# QCD Radiative Corrections for the LHC

Goran Duplancic
Rudjer Boskovic Institue

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# WHY?

Reliable and precise comparison between theoretical predictions and experimental data.

# WHY IS THAT IMPORTANT?

to better understand standard physics and to precisely measure fundamental quantities

To find New Physics

OK, it is important but not so exciting.

Really!
How is it possible to find New Physics by doing long and boring calculations in the Standard Physics?

# WE CAN SEE NEW PHYSICS IN TWO WAYS.

As small deviations when comparing precise measurements with detailed SM predictions.

Precise!? Detail!?
O my ...
I don’t want to do that!

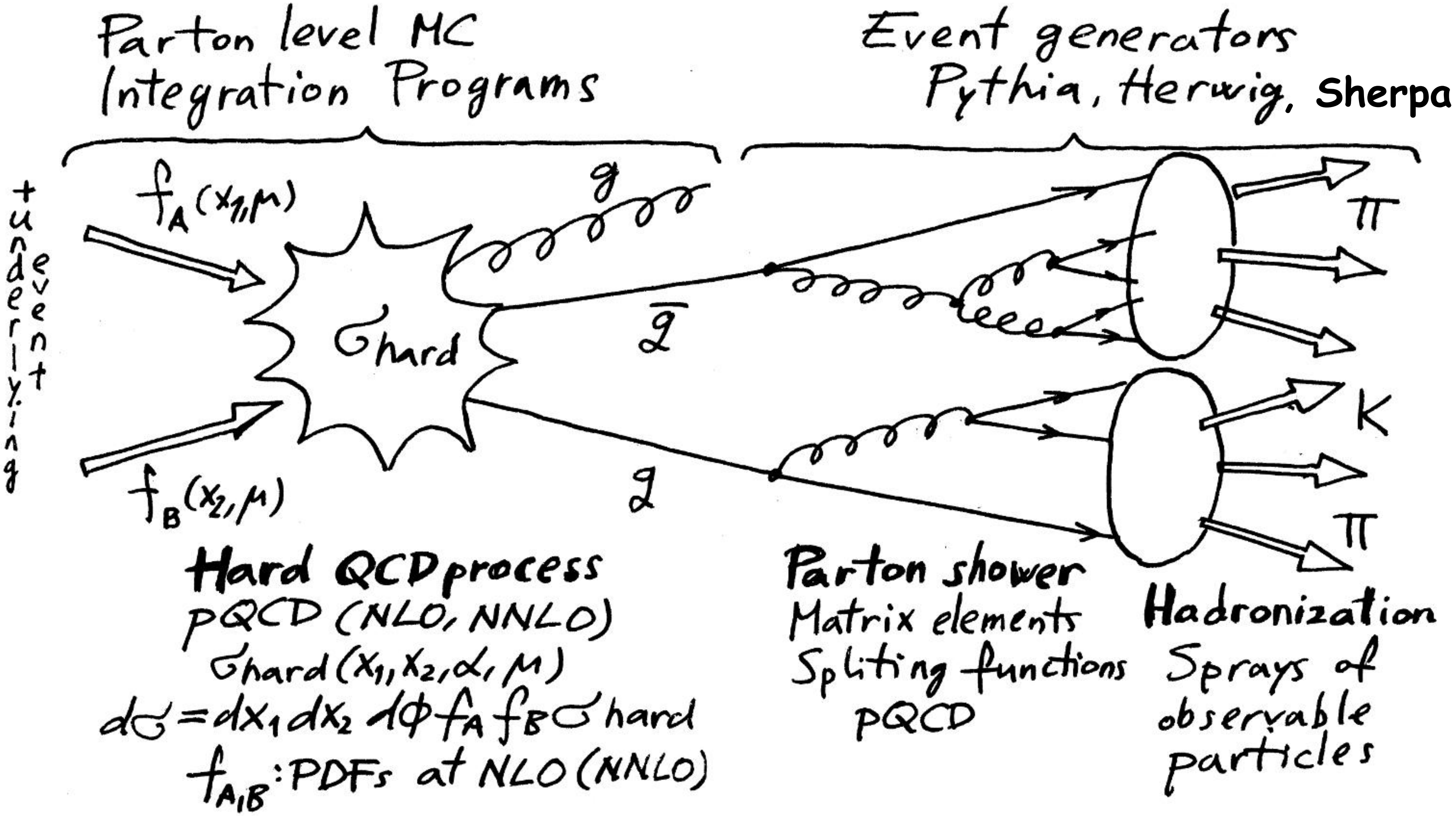
As direct production of new particles.

Yes! Let’s find it in that way!

Any new particle will be observed through its decay into several standard particles. This decay will be shadowed by much larger strong interaction background.

Oh, no! I quit!

# Anatomy of a QCD prediction at hadron colliders



From an experimental perspective, the ideal situation would be to have the matrix elements (NLO) interfaced to a parton shower Monte Carlo.

