### Neutrino Mass and Grand Unification of Flavor

#### R. N. Mohapatra



Goranfest, June 2010 Split.

### **Brief Remembrances**

#### CCNY-70's:



Look back, Look at, Look ahead Joy was doing physics...

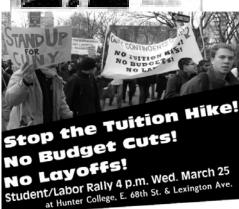
#### Turbulent times





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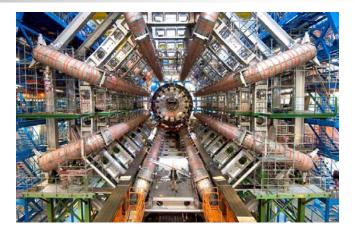


### Two Outstanding Problems in Particle Physics Today

## Origin of Mass:

--Higgs, SUSY, Extra D

--LHC will probe this:



#### Origin of Flavor: (this talk)

-- Many non-collider probes:



### Quark, Lepton flavor: Definitions

Masses and mixings- two aspects of flavor

Def. 
$$L_{mass} = \overline{Q}_L M_{q=u,d} Q_R + \overline{l}_L M_l l_R + v^T m_v v + h.c.$$

• Mass basis:  $U_L M_{q,l} U_R^+ = M_{q,l}^{diag}$  and for neutrinos

$$V_{CKM} = U_{u}U_{d}^{+} \qquad U_{PMNS} = U_{l}U_{v}^{+}$$

 $\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad \qquad \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$ 

#### Flavor Puzzle < 1998:

- Quark masses and mixings (at GUT scale)
   Up quarks: m<sub>u</sub>: m<sub>c</sub>: m<sub>t</sub> = 0.0008: 0.2:82
- **Down quarks:**  $m_d : m_s : m_b = 0.002 : 0.03 : 1$
- Mixings:  $V_{us} \approx 0.22; V_{cb} \approx 0.037; V_{ub} \approx 0.003$
- Leptons:  $m_e: m_\mu: m_\tau = 0.0005: 0.093: 1.58$
- Note:  $m_b \approx m_\tau; m_\mu \approx 3m_s$

#### WHY?

Attempts to Understand using texture zeros

• Relation:  $V_{us} \cong \sqrt{\frac{m_d}{m_s}}$   $\rightarrow$ d-s mass matrix  $\begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$ 

$$a \ll b \rightarrow m_s = b; m_d = -\frac{a^2}{b}; V_{us} = \frac{a}{b} = \sqrt{\frac{m_d}{m_s}}$$
(Weir

(Weinberg; Wilczek,Zee; Fritzsch)

Also GUT scale relations:  $m_b \cong m_\tau$ and  $m_e m_\mu \approx m_d m_s \Rightarrow Det[M^1] = Det[M^d]$ Finally at GUT scale,  $m_\mu \approx 3m_s$ This implies:  $M_d = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$  whereas  $M_l = \begin{pmatrix} 0 & a \\ a & -3b \end{pmatrix}$ (Georgi, Jarlskog)

# Neutrino mass discovery has added to this puzzle !

 $(\Delta m^2)_{so}$ 

 $(\Delta m^2)_{\text{star}}$ 

- What we know about neutrino masses ?
- Masses:  $\Delta m_{sol}^2 \cong 7.67 \times 10^{-5} eV^2$ ;  $\Delta m_{Atm}^2 \cong 2.39 \times 10^{-3} eV^2$
- Mixings:  $\sin^2 \theta_{12} \cong .312; \sin^2 \theta_{23} \cong .466 \quad \sin^2 \theta_{13} \le .04$
- Overall mass scale: < .1- 1 eV (roughly)</p>
- To be determined (expts in progress or planning)
   (i) Majorana or Dirac ?
   (ii) Mass ordering: normal or inverted?
   (iii) Value of θ<sub>13</sub>
   (iv) Any possible CP violation ?

