

Fluctuations of Conserved Charges (EX)

Toshihiro Nonaka, University of Tsukuba

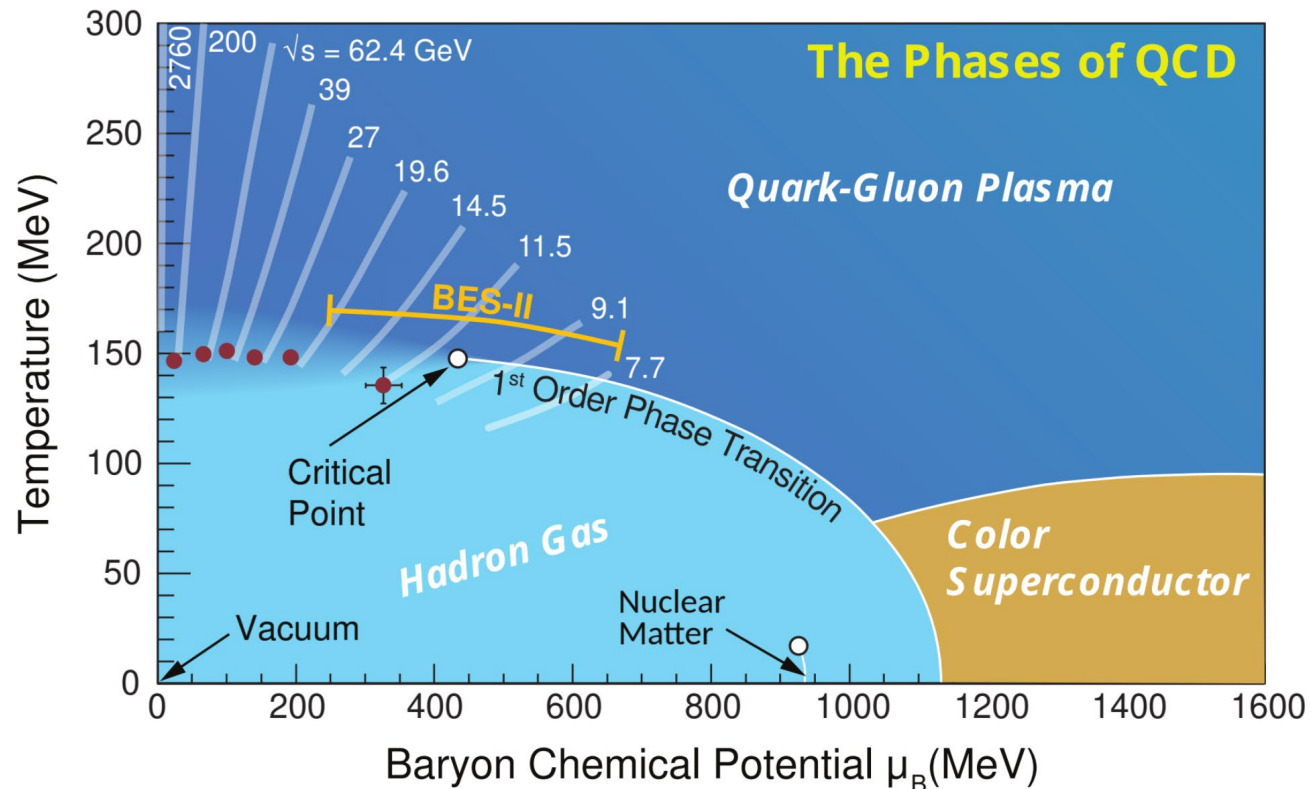


Outline

- Introduction
- Experimental challenges
- New results
- Future prospect
- Summary

Experimental results are mostly taken from published papers since QM2019, or from new results in QM2022

“Conjectured” QCD phase diagram



A. Bzdak et al, Phys.Rep.853 pp1-87 (2020)

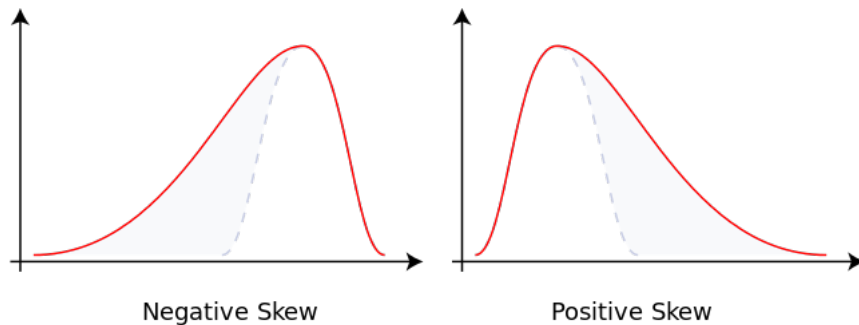
- Crossover at $\mu_B = 0$ MeV
 - Y. Aoki et al, Nature 443,675(2006)
- 1st-order phase transition at large μ_B ?
- Critical point?

Fluctuations of conserved charges are sensitive to the phase structure

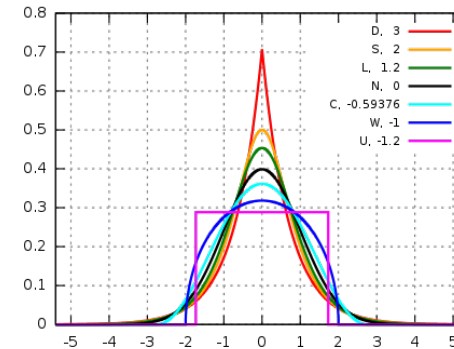
Higher-order fluctuation

- Moments and **cumulants** are mathematical measures of “shape” of a distribution, which probes fluctuations of an observable.

Skewness (S) \rightarrow asymmetry



Kurtosis (κ) \rightarrow sharpness



- Cumulant \leftrightarrow Central moment

$$C_1 = \langle N \rangle, \quad C_2 = \langle (\delta N)^2 \rangle \quad \delta N = N - \langle N \rangle$$

$$C_3 = \langle (\delta N)^3 \rangle \quad C_4 = \langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2$$

$$C_5 = \langle (\delta N)^5 \rangle - 10 \langle (\delta N)^2 \rangle \langle (\delta N)^3 \rangle$$

$$C_6 = \langle (\delta N)^6 \rangle + 30 \langle (\delta N)^2 \rangle^3 - 15 \langle (\delta N)^2 \rangle \langle (\delta N)^4 \rangle$$

- Cumulants have additivity :
proportional to the system volume

$$C_n(X + Y) = C_n(X) + C_n(Y)$$

Cumulants of conserved charges

- Measure event-by-event distributions of **net-baryon, net-charge, and net-strangeness** number

$$\Delta N_q = N_q - N_{\bar{q}}, \quad q = B, Q, S$$

(1) Sensitive to the correlation length

$$C_2 = \langle (\delta N)^2 \rangle_c \approx \xi^2 \quad C_5 = \langle (\delta N)^5 \rangle_c \approx \xi^{9.5}$$

$$C_3 = \langle (\delta N)^3 \rangle_c \approx \xi^{4.5} \quad C_6 = \langle (\delta N)^6 \rangle_c \approx \xi^{12}$$

$$C_4 = \langle (\delta N)^4 \rangle_c \approx \xi^7$$

M. A. Stephanov, PRL102.032301(2009), PRL107.052301(2011)

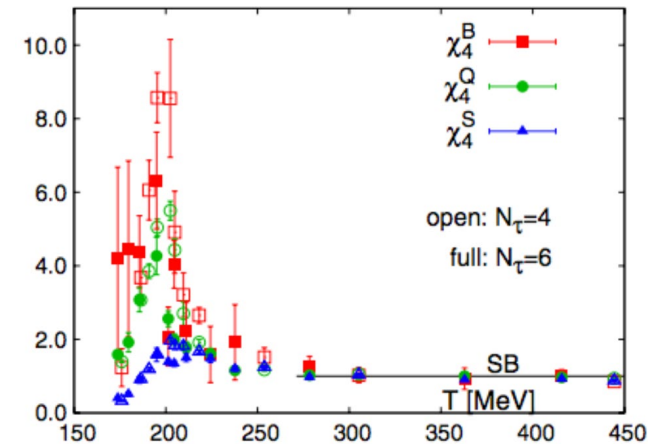
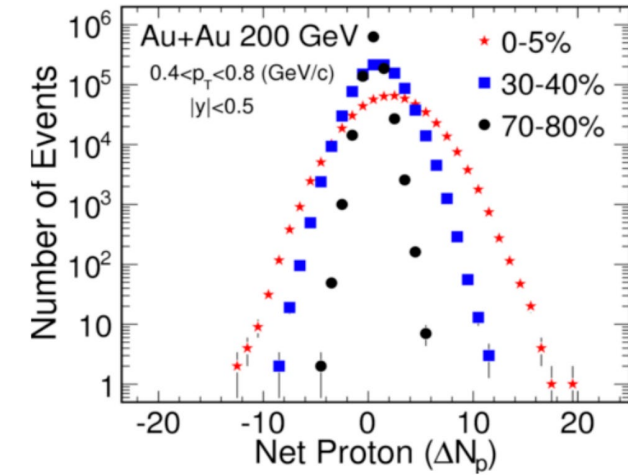
M. Asakawa, S. Ejiri, and M. Kitazawa, PRL103.262301(2009)

(2) Comparison with susceptibilities

$$S\sigma = \frac{C_3}{C_2} = \frac{\chi_3}{\chi_2} \quad \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\chi_4}{\chi_2}$$

$$\chi_n^q = \frac{1}{VT^3} \times C_n^q = \frac{\partial^n p/T^4}{\partial \mu_q^n}, \quad q = B, Q, S$$

STAR Collaboration, PRL105.022302(2010)



*M.Cheng et al,
PRD79.074505(2009)*

Baselines

- Skellam distribution $p(k; \mu_1, \mu_2) = \Pr\{K = k\} = e^{-(\mu_1 + \mu_2)} \left(\frac{\mu_1}{\mu_2}\right)^{k/2} I_k(2\sqrt{\mu_1\mu_2})$
 - “Statistical” baseline:
 - (Poisson) – (Poisson) = (Skellam)
 - $C_1 = C_3 = C_5 = \mu_1 - \mu_2$
 - $C_2 = C_4 = C_6 = \mu_1 + \mu_2$
- $C_3/C_1 = C_4/C_2 = C_6/C_2 = 1$

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- Non-critical baseline
 - Volume fluctuation, baryon number conservation...

P.Braun-Munzinger et al, NPA982.307(2019), NPA1008.122141(2021)

A. Bzdak et al, EPJC77(2017)5.288, A. Bhattacharyya et al, PRC90.034909(2014)

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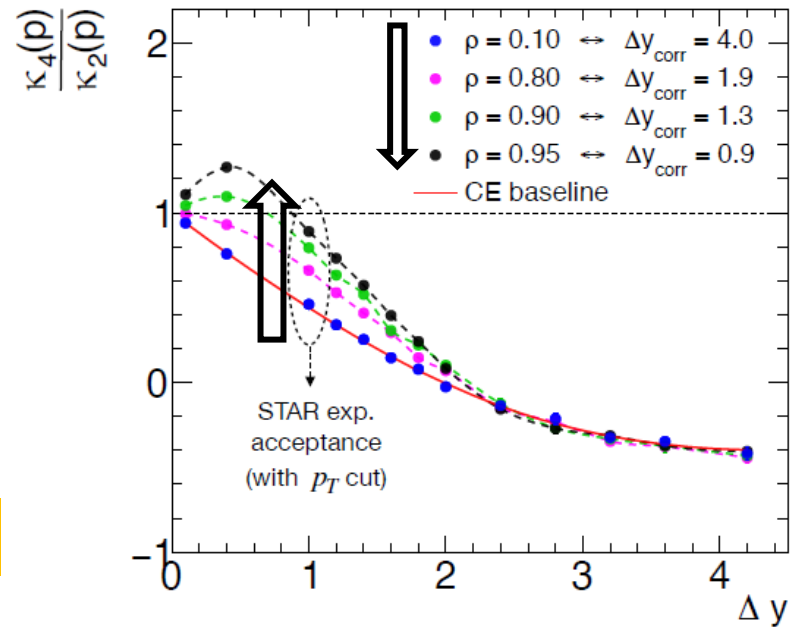
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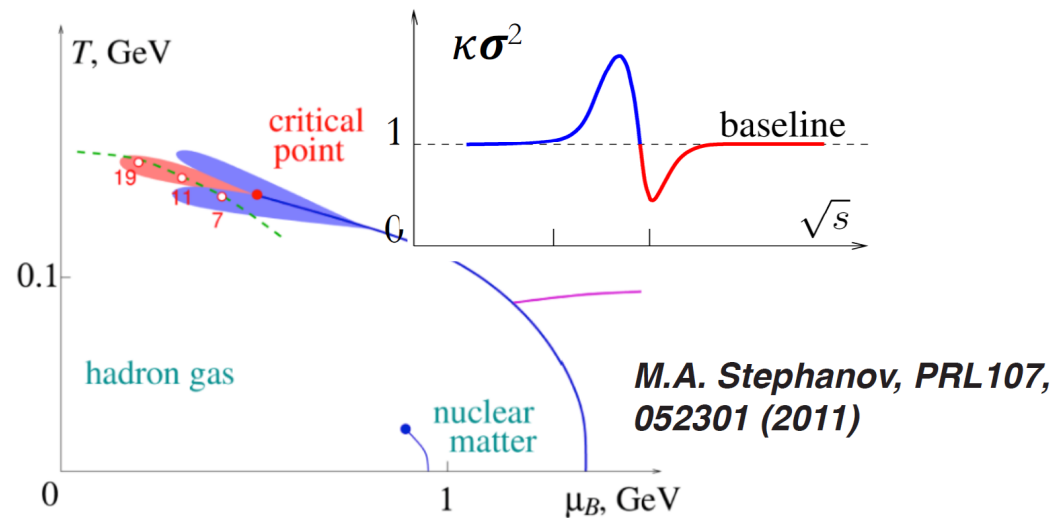
Apr. 6th, 4:40 pm, Anar Rustamov

predictions for $\kappa_4(p)/\kappa_2(p)$
 at $\sqrt{s_{NN}} = 8.8$ GeV

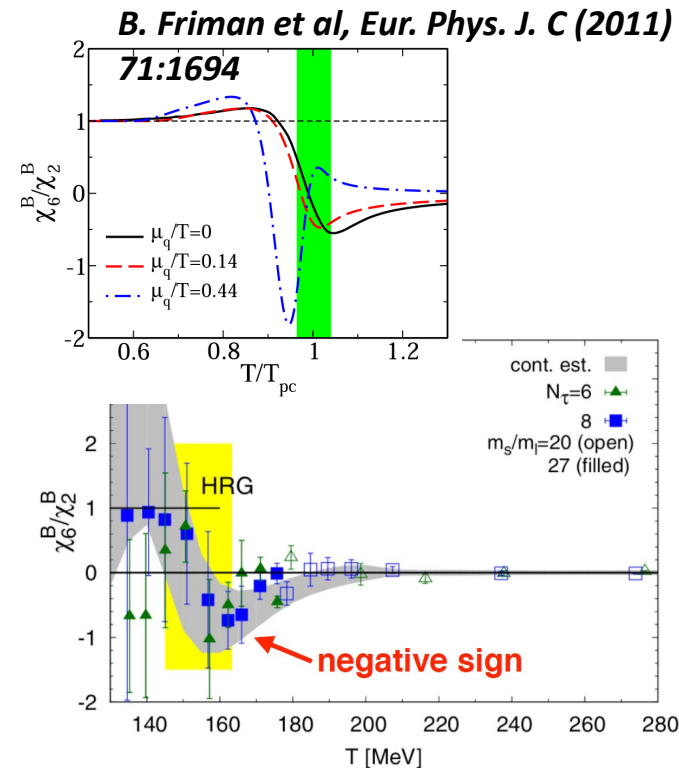


Probe the phase structure

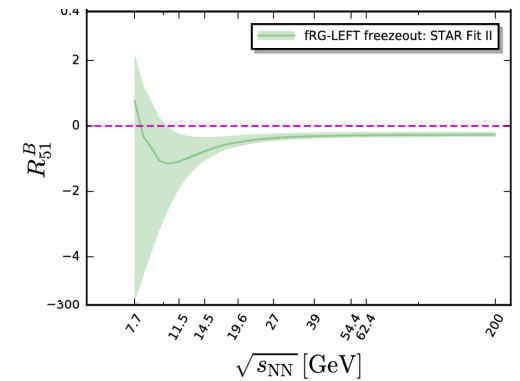
Critical point : nonmonotonic beam energy dependence of net-baryon C_4/C_2 w.r.t the baseline



Crossover : Negative sign of C_5 and C_6



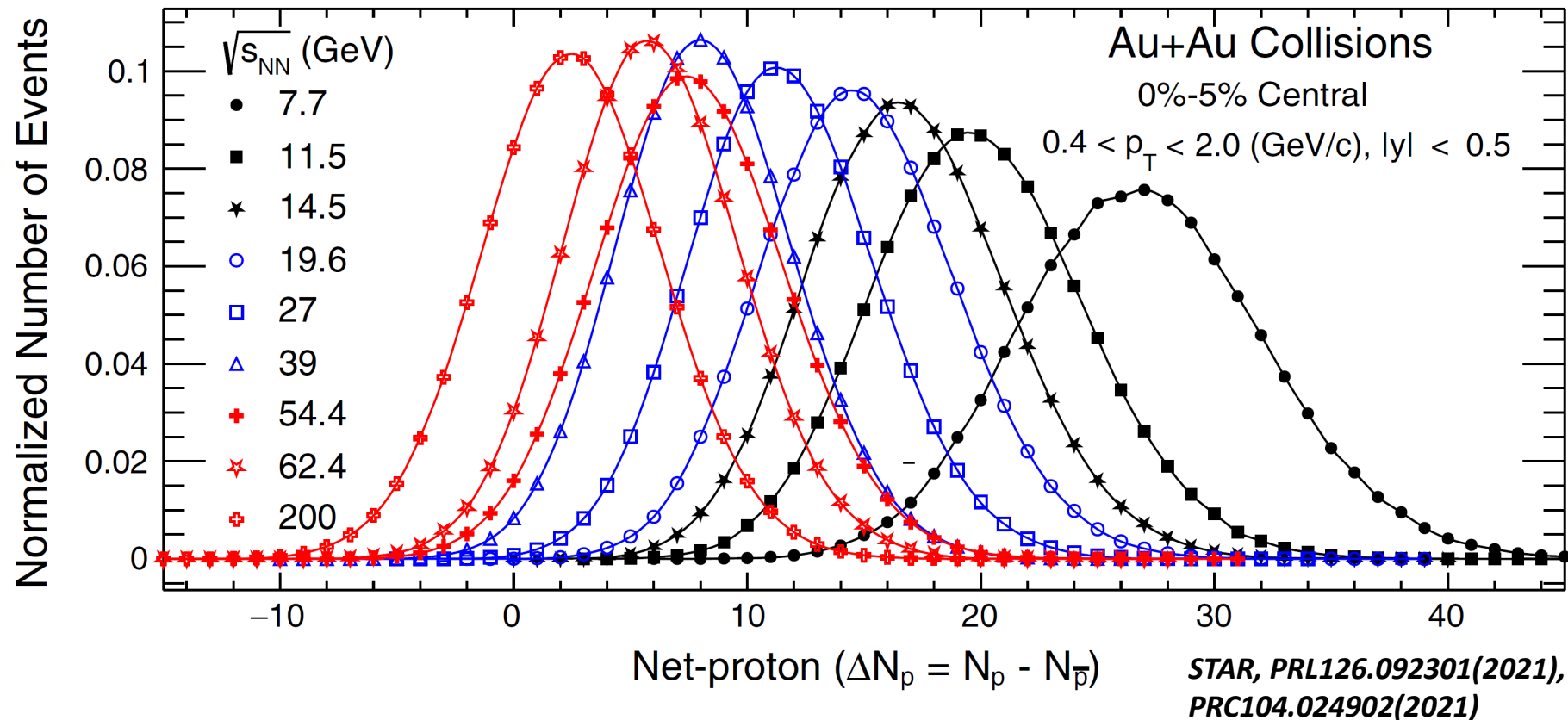
W. Fu et al, PRD104.094047(2021)



A. Bazavov et al, PRD95.054504(2017)

Raw net-proton multiplicity distribution

- Need to consider various experimental effects.



