Entropy production in spin hydrodynamics

A relativistic quantum-statistical approach

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(https://online.kitp.ucsb.edu/online/relfluids23/becattini/rm/jwvideo.html)

(arXiv: 2309.05789)

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INFN Istituto Nazionale di Fisica Nucleare

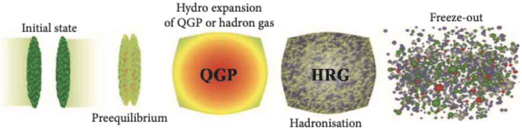
NAWA Narodowa Agencja Wymiany Akademickiej

Motivation: Why? Goal? Approach?

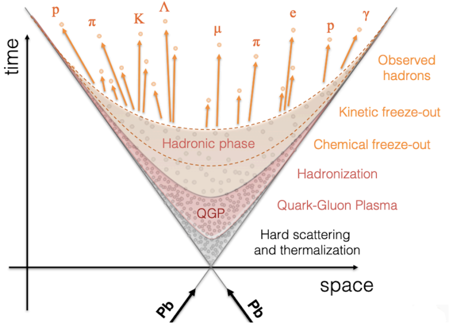
Why spin hydro?

Successes of relativistic hydrodynamics in heavy ion collisions

[W.Florkowski-Phenomenology of Ultra-relativistic Heavy-ion Collisions-World Scientific]



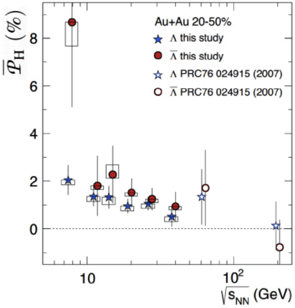
Various stages of relativistic heavy-ion collision [http://qgp.phy.duke.edu]



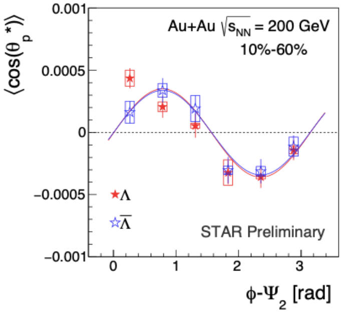
Spacetime diagram of relativistic heavy-ion collision [D.D. Chinellato]

Why spin hydro?

Growing interests in spin hydro as a potential theory to describe the polarization data



Average Λ global polarization [STAR, L. Adamczyk et al., Nature 548, 62 (2017)]



[T.Niida, NPA 982 (2019) 511514]

Why quantum-based approach?

Spin hydrodynamics stipulates that the description of a relativistic fluid requires the addition of a spin tensor, that is the mean value of a rank-3 spin tensor operator:

$$\boxed{\widehat{\mathcal{J}}^{\lambda\mu\nu} = x^{\mu}\widehat{\mathcal{T}}^{\lambda\nu} - x^{\nu}\widehat{\mathcal{T}}^{\lambda\mu} + \widehat{\mathcal{S}}^{\lambda\mu\nu}}$$

$\Longrightarrow$ Spin hydro involves “spin” and so quantum methods are hard to be ignored

Goal

One of the goals of spin hydro is to determine the dissipative corrections of the mean values in-terms of the system's 10 indep. variables: $\{T,\ u^\mu,\ S^{\mu\nu}\}$

1. Mean value energy-momentum tensor:

$$\underbrace{\boxed{\partial_\mu T^{\mu\nu} = 0}}_{\text{4 equations}} \quad \text{s.t} \quad \underbrace{\boxed{T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + T_s^{\mu\nu} + T_a^{\mu\nu}}}_{\text{4 unkowns}}$$

2. Mean value spin tensor:

$$\underbrace{\boxed{\partial_\lambda S^{\lambda\mu\nu} = -2T_a^{\mu\nu}}}_{\text{6 equations}} \quad \text{s.t} \quad \underbrace{\boxed{S^{\lambda\mu\nu} = u^\lambda S^{\mu\nu} + S_1^{\lambda\mu\nu}}}_{\text{6 unkowns}}$$

Approach

This goal (determining the dissipative corrections) can be achieved using:

"The second law of thermodynamics : Entropy production rate"

that logically should be derived or verified by a quantum description.