Each order in the perturbative series in $\alpha_S$ helps to increase the reliability of QCD corrections.

LO: large theoretical uncertainties due to renormalization/factorization scale dependence, order of magnitude estimate

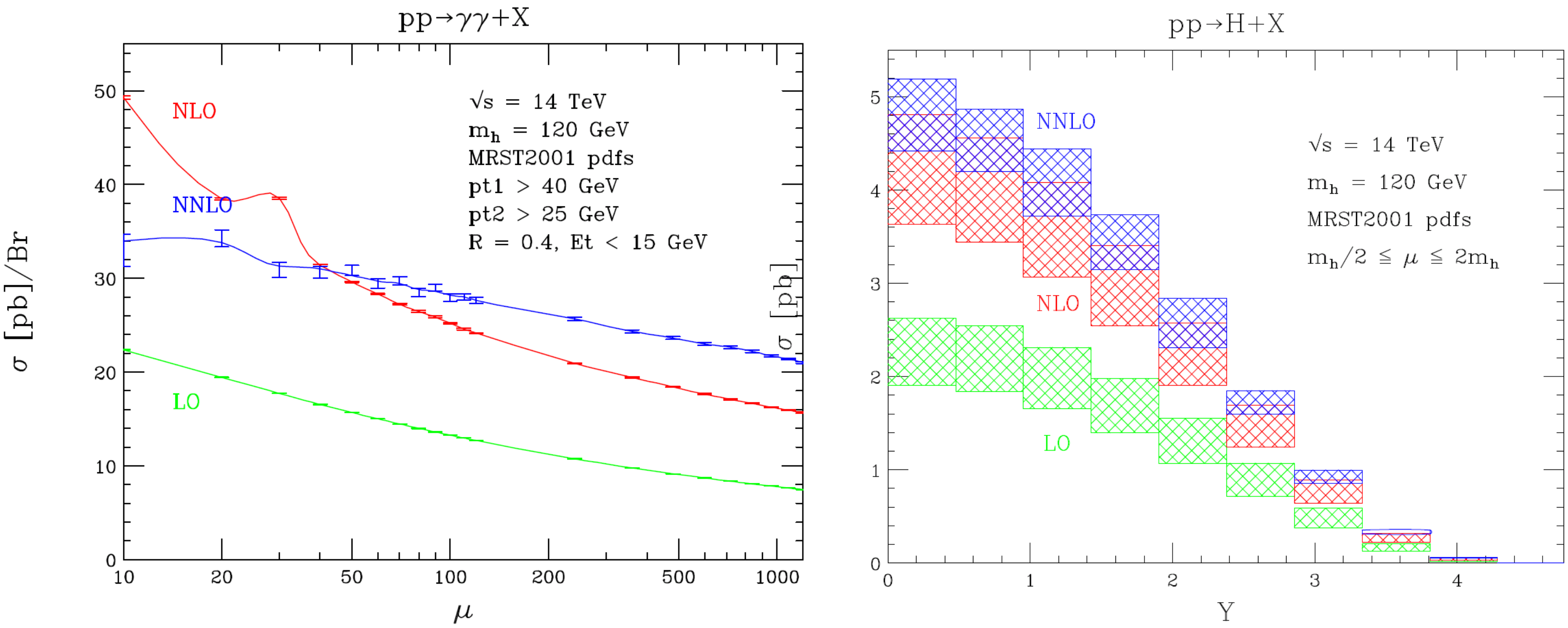
NLO: reduced scale dependence, reliabe predictions, considerably reduces/enhances the LO cross sections, may distort the shape of distribution

NNLO: reduced scale dependence, reliable cross section and error estimate

At the LHC experimental errors for many QCD processes will be typically smaller than intrinsic uncertainties off NLO predictions (10-20%).

# Examples

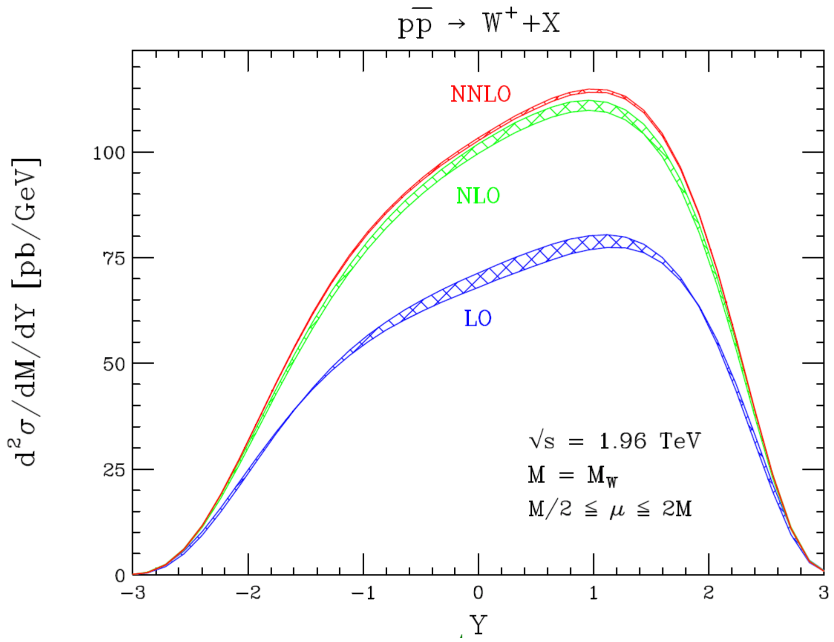
## Higgs production via gluon fusion with $H\to\gamma\gamma$ at the LHC

