The statistical (thermal) model and nuclear clusters

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The charge from the organizers:
"the basics of the thermal model and the implications for light (anti-)nuclei formation"

[if too basics, you can easily speed me up, roughly so: "we know all of this" (F.Zwicky)]

- The statistical (thermal) model and the thermal fits
- Thermal fits and the QCD phase diagram
- Thermal model for (hyper)nuclei - recent developments

TH Workshop *Origin of nuclear clusters in hadronic collisions* - CERN (online), 19-20.05.2020

Further reading (general):

Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561 (2018) 321

Decoding the phase structure of QCD via particle production at high energy

...and many dedicated reviews:

Dönigus, IJMPE 29 (2020) 20040001

Light nuclei in the hadron resonance gas

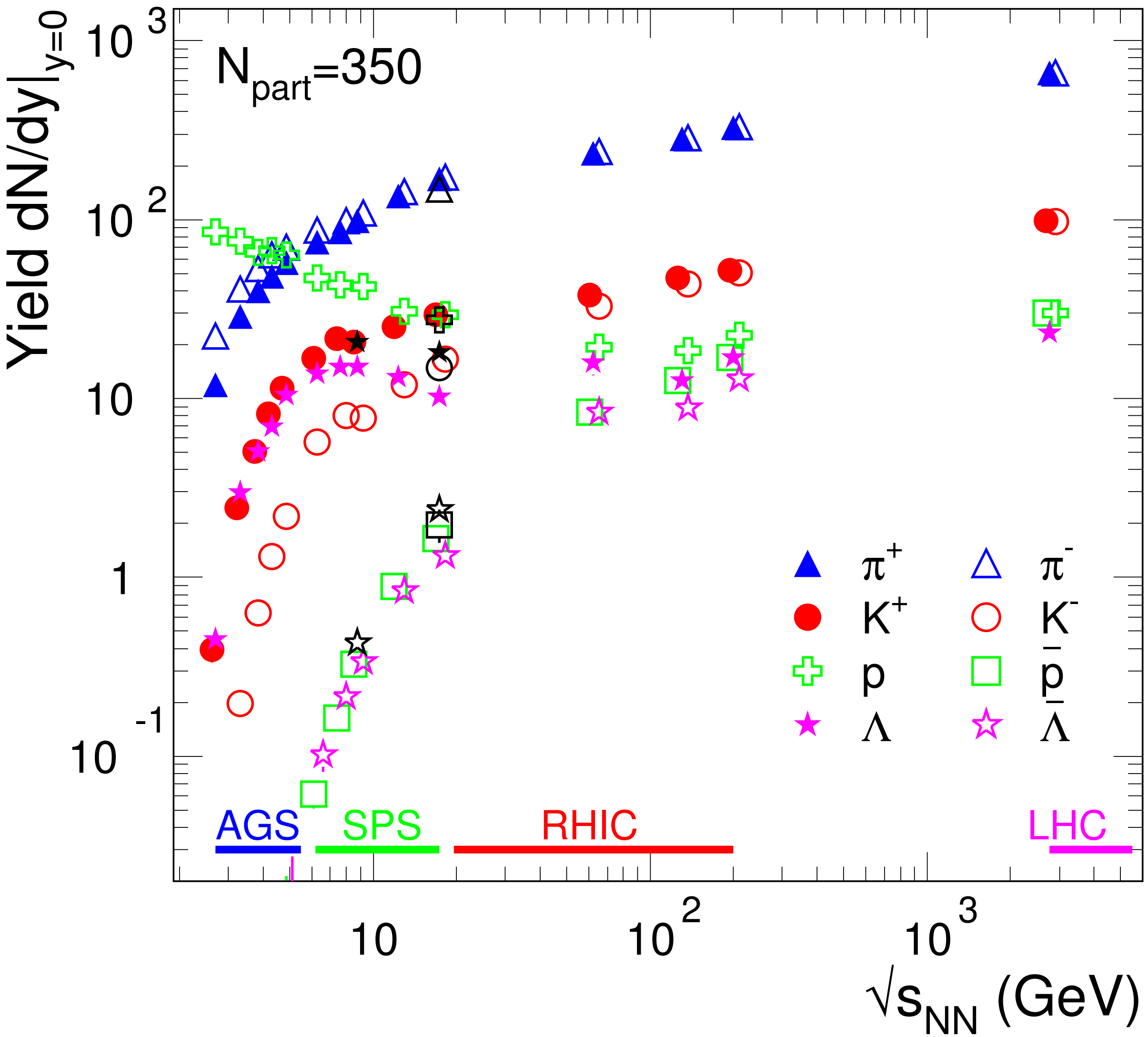
Braun-Munzinger, Dönigus, NPA 987 (2019) 144

Loosely-bound objects produced in nuclear collisions at the LHC

Chen, Keane, Ma, Tang, Xu, Phys.Rep. 760 (2018) 1

Antinuclei in Heavy-Ion Collisions

Hadron yields at midrapidity (central collisions)



A.Andronic, arXiv:1407.5003

- lots of particles, mostly newly created ($m = E/c^2$)
- a great variety of species:

  $\pi^\pm$ $(u\bar{d},\ d\bar{u})$, m=140 MeV
  $K^\pm$ $(u\bar{s},\ \bar{u}s)$, m=494 MeV
  p (uud), m=938 MeV
  Λ (uds), m=1116 MeV
  also: $\Xi(dss)$, $\Omega(sss)$...

- mass hierarchy in production (u, d quarks: remnants from the incoming nuclei)

...natural to think of the thermal (statistical) model ($\mathrm{e}^{-m/T}$)

The thermal model

also known as: statistical / hadron resonance gas / statistical hadronization model

...is in a way the simplest model

the analysis of hadron yields within the thermal model provides a "snapshot" of a nucleus-nucleus collision at chemical freeze-out
(the earliest in the collision timeline we can look with hadronic observables)
test hypothesis of hadron abundancies in equilibrium

...but the devil is in the details ...one needs:

- a complete hadron spectrum (all species of hadrons, see Particle Data Book)
- canonical approach at low energies (and smaller systems)
- to understand the data well (control fractions from weak decays)

microcanonical: describes an isolated system (E, V, T)

canonical and *grand canonical*: suppose a heat reservoir (temperature T)
the system (our collection of hadrons) exchange energy with that

canonical: no particles are exchanged (N, V, T)

grand canonical: the system exchanges particles ($\langle N \rangle$, V, T)
chemical potentials, or fugacities, are introduced to ensure conservation, on average, of particle numbers

here use *grand canonical*
(supplemented when needed with correction for canonical)

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2 \mathrm{d}p \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;
$E_i = \sqrt{p^2 + m_i^2}$ total energy; $(+)$ for fermions (–) for bosons
$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by "initial conditions"

i) isospin: $V_{cons} \sum_i n_i I_{3i} = I_3^{tot}$, with $V_{cons} = N_B^{tot} / \sum_i n_i B_i$

I_3^{tot}, N_B^{tot} isospin and baryon number of the system (=0 at high energies)

ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

Brief summary of cases

General ($\epsilon = +1$ for bosons, $\epsilon = -1$ for fermions; particle index i omitted):

$$N = -T\frac{\partial \ln Z}{\partial \mu} = \frac{g}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E-\mu)/T] \pm 1}$$

$$N = \frac{g}{2\pi^2} T V m^2 \epsilon \sum_{k=1}^{\infty} \frac{\epsilon^k}{k} \mathrm{e}^{\frac{\mu}{T}k} K_2\left(\frac{m}{T}k\right)$$

Classical statistics:

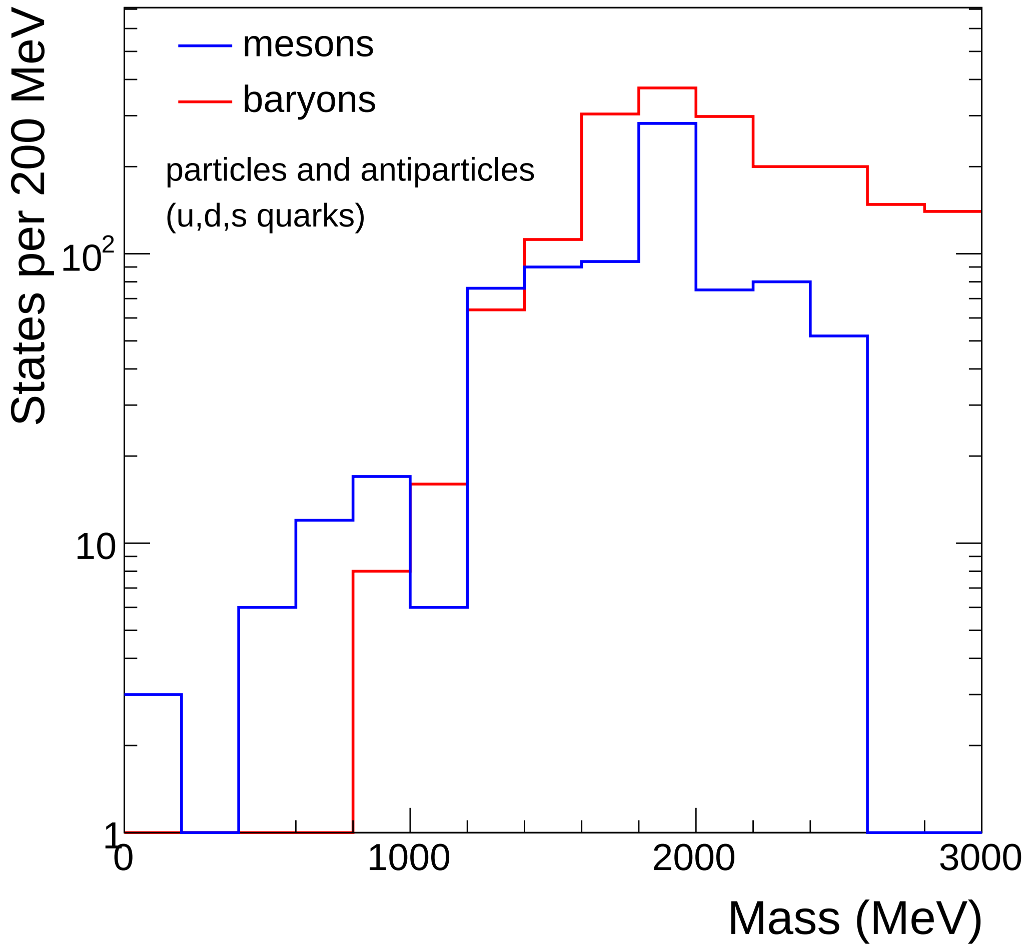
$$N = \frac{g}{2\pi^2} T V m^2 \mathrm{e}^{\frac{\mu}{T}} K_2\left(\frac{m}{T}\right) = g V \mathrm{e}^{\frac{\mu}{T}} \left(\frac{mT}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{m}{T}} \left(1 + \frac{15T}{8m} + O(T^2/m^2)\right)$$

$$K_2(x) = \sqrt{\frac{\pi}{2x}} \mathrm{e}^{-x} \left(1 + \frac{15}{8x} + O(1/x^2)\right)$$