# An Interesting mixing pattern ?

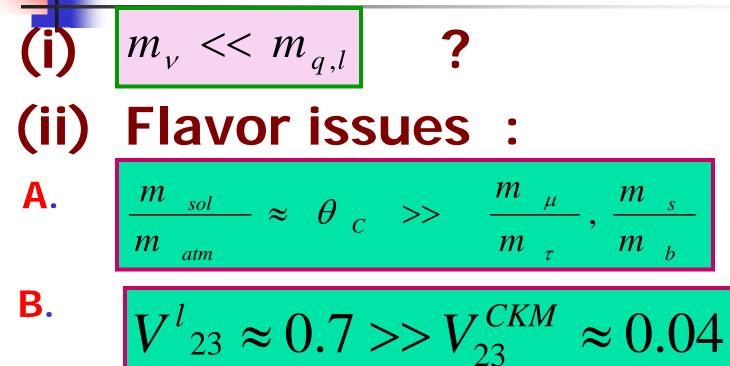
Tri-bi-maximal mixing for neutrinos:

$$\mathbf{U} = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0\\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

(Harrison, Perkins, Scott; Xing; He, Zee)

Is it exact ? If not how big are corrections ?

## New Challenges posed by neutrino masses



Quarks and leptons so differentis a unified description of Flavor possible ?

# Hints for a strategy for flavor

Small quark mixings:  $\rightarrow M_u^0 \propto M_d^0$ 

Mass hierarchy for quarks and charged leptons: suggests:

$$M^{0}_{u,d,l} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_{t,b,\tau} \end{pmatrix}$$

- Large mixings for leptons  $\rightarrow M_l, M_{\nu}$  unrelated.
- Unifying quark-lepton flavors: GUTs

#### **Basic strategy to unify** quark-lepton flavor:

#### Assumption (I): Suppose a theory gives:

$$M_{u} = M_{0} + \delta_{u}$$

$$M_{d} = rM_{0} + \delta_{d}$$

$$M_{1} = rM_{0} + \delta_{1}$$

$$m_{\nu} = f v_L$$

 $\delta_{u,d,l} << M_0$ 

- Choose basis sof diagonal. Then lepton mixings are given by the matrix that diagonalizes;  $M_{I}$
- For anarchic Mo, quark mixings are small while lepton mixings are large.

#### How to see that ?

- **Suppose:**  $U_0 M_0 U_0^+ = M^{diag}$
- Then  $VU_0(rM_0 + \delta_d)U_0^+V^+ = M_d^{diag}$
- Since  $\delta_{u,d,l} << M_0$  off-diagonal elements of V are small.

$$V_{CKM} = U_0 U_0^+ V^+ = V^+$$

 On the other hand, elements are large.

$$U_{PMNS} = U_0$$

#### whose matrix

# Rank One mechanism and mass hierarchy

Assumption (II): Mo has rank one i.e.

$$M_{0} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} (a \quad b \quad c)$$

gives mass to third gen fermions: t, b, tau + m<sub>b</sub> ≅ m<sub>τ</sub> others are massless. Turn on δ<sub>u,d,l</sub> << M<sub>0</sub>
 Other fermions c,s,mu pick up mass with
 M<sub>c,s,μ</sub> << M<sub>t,b,τ</sub> and relates mixings to masses

**Illustration for 2-Gen. case**  
Suppose 
$$M_0 = \begin{pmatrix} c \\ s \end{pmatrix} (c \ s)$$
 and  $f = diag(\varepsilon_2, \varepsilon_3) \propto \delta_{u,d}$ 

•  $\theta = Atm$ . angle; chosen large; f <<h.

