Studying polarization of spin-1/2 particles with an effective spacetime dependent mass

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At
XVI Polish Workshop on Relativistic Heavy-Ion Collisions, Kielce

December 03, 2023

Based On : 2307.12436

Collaborators : Arpan Das, Wojceich Florkowski, Gowthama K. K., Radoslaw Ryblewski

Features of Non-central Collisions :

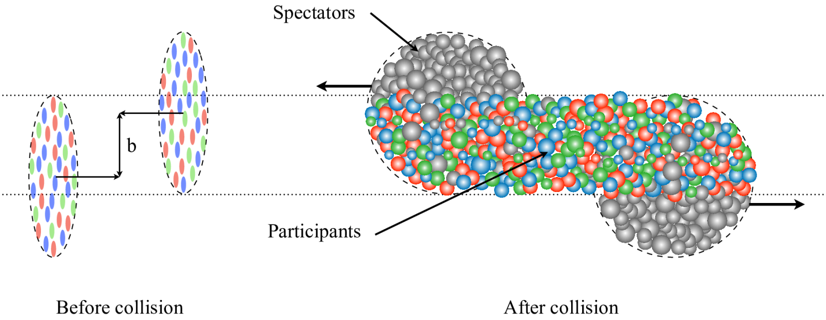

Figure 1: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

- Special feature of Non-Central Collisions :
  - Large Magnetic Field. [A. Bzdak and, V. Skokov, Phys. Lett. B 710 (2012) 171–174]
  - Large Angular Momentum. [F. Becattini et. al. Phys. Rev. C 77 (2008) 204906]
  - Particle polarization at small $\sqrt{S_{NN}}$. [STAR Collaboration, Nature 548 62-65, 2017]

Particle Polarization :

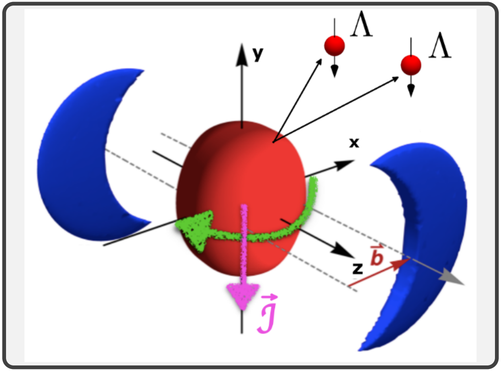

Figure 2: Origin of particle polarization. [W. Florkowski *et al*, PPNP 108 (2019) 103709]

- Large angular momentum → Local vorticities → spin alignment.

  [Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005); Phys. Lett. B 629, 20 (2005)]

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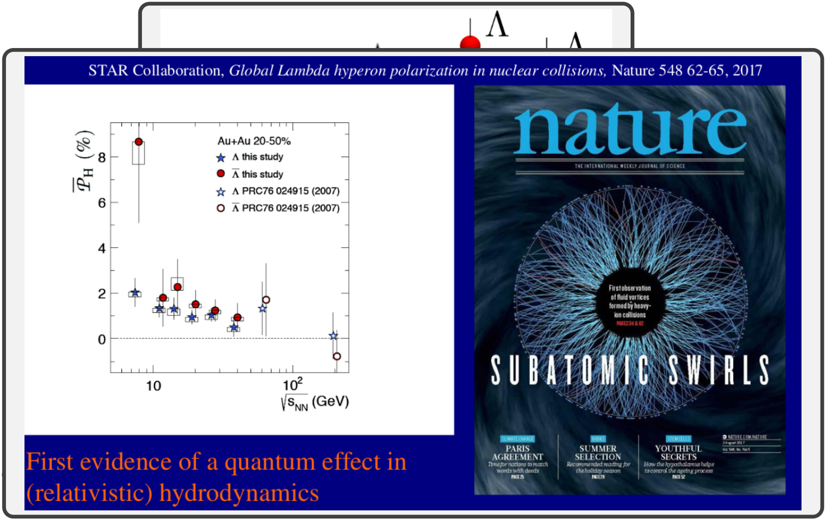

Experimental evidence, [STAR Collaboration, Nature 548, 62 (2017), Phys. Rev. Lett. 123, 132301 (2019), Phys. Rev. Lett. 126, 162301 (2021)]

- Theoretical models assuming equilibration of spin d.o.f. explains the data.

