Experimental and theory overview of QCD phase diagram

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Outline

- what we know

- theory vs. experiment

- cumulants, factorial cumulants, factorial moments

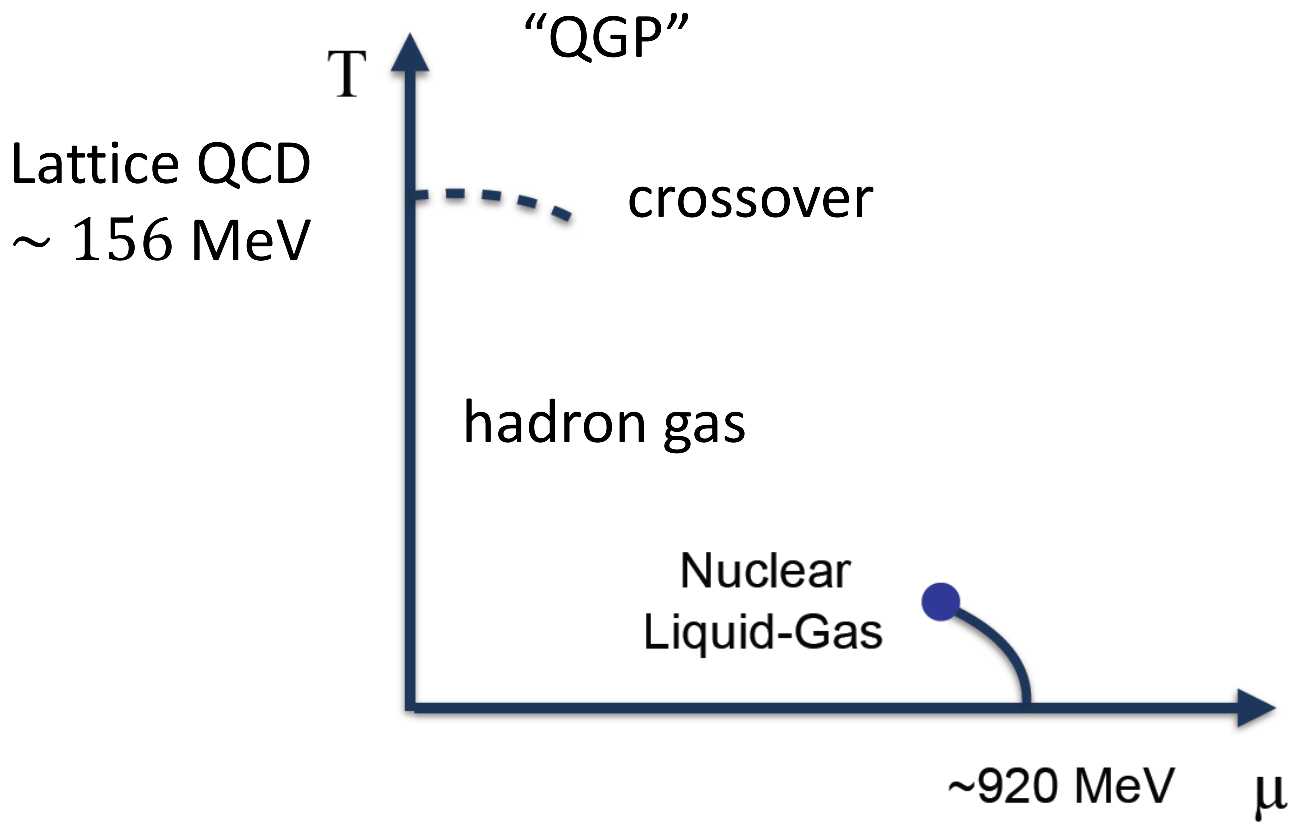
- long-range and short-range correlations

