Isospin breaking in kaon production in heavy ion collisions

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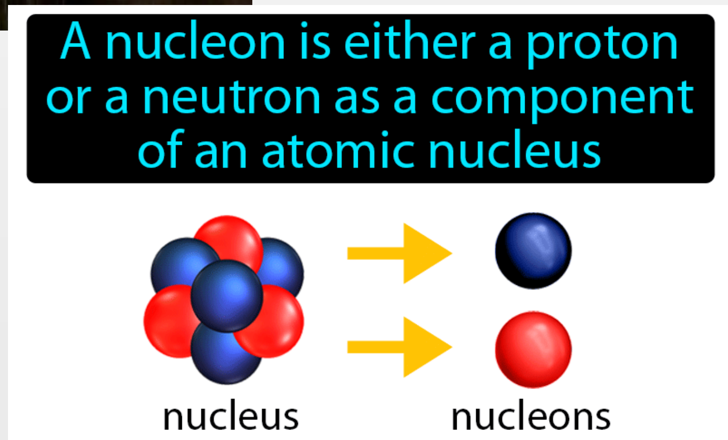
Outline

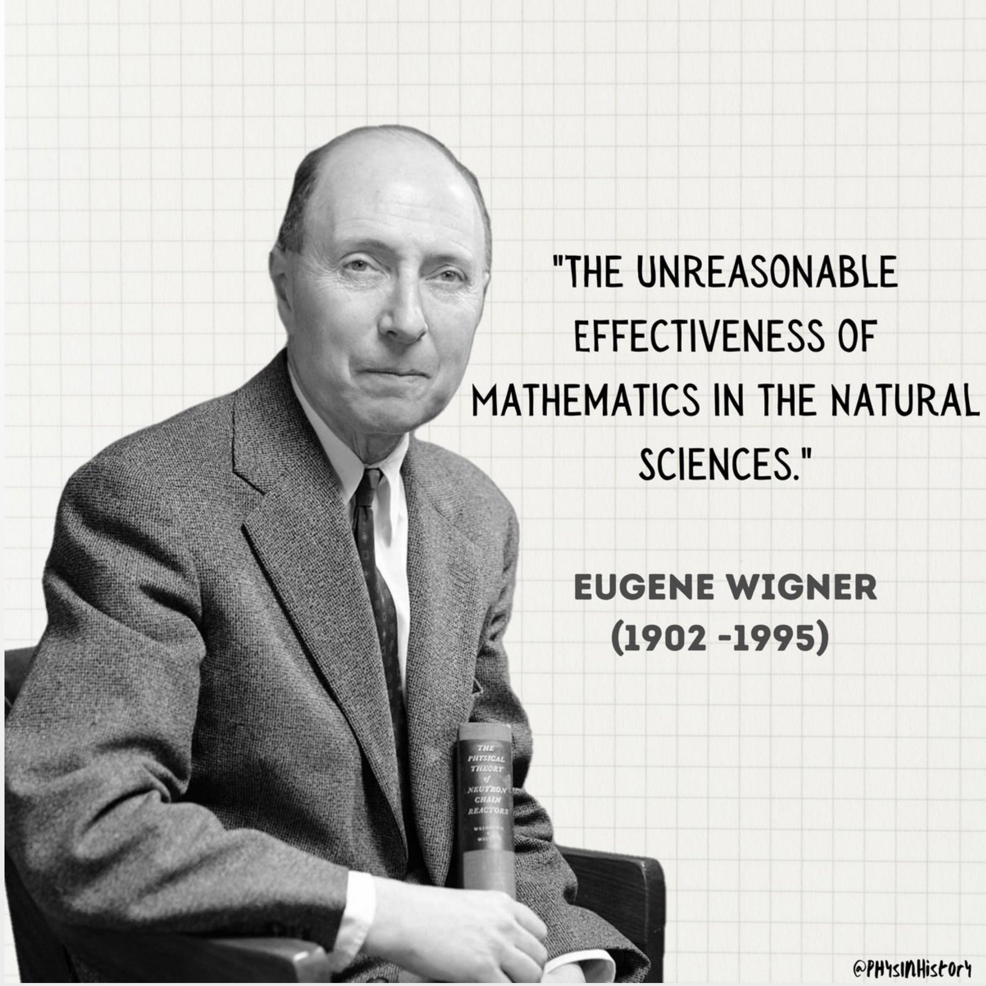
1. Isospin: brief recall
2. Kaon productions
3. Theory vs experiment
4. Conclusions

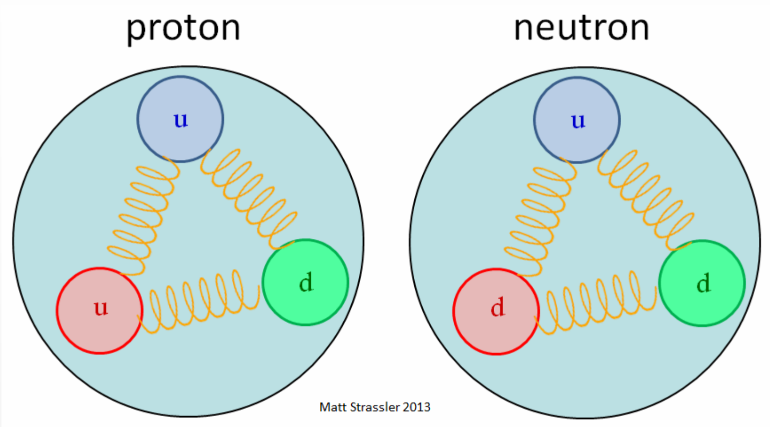
Heisenberg (1932)

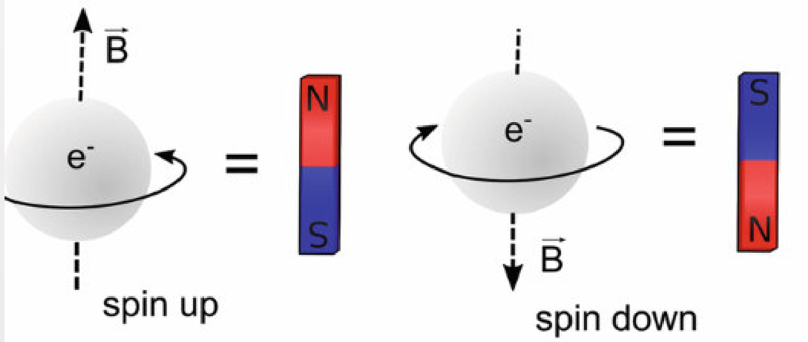


Proton and neutron merge into the nucleon
Masses very similar

Wigner (1932): isotopic spin, thus isospin







Nucleon doublet: I=1/2

$$\begin{pmatrix} p \\ n \end{pmatrix} \to \hat{O} \begin{pmatrix} p \\ n \end{pmatrix}$$

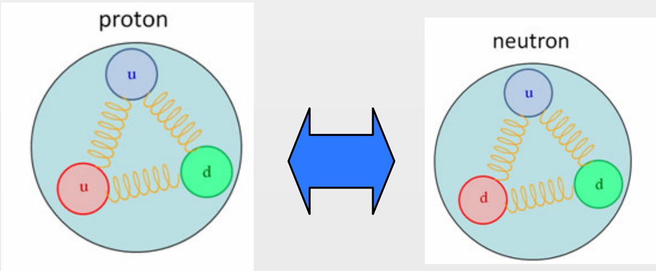
$\hat{O}$ is a 2×2 unitary matrix.