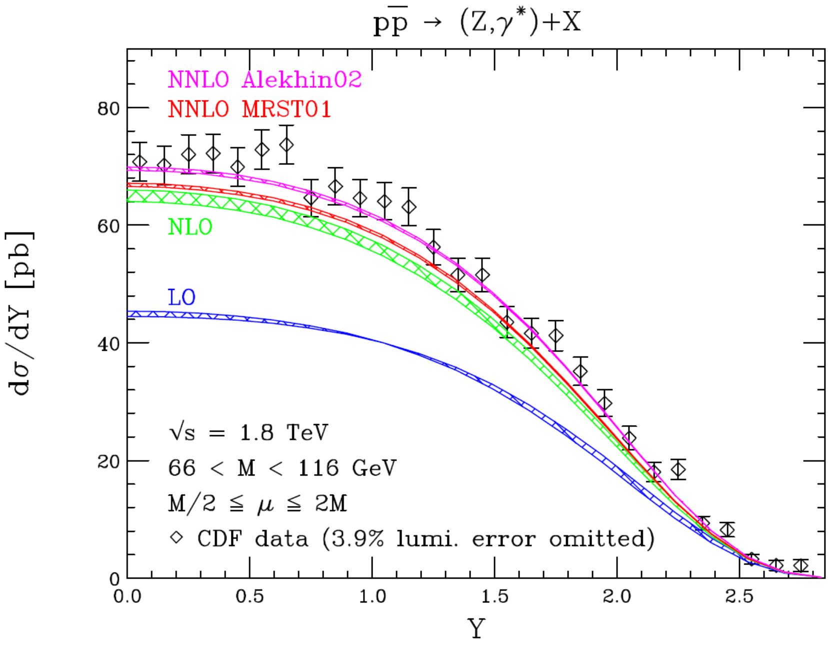


Anastasiou, Melnikov, Petriello (2005)

Large scale uncertainty at NLO which is considerably reduced at NNLO.

# W and Z boson production at the Tevatron





Anastasiou, Dixon, Melnikov, Petriello , PRD69 (2004)

NLO prediction not within LO uncertainty band. NNLO shows that perturbative prediction is reliable.

