The Hypertriton as an Efimov State

H.-W. Hammer

Institut für Kernphysik, TU Darmstadt and Extreme Matter Institute EMMI

Bundesministerium für Bildung und Forschung

Deutsche Forschungsgemeinschaft DFG

SFB 1245

“Origin of nuclear clusters in hadronic collisions”, May 19-20, 2020

Outline

TECHNISCHE UNIVERSITÄT DARMSTADT

- Threshold bound states and the unitary limit
- Limit cycles and Efimov physics
- Hypertriton
  - Structure
  - Lifetime
- Summary and Outlook

Collaborators: F. Hildenbrand (TU Darmstadt)

References:

Braaten, HWH, Phys. Rep. **428** (2006) 259

HWH, Nucl. Phys. **A705** (2002) 173

Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

Hildenbrand, HWH, in preparation

Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \implies i/k$$

  - Scattering amplitude scale invariant, saturates unitarity bound

Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \implies i/k$$

  - Scattering amplitude scale invariant, saturates unitarity bound
- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg \ell \sim r_e, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal dimer with energy $B_2 = -1/(ma^2)$ $(a > 0)$

    size $\langle r^2 \rangle^{1/2} \sim a$ $\Rightarrow$ halo state

Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \Longrightarrow i/k$$

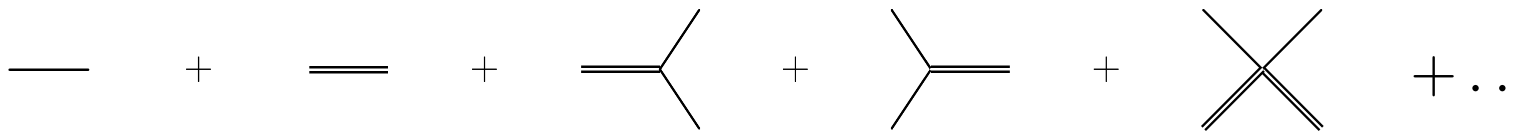
  - Scattering amplitude scale invariant, saturates unitarity bound
- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg \ell \sim r_e, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal dimer with energy $B_2 = -1/(ma^2)$ $(a > 0)$

    size $\langle r^2 \rangle^{1/2} \sim a$ $\Rightarrow$ halo state
- But Efimov effect in certain channels

EFT for the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$\mathcal{L}_{eff} =$ 

- 2-body amplitude: 

- 2-body coupling g_2 near fixed point $(1/a = 0)$

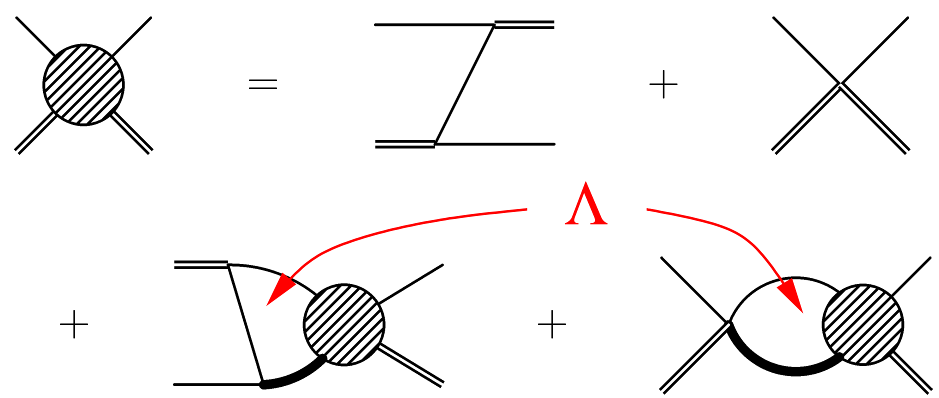
$\Rightarrow$ scale and conformal invariance $\Longleftrightarrow$ unitary limit

(Mehen, Stewart, Wise, 2000; Nishida, Son, 2007; ...)

- 3-body amplitude:

$g_3(\Lambda)$ $\Rightarrow$ limit cycle

$\Rightarrow$ discrete scale inv.



Three-Body Force: Limit Cycle

TECHNISCHE UNIVERSITÄT DARMSTADT

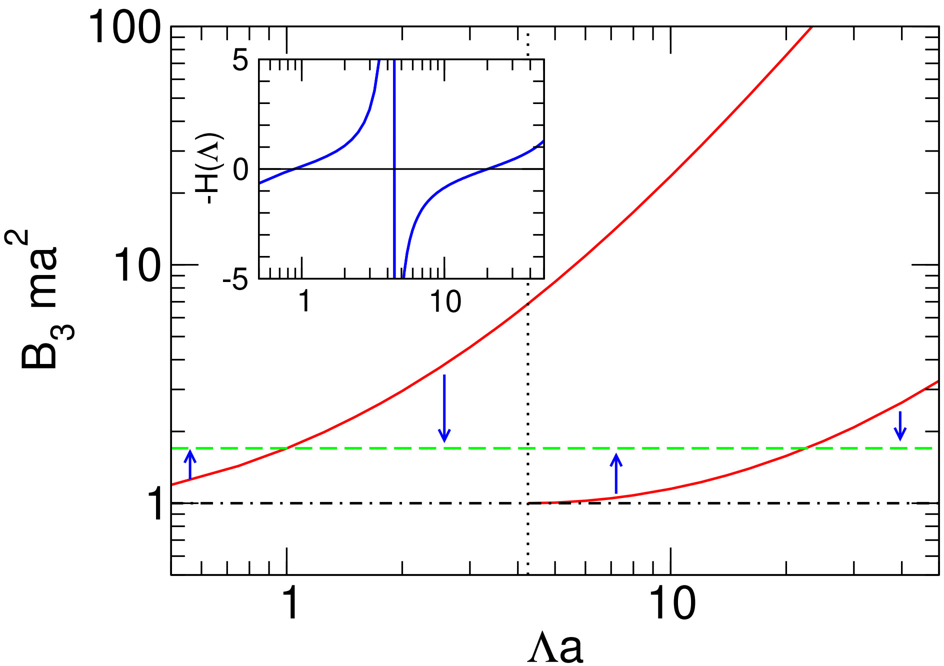
- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3\Lambda^2/(9g_2^2)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup



$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

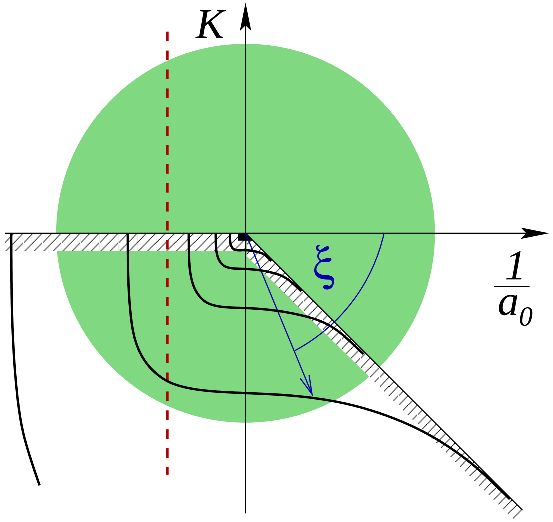
(Bedaque, HWH, van Kolck, 1999)

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance $\Longleftrightarrow$ Efimov physics

Limit Cycle: Efimov Physics

TECHNISCHE UNIVERSITÄT DARMSTADT

- Universal spectrum of three-body states (Efimov, 1970)



- Window of universality
- Discrete scale invariance for fixed angle ξ
- Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a\to 0} \quad e^{2\pi/s_0} = 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances
- Nuclei $\Longrightarrow$ universal correlations and scaling relations

Λd-System and the Hypertriton

TECHNISCHE UNIVERSITÄT DARMSTADT

- Hypertriton
  - $np\Lambda$ bound state with $J^P = \frac{1}{2}^+$, $I = 0$
  - Λd separation energy: $B^\Lambda = 0.13 \pm 0.05$ MeV
  - total binding energy: $B_3^\Lambda = 2.35$ MeV
- EFT for large scattering lengths

  $\Longrightarrow$ shallow hypertriton follows naturally
- Leading order EFT $\Rightarrow$ S-wave interactions
  - ${}^3S_1(NN) + \Lambda \longrightarrow a_d \sim 1/\gamma_d$
  - ${}^3S_1(\Lambda N) + N \longrightarrow a_3 \sim 1/\gamma_3$
  - ${}^1S_0(\Lambda N) + N \longrightarrow a_1 \sim 1/\gamma_1$
- Scattering lengths large compared to interaction range

  ($NN \to \pi$-exchange, $\Lambda N \to 2\pi$-exchange)

Low-Energy ΛN-System

- ΛN system unbound
- (Old) effective range analyses inconclusive (few data at relatively high energies)

$0 > a_1 > -15$ fm $\qquad 0 < r_1 < 15$ fm

-0.6 fm $> a_3 > -3.2$ fm $\qquad 2.5$ fm $< r_3 < 15$ fm

- Extractions using hyperon-nucleon potentials

$a_1 \approx -2.9$ fm, $\quad a_3 \approx -1.6$ fm, $\quad \gg R \sim 1/(2m_\pi)$

(chiral EFT: Haidenbauer et al., Nucl. Phys. A **915** (2013) 24)

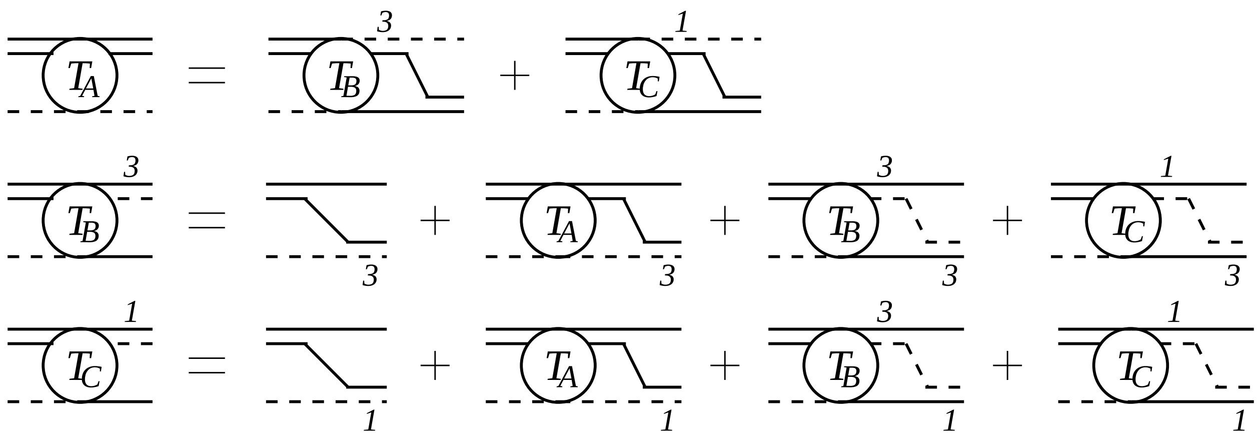
- Characteristic three-body momentum

$$\gamma_3^\Lambda \sim 2\sqrt{|MB_3^\Lambda - \gamma_d^2|/3} \approx 14 \text{ MeV} \ll \sqrt{m_\Lambda(m_\Sigma - m_\Lambda)} \approx 300 \text{ MeV}$$