Model input: hadron spectrum

...embodies low-energy QCD ...*vacuum masses*

well-known for $m < 2$ GeV; many confirmed states above 2 GeV, still incomplete



for high m, BR not well known, but can be reasonably guessed

T found to be robust in fits with spectrum truncated above 1.8 GeV

$$\rho(m) = c \cdot m^{-a} \exp\left(m/T_H\right)$$

$T_H \simeq 180$ MeV (max T for hadrons)

$(2J+1)$ counted in

Hadron mass spectrum and Hagedorn's bootstrap model

exponential form of mass spectrum (for large m): $\rho(m) = c \cdot m^{-a} \exp(m/T_H)$
in the statistical bootstrap model (Hagedorn, CERN-TH.3918/84)

"fireballs (hadrons) consist of fireballs, which consist of fireballs, which..."

consider Boltzmann statistics, non-relativistic approximation

$$\ln Z_i(T,V) = \frac{V}{(2\pi)^3}\int e^{-\sqrt{p^2+m^2}/T}\mathrm{d}^3p = \frac{VT}{2\pi^2} \simeq V\left(\frac{T}{2\pi}\right)^{2/3} m_i^{2/3} e^{-m_i/T}$$

$\ln Z = \sum_i \ln Z_i = V\left(\frac{T}{2\pi}\right)^{2/3}\sum_i m_i^{2/3} e^{-m_i/T}$ (sum over all hadrons)

$$\ln Z = V\left(\frac{T}{2\pi}\right)^{2/3}\int_0^\infty \rho(m) m^{3/2} e^{-m/T}\mathrm{d}m$$

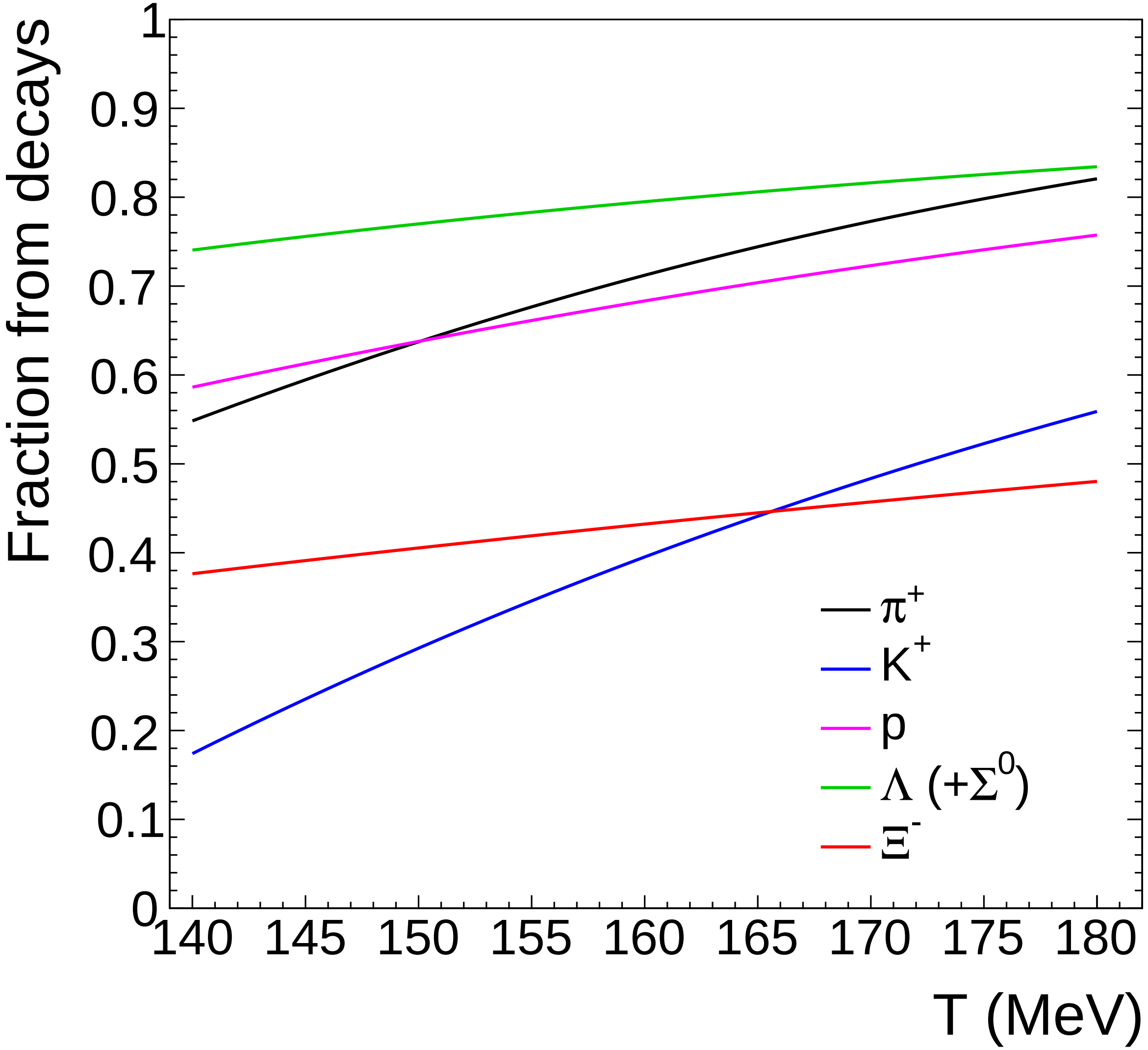
$$\varepsilon(T_H) = \frac{T^2}{V}\frac{\partial \ln Z}{\partial T}(T_H) \sim \frac{3T_H}{2}(m^{5/2-a})|_0^\infty + (m^{7/2-a})|_0^\infty + \mathrm{finite}$$

diverge at T_H for $a < 7/2$

T_H maximal temperature of hadronic systems; beyond: QGP (Cabibbo, Parisi)

Decays (feed-down)

(almost all) hadrons are subject to strong and electromagnetic decays



$\Delta \to p(n) + \pi$, $\rho \to \pi + \pi$

$\Sigma^0 \to \Lambda + \gamma$

weak decays can be treated as well ...to account for the exact experimental situation

contribution of resonances is significant (and particle-dependent)

(plot for μ_B=0)

Considering widths of resonances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$n_i = \frac{g_i}{2\pi^2}\frac{1}{N_{BW}}\int_{M_{thr}}^\infty \mathrm{d}m \int_0^\infty \frac{\Gamma_i^2}{(m - m_i)^2 + \Gamma_i^2/4} \cdot \frac{p^2 \mathrm{d}p}{\exp[(E_i^m - \mu_i)/T] \pm 1}$$

M_{thr} threshold mass for the decay channel.

Example: for $\Delta^{++} \rightarrow p + \pi^+$, M_{thr}=1.068 GeV ($m_{\Delta^{++}}$=1.232 GeV)

Important mainly at "low" temperatures ($T \lesssim$150 MeV)