- expectations

- measurements and interpretation

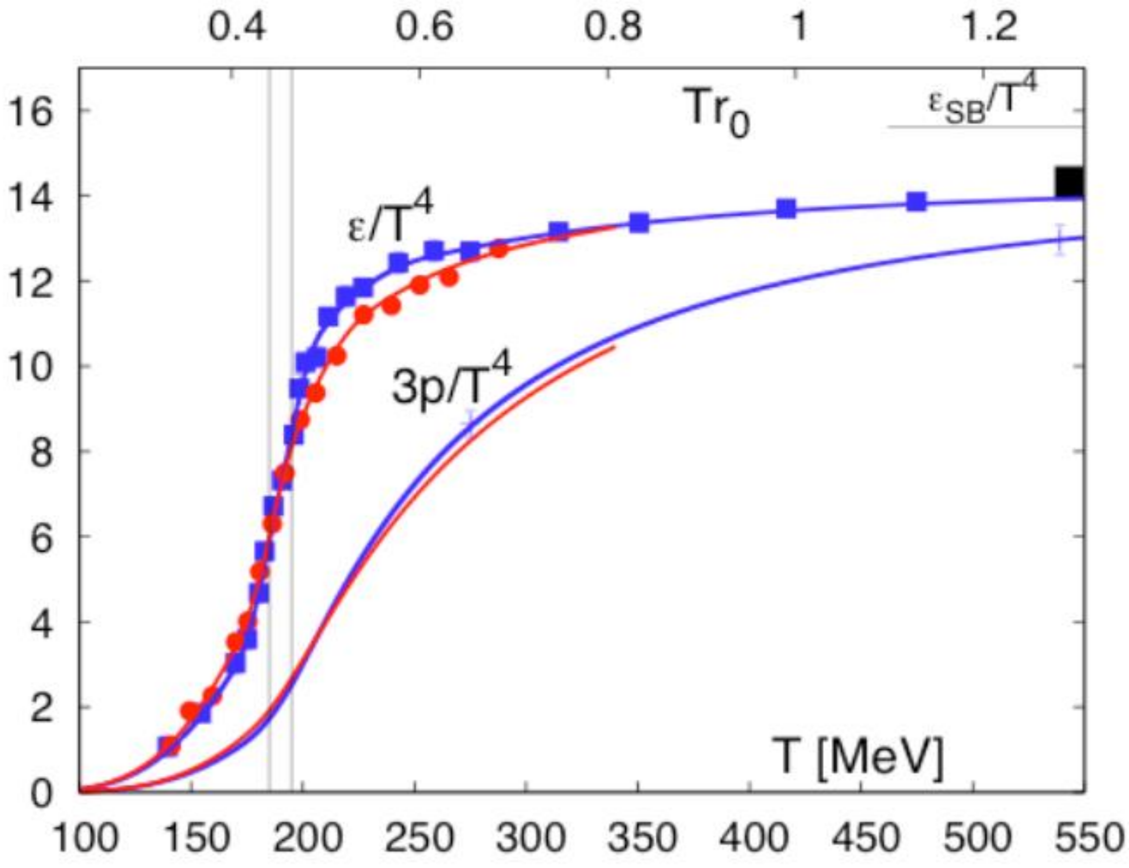
- summary

The QCD phase diagram



Figured taken from V.Koch

Lattice QCD



A smooth and wide crossover

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature 443 (2006) 675

Hopes

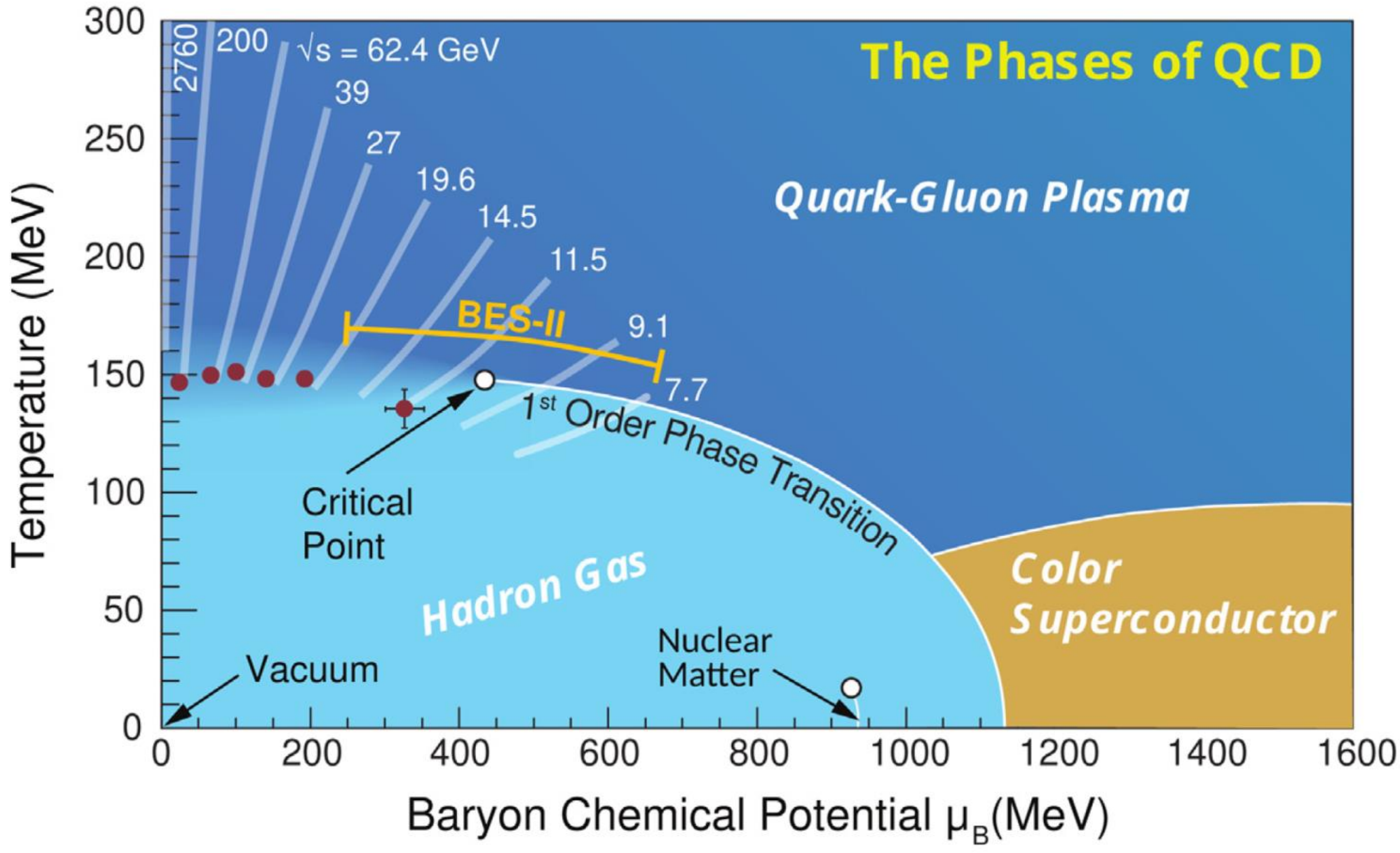
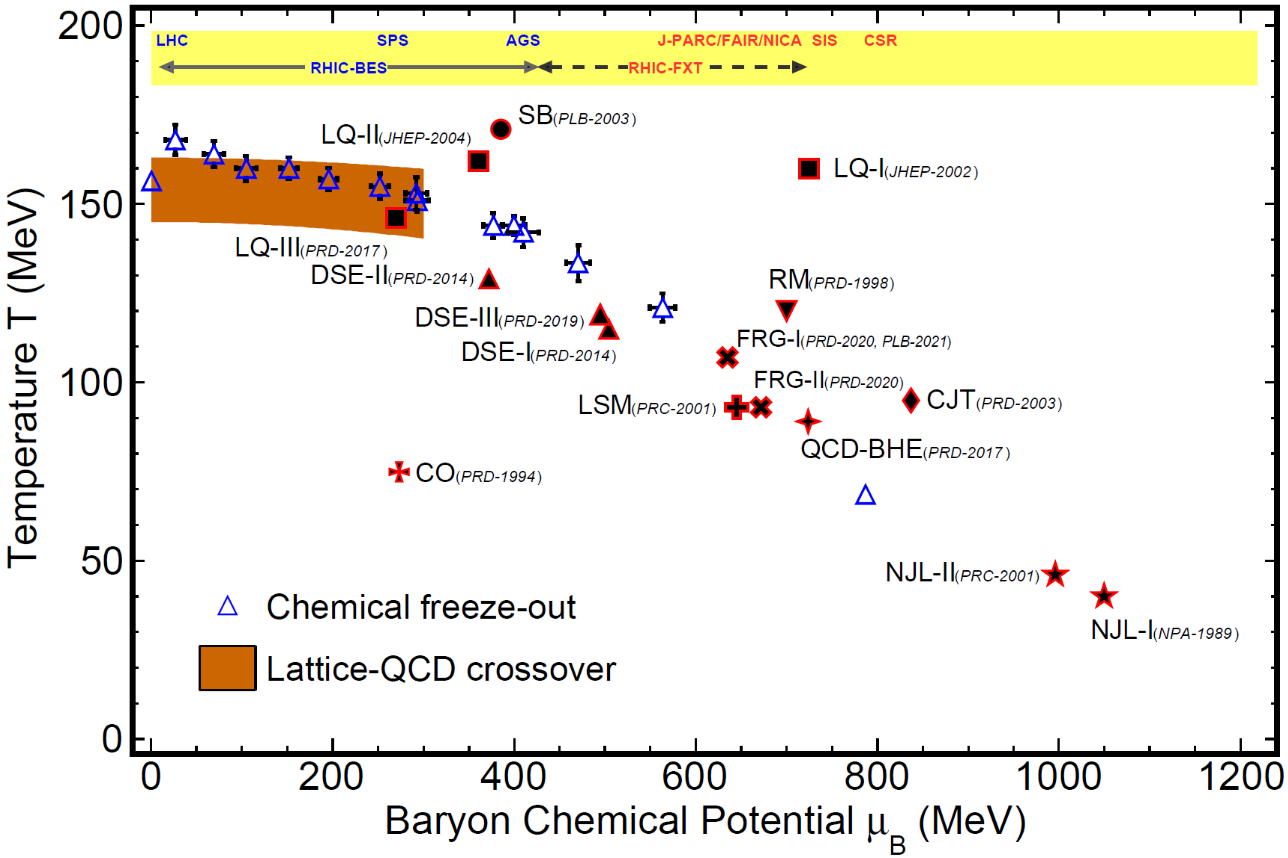


Figure from Phys. Rept. 853 (2020) (AB, S.Esumi, V.Koch, J.Liao, M.Stephanov, N.Xu)

Critical point?