• Predictions: 
$$m_{\tau} \cong m_{b}$$
  
 $\frac{m_{s}}{m_{b}} \approx -V_{cb} \tan \theta$ 

consistent with observations:

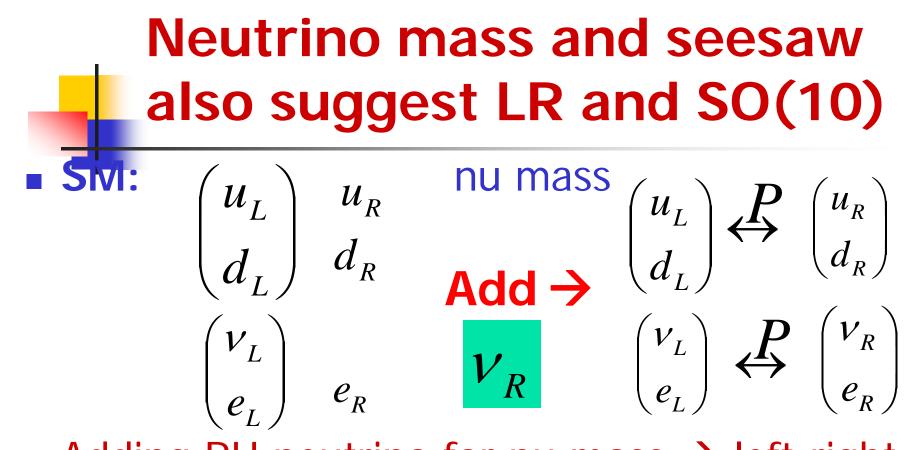
### Making model predictive

Key idea: SM sym for massless fermions :[SU(3)]^5;

- Choose subgroup: Discrete subgroup with 3-d. rep.
- Replace Yukawa's by scalar fields (flavons);
- Minima of the flavon theory → Yukawas:
- GUT theory that realizes the new ansatz for flavor

### What kind of GUT theory ?

- Recall ansatz:  $M_u = M_0 + \delta_u$  as  $\delta_{u,d} \to 0, M_u \propto M_d$  $M_d = rM_0 + \delta_d$
- In SM, *u<sub>R</sub> d<sub>R</sub>* singlets- so M<sub>u</sub>, M<sub>d</sub> unrelated.
   We need a theory where,  $\begin{pmatrix} u_R \\ d_R \end{pmatrix}$  are in a doublet.
- Left-Right symmetric  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ and SO(10) (which contains LR) are precisely such theories.



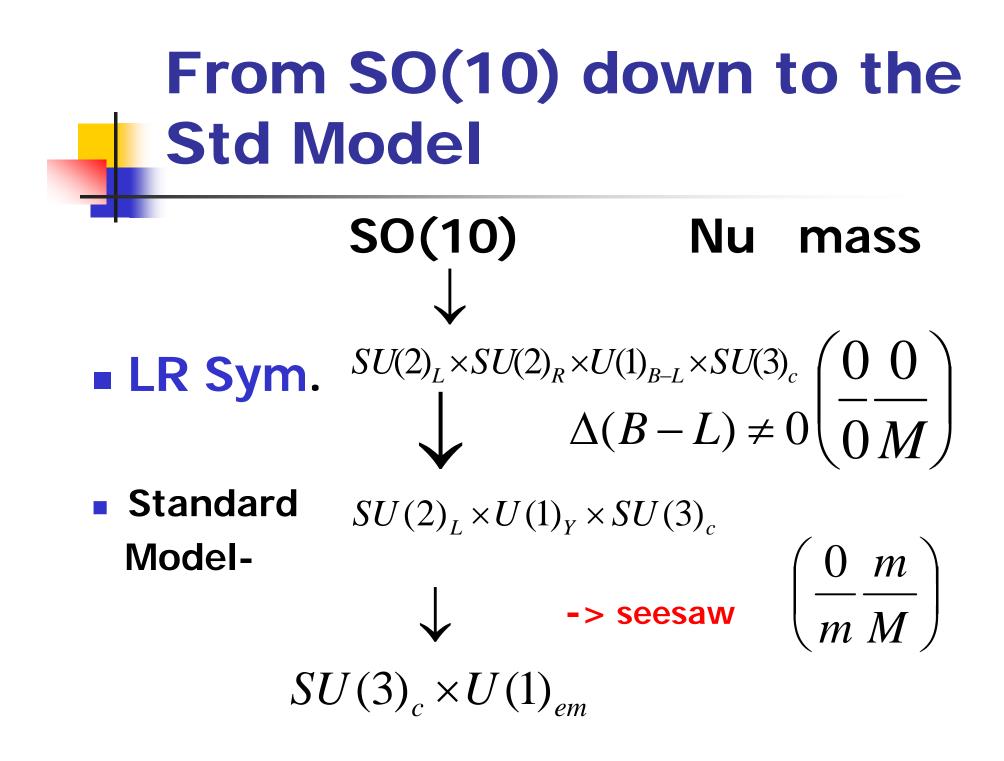
 Adding RH neutrino for nu mass → left-right sym.unification based on SU(2)LXSU(2)RXU(1)B-LXSU(3)c and SO(10)