Hadron densities

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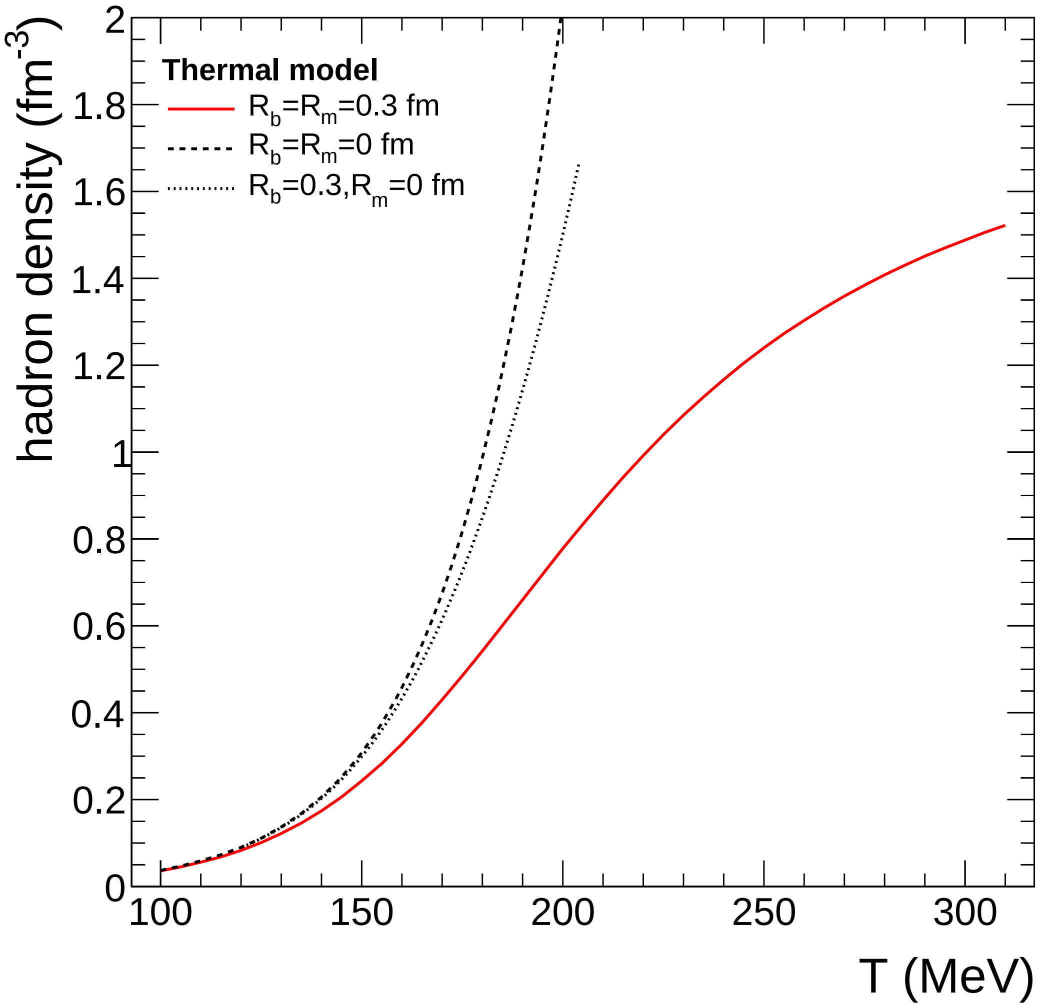
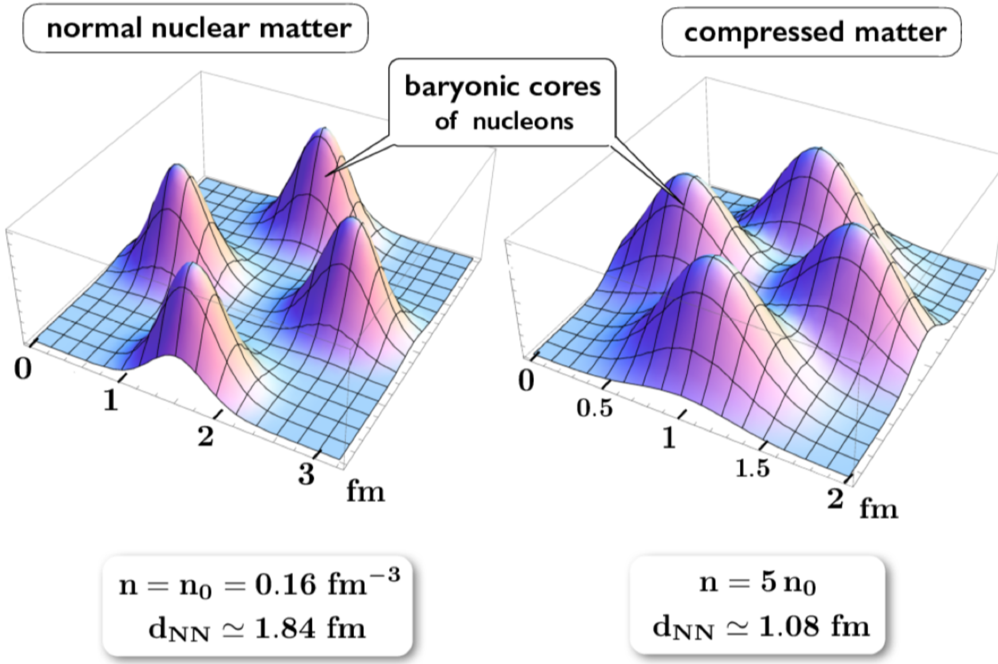




Weise, arXiv:1811.09682

(baryons: gaussians, r=0.5 fm)

"hadron gas": a dense system (also nuclear matter is rather a liquid than a gas)

(the usual case is $R_{baryon} = R_{meson} = 0.3$ fm ...hard-sphere repulsion)

Air at NTP: intermolecule distance $\simeq$50 $\times$ molecule size needed whenever the abundance of hadrons with a given quantum number is very small ...so that one needs to enforce exact quantum-number conservation

in AA collisions:

strangeness at low energies

$$n_{i,S}^{C} = n_{i,S}^{GC} \cdot \frac{I_s(N_S)}{I_0(N_S)}$$

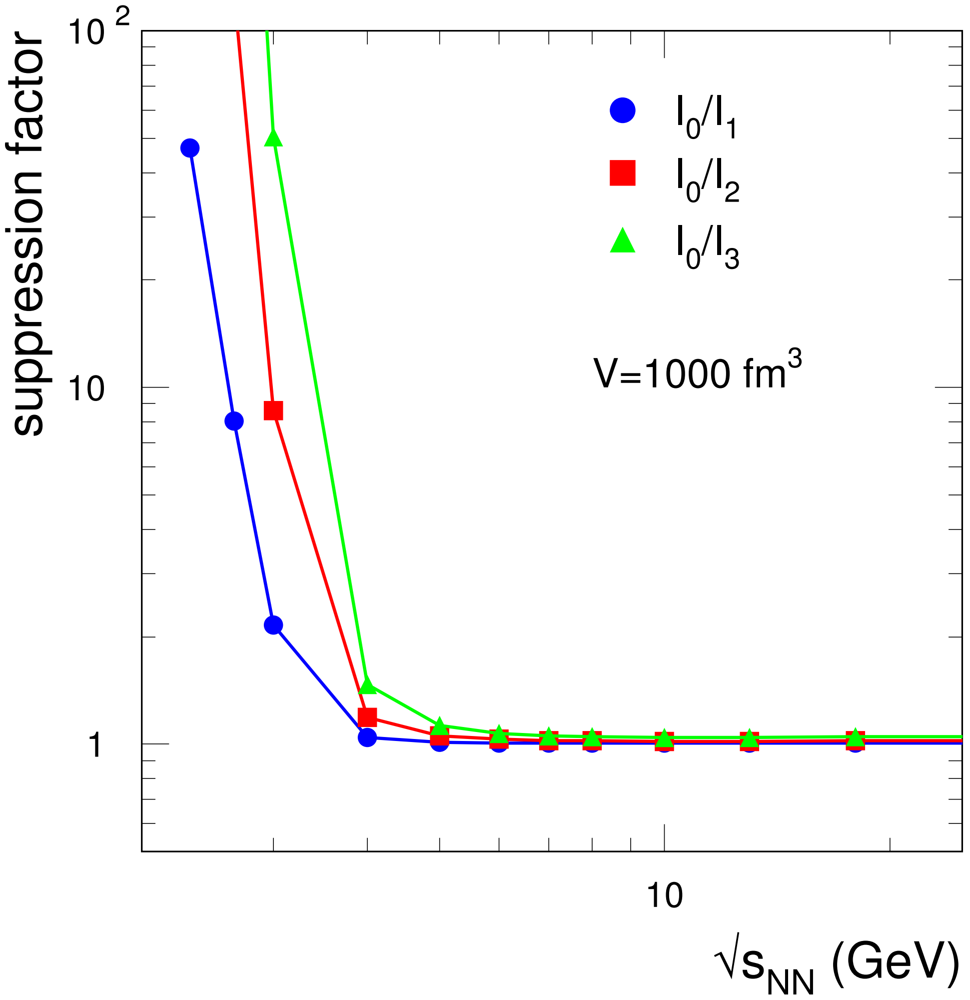
$$N_S = V_c \cdot \sum S \cdot n_{i,S},$$

total amount of strangeness-carrying hadrons (part.+antipart.)

$$n_{K,\Lambda}^{C} = n_{K,\Lambda}^{GC} \cdot \frac{I_1(N_S)}{I_0(N_S)},$$

$$n_{\phi}^{C} = n_{\phi}^{GC}$$

...negligible for $\sqrt{s_{NN}}$ >5 GeV



relevant for small volumes (peripheral AA, pp, p–Pb collisions)

...a non-thermal fit parameter, to check possible non-thermal production of strangeness

for a hadron carrying "absolute" strangeness $s = |S - \bar{S}|$: $n_i \rightarrow n_i \gamma_s^s$

Examples: $K^{\pm}$ $(u\bar{s}, \bar{u}s)$: $n_K \gamma_s$, Λ (uds): $n_\Lambda \gamma_s$,

$$\Xi(dss)\text{: } n_\Xi \gamma_s^2,\ \Omega(sss)\text{: } n_\Omega \gamma_s^3,\ \phi(s\bar{s})\text{: } n_\phi \gamma_s^2$$

in principle, usage of γ_s is to be avoided if one tests the basic thermal model

even as some models employ it ($\Rightarrow \gamma_s = 0.6 - 0.8$), all agree that it is not needed at RHIC, LHC energies (for central collisions)

here (central AA collisions) we fix γ_s=1

Thermal fits of hadron abundances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T] \pm 1}$$

Latest PDG hadron mass spectrum ...quasi-complete up to m=2 GeV;
our code: 555 species (including fragments, charm and bottom hadrons)

