Self-Completeness of Einstein Gravity.

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Pure Einstein gravity in 4D:

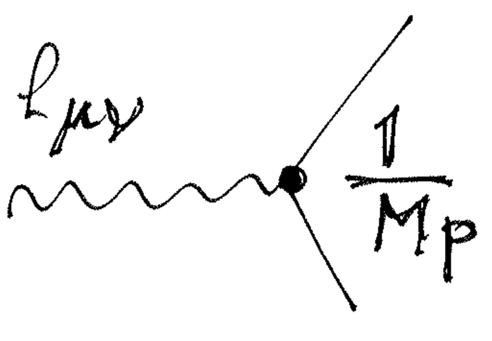
$$S_{EH} = \int d^4x\, M_p^2 \sqrt{-g}\, R.$$

$$L_p \equiv M_p^{-1} \sim 10^{-33}\,\mathrm{cm} \sim \left(10^{19}\,\mathrm{GeV}\right)^{-1}$$

Field theoretic meaning of M_p:

Sets the coupling of graviton with all the energy-momentum sources Universally:

$$h_{\mu\nu} \frac{T^{\mu\nu}}{M_p}$$



We shall argue, that pure-Einstein gravity is self-complete in deep-UV, although not Wilsonian.

In Einstein L_P is the shortest length of nature.

Any attempt of resolving physics at some distance $L \ll L_P$, will bounce us back to

$$\frac{L_P^2}{L} \gg L_P .$$

UV-Gravity = IR-Gravity.

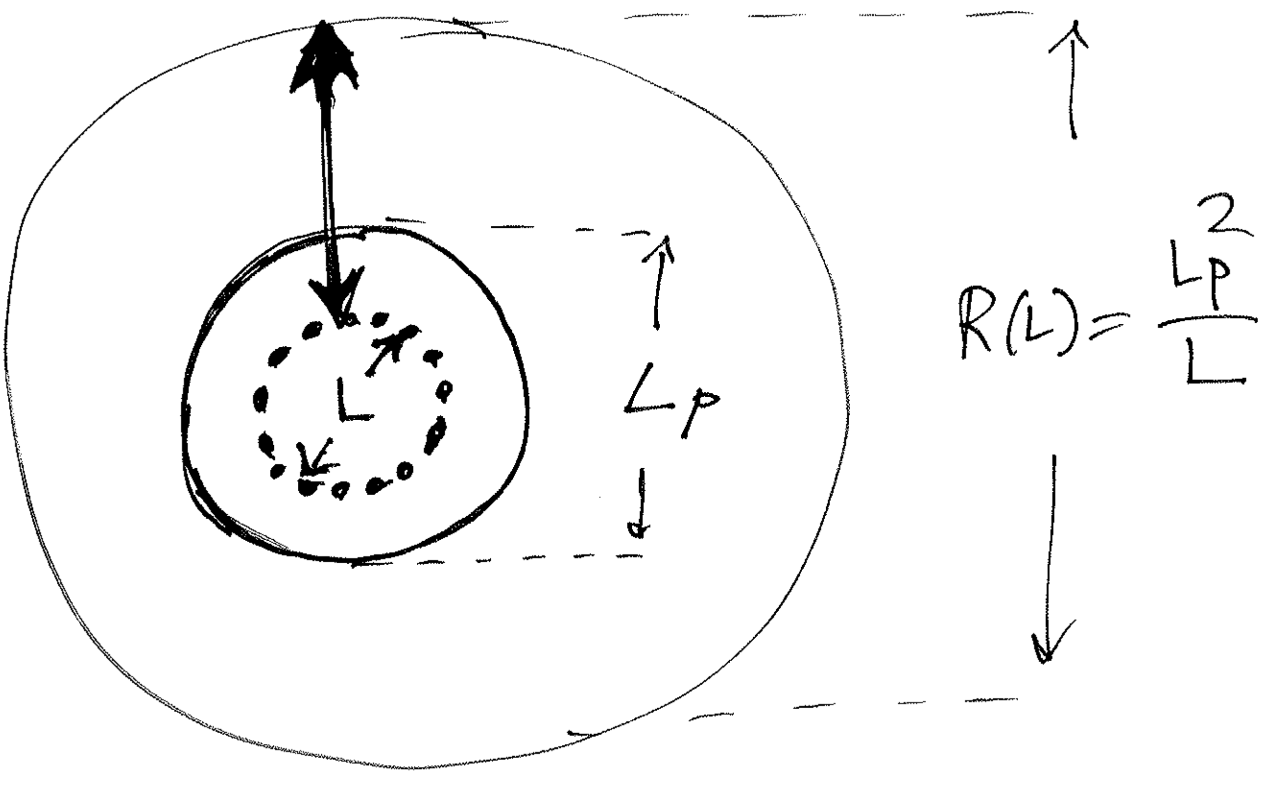
$$L \longleftrightarrow \frac{L_P^2}{L}$$