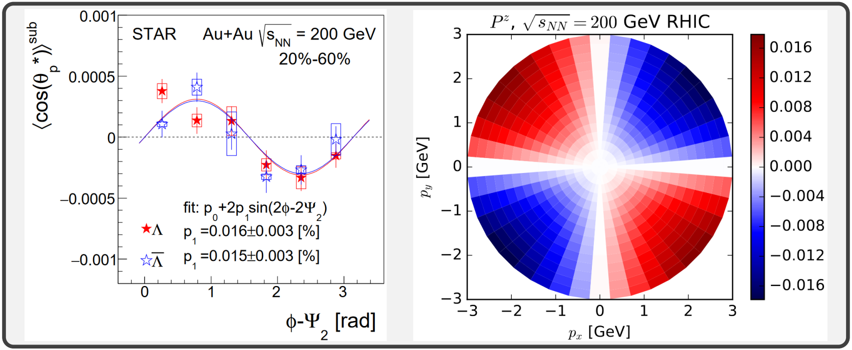

Figure 3: Observation (L) and prediction (R) of longitudinal polarization.
[Left: Phys. Rev. Lett. **123** 132301 (2019); Right: Phys. Rev. Lett. **120** 012302 (2018)]

- Theoretical models assuming equilibration of spin d.o.f. predict the opposite sign.

Recent Developments :

- Non-local collisions have been considered.

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◦ We would like to examine the effects of the equation of state.

Roadmap :

- A theory with spin should be built from Quantum Field Theory.

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$$\boxed{\text{QFT} \xrightarrow{\text{WF}} \text{Kinetic Equation} \xrightarrow{\int_{\text{p}}} \text{Macroscopic theory.}}$$

Lagrangian :

- Let us consider the Lagrangian:

$$\mathscr{L} = \bar{\psi}\left(i\,\partial\!\!\!/ + m_0\right)\psi + G\Big[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\psi\right)^2\Big]$$

[1] We have assumed $m_0 = 0$.

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- This leads to the equation of motion[1] :

$$\Big[i\,\partial\!\!\!/ - \sigma(x) - i\gamma_{5}\,\pi(x)\Big]\psi = 0.$$

$$\sigma = \langle\hat{\sigma}\rangle = -2G\,\left\langle\bar{\psi}\psi\right\rangle, \qquad \pi = \langle\hat{\pi}\rangle = -2G\,\left\langle\bar{\psi}\,i\gamma_{5}\psi\right\rangle.$$

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- The next step is to obtain kinetic equations of the system.

[W. Florkowski et. al., Annals Phys. **245** 445-463 (1996)]

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Wigner Function :

- The Wigner function is defined as:

$$\mathscr{W}_{\alpha\beta}(x,k) \equiv \int d^4y \, e^{ik\cdot y} \, G_{\alpha\beta}\left(x+\frac{y}{2}, x-\frac{y}{2}\right)$$

where, $G_{\alpha\beta}(x,y) = \langle \bar{\psi}_\beta(y)\psi_\alpha(x)\rangle$.

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- We can decompose the Wigner function as,

$$\mathscr{W} = \mathscr{F} + i\gamma_5\mathscr{P} + \gamma_\mu\mathscr{V}^\mu + \gamma^\mu\gamma_5\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}.$$

where, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

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- The components are obtained to be:

$$\mathscr{F} = \mathbf{Tr}\Big[\mathscr{W}\Big], \quad \mathscr{P} = -i\mathbf{Tr}\Big[\gamma_5\mathscr{W}\Big], \quad \mathscr{V}^\mu = \mathbf{Tr}\Big[\gamma^\mu\mathscr{W}\Big],$$
$$\mathscr{A}^\mu = \mathbf{Tr}\Big[\gamma_5\gamma^\mu\mathscr{W}\Big], \quad \mathscr{S}^{\mu\nu} = \mathbf{Tr}\Big[\sigma^{\mu\nu}\mathscr{W}\Big],$$

Semi-classical Expansion :

- The kinetic equations of the components are:

$$K^\mu \mathcal{V}_\mu - \sigma \mathcal{F} + \pi \mathcal{P} = -\frac{i\hbar}{2}\Big[(\partial_\nu \sigma)(\partial_k^\nu \mathcal{F}) - (\partial_\nu \pi)(\partial_k^\nu \mathcal{P})\Big]$$

$$-iK^\mu \mathcal{A}_\mu - \sigma \mathcal{P} - \pi \mathcal{F} = -\frac{i\hbar}{2}\Big[(\partial_\nu \sigma)(\partial_k^\nu \mathcal{P}) + (\partial_\nu \pi)(\partial_k^\nu \mathcal{F})\Big]$$