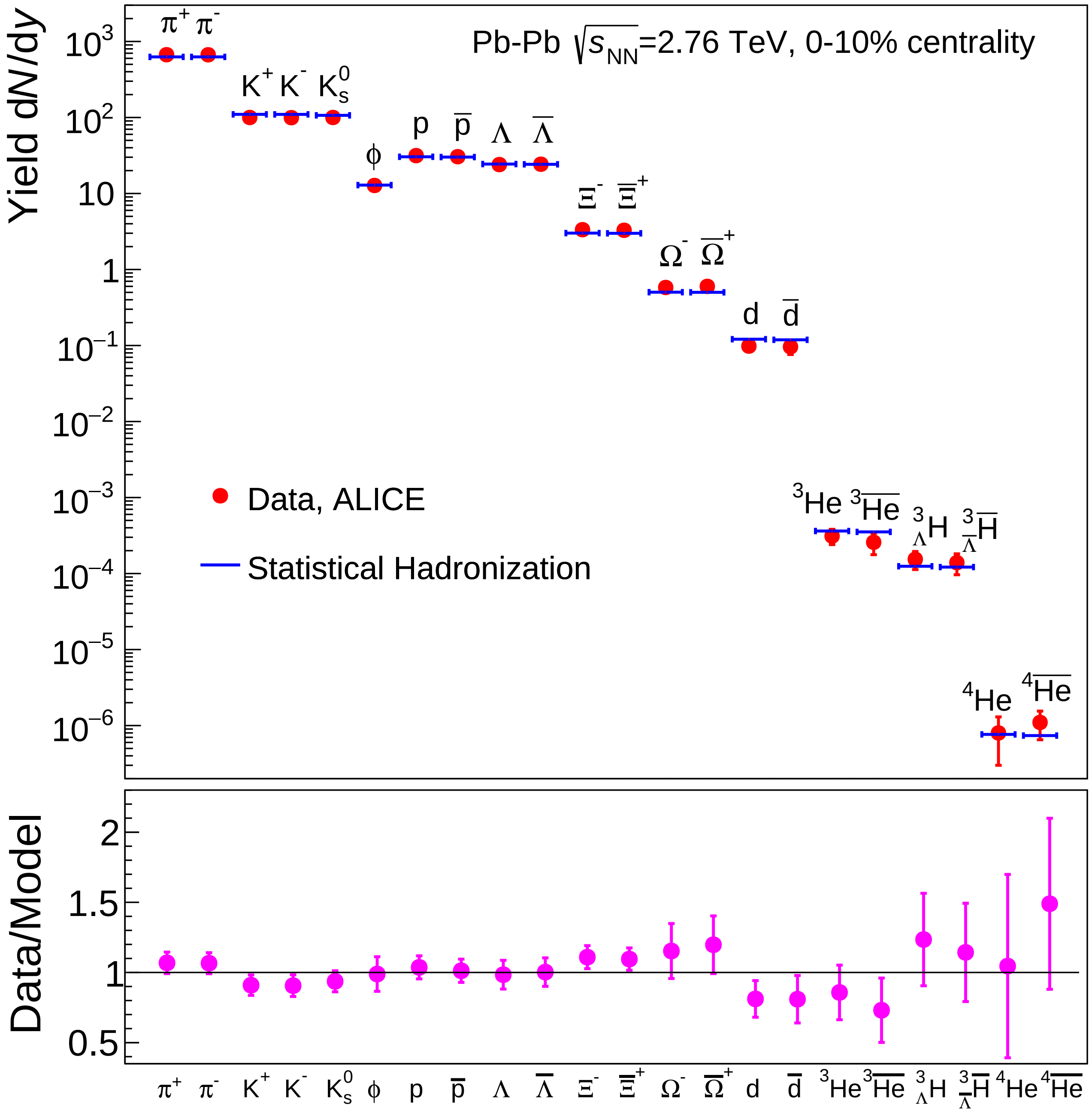
for resonances, the width is considered in calculations

canonical treatment whenever needed (small abundances)

Minimize: $\chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$

N_i hadron yield, σ_i experimental uncertainty (stat.+syst.)

$\Rightarrow$ (T, μ_B, V) *...tests chemical freeze-out* (chemical equilibrium)



matter and antimatter produced in equal amounts

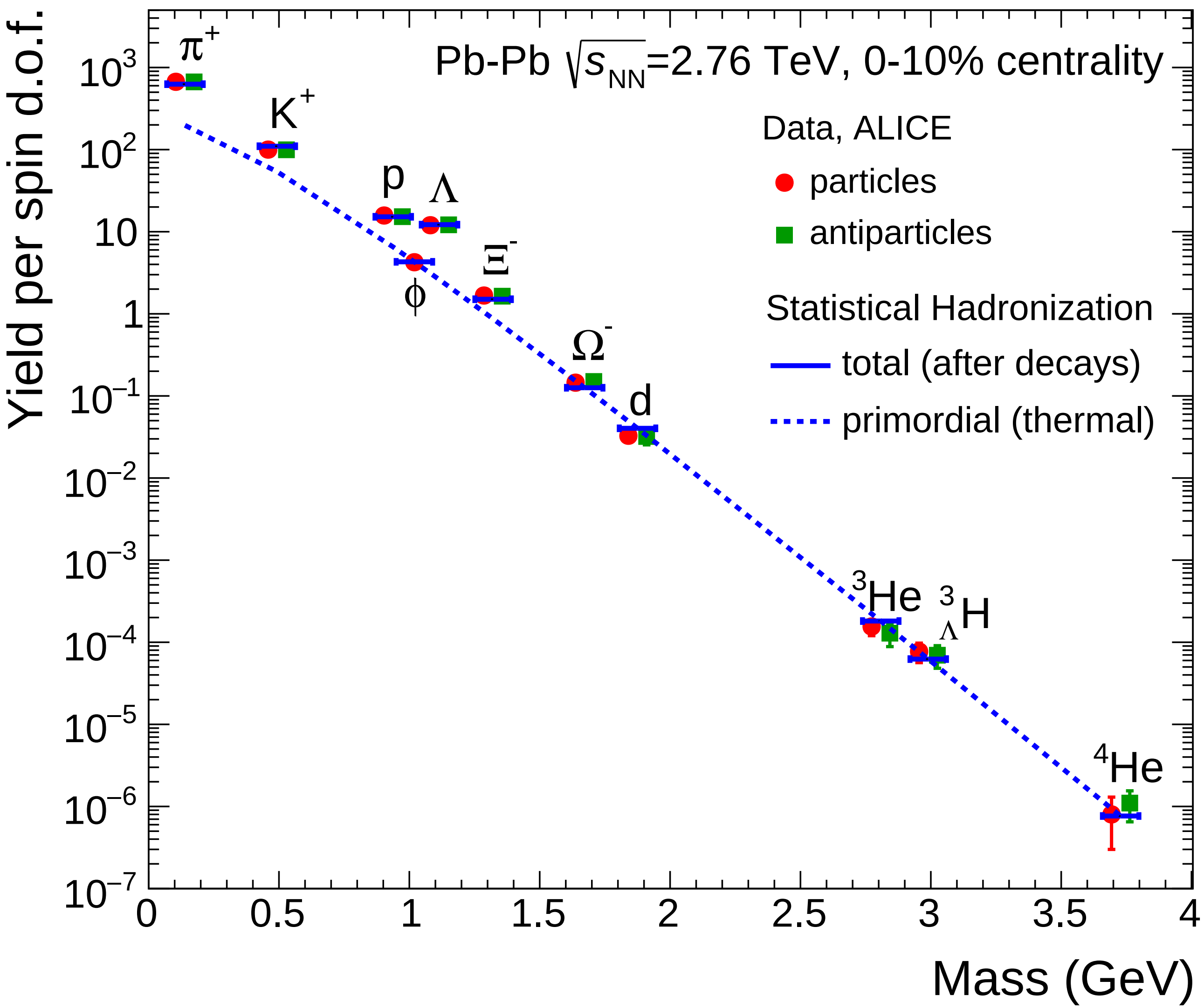
$T_{CF} = 156.6 \pm 1.7$ MeV
$\mu_B = 0.7 \pm 3.8$ MeV
$V_{\Delta y=1} = 4175 \pm 380$ fm^3

$\chi^2/N_{df} = 16.7/19$

remarkably, loosely-bound objects are also well described ($^3_\Lambda$H with 25% B.R.)

hadronization as bags of quarks and gluons?

Model uncertainties 1. Hadron spectrum



contribution of resonances is significant (and particle-dependent)

Fit of ϕ, Ω, d, ^{3}He, $^3_\Lambda$H, ^{4}He:

$T_{CF} = 156 \pm 2.5$ MeV

$(\chi^2/N_{df} = 7.4/8)$

Fit of nuclei (d, ^{3}He, ^{4}He):

$T_{CF} = 159 \pm 5$ MeV

3-4 MeV upper bound of systematic uncertainty due to hadron spectrum

hadron eigenvolumes ...to mimick interactions (beyond low-density, Dashen-Ma)

we consider that $R_{meson} = 0.3, R_{baryon} = 0.3$ *fm is a reasonable case*

point-like hadrons lead to same T, but volume larger by 20-25%

an extreme case, $R_{meson} = 0, R_{baryon} = 0.3$ fm leads to
$T = 161.0 \pm 2.0$ MeV, $\mu_B = 0$ *fixed*, $V = 3470 \pm 280$ fm^3

NB: in this case, the result is rather sensitive on the set of hadrons in the fit
for instance, using hadrons up to Ω, cannot constrain T (unphysically large)
Vovchenko, Stöcker (et al.), JPG 44 (2017) 055103, arXiv:1606.06350

...and anything else can be imagined, see (R dependent on mass & strangeness)
Alba, Vovchenko, Gorenstein, Stöcker, NPA 974 (2018) 22, etc.