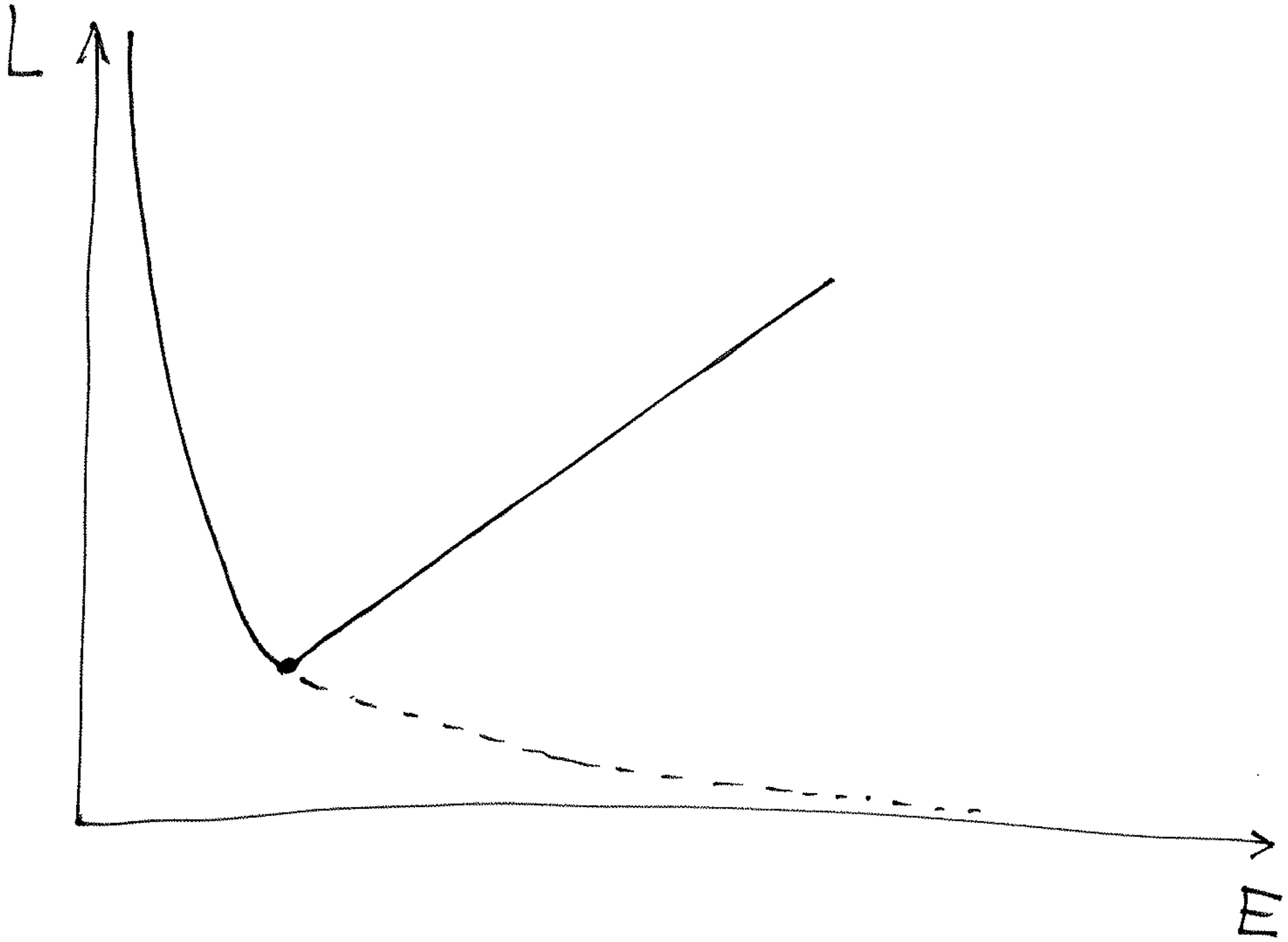
In order to resolve physics at a length-scale L one has to localize energy $E \geq 1/L$ within the space-time volume L.