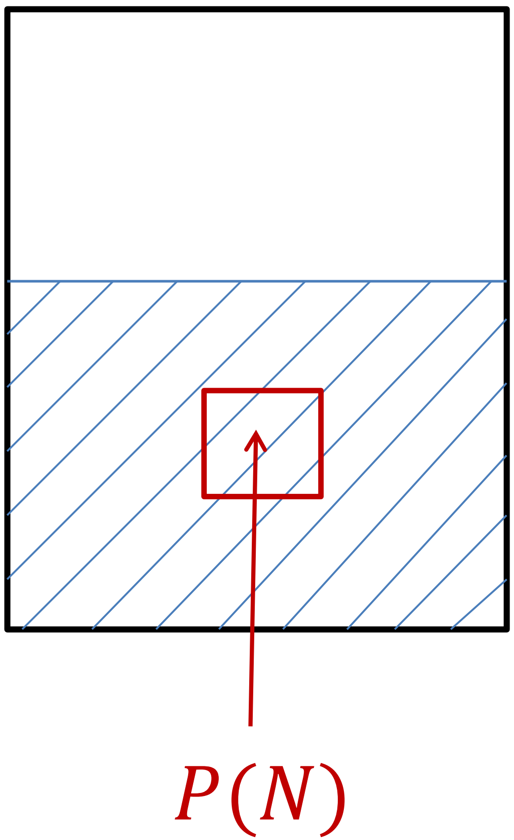


A. Pandav, D. Mallick, B. Mohanty, 2203.07817

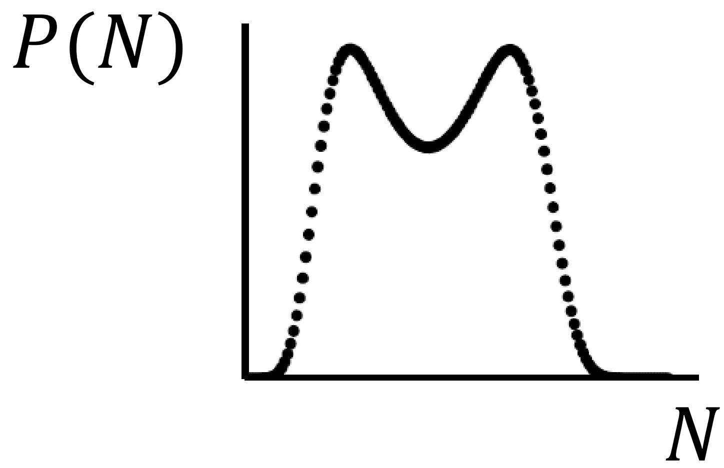
M. Stephanov, hep-lat/0701002

How to approach this problem?

Consider water vapour transition



right at the phase transition



number of H_2O molecules

so we measure multiplicity distributions

In QCD we use, e.g., net-baryon, net-charge, net-strangeness

Theory vs. experiment

Theory	**Experiment**
Coordinate space	Momentum space
Fixed volume	Expanding and fluctuating volume
Long-lived	Extremely short-lived
Conserved charges	Non-conserved numbers

Lots of detector problems + various corrections (e.g. volume fluctuation)

Some progress towards dynamical models.

HYDRO+ (hydrodynamics with critical modes)

M. Stephanov, Y. Yi, PRD 98, 036006 (2018)
K. Rajagopal, G. Ridgway, R. Weller, Y. Yin, PRD 102, 094025 (2020)
We consider … a generic extension of hydrodynamics by a parametrically slow mode or modes ("Hydro+") and a description of fluctuations out of equilibrium.

Equation of state with critical point

P.Parotto, M.Bluhm, D.Mroczek, M.Nahrgang, J. Noronha-Hostler, K.Rajagopal, C.Ratti, T.Schäfer, M. Stephanov, PRC 101, 034901 (2020)
We construct a family of equations of state for QCD in the temperature range 30 MeV ≤ T ≤ 800 MeV and in the chemical potential range 0 ≤ μ_B ≤ 450 MeV. These equations of state match available lattice QCD results up to O(μ_B^4) and in each of them we place a critical point in the three-dimensional (3D) Ising model universality class.

Molecular dynamics

V.A. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, V. Vovchenko, 2201.08486

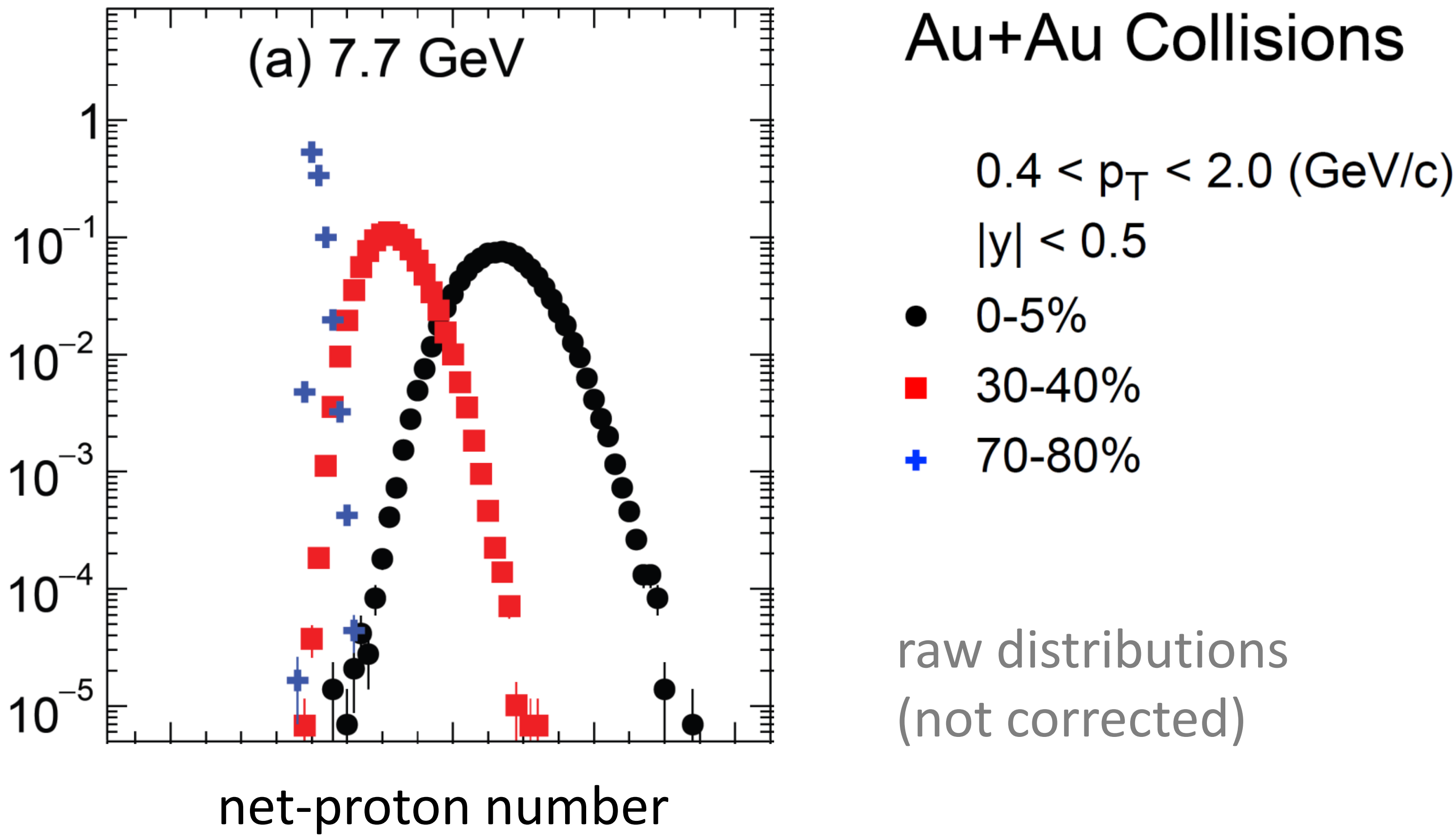
We find that large fluctuations associated with the critical point are observed when measurements are performed in coordinate subspace, but, in the absence of collective flow and expansion, are essentially washed out when momentum cuts are imposed instead.

Hydrodynamics with many non-critical contributions

V.Vovchenko, V.Koch, C.Shen, PRC 105 (2022) 1, 014904

The experimental data of the STAR Collaboration are consistent at $\sqrt{s_{NN}} \gtrsim 20$ GeV with simultaneous effects of global baryon number conservation and repulsive interactions in the baryon sector…

So we measure multiplicity distributions



raw distributions
(not corrected)

STAR Collaboration, PRC 104 (2021) 2, 024902

For baryons absolutely minimal goal is to see any deviations from Poisson (Skellam) distribution.

It is difficult to see something in multiplicity distributions. If there is any signal, it is likely very tiny.

We usually characterize $P(N)$ by:

- cumulants κ_n

- factorial cumualnts, C_n (or $\hat{C}_n$)

- factorial moments F_n (mean number of pairs, triplets, etc.)

