202.05061 [nucl-th]

- **Polyakov loop potential**

$$\Phi = \frac{1}{N_c} \text{Tr}_c \exp(i\beta A_0), \quad M_H = 1 - 6\bar{\phi}\phi + 4(\phi^3 + \bar{\phi}^3) - 3(\bar{\phi}\phi)^2$$

$$\frac{\mathcal{U}_\Phi}{T^4} = -\frac{1}{2}a\bar{\Phi}\Phi + b \log M_H + \frac{1}{2}c(\Phi^3 + \bar{\Phi}^3) + d(\bar{\Phi}\Phi)^2$$

$T$ -dependence of  $a, b, c$  &  $d$  is fitted to the pure SU(3) gauge lattice data

P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88, 074502 (2013)

# Expansion around mean-field solution

$$\mathcal{U}_\chi = \underbrace{\mathcal{U}_\chi^{MF}}_{0^{\text{th}} \text{ order}} + \underbrace{\bar{q} \hat{\Sigma} q - \langle \bar{q} \hat{\Sigma} q \rangle}_{1^{\text{st}} \text{ order}} \\ - \underbrace{\sum_{a,b=0,8} (\bar{q} \tau_a q - \langle \bar{q} \tau_a q \rangle) G_S^{ab} (\bar{q} \tau_b q - \langle \bar{q} \tau_b q \rangle) - G_{PS} \sum_{a=0,8} (\bar{q} i \tau_a \gamma_5 q)^2}_{2^{\text{nd}} \text{ order}} + \dots$$

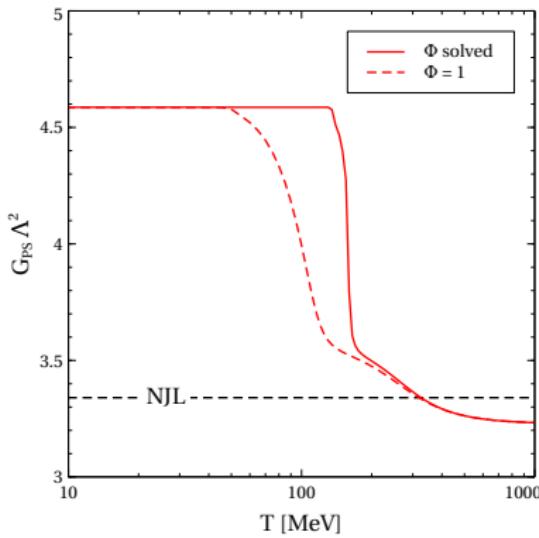
- Mean-field scalar self-energy

$$\hat{\Sigma} = \text{diag}(\Sigma_u, \Sigma_d, \Sigma_s), \quad \Sigma_f = \frac{\partial \mathcal{U}_\chi^{MF}}{\partial \langle \bar{f} f \rangle}$$

- Effective medium dependent couplings

$$G_S^{ab} = -\frac{1}{2} \frac{\partial^2 \mathcal{U}_\chi^{MF}}{\partial \langle \bar{q} \tau_a q \rangle \partial \langle \bar{q} \tau_b q \rangle}$$

$$G_{PS} = -\frac{1}{2} \frac{\partial^2 \mathcal{U}_\chi^{MF}}{\partial \langle \bar{q} i \gamma_5 \tau_a q \rangle^2}$$



# Comparison to NJL model ( $N_f = 2$ for simplicity)

$$\mathcal{L} = \bar{q}(i\cancel{\partial} - \underbrace{(m + \Sigma_S)}_{\text{effective mass } m^*})q + G_S(\bar{q}q)^2 + G_{PS}(\bar{q}i\vec{\tau}\gamma_5 q)^2 + \dots$$

$T = 0$

- **Similarities:**

- current-current interaction
- (pseudo)scalar, vector, diquark, ... channels

- **Differences:**

- high  $m^*$  at low  $T, \mu \Rightarrow \text{"confinement"}$

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 \Rightarrow m^* = m - \frac{2D_0}{3\alpha^{2/3} \langle \bar{q}q \rangle_0^{1/3}}$$