T-dependent Breit-Wigner resonance widths:
Vovchenko, Gorenstein, Stöcker, PRC 98 (2018) 034906

...for now only at the LHC ($\mu_B \simeq$0)

non-strange baryon sector treated in S-matrix formalism
(πN scattering phase shifts, *including non-resonant contributions*)

PLB 792 (2019) 304

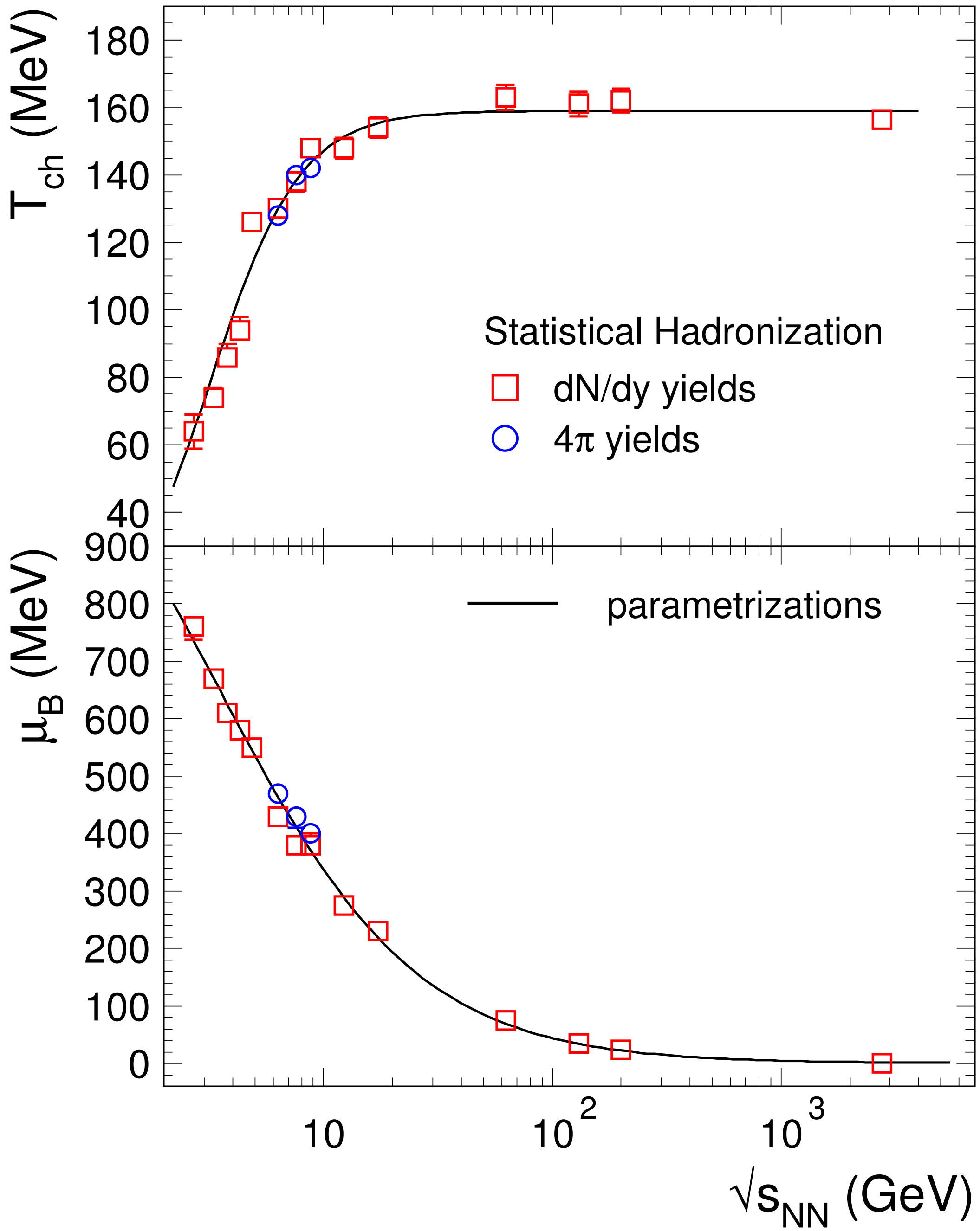
solved the so-called "proton puzzle" (too many protons in the statistical model)

for T=156 MeV, proton yield decreased by 17% compared to point-like

still missing: strange baryon sector ($\sim$7% of protons from Λ^*, Σ^*, T-dep.)
PRC 98 (2018) 044910

NB: presence of resonances implies interaction
(this is why moderate $R = 0.3$ fm is a reasonable choice)

Energy dependence of T, μ_B (central collisions)



thermal fits exhibit a limiting temperature:

$$T_{lim} = 158.4 \pm 1.4 \text{ MeV}$$

$$T_{CF} = T_{lim} \frac{1}{1+\exp(2.60-\ln(\sqrt{s_{NN}}(\mathrm{GeV}))/0.45)}$$

$$\mu_B[\mathrm{MeV}] = \frac{1307.5}{1+0.288\sqrt{s_{NN}}(\mathrm{GeV})}$$

NPA 772 (2006) 167, PLB 673 (2009) 142

μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

determined by the "stopping" of the colliding nuclei

The grand (albeit partial) view

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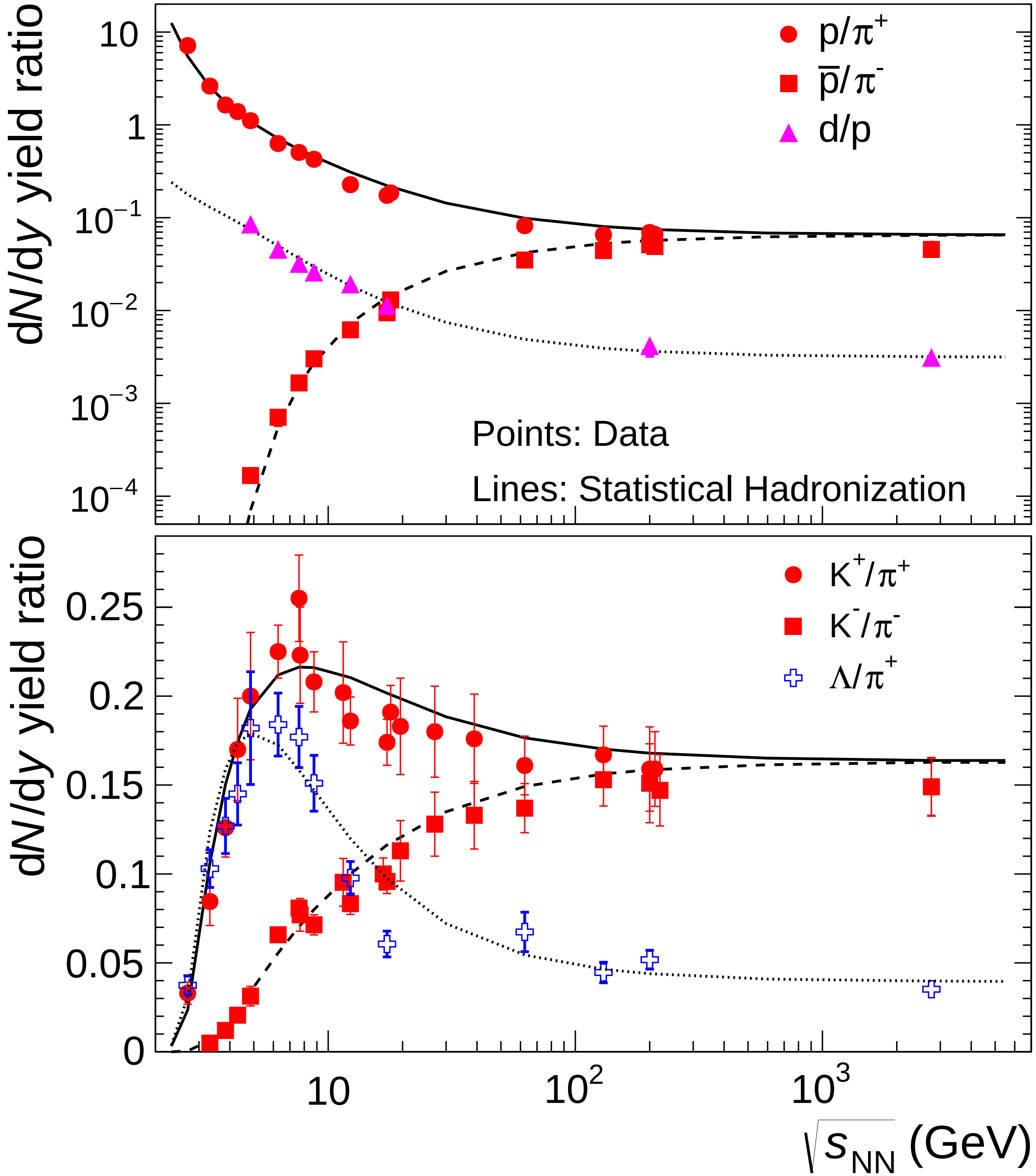


Data:

AGS: E895, E864, E866, E917, E877

SPS: NA49, NA44

RHIC: STAR, BRAHMS

LHC: ALICE

NB: no contribution from weak decays

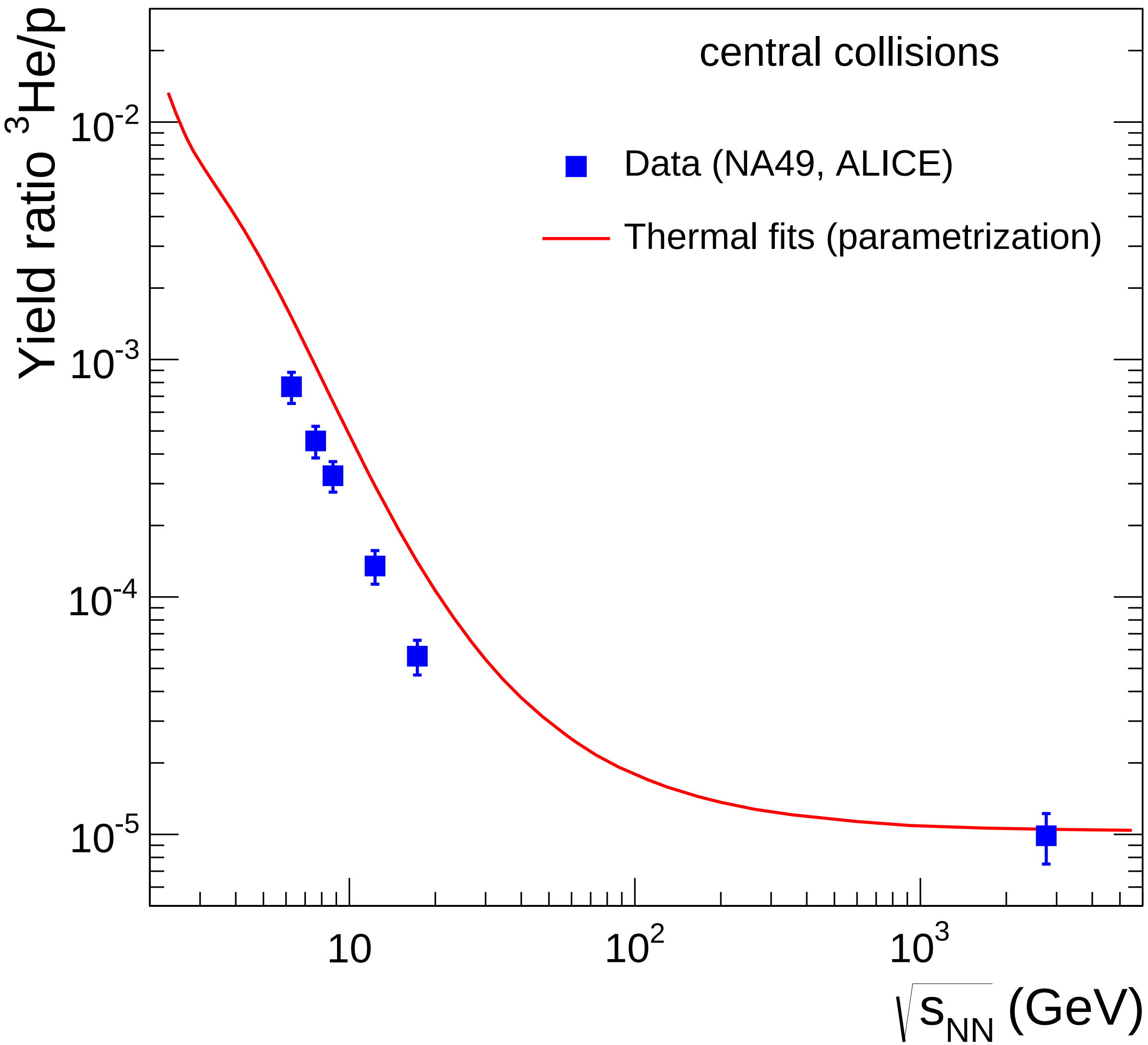
d/p ratio is well described for all energies