$\Rightarrow$ $\Lambda\Sigma$ conversion is short range $\quad \Longrightarrow \quad$ three-body force

Integral Equations for Hypertriton

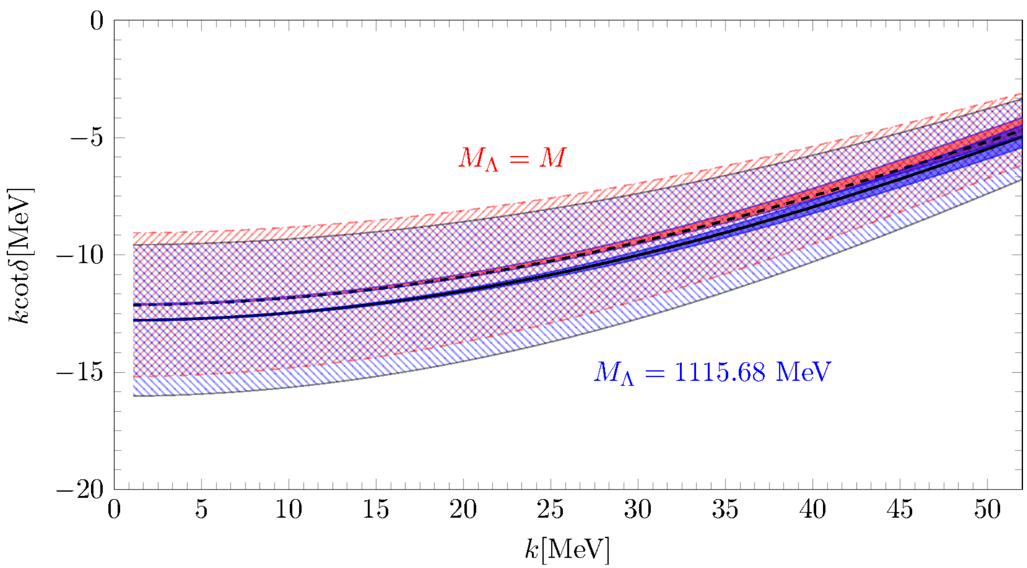
TECHNISCHE UNIVERSITÄT DARMSTADT



HWH, Nucl. Phys. **A705** (2002) 173 Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

- Strong cutoff dependence $\Longrightarrow$ renormalize with Λnp three-body force (cf. triton, bosons)
- Limit cycle with $s_0 = 1.0076$ (unequal masses)
- Scaling factor: $\exp \pi / s_0 \approx 22.60$
- Corrects error in original publication
- No room for excited states....

Λd Scattering



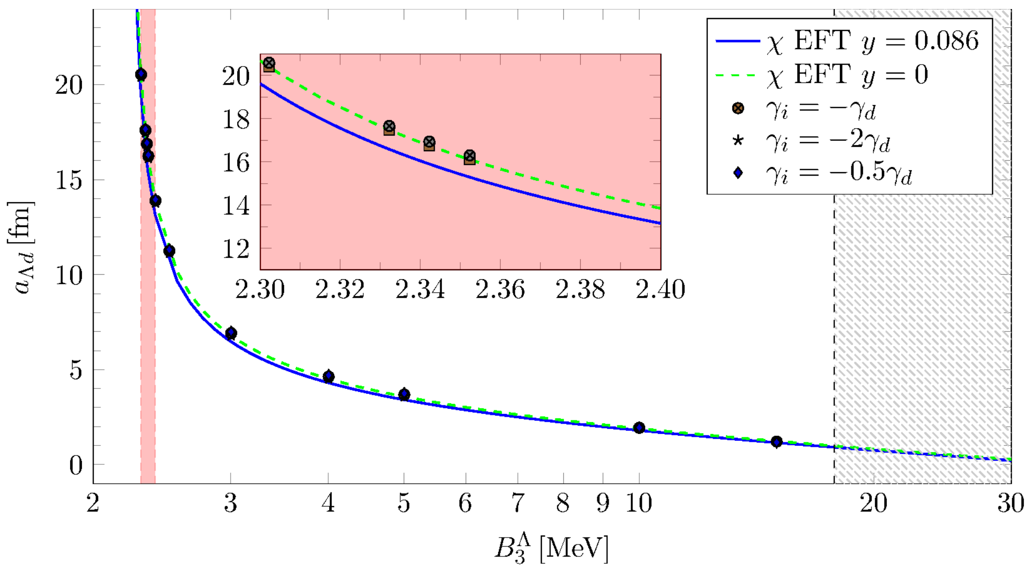
Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

- Exact value of γ_i not determined by B_3^Λ
- Phase shifts independent of γ_i $\Longleftarrow$ shallowness of hypertriton
- Low-energy parameters:

$$a_{\Lambda d} = 15.4 \text{ fm} \quad \text{and} \quad r_{\Lambda d} = 1.3 \text{ fm}$$