$$\hat{O} = e^{i\theta_i \sigma_i / 2}$$

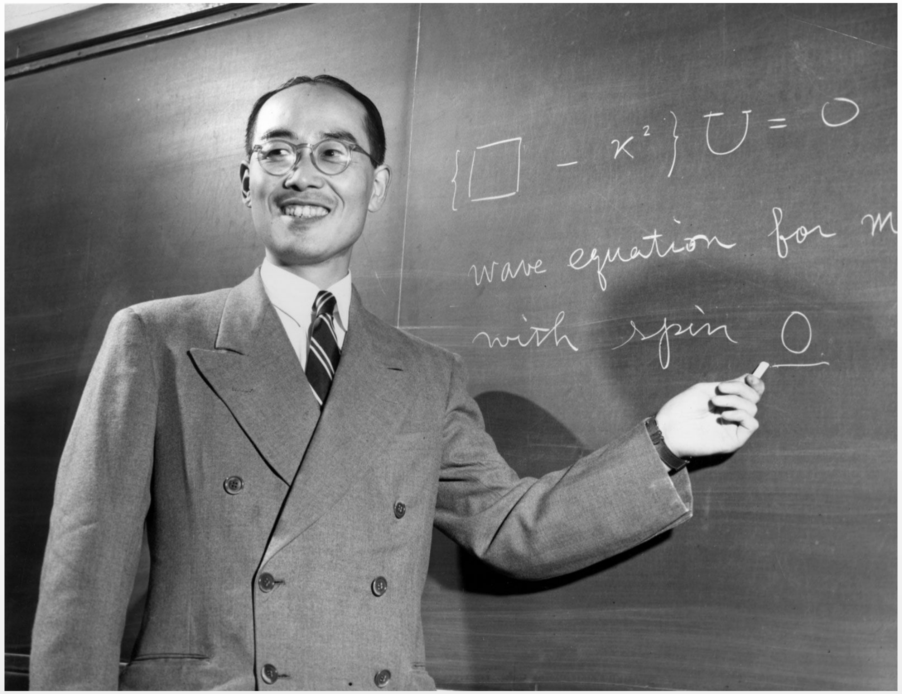
$$\hat{C} = e^{i\pi\sigma_2/2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Then under $\hat{C}$:

$$p \Longleftrightarrow n$$



Yukawa (1932) and Kemmer (1939): isospin triplet I=1



$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

under $\hat{C}$:

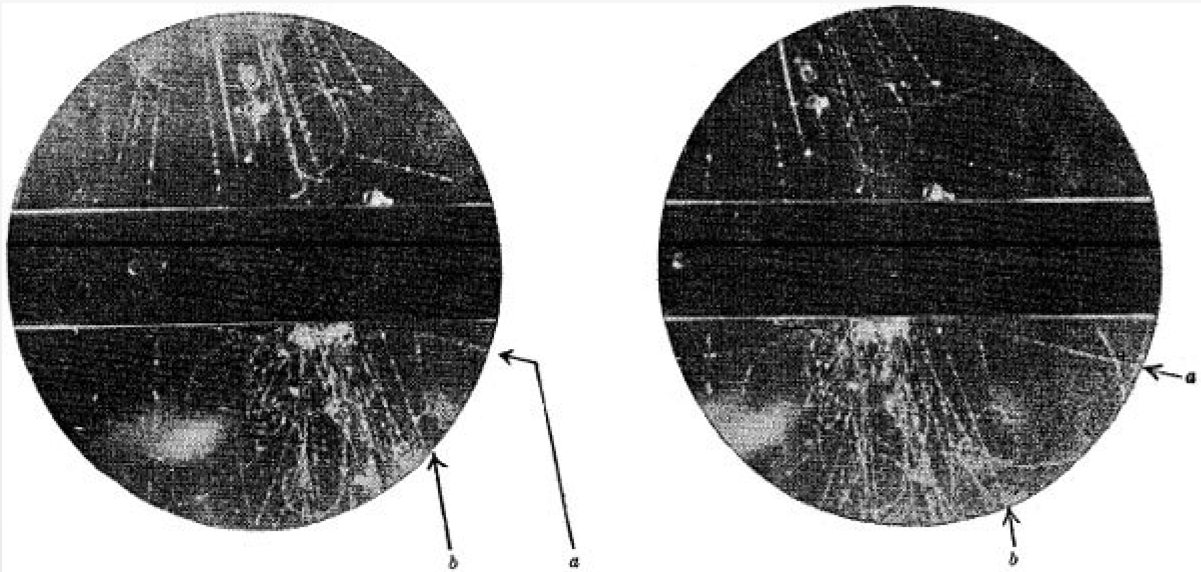
$$\pi^+ \Longleftrightarrow \pi^-$$

Kaons

20 DECEMBER 1947

Clifford Butler and George Rochester discover the kaon

first strange particle



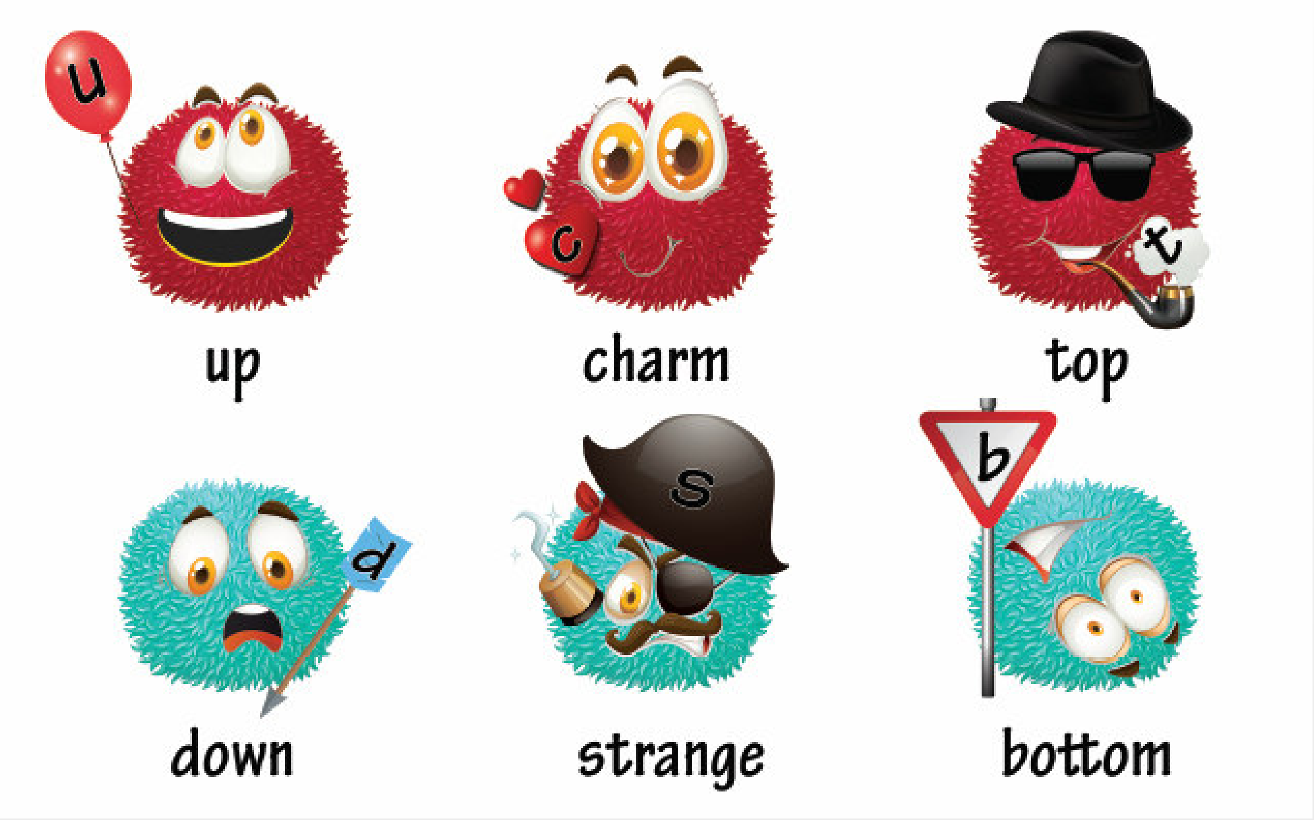
Kaons form isospin doublets, just as the nucleon