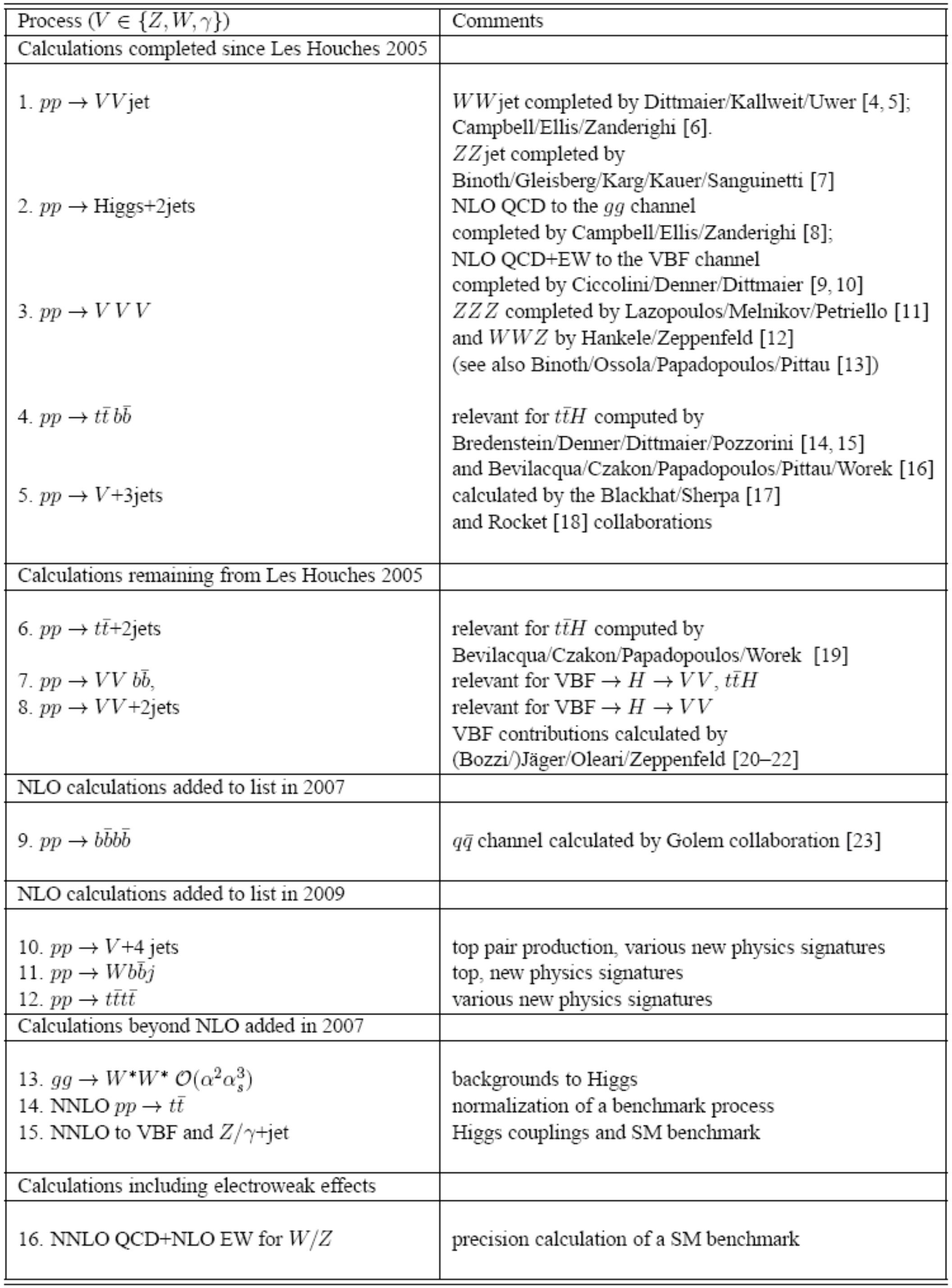
# Experimenter's wishlist for LHC processes

A prioritized list of cross sections which are experimentaly important and which are theoretically feasible (if difficult) to calculate (asembled in 2005 and added to in 2007 and 2009).

Basically all 2→3 cross sections of interest have been calculated

The frontier now extending to 2→4 calculations. Since 2007, two additional calculations have been completed: $t\bar{t}b\bar{b}$ and W+3 jets. In addition $b\bar{b}b\bar{b}$ has been calculated for the $q\bar{q}$ initial state with the gg calculation in progress.

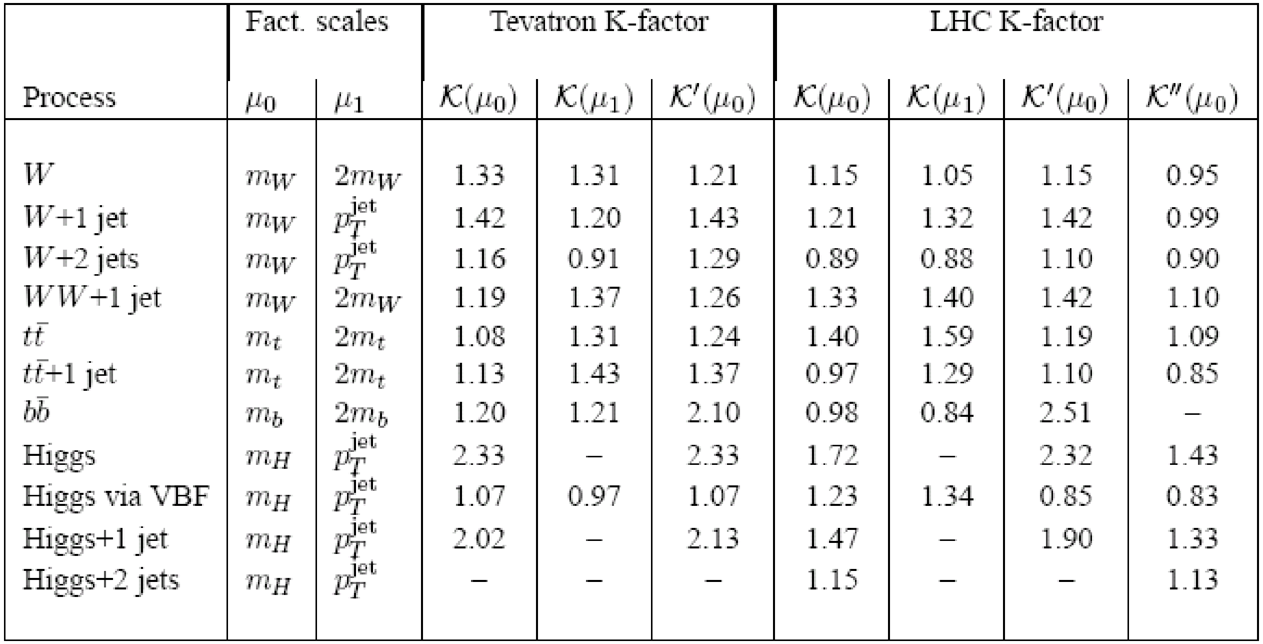
Often these calculations exist only as private codes.



