The Minimal Fine Tuning Principle and Observable new Physics

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THE MINIMAL FINE TUNING PRINCIPLE AND OBSERVABLE NEW PHYSICS

I. Anthropic fine tuning and the multiverse idea

II. Anthropic tuning of the Weak scale

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I. Anthropic fine tuning and the multiverse idea.

Very controversial! ("not science", "giving up", attempt to revive discredited teleology, … , not testable) But Steven Weinberg's suggestion in 1989 that the smallness of Λ may have anthropic explanation gave greater respectability to idea

Multiverse idea: There is one universe with many "domains" or "sub-universes". Some "constants of nature " are determined by fields whose values vary among the domains: i.e. they "scan". Parameters can only get measured in in domains where the values of those parameters allow life to evolve.

Other domains are outside our horizon $\Longrightarrow$ NOT TESTABLE

Point of this talk: "Minimal Fine Tuning Principle" (which is that there should be no unnatural fine tuning unless it can be anthropically explained) + other well-motivated assumptions can lead to testable scenarios.

II. Anthropic tuning of the Weak scale

ABDS: V. Agrawal, S.M. Barr, J.F. Donoghue, D. Seckel, Phys. Rev. D57, 5480 (1998),
BK: S.M. Barr and A. Khan, Phys. Rev. D76, 045002 (2007).

Motivation for looking for anthropic explanation of weak scale:

(1) $\mu^2, \ \Lambda$ are the most intractable of the fine tuning problems.

(2) They involve the greatest amount of fine tuning : $\mu^2 / M_{P\ell}^2 \approx 10^{-34}$ and $\Lambda / M_{P\ell}{}^4 \approx 10^{-120}$. (The next most tuning is of $\bar{\theta} \leq 10^{-10}$)

(3) They are the only dimensionful parameters of the SM + GR (except for $M_{P\ell}$).

“Living in the multiverse” (hep-th 0511037)
Steven Weinberg

“II. What constants scan?

…

We need to know what constants actually scan in this sense. Physicists would like to be able to calculate as much as possible, so we hope that not too many constants scan.

The most optimistic hypothesis is that the only constants that scan are the few whose dimensionality is a positive power of mass: the vacuum energy and whatever mass or masses set the scale of electroweak symmetry breaking. With all other parameters of the Standard Model fixed, the scale of electroweak symmetry breaking is bounded by about 1.4 to 2.7 of its value in our subuniverse, by the condition that the pion mass should be small enough to make the nuclear force strong enough to keep the deuteron stable against fission.[13] …

If the electroweak scale is anthropically fixed, then we can give up the decades long search for a natural solution to the hierarchy problem. This is a very attractive prospect, because none of the “natural” solutions that have been proposed, such as technicolor or low energy supersymmetry, were ever free of difficulties. In particular, giving up low energy supersymmetry can restore some of the most attractive features of the non-supersymmetric standard model: automatic conservation of baryon and lepton number in interactions up to dimension 5 and 4 respectively; natural conservation of flavors in neutral currents; and a small neutron electric dipole moment.”

13. V. Agrawal, S.M. Barr, J.F. Donoghue, and D. Seckel, Physical Review **D57**, 5480 (1998).

Summary of ABDS paper:

$$\mu^2 < 0: \qquad \langle H \rangle \propto \sqrt{|\mu^2|} \quad \Rightarrow \quad m_q \propto \sqrt{|\mu^2|} \quad \Rightarrow \quad m_\pi^2 \propto f_\pi \sqrt{|\mu^2|}$$

$$\Rightarrow r_{NN} \propto |\mu^2|^{-1/4} f_\pi^{-1/2}$$

So , for large negative μ^2 nuclei become unstable, starting with the deuteron.

$$\mu^2 > 0: \qquad V_{eff}(H) = +\tfrac{1}{2}\mu^2 H^2 + \sum_f h_f \bar{q}_f q_f H + \ldots$$

$$\Rightarrow \quad \langle H \rangle \propto f_\pi^3 / \mu^2 \quad \Rightarrow T_{chem} \sim \alpha^2 m_e \propto f_\pi^3 / \mu^2$$

So, for large positive μ^2 the universe must be very cold for chemistry-based life, and therefore (according to ABDS) very old → no stars left/perhaps baryons gone. Dark energy undercuts this argument, but (BK) cosmic expansion rips planets apart.