Approach

However, most approaches achieves the entropy production starting from an educational guess of:

"Local thermodynamic relations (**LTR**)"

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How? At homogeneous global equilibrium without spin, we know that:

$$\boxed{\epsilon + p = Ts}$$

At local equilibrium with spin, the educated guesses goes like:

$$\boxed{\varepsilon + p = Ts + \omega_{\alpha\beta} S^{\alpha\beta}}$$

$\omega_{\mu\nu}$: can be interpreted as the spin chemical potential conjugated to the spin density.

Approach

So in this talk, we're going to present in a pedagogical fashion a quantum statistical derivation (without the usage of **LTR**) of the:

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So in this talk, we're going to present in a pedagogical fashion a quantum statistical derivation (without the usage of **LTR**) of the:

1. Entropy current,
2. Entropy production rate,
3. Dissipative corrections (future work),
4. Solve the system and compare with the polarization data (future work).

Goal

Several groups are working on spin hydrodynamics/spin polarization:

- Kinetic approach (Rishkle et. al: PhysRevD.106.096014, PhysRevD.106.L091901)
- Macroscopic approach (Florkowski, Ryblewski et. al: PhysRevC.108.024902, PhysRevD.107.094022, PhysRevD.108.014024)
- Spin hydro with torsion (Hongo et. al. JHEP11(2021)150, Gallegos et. al. JHEP05(2023)139)

[1] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, "Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation," *JHEP* **11** (2021) 150, arXiv:2107.14231 [hep-th].

[2] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, "Fate of spin polarization in a relativistic fluid: An entropy-current analysis," *Phys. Lett. B* **795** (2019) 100–106, arXiv:1901.06615 [hep-th].

[3] K. Fukushima and S. Pu, "Spin hydrodynamics and symmetric energy-momentum tensors – A current induced by the spin vorticity –," *Phys. Lett. B* **817** (2021) 136346, arXiv:2010.01608 [hep-th].

[4] A. Daher, A. Das, W. Florkowski, and R. Ryblewski, "Canonical and phenomenological formulations of spin hydrodynamics," *Phys. Rev. C* **108** no. 2, (2023) 024902, arXiv:2202.12609 [nucl-th].

[5] D. She, A. Huang, D. Hou, and J. Liao, "Relativistic viscous hydrodynamics with angular momentum," *Sci. Bull.* **67** (2022) 2265–2268, arXiv:2105.04060 [nucl-th].

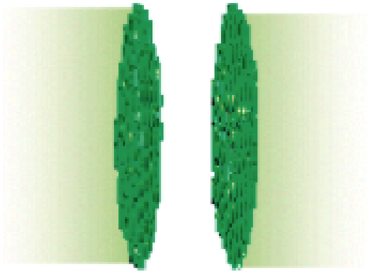
[6] A. D. Gallegos, U. Gürsoy, and A. Yarom, "Hydrodynamics of spin currents," *SciPost Phys.* **11** (2021) 041, arXiv:2101.04759 [hep-th].

[7] A. D. Gallegos, U. Gursoy, and A. Yarom, "Hydrodynamics, spin currents and torsion," arXiv:2203.05044 [hep-th].

[8] N. Weickgenannt, D. Wagner, E. Speranza, and D. H. Rischke, "Relativistic second-order dissipative spin hydrodynamics from the method of moments," *Phys. Rev. D* **106** no. 9, (2022) 096014, arXiv:2203.04766 [nucl-th].

