

# QGP physics from attractor perturbations

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# Motivation

# QGP evolution starts far from equilibrium

- Characteristics of heavy-ion collisions:

Speed  $\sim 1$  fast

Energy  $\sim 10 - 10^4$  GeV high

Collision time  $\sim 0.01 - 1$  fm short

Size  $\sim 10$  fm small

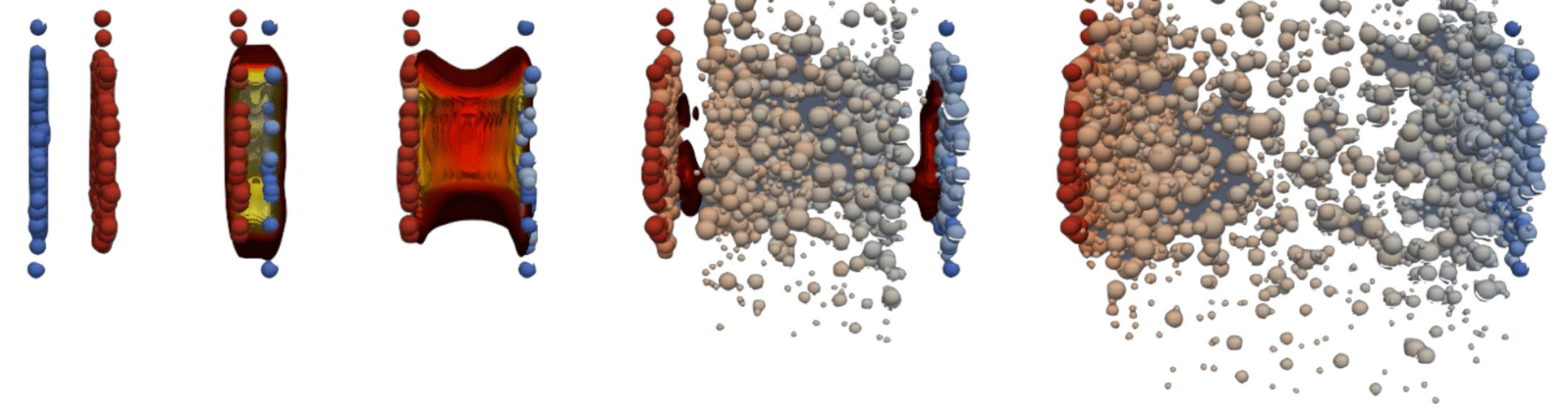
Particles  $\sim 10^2 - 10^4$  few

initial stage

QGP

hadronic

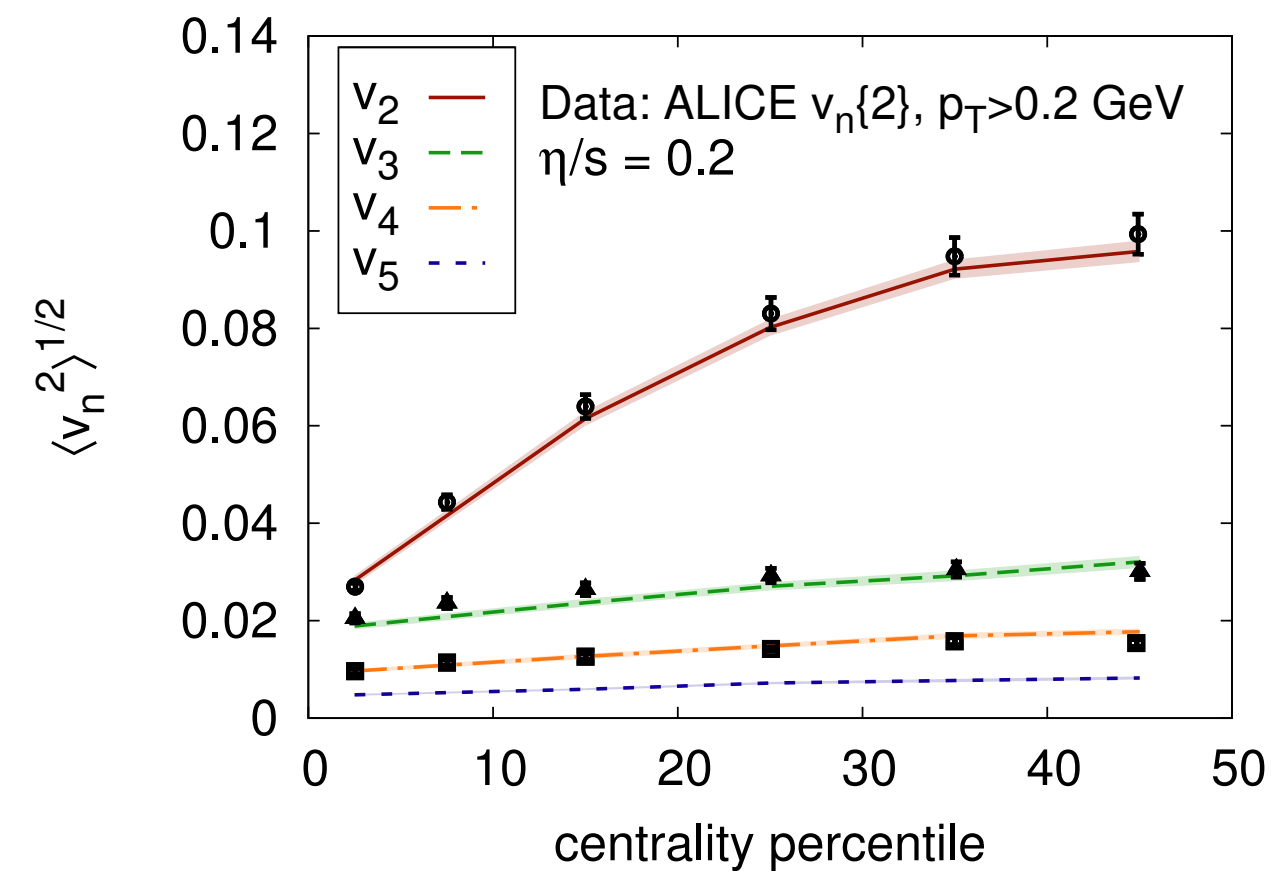
freezeout



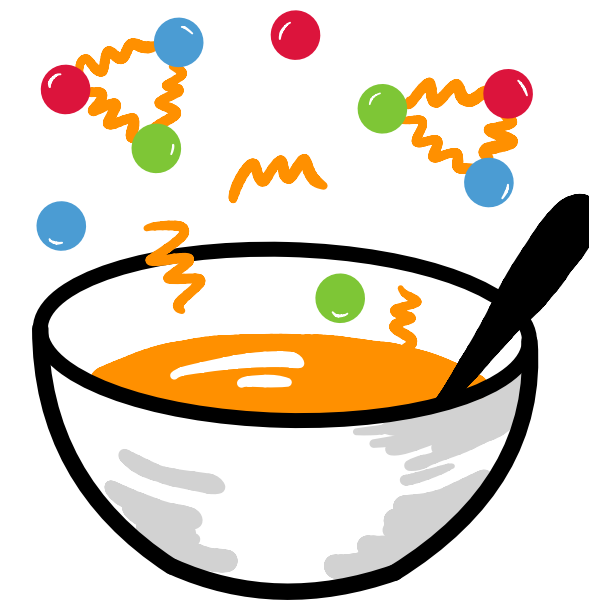
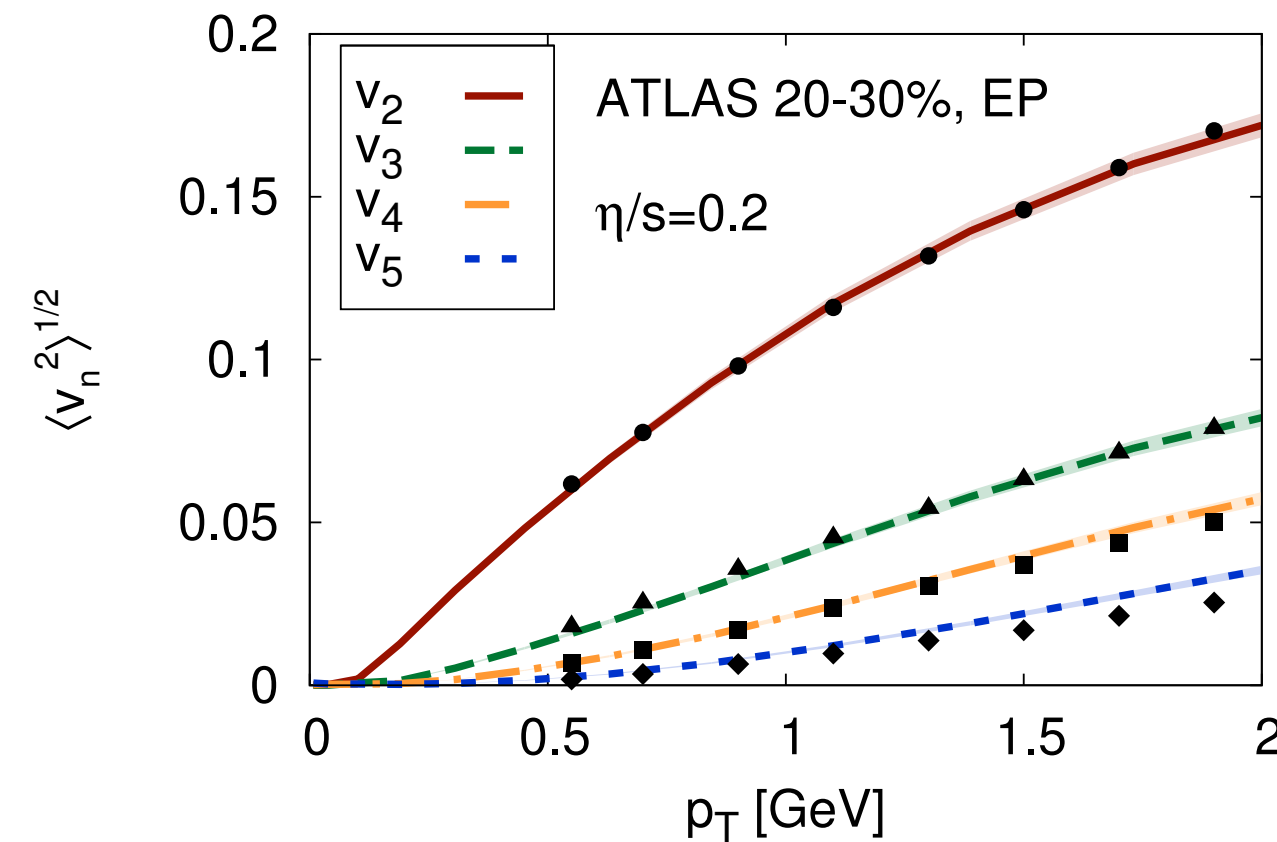
History of the little bang

# QGP is well described by hydrodynamics

- Flow collectivity manifests QGP as a *nearly perfect fluid*.