$$\begin{pmatrix} p \\ n \end{pmatrix} \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \begin{pmatrix} -\bar{K}^0 \\ K^- \end{pmatrix} \cdots$$

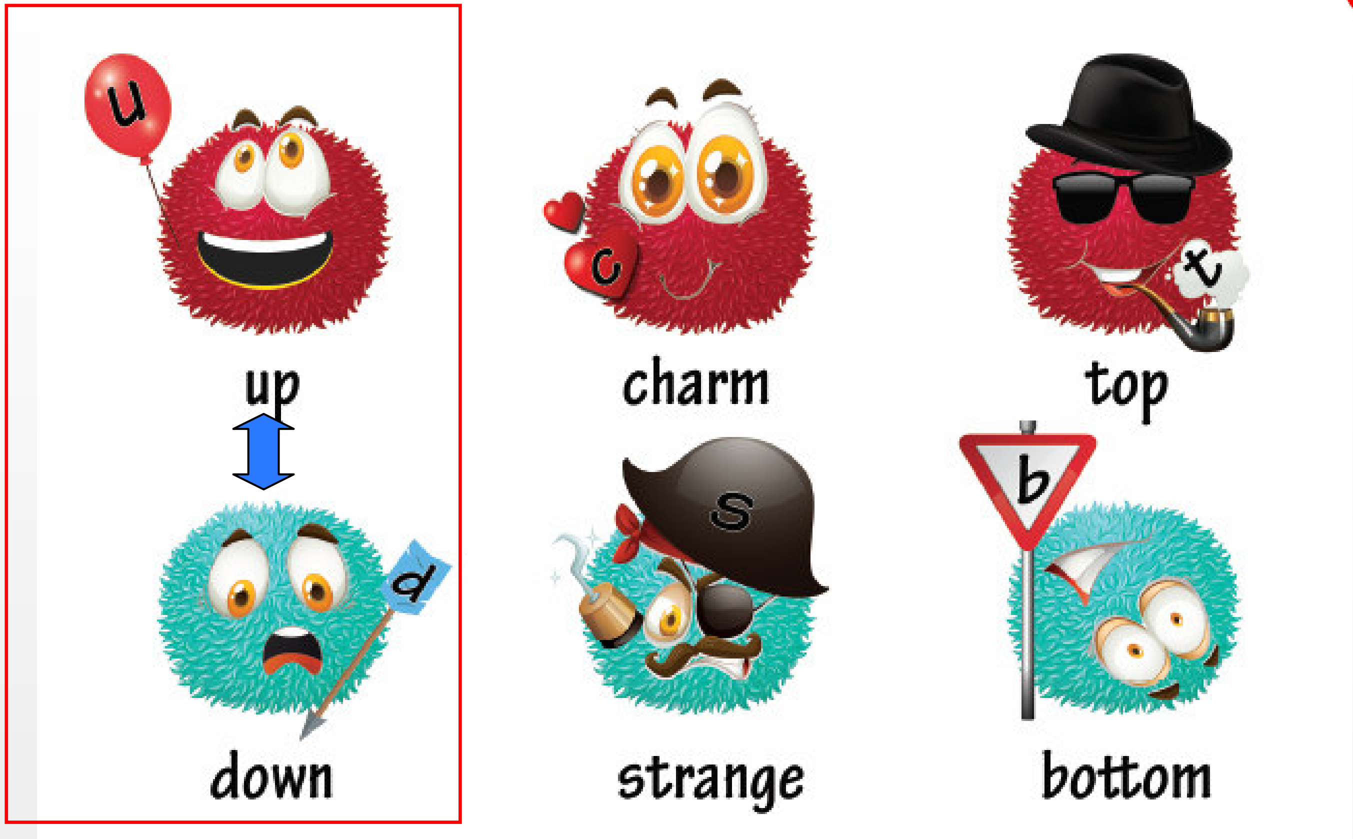
under $\hat{C}$:

$$\begin{array}{ccc} p & \Longleftrightarrow & n \\ K^+ & \Longleftrightarrow & K^0 \\ \bar{K}^0 & \Longleftrightarrow & K^- \end{array}$$

Quarks and QCD



Quarks and QCD, isospin:



In terms of quarks: $\begin{pmatrix} u \\ d \end{pmatrix} \to \hat{O} \begin{pmatrix} u \\ d \end{pmatrix}$

Then under $\hat{C}$: $u \Longleftrightarrow d$

Kaons form isospin doublets, just as the nucleon

$$K^+ \equiv u\bar{s}, K^0 \equiv d\bar{s} \qquad \bar{K}^0 \equiv s\bar{d}, K^- \equiv s\bar{u}$$

$$\begin{pmatrix} p \\ n \end{pmatrix} \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \begin{pmatrix} -\bar{K}^0 \\ K^- \end{pmatrix} \cdots$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

Isospin is an approximate symmetry of QCD

- Mesonic multiplets (nucleon doublet, pion trioplet, kaon doublets).

- Reactions: if an initial state has a certain (I,Iz), then the final state is also such. Indeed, pion-pion, pion-nucleon and nucleon-nulceon scattering conserve isospin.

  Example: $p + p \rightarrow \Lambda + K^+ + p$ (I=Iz=2)

- Isospin symmetry is good, but not exact. Masses of u and d not equal (explicit symmetry breaking).