Experimental Challenges

Experimental challenges

- Detector efficiency correction

- **Binomial distribution**

- *M. Kitazawa and M. Asakawa, PRC86.024904(2012), A. Bzdak and V. Koch, PRC86.044904(2012), X. Luo, PRC91.034907(2016),*
 - *T. Nonaka, M. Kitazawa, S. Esumi, PRC95.064912(2017), X. Luo and T. Nonaka, PRC99.044917(2019)*

- **Non-binomial distribution**

- *T. Nonaka, M. Kitazawa, S. Esumi, NIMA906.10-17(2018)*
 - *S. Esumi, K. Nakagawa, T. Nonaka, NIMA987.164802(2021)*

- Initial volume fluctuation

- *M. I. Gorenstein and M. Gaździcki, PRC84.014904 (2011), V. Skokov, B. Friman, and K. Redlich, PRC88.034911 (2013)*
 - *X. Luo, J. Xu, B. Mohanty, N. Xu, J. Phys. G40.105104 (2013), P. Munzinger, A. Rustamov, and J. Stachel, NPA960.114 (2017)*
 - *T. Sugiura, T. Nonaka, and S. Esumi, PRC100.044904 (2019)*

- Pileup events

- *S. Sombun et al, J.Phys.G45.025101(2018), P. Garg and D. Mishra, PRC96.044908(2017)*
 - *T. Nonaka, M. Kitazawa, S. Esumi, NIMA984.164632(2020), Y. Zhang, Y. Huang, T. Nonaka, X. Luo, NIMA1026.166246(2022)*

- Identity method

- *M. Gaździcki, K. Grebieszko, M. Maćkowiak, and S. Mrówczyński, PRC83.054907 (2011)*
 - *A. Rustamov and M. I. Gorenstein, PRC86.044906 (2012), M. I. Gorenstein, PRC84.024902 (2018)*
 - *M. Arslanovic and A. Rustamov, NIMA946.162622 (2019)*

- More to be resolved...

- Net-proton \neq net-baryon, purity correction, acceptance dependence for comparison with theory,

***Not all important studies are listed here**

Detector efficiency correction

- Efficiency correction depends on the efficiency distributions.
The simplest scenario is the **binomial** distribution.

$$B_{p,N}(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

p: efficiency
N: generated particles
n: reconstructed particles

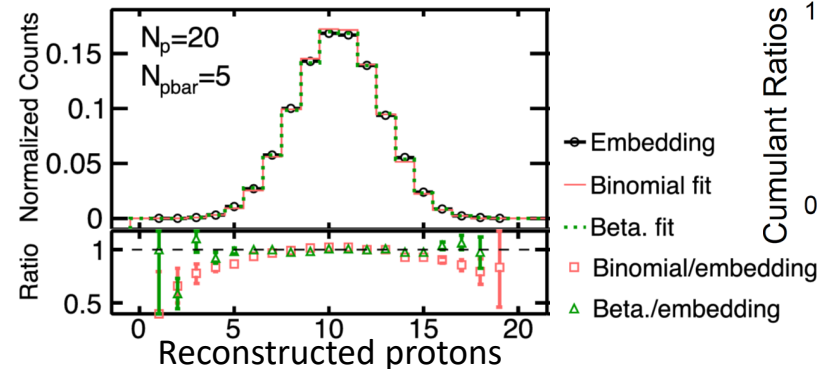
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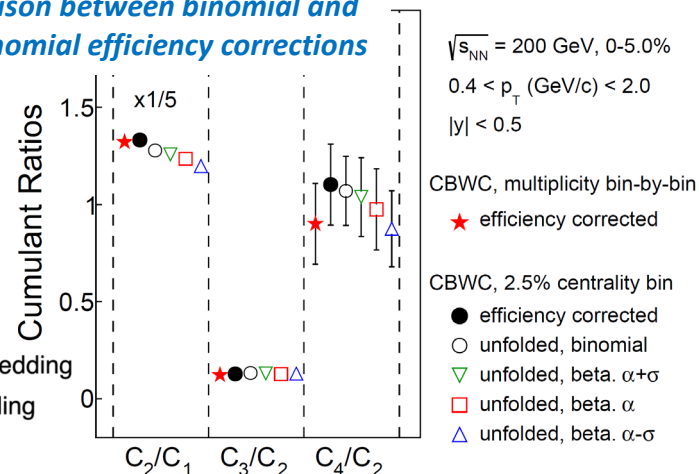
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Efficiency distribution from embedding simulations at STAR



STAR, PRC.104.024902(2021), PRL126.092301(2021)

Comparison between binomial and beta-binomial efficiency corrections



$\sqrt{s_{NN}} = 200 \text{ GeV}, 0-5.0\%$
 $0.4 < p_T \text{ (GeV/c)} < 2.0$
 $|y| < 0.5$

CBWC, multiplicity bin-by-bin
 * efficiency corrected

CBWC, 2.5% centrality bin
 ● efficiency corrected
 ○ unfolded, binomial
 ▽ unfolded, beta. $\alpha+\sigma$
 □ unfolded, beta. α
 △ unfolded, beta. $\alpha-\sigma$

Detector efficiency correction

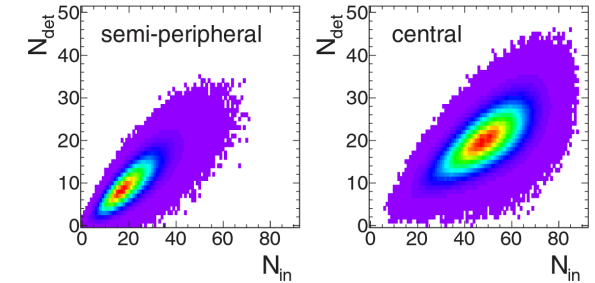
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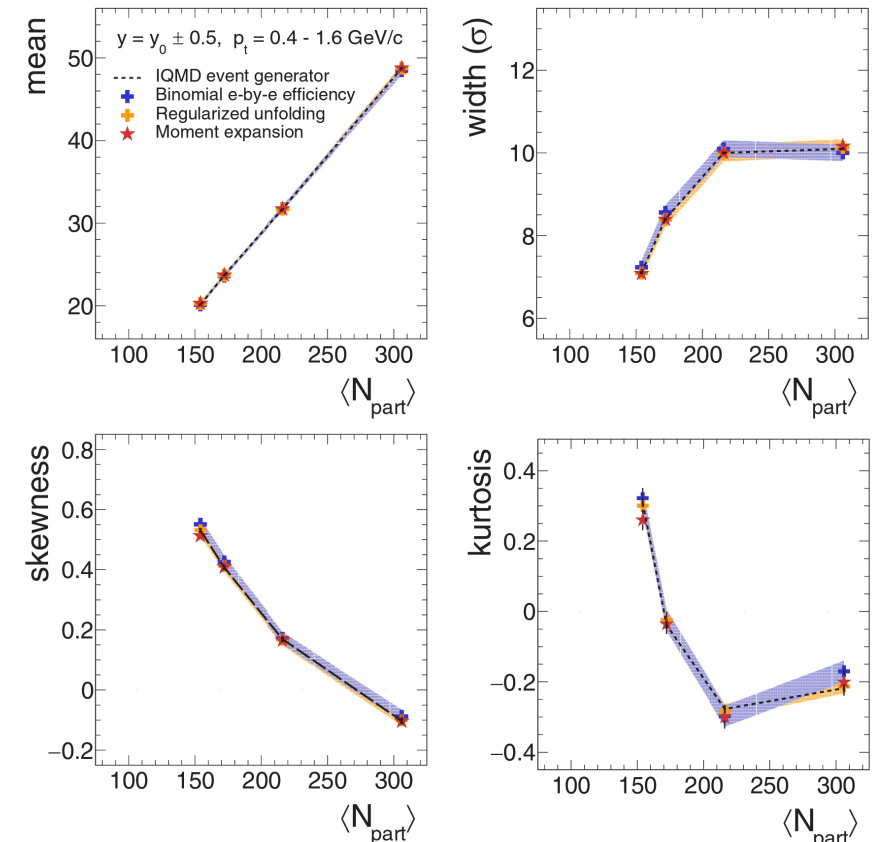
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- Effects of **non-binomial** efficiencies need to be carefully studied for each experimental group.

Response matrices constructed from HADES detector simulations with IQMD

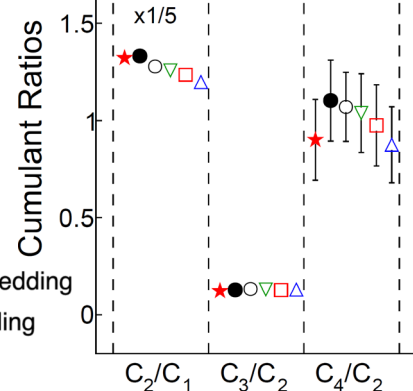
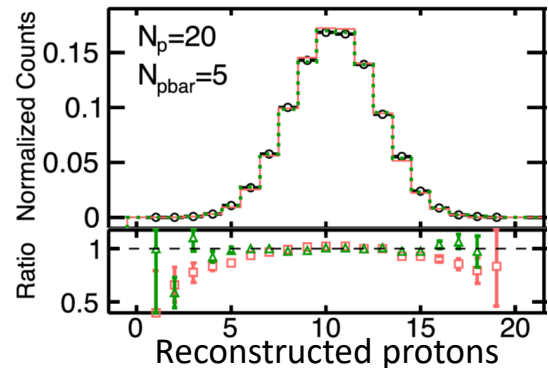


Results from different methods for efficiency corrections



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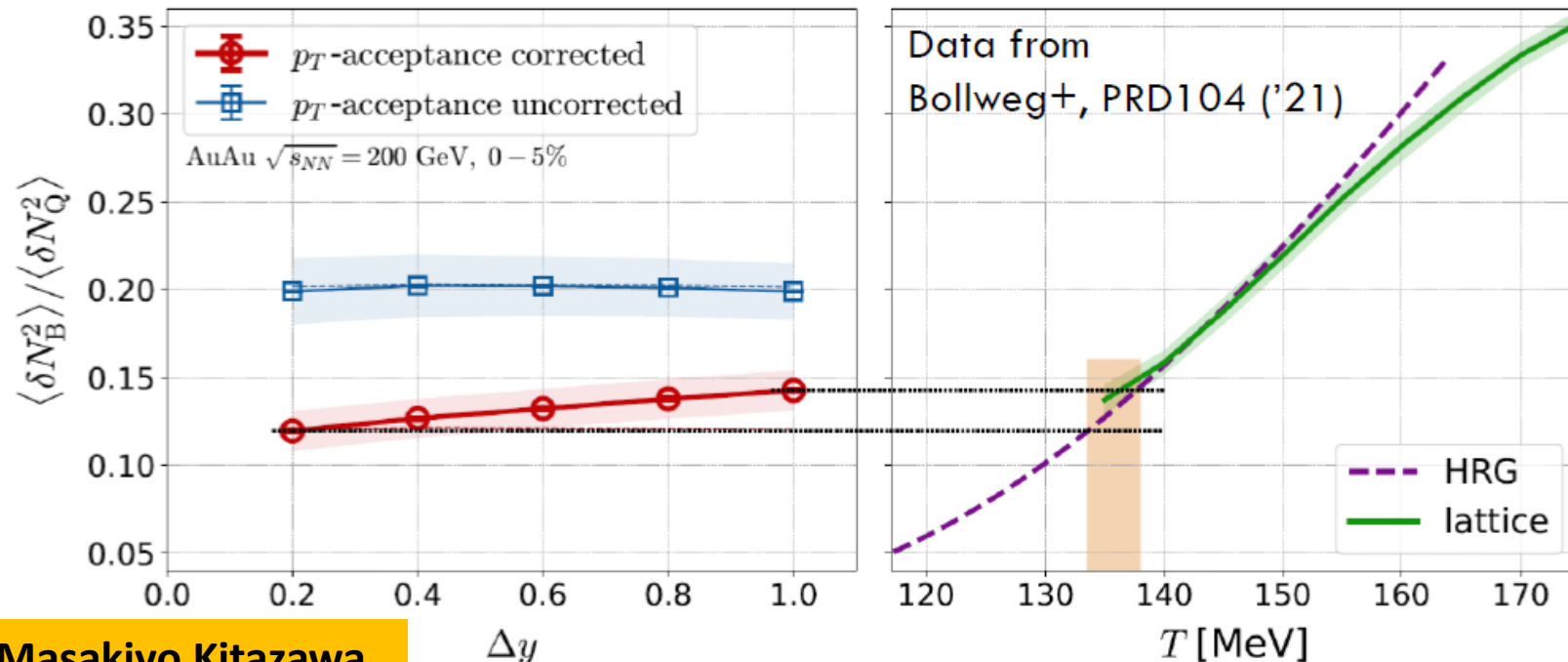
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STAR, PRC.104.024902(2021), PRL126.092301(2021)

HADES, PRC102.024914(2020)

Effect of p_T acceptance cut

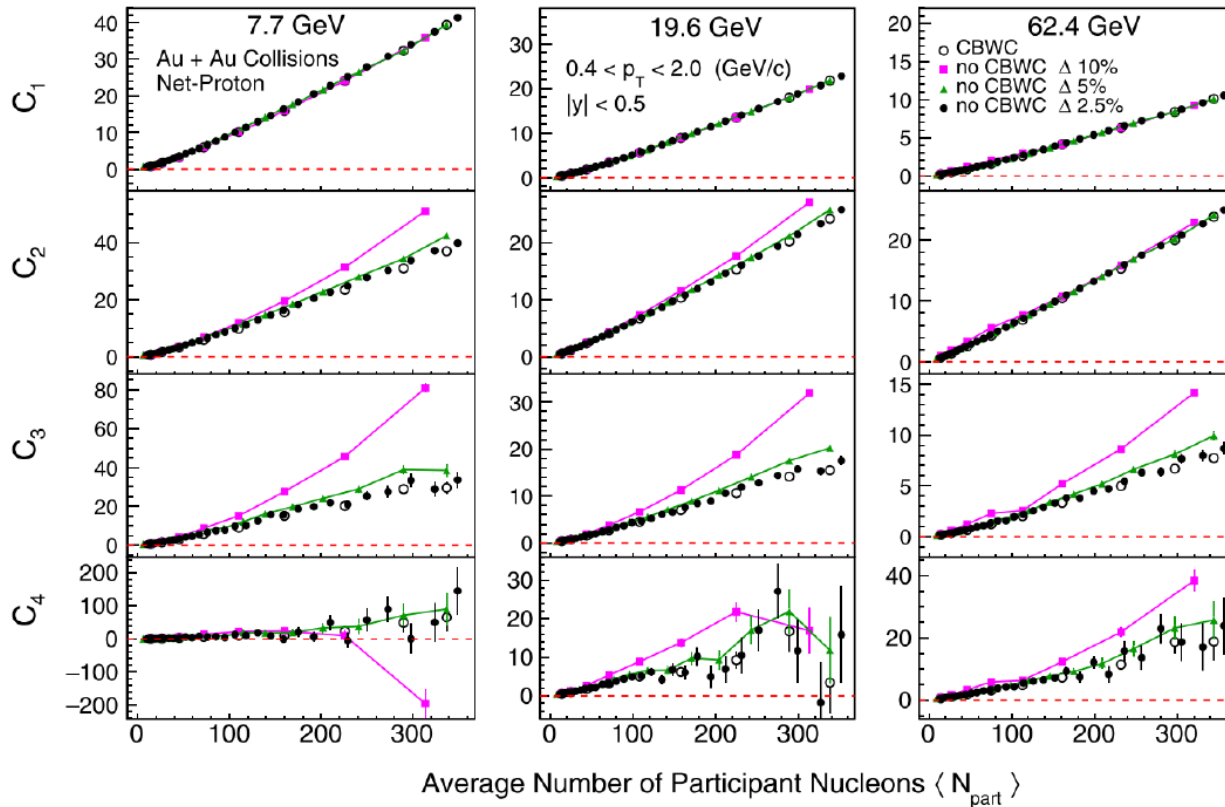
- Baryon-charge ratio at 2nd-order shows significantly lower temperature than chemical freeze-out temperature → effect of resonance decays?
- p_T acceptance correction plays an important role on baryon-charge ratio at 2nd-order.
- Effect on for higher-order fluctuations?



Apr. 6th, 3:40pm, Masakiyo Kitazawa

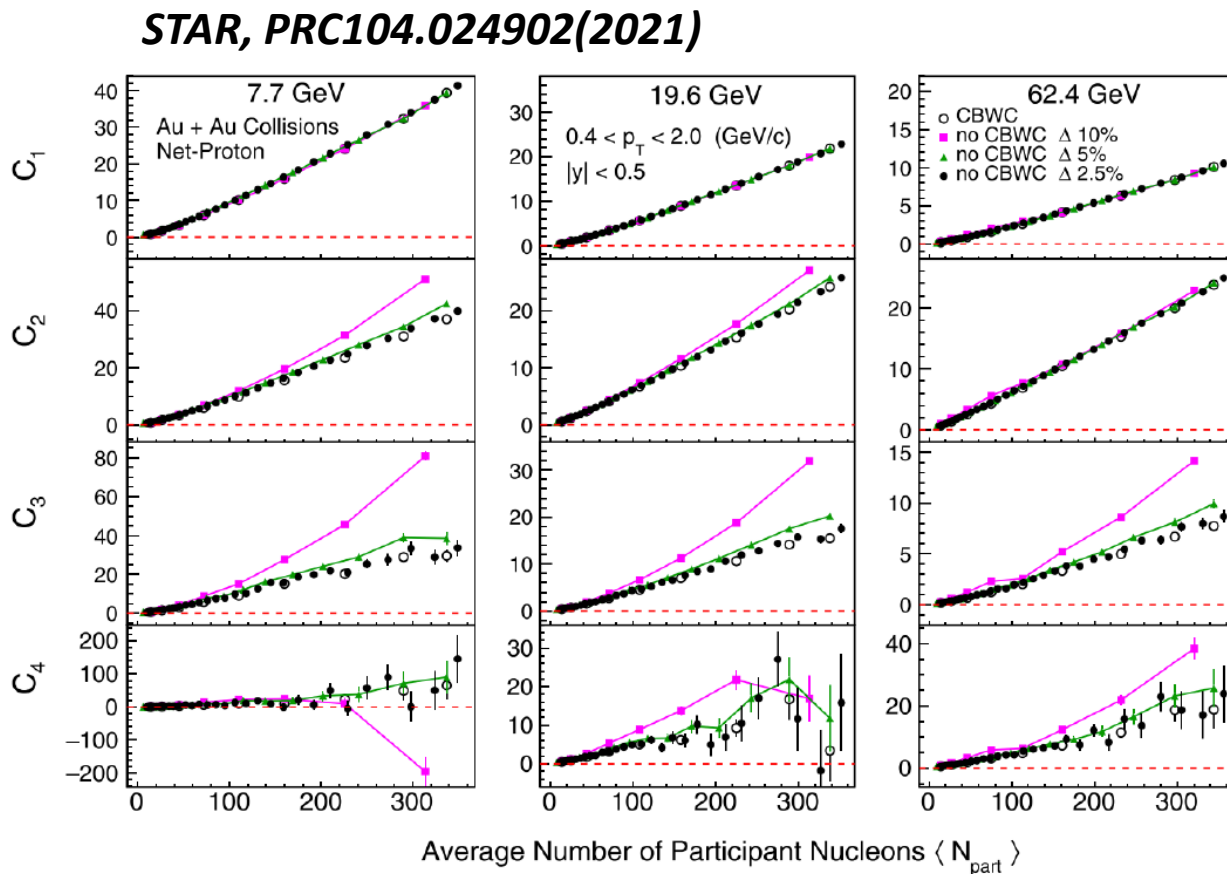
Initial volume fluctuation: Centrality Bin Width Correction (CBWC)

STAR, PRC104.024902(2021)



- Initial geometry and final-state multiplicity are not one-to-one corresponding: initial volume fluctuation.