"structures" described by SHM

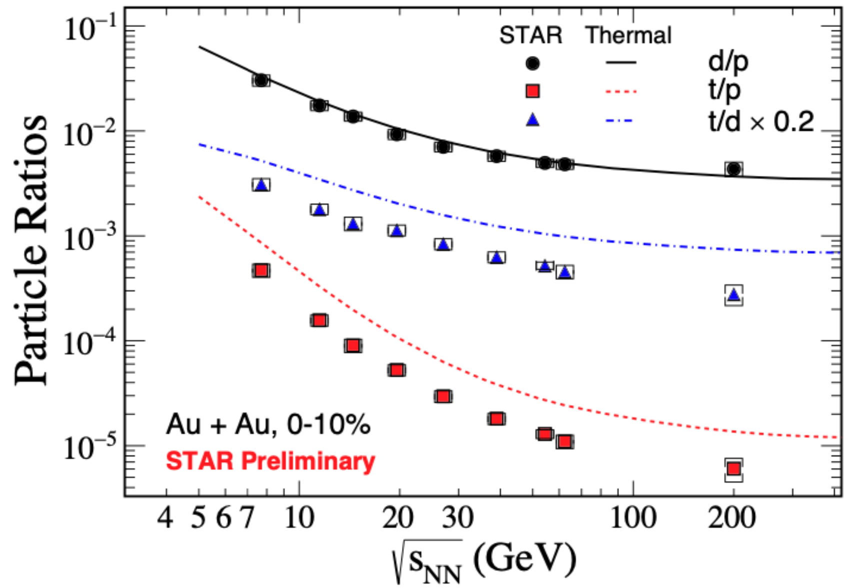
...determined by strangeness conservation

Λ/π peak reflects increasing T and decreasing μ_B

^{3}He/p ratio well described at the LHC, but not at lower energies



(prelim.) STAR BES data confirm this discrepancy

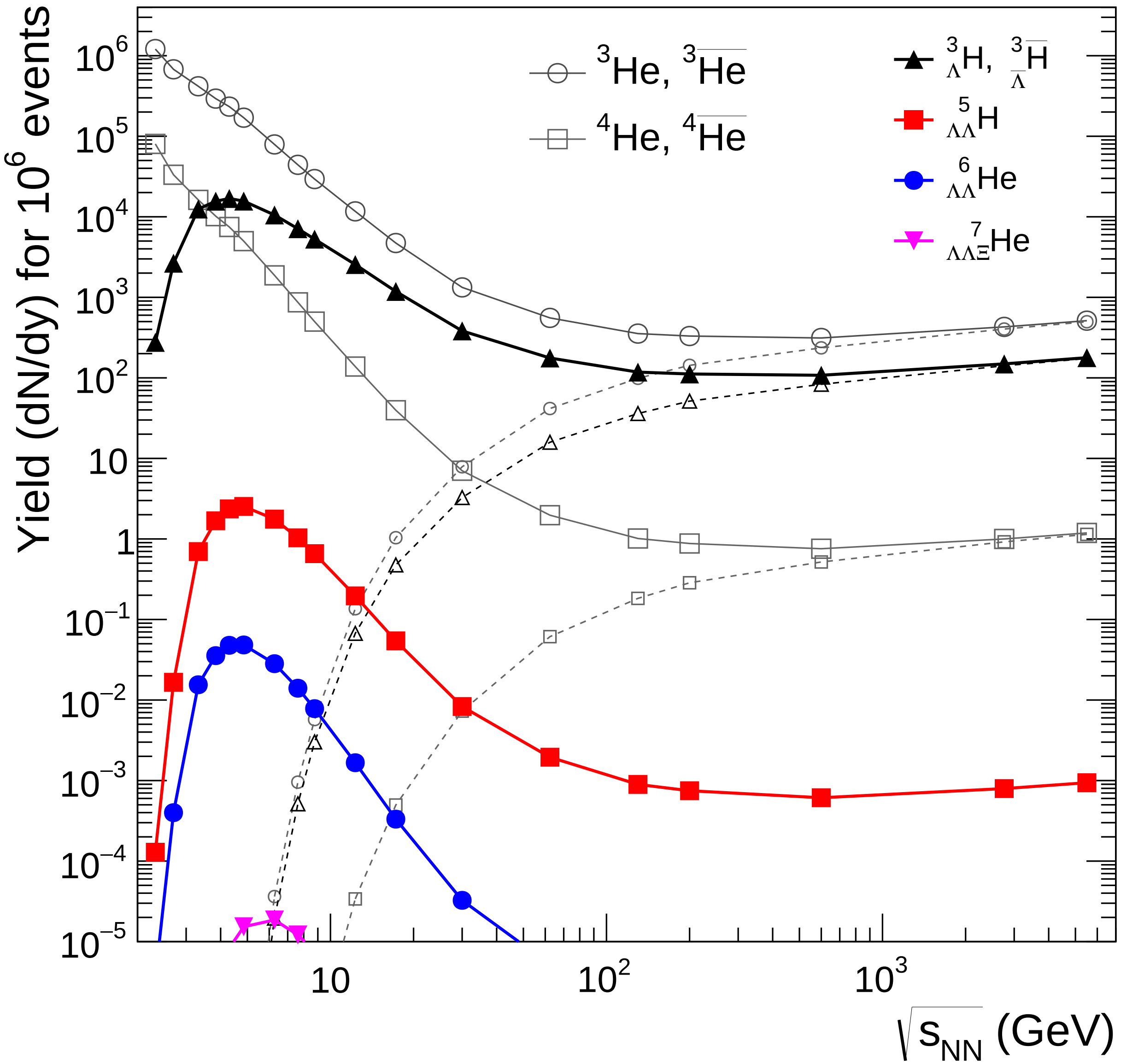


Zhang (STAR), QM'19, arXiv:2002.10677

does it have to do with y distributions?

Complex objects

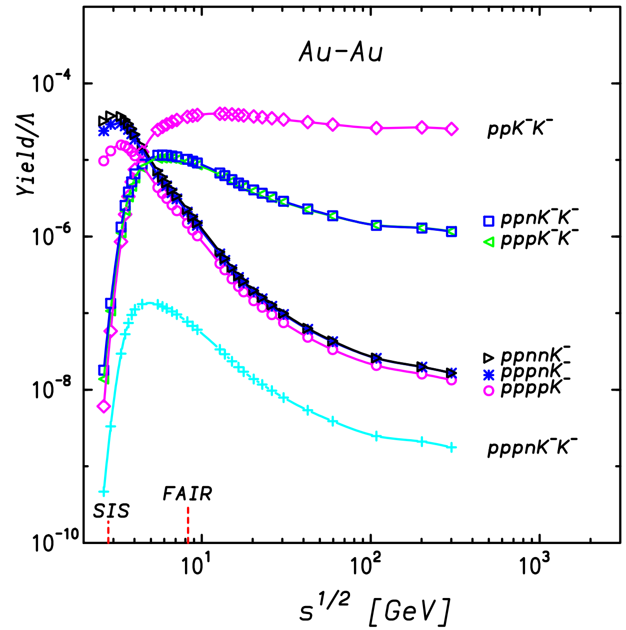
...are copiously produced at low (FAIR) energies