Λnp Phillips Line

TECHNISCHE UNIVERSITÄT DARMSTADT

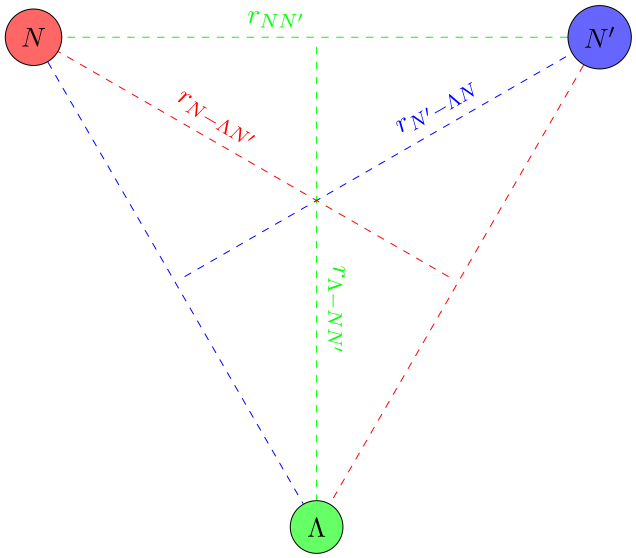


Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

- Correlation between hypertriton triton binding energy and $S = 1/2$ Λd scattering length (cf. Phillips '68)
- Sensitivity to specific values of γ_i only for deeper binding
- Hypertriton wave function can also be extracted

$\Rightarrow$ Matter radii

Hypertriton Radii



- Matter radii for the hypertriton ($B_3^\Lambda = 2.35$ MeV)

$\sqrt{\langle r^2_{\Lambda-NN'}\rangle}$[fm]	$\sqrt{\langle r^2_{N'-\Lambda N}\rangle}$[fm]	$\sqrt{\langle r^2_{N-N'\Lambda}\rangle}$[fm]	$\sqrt{\langle r^2_{NN'}\rangle}$[fm]
10.79	3.96	4.02	2.96
$+3.04/-1.53$	$+0.40/-0.25$	$+0.41/-0.25$	$+0.06/-0.05$
$+0.03/-0.02$	$+0.03/-0.03$	$+0.03/-0.03$	$+0.03/-0.04$

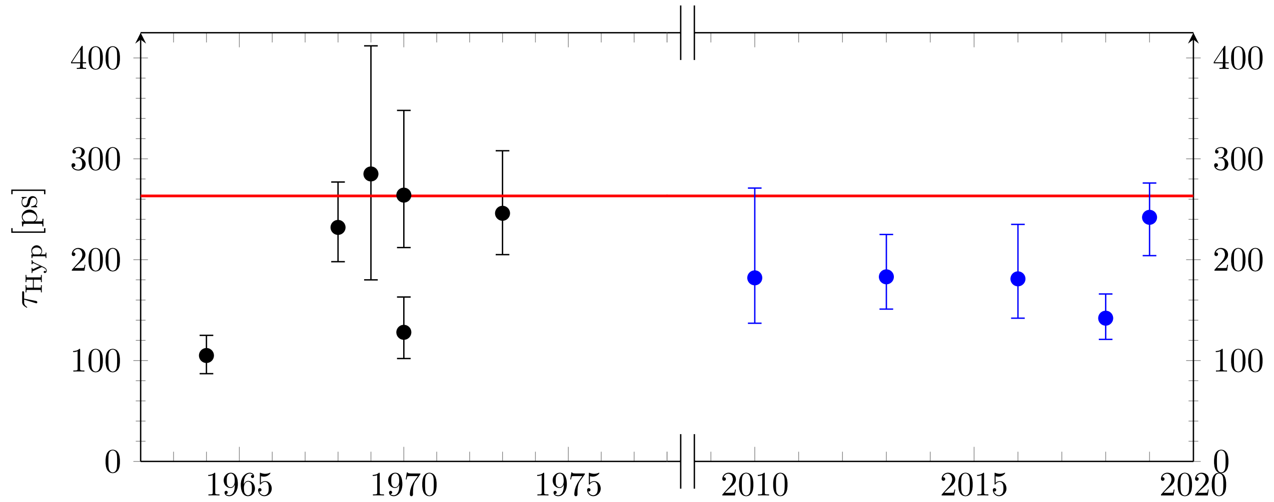
Hildenbrand, HWH, Phys. Rev. C **100** (2019) 034002

- Two-body Λd EFT: $\sqrt{\langle r^2_{\Lambda-NN'}\rangle} = 10.3$ fm

Hypertriton Lifetime

TECHNISCHE UNIVERSITÄT DARMSTADT

- Recent controversy regarding hypertriton lifetime (and binding energy)



- STAR measurement (J. Adam et al. (STAR), Nature Phys. **16** (2020) 409)