NOTE: We need the light quark masses to be smaller than f_π, Λ_{QCD} to get strong dynamics, dynamical symmetry breaking, pions as ps.-goldstone bosons, etc.

The weak scale needs to be near the strong scale!

$$M_{WEAK} / M_{P\ell} \approx 10^{-17} \qquad\qquad M_{STRONG} / M_{P\ell} \approx 10^{-19}$$

Summary of BK paper: **Consider two Higgs doublets (e.g. GUTs, SUSY, PQ, …):**

$$H_U = (1,\ 2,\ +\tfrac{1}{2}) \qquad H_D = (1,\ 2,\ -\tfrac{1}{2})$$

$$\rightarrow \left(H_U,\ H_D^* \right) \underbrace{\begin{pmatrix} M_U^2 & \Delta^2 \\ \Delta^{2*} & M_D^2 \end{pmatrix}}_{\mathrm{M}^2} \begin{pmatrix} H_U^* \\ H_D \end{pmatrix}, \qquad M_U^2, M_D^2, \Delta^2 \sim M_{GUT}^2$$

To get a light Higgs doublet, one only needs to tune $\det \mathrm{M}^2 = \mu^2 M_h^2$ **to be** $O(M_W^2 M_{GUT}^2)$

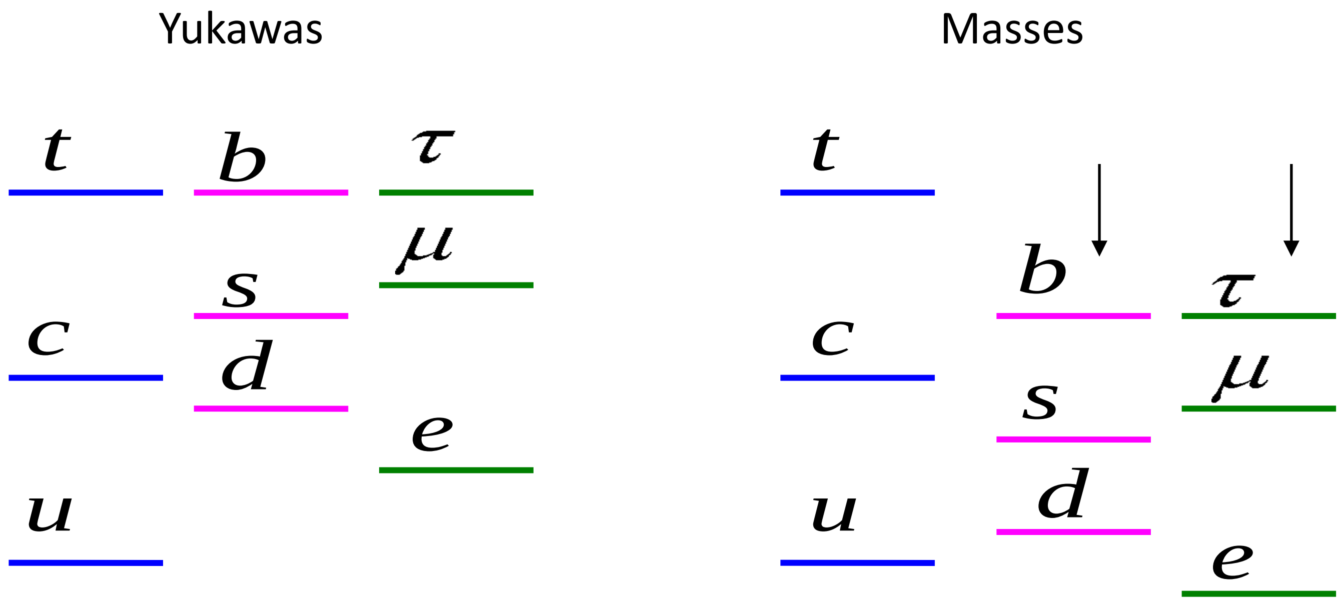
Then:

$$\begin{cases} H_{SM} = \cos\theta_H\ H_U + \sin\theta_H\ H_D^* \\ H_h = -\sin\theta_H\ H_U + \cos\theta_H\ H_D^* \end{cases}$$

where $\tan 2\theta_H = 2\Delta^2 / (M_D^2 - M_U^2) \sim O(1)$

$$\Rightarrow \ M_{down},\ M_{\ell ep} \propto \sin\theta_H \langle H_{SM} \rangle, \qquad M_{up} \propto \cos\theta_H \langle H_{SM} \rangle$$

If both M_U^2 **and** M_D^2 **"scan", then both** μ^2 **and** θ_H **scan**



Idea: **The *intra-family mass ratios* are just the ratios of *Yukawas*, and are set by symmetries (GUT, flavor, etc)**

but the *overall scale* of the *down quark & charged lepton masses relative to the up quark masses* is set by the ratio $\langle H_D \rangle / \langle H_U \rangle = \tan\theta_H$, i.e. the Higgs mixing angle θ_H , which is set "anthropically".

Note, that anthropic considerations require that the u and d masses be close and that the d mass be slightly larger than the u mass.

III. The Minimal Fine Tuning Principle and predictive schemes

MFTP: No "unnatural" fine tunings except those that can be understood anthropically (e.g. $\mu^2, \ \Lambda$)

Additional assumption: No low-energy SUSY (not needed for gauge hierarchy)

Unification of gauge couplings? (a) intermediate scale (→ split multiplets → extra fine tunings), or (b) weak scale incomplete fermion multiplets (also typically extra fine tunings), or (c) Extra weak-scale incomplete scalar multiplets.

6 light Higgs doublets (or even 5) can give unification of gauge couplings (see S. Willenbrock, Phys. Lett. B561, 130 (2003); J. Sayre, S. Wiesenfeldt and S. Willenbrock, Phys. Rev. D73, 035013 (2006)

Proton decay? Unification scale is only 10^{14} GeV $\Rightarrow SU(3)\times SU(3)\times SU(3)\times S_3$
(or orbifold breaking of GUT?)

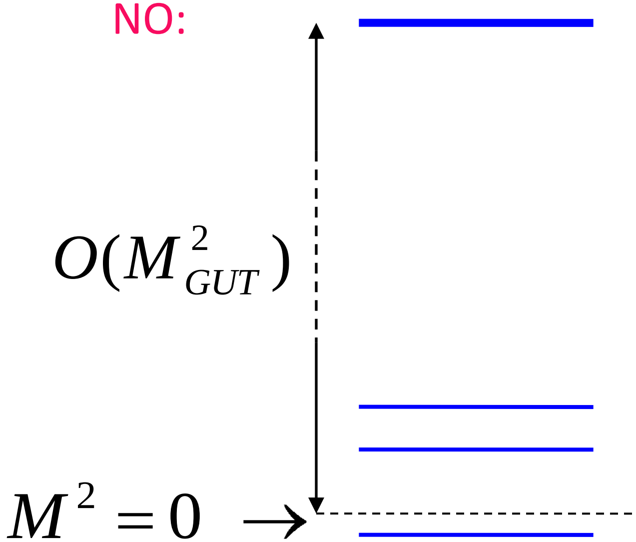
What makes the "extra" light Higgs doublets light, without "extra" fine tunings?
They must be in a multiplet with the SM Higgs of a non-abelian group.

Since the SM Higgs couples to quarks and leptons, the quarks and leptons must also be non-singlets under the non-abelian group → FAMILY SYMMETRY

This is an attractive idea: It is a strange feature of the Standard Model that there are so many quarks and leptons and so few Higgs. Moreover, a family group "explains" the multiplicity of families.