### **SUSY SO(10) Features**

 Minimal GUT group with complete fermion unification (per family) is SO(10)-its spinor rep contains <u>all 16</u> SM fermions (including RH nu) in single rep.

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$$

- Has B-L needed to understand why MR<< M\_PI</p>
- Theory below GUT scale is MSSM:
- B-L needed for naturally stable dark matter.



SUSY SO(10) and unified understanding of flavor

- Fermions in {16}: 16mx16m={10}н+{120}н+{126}н
- Only renorm. couplings for fermion masses:  $L_Y = h16 \cdot 16 \cdot 10_H + f16 \cdot 16 \cdot 126_H + h'16 \cdot 16 \cdot [12010]_H$
- Has SM doublets → contributes to fermion mass
- {126}
   H responsible for both neutrino masses and quark masses: → <u>helps to connect quark</u> <u>mixings to neutrino mixings</u>: Unifies quark and lepton flavors: (Babu, Mohapatra, 93)

## Fermion mass formulae in renormalizable SO(10)

- Define  $Y_f = M_f / v_{wk}$
- The mass formulae:

$$Y_u = h + r_2 f + r_3 h'$$

$$Y_d = r_1(h+f+h')$$

$$Y_e = r_1(h - 3f + c_e h')$$

$$Y_{\nu} = h - 3r_2f + c_{\nu}h'$$

#### **Compare with ansatz**

$$M_{u} = M_{0} + \delta_{u}$$
$$M_{d} = rM_{0} + \delta_{d}$$
$$M_{l} = rM_{0} + \delta_{l}$$

#### Both sets of formulae identical for f, h'<< h</p>

### Neutrino mass in Renormalizable SO(10):

- {1'26} has an SU(2) triplet with B-L=2:
  - New formula for nu-mass:  $m_{\nu} = fv_{\Delta} - M_{D} \frac{1}{fv_{BL}} M_{D}^{T}$   $v_{\Delta} = \lambda_{\Delta} \mu \frac{v_{wk}^{2}}{M_{\Delta}^{2}}$   $v_{L} = \frac{1}{V_{\Delta}} \frac{v_{wk}^{2}}{M_{\Delta}^{2}}$   $v_{L} = \frac{1}{V_{\Delta}} \frac{v_{wk}^{2}}{M_{\Delta}^{2}}$
- Type II seesaw:  $M_{\Delta} \approx M_U$  gives naturally small v • Two independent parameters:  $M_{\Delta}^2, v_R$

Lazaridis, Shafi, Wetterich; R.N.M., Senjanovic; Schecter, Valle'81

**Type II dominance:**  
If 
$$M_{\Delta} \ll fv_{BL}$$
, first term dominates  
Then the fermion mass formula become:  
 $Y_u = h + r_2 f + r_3 h'$ 

$$Y_d = r_1(h + f + h')$$

$$Y_e = r_1(h - 3f + c_e h')$$

$$m_{\nu} \cong f v_{\Delta}$$

(Bajc, Senjanovic, Vissani'02)

$$Y_{\nu} = h - 3r_2f + c_{\nu}h'$$

(Babu, Mohapatra'92)

Neutrino mass and quark and charged lepton masses connected and all ingredients of our ansatz are realized in SO(10).