Example of isospin breaking

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/84-27

March 8th, 1984

THE ISOSPIN-VIOLATING DECAY $\eta' \to 3\pi^0$

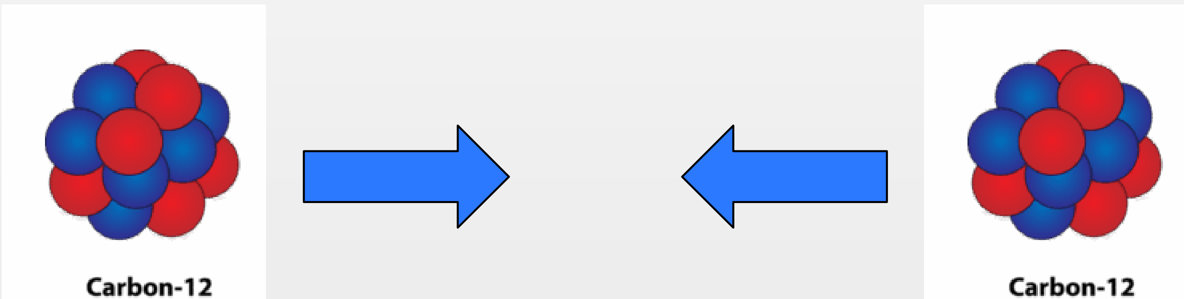
IHEP[1]-IISN[2]-LAPP[3] Collaboration

$$BR(\eta' \to 3\pi^0) = 5.2\left(1 - \frac{m_u}{m_d}\right)^2 10^{-3}$$

Nucleus-nucleus collion with equal numbers of protons and neutrons

$$Z = N = A/2 \qquad Q/B = 1/2$$

$$|A + A\rangle$$



Iz = 0 (typically also I =0 for each nucleus, thus total isospin also vanishing)

Expected kaon multiplicities

Without referring to a detailed mathematical formalism, charge symmetry means that strong interactions are invariant under the inversion of the third component of the isospin of every nucleus and hadron of the initial and final states. Let us consider an ensemble of initial states being invariant under the charge transformation $\hat{C}$ - probabilities of having initial states related by this transformation are equal. This is indeed the case of nucleus-nucleus collisions where each nucleus has an equal number of protons and neutrons (thus, $I_z = 0$). Then, the invariance under $\hat{C}$-transformation holds also for the final state ensemble, implying that the mean multiplicities of $\hat{C}$-related quantities, such K^+ and K^0 and $\bar{K}^0$ and K^-, coincide:

$$\langle K^+ \rangle = \langle K^0 \rangle \qquad (1)$$

$$\langle K^- \rangle = \langle \bar{K}^0 \rangle \qquad (2)$$

Shmushkevich rule

The relations (1) and (2) are in agreement with the result first formulated in 1955 by Shmushkevich [15, 16], see also Refs. [17–19]. This result states that an initial 'uniform' ensemble of hadronic state (that is, one with an equal mean number of each member of any isospin multiplet, such as the scattering of two isosinglet nuclei) evolves into a uniform final-state ensemble.

Uniform stays uniform

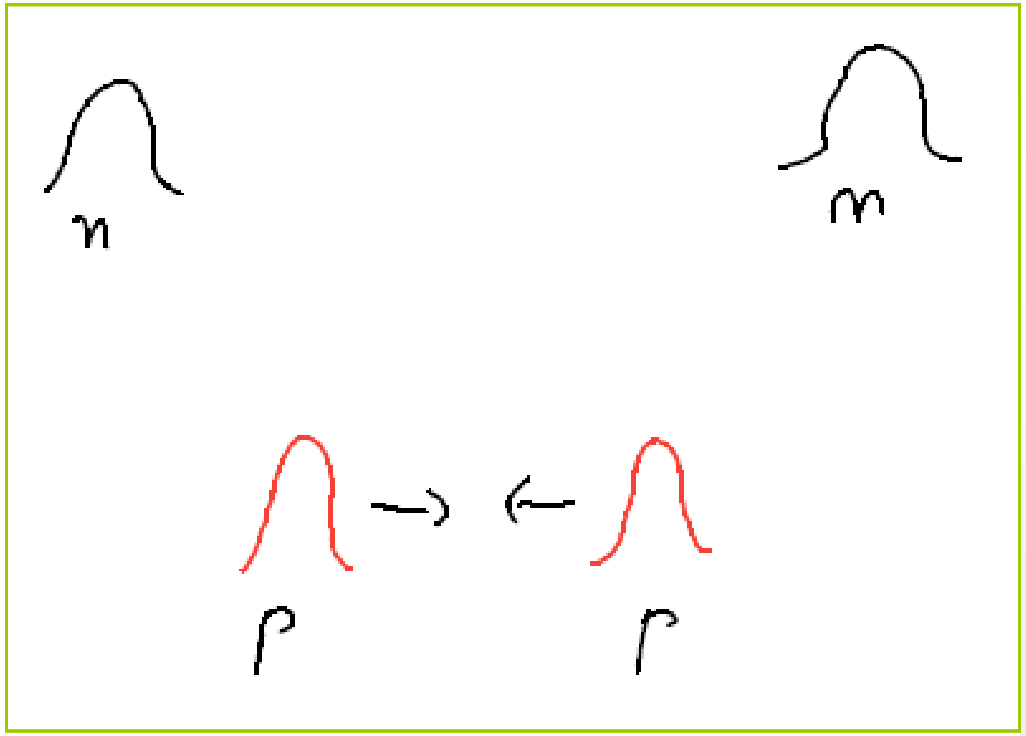
[15] Shmushkevich, I.: . Dokl. Akad. Nauk SSSR **103**, 235 (1955)

[16] Dushin, N., Shmushkevich, I.: . Dokl. Akad. Nauk SSSR **106**, 801 (1956)

[17] MacFarlane, A.J., Pinski, G., Sudarshan, G.: Shmushkevich's method for a charge independent theory. Phys. Rev. **140**, 1045 (1965) https://doi.org/10.1103/PhysRev.140.B1045

[18] Wohl, C.G.: Isospin relations by counting. American Journal of Physics **50**(8), 748–753 (1982) https://doi.org/10.1119/1.12743

$pp\,mm \longmapsto\ ?$



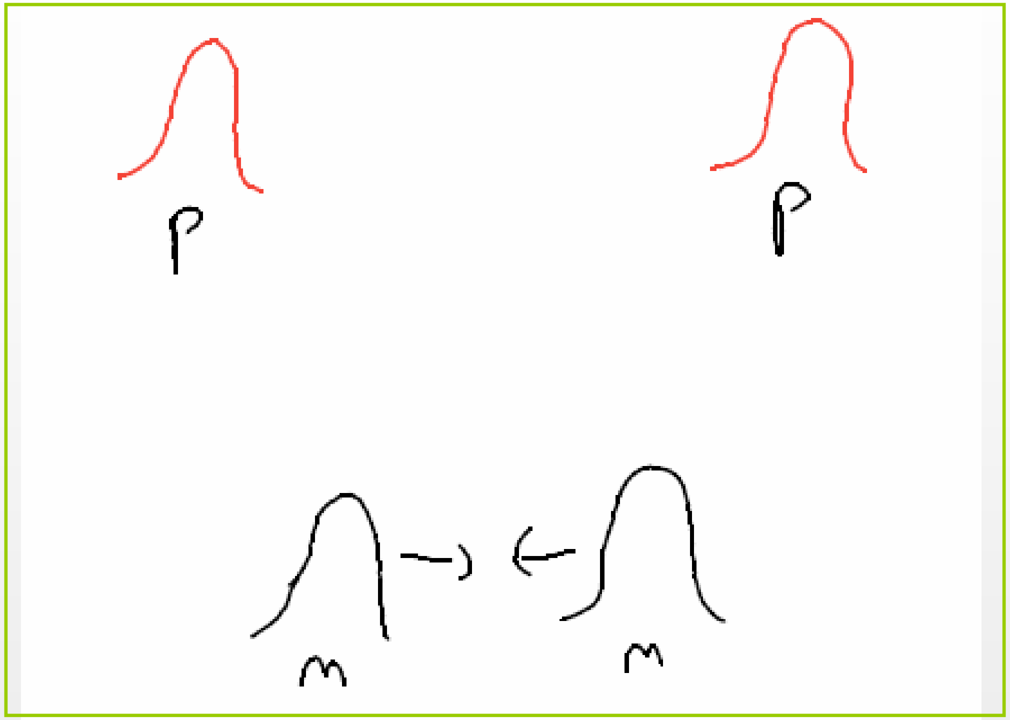
Just as pp!