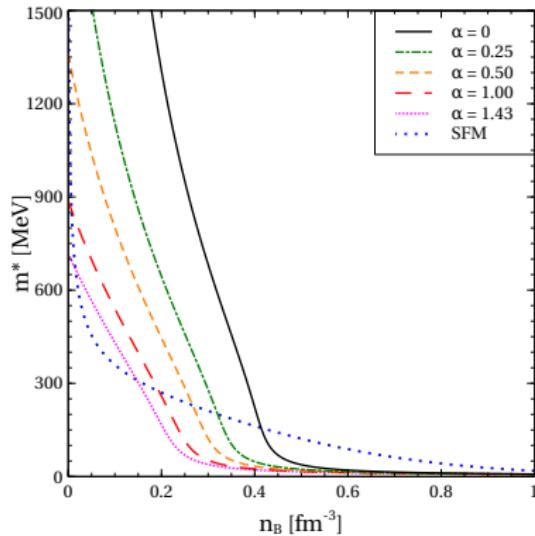


$$m^* \rightarrow \infty \text{ at } \alpha \rightarrow 0$$

- medium dependent couplings:

low  $T, \mu, \Rightarrow G_S \neq G_{PS} \Rightarrow \chi\text{-broken}$

high  $T, \mu, \Rightarrow G_S = G_{PS} \Rightarrow \chi\text{-symmetric}$



O.Ivanystkyi, D. Blaschke, PRD 105, 114042 (2022)

# Model setup

- Relativistic density functional for quark matter

$$M_\pi = 140 \text{ MeV}, F_\pi = 93 \text{ MeV}, M_K = 494 \text{ MeV}, F_K = 112 \text{ MeV}, T_c = 156.5 \text{ MeV}$$



$m_u = m_d$  and  $m_s$ , momentum cutoff  $\Lambda$ , parameters  $D_0, \alpha$  of the RDF

- Perturbative correction

$$\alpha_s = \frac{4\pi}{9} \left( \ln^{-1} \frac{T^2}{T_{pert}^2} - \frac{T_{pert}^2}{T^2 - T_{pert}^2} \right), \quad T_{pert} = 94 \text{ MeV}$$

- Hadrons - 62 meson & 60+60 baryon-antibaryon states with  $M < 2.6 \text{ GeV}$
- Colored multiquark states - diquarks, tetraquarks & pentaquarks

$$M^{vacuum} = \sum_f M_f^{vacuum} N_f - B \left( \sum_f N_f - 1 \right)$$

$B$  is defined by fitting this expression to the hadron masses

# Thermodynamic potential

$$\Omega = \Omega_{\text{quarks}} + \mathcal{U}_\chi - \langle \bar{\mathbf{q}} \hat{\Sigma} \mathbf{q} \rangle + \mathcal{U}_\phi + \underbrace{\Omega_{\text{hadrons}} + \Omega_{\text{colored clusters}}}_{\text{multiquark clusters}}$$

- **Quarks**

- Non-perturbative states at low momenta  $k < \Lambda_{QCD}$

$$\Omega_{\text{quarks}}^{k < \Lambda_{QCD}} = -\frac{1}{\beta V} \text{Tr} \ln(\beta S_{\text{quarks}}^{-1})$$

$S_{\text{quarks}}$  - quark propagator @ mean-field

- Perturbative states at high momenta  $k > \Lambda_{QCD}$

$$\Omega_{\text{quarks}}^{k > \Lambda_{QCD}} = \frac{1}{2\beta V} \quad \text{Diagram: A circle with a wavy line inside, and two arrows pointing clockwise around the circle.}$$

J.I. Kapusta, Finite Temperature Field Theory, Cambridge (1989)