Initial volume fluctuation: Centrality Bin Width Correction (CBWC)

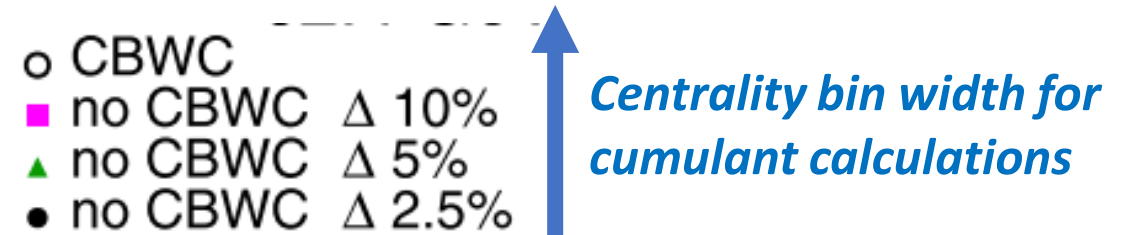
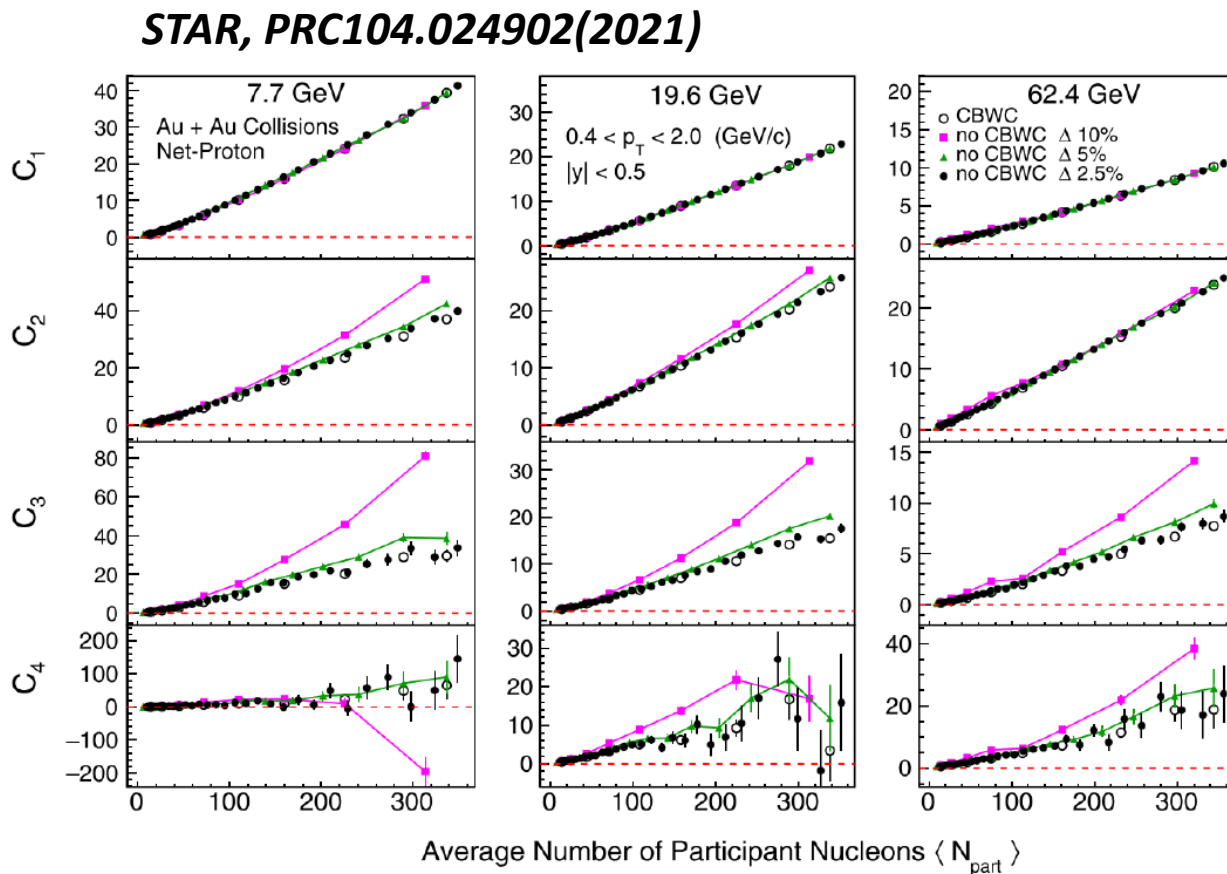


○ CBWC
■ no CBWC Δ 10%
▲ no CBWC Δ 5%
● no CBWC Δ 2.5%

↑ Centrality bin width for cumulant calculations

- Initial geometry and final-state multiplicity are not one-to-one corresponding: initial volume fluctuation

Initial volume fluctuation: Centrality Bin Width Correction (CBWC)



- Initial geometry and final-state multiplicity are not one-to-one corresponding: initial volume fluctuation
- Volume fluctuations can be partly suppressed by CBWC.
- Purely data-driven, but cannot eliminate volume fluctuations.

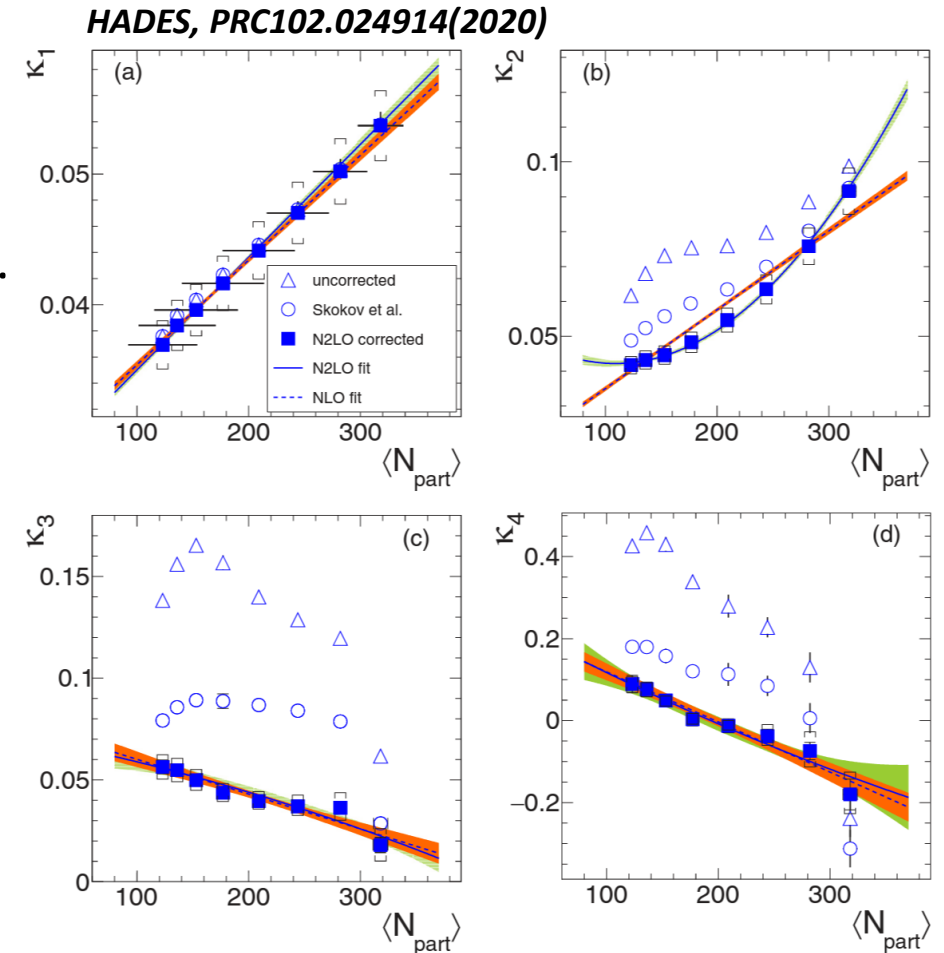
Initial volume fluctuation: Volume Correction

- One can define/determine **initial volume distributions and their cumulants** by using model simulations, which can be eliminated from measurements based on correction formulas.

$\kappa_n = \text{const. w.r.t. } N_{\text{part}}$:
independent particle production

$$\begin{aligned} \kappa_1 &= \tilde{\kappa}_1, \\ \kappa_2 &= \tilde{\kappa}_2 - \kappa_1^2 v_2, \\ \kappa_3 &= \tilde{\kappa}_3 - 3\kappa_1 \kappa_2 v_2 - \kappa_1^3 v_3, \end{aligned}$$

κ_n : cumulant normalized by N_{part}
 $\tilde{\kappa}_n$: volume-affected cumulant

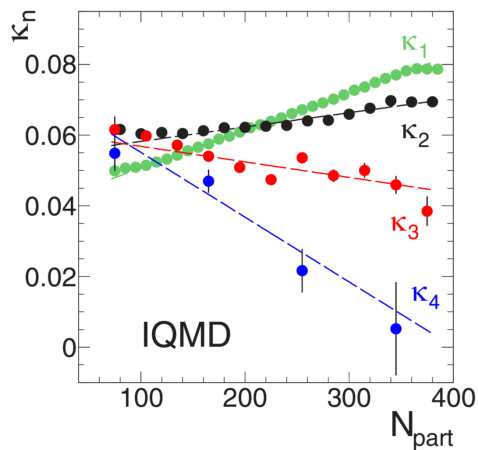


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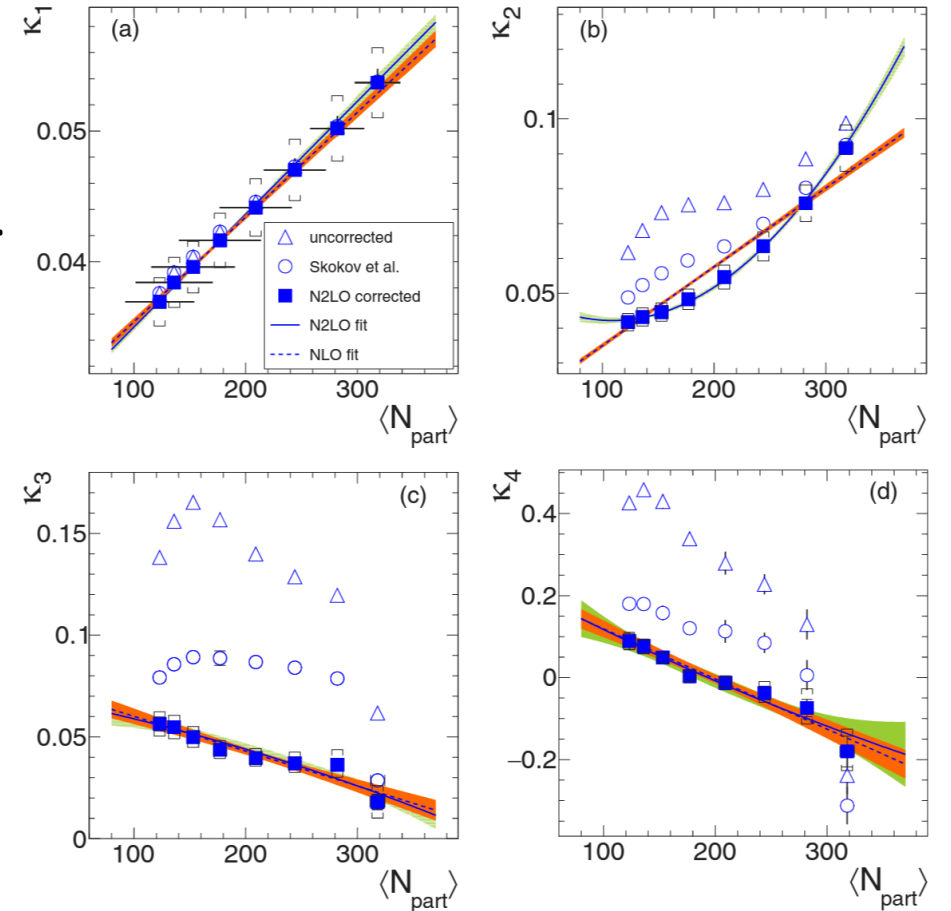


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HADES, PRC102.024914(2020)

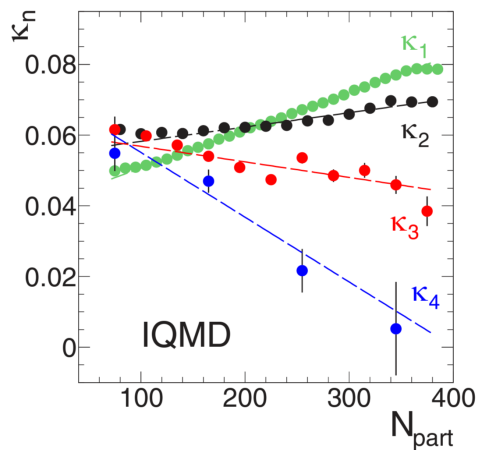


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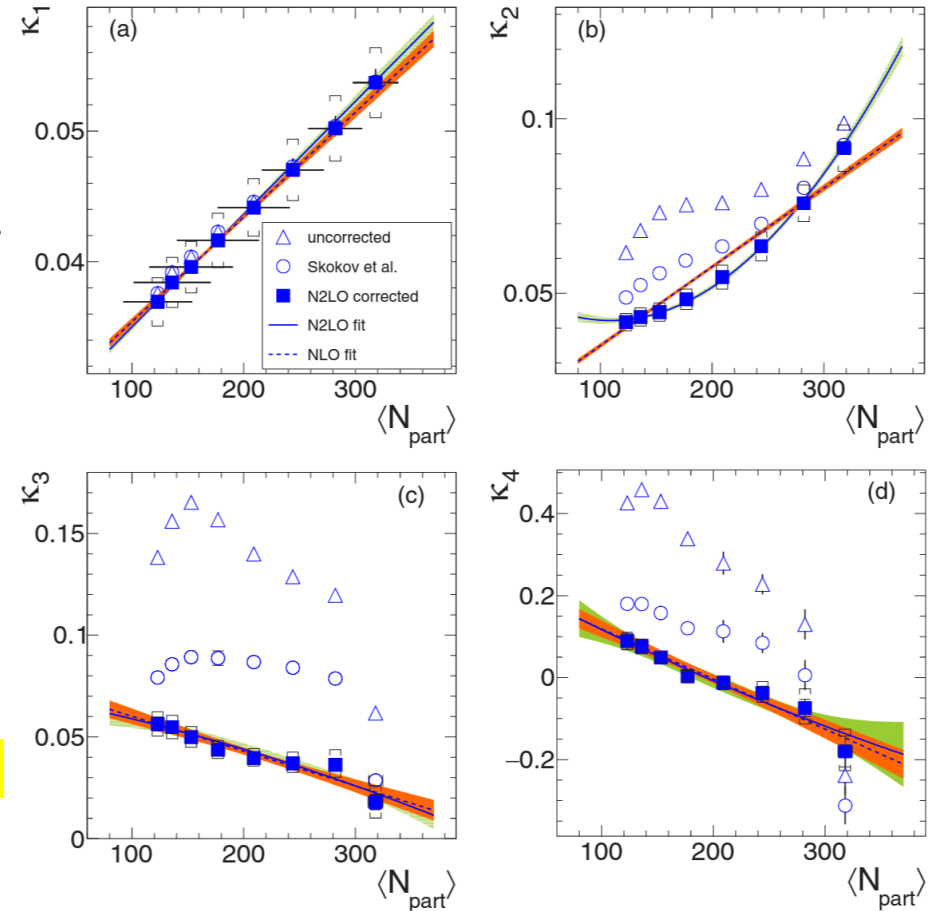
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There is no established way to correct for initial volume fluctuations.

$$\begin{aligned} &+ 3(\kappa_1 \kappa_2' + \kappa_1' \kappa_2) v_2 + 3\kappa_1' v_2 v_2 + 3\kappa_1' v_1 v_2 \\ &+ 10\kappa_1'^3 v_3 v_3 + \kappa_1'^3 v_1^2 v_3 + 24v_2 v_3 \kappa_1'^3 + 3\kappa_1'^3 v_1 v_4 \\ &+ 12\kappa_1'^3 v_2 v_4 + 3\kappa_1'^3 v_5 + \kappa_1'^2 v_6 + 3\kappa_1' \kappa_2' v_1 v_2, \end{aligned}$$

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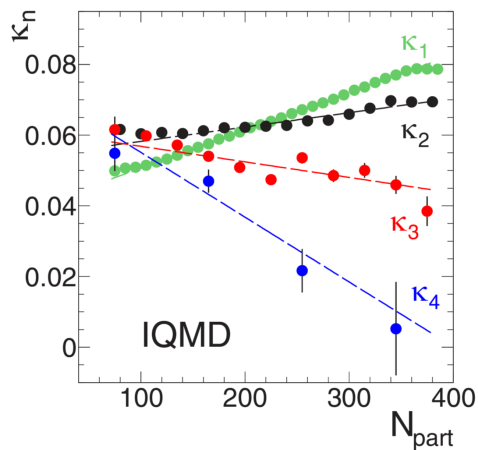


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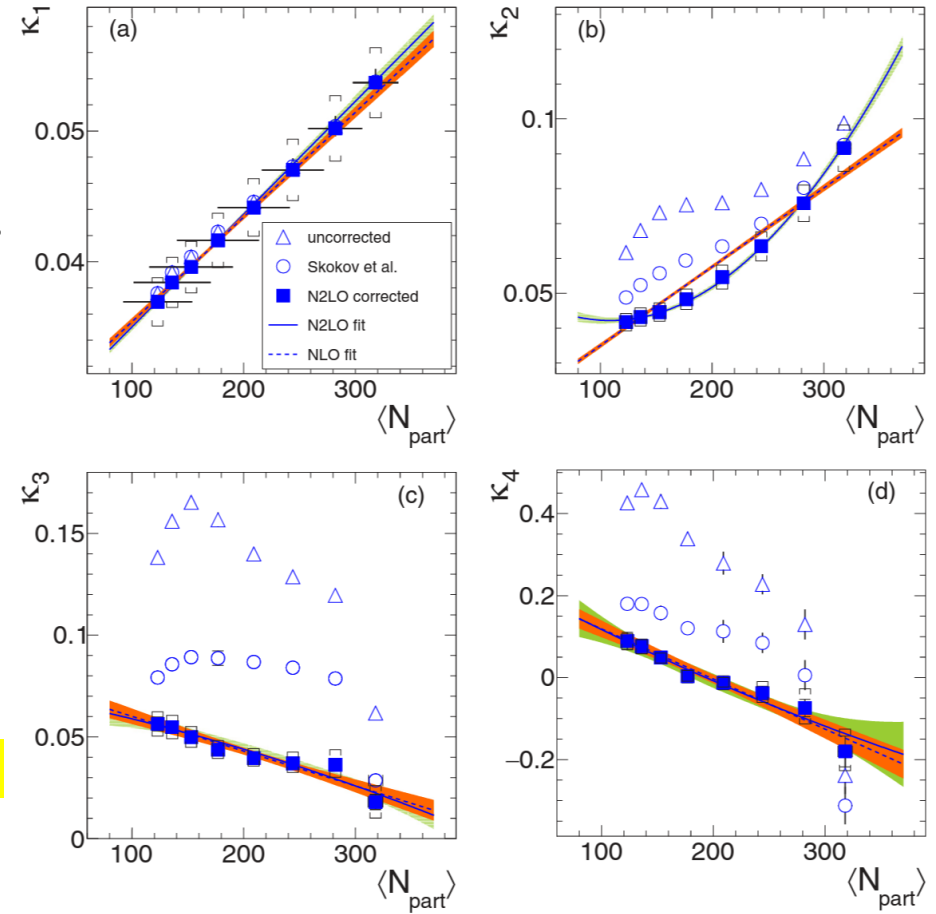
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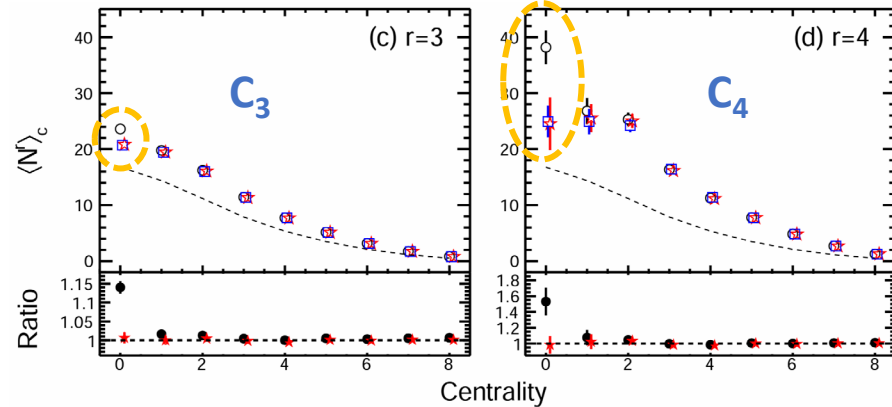
HADES, PRC102.024914(2020)



Cumulants are least affected for most central collisions because of the maximum number of nucleons (394 for Au+Au).