More K^+ than K^0

Is then the previous Argumentation wrong?

No.
One needs to average.

But... $\hat{C}$ transform



This is the C-transformed version fo the previous reaction.

Here, the protons are spectactors and the neutrons interact.

Just as nn scattering!

More K^0 than K^+

Averaging leads to…

If both initial states are equally probable

$$\langle K^+ \rangle = \langle K^0 \rangle$$

holds!

This is a general result!

Formally:

$$\hat{\rho} = \sum_n p_n \left|\Psi_n\right\rangle \left\langle\Psi_n\right|$$

$$\hat{C}\hat{\rho}\hat{C}^\dagger = \hat{\rho}$$

Neutral kaons and the ratio RK

$$\begin{pmatrix} |K_S^0\rangle \\ |K_L^0\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

$$\langle K_S^0\rangle = \frac{1}{2}\langle K^0\rangle + \frac{1}{2}\langle \bar{K}^0\rangle = \langle K_L^0\rangle$$

$$\langle K^+\rangle + \langle K^-\rangle = 2\langle K_S^0\rangle$$

$Q/B = 1/2$

+ isospin exact…

$$\boxed{R_K \equiv \frac{\langle K^+\rangle + \langle K^-\rangle}{\langle K^0\rangle + \langle \bar{K}^0\rangle} = \frac{\langle K^+\rangle + \langle K^-\rangle}{2\langle K_S^0\rangle} = 1}$$

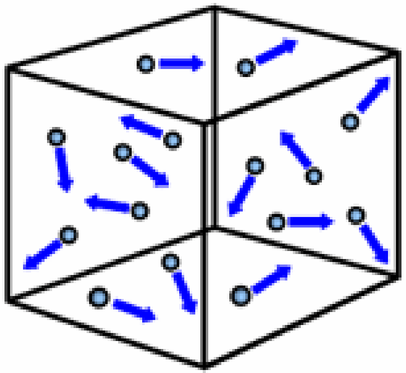
NA61 (ongoing, preliminary)

$$R_K = 1.233 \pm 0.057$$

Seems far from 1.
Yet, Q/B<1/2 and, of course, isospin is not exact.

Theoretical description of a thermal gas

$$\ln Z = \sum_k \ln Z_k^{\text{stable}} + \sum_k \ln Z_k^{\text{res}}$$



$$\ln Z_k^{\text{stable,}} = f_k V \int \frac{d^3p}{(2\pi)^3} \ln\left[1 \pm e^{-E_p/T}\right]^{\pm 1}$$