- **Multi-quark clusters  $n > 1$  – generalized Beth-Uhlenbeck approach**

$$\Omega_n = \frac{d_n}{\kappa_n} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} (tr_D)^{2-\kappa_n} \ln(\beta^{\kappa_n} S_n^{-1}) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}, \quad \kappa_n = \begin{cases} 1, & \text{fermions} \\ 2, & \text{bosons} \end{cases}$$

$$S_n = |S_n| e^{i\delta_n}, \quad \delta_n - \text{phase shift}$$

G. Röpke, N.U. Bastian, D. Blaschke, T. Klähn, S. Typel, H. Wolter, NPA 897 (2013)

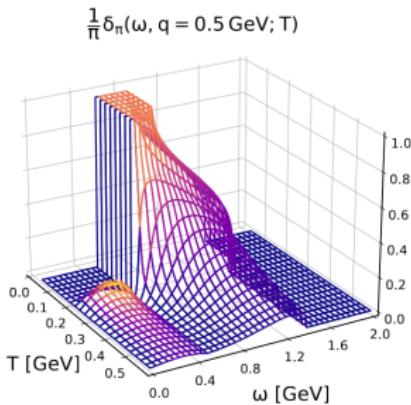
# Phase shifts of multiquark states

$$\delta_n = \Im \ln S_n$$

## • Microscopic calculations for pions

K. Maslov, D. Blaschke, PRD 107, 094010 (2023)

- ① discontinuous jump at the on-shell energy below the dissociation temperature
- ② continuous growth at small energies above the dissociation temperature
- ③ continuous fall above the decay threshold
- ④ vanishing at high energies

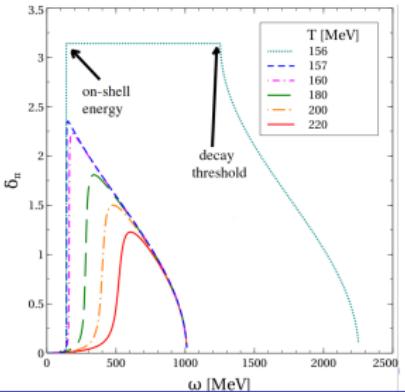


## • Parametric model of $\delta = \delta(T, \omega)$

D. Blaschke, M. Cierniak, O. Ivanytskyi, G. Röpke, arxiv:2308.07950 [nucl-th]

- ① parametric expression reproduces all the properties of the microscopic calculations
- ② T-dependence of the hadron masses & widths agree with the microscopic calculations
- ③ requires hadron decay threshold given by quark masses  $M_{u,d,s}$

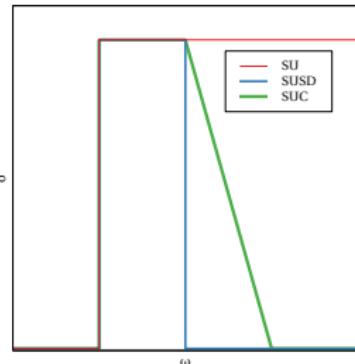
$$M_h^{\text{Th}} = N_h^u M_u + N_h^d M_d + N_h^s M_s$$



# Beth-Uhlenbeck vs Hadron resonance gas

## • Step-up

- ① is generated by the pole of  $S_{n>1}$
- ② corresponds to a bound multiquark state
- ③ is present only below the dissociation temperature
- ④ generates a HRG-like term in  $\Omega$

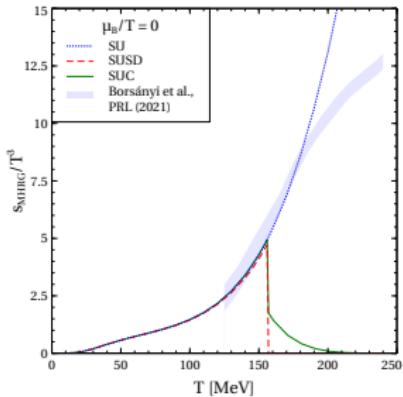


## • Step-down

- ① rough account of the decay threshold
- ② partially/totally compensates HRG-like term in  $\Omega$