Simple possibilities:
Three families of quarks/leptons in 3 of SU(3) or SO(3).
Six Higgs doublets in 3 + 3 of SU(3) or SO(3) or 6 of SU(3)
or Five Higgs doublets in 5 of SO(3)

BUT: The extra Higgs doublets must be very heavy (to avoid FCNC). Doesn't That contradict them being in a multiplet with the SM Higgs doublet?
NO:



Family-invariant mass of entire Higgs multiplet tuned until lightest Higgs doublet in multiplet has small negative mass-squared. The mass of "extra" Higgs doublets will then be set by the breaking scale of the family symmetry.

$$\Delta M^2 \sim M_F^2$$

(Due to dynamical breaking of family group)

IV. MODEL

WHICH GROUP for G_F ?

SU(3) has difficulties:

(a) Anomaly cancellation.

(b) Δ^2 is typically forbidden: e.g.

$$(R,3)_H \to \underbrace{(1,\ 2,\ +\tfrac{1}{2};\ 3)_H}_{H_U^\alpha} + \underbrace{(1,\ 2,\ -\tfrac{1}{2};\ 3)_H}_{H_D^\alpha}$$

$\Rightarrow \Delta^2 H_U H_D$ would have to come from $\langle S_{\alpha\beta} \rangle H_U^\alpha H_D^\beta$, which breaks the family group, and must therefore be near the Weak scale. That would mean that both M_U^2 and M_D^2 would need to be fine tuned.

It seems simpler to construct models based on orthogonal groups, such as SO(3), SO(4). Maybe non-abelian discrete groups could give interesting models.

SO(3) TOY MODEL

$$G = G_U \times G_F \times G_{DSB}$$

$$\begin{cases} G_U = [SU(3)]^3 \times S_3 \quad (or\ orbifold-broken\ GUT?) \\ G_F = SO(3) \\ G_{DSB} = SU(N) \end{cases}$$

FAMILIES:

$$(27,\ 3,\ 1)_L = \psi^i, \qquad i = 1,2,3$$

HIGGS:

$$(27,\ 5,\ 1)_H = \Phi^{(ij)}$$

G_F BREAKING SECTOR:

$$\begin{cases} (1,\ 3,\ N)_L = \chi^{\mu\ i} & \mu = 1,\ldots,N & (SU(N)\ index) \\ 3 \times (1,\ 1,\ \overline{N}) = \overline{\chi}_{\mu\ I} & I = 1,\ldots,N & (just\ a\ label) \\ k \times (1,\ 3,\ 1)_H = \eta^i_J & J = 1,\ldots,k & (just\ a\ label) \end{cases}$$

COUPLINGS:

Q & L Yukawas: $y(\psi^i\psi^j)\Phi^{(ij)}$

Higgs mass: $M_\Phi^2\ \Phi^{(ij)*}\Phi^{(ij)} + \Delta^2\ \Phi^{(ij)}\Phi^{(ij)} + \Delta^{2*}\ \Phi^{(ij)*}\Phi^{(ij)*}$

$M^2 + \lambda\langle Adj\rangle^2$ can break G_U

G_F **breaking:**

$$[f_{IJ}(\chi^{\mu\ i}\overline{\chi}_{\mu\ I})\ \eta_J^i + h.c.] + (M_\eta^2)_{JJ'}\eta_{J'}^{i*}\ \eta_J^i$$

$$\Rightarrow \langle \eta_J^i \rangle^* = (M_\eta^2)_{JK}^{-1}\ f_{IK}\langle \chi^{\mu\ i}\overline{\chi}_{\mu\ I}\rangle = (M_\eta^2)_{JK}^{-1}\ f_{IK}\delta_I^i\ f_N^3$$

$$= (M_\eta^2)_{JK}^{-1}\ f_{iK}\ f_N^3 \sim f_N^3 / M_{GUT}^2$$

$\not{G}_F$ **Higgs masses:**

$$\lambda_{JK}(\Phi^{(ij)*}\Phi^{(jk)}\eta_J^k\ \eta_K^{i*})$$

$$\rightarrow \Phi^{(ij)*}\Phi^{(jk)}(m^2)^{ki}, \quad where \quad (m^2)^{ki} \equiv \lambda_{JK}\eta_J^k\ \eta_K^{i*}$$

$$= tr\left\{\Phi^+\ \Phi\ m^2\right\}$$

H_{SM} is the lightest of the 5 Higgs doublets and is a mixture of the $\Phi^{(ij)}$:

$$H_{SM} = \sum_{ij} a_{(ij)} \Phi^{(ij)} \;\; for \;\; some \;\; a_{(ij)} \quad \Rightarrow \quad \left\langle \Phi^{(ij)} \right\rangle \propto a_{(ij)}$$