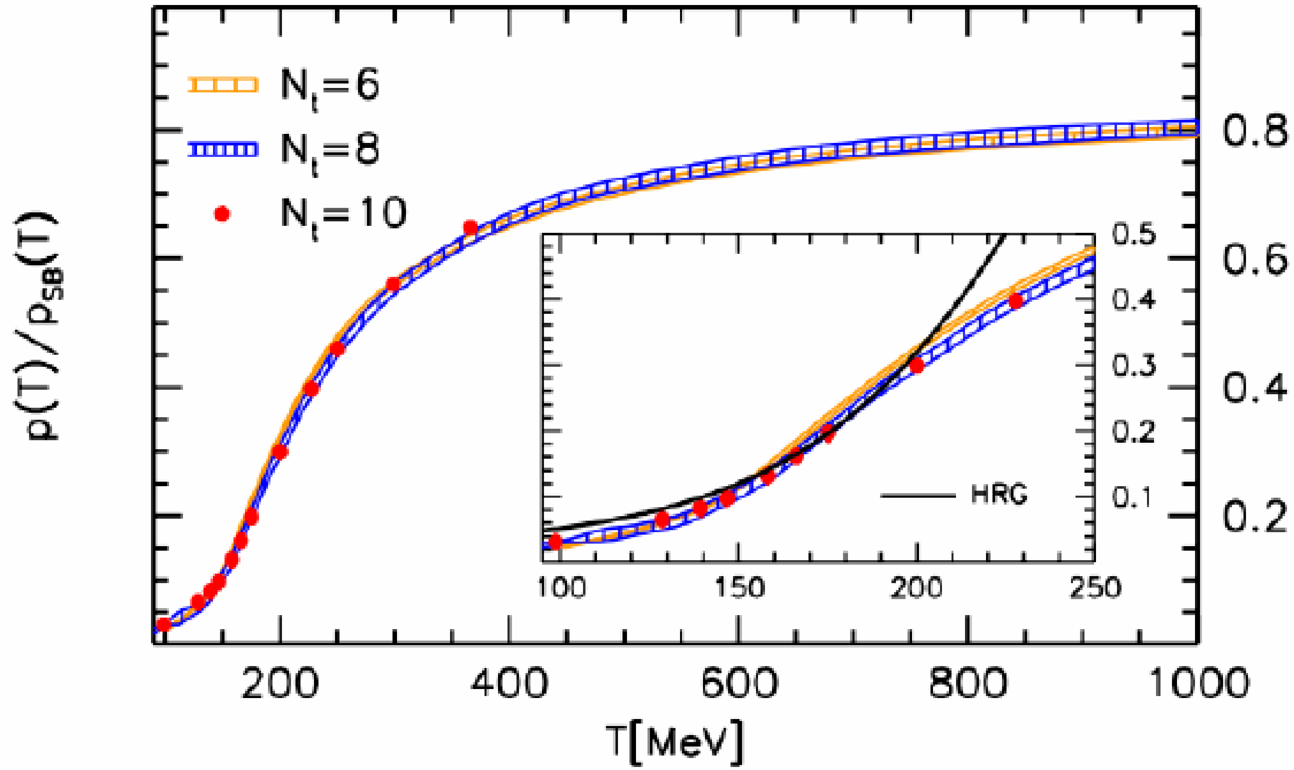
$$E_p = \sqrt{\vec{p}^2 + M_k^2}$$

$$P = \frac{T}{V} \ln Z$$

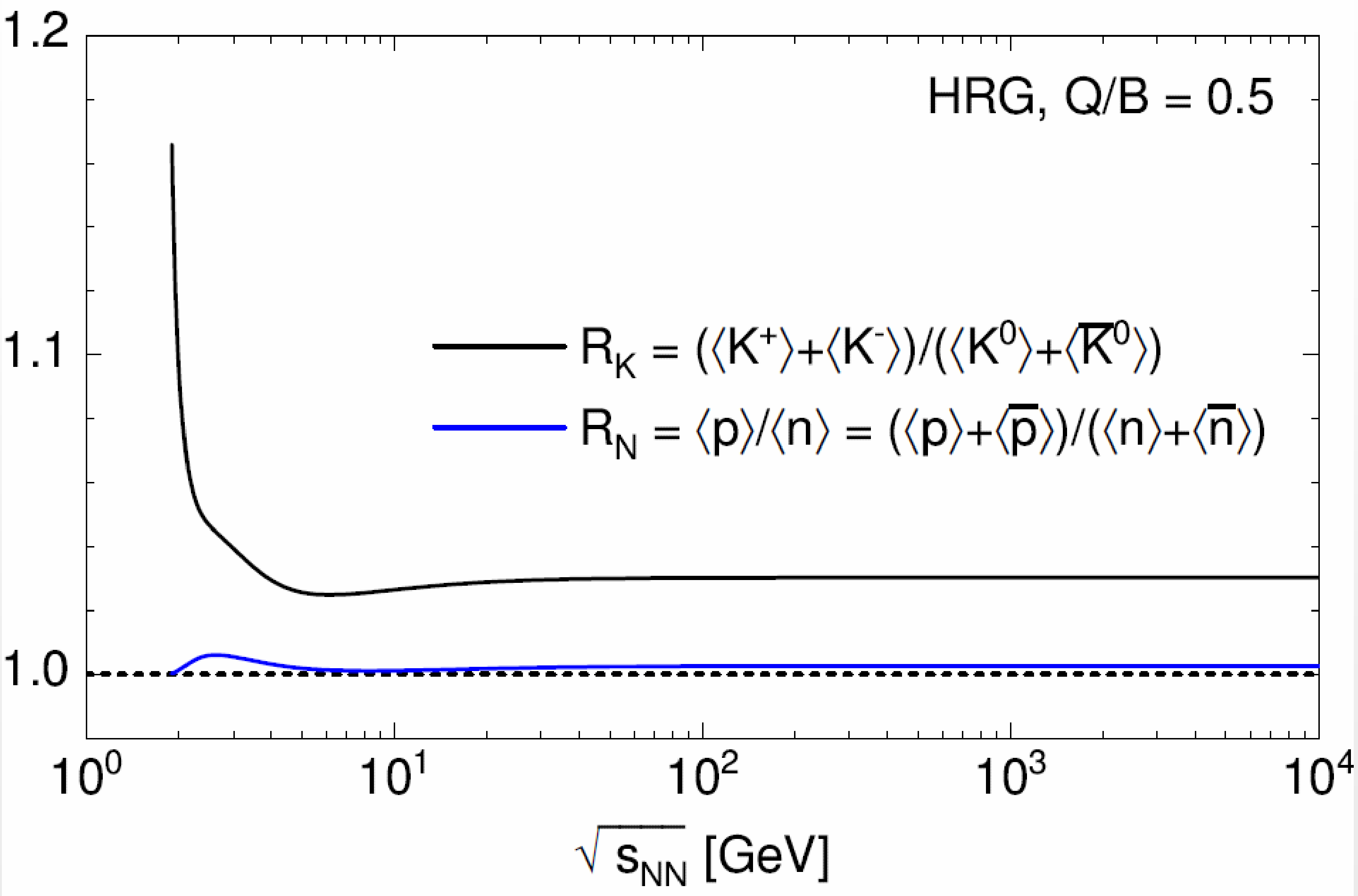
The stable part is “easy”: In first approximation, resonance as stable.
Then, correcting for width, itneraction,…

Hadron resonance gas vs lattice results

- All baryons and mesons (m < 2.5 GeV) from PDG [Borsnayi et al.JHEP11(2010)077]

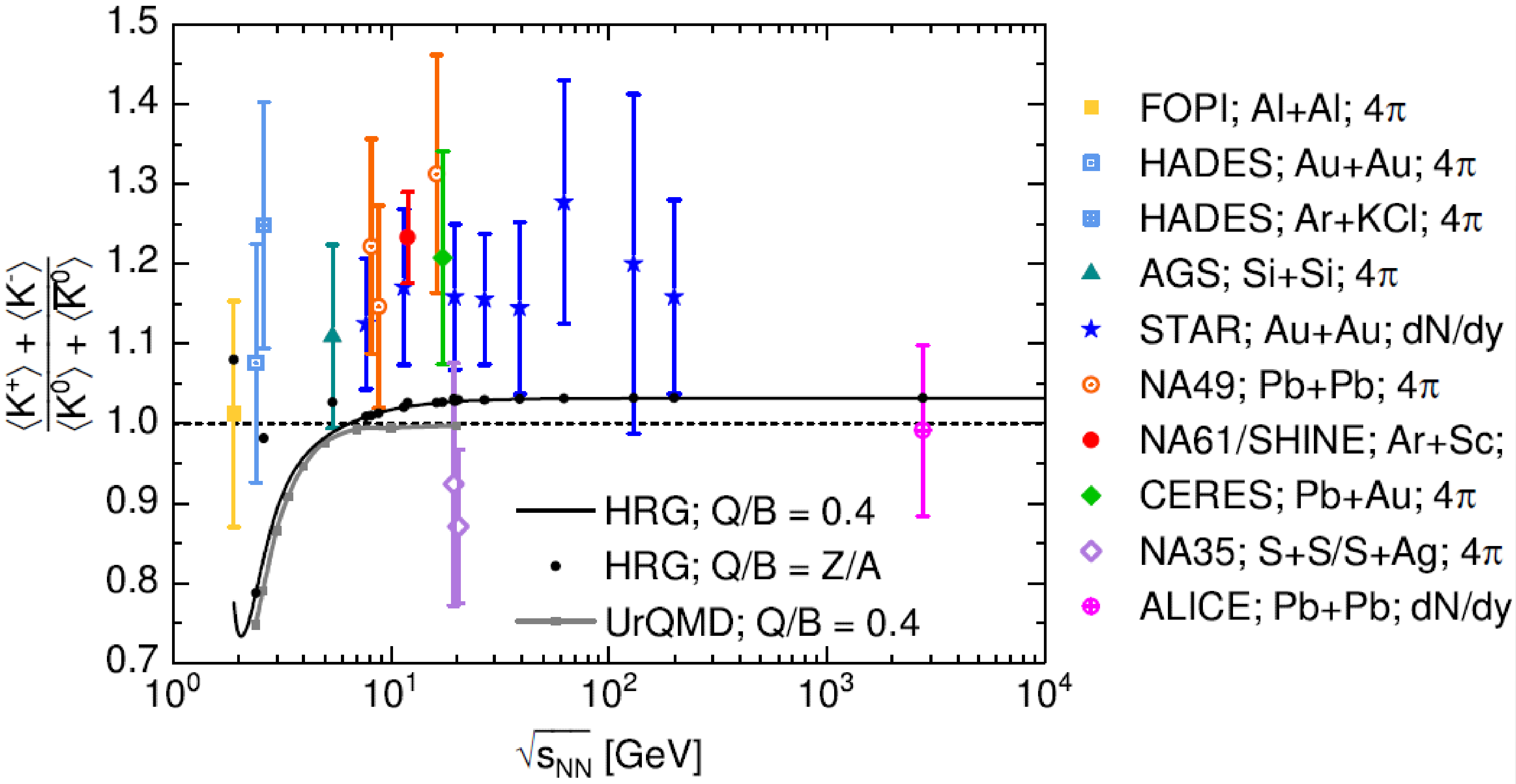


HRG for Q/B=1/2



If we enforce isospin symmetry to be exact, $R_K = 1$ for any energy.