### Rank One mechanism for Flavor

Generic case does not explain mass hierarchies

$$Y_u = h + r_2 f + r_3 h'$$
  

$$Y_d = r_1(h + f + h')$$
  

$$Y_e = r_1(h - 3f + c_e h')$$
  
**Assume h is rank 1**  

$$m_v \cong fv_\Delta$$
  

$$h = \begin{pmatrix} a \\ b \\ c \end{pmatrix} (a \ b \ c) + f, h' << h$$

- For f, h'=0, only 3<sup>rd</sup> gen. pick up mass.
- Leads to  $m_{s,d} \ll m_b; m_{e,\mu} \ll m_\tau$  with f, h' < <h

• Gives 
$$m_{\tau} \cong m_b$$
 and  $m_{\mu} = -3m_s$ ;  $\frac{m_{sol}}{m_{atm}} \sim \theta_c$ 

### Origin of Rank one SO(10)

- Rank one model as an effective theory at GUT scale:
- Add one vector like matter  $\Psi_V \{16\} + \overline{\Psi}_V \{\overline{1}\overline{6}\}$ and singlets:  $\phi_i$
- Superpotential:  $W = \phi_i \psi_i \overline{\Psi}_V + \overline{\Psi}_V \overline{\Psi}_V H + M \overline{\Psi}_V \Psi_V$

$$\underbrace{\psi \quad \overline{\Psi_{V}} \quad \Psi_{V} \Psi_{V} \quad \overline{\Psi_{V}} \quad \psi}_{H}$$

Flavor texture depends on  $< \phi_i >$ ; with symmetries it can be predicted.

### VEV alignment from flat directions in an S4 model

Examples: S4 triplet flavon case:

$$W = \frac{1}{2}m\phi^2 - \lambda\phi^3 = \frac{1}{2}m(x^2 + y^2 + z^2) - \lambda xyz.$$

$$\phi = \frac{m}{\lambda} \{ (1, 1, 1) \text{ or } (1, -1, -1) \text{ or } (-1, 1, -1) \text{ or } (-1, -1, 1) \}.$$

• While for 
$$W = \frac{1}{2}m\phi^2 - \frac{\kappa_1}{M}(\phi^4)_1 - \frac{\kappa_2}{M}(\phi^4)_2$$

 $\vec{a} = (0, 0, \pm 1), (0, \pm 1, 0), (\pm 1, 0, 0), \vec{b} = (\pm 1, \pm 1, \pm 1), \text{ and } \vec{c} = (0, \pm 1, \pm 1),$ 

# A specific realization with predictive textures:

- Group: SO(10)xS<sub>4</sub>  $\supset 3_1 + 3_2 + 2 + 1_1 + 1_2$
- Consider flavons  $\phi_{1,2,3} \subset 3_{1,2}$ ; matter {16}  $\subset 3_2$
- Inv effective superpotential at GUT scale:

 $W = (\phi_1 \psi)(\phi_1 \psi)H + (\phi_2 \psi)(\phi_2 \psi)\overline{\Delta} + \phi_3 \psi \psi \overline{\Delta} + \phi_2 \psi \psi H'$ 

• The flavon vevs align as:  $\phi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \phi_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \phi_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$ 

• Leading to  $\mathbf{f} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$  and  $\mathbf{h'} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ 

Gives realistic model for fermion masses and mixings



• Solar mass  $\frac{m_{solar}}{m_{atm}} \cong \lambda \cong \theta_c$ • Bottom-tau:  $m_b \approx m_{\tau}$  and  $m_{\mu} = -3m_s$ 

Leading order PMNS: U =

$$\begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0\\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

#### Testable prediction:

Bjorken, King, Pakvasa Ferrandis (2004-05)

• Double beta mass 3 meV.

