Time Difference Measurement with Shape Information

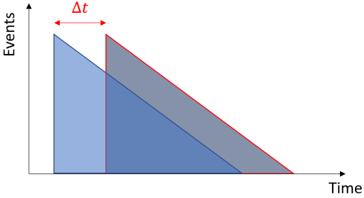
Jia-Shian Wang

SNEWS 2.0 Meeting

20 June 2020

Comparing Shapes

- Assumption: detected light curves in different detectors should be similar, since they originate from the same SN flux
  - Detector response could lead to some shape difference and, hence, biases to the measurement
- Shape difference parameters should be minimised when the distributions are shifted by the correct time difference Δt_{true}



Difference Metric

- We tested several different metrics including χ^2
- Observed resolution power dominated by the leading edge
- Can we gain more resolution out of the leading edge?
  - we considered metrics of the form $|b - b'|\, w(b, b')$ where b and b' are the bin contents from the two time distributions; and $w(b, b')$ is just some weight based on the bin contents

Simple Metric

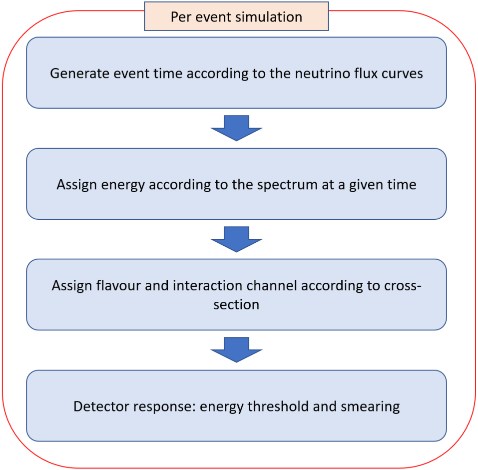
- A visible separation in the time distributions is a more obvious mismatch so we will weight the leading edge more
- Notation: for some bin i in the time distribution, we define
  - η_i is the metric contribution from bin i
  - b_i, b'_i are the bin contents of the normalised time distribution
  - $b_{max} = max\{b_i\}$, $b'_{max} = max\{b'_i\}$ are the maximum bins
- Here we devise a simple case of weighted difference (not optimised just a simple case to demonstrate how we handle the edge):

$$\begin{cases} \eta_i = |b_i - b'_i|(b_i + b'_i) & \text{if both bins are non-zero} \\ \eta_i = |b_i - b'_i|(b_{max} + b'_{max}) & \text{if one of the bin is empty} \\ \eta_i = 0 & \text{if both bins are empty} \end{cases}$$

- At the end, we compare the total metric $\eta = \sum_i \eta_i$

Testing the Metric

- Simulating trial signal light curves to test the performance of the metric
- 1000 trials simulated
- Total event yield calculated by `SNOwGLoBES`
- Only simulated for JUNO and SuperK
- Energy from pinched spectrum, and applied energy smearing and threshold



Per event simulation

Generate event time according to the neutrino flux curves

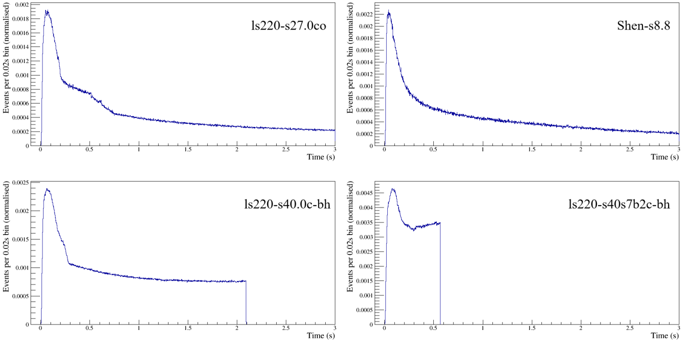
Assign energy according to the spectrum at a given time

Assign flavour and interaction channel according to cross-section

Detector response: energy threshold and smearing

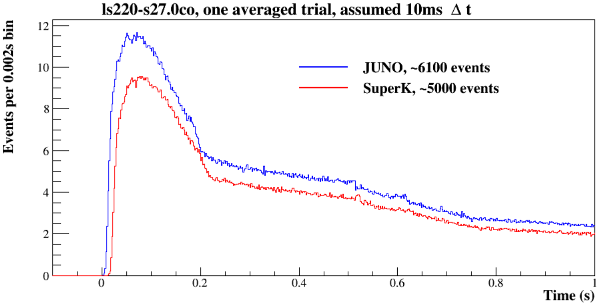
Signal Models

- Four SN models tested:
  - Full SN burst: shen-s8.8 ($8.8M_\odot$, electron capture SN), ls220-s27.0co ($27M_\odot$, Fe core SN)
  - SN with black hole formation: ls220-s40.0c-bh (2s), ls220-s40s7b2c-bh (0.5s)