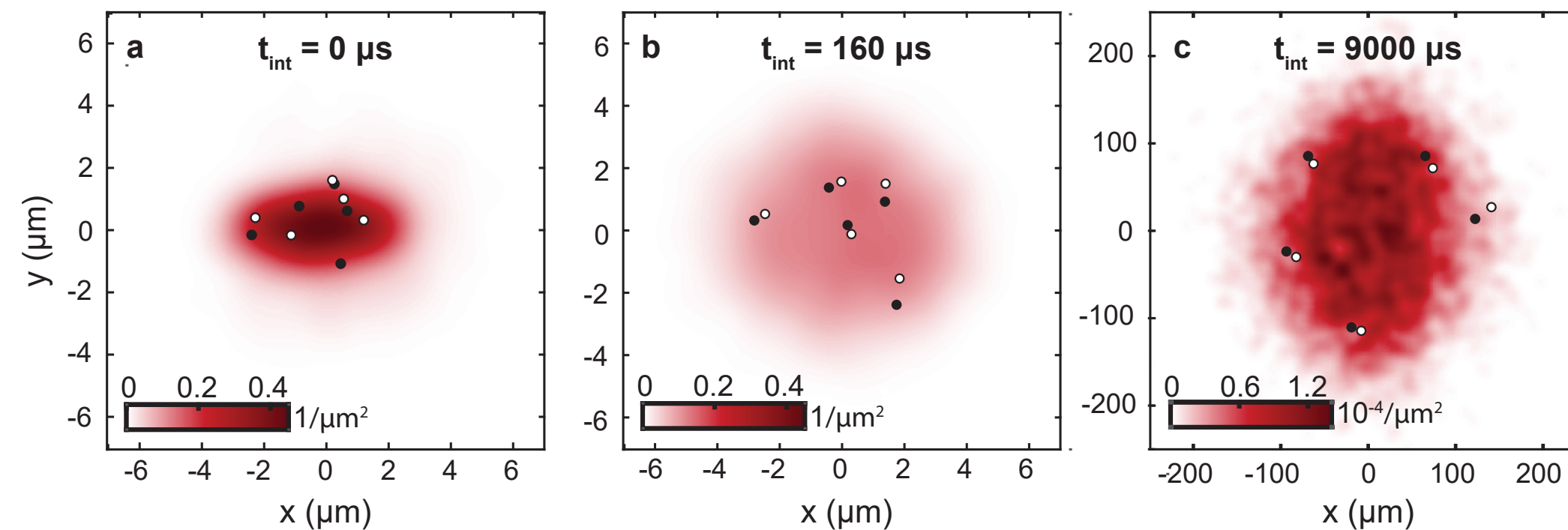


Gale et al, 1301.5893



Hydrodynamics is believed to be applicable near equilibrium

- And even more:



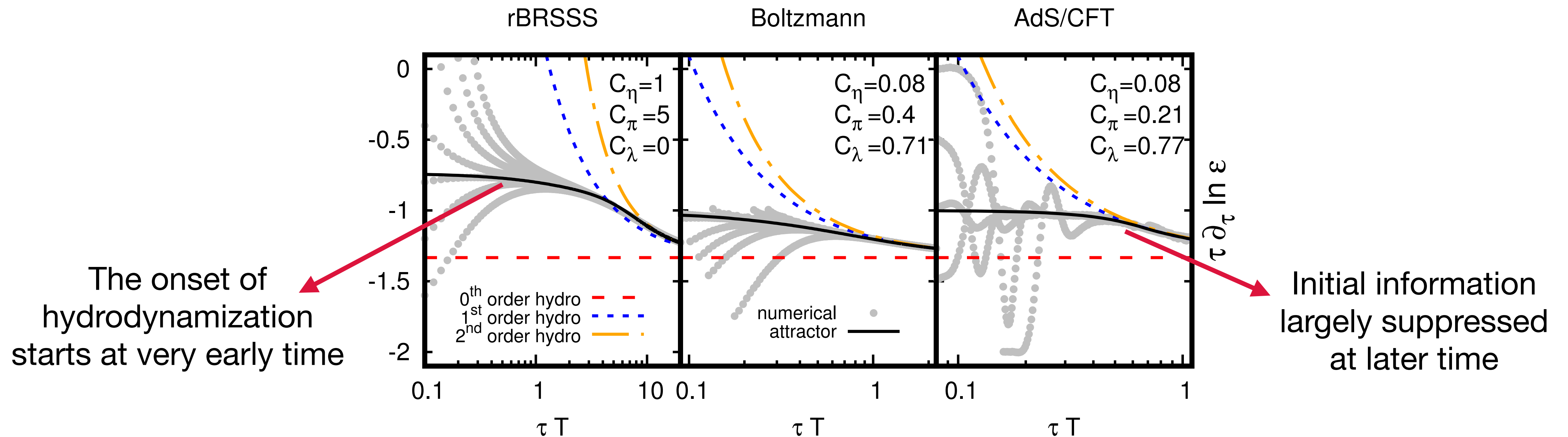
Density distribution in position space

5-particle  
hydrodynamics  
Brandstetter et al, 2308.09699



# Hydrodynamic attractor

- *Attractor* plays an important role to explain the success of hydrodynamics even far from equilibrium.



Florkowski et al, 1707.02282, Romatschke, 1712.05815

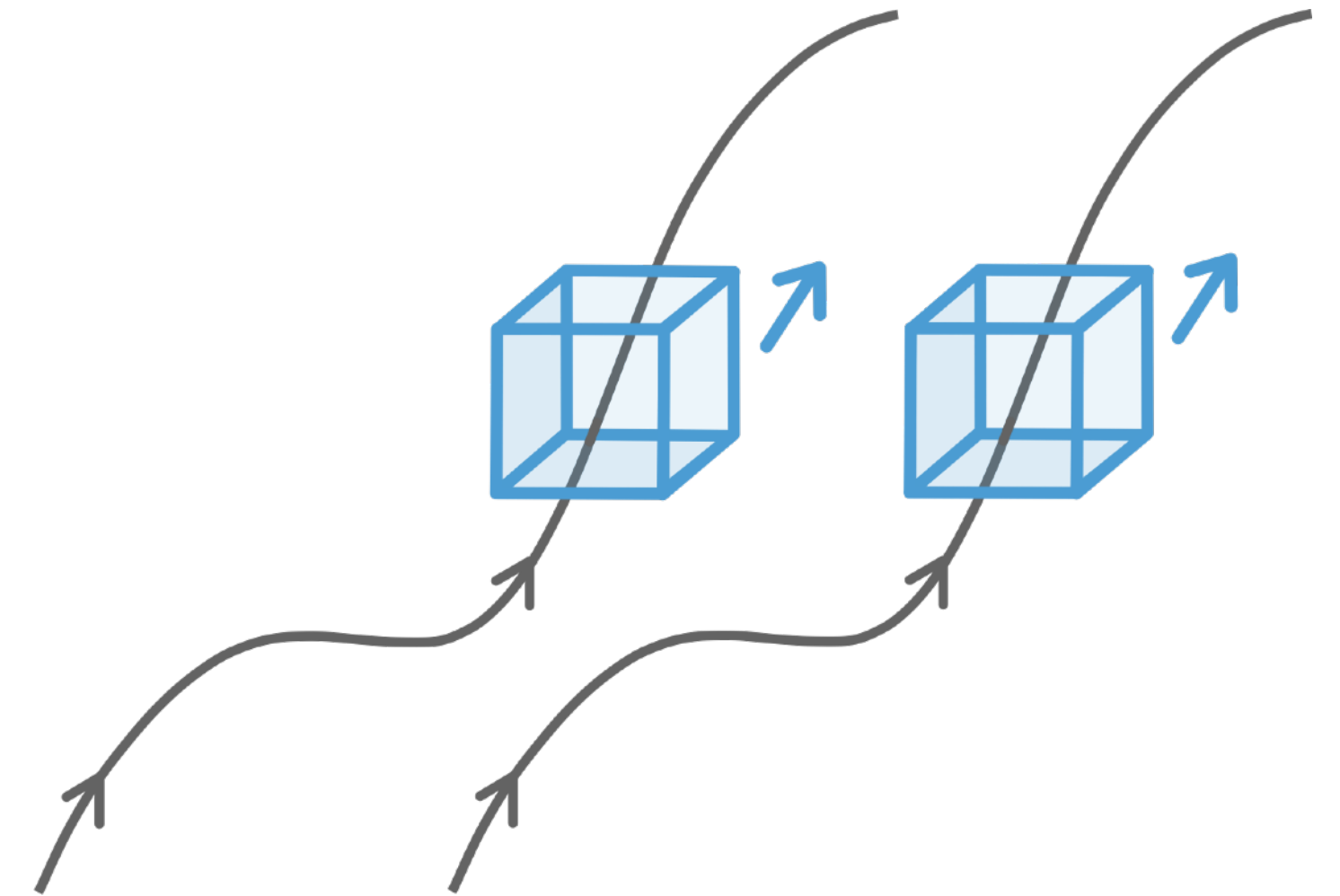
- Does attractor wash out everything? Does attractor exist with less symmetries? Can we understand it better analytically?

# Attractors

# Fluids in equilibrium: Euler equation

- Stress tensor is homogeneous in LRF.

$$T_{(0)\text{LRF}}^{\mu\nu} = \begin{pmatrix} \varepsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \xrightarrow{\text{boost}} T_{(0)}^{\mu\nu} = \varepsilon u^\mu u^\nu + p \Delta^{\mu\nu}$$



- Euler equation:

$$\partial_\mu T_{(0)}^{\mu\nu} = 0 \quad \implies \quad \partial_t \psi = \nabla \cdot J_{(0)}[\psi] \quad \text{where} \quad \psi = (n, \varepsilon, \pi, \dots)$$

Conserved quantities evolve via advection & expansion.