Pileup correction

Toy model : 5% pileup

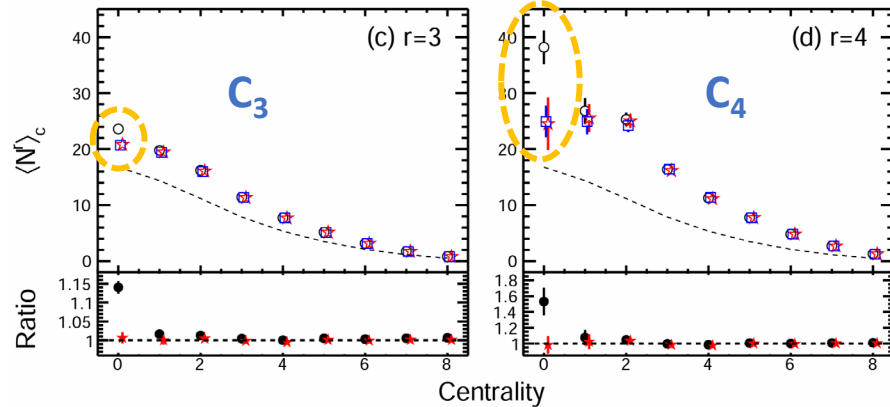


Event pileups: superposition of more than one single-collision events. It is difficult to remove pileup events completely in experiment.

- Pileup events are crucial in fixed-target experiments, which could make an **artificial enhancement of higher-order cumulants in central collisions.**

Pileup correction

Toy model : 5% pileup



$$\langle N^r \rangle_m^t = \frac{\text{Measured moment } \langle N^r \rangle_m - \alpha_m C_m^{(r)}}{1 - \alpha_m + 2\alpha_m w_{m,0}}$$

with

$$C_m^{(r)} = \mu_m^{(r)} + \sum_{i,j>0} \delta_{m,i+j} w_{i,j} \langle N^r \rangle_{i,j}^{\text{sub}},$$

m : reference multiplicity

and

✓ Solvable recursively starting from m=0 and r=1

$$\mu_m^{(r)} = \begin{cases} 2w_{m,0} \sum_{k=0}^{r-1} \binom{r}{k} \langle N^{r-k} \rangle_0^t \langle N^k \rangle_m^t & (m > 0), \\ \sum_{k=1}^{r-1} \binom{r}{k} \langle N^{r-k} \rangle_0^t \langle N^k \rangle_0^t & (m = 0). \quad \text{Initial condition} \end{cases}$$

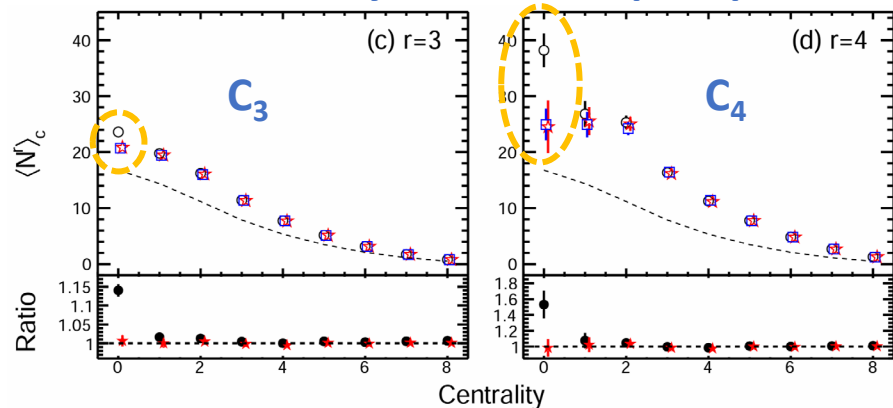
T. Nonaka, M. Kitazawa, S. Esumi, NIMA984.164632(2020)

Event pileups: superposition of more than one single-collision events. It is difficult to remove pileup events completely in experiment.

- Pileup events are crucial in fixed-target experiments, which could make an **artificial enhancement of higher-order cumulants in central collisions.**
- Pileup correction can be applied for residual pileups.

Pileup correction

Toy model : 5% pileup



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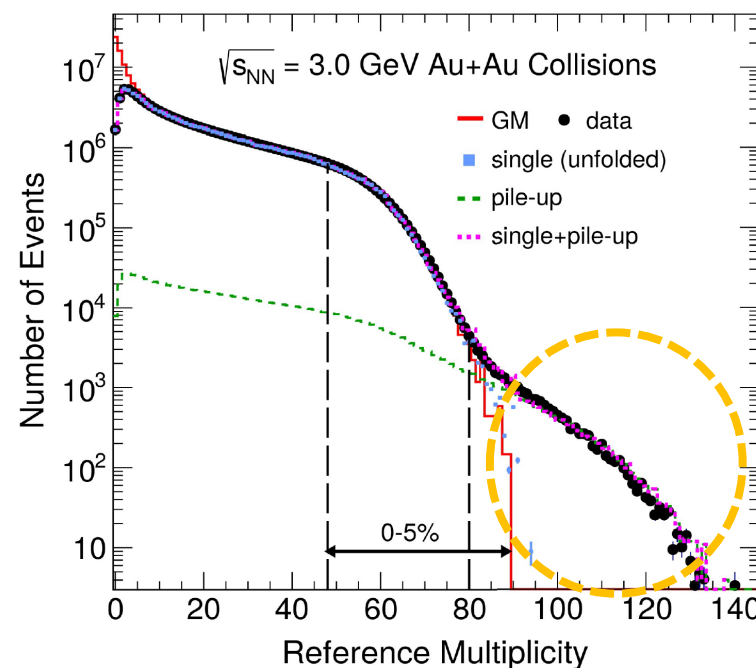
Initial condition

T. Nonaka, M. Kitazawa, S. Esumi, NIMA984.164632(2020)

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- Pileup correction can be applied for residual pileups.

STAR data : 0.46% pileup

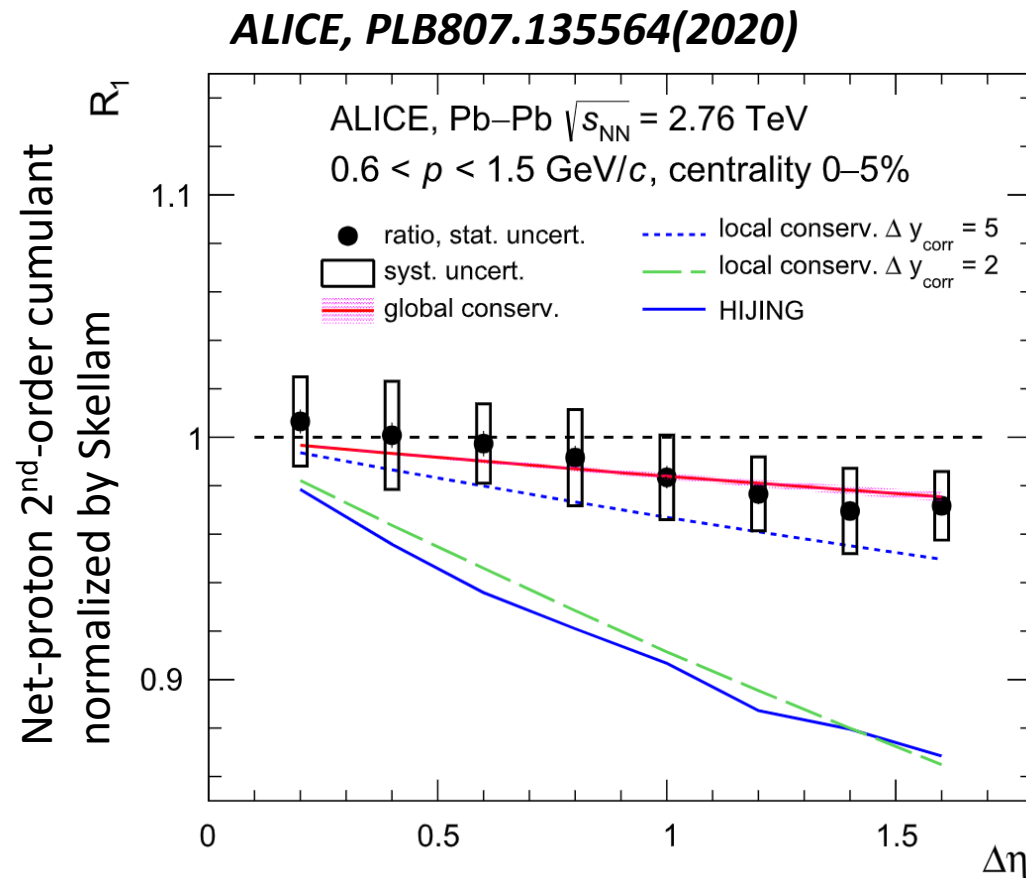


STAR Collaboration,
arXiv:2112.00240

Experimental Results

Experimental results are mostly taken from published papers since QM2019, or from new results in QM2022

Net-proton fluctuations from ALICE

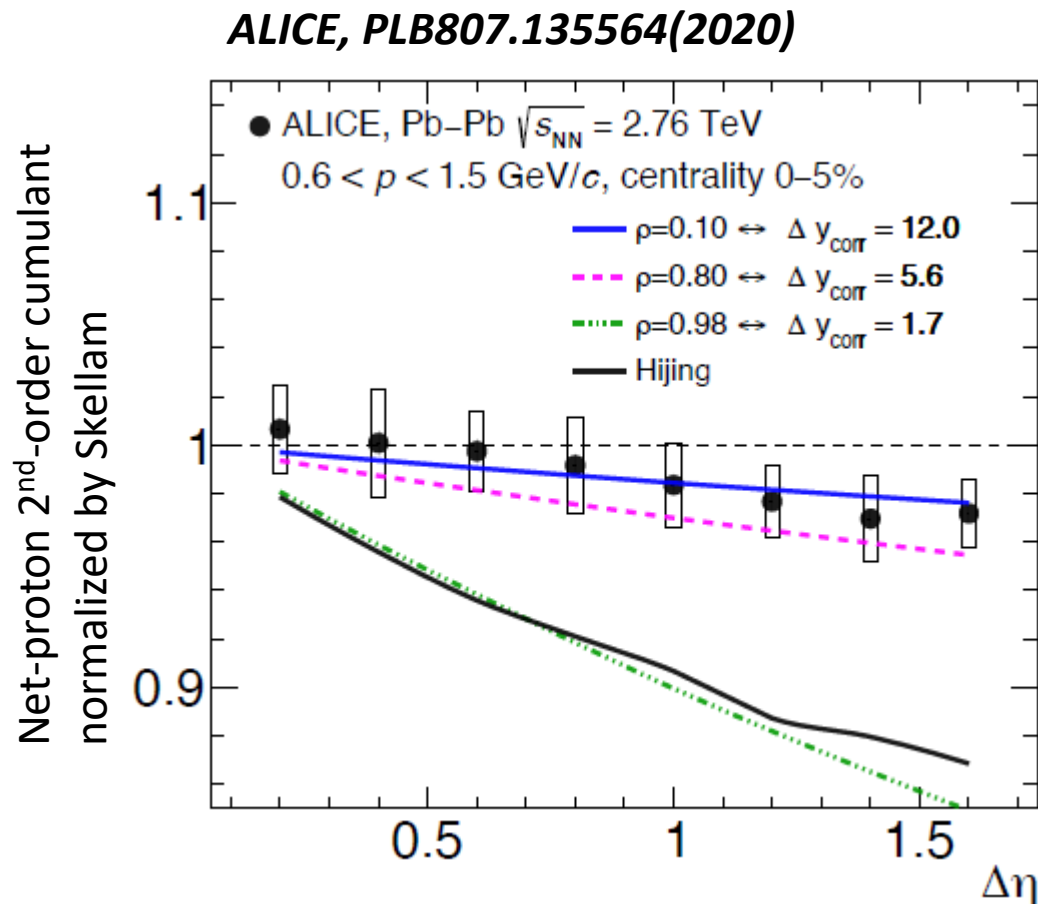


- Net-proton 2nd-order (normalized) cumulant decreases with widening the rapidity acceptance.
- Results are explained by global baryon number conservation, which implies **long range correlation in data**.

$$R_1 = 1 - \alpha, \quad \alpha = \langle n_p \rangle / \langle N_B^{4\pi} \rangle$$

P.B.Munzinger, A. Rustamov, J. Stachel, NPA960.114130(2017)
A. Bzdak, V. Koch, V. Skokov, PRC87.014901(2013)

Canonical ensemble + rapidity correlations



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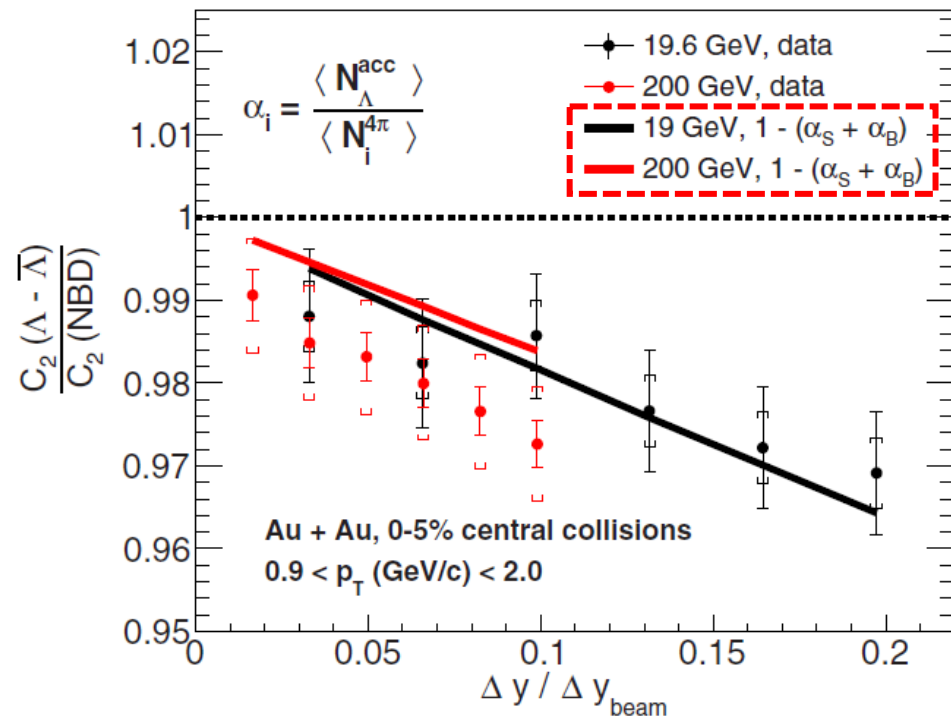
P.B.Munzinger, A. Rustamov, J. Stachel, NPA960.114130(2017)
A. Bzdak, V. Koch, V. Skokov, PRC87.014901(2013)

- The data are best described with new implementation of rapidity correlation between baryons and antibaryons with $\Delta y_{\text{corr}}=12$.

Apr. 6th, 4:40 pm, Anar Rustamov

Net-lambda fluctuation

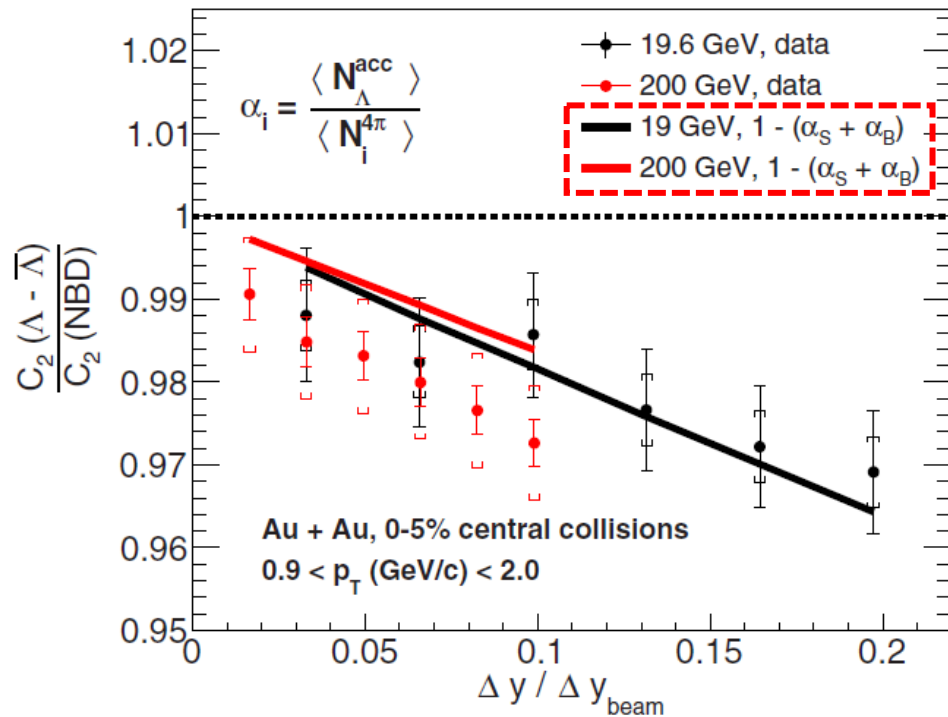
- Both baryon and strangeness conservations play an important role for net-lambda fluctuations.



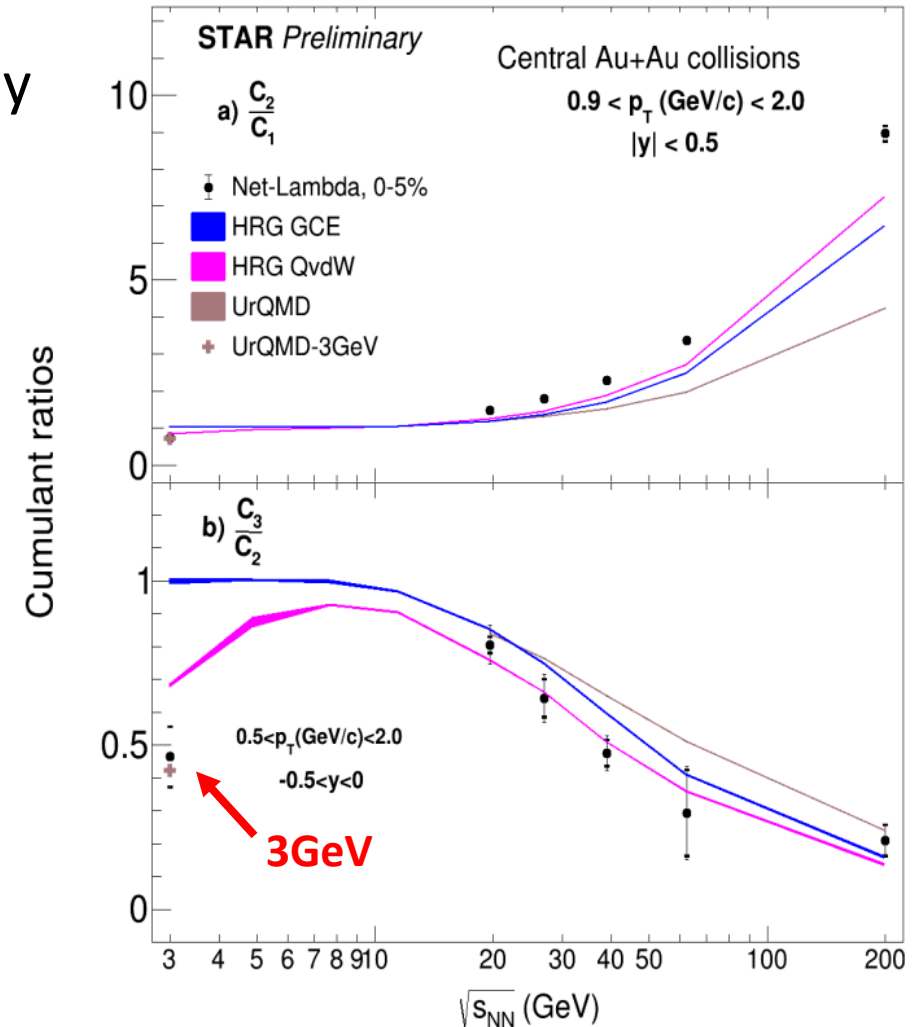
STAR, PRC102.024903(2020)

Net-lambda fluctuation

- Both baryon and strangeness conservations play an important role for net-lambda fluctuations.



STAR, PRC102.024903(2020)



Deuteron number fluctuation

Apr. 6th, 3pm, Debasish Mallick

Coalescence Toy Model

Z. Fecková, J. Steinheimer, B. Tomášik and M. Bleicher: *Phys. Rev. C* 93, 054906 (2016)

Probability of deuteron formation, $\lambda_d = B_2 n_p n_n$

Assume, proton (n_p) and neutron (n_n) follow Poisson distributions,

- At low $\sqrt{s_{NN}}$, B_2 increases. *STAR: Phys. Rev. C* 99, 064905 (2019)
- Larger value of n_p and n_n at low $\sqrt{s_{NN}}$.

- Results in rise of scaled moments of deuteron number.