$$K_\mu \mathcal{F} + iK^\nu \mathcal{S}_{\nu\mu} - \sigma \mathcal{V}_\mu + i\pi \mathcal{A}_\mu = -\frac{i\hbar}{2}\Big[(\partial_\nu \sigma)(\partial_k^\nu \mathcal{V}_\mu) - (\partial_\nu \pi)(\partial_k^\nu \mathcal{A}_\mu)\Big]$$

$$iK^\mu \mathcal{P} - K_\nu \tilde{\mathcal{S}}^{\nu\mu} - \sigma \mathcal{A}^\mu + i\pi \mathcal{V}^\mu = -\frac{i\hbar}{2}\Big[(\partial_\nu \sigma)(\partial_k^\nu \mathcal{A}^\mu) - (\partial_\nu \pi)(\partial_k^\nu \mathcal{V}^\mu)\Big]$$

$$2iK^{[\mu}\mathcal{V}^{\nu]} - \varepsilon^{\mu\nu\alpha\beta} K_\alpha \mathcal{A}_\beta - \pi \tilde{\mathcal{S}}^{\mu\nu} + \sigma \mathcal{S}^{\mu\nu} = \frac{i\hbar}{2}\Big[(\partial_\gamma \sigma)\left(\partial_k^\gamma \mathcal{S}^{\mu\nu}\right) - (\partial_\gamma \pi)\left(\partial_k^\gamma \tilde{\mathcal{S}}^{\mu\nu}\right)\Big]$$

where, $K^\mu = k^\mu + \frac{i\hbar}{2}\partial^\mu$ and $\tilde{\mathcal{S}}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\mathcal{S}_{\alpha\beta}$.

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K_{\mu}\mathcal{F} + iK^{\nu}\mathcal{S}_{\nu\mu} - \sigma\mathcal{V}_{\mu} + i\pi\mathcal{A}_{\mu} &= -\frac{i\hbar}{2}\Big[(\partial_{\nu}\sigma)(\partial_k^{\nu}\mathcal{V}_{\mu}) - (\partial_{\nu}\pi)(\partial_k^{\nu}\mathcal{A}_{\mu})\Big] \\
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- In the following we will set, $\pi = 0$ and $M(x) = \sigma_{(0)}(x)$.

Kinetic Equations :

- We can obtain the Kinetic equation for Axial current as:

$$k^{\alpha}\left(\partial_{\alpha}\mathscr{A}^{\mu}\right)+M\left(\partial_{\alpha}M\right)\left(\partial_{(k)}^{\alpha}\mathscr{A}^{\mu}\right)+\left(\partial_{\alpha}\ln M\right)\left(k^{\mu}\mathscr{A}^{\alpha}-k^{\alpha}\mathscr{A}^{\mu}\right)=0.$$

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- Leading order ansatz:

$$\mathcal{A}^\mu(x,k) = 2M \int dP\, dS\, s^\mu \left[f^+(x,p,s)\delta^{(4)}(k-p) + f^-(x,p,s)\delta^{(4)}(k+p) \right].$$

$\longrightarrow s^\mu = (s^0, \mathbf{s}) \implies$ Spin 4-vector,
$\longrightarrow p^\mu = (p^0, \mathbf{p}) \implies$ On-shell momentum 4-vector i.e. $p^2 = M^2(x)$.
$\longrightarrow f^\pm(x,p,s) \implies$ Phase-space distribution functions.

- This ansatz satisfies, $k \cdot \mathcal{A}(x,k) = 0$.

Spin Tensors :

- The canonical spin tensor is defined as:

$$S_{\text{can}}^{\lambda\mu\nu}(x) = \frac{1}{2}\,\varepsilon^{\lambda\mu\nu\alpha}\int d^4k\,\mathcal{A}_{\alpha}(x,k).$$

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- The GLW (Groot, Leeuwen, Weert) spin tensor is defined as:

$$S^{\lambda,\mu\nu}(x) = \int dP\,dS\,p^\lambda s^{\mu\nu} f(x,p,s).$$

where, $s^{\alpha\beta} = \frac{1}{M}\varepsilon^{\alpha\beta\mu\nu}p_\mu s_\nu$, $\quad dP = \frac{d^3p}{E_{\mathrm{p}}}$, $\quad dS = \left(\frac{M}{\pi\mathfrak{s}}\right) d^4s\,\delta(s\cdot s + \mathfrak{s}^2)\delta(p\cdot s)$.