| Process ($V \in \{Z, W, \gamma\}$) | Comments |
|---|---|
| Calculations completed since Les Houches 2005 | |
| 1. $pp \to VV$jet | $WW$jet completed by Dittmaier/Kallweit/Uwer [4, 5];<br>Campbell/Ellis/Zanderighi [6].<br>$ZZ$jet completed by<br>Binoth/Gleisberg/Karg/Kauer/Sanguinetti [7] |
| 2. $pp \to$ Higgs+2jets | NLO QCD to the $gg$ channel<br>completed by Campbell/Ellis/Zanderighi [8];<br>NLO QCD+EW to the VBF channel<br>completed by Ciccolini/Denner/Dittmaier [9, 10] |
| 3. $pp \to VVV$ | $ZZZ$ completed by Lazopoulos/Melnikov/Petriello [11]<br>and $WWZ$ by Hankele/Zeppenfeld [12]<br>(see also Binoth/Ossola/Papadopoulos/Pittau [13]) |
| 4. $pp \to t\bar{t}\,b\bar{b}$ | relevant for $t\bar{t}H$ computed by<br>Bredenstein/Denner/Dittmaier/Pozzorini [14, 15]<br>and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [16] |
| 5. $pp \to V$+3jets | calculated by the Blackhat/Sherpa [17]<br>and Rocket [18] collaborations |
| Calculations remaining from Les Houches 2005 | |
| 6. $pp \to t\bar{t}$+2jets | relevant for $t\bar{t}H$ computed by<br>Bevilacqua/Czakon/Papadopoulos/Worek [19] |
| 7. $pp \to VV\,b\bar{b}$, | relevant for VBF $\to H \to VV$, $t\bar{t}H$ |
| 8. $pp \to VV$+2jets | relevant for VBF $\to H \to VV$<br>VBF contributions calculated by<br>(Bozzi/)Jäger/Oleari/Zeppenfeld [20–22] |
| NLO calculations added to list in 2007 | |
| 9. $pp \to b\bar{b}b\bar{b}$ | $q\bar{q}$ channel calculated by Golem collaboration [23] |
| NLO calculations added to list in 2009 | |
| 10. $pp \to V$+4 jets | top pair production, various new physics signatures |
| 11. $pp \to Wb\bar{b}j$ | top, new physics signatures |
| 12. $pp \to t\bar{t}t\bar{t}$ | various new physics signatures |
| Calculations beyond NLO added in 2007 | |
| 13. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$ | backgrounds to Higgs |
| 14. NNLO $pp \to t\bar{t}$ | normalization of a benchmark process |
| 15. NNLO to VBF and $Z/\gamma$+jet | Higgs couplings and SM benchmark |
| Calculations including electroweak effects | |
| 16. NNLO QCD+NLO EW for $W/Z$ | precision calculation of a SM benchmark |

# K-factors

Experimentalists typically deal with LO calcultions, especially in the context of parton shower Monte Carlos. Some of the information from a NLO calculation can be encapsulated in the K-factor (ratio of NLO to LO cross section).