Exp vs data



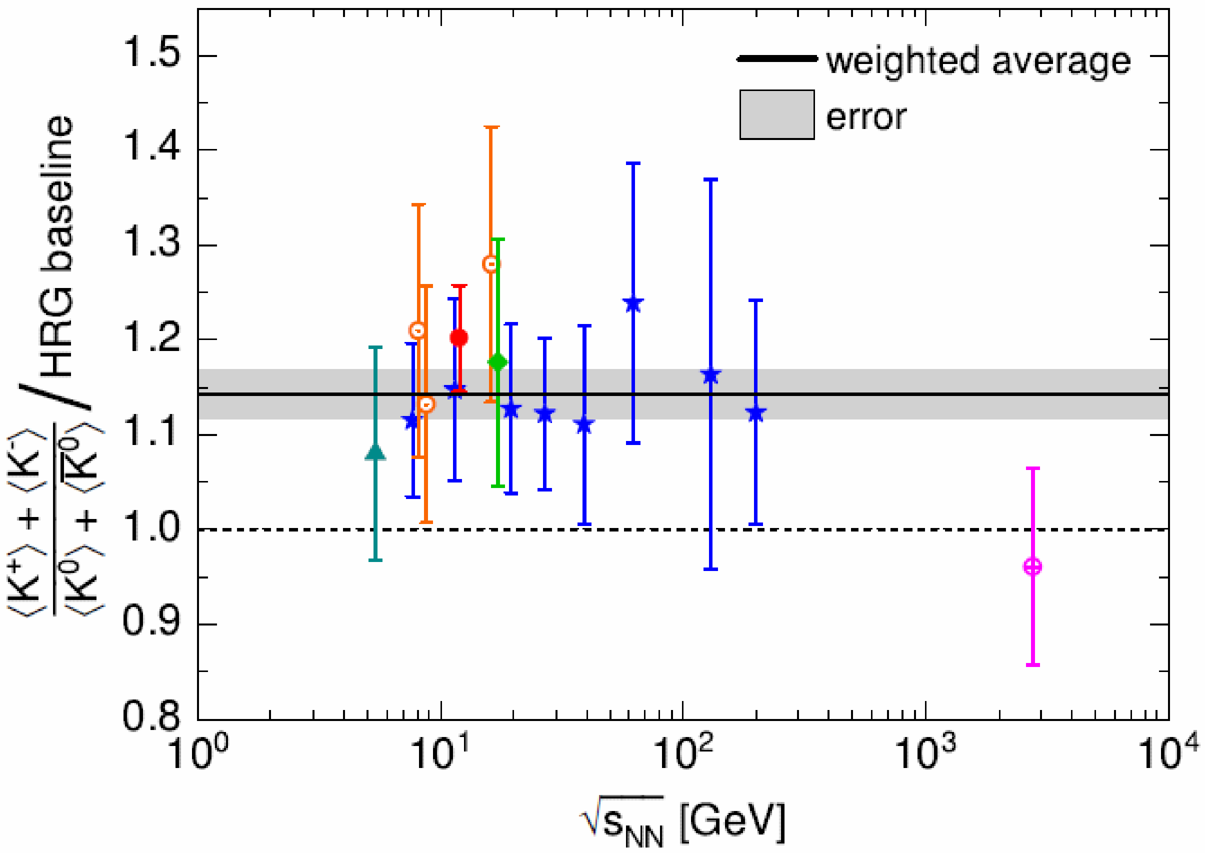
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
$\frac{\langle K^+\rangle + \langle K^-\rangle}{\langle K^0\rangle + \langle \bar{K}^0\rangle}$
10^0
10^1
10^2
10^3
10^4
$\sqrt{s_{NN}}$ [GeV]
HRG; Q/B = 0.4
HRG; Q/B = Z/A
UrQMD; Q/B = 0.4
FOPI; Al+Al; 4π
HADES; Au+Au; 4π
HADES; Ar+KCl; 4π
AGS; Si+Si; 4π
STAR; Au+Au; dN/dy
NA49; Pb+Pb; 4π
NA61/SHINE; Ar+Sc;
CERES; Pb+Au; 4π
NA35; S+S/S+Ag; 4π
ALICE; Pb+Pb; dN/dy

Considerations

- Q/B<1/2 favours neutral kaons
- charged kaons are lighter than neutral ones: this favours charged kaons
- UrQMD (Hadron-String transport model, fully integrated Monte Carlo simulation of nucleus-nucleus simulations) agrees with HRG

Theory vs experiment: ratio

$$1.143 \pm 0.026$$



$$\chi^2_{\min}/\mathrm{dof} = 0.47$$

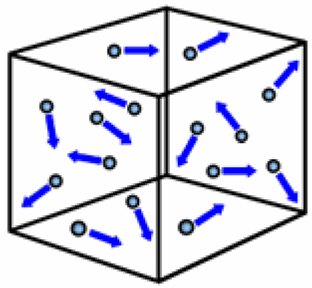
Considerations and conclusions

- Theory cannot explain experiment
- Scattering of nuclei with Z=N=A/2 highly desired…
- Easier but equally good? Average over: $\pi^- + C \text{ and } \pi^+ + C$
- Study other isospin multiplets
- Non-pertrubative effects? Chiral anomaly (Pisarski&Wilczek,…)

Thanks!

Theoretical description of a thermal gas

$$\ln Z = \sum_k \ln Z_k^{\mathrm{stable}} + \sum_k \ln Z_k^{\mathrm{res}}$$



The stable part is “easy”:

$$\ln Z_k^{\mathrm{stable},} = f_k V \int \frac{d^3p}{(2\pi)^3} \ln\left[1 \pm e^{-E_p/T}\right]^{\pm 1}$$

$$E_p = \sqrt{\vec{p}^2 + M_k^2}$$

$$P = \frac{T}{V} \ln Z$$

(Simplified) Thermal gas in QCD

It is well known that the ‘dominant’ term is given by pions (since these are the lightest mesons).

For simplicity, let us first consider only the pions and the rho meson.

The question is: how to treat the rho meson?

(Simplified) Thermal gas in QCD – the pions

$$\ln Z = \ln Z_\pi + \ln Z_\rho$$

$$\ln Z_\pi = 3V \int \frac{d^3p}{(2\pi)^3} \ln\left[\frac{1}{1-e^{-\beta E_\pi}}\right]$$