PROBLEM: $\Phi^{(ij)}$ IS TRACELESS: Mass matrices not realistic. ONE family can be made very light (but not two) as follows:

Assume that m^2 has the form

$$m^2 \propto \begin{pmatrix} 1+\delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{12}^* & \delta_{22} & \delta_{23} \\ \delta_{13}^* & \delta_{23}^* & \delta_{33} \end{pmatrix}, \;\; where \; \left|\delta_{ij}\right| << 1$$

Then, from the form of the mass matrix $tr\left(\Phi^* \Phi m^2\right)$ one sees that three Doublets become much heavier than the other two, and these are approximately

$\frac{1}{\sqrt{6}}\left(2\Phi^{(11)} - \Phi^{(22)} - \Phi^{(33)}\right), \; \Phi^{(12)}, \; and \; \Phi^{(13)}$. The lightest two doublets are approximately linear combinations of $\frac{1}{\sqrt{2}}\left(\Phi^{(22)} - \Phi^{(33)}\right) and \; \Phi^{(23)}$. The SM Higgs is

$$H_{SM} = \cos\theta \frac{1}{\sqrt{2}}\left(\Phi^{(22)} - \Phi^{(33)}\right) + \sin\theta \; \Phi^{(23)}$$

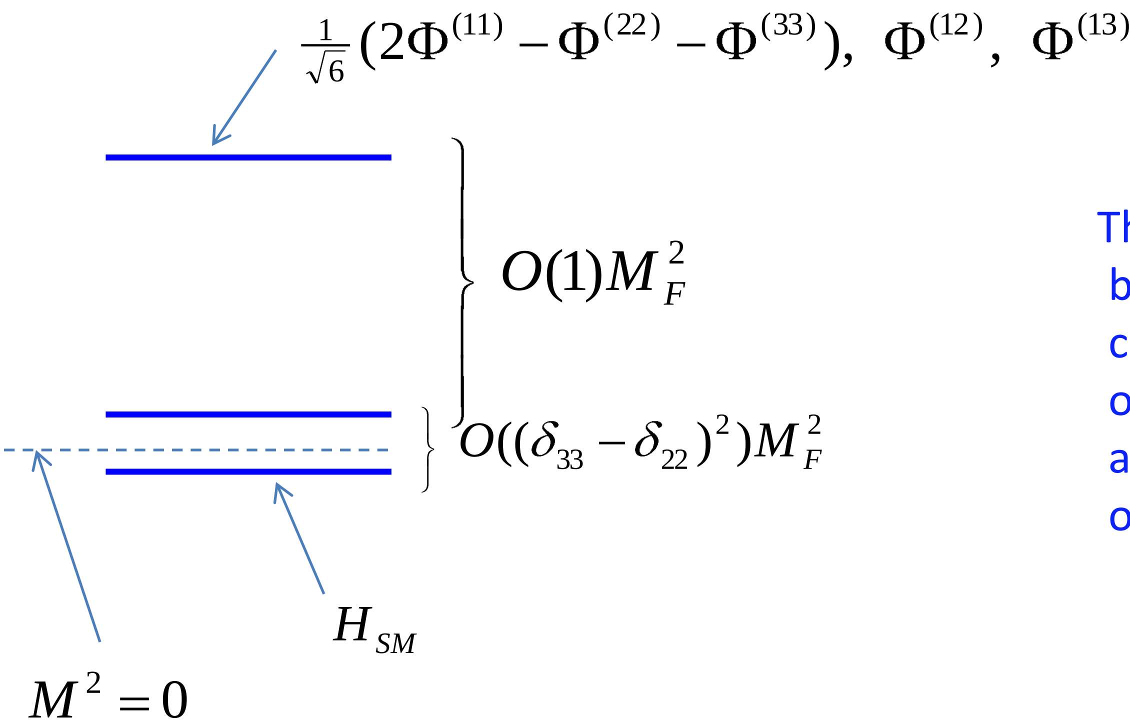
$$+ O(\delta_{22} - \delta_{33}) \frac{1}{\sqrt{6}}\left(2\Phi^{(11)} - \Phi^{(22)} - \Phi^{(33)}\right)$$

$$+ O\left(\delta_{12}\right)\Phi^{(12)} + O\left(\delta_{13}\right)\Phi^{(13)}$$