Scaled Moments: $\sigma^2/M = C_2/C_1$, $S\sigma = C_3/C_2$, $\kappa\sigma^2 = C_4/C_2$

Two assumptions in the model:

Model A: Correlated p and n ($n_p=n_n$). Model B: Independent p and n.

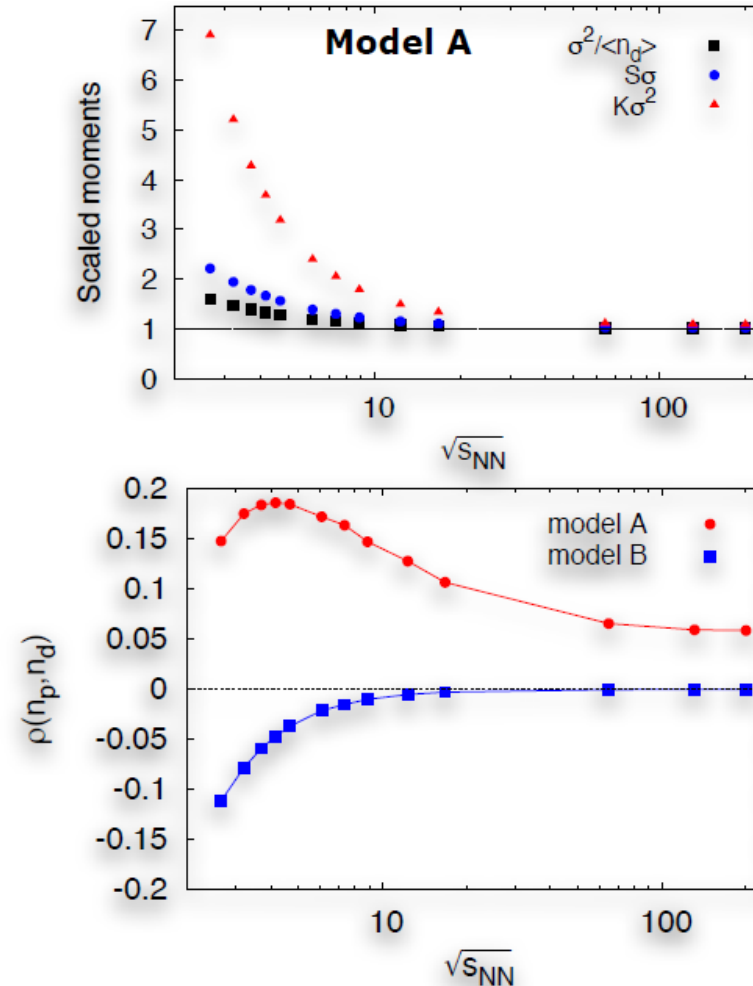
$$\lambda_d = B_2 n_p^2$$

$$\lambda_d = B_2 n_p n_n$$

$$\rho(n_p, n_d) = \frac{\langle (n_p - \langle n_p \rangle)(n_d - \langle n_d \rangle) \rangle}{\sigma_p \sigma_d}$$

- Model A: $\rho > 0$

Model B: $\rho < 0$



Deuteron number fluctuation

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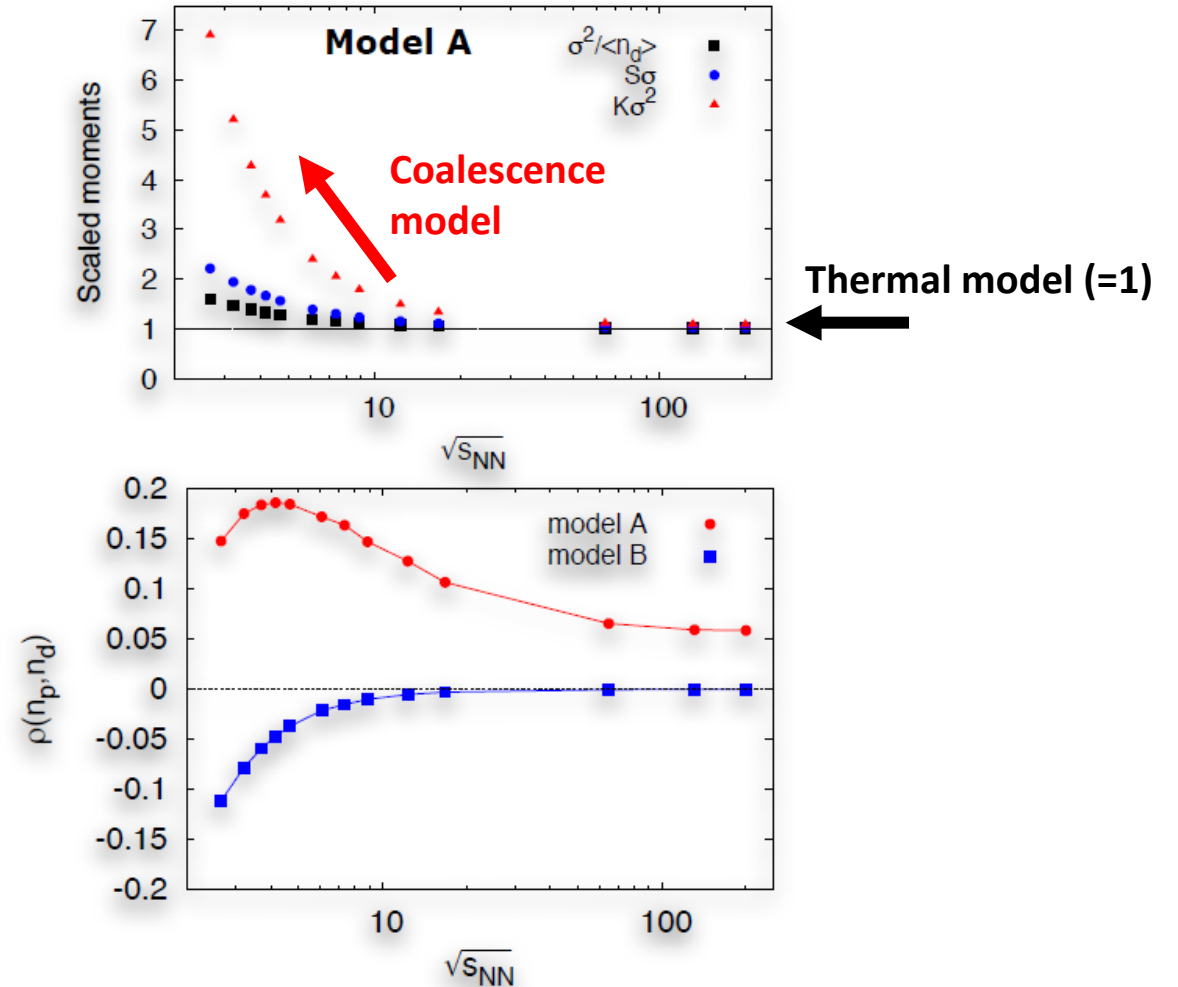
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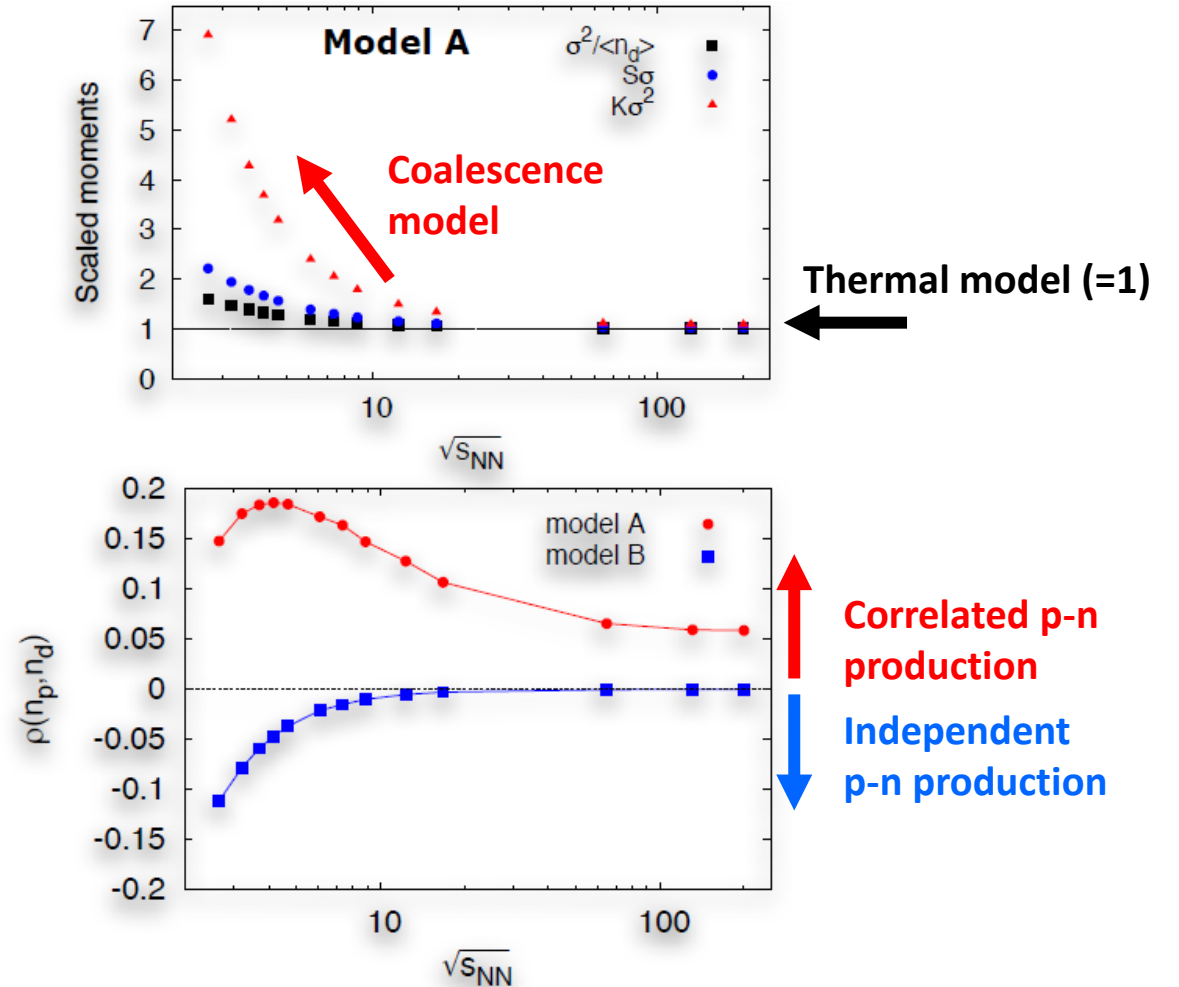
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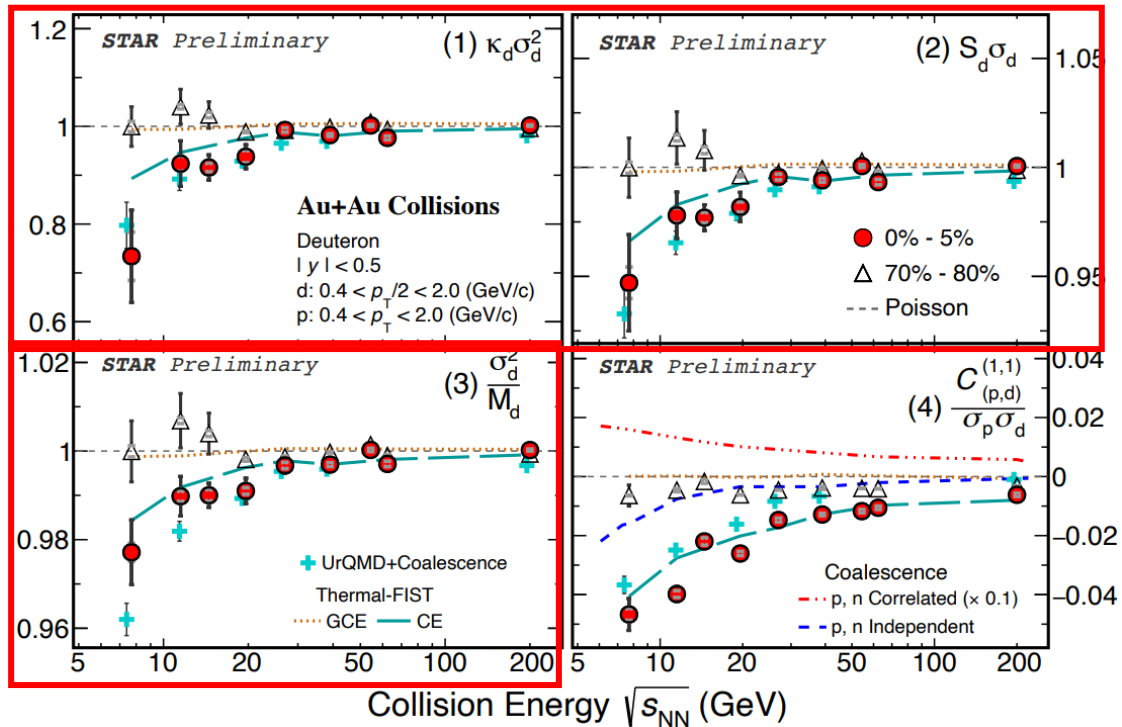
Model B: $\rho < 0$



(Anti)Deuteron number fluctuation

- Collision energy dependence is qualitatively reproduced by UrQMD+coalescence and CE thermal models.

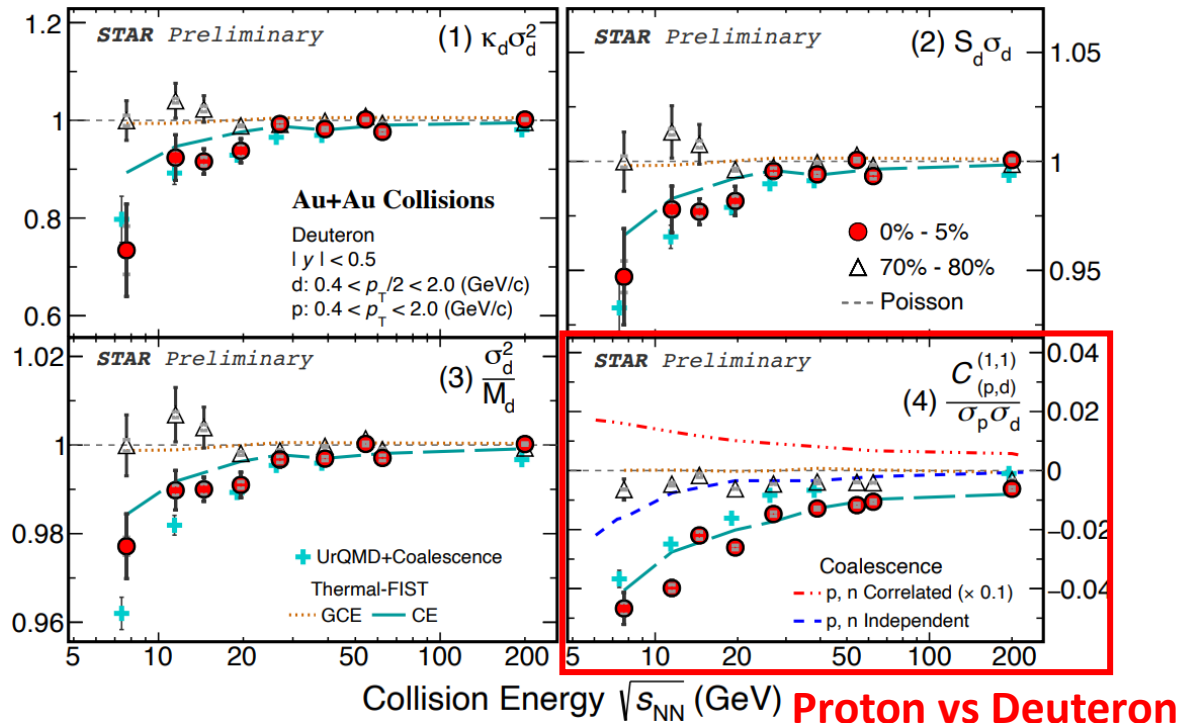
Deuteron



Apr. 6th, 3pm, Debasish Mallick

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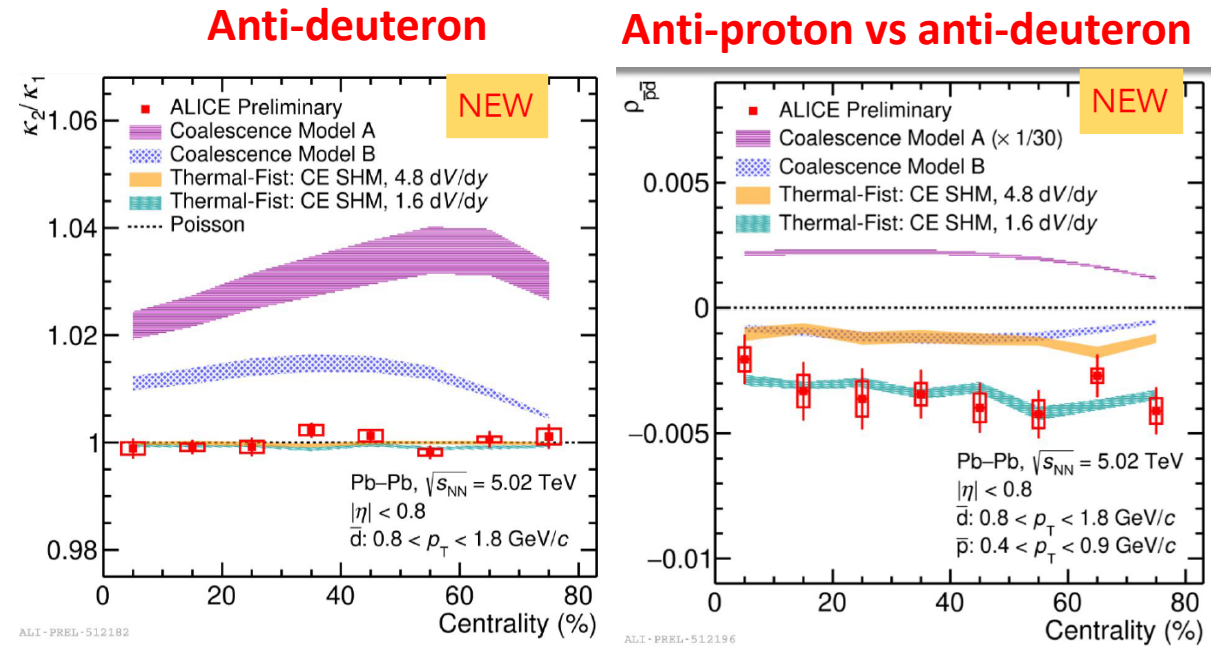
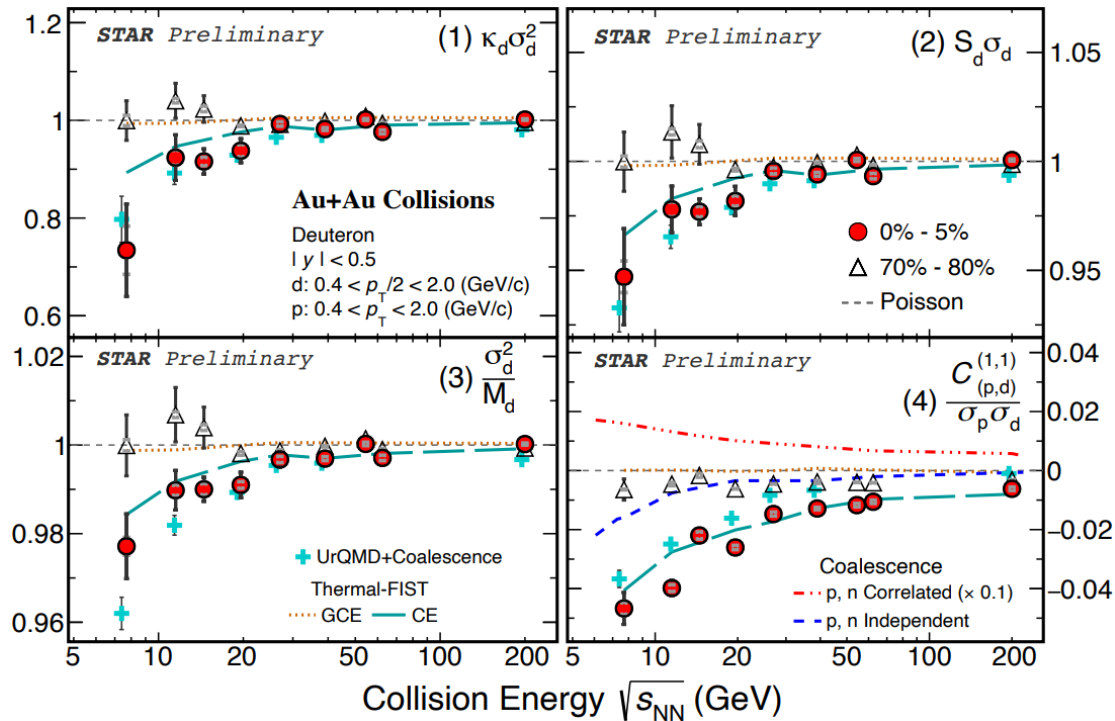