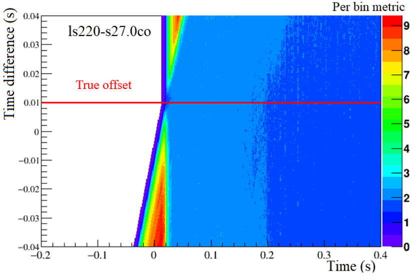
Signal Time Series

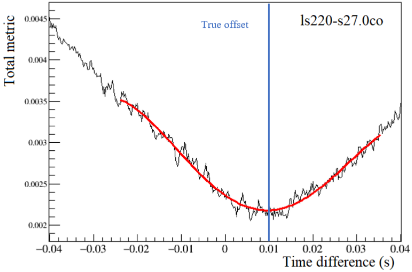
- Considering only IBD events, taking prompt time as event time
- Distance set at 10kpc
- Backgrounds not included
- JUNO and SuperK only (maximum possible Δt ≈10ms)



Applying Metric

- Time window of comparison is (-0.2, 0.4)s with 1.2ms bins (tuned for ls220-s27.0co)
- The bin size can be set as a variable dependent on the event yield
- The $\widehat{\Delta t}$ is scanned through (-60, 60)ms in 0.2ms steps, and in each step we rebin the time distribution
- Since the times are actually discrete values, the metric profile is then smoothed with a 4th order polynomial





Performance

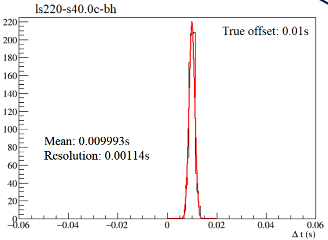
- Tested with offsets -30, -10, 5, 10, 30ms
- Bias ($|\Delta t_{true} - \text{Mean}|$) and resolution (standard deviation) stable
- Results obtained assuming no backgrounds

N.B. Linzer and K. Scholberg (2019)

shen-s8.8, $\delta t = 5.7$ms

Coleiro et al. (2020)

$27M_{\odot}$ model, $\delta t = 2.7$ms



Shape Comparison

Models	Resolution (ms)	Bias (ms)
Shen-s8.8	3.4	0.6 ± 0.17
ls220-s27.0co	1.83	0.04 ± 0.15
ls220-s40.0c-bh	1.14	0.007 ± 0.15
ls220-s40s7b2c-bh	1.28	0.47 ± 0.16

Background Effects

- The expected background IBD rate in SuperK and JUNO are far less than 1 in the 0.6s time window, so the worst case scenario would be one or two events occurring at the early edge
- To simulate this worst case, we randomly generate one or two events in the time region (-0.2, 0.2)s
- Testing on the ls220-s27.0co model:

  No background: $\delta t =$1.83ms
  One background event: $\delta t =$1.85ms
  Two background events: $\delta t =$1.86ms
- Bias not affected

Conclusion

- We have tried some variant metrics to see if we can improve the resolution
- One metric based on $|b_i - b'_i|(b_i + b'_i)$ shows promising results:
    - Improved bias and resolution compared with previous results
    - Robust against backgrounds
- There may be further mileage to be gained from "unconventional" metrics

Backup

Cramer-Rao Limit

- In *[astro-ph/9811350]*, the Cramer-Rao theorem was used to estimate the theoretical limit on the resolution:

$$\frac{1}{(\delta t)^2_{min}} = N \int \mathrm{d}t \, \frac{[\partial f(t, t_0)/\partial t]^2}{f(t, t_0)}$$

  where $(\delta t)_{min}$ is the resolution limit on the time shift of the time distribution, N is the number of detected events, and $f(t, t_0)$ is the underlying model with t_0 being the true offset

- For rise time far shorter than decay time $\tau_1 \ll \tau_2$:

$$(\delta t)_{min} \approx \sqrt{\frac{\tau_1 \tau_2}{N}}$$

- The limits on the models used (considering the JUNO and SuperK case):

  shen-s8.8: $\delta t = 1.6$ms
  ls220-s27.0co: $\delta t = 0.7$ms
  ls220-s40.0c-bh: $\delta t = 0.37$ms
  ls220-s40-s7b2c-bh: $\delta t = 0.8$ms

Bin Size

- The 1.2ms bin size (500 bins) is actually tuned only for ls220-s27.0co
- Optimised bin sizes for other models are (number of bins in time window (-0.2,0.4)s):

  shen-s8.8: 150 bins, δt =2.9ms
  ls220-s40.0c-bh: 1000 bins, δt =1.01ms
  ls220-s40-s7b2c-bh: 1100 bins, δt =1.15ms

Energy Formulae

- Pinched spectrum:

$$f(E,t) = \frac{(\alpha(t)+1)^{(\alpha(t)+1)}}{\langle E\rangle(t)\,\Gamma(\alpha+1)}\left(\frac{E}{\langle E\rangle(t)}\right)^{\alpha(t)} exp\left[-(\alpha(t)+1)\frac{E}{\langle E\rangle(t)}\right]$$

  where $\langle E\rangle$ and α are inputted from the models

- IBD visible energy:

$$E_{vis} = E_{\bar{\nu}_e} + 2\times 511\text{keV} - 1806\text{keV}$$

- Energy resolution: Gaussian standard deviation $\sigma(E)$ =resolution$\times\sqrt{E}$
- Detector response:

  JUNO: 20kt, 3% energy resolution, 0.2MeV threshold
  SuperK: 22.5kt, 4.5% energy resolution, 3.5MeV threshold

Δt Scan

- The discrimination power of the χ^2 really comes from the leading edge (or the visible separation)