Local Equilibrium Quantum State

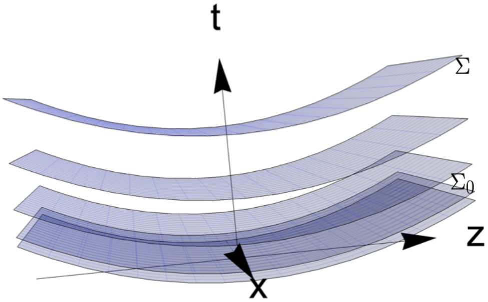
Typical Problem in QFT



Heisenberg Picture	$\widehat{\mathcal{T}}^{\mu\nu}(x,t) = e^{i\widehat{\mathcal{H}}t}\ \widehat{\mathcal{T}}^{\mu\nu}(x,0)\ e^{-i\widehat{\mathcal{H}}t}$	$\widehat{\rho}(0)$ fixed
Schrodinger Picture	$\widehat{\rho}_{(t)} = e^{i\widehat{\mathcal{H}}t}\ \widehat{\rho}_{(0)}\ e^{-i\widehat{\mathcal{H}}t}$	$\widehat{\mathcal{T}}^{\mu\nu}(x,0)$ fixed

As the system evolves, maybe we can find the form of the density operator (yet not the exact). This requires a physical assumption:

“At Σ_0 local thermodynamic equilibrium is achieved”



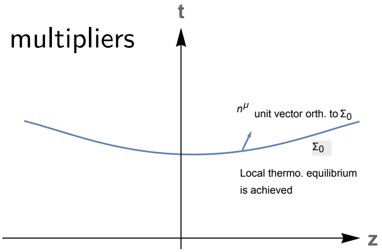
3D space-like hypersurfaces defining foliation

A LE state is obtained by looking for maximum entropy with specific constraints:

$$F[\widehat{\rho}] = -Tr[\widehat{\rho}\log\widehat{\rho}] - \int d\Sigma\ n_\mu \left(T^{\mu\nu}_{\mathrm{LE}} - T^{\mu\nu}\right)\beta_\nu(x) - \int d\Sigma\ n_\mu \left(S^{\mu\lambda\nu}_{\mathrm{LE}} - S^{\mu\lambda\nu}\right)\Omega_{\lambda\nu}(x)$$

$$T^{\mu\nu}_{\mathrm{LE}} \sim Tr\left[\widehat{\rho}_{(\Sigma_0)\mathrm{LE}}\widehat{T}^{\mu\nu}\right]$$

where : $\beta_\nu = u_\nu/T, \Omega_{\lambda\nu} = \omega_{\lambda\nu}/T$ Lagrange multipliers



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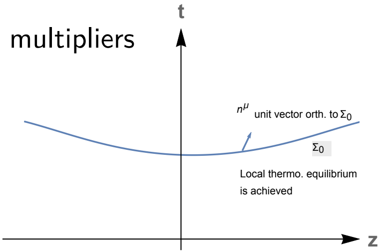

$$\frac{\delta F[\widehat{\rho}]}{\delta\widehat{\rho}} = 0$$

$$\Longrightarrow\; \widehat{\rho}_{(\Sigma_0)\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}}\exp\left[-\int_{\Sigma_0} d\Sigma_\mu\left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{S}^{\mu\lambda\nu}\right)\right]$$

Indeed demanding that $\widehat{\rho}_{(\Sigma_0)\mathrm{LE}}$ to be independent of the hypersurface (the integrant is divergence-less), then we obtain a $\mathrm{G.E}$ situation s.t:

$$\boxed{\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0}$$

$$\boxed{\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu \quad \text{with} \quad b, \varpi = \text{const}}$$

$$\boxed{\Omega_{\mu\nu} = \varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)}$$

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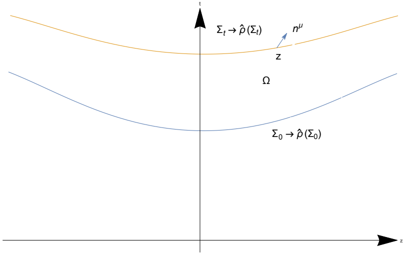
$$\boxed{\partial_\mu \Omega_{\lambda\nu} = 0}$$

Now we have full info. of our state $\widehat{\rho}_{(\Sigma_0)\mathrm{LE}}$. So we are able to calculate the mean value of any operator $\widehat{\mathcal{T}}^{\mu\nu}$, $\widehat{\mathcal{S}}^{\lambda\mu\nu}$ at Σ_0. But what about Σ_t...dissipation?