- These are related by:

$$S^{\lambda\mu\nu}_{\text{can}} = S^{\lambda,\mu\nu} + S^{\mu,\nu\lambda} + S^{\nu,\lambda\mu}.$$

[W. Florkowski et. al., Prog. Part. Nucl. Phys. **108** 103709 (2019)]

Evolution of Spin Tensor :

- Recall the kinetic equation for the Axial current,

$$k^\alpha \left(\partial_\alpha \mathscr{A}^\mu\right) + M \left(\partial_\alpha M\right) \left(\partial^\alpha_{(k)} \mathscr{A}^\mu\right) + \left(\partial_\alpha \ln M\right) \left(k^\mu \mathscr{A}^\alpha - k^\alpha \mathscr{A}^\mu\right) = 0.$$

- Multiplying this by $k_\beta \varepsilon_\mu^{\ \beta\gamma\delta}$ and integrating over k-momenta we get:

$$\boxed{\partial_\alpha S^{\alpha,\gamma\delta} = \left(\partial_\alpha \ln M\right) \left(S^{\gamma,\delta\alpha} - S^{\delta,\gamma\alpha}\right)} \neq 0$$

for $M = M(x)$.

[SB et. al. arXiv:2307.12436 [hep-ph]]

- As expected, the spin tensor is conserved when M is constant.

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- Then we can find:

$$M\partial_\lambda S^{\lambda,\mu\nu}_{\text{can}} = \partial_\lambda \left(MS^{\nu,\lambda\mu} \right) - \partial_\lambda \left(MS^{\mu,\lambda\nu} \right)$$

- Thus our approach is consistent with *“conservation of angular momentum.”*

Analytical Solutions :

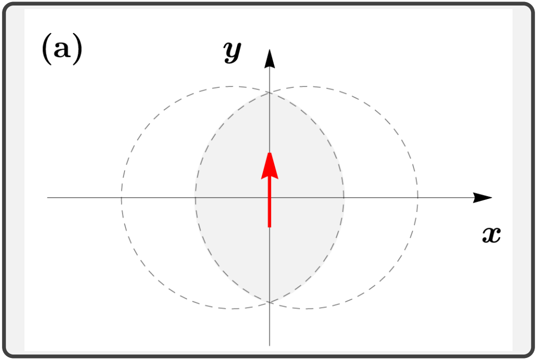

Figure 4: Transverse view of non-central collisions. [SB et. al. arXiv:2307.12436 [hep-ph]]

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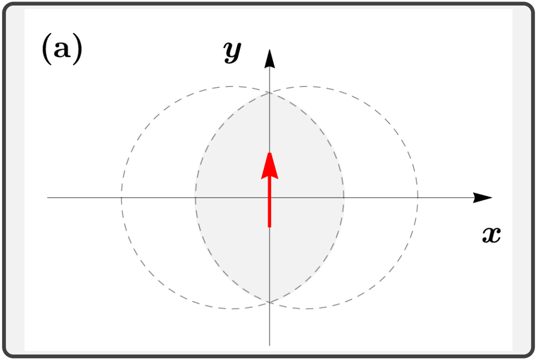

Figure 4: Transverse view of non-central collisions. [SB et. al. arXiv:2307.12436 [hep-ph]]

- Consider a system expanding boost-invariantly along the z-axis:

$$f(x,p,s) = g(x,p,s)\delta(p_x)\delta(p_y)$$

$$\implies S^{\lambda,\mu\nu}(x) = \int dP\, dS\, p^\lambda s^{\mu\nu} g(x,p,s)\delta(p_x)\delta(p_y)$$

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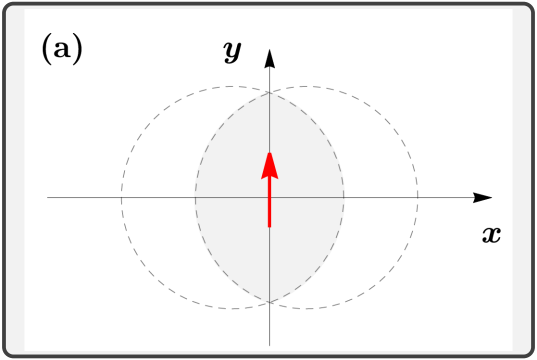

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- So, we have: $S^{1,\mu\nu} = S^{2,\mu\nu} = 0.$

Analytical Solutions - I (Transverse Polarization) :