For $L \ll L_p$, the corresponding Schwarzschild radius is

$$R(L) = L_p^2 / L.$$

So we shall form a _classical_ black hole way before we have any chance of probing distance L!





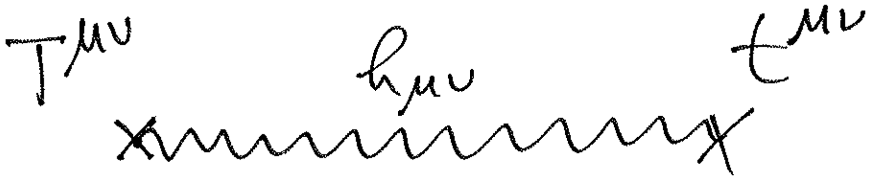
Existence of the minimal length in gravity: generalized uncertainty principle.

The key point for the UV-completness:

In Einstein gravity trans-Planckian propagating degrees of freedom cannot exist. Instead they become classical <u>states</u>, fully described by light (IR) propagating degrees of freedom!

For example consider a modification of the graviton propagator in UV:

$$\frac{1}{M_P^2} T^{\mu\nu} \langle h_{\mu\nu} h_{\alpha\beta} \rangle t^{\alpha\beta} = \frac{1}{M_P^2} \left\{ \frac{T_{\mu\nu} t^{\mu\nu} - \frac{1}{2} T_\mu^\mu t_\nu^\nu}{p^2} + \frac{T_{\mu\nu} t^{\mu\nu} - \frac{1}{3} T_\mu^\mu t_\nu^\nu}{p^2 + m^2} \right\}$$



As long as $m \ll M_P$, the massive pole gives a measurable correction to the observables, e.g., gravitational potential, at large distances.

Such measurements would probe physics at distance $L \equiv m^{-1}$.

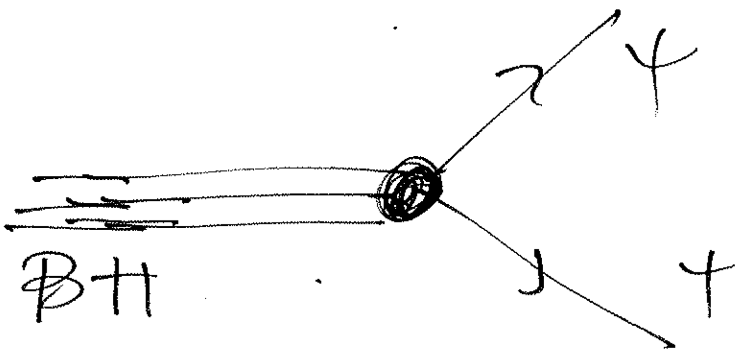
However, for $m \gg M_P$, the massive pole no longer describes a propagating quantum degree of freedom. Instead it becomes a BH!

A classical object of minimal size $R_g \sim m L_P^2 \gg L_P$.

Such a state no longer carries any information about the short distance physics but only about the long distance one.