1. Choose a family of 3D space-like hypersurfaces (foliation)
2. We want to build the local equilibrium density operator at point $x \in \Sigma_t$
3. Use Gauss theorem, and linear response theory



$$\widehat{\rho}(\Sigma_t) = \frac{1}{Z}\exp\Bigg[\underbrace{-\int_{\Sigma_t} d\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{S}^{\mu\lambda\nu}\right)}_{\widehat{\rho}_{(\Sigma_t)\mathrm{LE}}}$$

$$\underbrace{+\int_\Omega d\Omega\, \widehat{T}_s^{\mu\nu}\xi_{\mu\nu} + \widehat{T}_a^{\mu\nu}(\Omega_{\mu\nu} - \varpi_{\mu\nu}) - \frac{1}{2}\widehat{S}^{\mu\lambda\nu}\partial_\mu\Omega_{\lambda\nu}\Bigg]}_{\text{dissipative corrections}}$$

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right) \equiv \text{ thermal shear}$$

This implies that dissipation occurs when,

$$\boxed{\xi \neq 0\ ,\ \Omega \neq \varpi\ ,\ \partial\Omega \neq 0}$$

Entropy Current

Near local equilibrium at a given hypersurface Σ, the total entropy:

$$\begin{aligned} S &= -Tr(\widehat{\rho}_{\mathrm{LE}} \log \widehat{\rho}_{\mathrm{LE}}) \\ &= \log Z_{\mathrm{LE}} + \int_{\Sigma} d\Sigma_{\mu} \left[Tr(\widehat{\rho}_{\mathrm{LE}} \widehat{\mathcal{T}}^{\mu\nu}) \beta_{\nu} - \frac{1}{2} \Omega_{\lambda\nu} Tr(\widehat{\rho}_{\mathrm{LE}} \widehat{\mathcal{S}}^{\mu\lambda\nu}) \right] \end{aligned}$$

Near local equilibrium at a given hypersurface Σ, the total entropy:

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$$= \log Z_{\rm LE} + \int_\Sigma d\Sigma_\mu \left[Tr(\widehat{\rho}_{\rm LE} \widehat{\mathcal{T}}^{\mu\nu})\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu} Tr(\widehat{\rho}_{\rm LE} \widehat{\mathcal{S}}^{\mu\lambda\nu}) \right]$$

Can we define an entropy current out of S ? In other words, is it possible to show that $\log Z_{\rm LE}$ is an extensive quantity?

$$\log Z_{\rm LE} \sim \int_\Sigma d\Sigma_\mu \; \phi^\mu$$

Therefore entropy current exists,

$$S = \int_{\Sigma} d\Sigma_{\mu} \left(\phi^{\mu} + T^{\mu\nu}\beta_{\nu} - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu} \right)$$

$$s^{\mu} = \phi^{\mu} + T^{\mu\nu}\beta_{\nu} - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu}$$

$$\phi^{\mu} = \int_{0}^{T} \frac{dT'}{T'^{2}} \left[T^{\mu\nu}(T')u_{\nu} - \frac{1}{2}\omega_{\lambda\nu}S^{\mu\lambda\nu}(T') \right]$$

But what is the physics of ϕ^μ

$\phi^\mu :=$ Thermodynamic potential vector field.