Warning. STAR uses opposite notation $\kappa_n \leftrightarrow C_n$

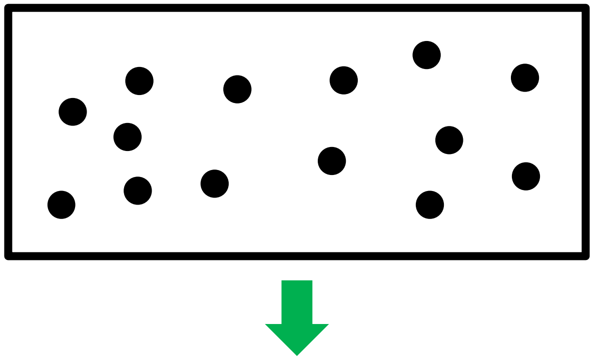
On the experimental side we need to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,
Stephanov, Rajagopal, Shuryak, PRL (1998)
Stephanov, PRL (2009)
Skokov, Friman, Redlich, PRC (2011)

There are many results:

ALICE, STAR, HADES	NA61/SHINE
Cumulants, factorial cumulamts	Intermittency, cumulamnts
Proton v_1 (STAR)	Scaled variance
HBT radii (STAR) R.A. Lacey, PRL 114 (2015) 142301	Strongly intensive variables

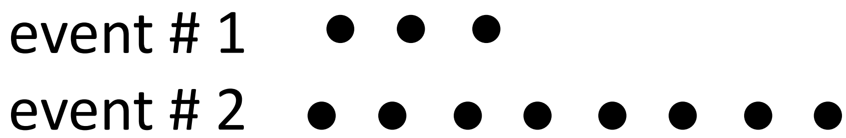
Poisson distribution (no correlations)



$N = 10^{10}$

$p = 10^{-9}$

$\langle n \rangle = Np = 10$



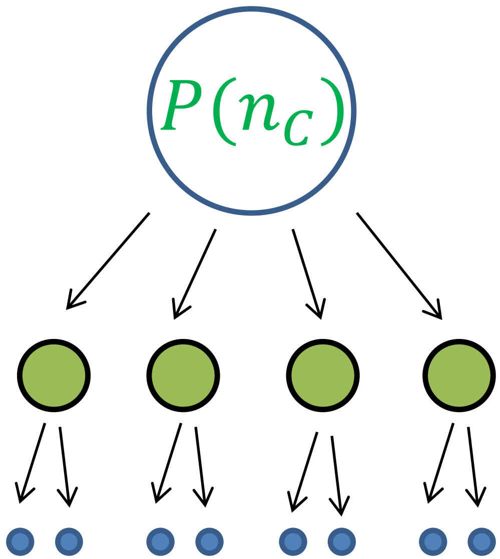
$P(n) =$ Poisson if $N \to \infty, \; p \to 0, \; Np = \langle n \rangle$

cumulants $\kappa_i = \langle n \rangle$

factorial cumulants $C_i = 0$

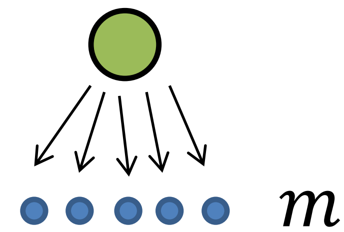
factorial moments $F_i = \langle n \rangle^i$

Factorial cumulants – example



Poisson

m particle cluster



$$C_2 \neq 0$$

$$C_k = 0, k > 2$$

$$C_{2,3,\dots,m} \neq 0$$

$$C_k = 0, k > m$$

factorial cumulants

$$C_k = \frac{d^k}{dz^k} \ln\left(\sum_n P(n) z^n\right)\Big|_{z=1}$$

Two-particle correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + C_2(y_1, y_2)$$

Integrating both sides over some bin in rapidity

$$\langle n(n-1)\rangle = \langle n\rangle^2 + C_2$$

$$C_2 = \int C_2(y_1, y_2)\, dy_1 dy_2$$

Same with multiparticle correlations.

Factorial cumulants are integrated multiparticle correlation functions

Factorial cumulants vs cumulants

factorial cumulant

$$C_i = \frac{d^i}{dz^i} \ln\left(\sum_n P(n) z^n\right) |_{z=1}$$

cumulant

$$\kappa_i = \frac{d^i}{dt^i} \ln\left(\sum_n P(n) e^{tn}\right) |_{t=0}$$

Poisson

$C_i = 0,\ \kappa_i = \langle n \rangle$

cumulants naturally appear in statistical physics

$$\ln(Z) = \ln\left(\sum_i e^{-\beta(E_i - \mu N_i)}\right)$$

Cumulants (one species of particles)

$$\kappa_2 = \langle N \rangle + C_2$$

$$\kappa_3 = \langle N \rangle + 3C_2 + C_3$$

$$\kappa_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Cumulants mix integrated correlation functions of different orders

They might be dominated by $\langle N \rangle$.

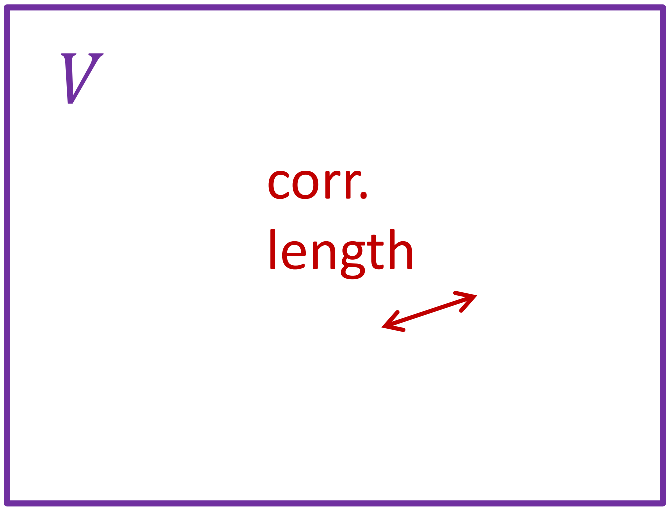
See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) 034915
AB, V.Koch, N.Strodthoff , PRC 95 (2017) 054906

"Cumulant ratios do not depend on volume"

but depend on volume fluctuation

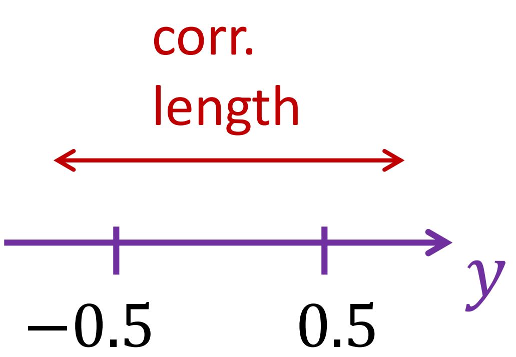
It is true if a correlation length is much smaller than the system size

coordinate space



Here this condition is satisfied

momentum rapidity space



Correlation length is usually larger than one unit of rapidity.