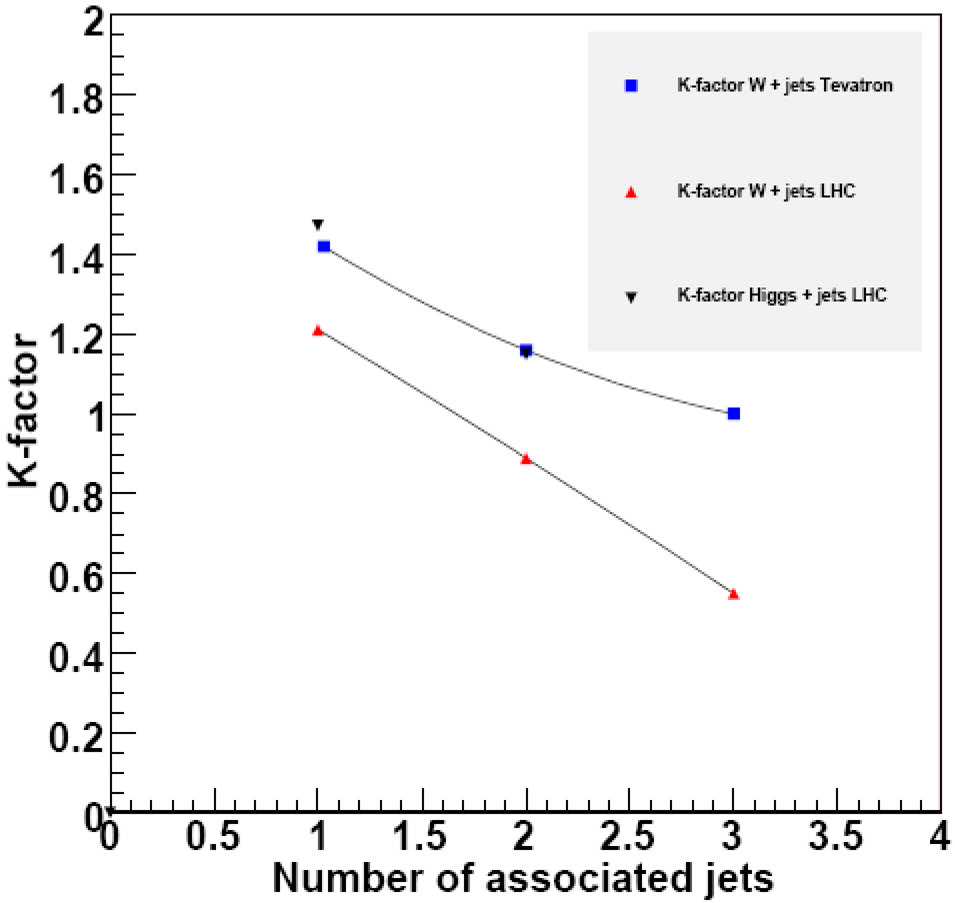
| | Fact. scales | | Tevatron K-factor | | | LHC K-factor | | | |
|---|---|---|---|---|---|---|---|---|---|
| Process | $\mu_0$ | $\mu_1$ | $\mathcal{K}(\mu_0)$ | $\mathcal{K}(\mu_1)$ | $\mathcal{K}'(\mu_0)$ | $\mathcal{K}(\mu_0)$ | $\mathcal{K}(\mu_1)$ | $\mathcal{K}'(\mu_0)$ | $\mathcal{K}''(\mu_0)$ |
| $W$ | $m_W$ | $2m_W$ | 1.33 | 1.31 | 1.21 | 1.15 | 1.05 | 1.15 | 0.95 |
| $W$+1 jet | $m_W$ | $p_T^{\rm jet}$ | 1.42 | 1.20 | 1.43 | 1.21 | 1.32 | 1.42 | 0.99 |
| $W$+2 jets | $m_W$ | $p_T^{\rm jet}$ | 1.16 | 0.91 | 1.29 | 0.89 | 0.88 | 1.10 | 0.90 |
| $WW$+1 jet | $m_W$ | $2m_W$ | 1.19 | 1.37 | 1.26 | 1.33 | 1.40 | 1.42 | 1.10 |
| $t\bar{t}$ | $m_t$ | $2m_t$ | 1.08 | 1.31 | 1.24 | 1.40 | 1.59 | 1.19 | 1.09 |
| $t\bar{t}$+1 jet | $m_t$ | $2m_t$ | 1.13 | 1.43 | 1.37 | 0.97 | 1.29 | 1.10 | 0.85 |
| $b\bar{b}$ | $m_b$ | $2m_b$ | 1.20 | 1.21 | 2.10 | 0.98 | 0.84 | 2.51 | – |
| Higgs | $m_H$ | $p_T^{\rm jet}$ | 2.33 | – | 2.33 | 1.72 | – | 2.32 | 1.43 |
| Higgs via VBF | $m_H$ | $p_T^{\rm jet}$ | 1.07 | 0.97 | 1.07 | 1.23 | 1.34 | 0.85 | 0.83 |
| Higgs+1 jet | $m_H$ | $p_T^{\rm jet}$ | 2.02 | – | 2.13 | 1.47 | – | 1.90 | 1.33 |
| Higgs+2 jets | $m_H$ | $p_T^{\rm jet}$ | – | – | – | 1.15 | – | – | 1.13 |

Table 2: K-factors for various processes at the LHC (at 14 TeV) calculated using a selection of input parameters. In all cases, for NLO calculations, the CTEQ6M PDF set is used. For LO calculations, $\mathcal{K}$ uses the CTEQ6L1 set, whilst $\mathcal{K}'$ uses the same PDF set, CTEQ6M, as at NLO, and $\mathcal{K}''$ uses the LO-MC (2-loop) PDF set CT09MC2. For Higgs+1 or 2 jets, a jet cut of 40 GeV$/c$ and $|\eta| < 4.5$ has been applied. A cut of $p_T^{\rm jet} > 20$ GeV$/c$ has been applied to the $t\bar{t}$+jet process, and a cut of $p_T^{\rm jet} > 50$ GeV$/c$ to the $WW$+jet process. In the $W$(Higgs)+2 jets process, the jets are separated by $\Delta R > 0.4$ (with $R_{sep} = 1.3$), whilst the vector boson fusion (VBF) calculations are performed for a Higgs boson of mass 120 GeV. In each case the value of the K-factor is compared at two often-used scale choices, $\mu_0$ and $\mu_1$.