$$E_\pi = \sqrt{\mathbf{p}^2 + M_\pi^2}$$

In the pion gas one performs the sum over all occupation number

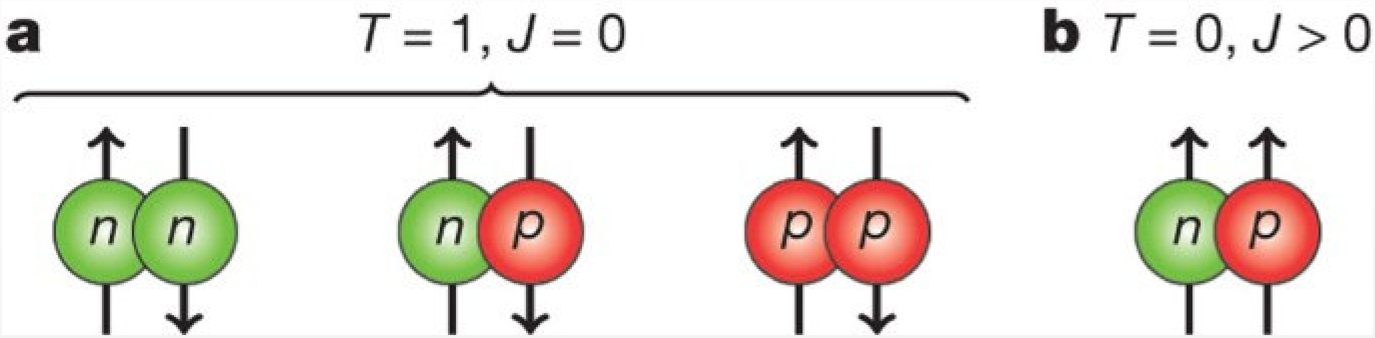
Conclusions

dfjgbjdkfg

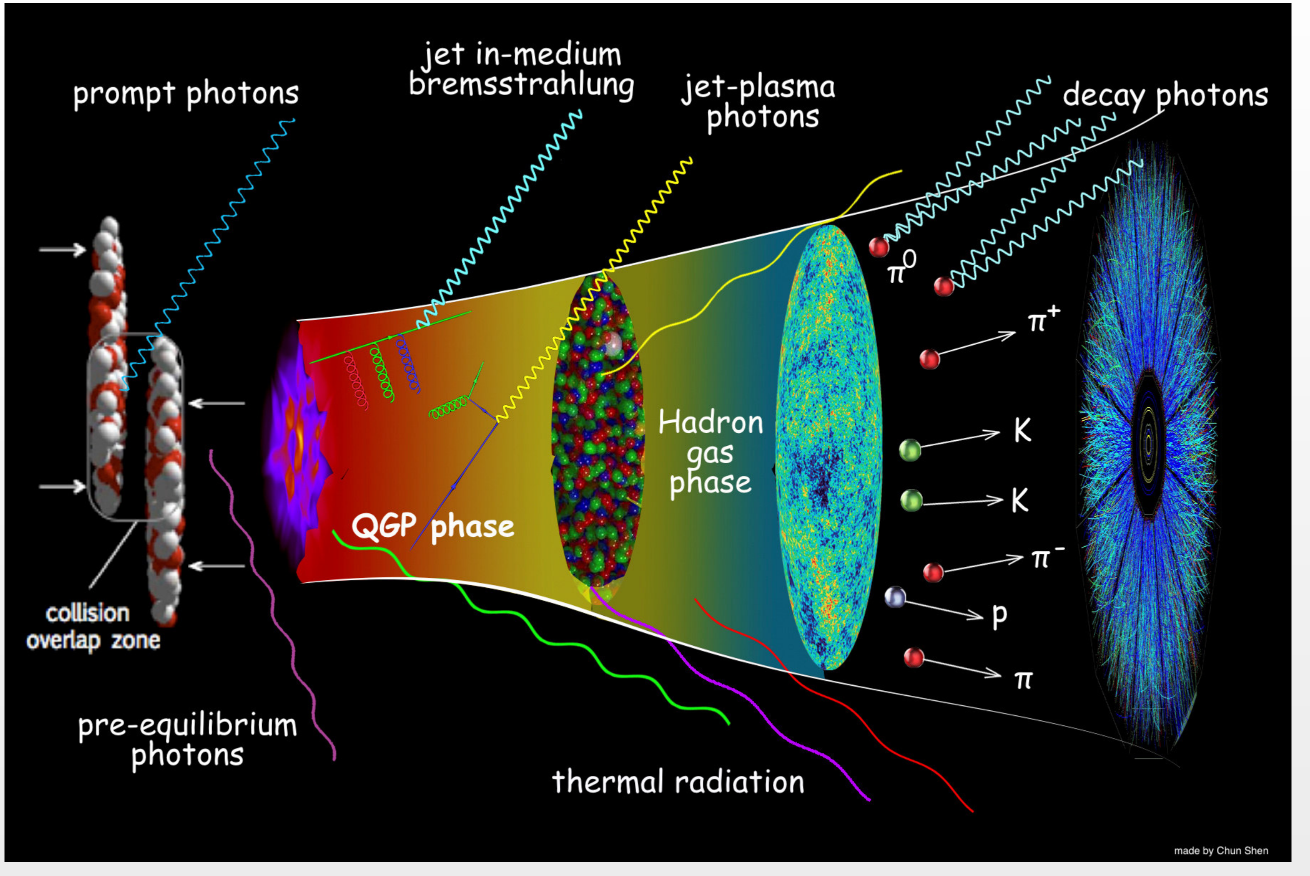
rfgsdg

raGTARG

Thank You


a
T = 1, J = 0
b
T = 0, J > 0
n
n
n
p
p
p
n
p

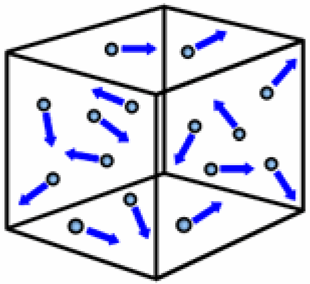
Heavy-ion collisions



At the freeze-out, the emission of hadrons is well described by thermal models.
Question: how to include resonances, such as the rho meson?
Does the $f_0(500)$ and his brother, the light k, play a role?

Theoretical description of a thermal gas

$$\ln Z = \sum_k \ln Z_k^{\text{stable}} + \boxed{\sum_k \ln Z_k^{\text{res}}}$$



The stable part is "easy":

$$\ln Z_k^{\text{stable,}} = f_k V \int \frac{d^3p}{(2\pi)^3} \ln\left[1 \pm e^{-E_p/T}\right]^{\pm 1}$$

$$E_p = \sqrt{\vec{p}^2 + M_k^2}$$

Yet, how to threat the unstable states (the resonances) ?

QM derivation of the phase-shift formula

As a last step, we discuss a simple way based on Quantum Mechanics. The radial wave function with angular momentum l of a particle scattered by central potential $U(r)$ is

$$\psi_l(r) \propto \sin[kr - l\pi/2 + \delta_l] \ , \tag{17}$$

where $k = \left|\vec{k}\right|$ is the length of the three-momentum, and δ_l is the phase shift due the interaction with the potential. If we confine our system into a sphere of radius R, the condition $kR - l\pi/2 + \delta_l = n\pi$ with $n = 0, 1, 2, ...$must be met, since $\psi_l(r)$ has to vanish at the boundary. Conversely, the number of states n_0 that one can have by limiting k in the range $(0, k_0)$ is given by $n_0 = (k_0 R - l\pi/2 + \delta_l)/\pi$. Then, the density of state that one can place between k and $k + dk$ is given by

$$\frac{dn_l}{dk} = \frac{R}{\pi} + \frac{1}{\pi}\frac{d\delta_l}{dk} \ , \tag{18}$$

where the first term describes the density of states $\frac{dn_l^{free}}{dk}$ in absence of interactions, while the second term $\frac{1}{\pi}\frac{d\delta_l}{dk}$ describes the effect of the interacting potential. When translating the discussion from Quantum Mechanics to Quantum Field Theory, we replace the momentum k with the invariant mass M, the angular momentum l with the pair (I, J). Upon summing over the latter, one obtains the full density of states of an interacting pion gas as

$$\frac{dn}{dM} = \delta(M - M_\pi) + \sum_{I,J} \frac{1}{\pi} \frac{d\delta_{(I,J)}(M)}{dM} \ . \tag{19}$$