Cumulant ratios are expected to depend on acceptance in rapidity

Short-range correlations

$$C_i \sim \langle N \rangle \sim \Delta y$$

$$\kappa_i \sim \langle N \rangle \sim \Delta y$$

Long-range correlations (expected in rapidity)

$$C_i \sim \langle N \rangle^i \sim (\Delta y)^i$$

κ_i is complicated, for example

$$\kappa_4 = \langle N \rangle + \left(\sim \langle N \rangle^2\right) + \left(\sim \langle N \rangle^3\right) + \left(\sim \langle N \rangle^4\right)$$

$$\kappa_2 = \langle N \rangle + \left(\sim \langle N \rangle^2\right)$$

polynomial in Δy

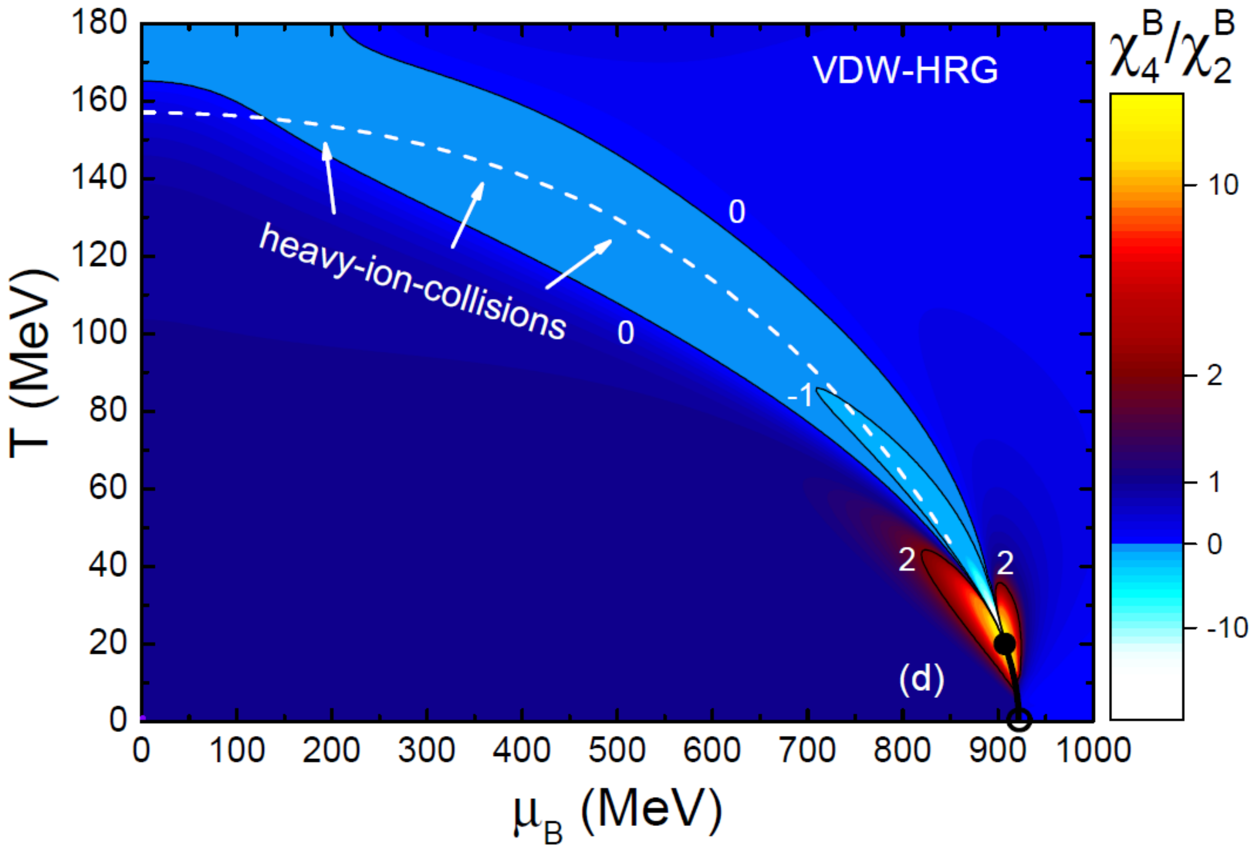
Cumulant ratios may strongly depend on acceptance in rapidity and in transverse momentum

Comparison with models which do not have experimental acceptance is questionable

Comparison with lattice QCD calculations is very tricky

With long-range rapidity correlations the cleanest observable is $\frac{C_i}{\langle N \rangle^i}$

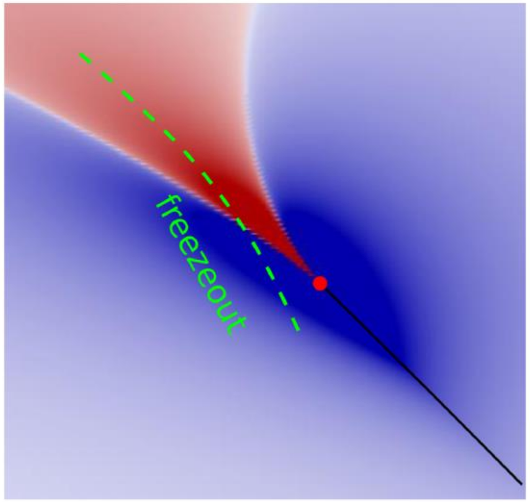
HRG with attractive and repulsive Van der Waals interactions between (anti)baryons



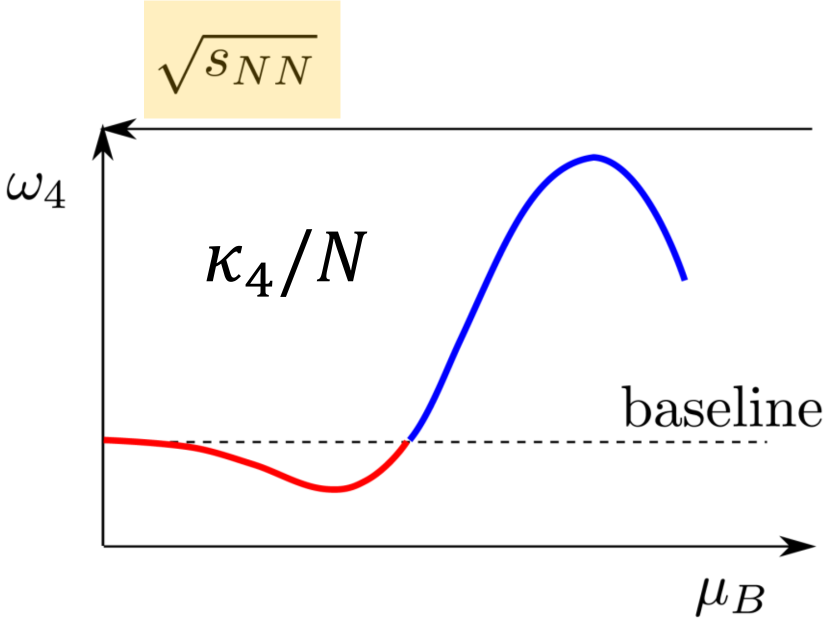
κ_4/κ_2

V. Vovchenko, M.I. Gorenstein, H. Stoecker, PRL 118 (2017) 182301

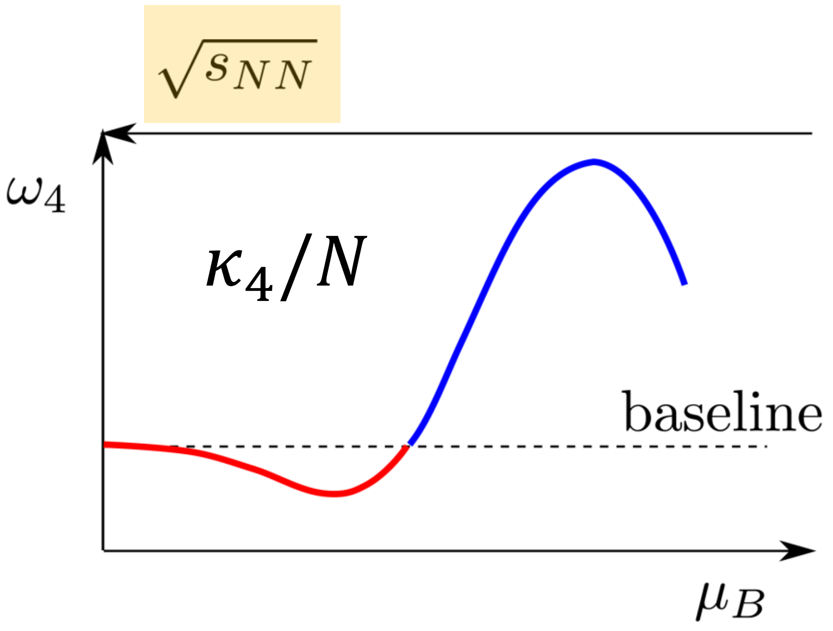
M.A. Stephanov PRL 107 (2011) 052301



Density plot of the quartic cumulant obtained by mapping the Ising model into QCD.
Freezeout line is for demonstration only.