NLO corrections appear to be larger for processes in which there is a great deal of color annihilation.

NLO corrections also tend to decrease as more final-state legs are added.

The K-factors at the LHC are similar to the K-factors for the same processes at the Tevatron, but have a tendency to be smaller.

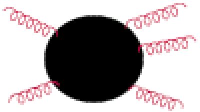


The K-factors for W production at the Tevatron and LHC and for Higgs production at the LHC as a function of the number of accompanying jets. The kT jet algorithm with a D parameter of 0.4 has been used.

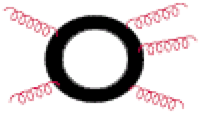
To understand the pattern jet algorithms at LO and NLO are imortant.

# Structure of One-Loop (NLO) computations and MC/NLO interface

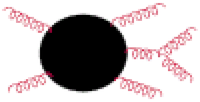
$\mathscr{A}_{\mathrm{LO}}$:



$\mathscr{A}_{\mathrm{NLO,\,virtual}}$:



$\mathscr{A}_{\mathrm{NLO,\,real}}$:



$$\sigma_{had}(p_1,p_2) = \sum_{a,b} \int dx_1\, f_{a/H_1}(x_1,\mu_{\mathrm{F}}^2) \int dx_2\, f_{b/H_2}(x_2,\mu_{\mathrm{F}}^2) \times \left[ d\sigma_{ab}^{\mathrm{LO}}(x_1p_1,x_2p_2;\mu_{\mathrm{R}}^2) + d\sigma_{ab}^{\mathrm{NLO}}(x_1p_1,x_2p_2;\mu_{\mathrm{R}}^2,\mu_{\mathrm{F}}^2) \right]$$

$$\sigma_{ab}^{\mathrm{LO}} = \int_m \boxed{d\sigma_{ab}^{B}},$$

**Born term**

$$\sigma_{ab}^{\mathrm{NLO}} = \int_{m+1} \boxed{d\sigma_{ab}^{R}} + \int_m \boxed{d\sigma_{ab}^{V}} + \int_m \boxed{d\sigma_{ab}^{C}(\mu_{\mathrm{F}}^2,\mathrm{F.S.})}$$

**real corr.**

**virtual corr.**

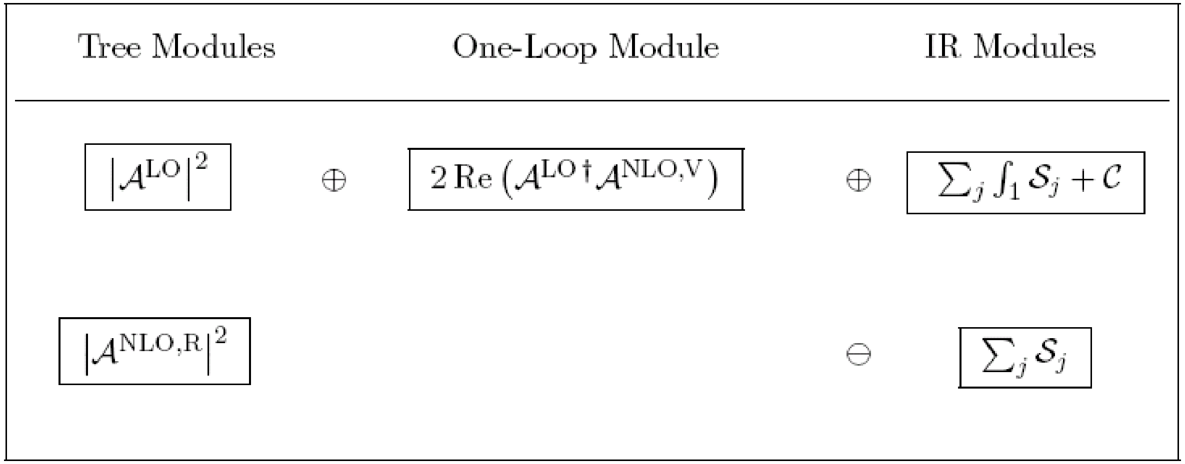
**collinear counterterm**

Born, real emission and collinear terms are defined by tree amplitudes,which can be efficiently evaluated with existing matrix element generators (MadGraph/MadEvent, Helac/Phegas,Comix,WHIZARD,AMEGIC++,ALPGEN,HELAC,CompHEP,CalcHEP,GRACE).