Apr. 6th, 3pm, Debasish Mallick

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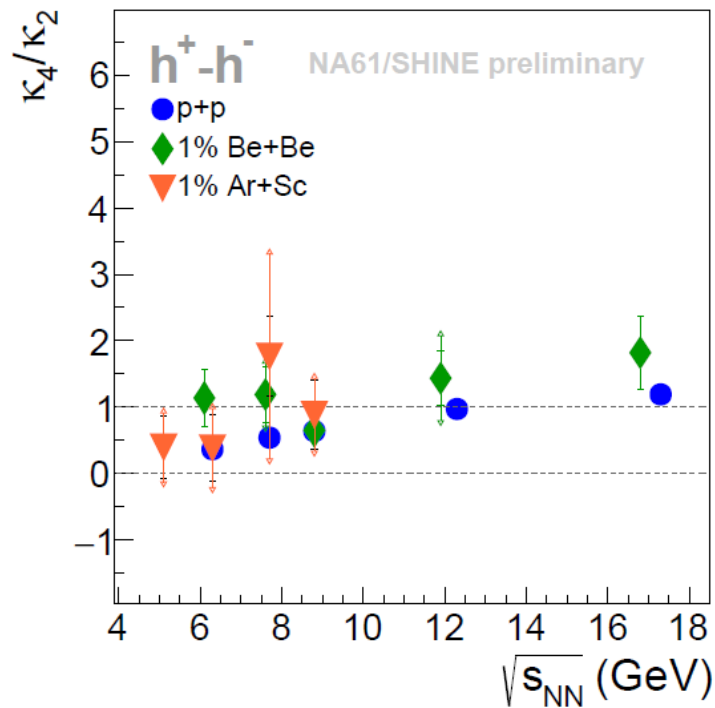
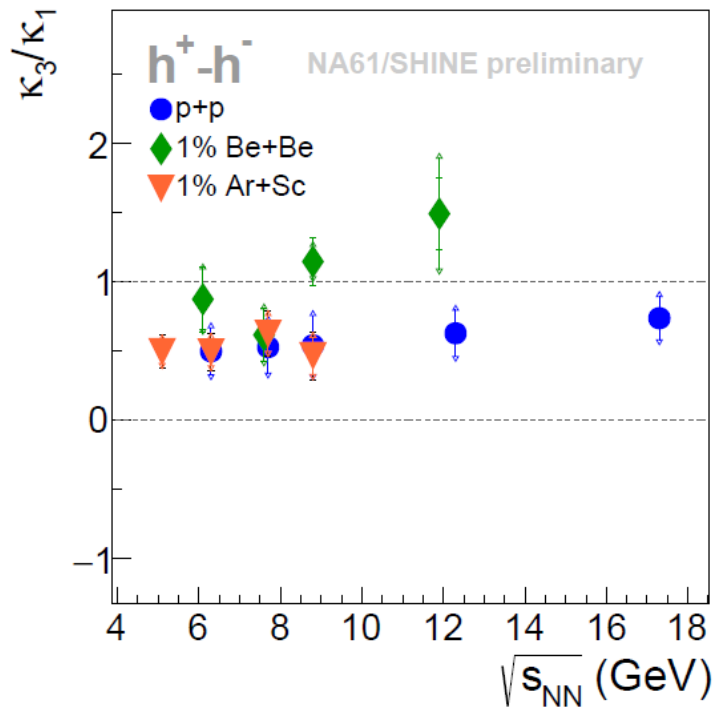
- Cumulant ratio is consistent with Poisson baseline and SHM.
- Negative correlation rules out a coalescence model with correlated production of (anti)protons and (anti)neutrons.**



Apr. 6th, 3pm, Debasish Mallick

Apr. 7th, 6:50pm, Sourav Kundu

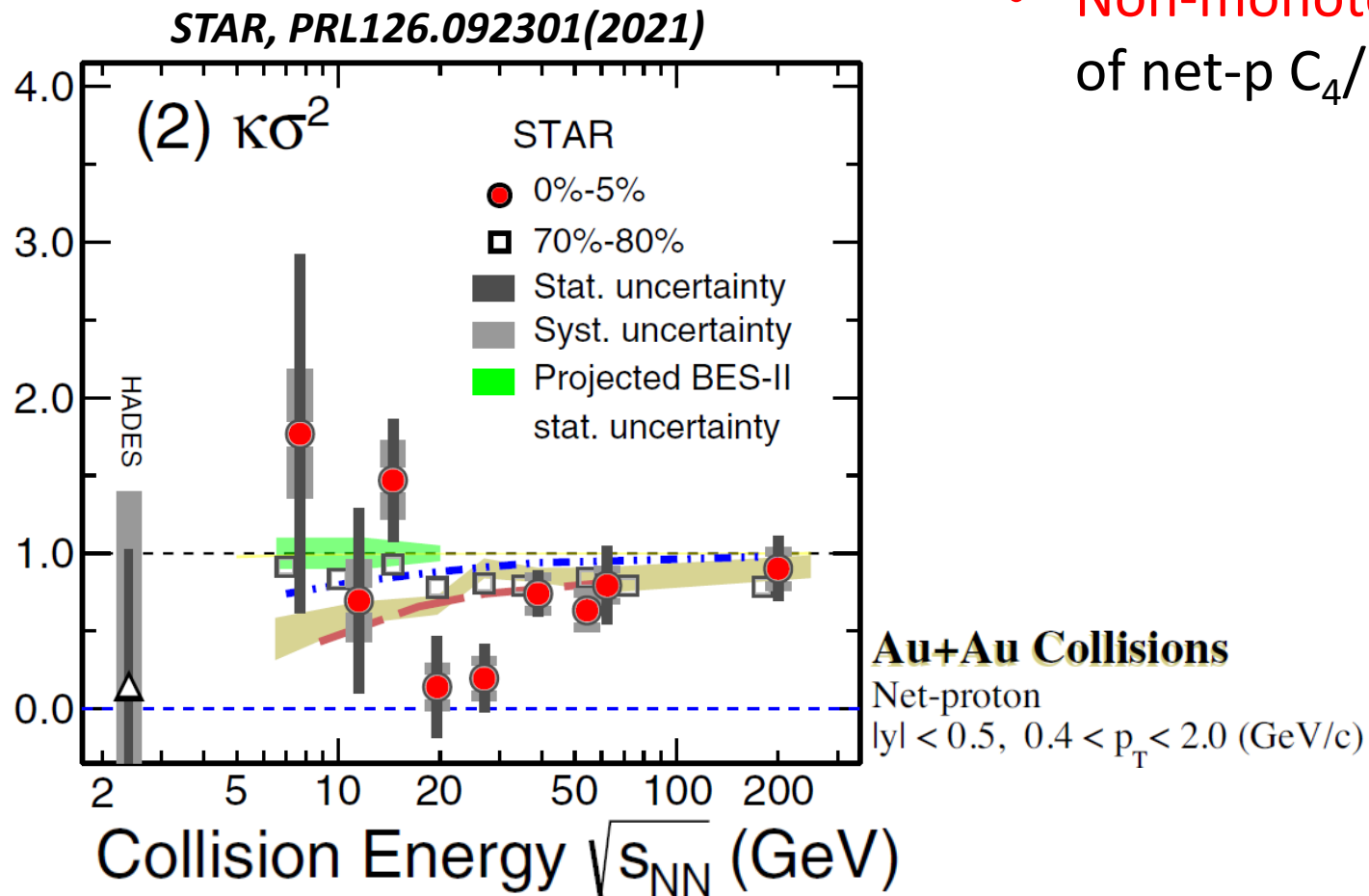
Net-charge fluctuation from NA61/SHINE



- Two-dimensional scan in system size and collision energy.
- No structure indicating critical point.

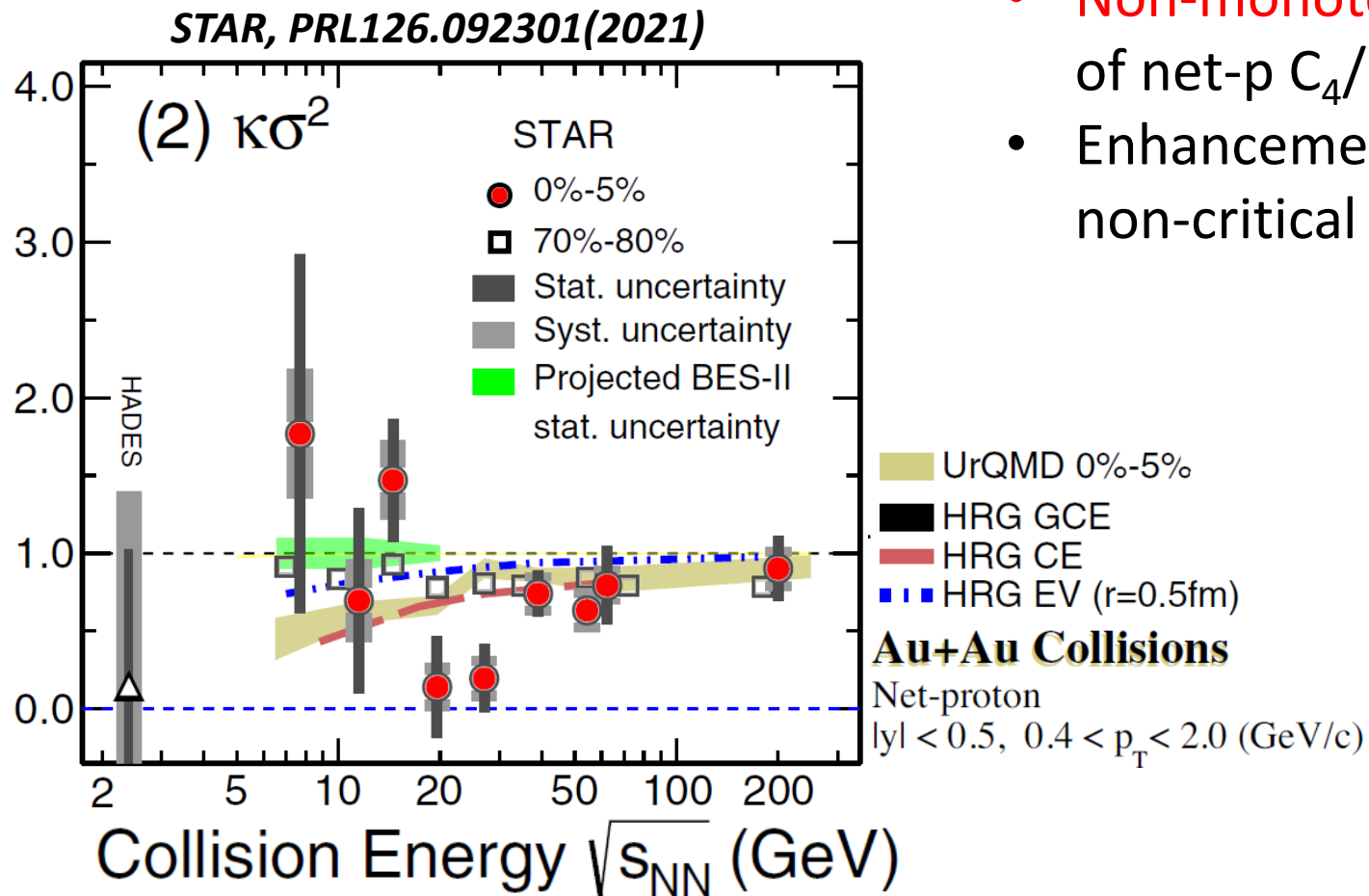
Apr. 5th, 2:45pm, Antoni Marcinek

Net-proton C_4/C_2 : critical point search in BES-I



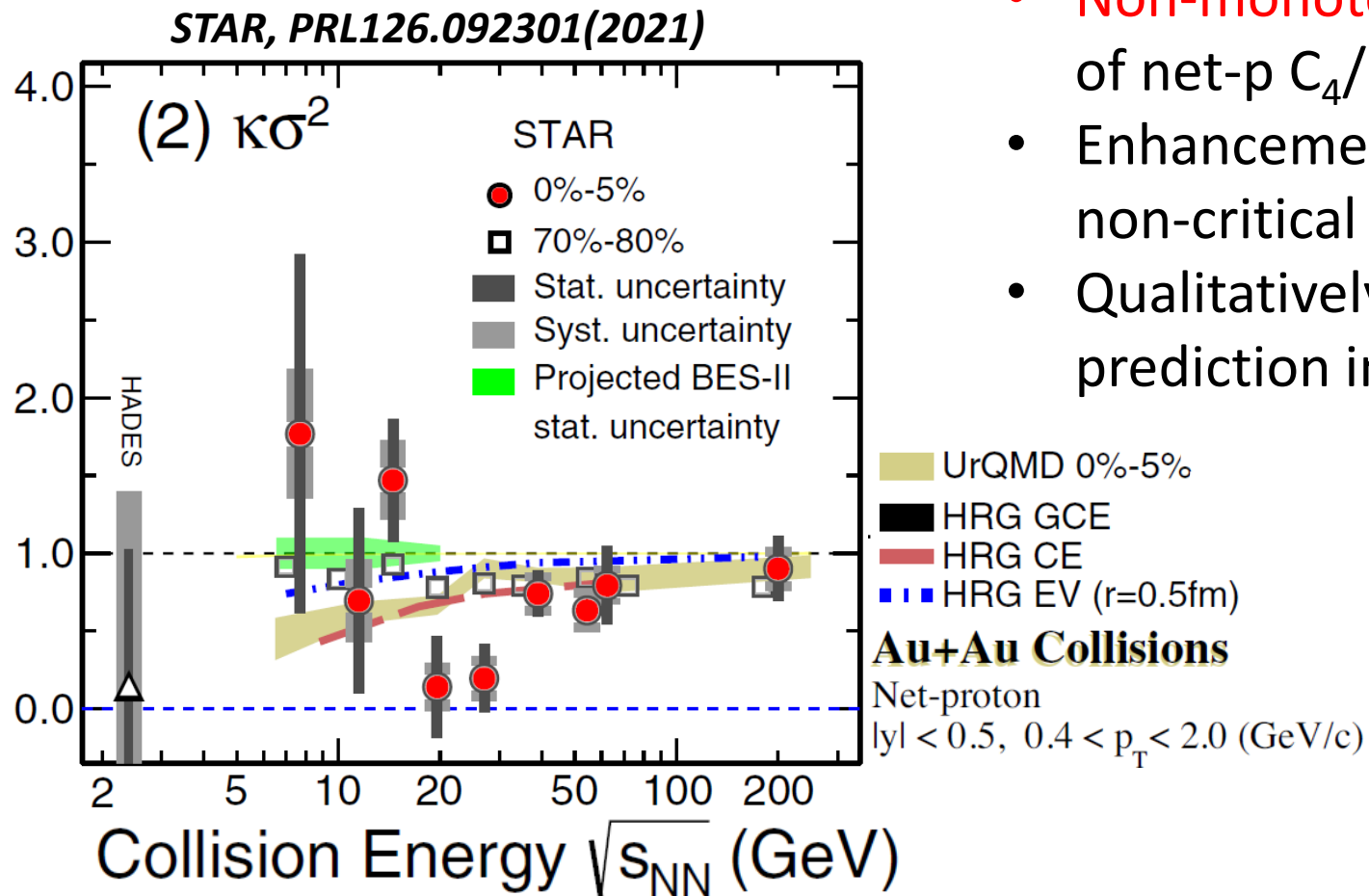
- Non-monotonic beam energy dependence (3.1σ) of net-p C_4/C_2 in Au+Au central collisions.

Net-proton C_4/C_2 : critical point search in BES-I

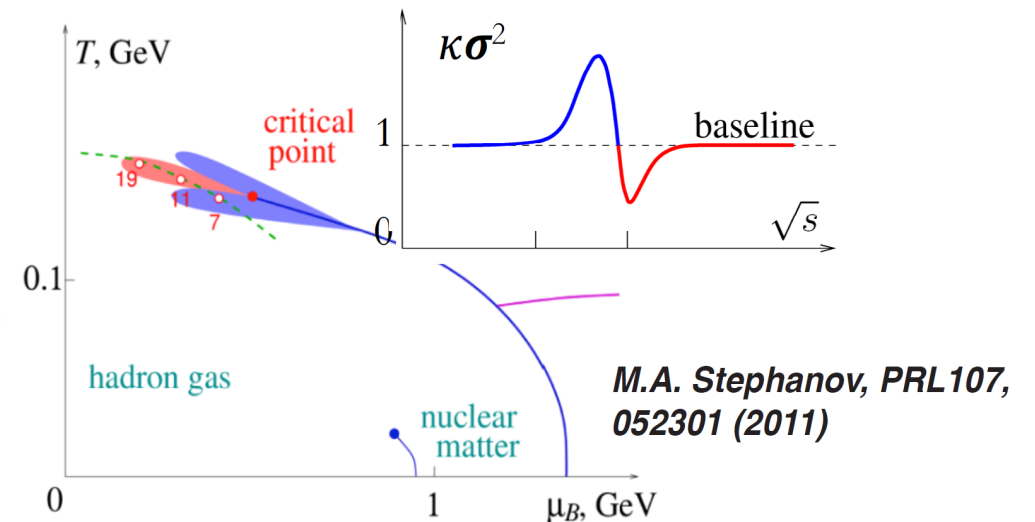


- **Non-monotonic beam energy dependence (3.1σ)** of net-p C_4/C_2 in Au+Au central collisions.
- Enhancement at ~ 7.7 GeV is not reproduced by non-critical baselines.

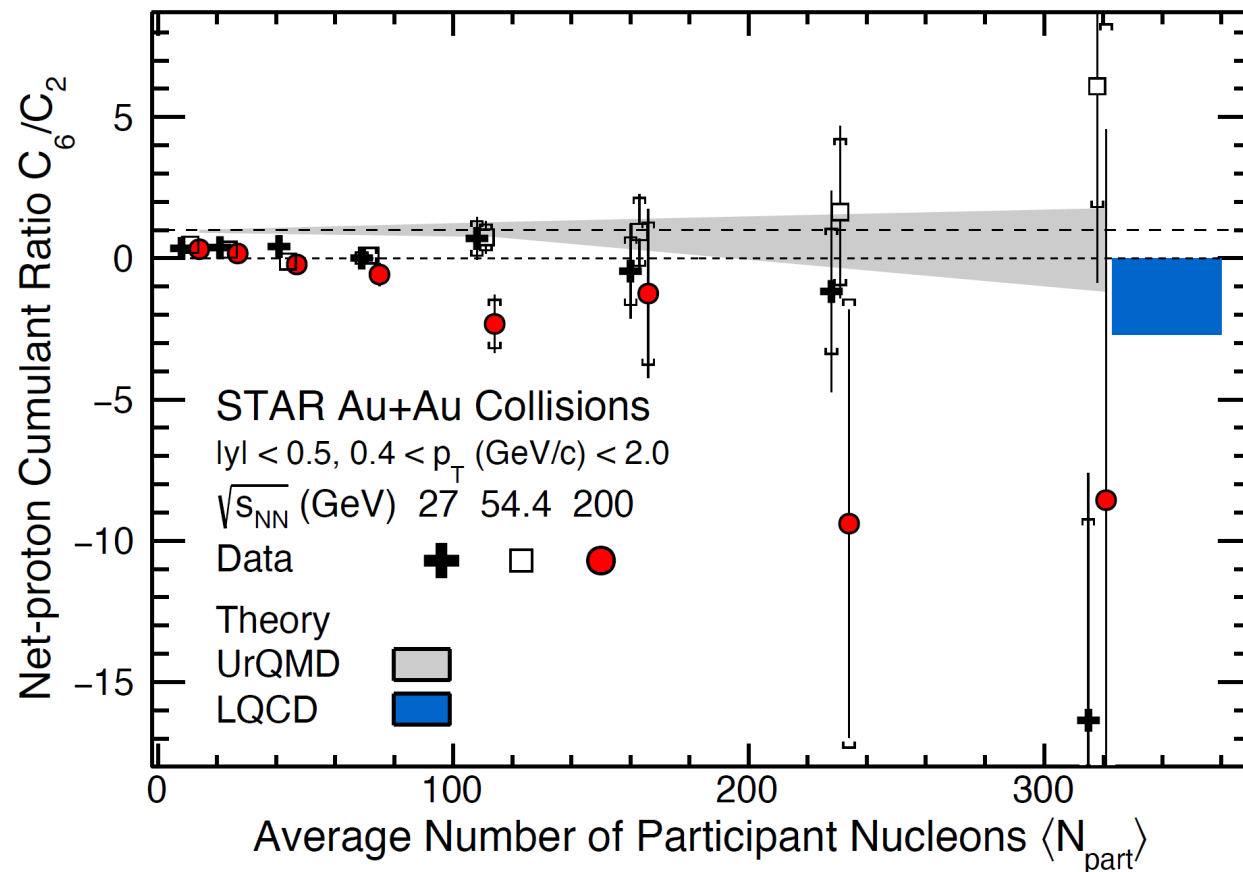
Net-proton C_4/C_2 : critical point search in BES-I



- **Non-monotonic beam energy dependence (3.1σ)** of net-p C_4/C_2 in Au+Au central collisions.
- Enhancement at ~ 7.7 GeV is not reproduced by non-critical baselines.
- Qualitatively consistent with the model prediction incorporating a critical point.