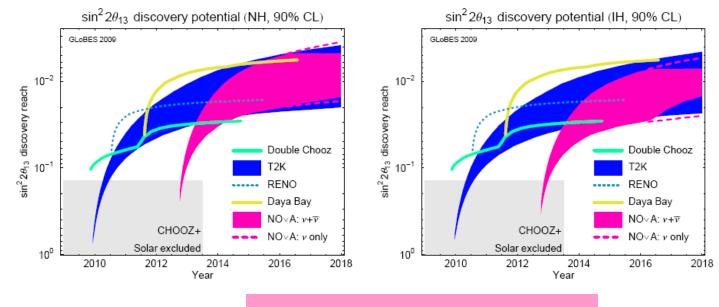
$$\theta_{13} = \frac{\theta_c}{3\sqrt{2}} \cong 0.05$$

Dutta, Mimura, RNM arXiv:0911.2242

### Prospects for measuring $\theta_{13}$

#### Reactor, Long base line e.g. T2K, NoVA:

#### (Lindner, Huber, Schwetz, Winter'09)



**Our prediction** 

 $\sin^2 2\theta_{13} > 0.01$ 

#### **GUTs and Proton decay**

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- Proton decay in SUSY GUTs have two generic sources:
- (i) Gauge exchange:

$$p \to e^+ \pi^0$$
,  $\tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{36 \pm 1} yr]^-$ 

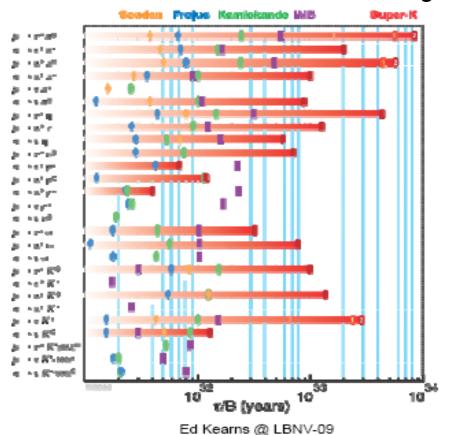
(ii) Higgsino exchange:

$$p \to \bar{\nu}K^+ \\ \tau_p^{-1} \approx \left[\frac{f^2}{M_{H_c}M_{SUSY}}\right]^2 (\frac{\alpha}{4\pi})^2 m_p^5 \approx [10^{28} - 10^{32}yr]^{-1}$$

Present limit:  $\tau_{\overline{v}K^+} > 2.3 \times 10^{33} yrs$ 

## Present experimental limits

#### Super-K, Soudan, IMB, Frejus

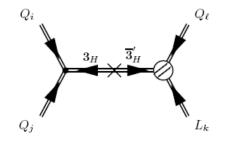


# Rank one also solves the proton decay problem

Proton decay problem in SU(5): one Higgs pair s

$$\xrightarrow{_{3_H}} A_p \propto Y_u Y_d$$

 In SO(10), there are more Higgs fields and if flavor structure is such that triplet Higgs do not connect, no p-decay problem:



Choice flavor structure that does it (Dutta, Mimura, RNM'05)

$$h_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; h_{126} = \begin{pmatrix} 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix};$$
$$h_{120} = \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix};$$

#### **Conclusion:**

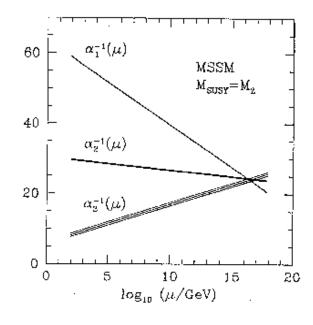
(i) New ansatz to unify diverse profiles of quark and lepton flavor patterns.
(ii) SO(10) GUT with type II seesaw provides a natural framework for realization of this ansatz.
(iii) Predicts measurable θ<sub>13</sub> and solves

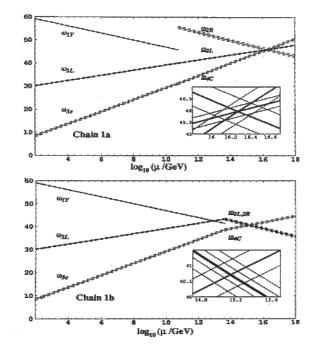
proton decay problem of susy GUTs.