## • Continuum

- ① corresponds to a scattering multiquark state
- ② partially compensates HRG-like term in  $\Omega$



# Mass-spectrum

$$M_{n>1} = M_{n>1}^{\text{vacuum}} + A(T - T_c)\theta(T - T_c)$$

$$\Gamma_{n>1} = B\theta(T - T_c)$$

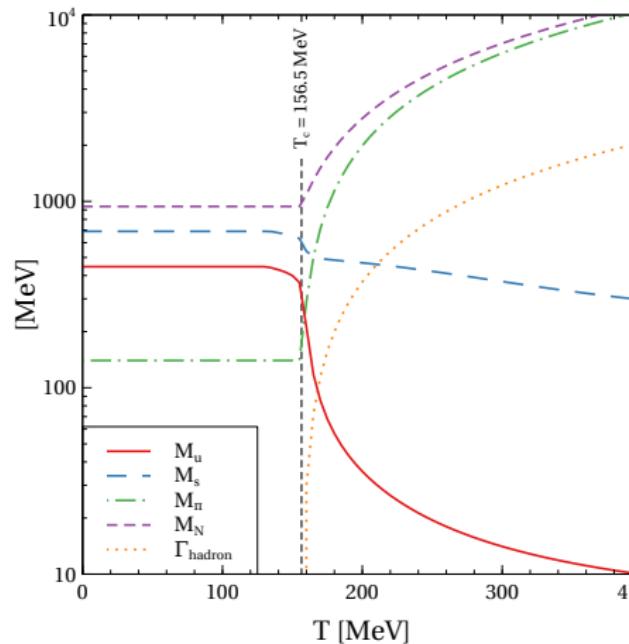
$T_c = 156.6$  MeV,  $A, B$  - fitted to IQCD

## • Low T ( $\chi$ -broken matter)

- ① heavy quarks
- ② stable multiquark clusters
- ③ constant mass of multiquark states
- ④ zero width of multiquark states

## • High T ( $\chi$ -symmetric matter)

- ① light quarks
- ② unstable multiquark clusters
- ③ growing mass of multiquark states
- ④ growing width of multiquark states



# Entropy density

$$s = -\frac{\partial \Omega}{\partial T}$$

- **Low T**

- ➊ hadron dominance

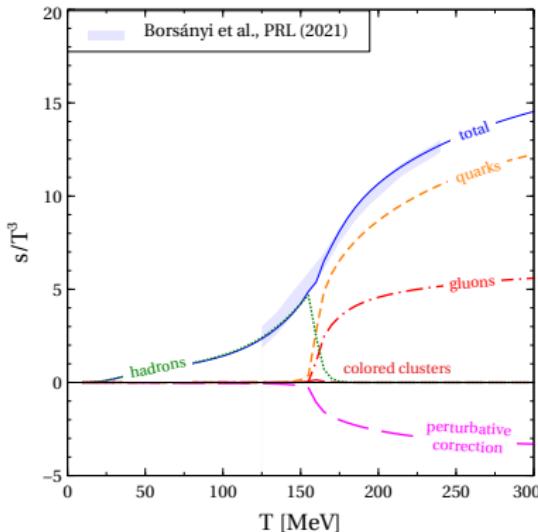
- **High T**

- ➊ quark-gluon dominance
  - ➋ negative perturbative contribution

- **Colored multiquark states**

$(\mu_B = 0 \text{ only})$

- ➊ suppressed by the Polyakov loop at high  $T$
  - ➋ suppressed by high mass at high  $T$



# Chiral condensate

$$\begin{aligned}\langle \bar{f}f \rangle = -\frac{\partial \Omega}{\partial m_f} &= \underbrace{2N_c \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{M_f}{E_f} (f_f^+ + f_f^- - 1)}_{\text{quarks}} \\ &+ \underbrace{\sum_{n>1} \frac{d_n \sigma_n^f}{m_f} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} \frac{M_n}{\omega} (f_n^+ + f_n^-) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}}_{\text{multiquark clusters}}\end{aligned}$$