- Transverse polarization implies:

$$g(x, p, s) = h(x, p, s)\delta(s_x)\delta(s_z)$$

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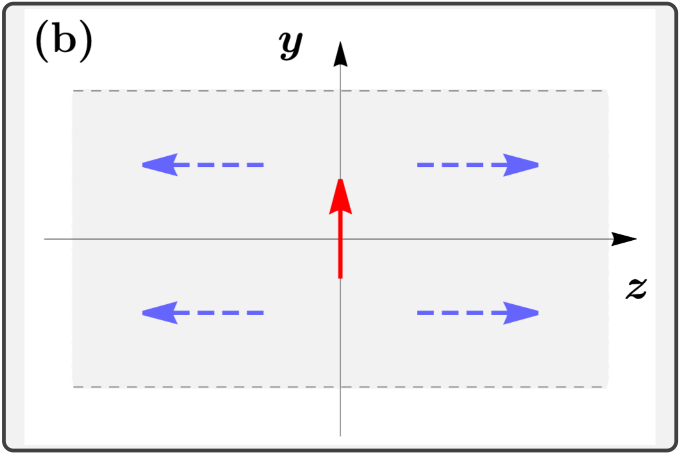

Figure 5: Transverse polarization schematic diagram. [SB et. al. arXiv:2307.12436 [hep-ph]]

Analytical Solutions - I (Transverse Polarization) :

- The spin tensor under transverse polarization becomes :

$$S^{\lambda,\mu\nu}(x) = \int dP\, dS\, p^{\lambda} s^{\mu\nu} h(x,p,s)\delta(s_x)\delta(s_z)\delta(p_x)\delta(p_y).$$

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- Furthermore, if $M = M(t,z)$, then the only non-zero components of spin-tensor are $S^{0,01},\ S^{3,01},\ S^{0,31},\ S^{3,31}$.

Analytical Solutions - I (Transverse Polarization) :

- The spin tensor under transverse polarization becomes :

$$S^{\lambda,\mu\nu}(x) = \int dP\, dS\, p^{\lambda} s^{\mu\nu} h(x,p,s)\delta(s_x)\delta(s_z)\delta(p_x)\delta(p_y).$$

- Furthermore, if $M = M(t,z)$, then the only non-zero components of spin-tensor are $S^{0,01},\ S^{3,01},\ S^{0,31},\ S^{3,31}$.

- The dynamics of spin is described by:

$$\partial_0 S^{0,01} + \partial_3 S^{3,01} = \frac{\partial_0 M}{M} S^{0,10} + \frac{\partial_3 M}{M} S^{0,13},$$
$$\partial_0 S^{0,31} + \partial_3 S^{3,31} = \frac{\partial_0 M}{M} S^{3,10} + \frac{\partial_3 M}{M} S^{3,13}.$$

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- Let us consider the following basis vector :

$$u^\mu = \begin{pmatrix} \frac{t}{\tau} \\ 0 \\ 0 \\ \frac{z}{\tau} \end{pmatrix}, \quad S_x^\mu = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad S_y^\mu = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad S_z^\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

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$$\frac{d\sigma}{d\tau} + \frac{\sigma}{\tau} = 0.$$
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- The solution is equivalent to conservation law in Bjorken model.

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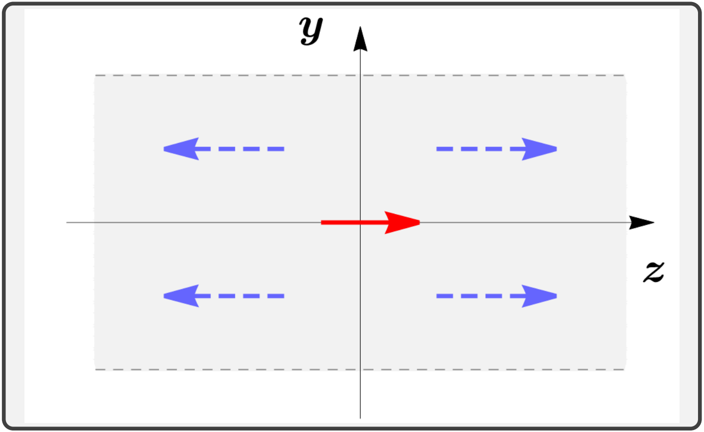

Figure 6: Longitudinal polarization schematic diagram. [SB et. al. arXiv:2307.12436 [hep-ph]]

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- Similar to transverse case, the spin decouples from the gradient of $M(x)$ and we have a similar solution.

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- Gradients of effective mass can act like a source of spin polarization.
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Summary and Outlook :

- Gradients of effective mass can act like a source of spin polarization.
- Spin evolution decouples from the source term in a highly symmetric system.
- More general geometries should be studied to observe the non-trivial effects.
- A self-consistently determined $M(x)$ should be used to study the evolution.
- Consequence of non-zero π should be explored.

Thank you.