$$B_\Lambda = 0.41 \pm 0.12(stat.) \pm 0.11(syst.)\ \text{MeV}$$

- Investigate lifetime dependence on B_Λ in two-body EFT with Λd dof

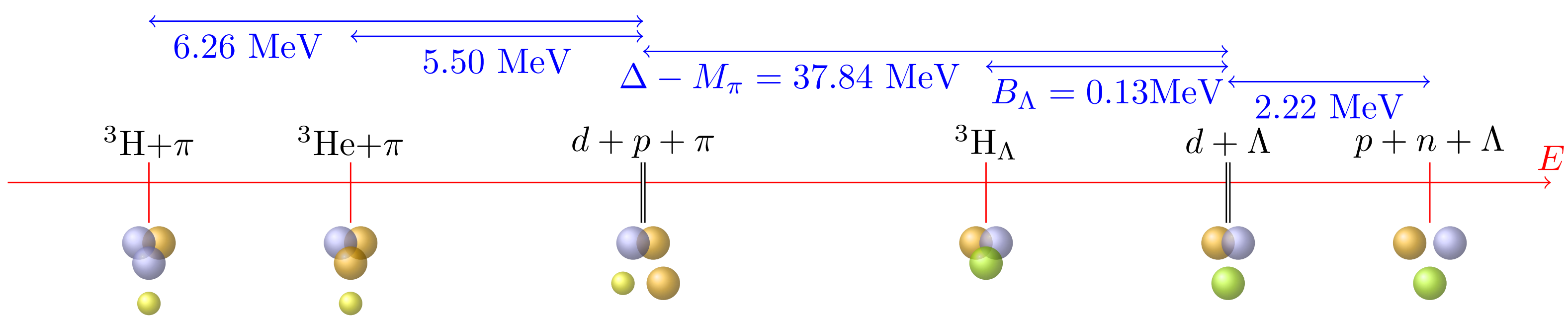
Hypertriton Lifetime

TECHNISCHE UNIVERSITÄT DARMSTADT

- Decay channels in hypertriton decay:

  $^3_\Lambda$H $\rightarrow \pi^- + ^3$ He, $^3_\Lambda$H $\rightarrow \pi^0 + ^3$ H,
  $^3_\Lambda$H $\rightarrow \pi^- + d + p$, $^3_\Lambda$H $\rightarrow \pi^0 + d + n$,
  ~~$^3_\Lambda$H $\rightarrow \pi^- + p + n + p$, $^3_\Lambda$H $\rightarrow \pi^0 + p + n + n$,~~
  ~~leptonic decay channels.~~

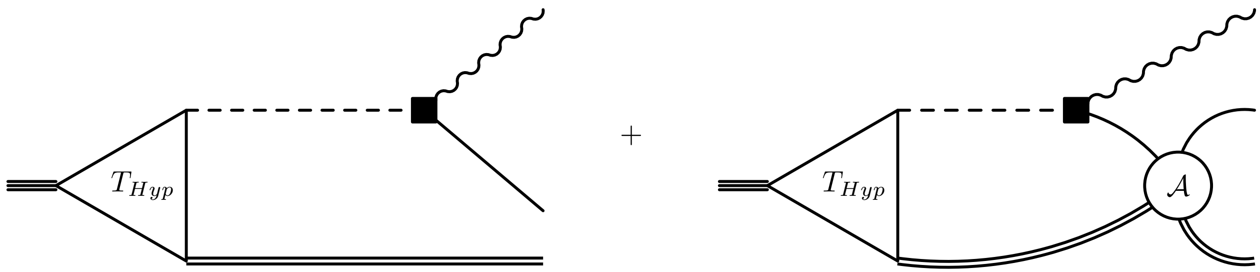
- Leptonic decays account for about 1.5% of free Λ width
- Thresholds