Thus $M_{down}, \ M_{down}, \ M_{lepton} \propto \begin{bmatrix} O(\delta_{33}-\delta_{22}) & O(\delta_{12}) & O(\delta_{13}) \\ O(\delta_{12}) & A & B \\ O(\delta_{13}) & B & -A \end{bmatrix}, \ A, B = O(1)$

So $\dfrac{m(1st \ \ family)}{m(2nd \ \ family)} \sim O(\delta_{33}-\delta_{22})$. But also, the Higgs doublets have a spectrum

Controlled by the same parameters, in the following way:



This SO(3) model is not realistic, but it illustrates that there is a close connection between the spectrum of Higgs doublets and the quark and lepton spectrum in this type of model.

There are several problems with this SO(3) model: (a) the tracelessness of the quark/lepton mass matrices prevents a realistic fermion mass hierarchy. (b) There is no breaking of G_U in the quark/lepton mass matrices. Consider a d > 4 effective Yukawa term with $\not{G}_U$ VEV: $(\psi^i \psi^j)\Phi^{(ij)}\langle Adj \rangle / M_{GUT}$
The $\not{G}_U$ VEV is GUT-scale and so must be invariant under the family group. Thus the form of the "textures" are not affected by the breaking of the unified group → all the mass fermion matrices are proportional → no CKM mixing.

One can solve these problems by adding to the SO(3) model extra families + mirror families that mix with the triplet of families.

Simpler: enlarge group to SO(4)

quarks and leptons: $(27,\ 4,\ 1) = \psi^i,\ \ i = 1,..,4$

$(27,\ 1,\ 1) = \overline{\psi}$

Yukawas: $y(\psi^i \psi^j)\Phi^{(ij)} + y_J^{(f)}(\psi^i \overline{\psi})\eta_J^i$ **(nine doublets)**

Fermion mass matrices:
$$(f_1, f_2, f_3, f_4, \bar{f}^c)\begin{pmatrix} \sim\delta & \sim\delta & \sim\delta & \sim\delta & \eta^1_{(f)} \\ \sim\delta & \sim\varepsilon & \sim\varepsilon & \sim\varepsilon & \eta^2_{(f)} \\ \sim\delta & \sim\varepsilon & A & B & \eta^3_{(f)} \\ \sim\delta & \sim\varepsilon & B & -A & \eta^4_{(f)} \\ \eta^1_{(f^c)} & \eta^2_{(f^c)} & \eta^3_{(f^c)} & \eta^4_{(f^c)} & 0 \end{pmatrix}\begin{pmatrix} f_1^c \\ f_2^c \\ f_3^c \\ f_4^c \\ \bar{f} \end{pmatrix}$$

Doublet Higgs masses:

4 ——— $\sim M_F$

3 ——— $\sim (m_1 / m_2) M_F$

1 ——— $\sim (m_1 / m_3)\ M_F$

1 ———

V. SUMMARY

There are some good reasons to take seriously that the Higgs mass(es) are anthropically tuned: It can explain why the weak scale is near the strong scale and why the d quark mass is similar to but slightly larger than the u quark mass.

Anthropic fine tuning itself is probably untestable.

However, the requirement of minimal fine-tuning can lead to testable scenarios when combined with other assumptions;

e.g. the assumption of no SUSY + multiple light Higgs doublets to get unification of gauge couplings leads to non-abelian family symmetries

The kinds of models that result in this case are highly constrained. One can hope that by fitting the quark and lepton masses and mixings, one can constrain the couplings of the lightest extra Higgs doublet, and get definite predictions.