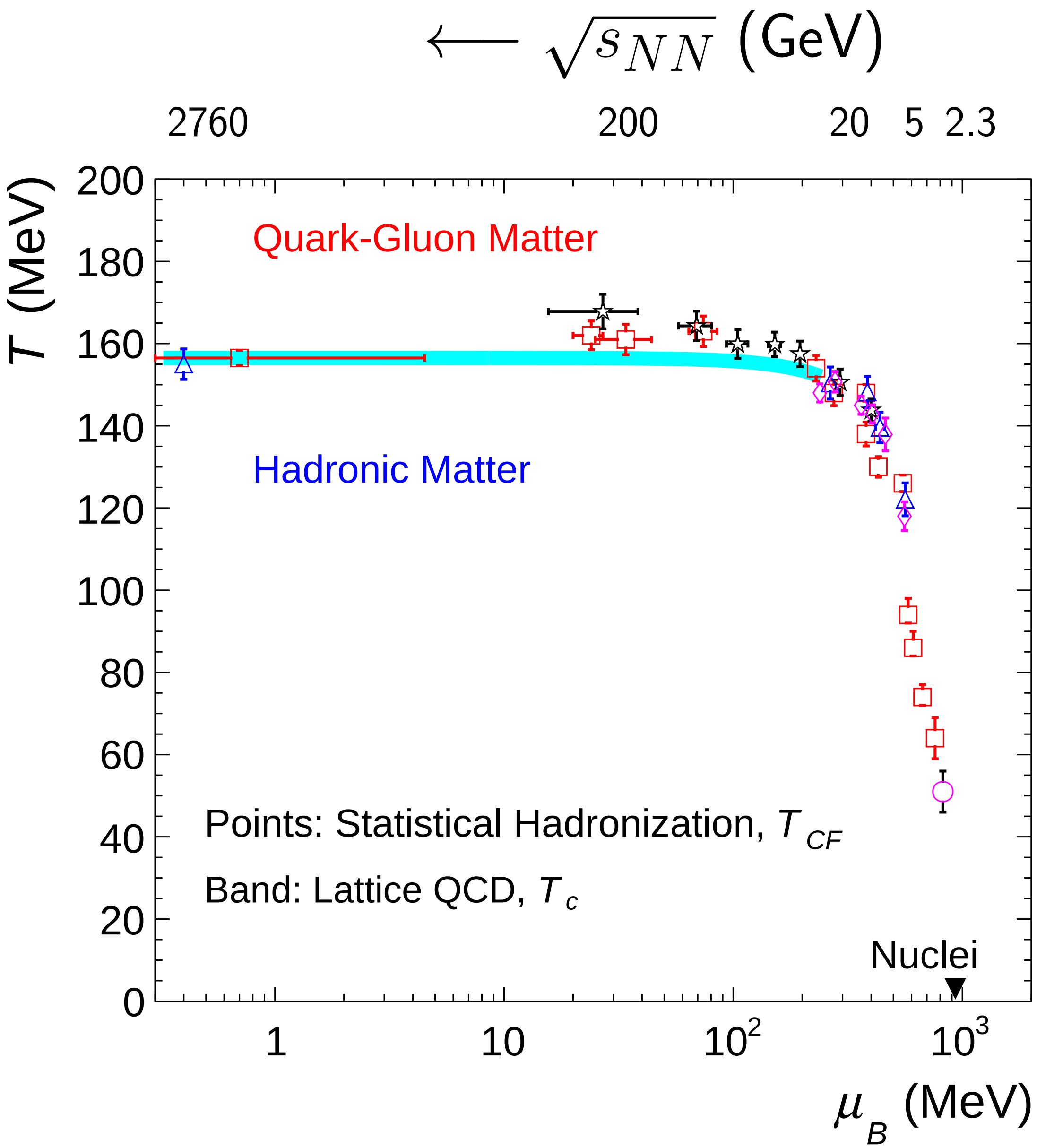


...some to be discovered

maybe also nucleon-K^- clusters?

AA, PBM, K.Redlich, NPA 765 (2006) 211





at LHC, remarkable "coincidence" with Lattice QCD results

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

$\mu_B > 0$: more matter, from "remnants" of the colliding nuclei

$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery* (at FAIR?)

μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry a subject of quite some debate

hadrons are not necessarily point-particles

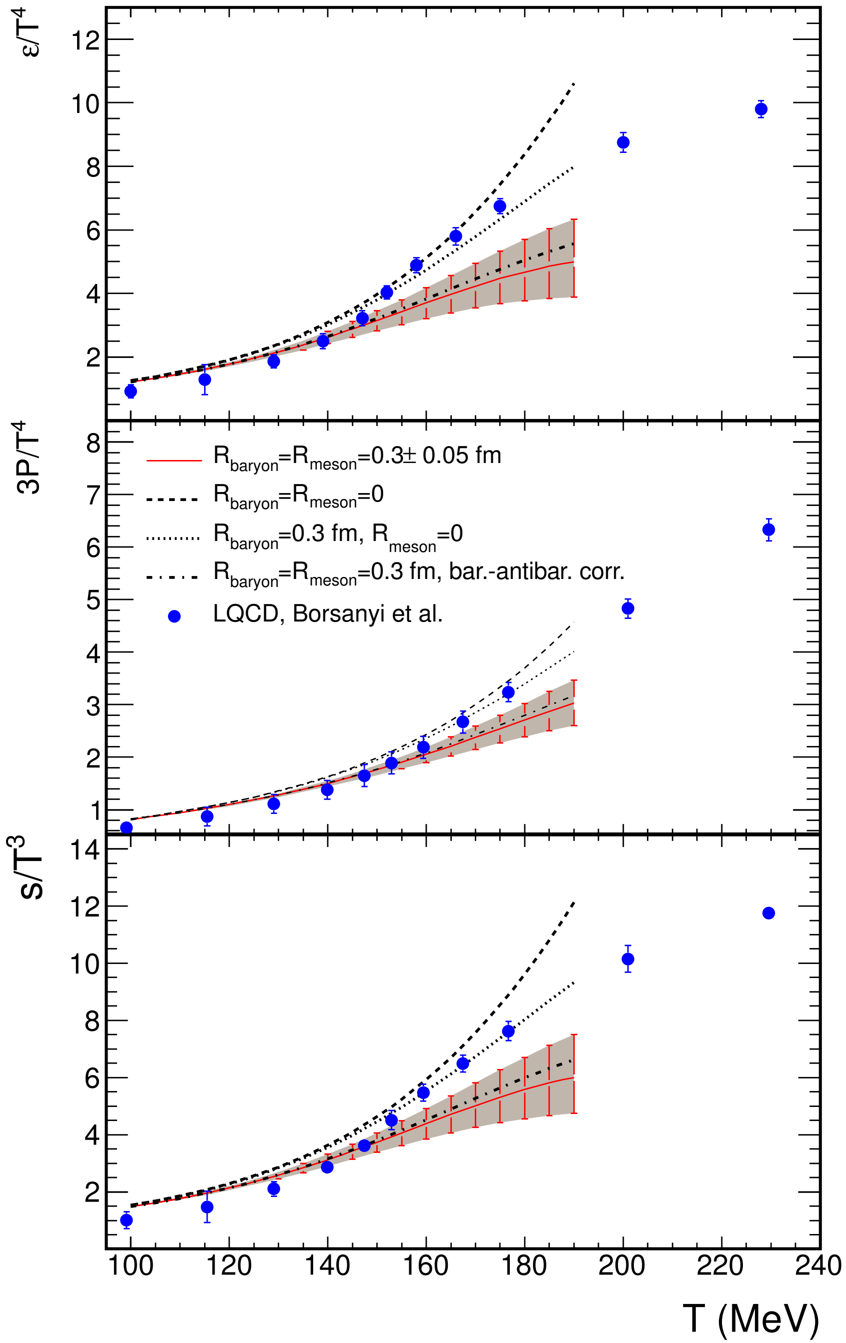
an "excluded-volume" (hard sphere re-pulsion) is often employed

AA et al., PLB 718 (2012) 80

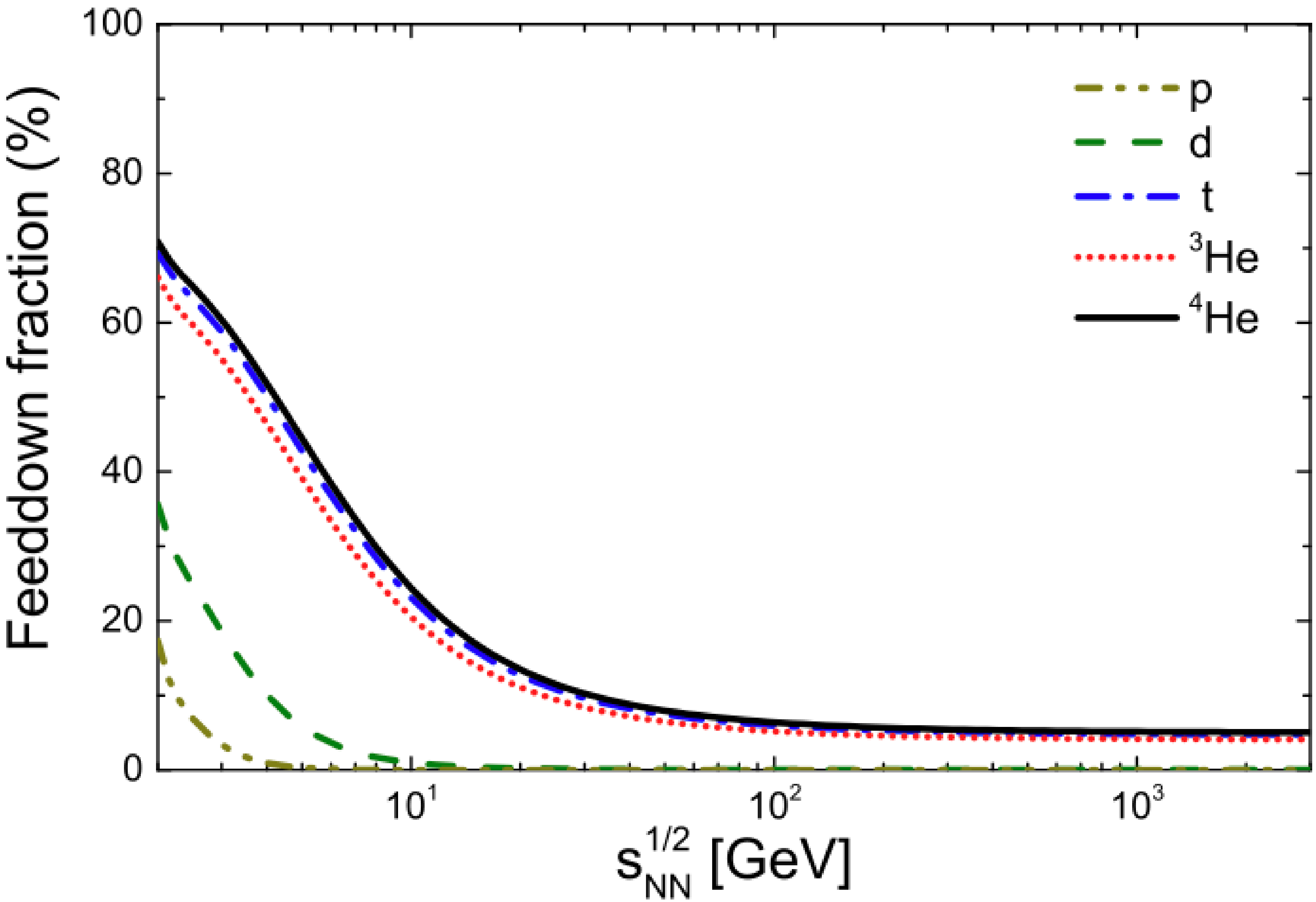
Several versions exist
for instance via van de Waals int.