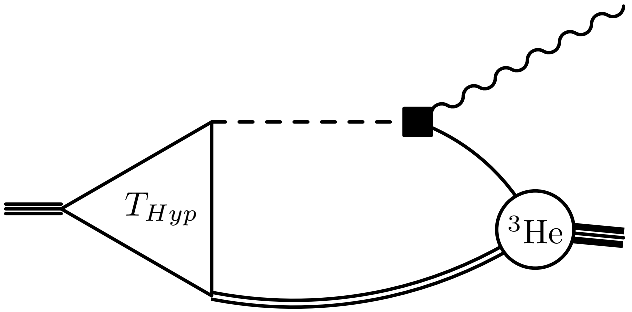
Feynman Diagrams

TECHNISCHE UNIVERSITÄT DARMSTADT

- Deuteron final state



- Trinucleon final state

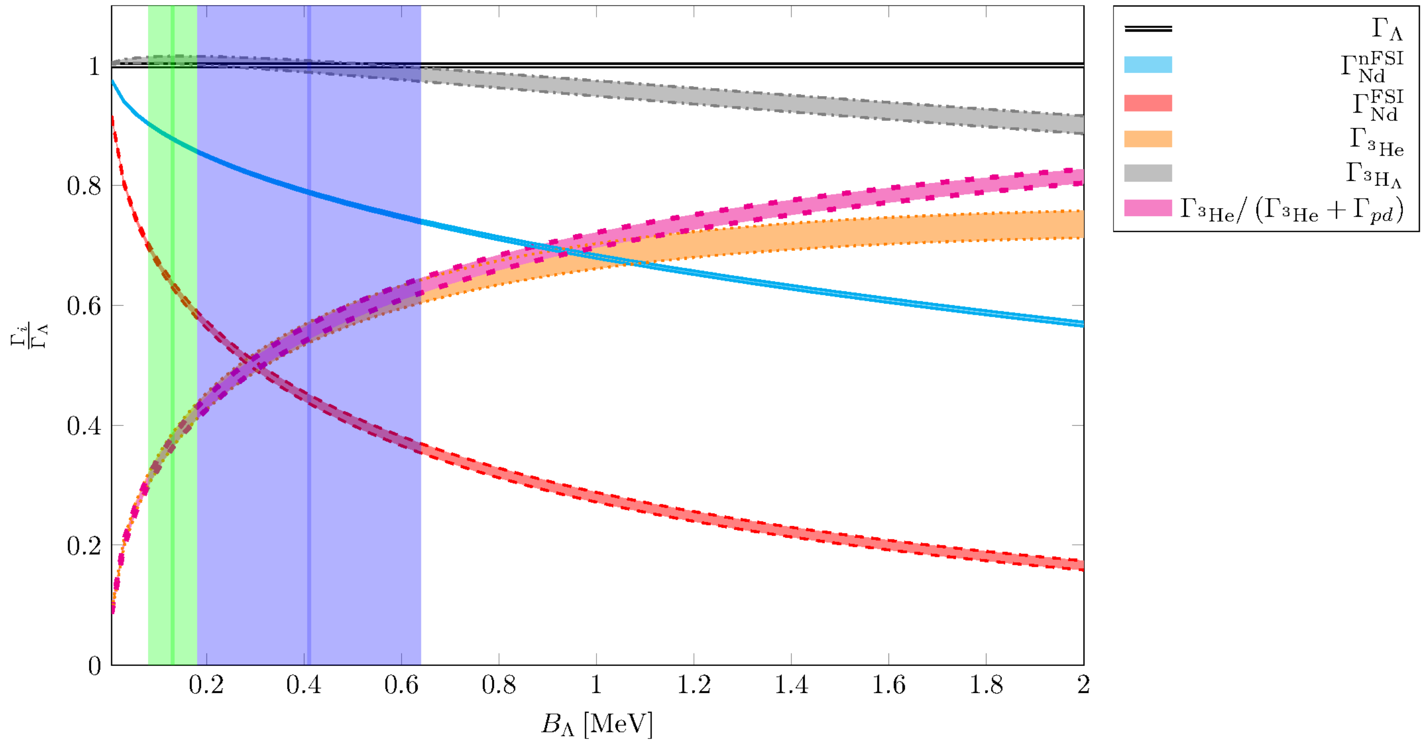


- Use $\Delta I = 1/2$ rule to relate decays into π^0 and π^-
- Neglect Coulomb interaction

Hypertriton Widths

TECHNISCHE UNIVERSITÄT DARMSTADT

- Test sensitivity of partial widths to B_Λ and Λ decay parameter α_-



Hildenbrand, HWH, in preparation

- Large sensitivity of partial widths to B_Λ
- Emulsion experiments: $\Gamma_{^3\mathrm{He}}/(\Gamma_{^3\mathrm{He}}+\Gamma_{pd}) = 0.3 \ldots 0.4$

Summary

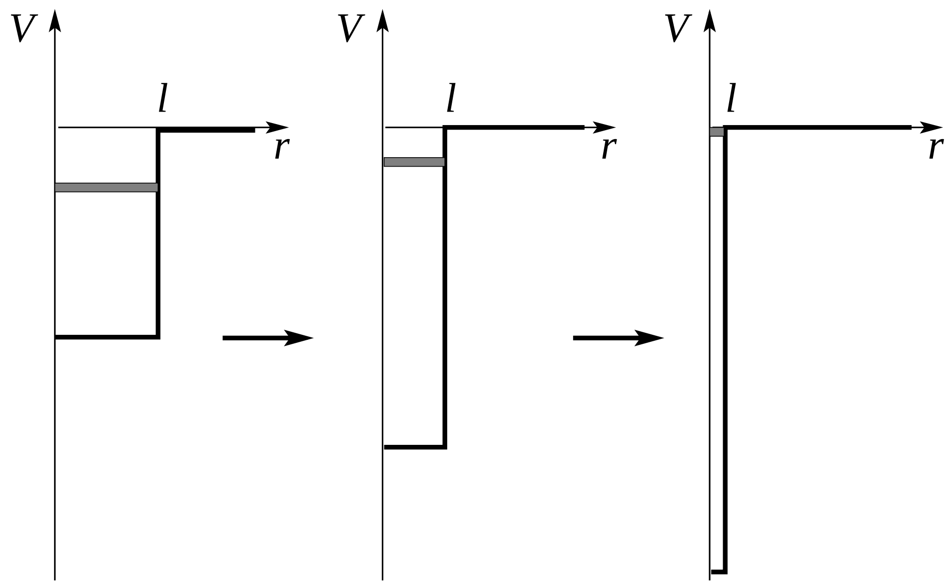
TECHNISCHE UNIVERSITÄT DARMSTADT

- Effective field theory for unitary limit
- Universal aspects of (Discrete) Scale Invariance $\Leftrightarrow$ Efimov physics
  - Effective field theory for hypertriton
  - ....
- Three-body calculation of hypertriton
  - Little sensitivity to exact values of ΛN scattering lengths
  - $\Lambda\Sigma$ conversion $\Longrightarrow$ ΛNN three-body force
  - Matter radius well described in EFT with Λd dof
- Two-body calculation of hypetriton lifetime
  - Large sensitivity of partial widths to B_Λ
  - Emulsion experiments for $\Gamma_{^3\mathrm{He}}/(\Gamma_{^3\mathrm{He}}+\Gamma_{pd})$ disfavor STAR value for B_Λ