Net-proton C_6/C_2 for crossover search

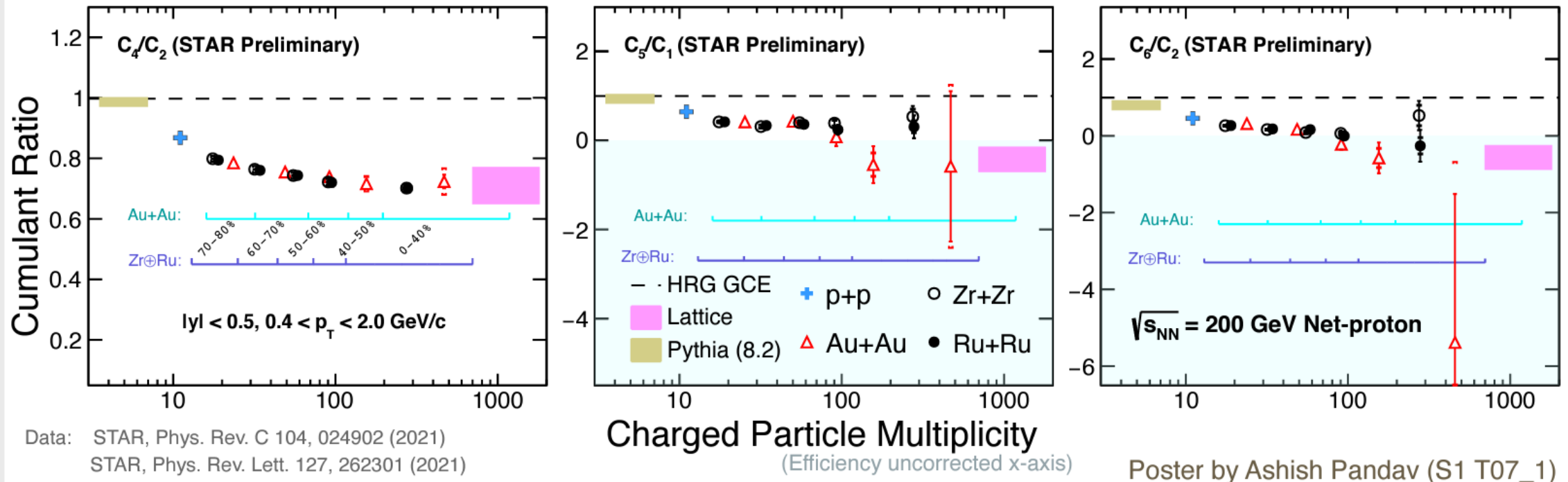


- C_6/C_2 values are progressively negative from peripheral to central collisions at 200 GeV, which is consistent with LQCD calculations.
- Could suggest a smooth crossover transition at top RHIC energy.

STAR, PRL127.262301(2021)

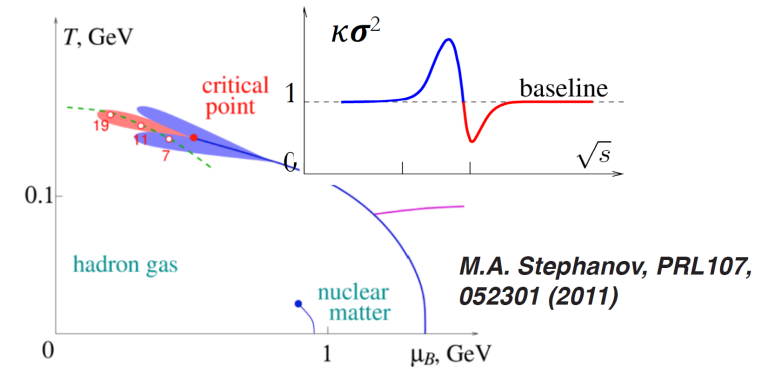
Isobaric collisions

- Ratios approach LQCD calculations with increasing the multiplicity, which imply that the **created system approach thermalized medium at high multiplicity region.**

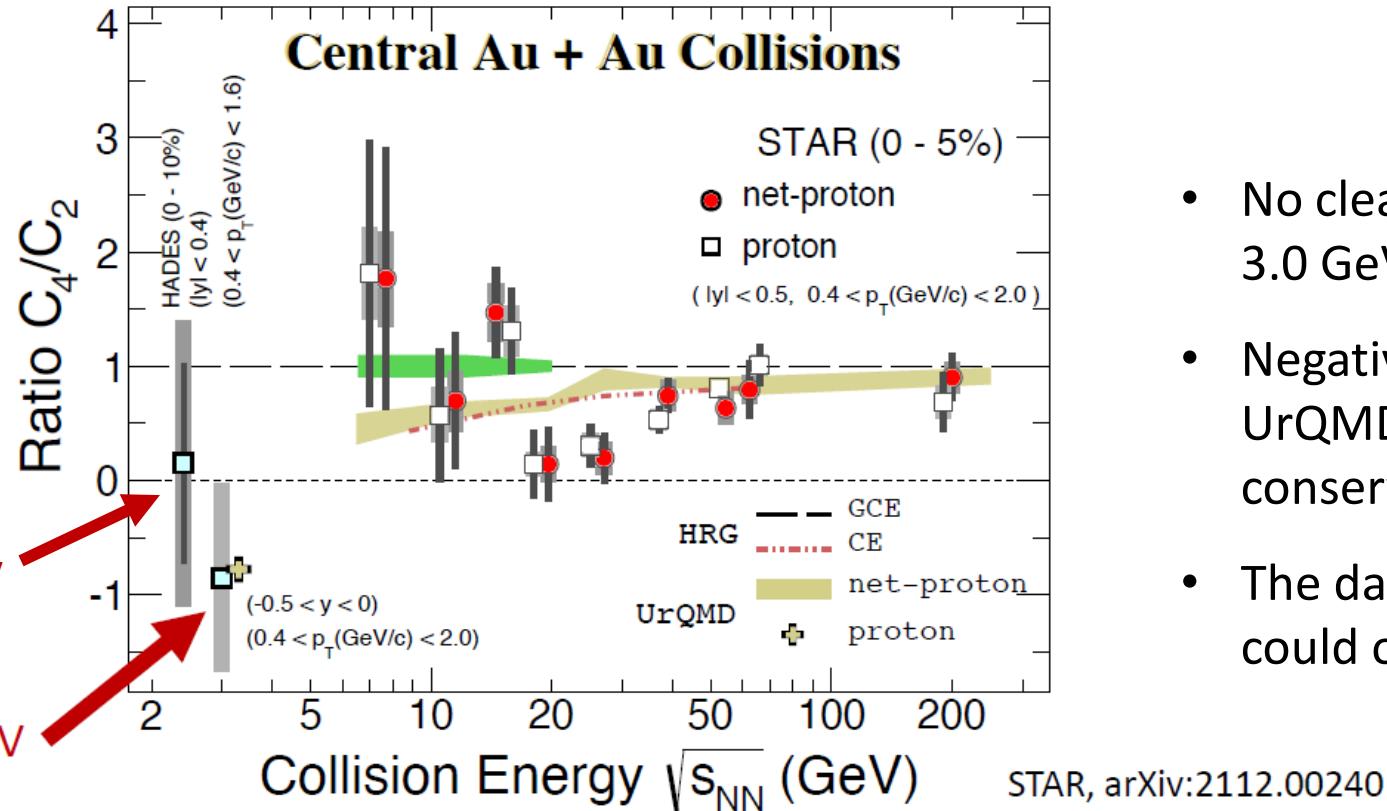


Apr. 7th, 10:20am, Ho-San Ko

FXT from HADES and STAR



- No clear enhancement is observed for 2.4 and 3.0 GeV data from HADES and STAR.
- Negative value at 3 GeV is reproduced by UrQMD, which incorporates baryon number conservation.
- The data implies that the QCD critical region could only exist at energies > 3 GeV.



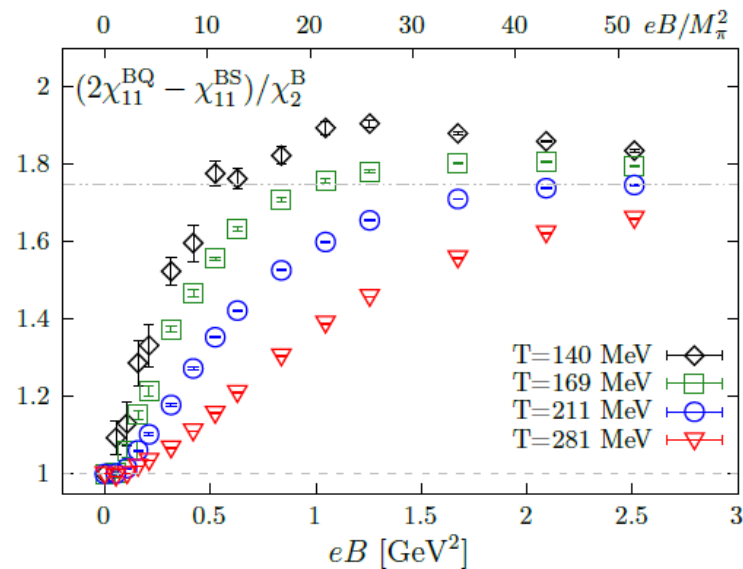
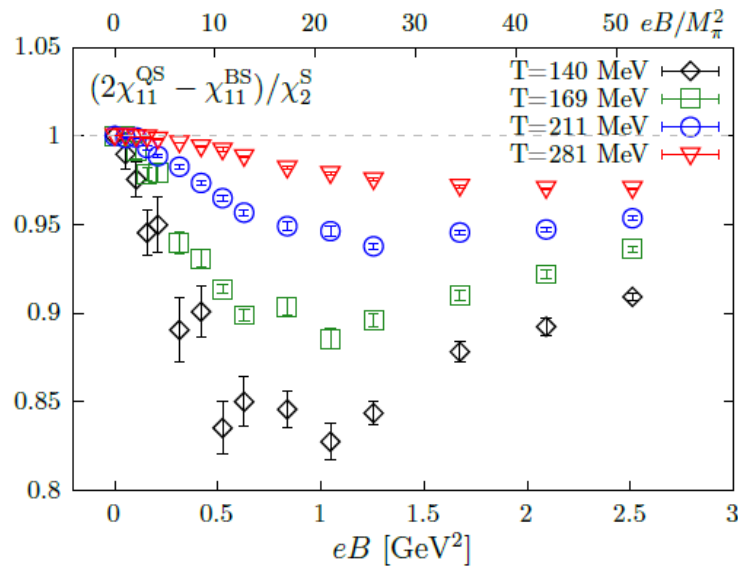
HADES, PRC102.024914(2020)
STAR, arXiv:2112.00240

Apr. 5th, 4:30pm, Yu Zhang

Future Prospect

Probe magnetic field in HIC

- 2nd order mix-cumulants among conserved charges are suggested to be sensitive to the magnetic field.
- Difference between Zr+Zr and Ru+Ru?

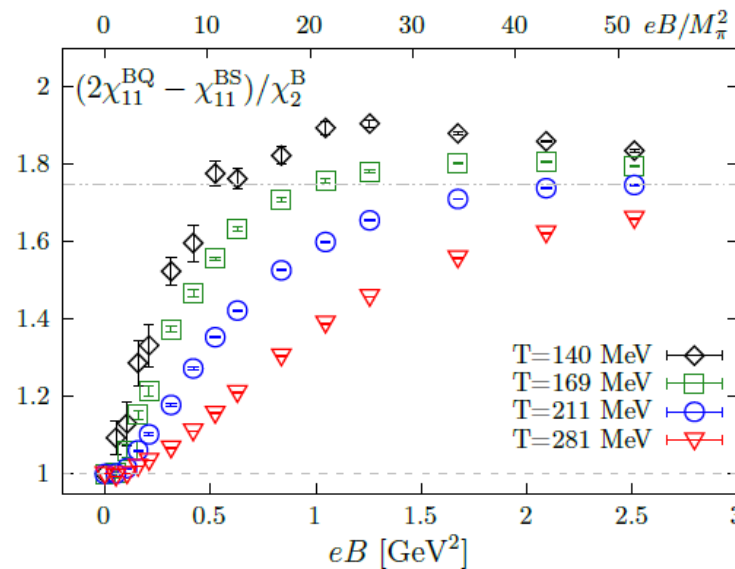
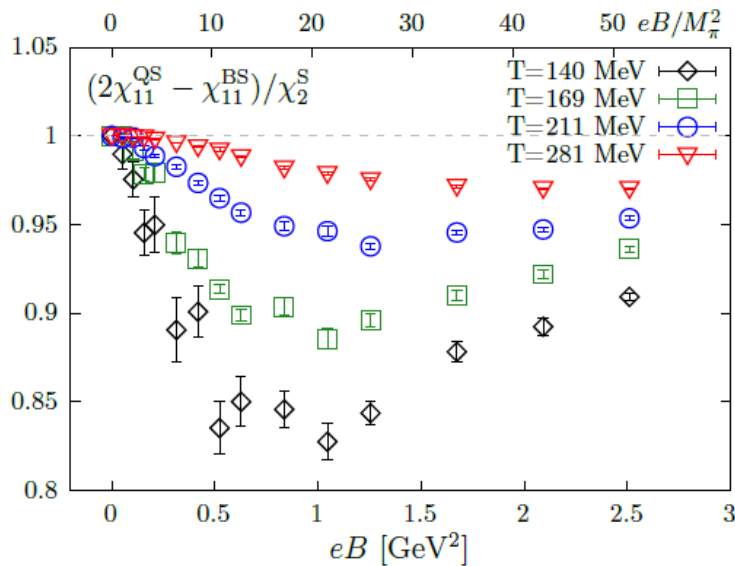


LQCD: H-T.Ding et al, EPJA57.202(2021)

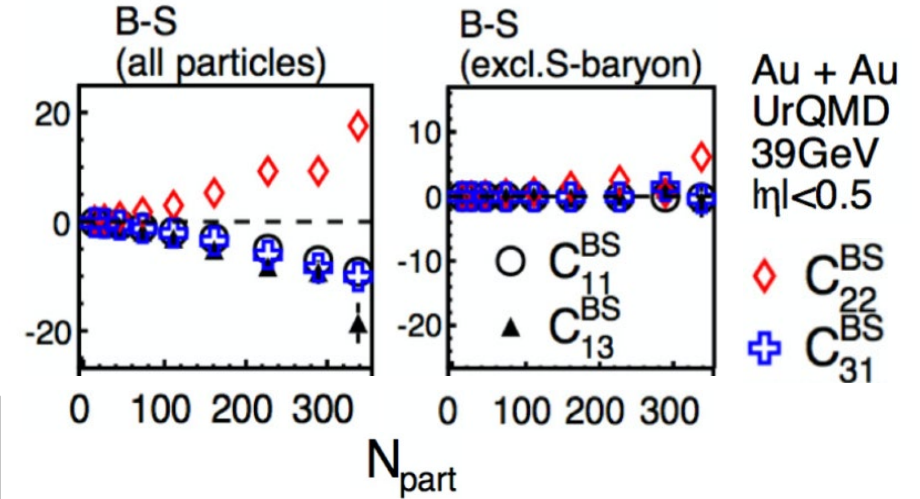
Apr. 7th, 3:40pm, Jun-Hong Liu

Probe magnetic field in HIC

- 2nd order mix-cumulants among conserved charges are suggested to be sensitive to the magnetic field.
- Difference between Zr+Zr and Ru+Ru?
- **Hyperons have an important role for B-S correlations.**



UrQMD: Z. Yang et al, PRC95.014914(2017)

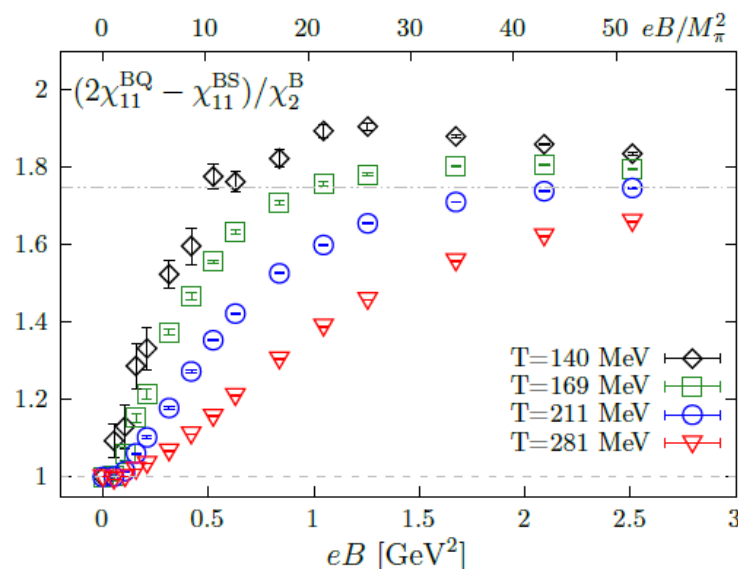
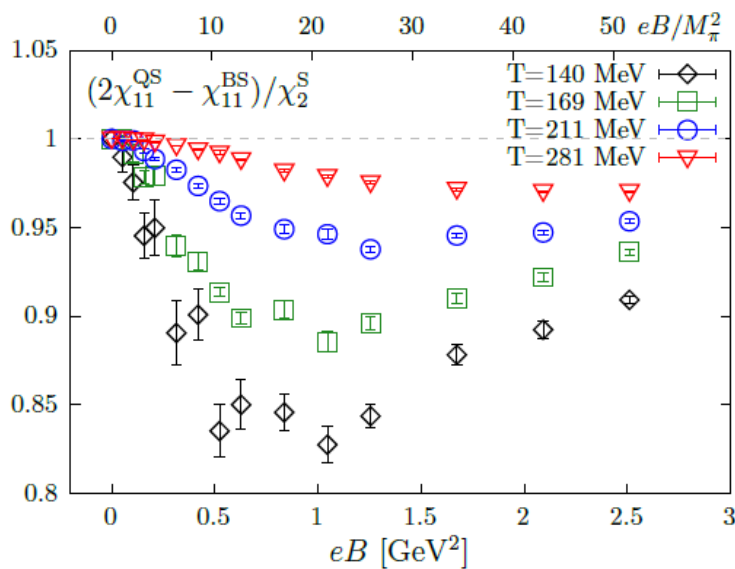


LQCD: H-T.Ding et al, EPJA57.202(2021)