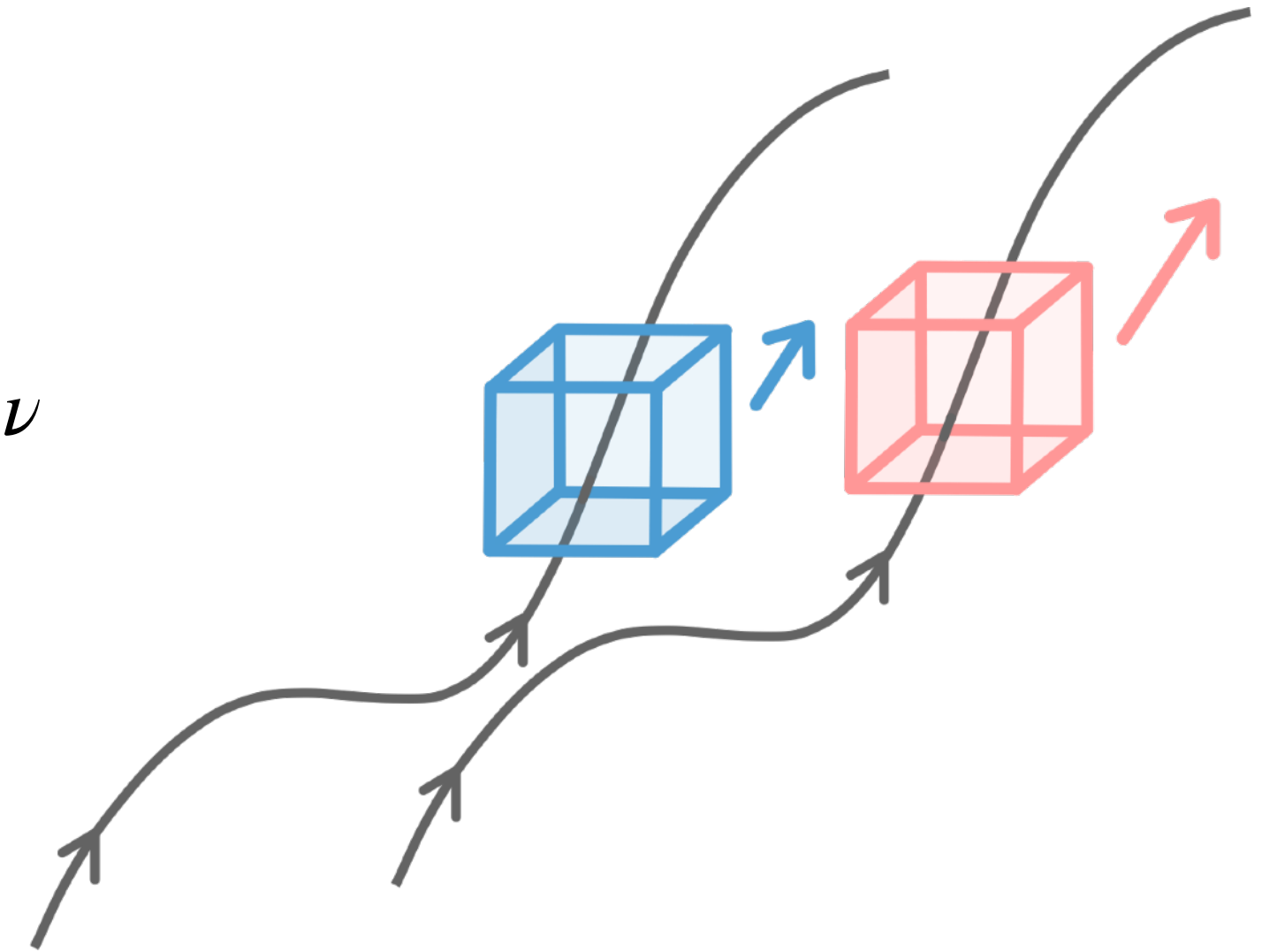
# Fluids near equilibrium: NS-like equations

- Stress tensor approximated by gradient expansion.

$$T^{\mu\nu} = T_{(0)}^{\mu\nu} + T_{(1)}^{\mu\nu} + \dots$$

$$T_{(1)}^{\mu\nu} = -2\eta\sigma^{\mu\nu}, \quad \sigma^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \frac{1}{3}\partial \cdot u \Delta^{\mu\nu}$$

NB: there are infinite many equilibrium proxies for a non-equilibrium state.



- Navier-Stokes(NS)-like (e.g., Burnett, BRSSS, etc.) equations:

$$\partial_\mu T^{\mu\nu} = 0 \quad \implies \quad \partial_t \psi = \nabla \cdot J[\psi, \nabla \psi, \dots] \quad \text{where} \quad \psi = (n, \varepsilon, \pi, \dots)$$

Conserved quantities evolve via advection & expansion, as well as dissipation & diffusion.

# Fluids far from equilibrium: MIS-like equations

- Stress tensor involves non-hydrodynamic DOFs for UV completion.  
E.g., 0+1D boost-invariant conformal fluids:

$$T^{\mu\nu} = T_{(0)}^{\mu\nu} + \pi^{\mu\nu} + \dots = \begin{pmatrix} \varepsilon & & \\ & p_T & \\ & & p_T \\ & & & p_L \end{pmatrix} \quad \begin{aligned} p_T &= p + \pi_T = p - \pi_\eta^\eta/2, \\ p_L &= p + \pi_\eta^\eta \end{aligned}$$

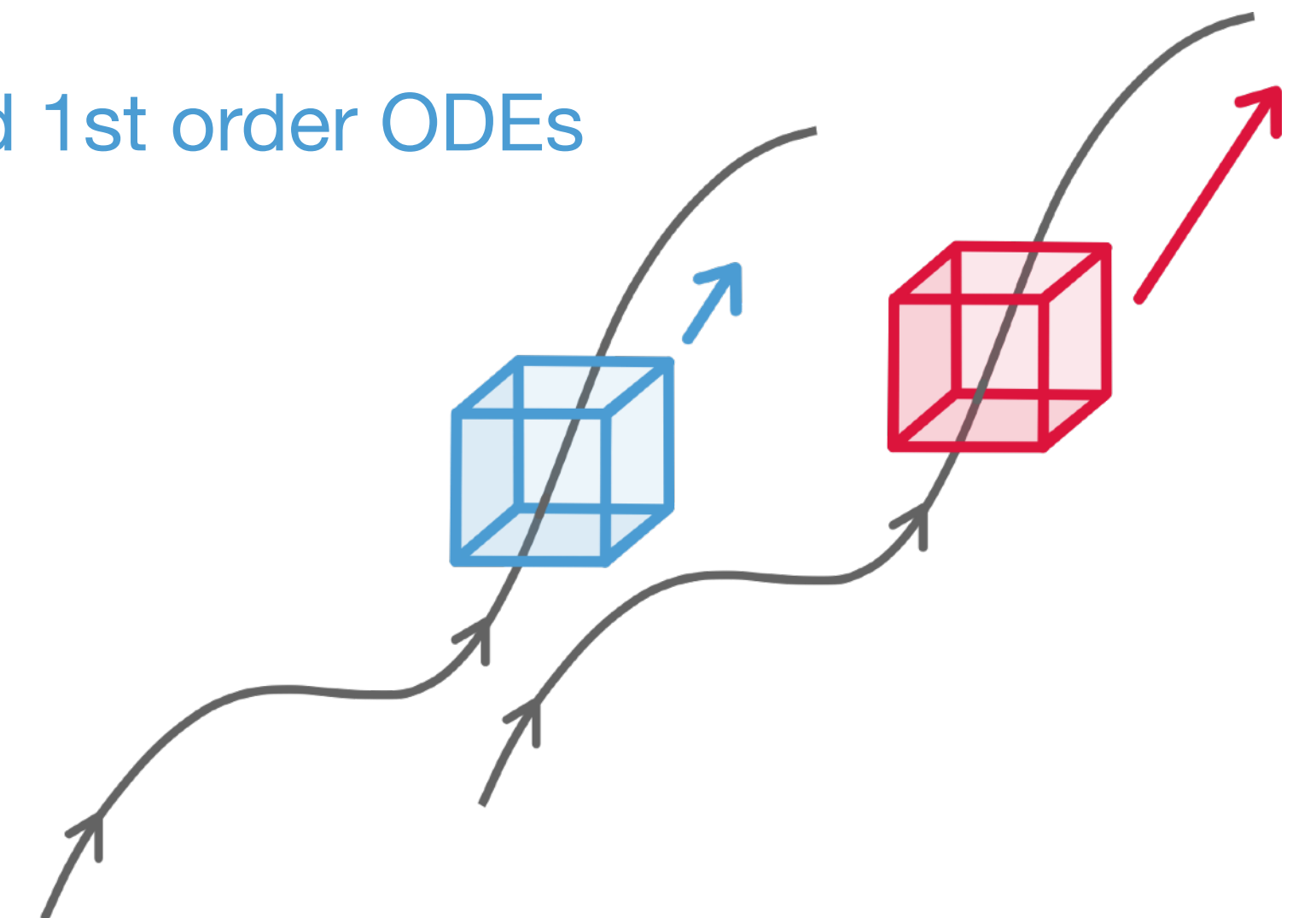
NB:  $\pi_\eta^\eta$  vanishes in equilibrium

- Muller-Israel-Stewart(MIS)-like (e.g., Maxwell-Cattaneo, DNMR, BDNK etc.) equations:

$$\partial_\mu T^{\mu\nu} = 0 \quad \implies \quad (\tau \partial_\tau + 1) \varepsilon + p + \pi_\eta^\eta = 0 \quad \text{coupled 1st order ODEs}$$

$$\text{MIS} \quad \left( \tau_\pi \partial_\tau + 1 + \frac{4\tau_\pi}{3\tau} \right) \pi_\eta^\eta + \frac{4\eta}{3\tau} = 0$$

$$\text{where } \varepsilon = 3p = C_e T^4, \eta = \frac{4}{3} C_e C_\eta T^3, \tau_\pi = C_\tau T^{-1}.$$



# Hydrodynamic attractors

- In terms of  $w = \tau T$ , equation for pressure anisotropy  $A(w) \equiv (P_T - P_L)/P$  decouples:

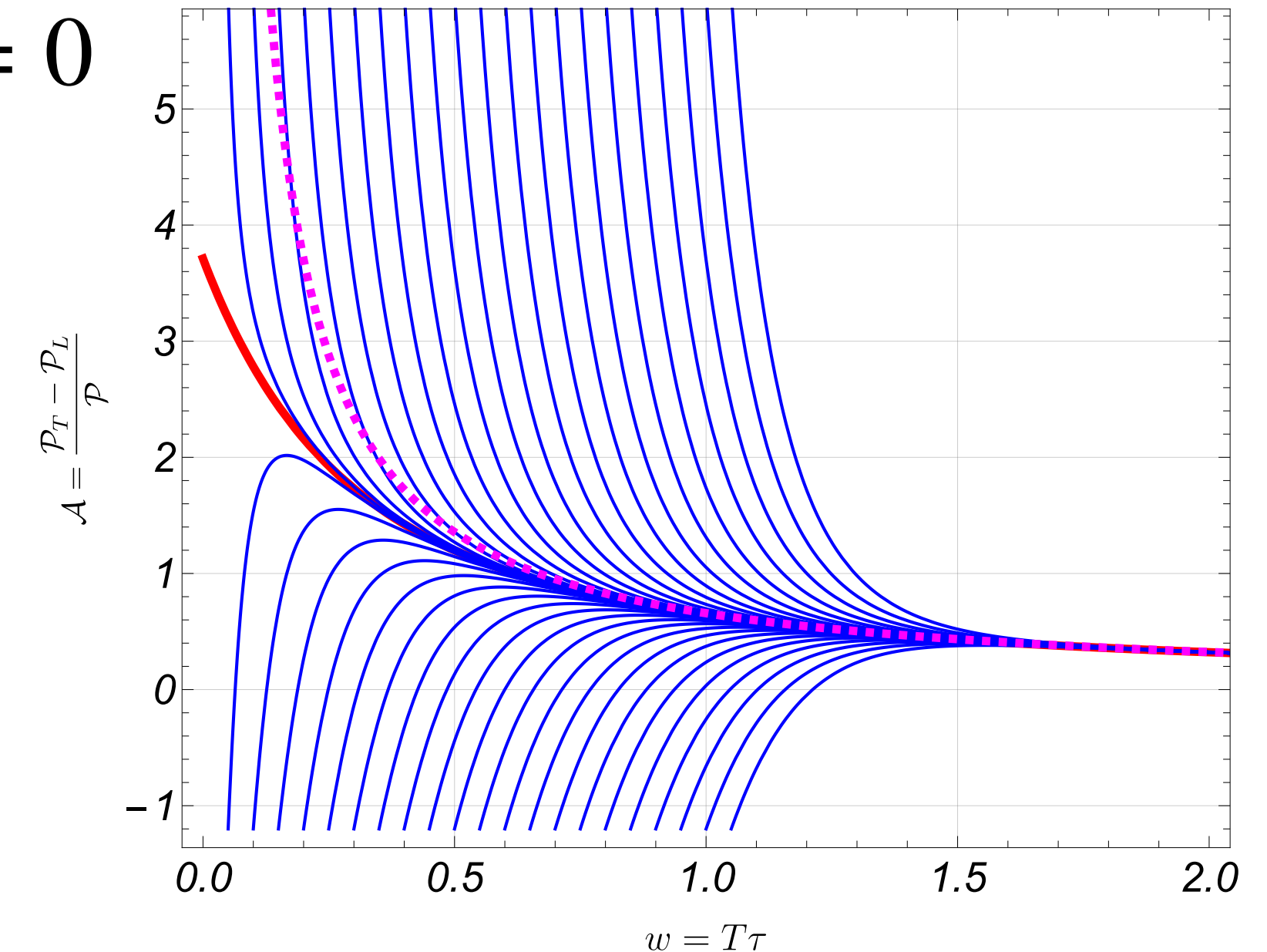
$$C_\tau \left( 1 + \frac{A(w)}{12} \right) w A'(w) + \frac{1}{3} C_\tau A(w)^2 + \frac{3}{2} w A(w) - 12 C_\eta = 0$$

*decoupled 1st order nonlinear & inhomogeneous ODE*

with asymptotic solutions

$$A(w) = \frac{C_0}{w^4} (1 + \mathcal{O}(w)) + 6\sqrt{C_\eta/C_\tau} + \mathcal{O}(w), \quad w \rightarrow 0$$

*longitudinal expansion dominates* + *early-time attractor*



Heller et al, 1503.07514; Jankowski et al, 2303.09414

$$A(w) = \frac{8C_\eta}{w} \left( 1 + \frac{2C_\tau}{3w} + \mathcal{O}(w^{-2}) \right) + C_\infty e^{-\frac{3w}{2C_\tau}} w^{\frac{C_\eta}{C_\tau}} (1 + \mathcal{O}(w^{-1})) + \dots, \quad w \rightarrow \infty$$

*hydrodynamic (late-time) attractor* + *non-hydrodynamic (transseries) modes*.



# Alternative formulation of attractors

- In the presence of additional scales other than  $T$ ,  $\tau$  is more convenient as dynamic variable than  $w = \tau T$ .

$$\tau T'(\tau) + T(\tau) \left( \frac{1}{3} - \frac{A(\tau)}{18} \right) = 0$$

two coupled 1st order ODEs  
(one 2nd order ODE)

$$C_\tau \tau A'(\tau) + \frac{2}{9} C_\tau A(\tau)^2 + \tau T(\tau) A(\tau) - 8 C_\eta = 0$$

- System of  $n$  coupled linear ODEs  $\longrightarrow$  one  $n$ th order ODE:

$$\tau T''(\tau) + \frac{3\tau T'(\tau)^2}{T(\tau)} + \left( \frac{11}{3} + \frac{\tau T(\tau)}{C_\tau} \right) T'(\tau) + \frac{T(\tau)^2}{3C_\tau} + \frac{4}{9\tau} \left( 1 - \frac{C_\eta}{C_\tau} \right) T(\tau) = 0$$

similar equation can be obtained for  $A(\tau)$

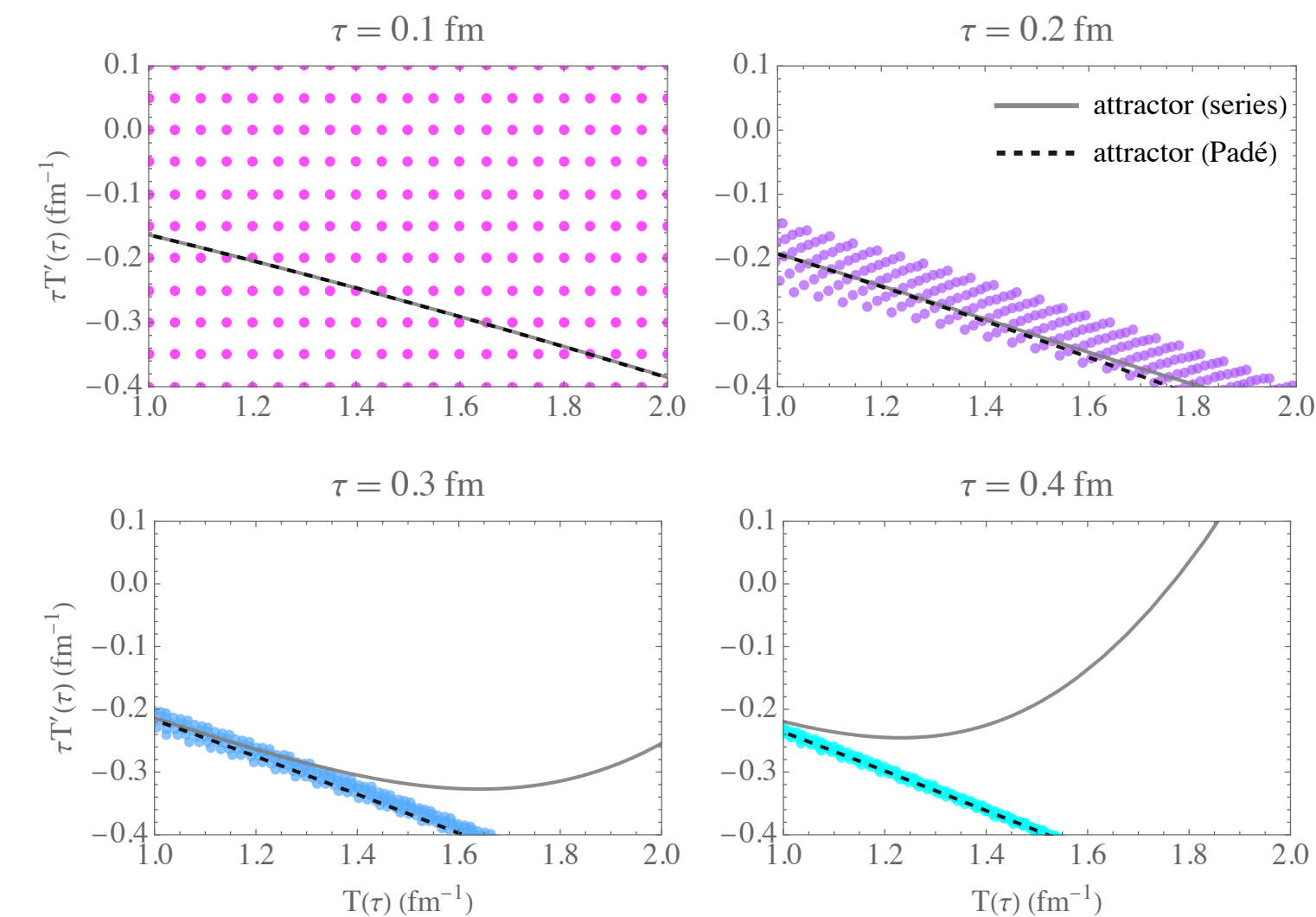
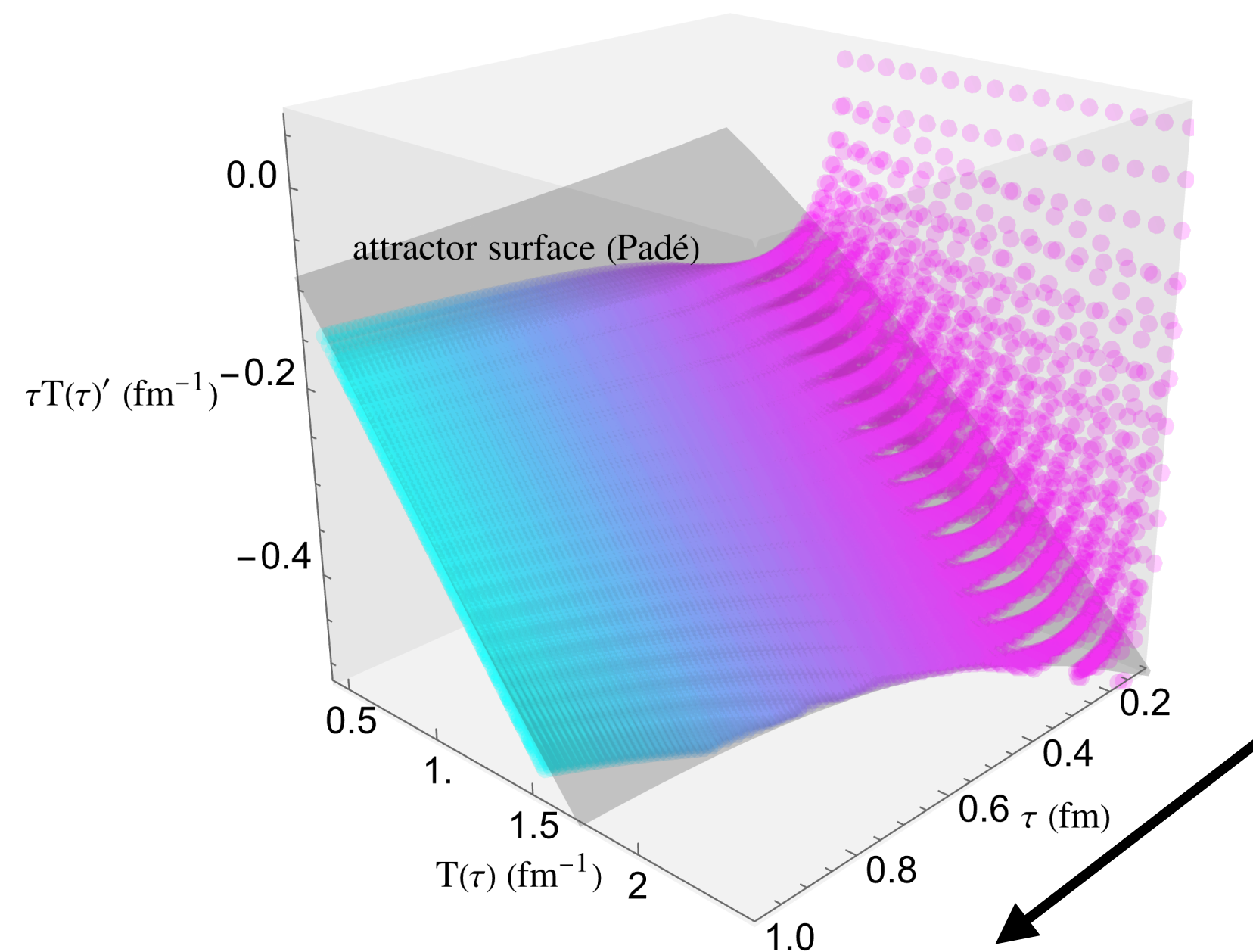
# Early-time attractor