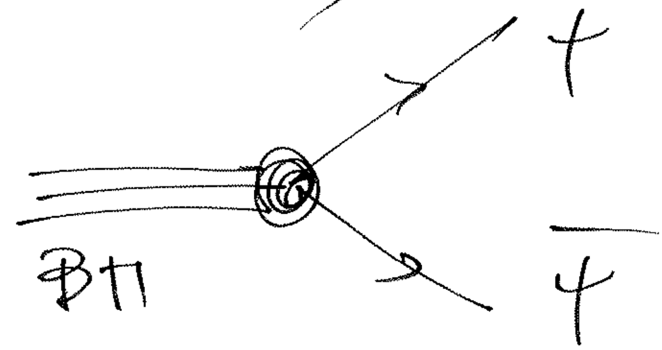
For example, integrating out such an object we never get any operator that carries information about the trans-Planckian physics.

Moreover coupling of such an object to quantum particles must be exponentially suppressed.



The exponential suppression can be understood as follows:

The vertex;



describes ~~ex~~ decay (evaporation) of a semi-classical BH of mass $M \gg M_P$ ~~ai~~ and temperature $T \ll M_P$, into two particle state of energy $E = M$. So the process must be Boltzmann-suppressed:

$$\boxed{\sim e^{-\frac{M}{T}} = e^{-\frac{M^2}{M_P^2}}.}$$

This can also be understood as suppression by the Bekenstein-Hawking entropy.

Because of this operators obtained by exchange of trans-Planckian states are:

1. Suppressed by

$$\sim e^{-\left(\frac{M}{M_P}\right)^2} \equiv e^{-\left(\frac{L_P}{L}\right)^2}$$

2. Not any different from ~~any~~ other operators obtained by integrating out any other classical BH of the same mass $M_{BH} = M$.

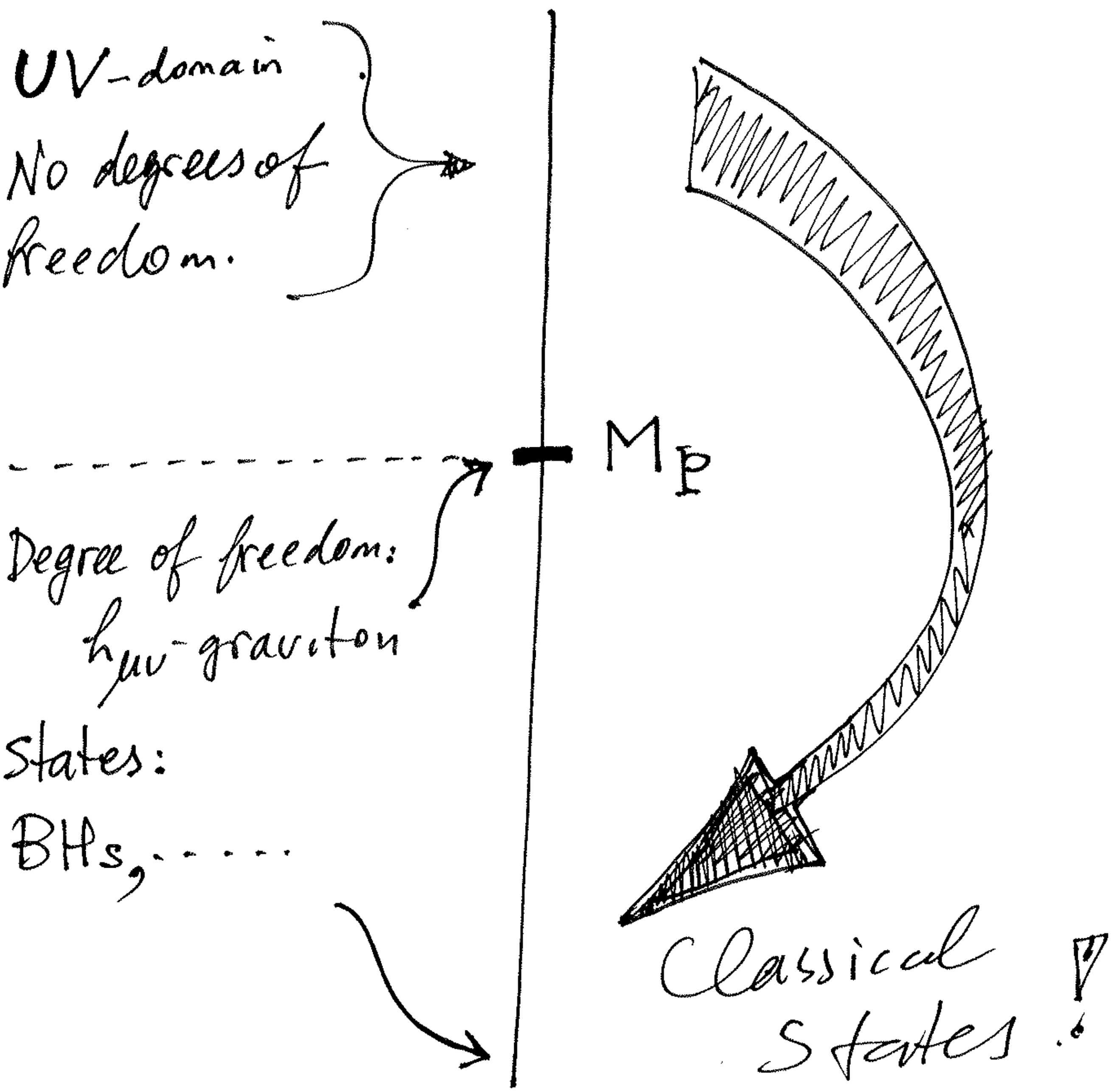
Thus, such states carry no information about the distance