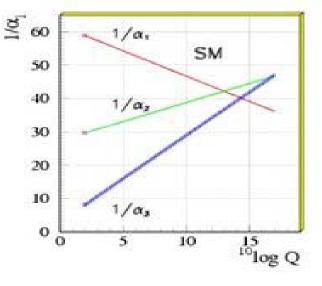
Happy Birthday, Goran !



#### with seesaw







## Simplest example: SUSY SU(5)

☞ The simplest GUT model (circa 1980s)

$$\succ \text{Fermions: } 5 = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e^- \end{pmatrix} \text{ and } 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & u_3 \\ & & 0 & u_3 & d_3 \\ & & & e^+ \\ & & & 0 \end{pmatrix}$$

 $\succ$ : Higgs 5 $\oplus$ 5  $\oplus$  24.

> Predicts: at  $M_U$ ,  $m_b = m_\tau$ ; very good prediction

Also predicts  $m_s = m_{\mu}; m_d = m_e; \text{VERY BAD}$ PREDICTION!!

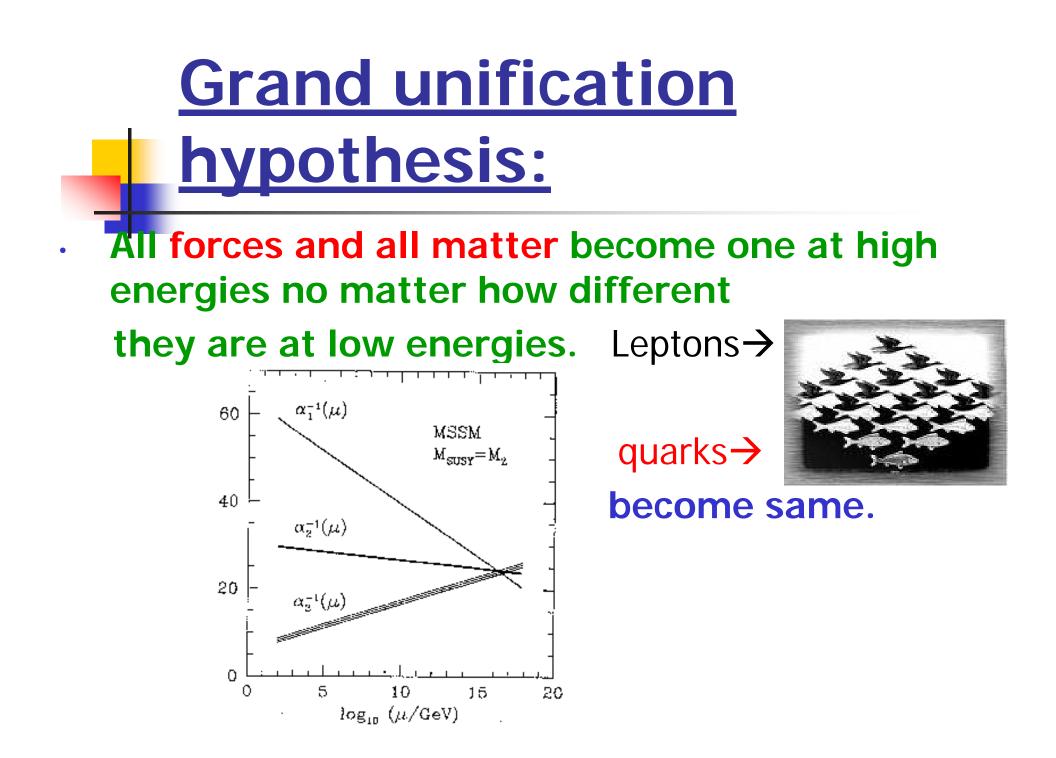
No explanation of neutrino mass:

Why minimal SU(5) not satisfactory

Minimal model ruled out by proton decay !