- Early-time *attractor* solutions:

$\mu$ : integration constant;  $\alpha = \sqrt{C_\eta/C_\tau}$

$$T(\tau) \sim \mu(\mu\tau)^{-\frac{1-\alpha}{3}} \left( 1 + \sum_{n=1}^{\infty} t_n^{(0)} (\mu\tau)^{\frac{n}{3}(2+\alpha)} \right), \quad A(\tau) \sim 6\alpha \left( 1 + \sum_{n=1}^{\infty} a_n^{(0)} (\mu\tau)^{\frac{n}{3}(2+\alpha)} \right)$$

- Generic solutions rapidly approach the *attractor surface* in phase space  $(\tau T', T, \tau)$  at early time.



snapshot of  $(\tau T', T)$  plane at different  $\tau$

# Later-time attractor

- Later-time asymptotic solutions

$$T(\tau) \sim \Lambda(\Lambda\tau)^{-\frac{1}{3}} \left( 1 + \sum_{n=1}^{\infty} t_n^{(\infty)} (\Lambda\tau)^{-\frac{2}{3}n} \right) + C_{\infty} (\Lambda\tau)^{-\frac{2}{3}(1-\alpha^2)} e^{-\frac{3}{2C_{\tau}}(\Lambda\tau)^{2/3}} \left( 1 + \mathcal{O}((\Lambda\tau)^{-2/3}) \right) + \dots$$

$$A(\tau) \sim 8C_{\eta}(\Lambda\tau)^{-\frac{2}{3}} \left( 1 + \sum_{n=1}^{\infty} a_n^{(\infty)} (\Lambda\tau)^{-\frac{2}{3}n} \right) + C'_{\infty} (\Lambda\tau)^{-\frac{1}{3}+\alpha^2} e^{-\frac{3}{2C_{\tau}}(\Lambda\tau)^{2/3}} \left( 1 + \mathcal{O}((\Lambda\tau)^{-2/3}) \right) + \dots$$

hydrodynamic attractor + non-hydrodynamic (transseries) modes.

$\Lambda, C_{\infty}$ : independent integration constant

NB: suppression is mild since  $\Lambda\tau \nrightarrow \infty$ ; integration constants  $C_{\infty} = C_{\infty}(\tau)$ .

# Perturbations

# Linearized modes

- Linearization of MIS theory around the attractor for 6 independent fields:

$$(\delta T, \delta\theta, \delta\omega, \delta\pi_{11}, \delta\pi_{22}, \delta\pi_{12})(\tau, \mathbf{x})$$

where  $\delta\theta \equiv \partial_i \delta u_i$  and  $\delta\omega \equiv \epsilon_{ij} \partial_i \delta u_j$ ,  $i = 1, 2$ .

The translation invariance symmetry in transverse plane is broken.

- The dynamic system reduces to a set of linear 2nd order ODEs for  $\phi(\tau, \mathbf{x}) = (\delta T, \delta\theta, \delta\omega)(\tau, \mathbf{x}) \longrightarrow \hat{\phi}(\tau, \mathbf{k}) = (\delta\hat{T}, \delta\hat{\theta}, \delta\hat{\omega}) \equiv (\delta T/T, \delta\theta/k, \delta\omega/k)(\tau, \mathbf{k})$ :

$$\hat{\phi}''(\tau, \mathbf{k}) + P_1(\tau, \mathbf{k})\hat{\phi}'(\tau, \mathbf{k}) + P_0(\tau, \mathbf{k})\hat{\phi}(\tau, \mathbf{k}) = 0$$

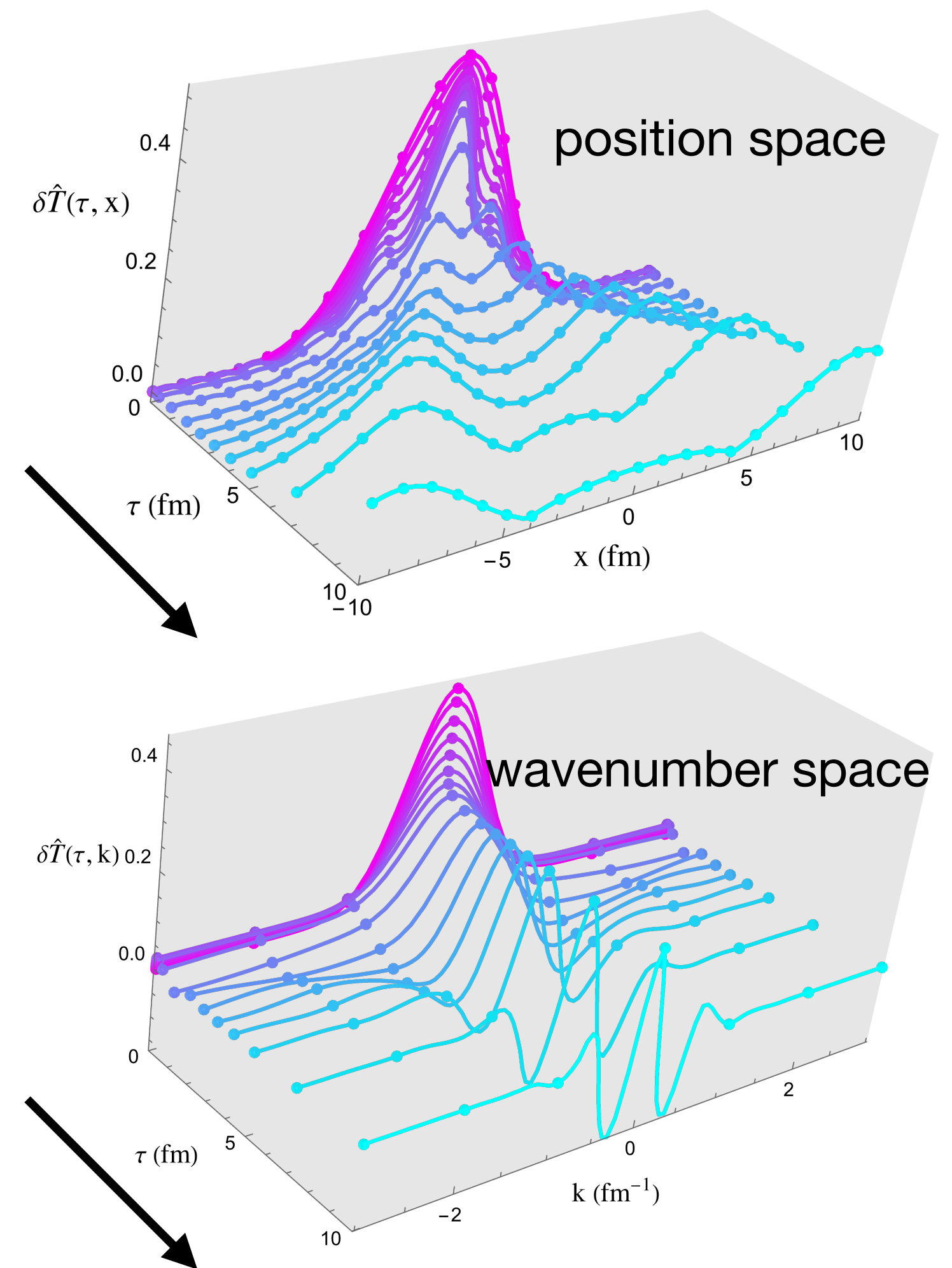
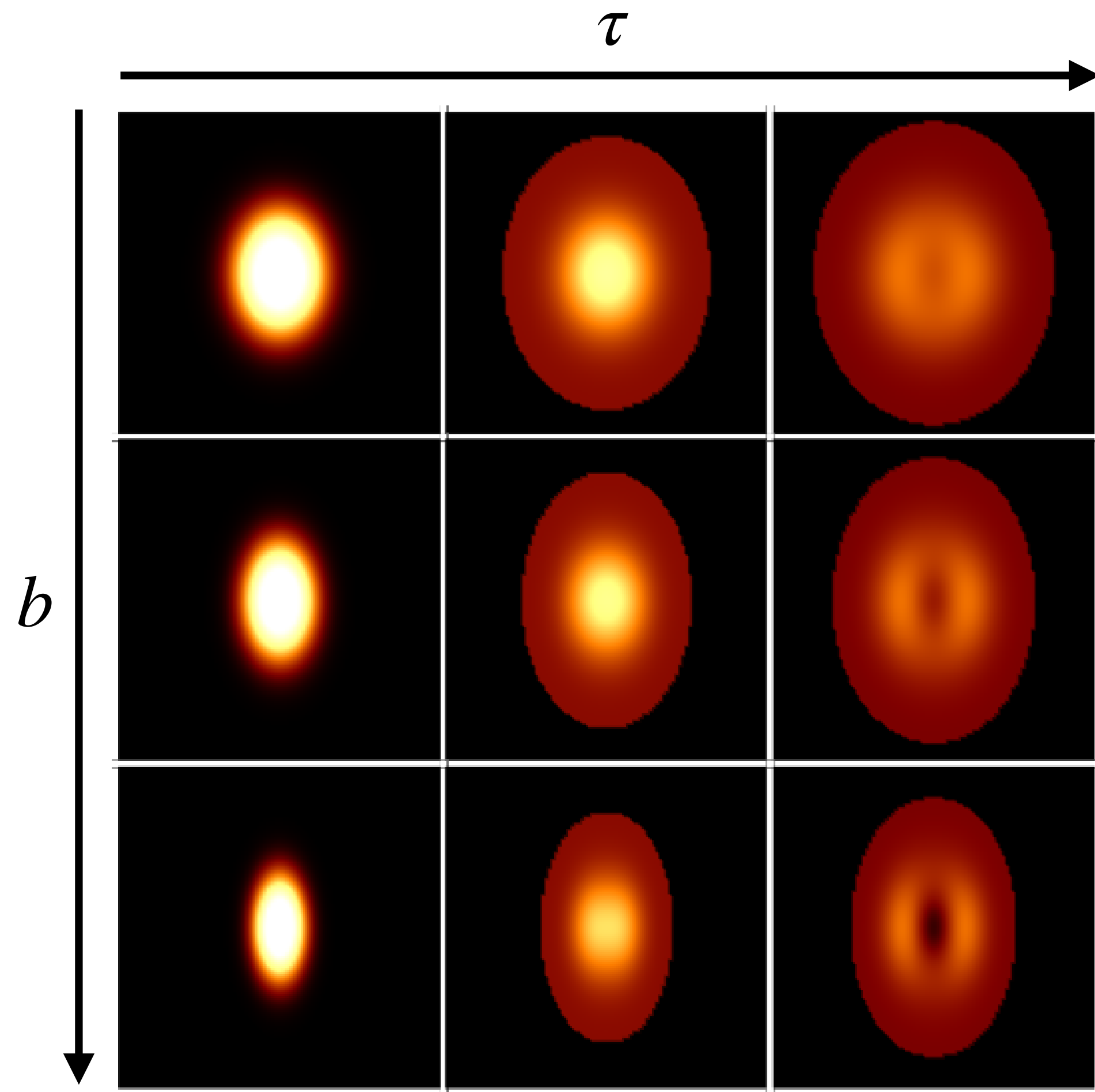
where  $P_1, P_0$  are block-diagonal-matrix coefficients.