Vovchenko, Gorenstein, Stöcker, PRL 118 (2017) 182301

up to T=200 MeV
(does it make sense to extend HRG to 200 MeV?)

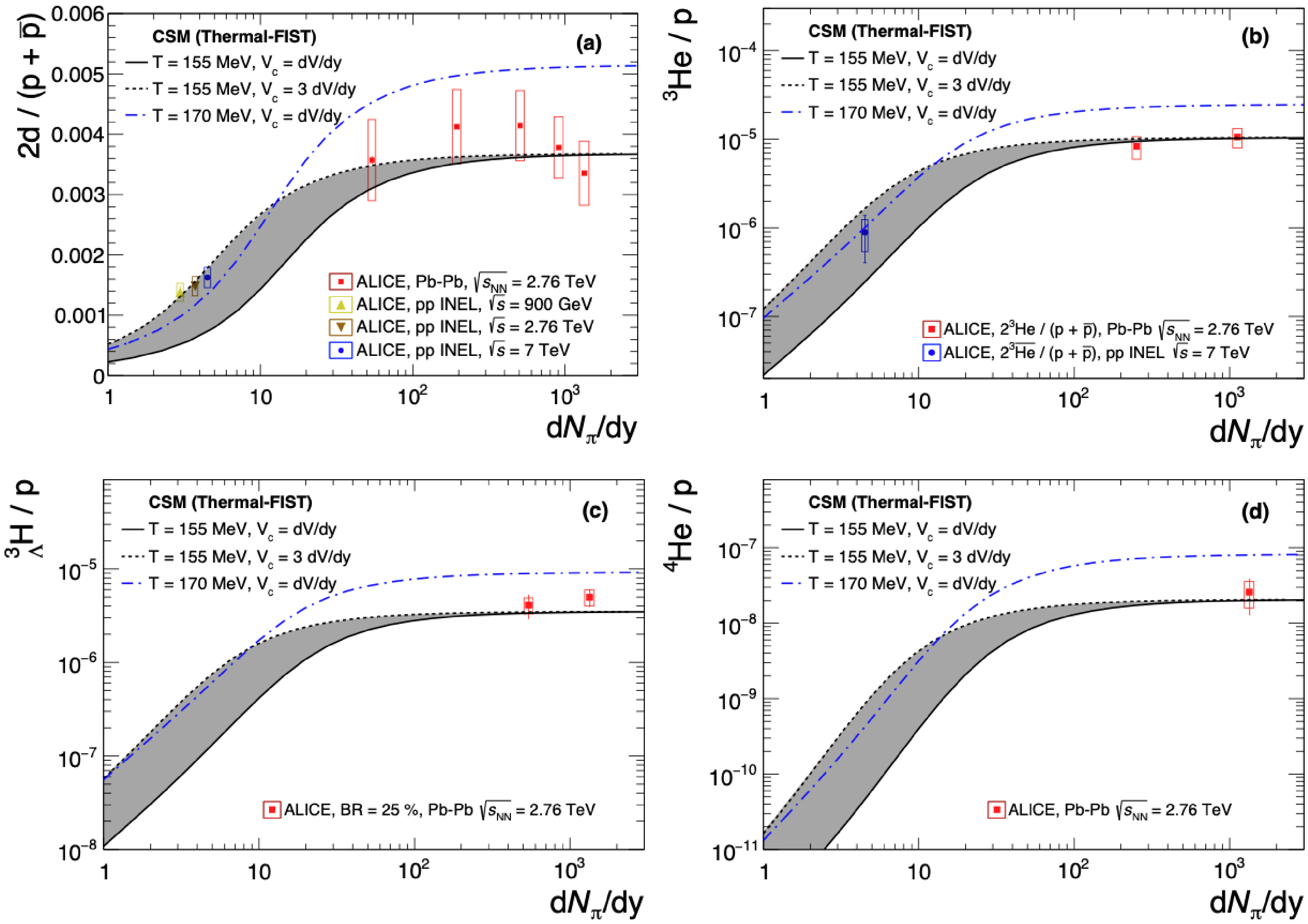


Feed-down from excited nuclear levels



Levels up to 30 MeV (15 for ^{4}He)

Vovchenko et al., arXiv:2004.04411



Vovchenko, Dönigus, Stöcker, PLB 785 (2018) 171

Summary

- abundance of hadrons with light quarks consistent with chemical equilibration
- there is a variety of approaches ... *a personal bias: the "minimal model"*

  a minimal set of parameters, means a well-constrained model
- the thermal model provides a simple way to access the QCD phase boundary

  ...at high energies (at low energies canonical suppression needs more care)

  ...but is it more than a 1st order description (of loosely-bound objects)?

  ...and what fundamental point does it make about hadronization?

  (statistical features dominate, but understanding still missing as a dynamical process)
- more insights from higher moments and from heavier (charm) quarks

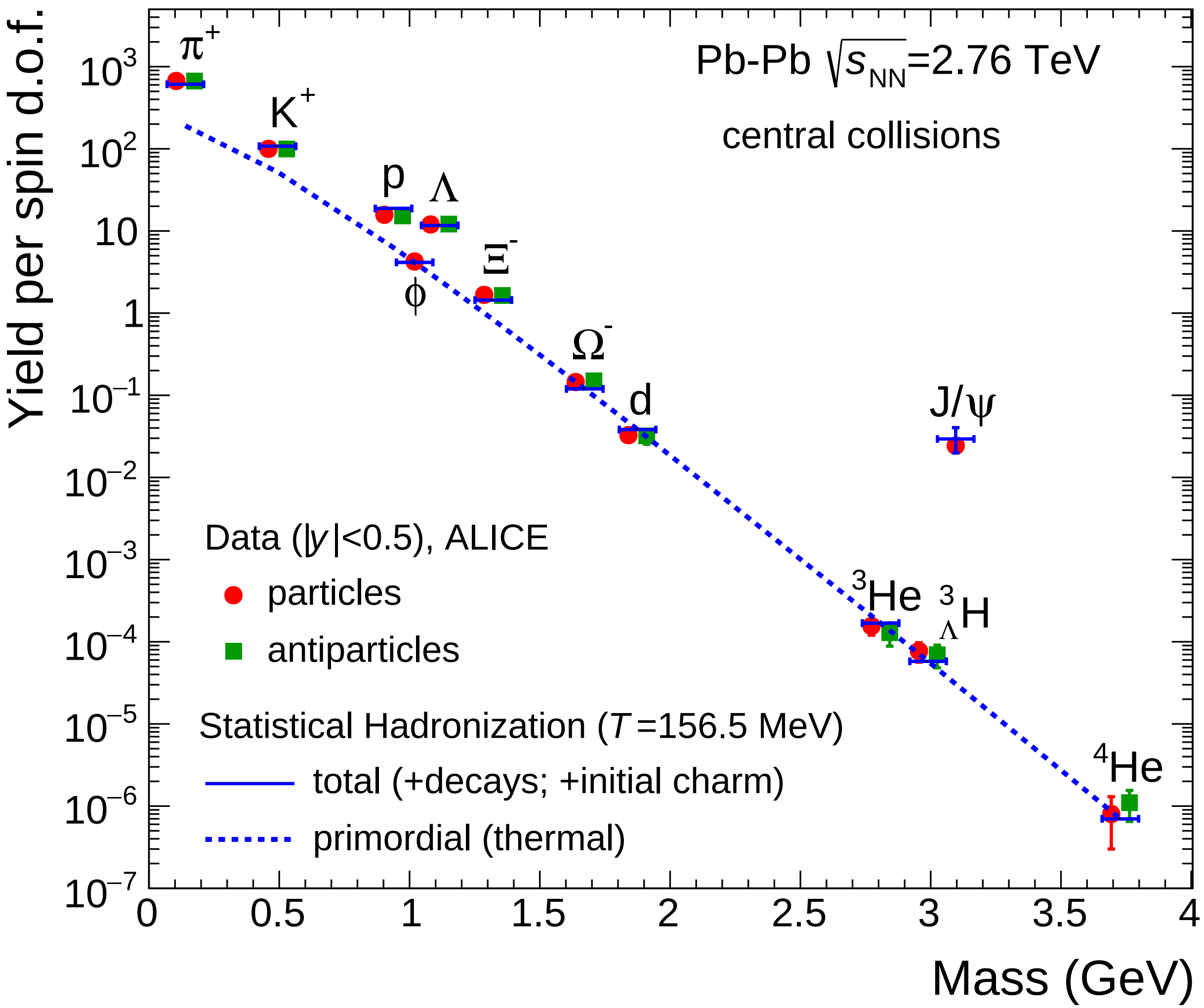
SHM for charm

"throw in" (pQCD production)

$N_{c\bar{c}} = 9.6$ ($I_1/I_0 = 0.974$)

$g_c = 30.1$

Model predicts $\psi(2S)$ and $X(3872)$



π, $K^{\pm}$, K^0 from charm included in the thermal fit (0.7%, 2.9%, 3.1% for T=156.5 MeV)

PLB 797 (2019) 134836