Normalized quartic cumulant of proton multiplicity



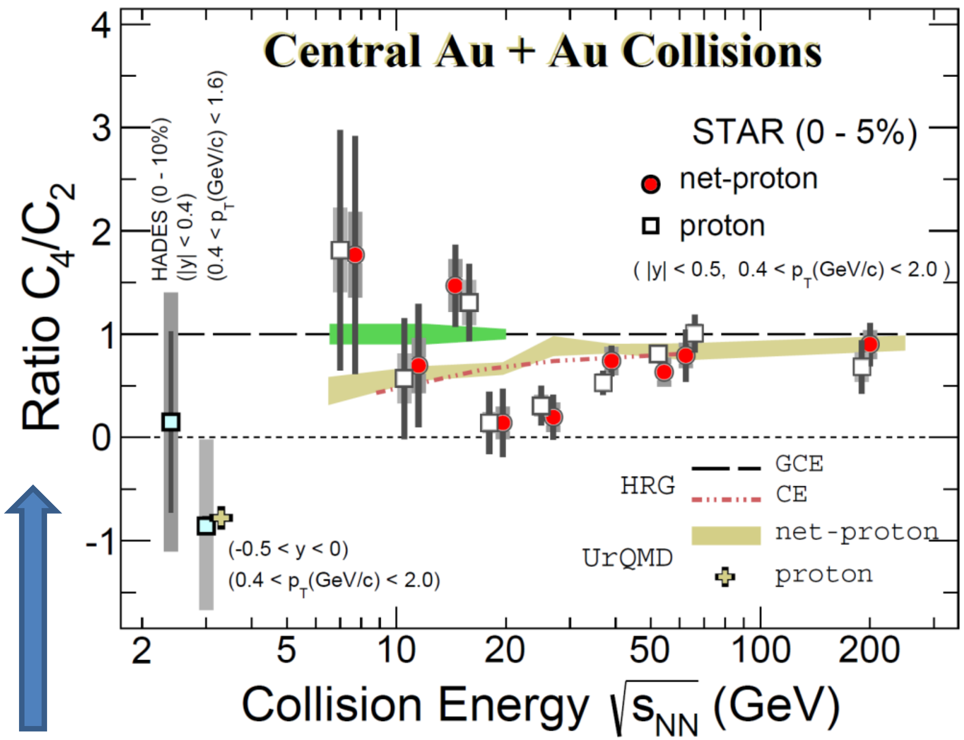
D. Mroczek, A.R. Nava Acuna, J. Noronha-Hostler, P. Parotto, C. Ratti, M.A. Stephanov, PRC 103 (2021) 3, 034901

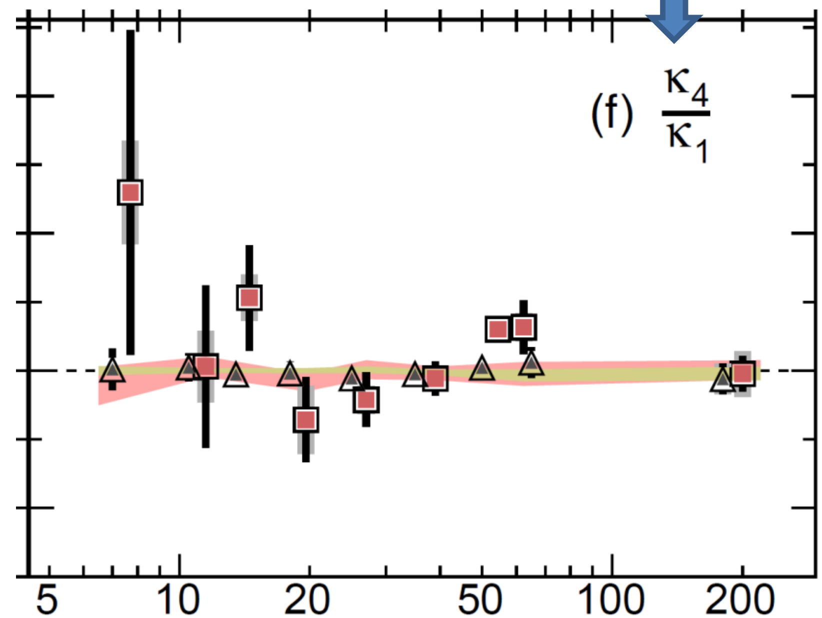
We find that, while the peak remains a solid feature, the presence of the critical point does not necessarily cause a dip in χ_4^B *on the freezeout line below the transition temperature.*

STAR Collaboration

2112.0024
PRC 104 (2021) 2, 024902

kindly read C_4/C_1





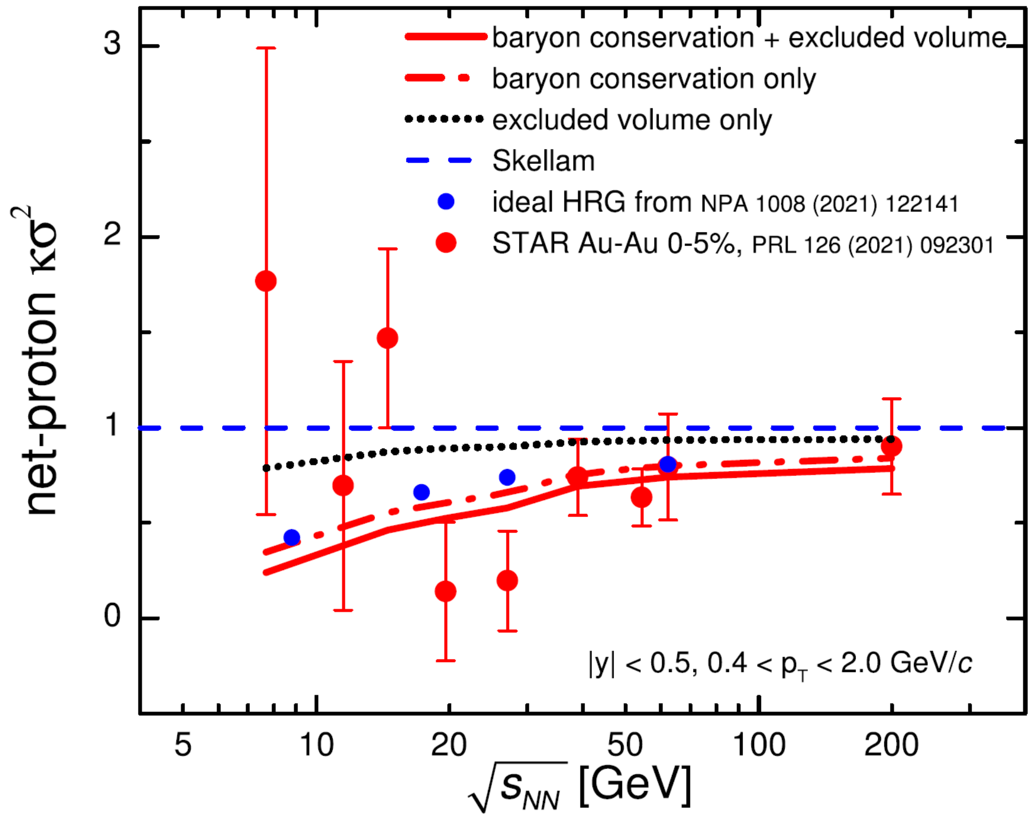
kindly read κ_4/κ_2

Visible four-proton correlations at 7.7 GeV (large errors)