For a fluid at global equilibrium with vanishing thermal vorticity $\varpi_{\mu\nu} = 0$:

$$\boxed{\phi^\mu = p\,\beta^\mu = p\,\frac{u^\mu}{T}}$$

where "p" is the hydrostatic pressure that is the diagonal spatial component of the mean value of the energy-momentum operator.

Entropy Production Rate

Using the entropy current $s^\mu = \phi^\mu + T^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}S^{\mu\lambda\nu}$, we obtain:

$$
\begin{aligned}
\partial_\mu s^\mu \;&=\; \partial_\mu\beta_\nu\left[T_s^{\mu\nu} - T_{s(\mathrm{LE})}^{\mu\nu}\right] + \left(\Omega_{\mu\nu} - \varpi_{\mu\nu}\right)\left[T_a^{\mu\nu} - T_{a(\mathrm{LE})}^{\mu\nu}\right] \\
&\quad - \frac{1}{2}\partial_\mu\Omega_{\lambda\nu}\left[S^{\mu\lambda\nu} - S_{(\mathrm{LE})}^{\mu\lambda\nu}\right]
\end{aligned}
$$

$\varpi_{\mu\nu}$: *is the thermal vorticity*

1. This formula is a generalization of what was obtained *C. Van Weert* without spin:

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2. We stress that the formula is exact and not an approximation at some order of a gradient expansion.

3. A novel feature is apparently the simultaneous appearance of the last two terms of the right hand side.

Conclusion and Outlook

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- We've derived the entropy current, and entropy production rate using a quantum statistical approach without assuming the **LTR**,
- We expect to reproduce all the dissipative currents along side with what this approach might add,
- Finally (later on...) to solve and compare with the spin polarization data.

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Thank You!

Discussion on Local Thermodynamic Relations

1st pt

At global thermodynamic equilibrium with "$\beta =$cnst" (vanishing thermal vorticity), we know that:

$$\boxed{\epsilon + p = Ts}$$

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$\boxed{\text{1st pt}}$

At global thermodynamic equilibrium with "$\beta =$cnst" (vanishing thermal vorticity), we know that:

$$\boxed{\epsilon + p = Ts}$$

Is this verified by our quantum-statistical approach? Yes, because

$$\boxed{u_\mu \phi^\mu = \frac{p}{T}} \implies \boxed{s = u_\mu \phi^\mu + \frac{\epsilon}{T}} \implies \boxed{\epsilon + p = Ts}$$

Therefore $\epsilon + p = Ts$ is verified by quantum mechanics at homogeneous GE.

2nd pt

However at local equilibrium, using the form of ϕ^μ we have:

$$\boxed{u_\mu \phi^\mu = \int_0^T \frac{dT'}{T'^2} \left[\epsilon[T', \omega] - \frac{1}{2}\omega_{\lambda\nu} S^{\lambda\nu}[T', \omega] \right]}$$

Therefore, in general, this is the true exact form of **LTR**

$$s = u_\mu \phi^\mu + \frac{\varepsilon}{T} - \frac{1}{2}\Omega_{\lambda\nu} S^{\lambda\nu}$$

3rd pt

Yet one can argue that we can still call this as generalize "$P_{Generalized}$" and obtain

$$P_{Generalized} = Tu_\mu \phi^\mu = \int_0^T \frac{dT'}{T'^2} \left[\epsilon[T', \omega] - \frac{1}{2}\omega_{\lambda\nu} S^{\lambda\nu}[T', \omega] \right]$$

$$\Longrightarrow \boxed{\varepsilon + P_{Generalized} = Ts + \omega_{\alpha\beta} S^{\alpha\beta}}$$

$$\boxed{dP_{Generalized} = sdT + S^{\alpha\beta} d\omega_{\alpha\beta}}$$

If this assumption is correct, then the following must hold:

$$\frac{\partial P_{Generalized}}{\partial \omega_{\lambda\nu}}|_T = S^{\lambda\nu}$$

However it is not trivial to verify.