$$L = M^{-1}\,!$$

The key to non-Wilsonian UV-completeness of gravity is in crossover between the Quantum Degrees of Freedom

and

Classical States



Compare gravity to any other field theory with a cut-off.

E.g., $O(N)$ - sigma model.

$$\mathcal{L} = V^2 \partial_\mu O^T_{(x)} \partial^\mu O_{(x)}$$

$$O(x)_a \equiv O(x)_{ab} n_b \qquad n_b \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

(1) Despite the fact that this theory has a scale V, propagating degrees of freedom (e.g. heavy quarks) of arbitrarily high mass can be integrated in.

(2) We can UV-complete this theory by integrating in a radial degree of freedom (Higgs)

$$\Phi_a(x) = \left(1 + \frac{\rho(x)}{V}\right) \Theta(x)_a$$

$$\Downarrow$$

$$\mathcal{L} = \partial_\mu \phi_a \partial^\mu \phi^a - \lambda\left(\phi_a \phi_a - V^2\right)^2$$

The radial mode $\rho(x)$ is crucial for restoring unitarity at arbitrarily high energies above V.

Points (1) and (2) are very different in gravity.

Because of cross-over between the classical states and quantum propagating degrees of freedom, in gravity there must exist quantum degrees of freedom in narrow window around M_P.

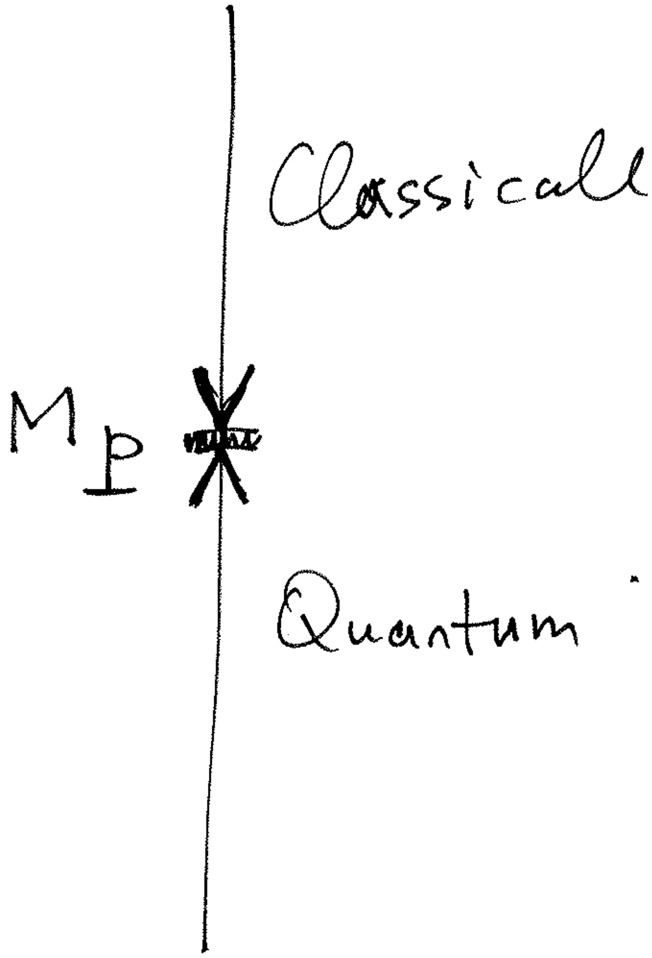
Lifetime of a classical BH:

$$\tau_{BH} = c L_P (M_{BH} L_P)^3$$

where $c \ll 1$.

Because $c \ll 1$, BH cannot cross-over from a long-lived classical state directly into a broad quantum resonance.

- An intermediate state of BH evolution must be a narrow quantum resonance: propagating degree of freedom.

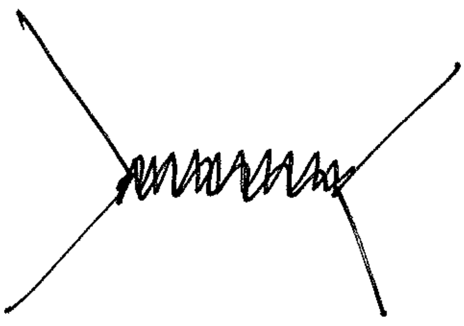


Implications for Asymptotically Weak or Safe Gravity.

Can gravity be asymptotically weak or safe?

Parameterizing the strength of gravity at given momentum-transfer P.

$$\alpha_{grav}(p^2) \equiv 16\pi G_N p^2 .$$



$$A(p) = \frac{\alpha_{grav}(p)}{(p^2)^2}\left(T_{\mu\nu}\, t^{\mu\nu} + b(p)\, T_\mu^\mu\, t_\nu^\nu\right)$$

Strong coupling scale.

$$\alpha_{grav}(p = M_*) = 1.$$

E.g. in Einstein $M_* = M_P$.

In general.

$$\underline{\underline{M_* \leq M_P.}}$$

Absolute bound!

We can now prove that:

(*) $$\frac{\alpha_{grav}(p)}{\alpha_{Ein}(p)} \geq 1.$$

(*)(*) $$\frac{d}{d(p^2)}\left(\frac{\alpha_{grav}(p)}{\alpha_{Ein}(p)}\right) \geq 0.$$

These relations follow from the Spectral represantation of the graviton propagator:

$$T^{\mu\nu} \Delta_{\mu\nu,\alpha\beta}\, t^{\alpha\beta} =$$

$$= \frac{1}{M_P^2} \left\{ \frac{T_{\mu\nu} t^{\mu\nu} - \frac{1}{2} T_\mu^\mu t_\nu^\nu}{p^2} + \right.$$

$$+ \int_0^\infty ds\, \rho_2(s)\, \frac{T_{\mu\nu} t^{\mu\nu} - \frac{1}{3} T_\mu^\mu t_\nu^\nu}{p^2 + s}$$

$$\left. + \int_0^\infty ds\, \rho_0(s)\, \frac{T_\mu^\mu t_\nu^\nu}{p^2 + s} \right\}$$

Absence of ghosts: $\rho_0(s) \geqslant 0$, $\rho_2(s) \geqslant 0$.