- Not predictive for neutrinos- so no advantage of GUTs except scale !
- However one nice feature:  $\mathcal{M}_b = \mathcal{M}_{\tau}$

A small piece of the flavor puzzle !!





# Unified understanding of Flavor in SO(10)

Fermion masses depend on 3 matrices: h, f, h'

$$Y_u = h + r_2 f + r_3 h'$$

$$Y_d = r_1(h+f+h')$$

$$Y_e = r_1(h - 3f + c_e h')$$

$$m_{\nu} \cong f v_{\Delta}$$

Suppose, h >> f and h'=0 and h is anarchic:

• Choose basis so  $f = diag(\varepsilon_1, \varepsilon_2, \varepsilon_3)$  with  $\varepsilon_{1,2} << \varepsilon_3 << h_{ab}$ 

**Dominant contributions to VCKM cancel out explaining** why CKM angles are small VCKM coming from  $\mathcal{E}_{1,2,3}$ .

→ large neutrino mixngs come entirely from charged lepton sector;  $U_{PMNS} = U_l^+ U_v \equiv U_l^+$  and hence are large!!

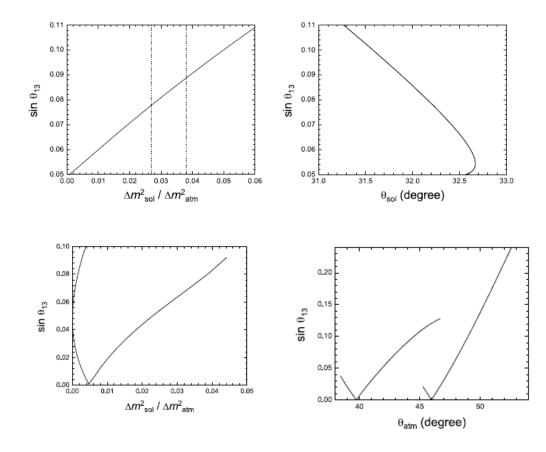
## Realistic 3-generation model for Flavor:

Our proposal after diagonalization of h
  $h \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  with appropriately rotated f and h'.

Different ansatzes for f and h' lead to different realizations of this idea:



Depends on solar and atm masses:

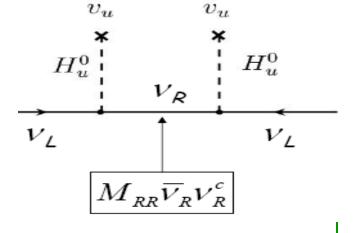


 $\theta_{13} > 0.05$ 

# Why GUT theory for neutrinos ?

- Seesaw paradigm to explain  $m_{\nu} << m_{q,l}$
- Add right handed neutrinos  $N_R$  to SM with Majorana mass:  $L_V = h_V \overline{L} H N_R + M_R NN$
- $M_R$  Breaks B-L : New scale and new physics beyond SM.
- After EWSB

$$m_{\nu} \cong -\frac{h_{\nu}^2 v_{wk}^2}{M_R}$$



-neutrino mass tiny

and neutrino Majorana

Minkowski,Gell-Mann, Ramond, Slansky,Yanagida, Mohapatra, Senjanovic,Glashow

#### **Seesaw scale**

- Neutrino masses → seesaw scale much lower than Planck scale → New symmetry (B-L).
- $m_D \approx m_t$  Type I seesaw +  $\Delta m_{atm}^2$  $\rightarrow M_R \approx 10^{14} GeV$  GUT scale  $10^{16}$  GeV-

-Small neutrino mass strong indication for SUSYGUT;

$$m_D \approx m_e$$

Seesaw scale is around **TeV** 

• Accessible at LHC, other signals,  $\mu \rightarrow e + \gamma$ ,  $\beta \beta_{0\nu}$ 

### Why Supersymmetry ?

#### Simple picture of force Unification:

- Predicts correctWeinberg angle
- Candidate for Dark matter