A hint of non-monotonic dependence

STAR data vs. hydrodynamics with baryon conservation and excluded volume

κ_4/κ_2

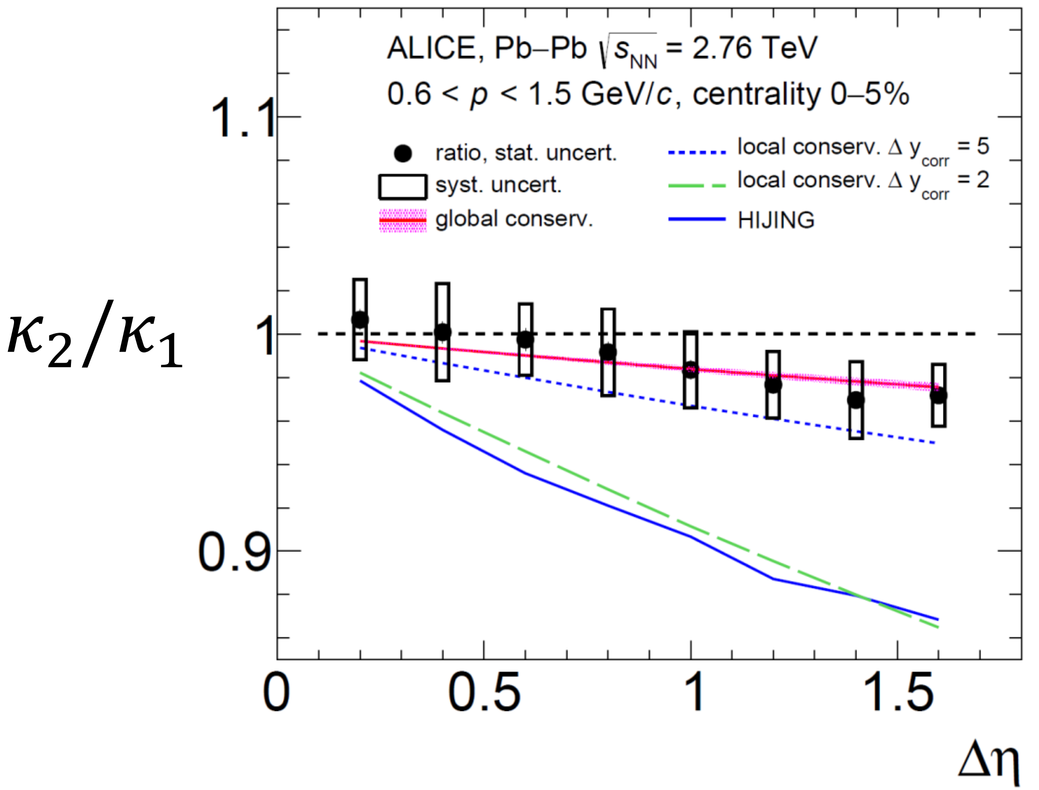


Baryon conservation for $\sqrt{s} > 20$ GeV

V.Vovchenko, V.Koch, C.Shen, PRC 105, 014904 (2022)
P.Braun-Munzinger, B.Friman, K.Redlich, A.Rustamov, J.Stachel, NPA 1008 (2021) 122141
AB, V.Koch, V.Skokov, PRC 87 (2013) 1, 014901

ALICE Collaboration

PLB 807 (2020) 135564

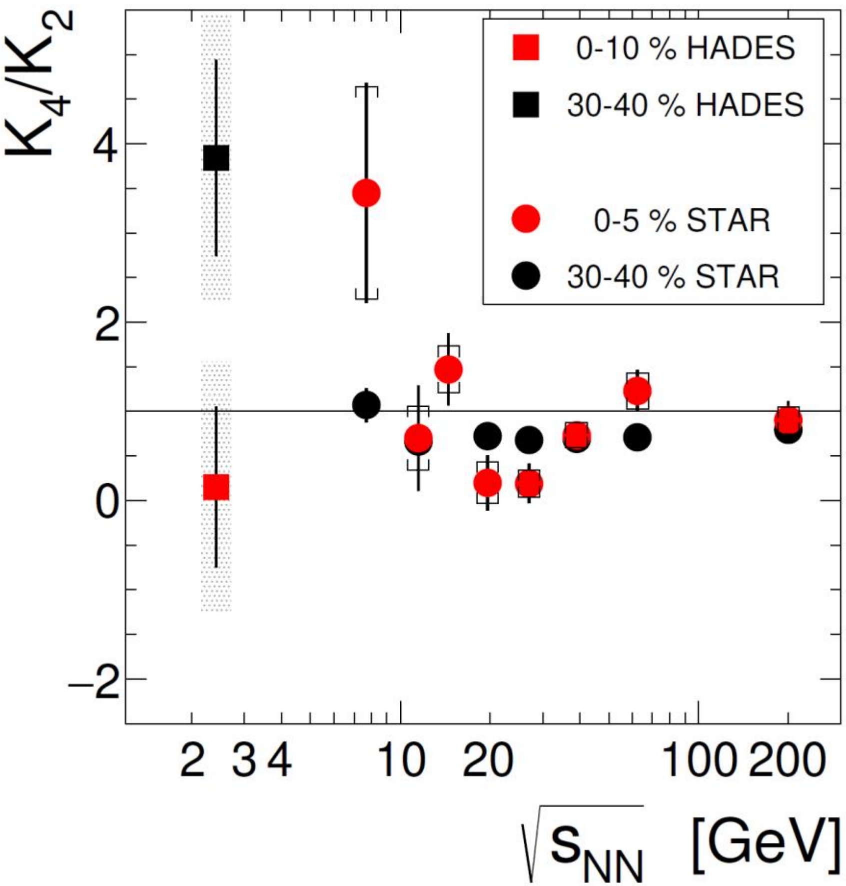


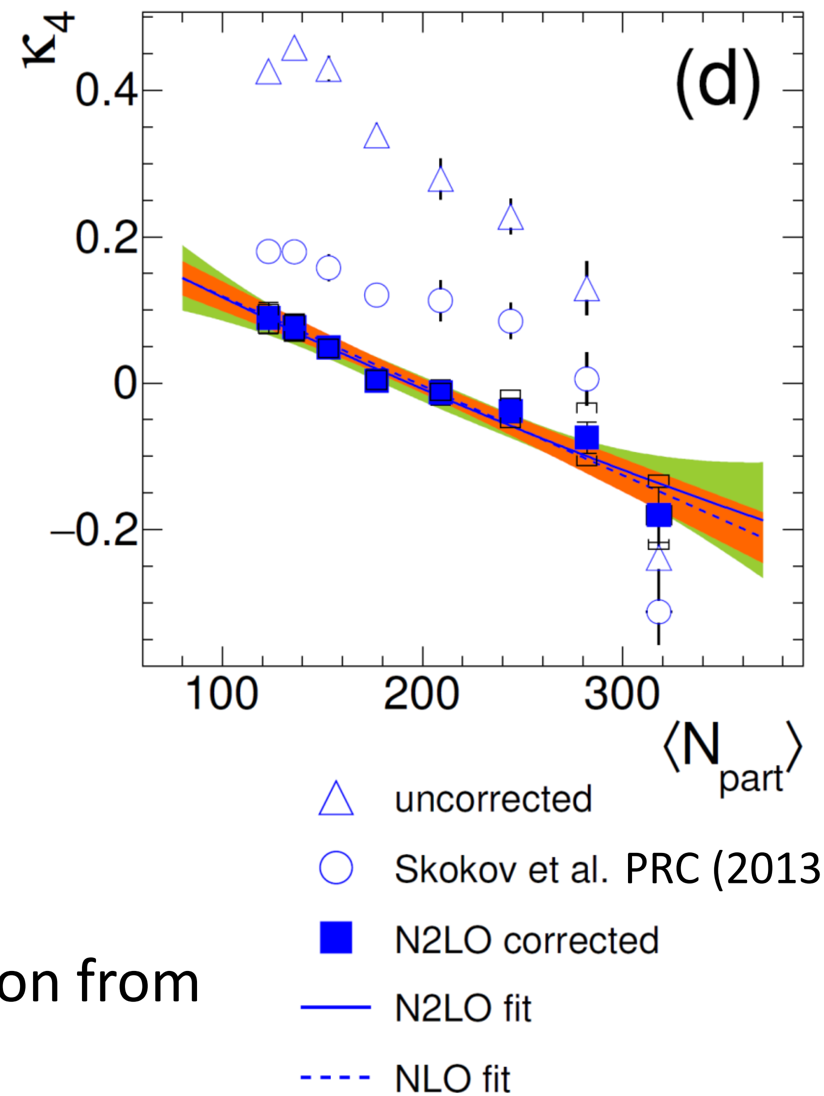
Global (not local) baryon conservation! Something to understand.