(Also note $\rho_{2,0}(s)\Big|_{\sqrt{s} \gg M_*} \propto e^{-\sqrt{s} R_H(s)}$

wher $R_H(s)$ in horizon of BH of mas $\sqrt{s}$)

From the above it follows that

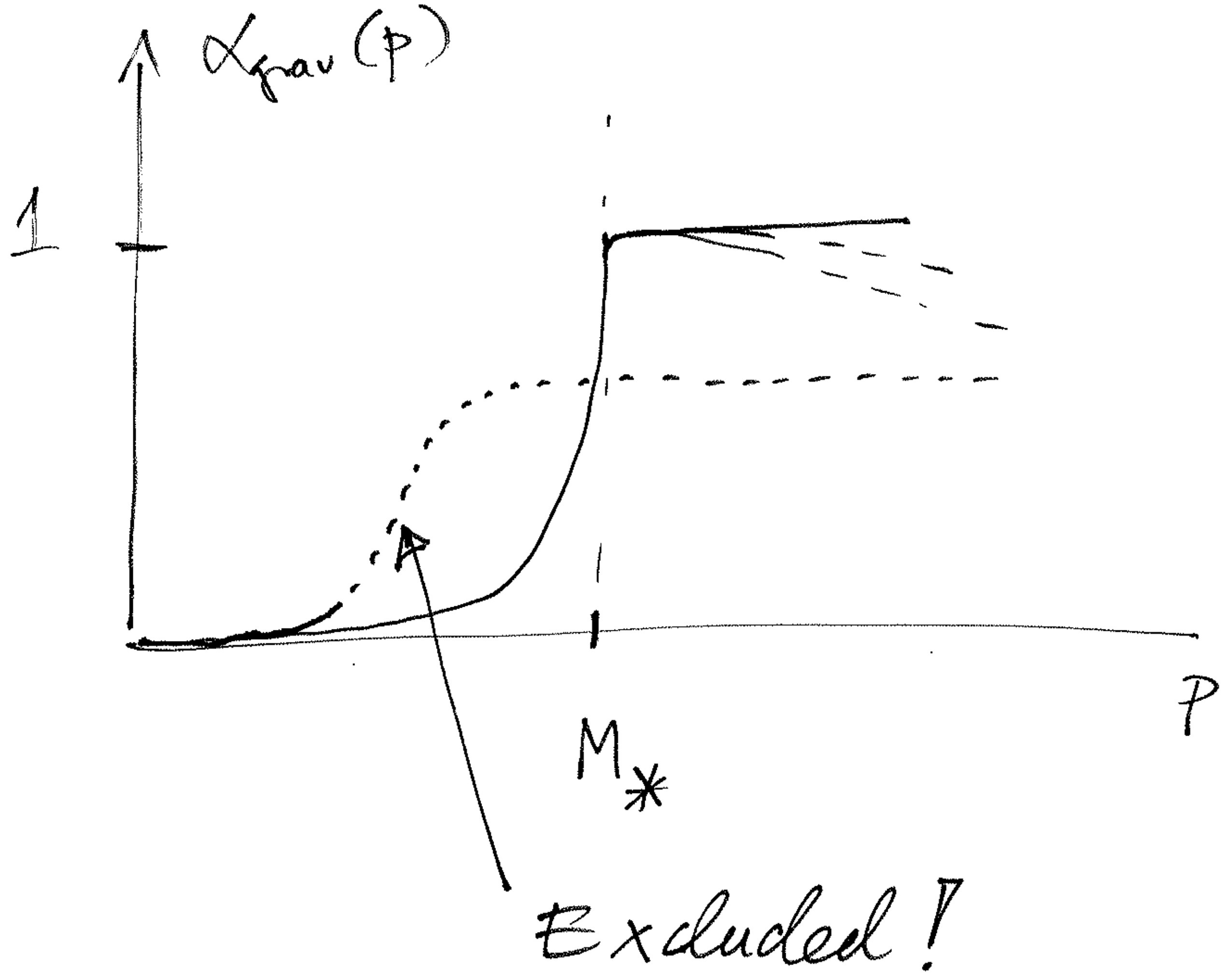
$$\frac{\mathcal{L}_{grav}(p)}{\mathcal{L}_{Ein}(p)} = 1 + p^2 \int_0^\infty \frac{\rho_2(s)}{p^2+s}$$

Thus,

Einstein gravity is the weakest of all gravities that flow to Einstein (in IR) with a given G_N!

Any modification of Einstein will result into larger BH horizon for the same mass!

Thus, gravity cannot become weak before becoming strong.



Can gravity become weak above $p > M_*$?

But, the scale M_* is the upper bound for the BH-formation.