- **$\sigma$ -factor**

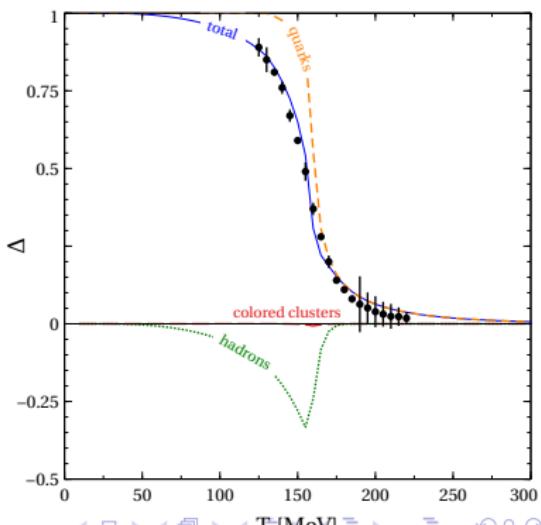
- ①  $\sigma_\pi^f, \sigma_K^f$  – defined from the GMOR relations
- ② other multiquark clusters

$$\sigma_n^f = m_f \frac{\partial M_n}{\partial m_f} = m_f N_n^f$$

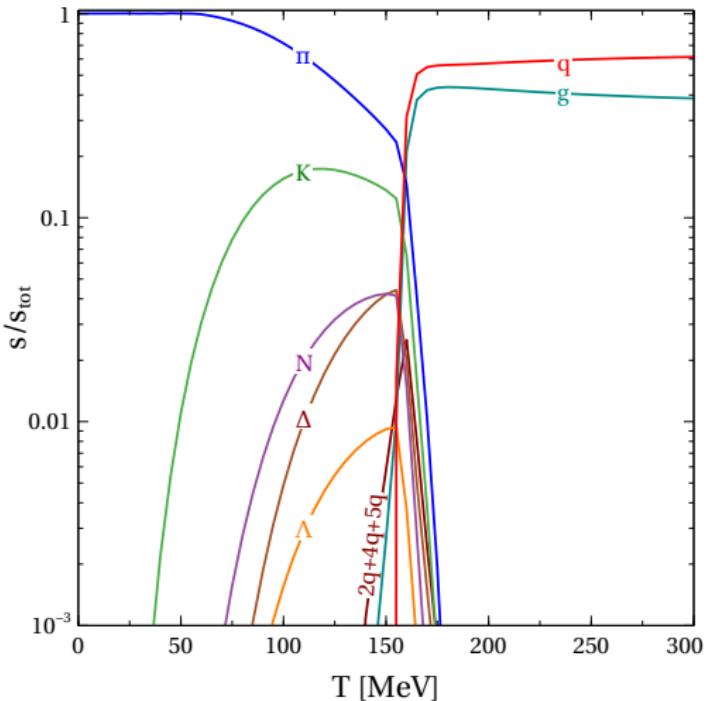
J. Jankowski, D. Blaschke, M. Spalinski, PRD 87, 10 (2013)

- Almost constant quark term below  $T_c$

- Hadrons are necessary to reproduce the IQCD data



# Composition



Sharp switching between partonic & hadronic degrees of freedom

# Conclusions

- A unified EoS of strongly interacting matter based on a cluster decomposition approach
- Agreement with the lattice QCD data on entropy density and chiral condensate (see arxiv:2308.07950 [nucl-th] for baryon density and stay tuned for more)
- Sudden switching between partonic and hadronic degrees of freedom

# Speculation

1. Sharp switching between hadrons and partons
2. Absence of colored mediators of interaction below  $T_c$



Narrow range of the freeze-out temperatures for all hadrons?