Additional Slides

Physics Near the Unitary Limit

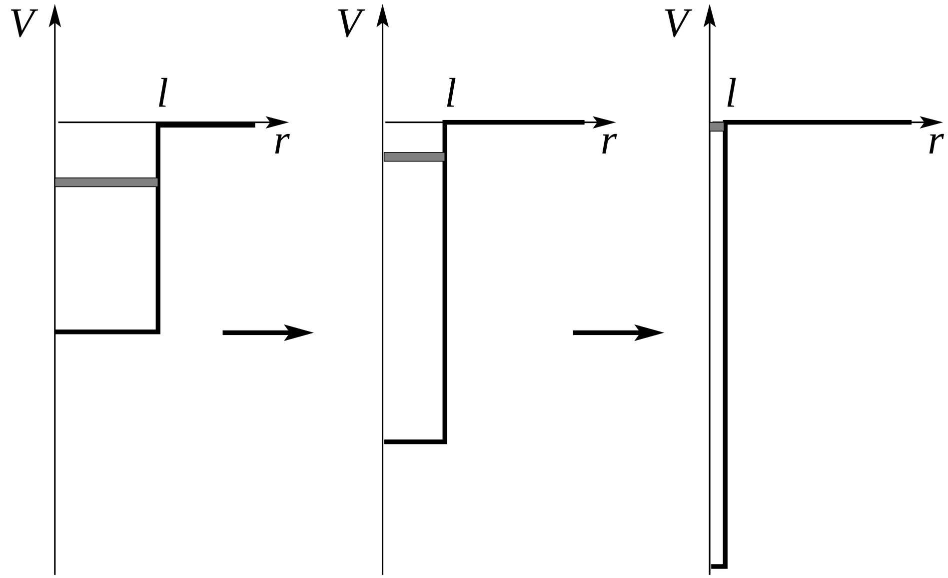
- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty,\ \ell \sim r_e \to 0$ (cf. Bertsch problem, 2000)



Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty, \ell \sim r_e \to 0$ (cf. Bertsch problem, 2000)



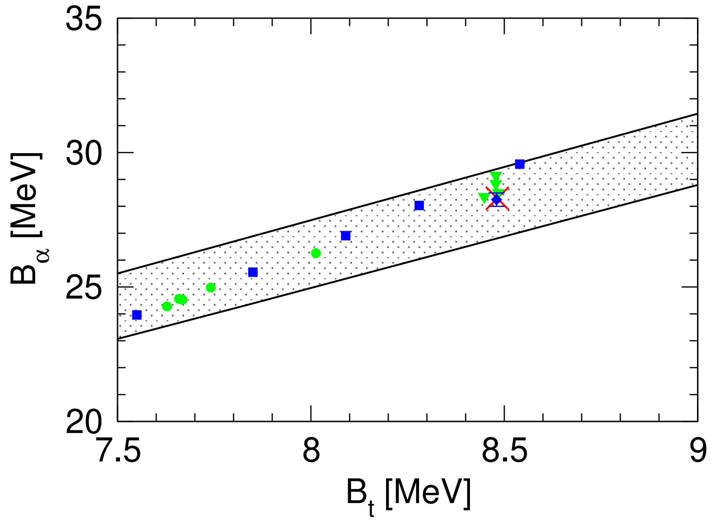
$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \implies i/k$$

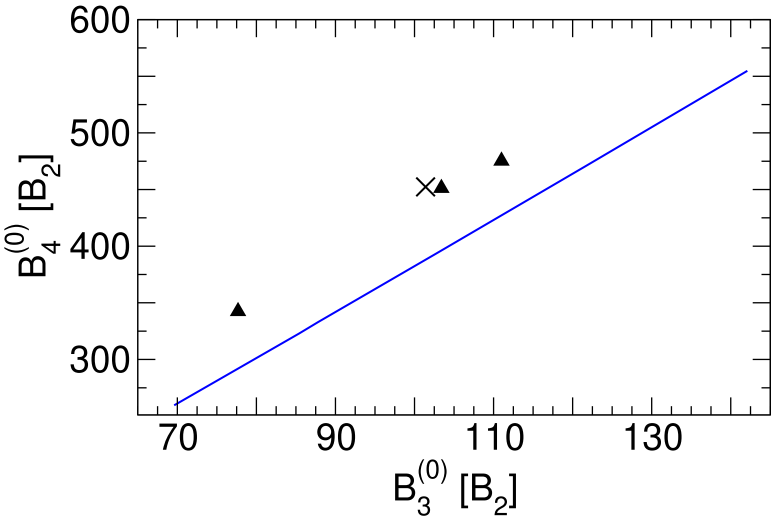
- Scattering amplitude scale invariant, saturates unitarity bound

Universal Correlations

TECHNISCHE UNIVERSITÄT DARMSTADT

- 2 Parameters at LO $\Rightarrow$ 3-body observables are correlated
  $\Longrightarrow$ Phillips line (Efimov, Tkachenko, 1985; Bedaque, HWH, van Kolck, 2000)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
  $\Rightarrow$ 4-body observables are correlated $\Longrightarrow$ Tjon line