Thus, distances $L \ll M_*^{-1}$ cannot be probed.

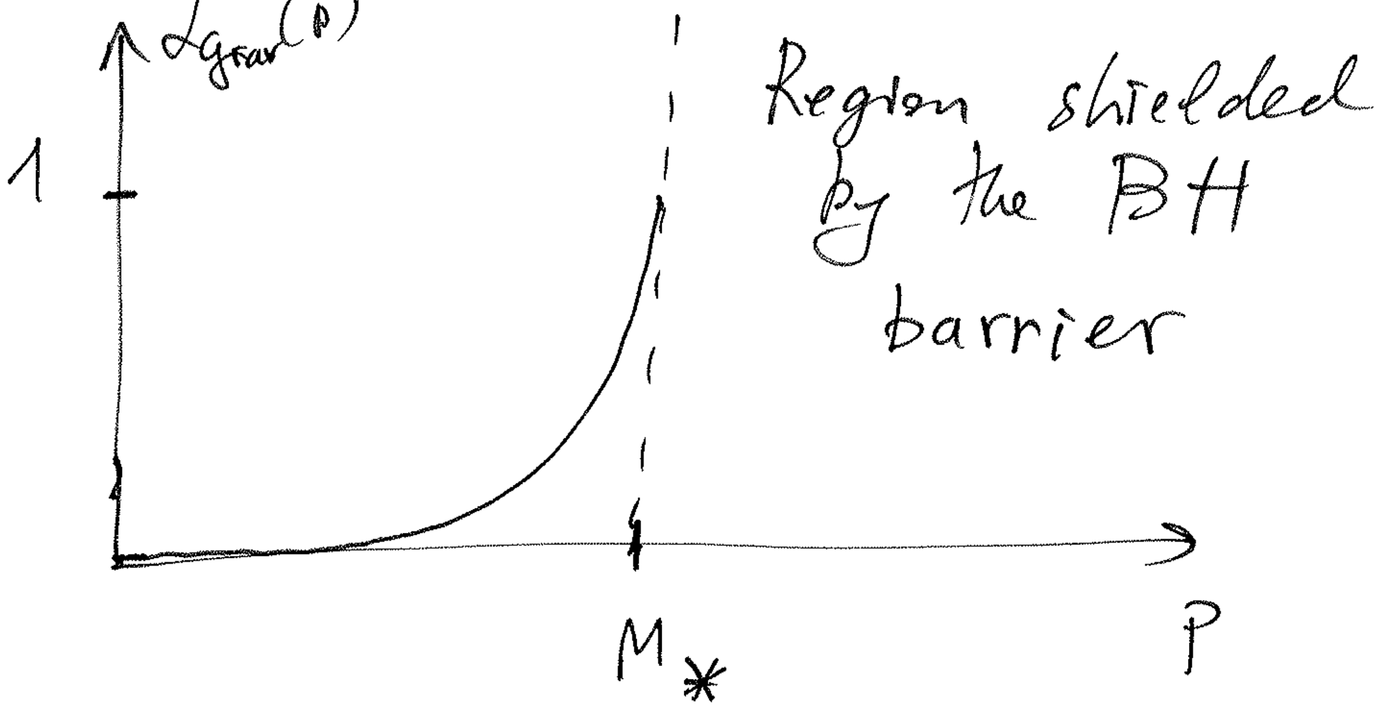
This follows from the fact that

$$\alpha_{grav}(M_*) \sim \frac{h_{00}(M_*^{-1})}{M_P} \sim 1$$

$$\Downarrow$$

$$\alpha_{grav}(M_*) = \frac{M_*^2}{M_P^2} \int_0^\infty \frac{\rho(s)}{1+s/M_*^2}\, ds$$

$$\frac{h_{00}(M_*^{-1})}{M_P^2} = \frac{M_*^2}{M_P^2} \int_0^\infty e^{-\sqrt{s}/M_*} \rho(s)\, ds$$



Thus, there is no well-defined window of distances (energies) in which gravity can become asymptotically weak or safe.

But, in the light of self-completeness is it even needed?