Apr. 7th, 3:40pm, Jun-Hong Liu

Probe magnetic field in HIC

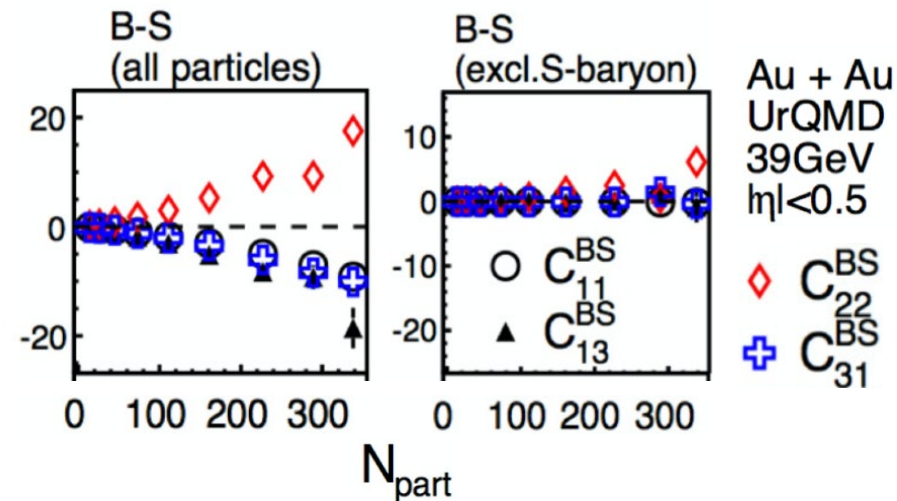
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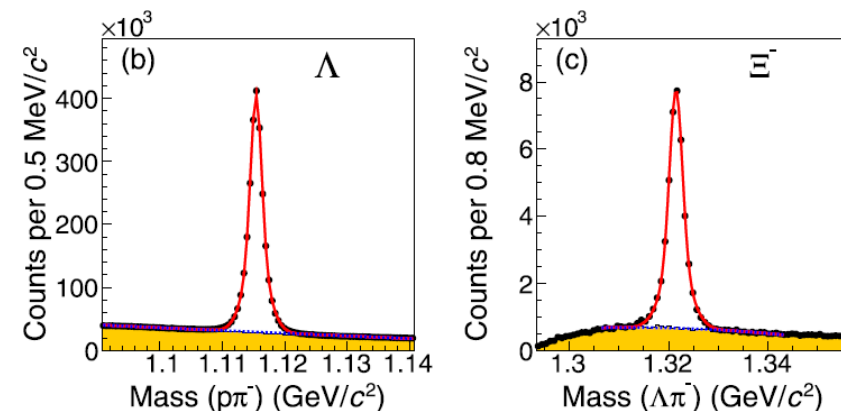
LQCD: H-T.Ding et al, EPJA57.202(2021)

Apr. 7th, 3:40pm, Jun-Hong Liu

UrQMD: Z. Yang et al, PRC95.014914(2017)



Invariant mass reconstruction: Signal and BG particles cannot be identified for track-by-track basis

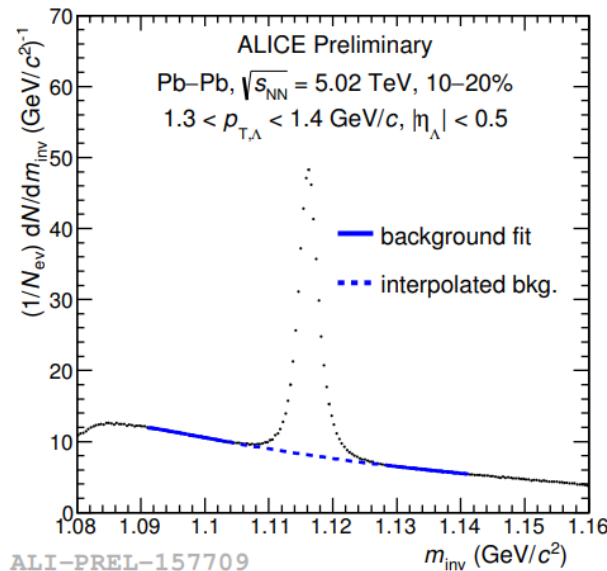


STAR PRC102.034909(2020)

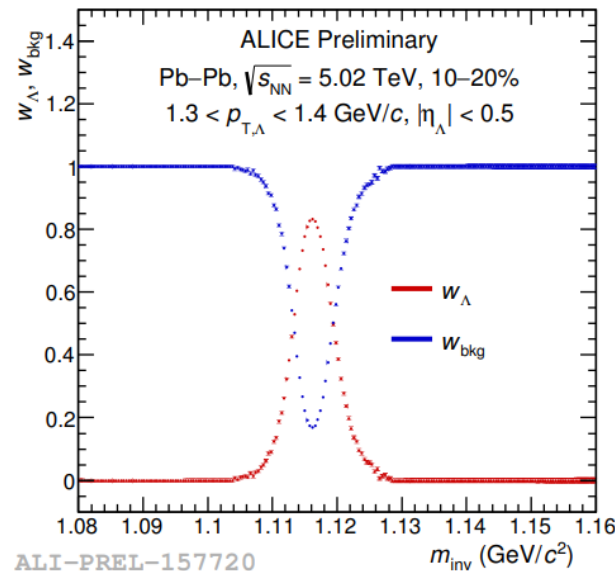
Hyperon number fluctuation

- Direct measurement of event-by-event fluctuation is not possible due to backgrounds under the signal peak.
- Identity method and purity corrections are available.

Identity method



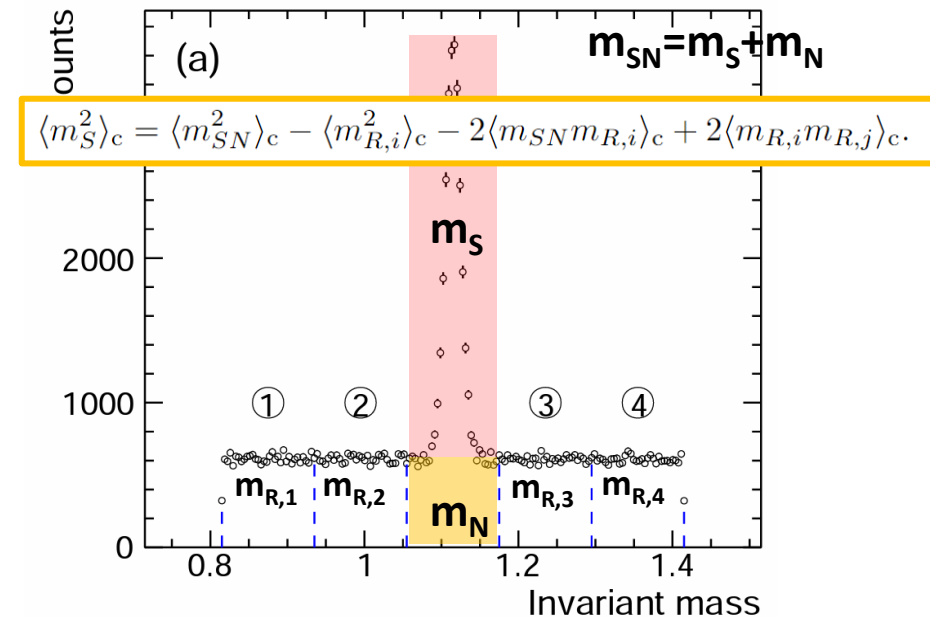
ALI-PREL-157709



ALI-PREL-157720

A. Ohlson, QM2018, NPA982.299(2019)

Purity correction

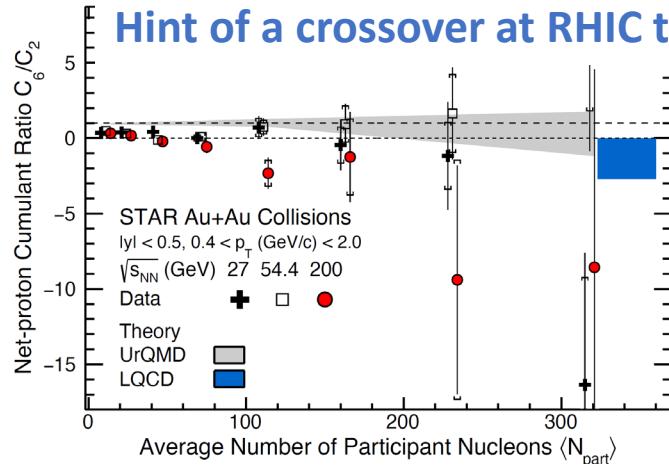


T. Nonaka, arXiv:2202.06953

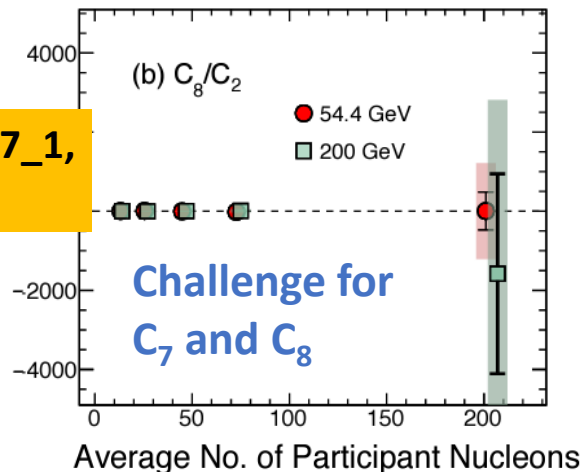
Crossover search

STAR, PRL127.262301(2021)

Hint of a crossover at RHIC top energy

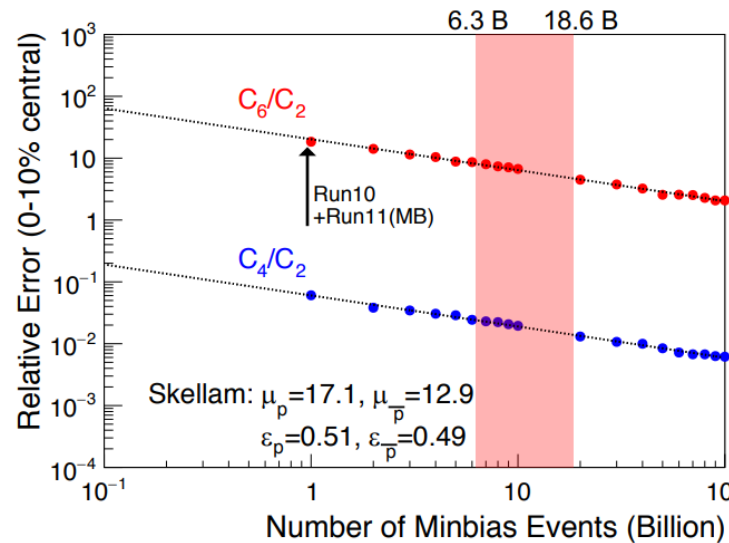


Poster session 1 T07_1,
Ashish Pandav

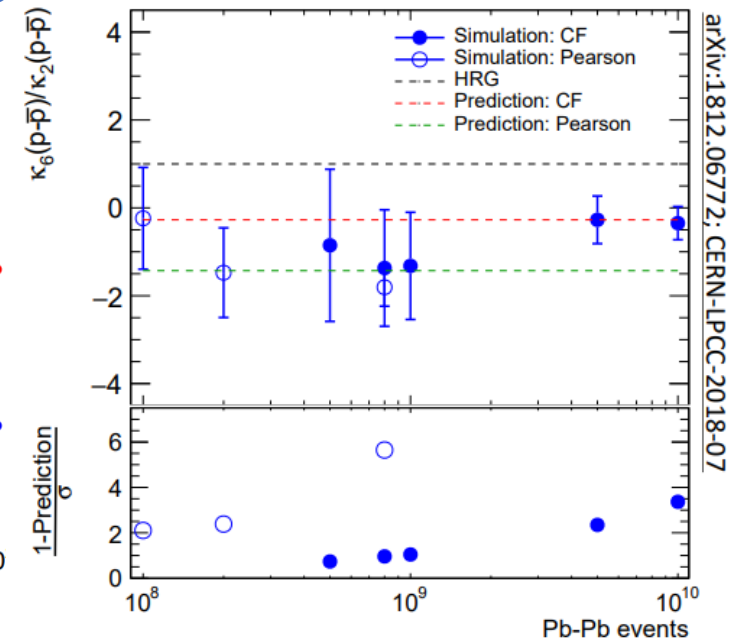


- There will be several opportunities at RHIC and LHC to establish the nature of the phase transition at small μ_B region.

STAR BUR for Run22, STAR NOTE #0773
 RHIC Au+Au 200 GeV 2023-2025



ALICE, LHC Run3

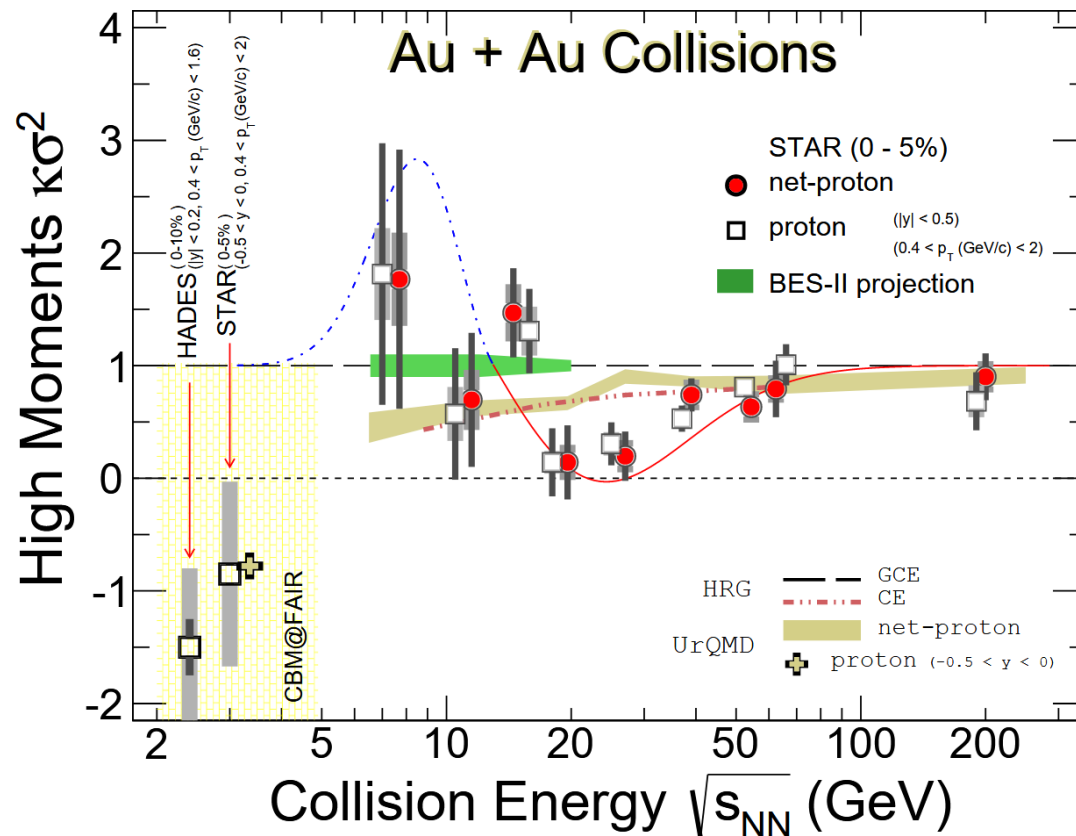


arXiv:1812.06772; CERN-LPCC-2018-07

Critical point search

HADES, PRC102.024914(2020)

STAR, arXiv:2112.00240

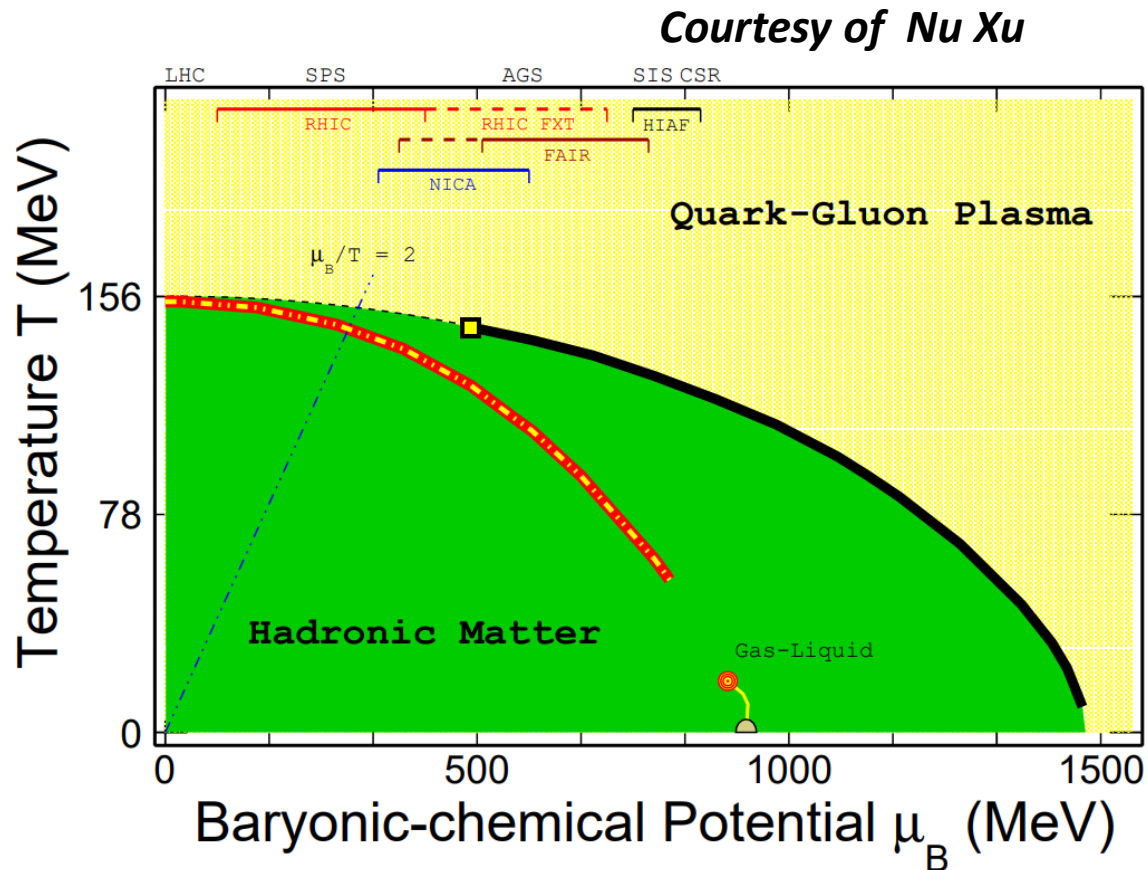


- More data will come from BES-II and FXT at STAR to fill the gap in $3 < \sqrt{s_{NN}} < 20$ GeV.
- More precise study will be carried out by CBM@FAIR, MPD@NICA, HIAF, and JPARC-HI.

| $\sqrt{s_{NN}}$ (GeV) | Beam Energy (GeV/nucleon) | Collider or Fixed Target | $y_{center\ of\ mass}$ | μ^B (MeV) | Run Time (days) | No. Events Collected (Request) | Date Collected |
|-----------------------|---------------------------|--------------------------|------------------------|---------------|-----------------|--------------------------------|----------------|
| 19.6 | 9.8 | C | 0 | 206 | 36 | 582 M (400 M) | Run-19 |
| 17.3 | 8.65 | C | 0 | 230 | 14 | 256 M (250 M) | Run-21 |
| 14.6 | 7.3 | C | 0 | 262 | 60 | 324 M (310 M) | Run-19 |
| 13.7 | 100 | FXT | 2.69 | 276 | 0.5 | 52 M (50 M) | Run-21 |
| 11.5 | 5.75 | C | 0 | 316 | 54 | 235 M (230 M) | Run-20 |
| 11.5 | 70 | FXT | 2.51 | 316 | 0.5 | 50 M (50 M) | Run-21 |
| 9.2 | 4.59 | C | 0 | 372 | 102 | 162 M (160 M) | Run-20+20b |
| 9.2 | 44.5 | FXT | 2.28 | 372 | 0.5 | 50 M (50 M) | Run-21 |
| 7.7 | 3.85 | C | 0 | 420 | 90 | 100 M (100 M) | Run-21 |

H. Caines, "RHIC BES and Beyond"

Summary : Where we are?



- Several hints on QCD phase structures have been obtained through measurements of fluctuations of conserved charges.
- LQCD indicates critical point is not likely to exist for $\mu_B/T < 2.5$.

Apr. 6th, 11:30am, Dennis Bollweg

- Precise data will come from CBM@FAIR, MPD@NICA, HIAF, and JPARC-HI for large μ_B region.

2022 Yagi Award

<https://ithems.riken.jp/en/about/yagi-award>

“Kohsuke Yagi Quark Matter Award” (Yagi Award) is based on the donation to iTHEMS from bereaved family of late Professor Kohsuke Yagi who was a renowned Japanese nuclear physicist. Responding to the family request, the award aims to support early career scientists with Japanese nationality, to promote and expand country's nuclear physics research field. It will be awarded to **junior Japanese physicists under age of 40 who give plenary talk at the “Quark Matter: International Conference on Ultra-relativistic Nucleus-Nucleus Collisions”** held in every 1.5 years.



Prof. Kohsuke Yagi (1934-2014)
Quark Matter 1997, Chair

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Thank you for your attention