NB: the 2nd order ODE for  $\delta\hat{\omega}$  decouples from that for  $\delta\hat{T}$  and  $\delta\hat{\theta}$ , the latter can also be converted to a single 4th order ODE.



# Transverse dependence

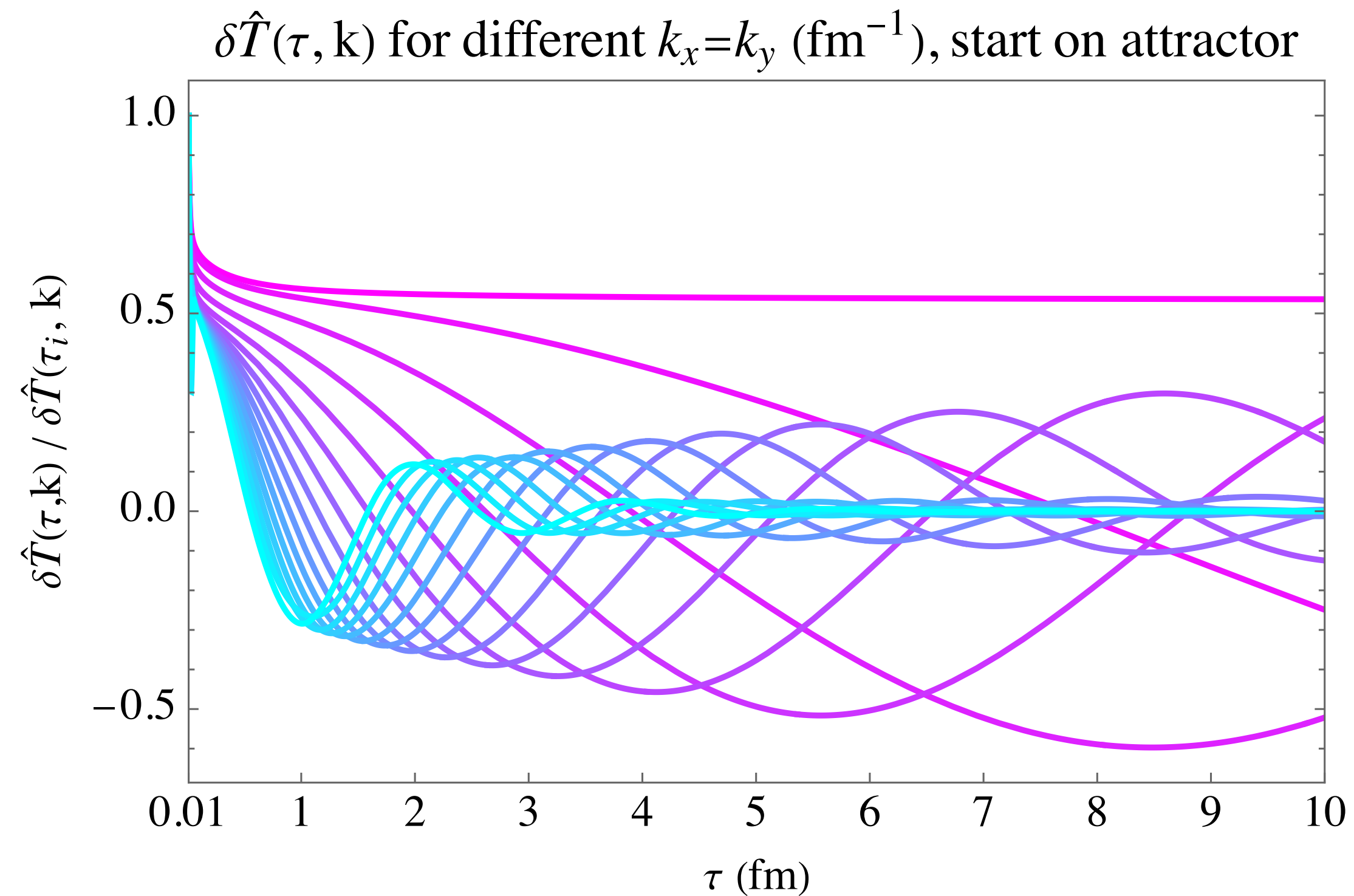
- Transverse information is encoded in a finite set of Fourier modes (FFT).



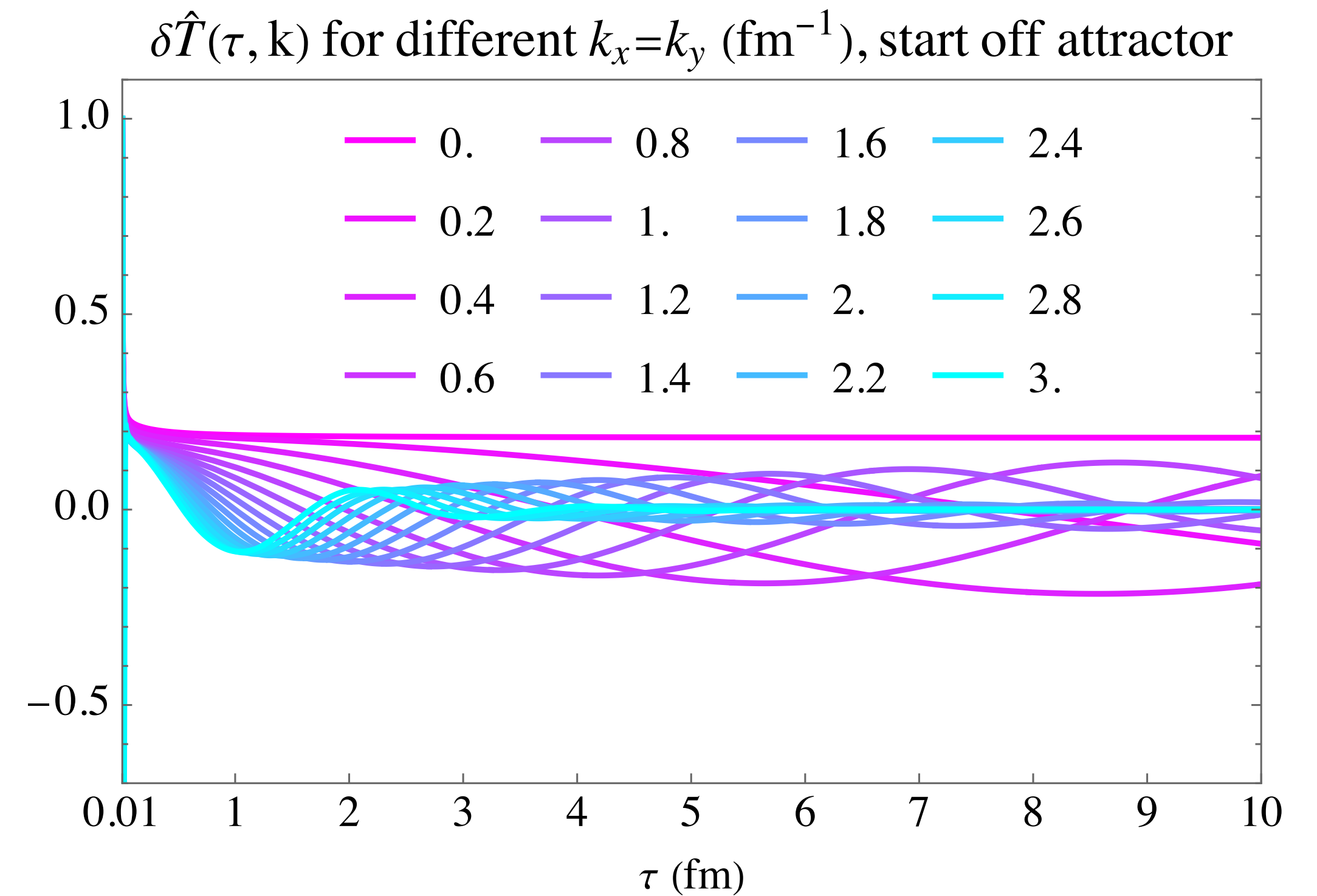


# Transverse dependence

- Suppression for large  $k$  modes and off-attractor perturbations:



large  $k$  suppression



off-attractor suppression

**Observables**

# Late-time asymptotics

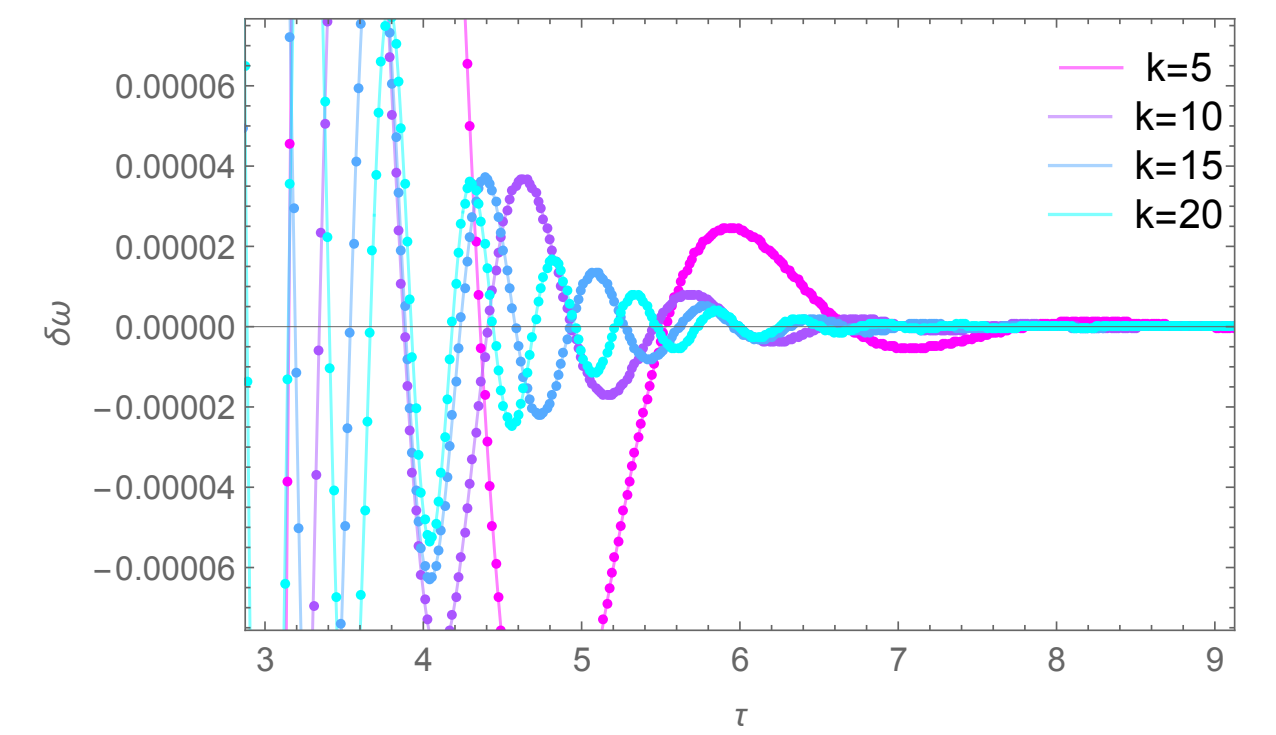
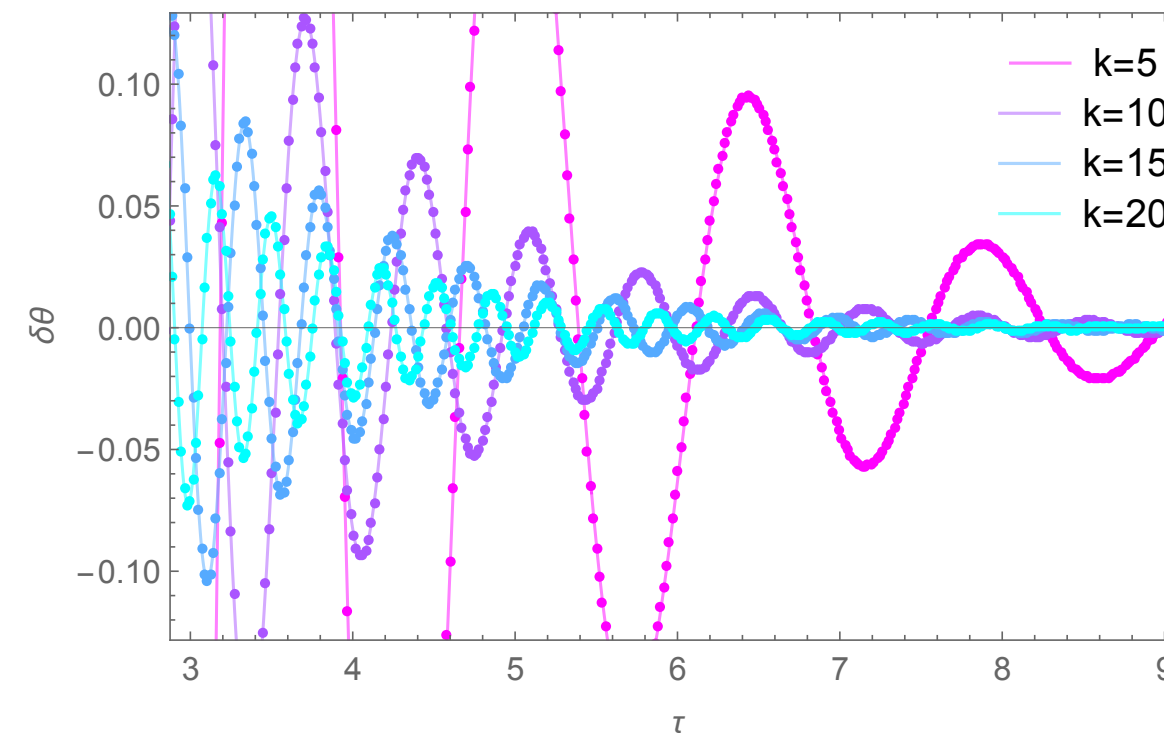
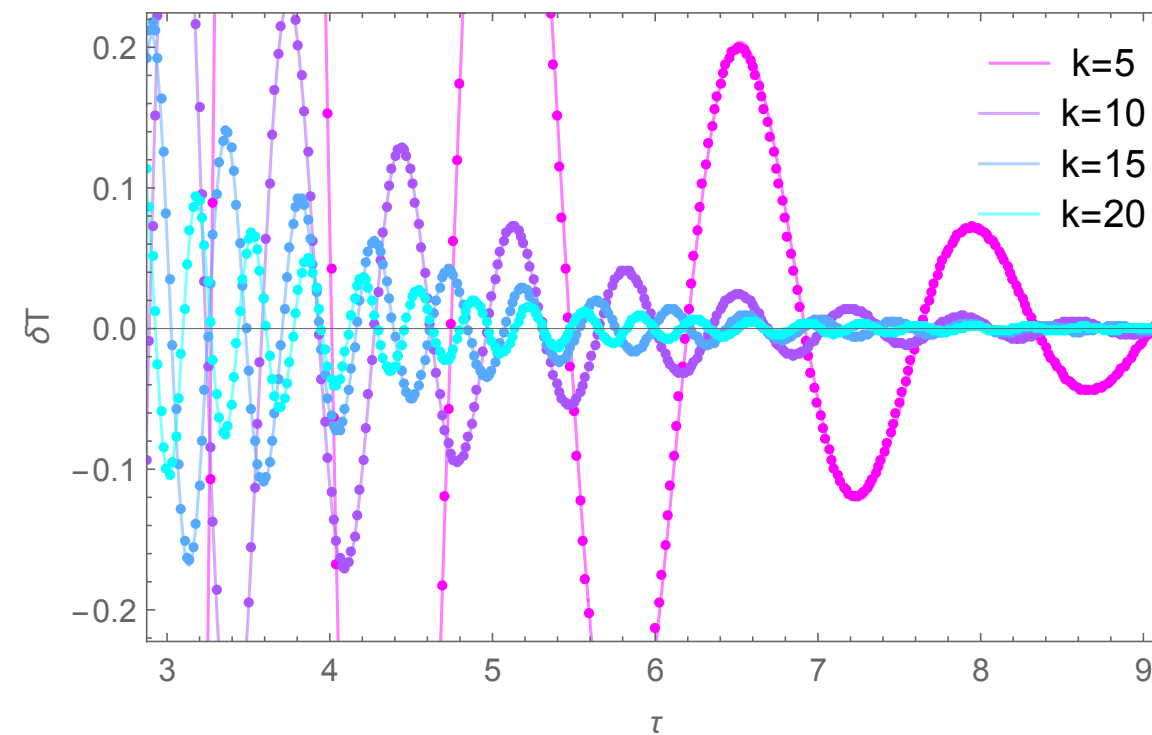
- Late-time asymptotic solutions perturbed around attractor:

$$\delta\hat{T} = C_1(\Lambda\tau)^{a_1} e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}} + C_2(\Lambda\tau)^{a_2} e^{-\frac{1}{2c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}} + e^{-\frac{\alpha^2}{c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}} \left( C_3 e^{-ic_\alpha k\tau} + C_4 e^{ic_\alpha k\tau} \right)$$

$$\delta\hat{\theta} = C'_1(\Lambda\tau)^{a_1-1} e^{-\frac{3}{2C_\tau}(\Lambda\tau)^{2/3}} + C'_2(\Lambda\tau)^{a_2-\frac{1}{3}} e^{-\frac{1}{2c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}} + e^{-\frac{\alpha^2}{c_\alpha^2 C_\tau}(\Lambda\tau)^{2/3}} \left( C'_3 e^{-ic_\alpha k\tau} + C'_4 e^{ic_\alpha k\tau} \right)$$

$$\delta\hat{\omega} = e^{-\frac{3}{4C_\tau}(\Lambda\tau)^{2/3}} \left( C_5 e^{-i\alpha k\tau} + C_6 e^{i\alpha k\tau} \right) \quad \Lambda, C_1, \dots, C_6: \text{integration constants}$$

NB: the solutions for  $\delta\hat{\pi}_{ij} \equiv \delta\pi_{ij}/T^4$  can be determined accordingly from  $\delta\hat{T}$ ,  $\delta\hat{\theta}$  and  $\delta\hat{\omega}$ .



The attractor is stable against transverse dynamics; observables extracted from the asymptotic data of  $(\delta\hat{T}, \delta\hat{\theta}, \delta\hat{\omega}, \delta\hat{\pi}_{ij})$  determined by  $(C_1, \dots, C_6)(\mathbf{k})$ .