It would be good to measure proton, antiproton and mixed proton-antiproton factorial cumulants. See a poster by **Michał Barej**

HADES Collaboration

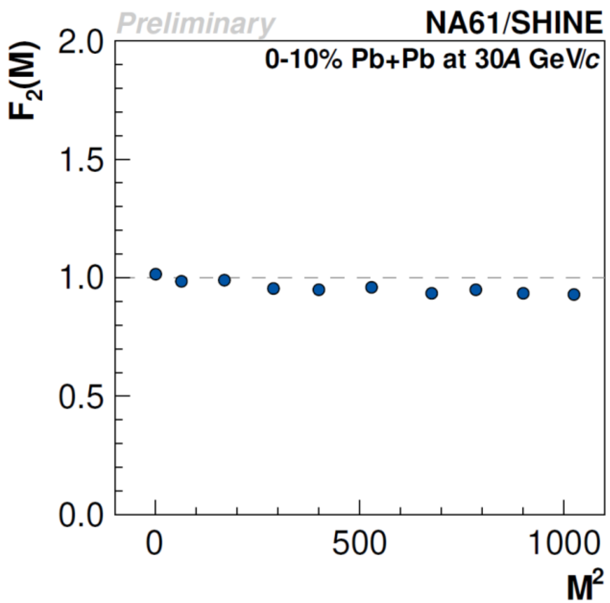
PRC 102 (2020) 2, 024914

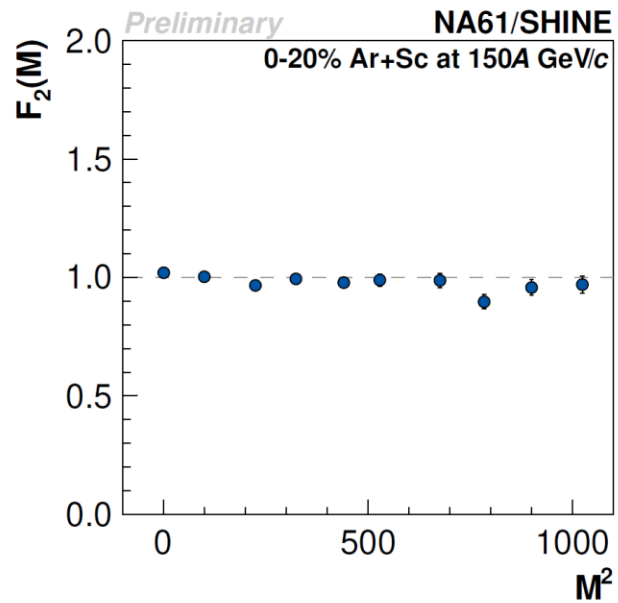




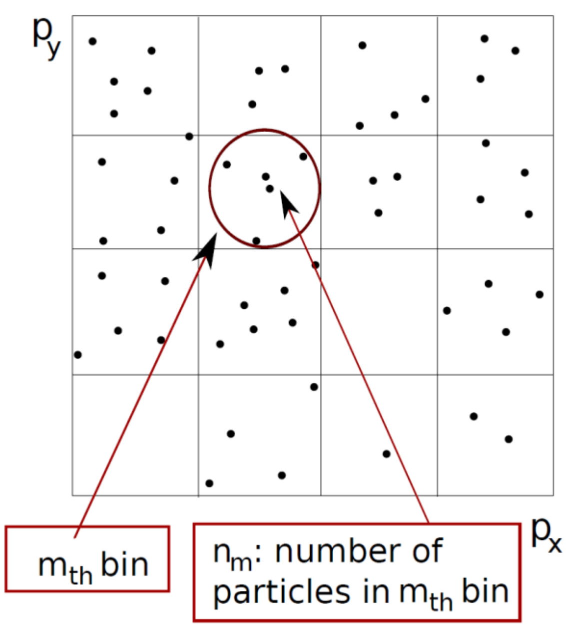
Significant correction from volume fluctuation

NA61/SHINE Collaboration





statistical uncertainties only

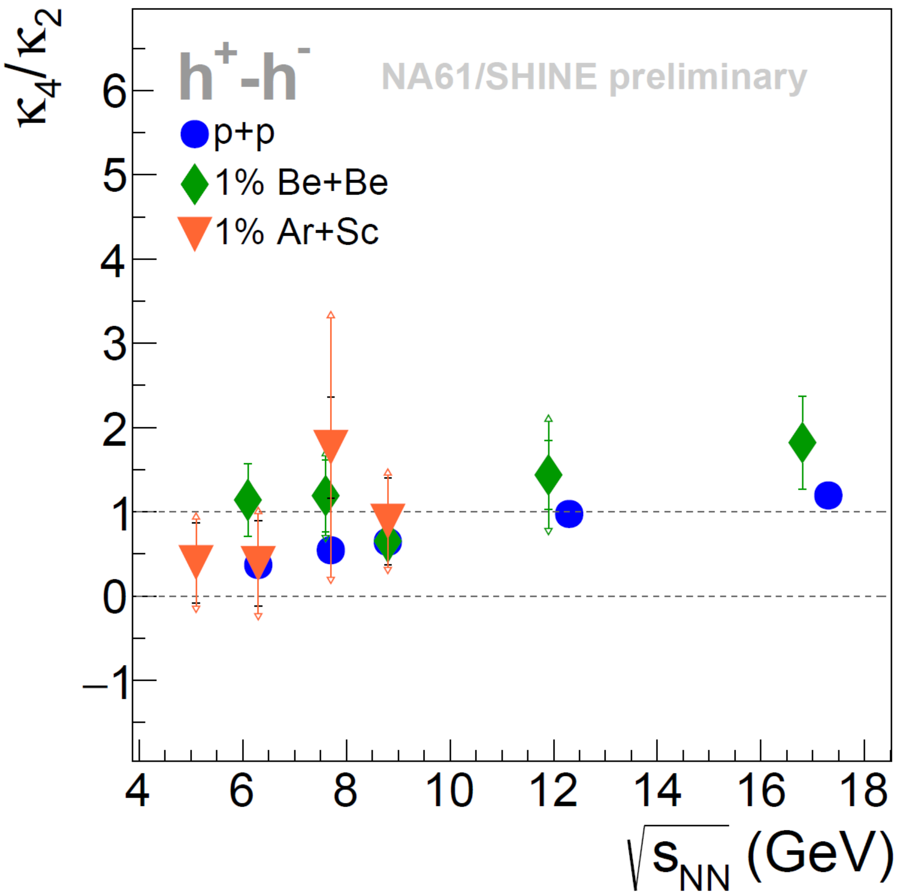


$$F_2(M) = \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i (n_i - 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2}$$

$$F_2(M) \sim (M^2)^{5/6}$$

N.G. Antoniou, F.K. Diakonos, A.S. Kapoyannis, K.S. Kousouris, PRL 97, 032002 (2006)

NA61/SHINE Collaboration



No critical signal. Consistent with p+p.

Conclusions

Interesting and important physics but so far no success

Clear signal of (global?) baryon conservation

Interesting STAR point at 7.7 GeV. Four-proton correlations (physics?)

We definitely need better statistics

Hopefully some progress will come from lattice QCD