$$d\sigma_{ab}^{V} = d\,\mathrm{LIPS}(\{k_j\})\,\mathcal{I}(\{k_j\})$$

$$\mathcal{I}(\{k_j\}) = \sum_{h,c,c'} \left( \mathcal{A}_{h,c}^{\mathrm{LO}\dagger} \langle c|c'\rangle \mathcal{A}_{h,c'}^{\mathrm{NLO},V} + \mathcal{A}_{h,c}^{\mathrm{NLO},V\dagger} \langle c|c'\rangle \mathcal{A}_{h,c'}^{\mathrm{LO}} \right)$$

$$\mathcal{I}\big(\{k_j\}, \mathrm{R.S.}, \mu_{\mathrm{R}}^2, \alpha_{\mathrm{S}}(\mu_{\mathrm{R}}^2), \alpha, \ldots\big) = C(\epsilon)\left(\frac{A_2}{\epsilon^2} + \frac{A_1}{\epsilon} + A_0\right)$$



| Tree Modules | | One-Loop Module | | IR Modules |
|---|---|---|---|---|
| $\left\lvert\mathcal{A}^{\mathrm{LO}}\right\rvert^2$ | $\oplus$ | $2\,\mathrm{Re}\left(\mathcal{A}^{\mathrm{LO}\,\dagger}\mathcal{A}^{\mathrm{NLO,V}}\right)$ | $\oplus$ | $\sum_j \int_1 \mathcal{S}_j + \mathcal{C}$ |
| $\left\lvert\mathcal{A}^{\mathrm{NLO,R}}\right\rvert^2$ | | | $\ominus$ | $\sum_j \mathcal{S}_j$ |

**Figure 1:** Modular structure of next-to-leading order computations for partonic processes. All structures related to tree amplitudes can be evaluated using LO MC tools. The one-loop module contains the UV renormalised interference term. The treatment of IR subtraction should be kept separate to allow for flexibility. The IR modules contain subtraction terms, $\mathcal{S}_j$, for the real-emission part and their integrated variants which compensate IR divergences in the One-Loop Module. In case of collinear initial-state divergences, collinear subtraction terms, $\mathcal{C}$, have to be provided too. Subsequently, the contributions in each horizontal line are independently finite after summation.

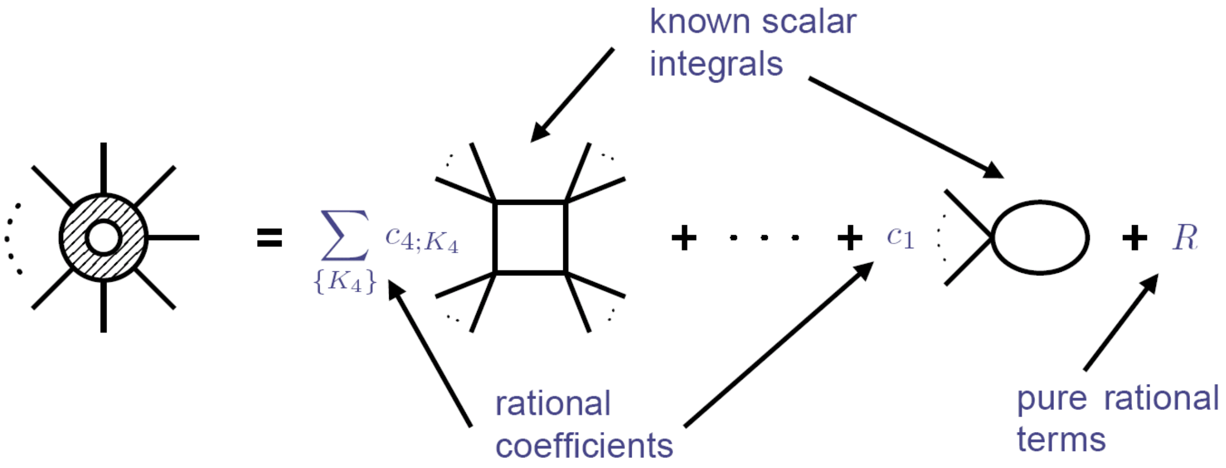
We reached the point where many of NLO calculations can be done almost automatically. In many cases two separate codes are needed for a full NLO generator.

1. **the One-Loop Program (OLP) which calculates the virtual contributions to the process**
2. **the Monte Carlo (MC) tool which takes care of the real emission, the subtraction terms and phase-space integrtion**

Only together OLP and MC can provide total cross sections and distributions at NLO accuracy.

A complete proposal for a standard interface between MC tols and one-loop matrix element programs can be found in Binoth Les Houches Accord.

# Methods to compute One-Loop Amplitudes



Two religions

Agnosticism

Feynmanians

Unitarians

Experimentes

Automated approaches

analytical

seminumerical

numerical

[ BlackHat, Rocket, CutTools/Helac-1loop, GOLEM, Denner et al., samurai, … ]

There is no preferred method.

# Conclusion

LHC needs and deserves an effort to predict prominent signal and background processes at NLO in QCD

Absolute rates and shapes cannot be predicted reliably with leading order Monte-Carlo tools and eventually this will hamper the understanding of LHC data and the discovery of new physics.

Many relevant SM processes are meanwhile available in the literature beyond the leading order.