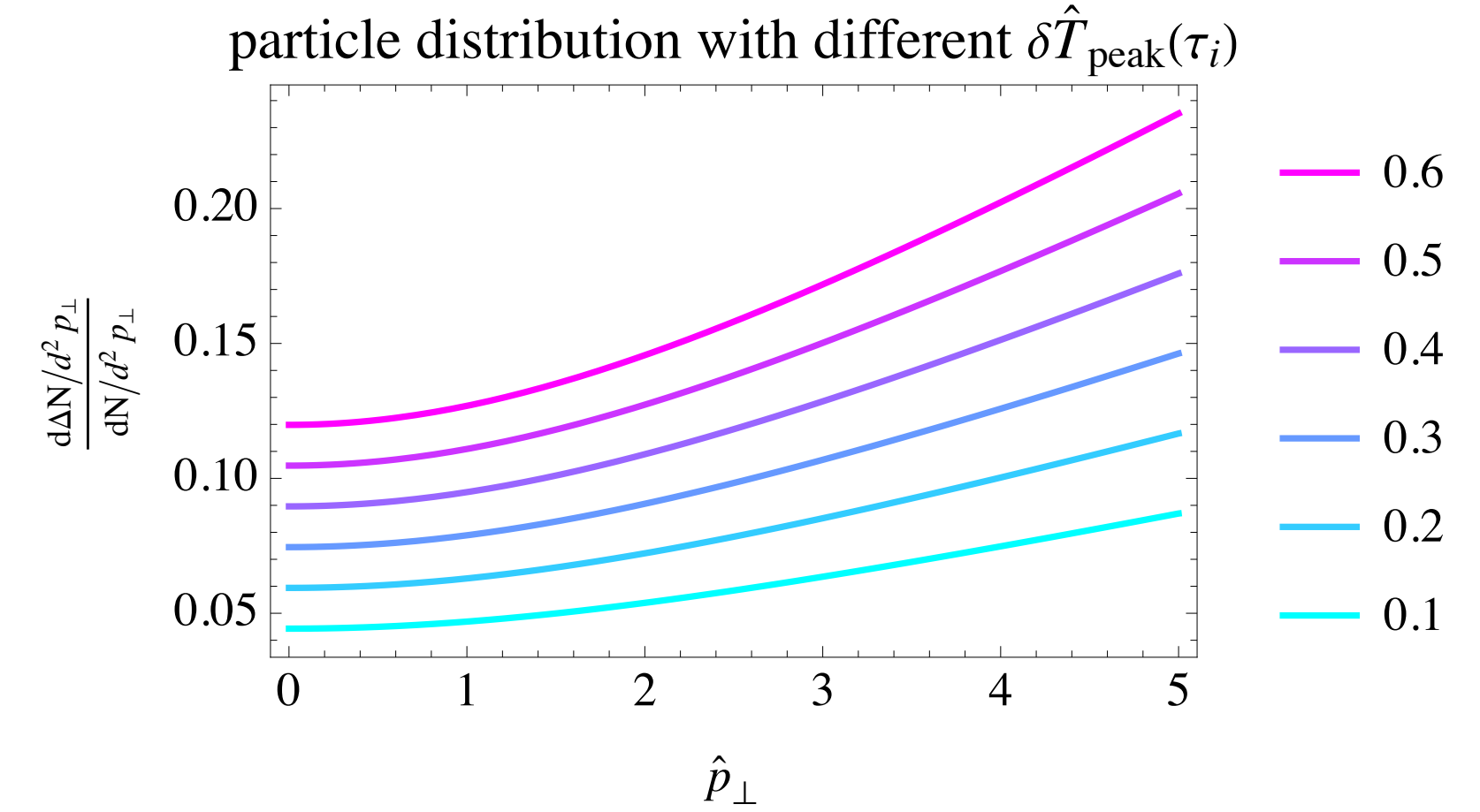
# Observables

- Linearized Cooper–Frye freezeout formula:

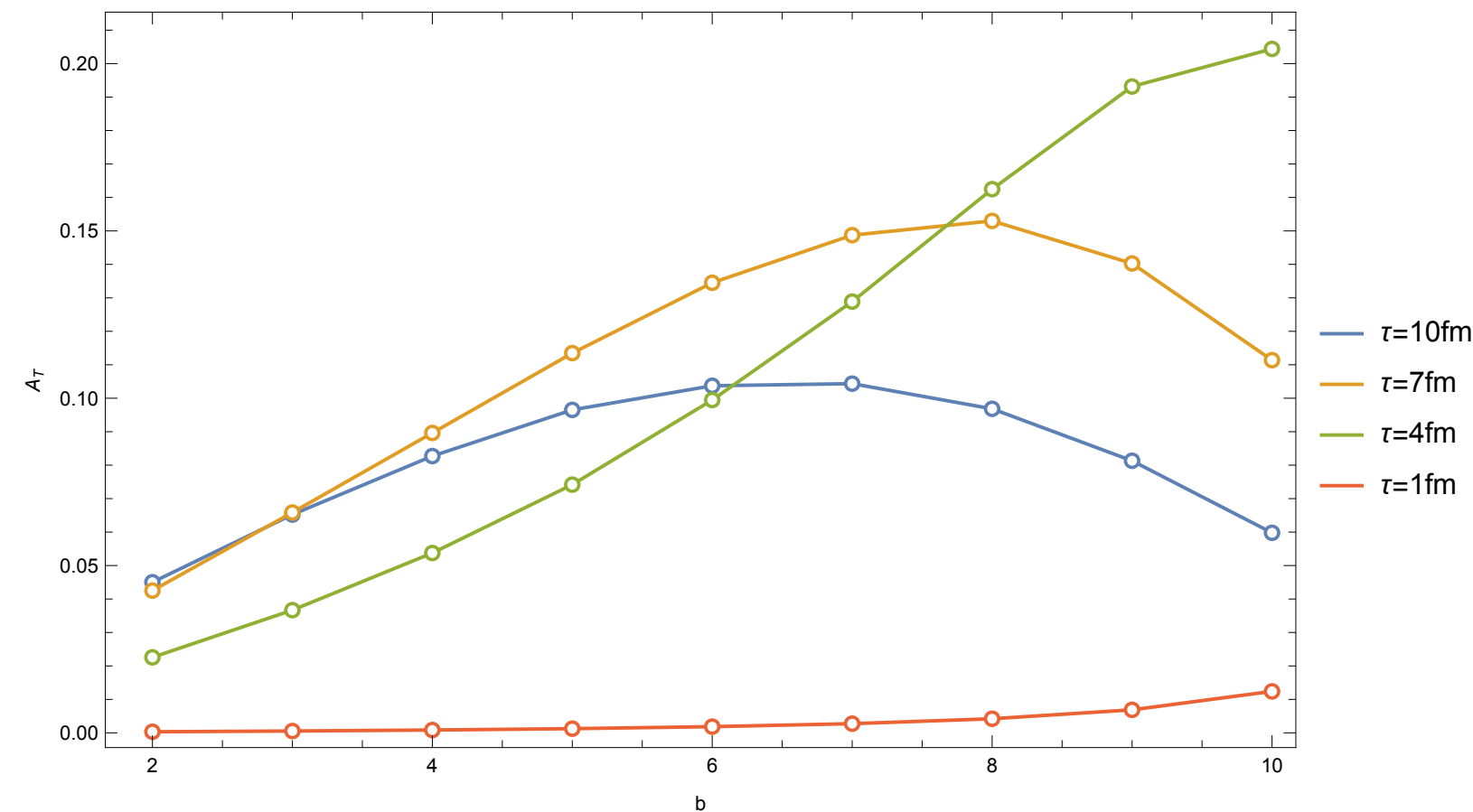
$$\frac{d\Delta N/d^2\mathbf{p}_\perp}{dN/d^2\mathbf{p}_\perp} = \left(1 + \hat{m}_\perp \frac{K_0(\hat{m}_\perp)}{K_1(\hat{m}_\perp)}\right) \langle \delta \hat{T} \rangle_\perp + \hat{\mathbf{p}}_\perp \cdot \langle \delta \mathbf{u} \rangle_\perp$$

$$\hat{m}_\perp = \frac{\sqrt{m^2 + \mathbf{p}_\perp^2}}{T}, \quad \hat{\mathbf{p}}_\perp = \frac{\mathbf{p}_\perp}{T}, \quad K_n : \text{Bessel function}$$

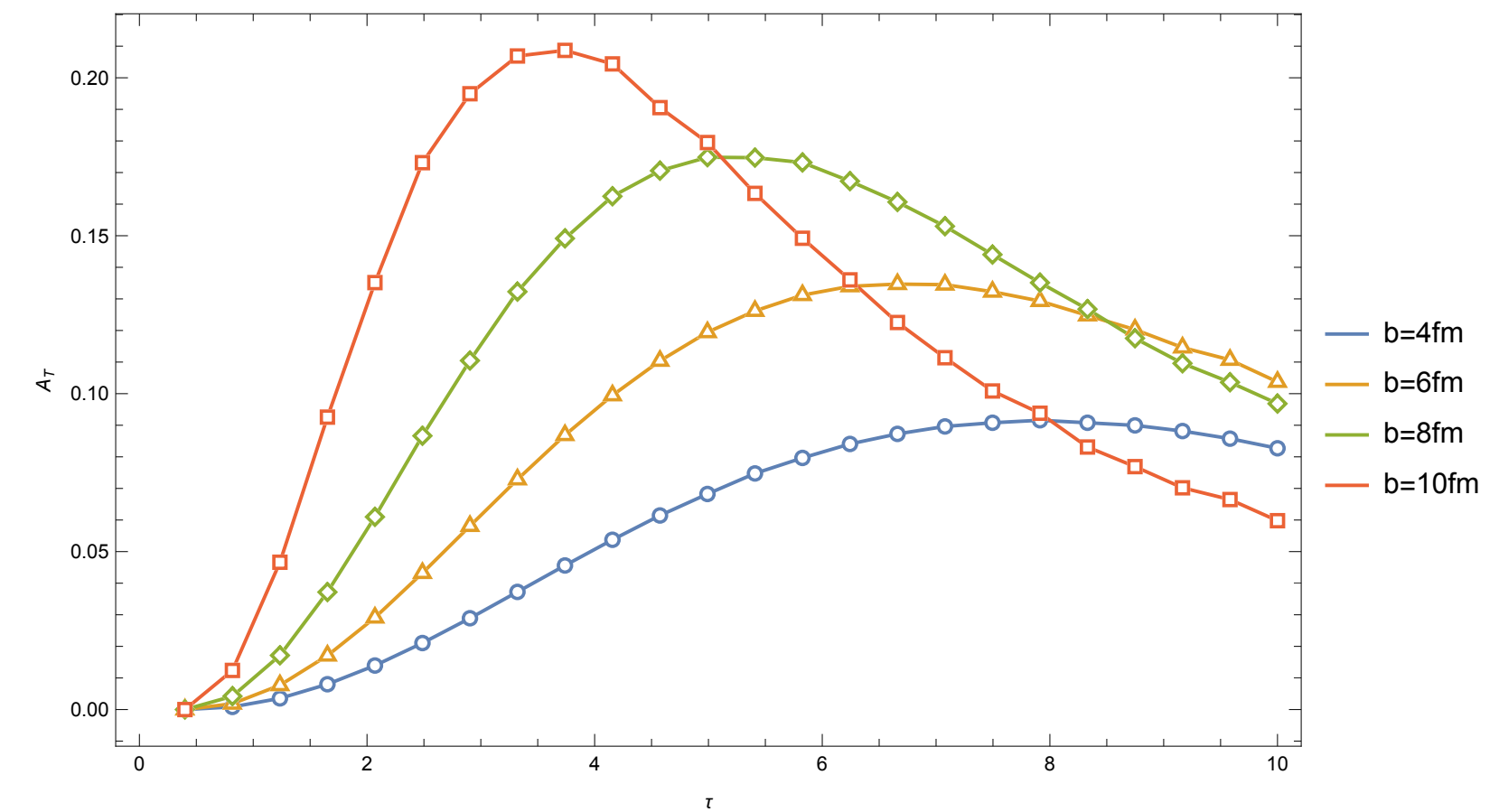
$$\parallel \delta \mathbf{u}(\delta \hat{\theta}, \delta \hat{\omega})$$



- Momentum anisotropy  $A_T \equiv \frac{\langle T_{11} - T_{22} \rangle_\perp}{\langle T_{11} + T_{22} \rangle_\perp} = \frac{12C_e \langle \delta u_1^2 - \delta u_2^2 \rangle_\perp + 9 \langle \delta \hat{\pi}_{11} - \delta \hat{\pi}_{22} \rangle_\perp}{2C_e(3 + A)}$



centrality dependence at different time



time dependence at different centralities

# Conclusion

# Recap

- Transverse dynamics can be described by perturbations around the attractor background.
- The problem reduces to a set of linear ODEs which can be analyzed semi-analytically.
- Physical observables are calculated by finite asymptotic data and FFT.

# Outlook

- Systems with lesser symmetries.
- Systems with additional scales/DOF: jets, spin polarization, and noises.
- More...