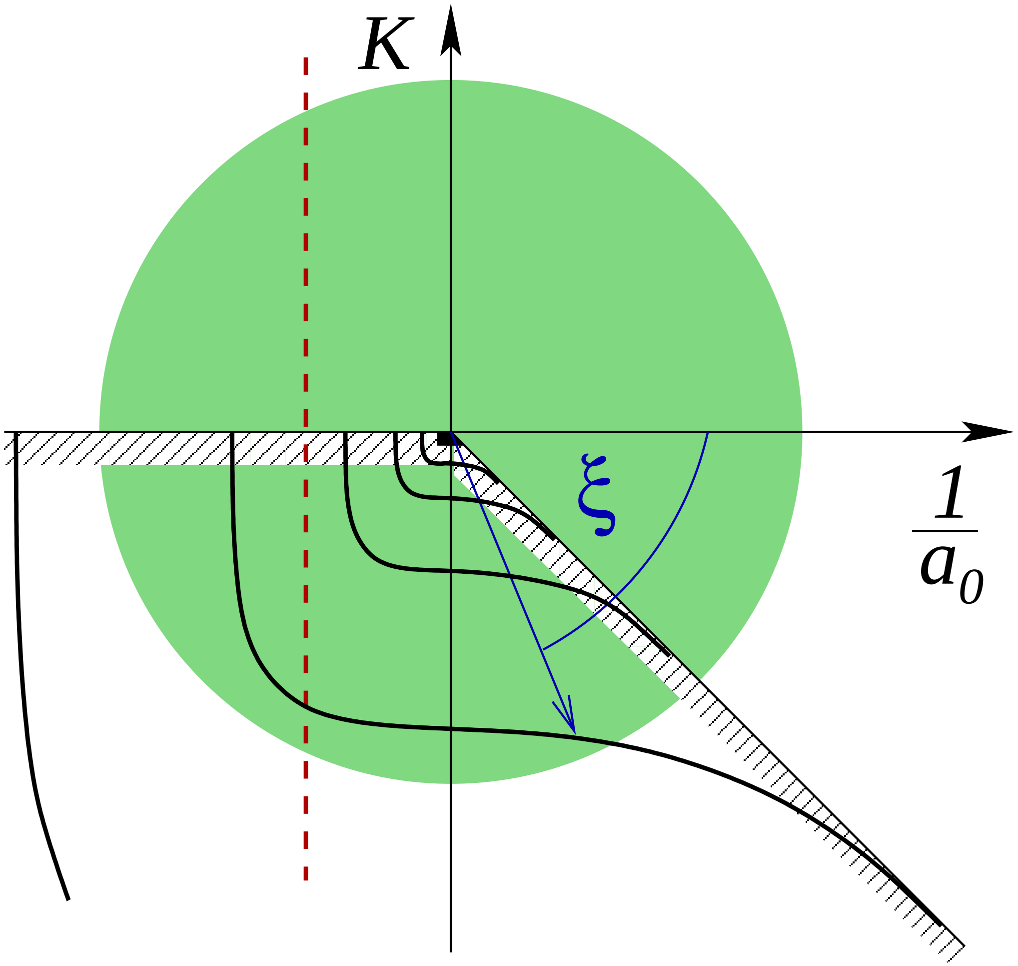




- Variation of 3-body parameter generates correlations
- RG-evolved interactions: Λ dependence traces correlations
  (cf. Nogga, Bogner, Schwenk, 2004)

Efimov Plot

- Window of universality



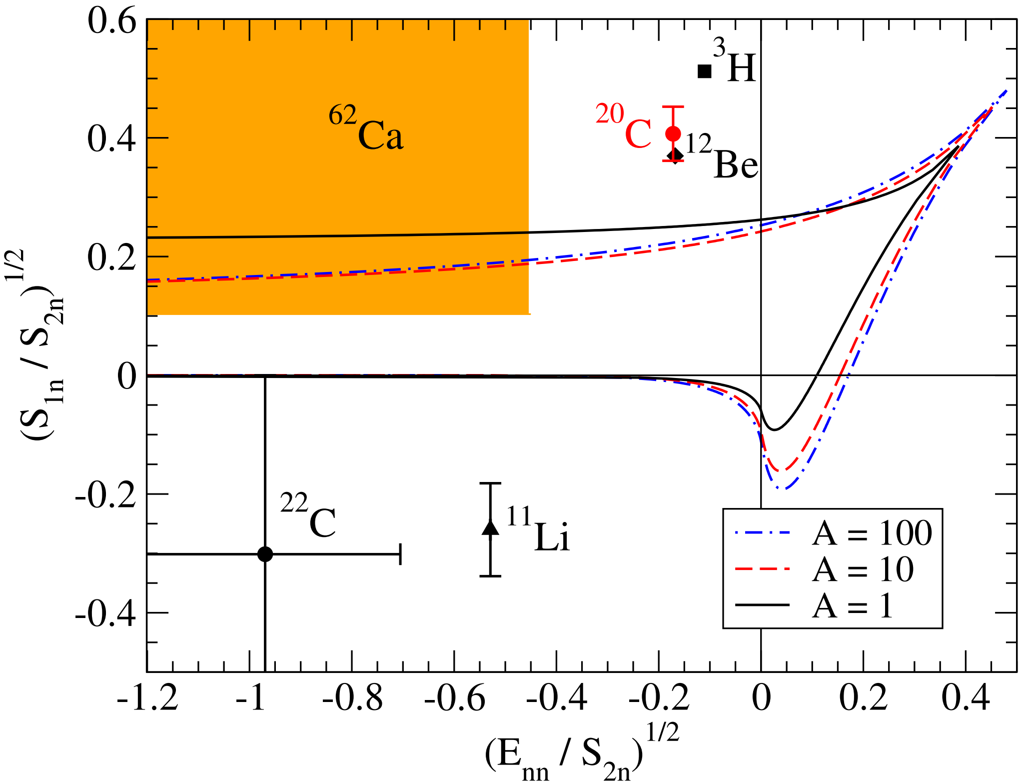
Efimov Physics in Halo Nuclei

TECHNISCHE UNIVERSITÄT DARMSTADT

- Efimov effect in halo nuclei? (Fedorov, Jensen, Riisager, 1994)

  $\Longrightarrow$ excited states obeying scaling relations

- Correlation plot: $E_{nn} \leftrightarrow S_{1n}$ (Amorin, Frederico, Tomio, 1997)



adapted from Canham, HWH, Eur. Phys. J. A **37** (2008) 367