

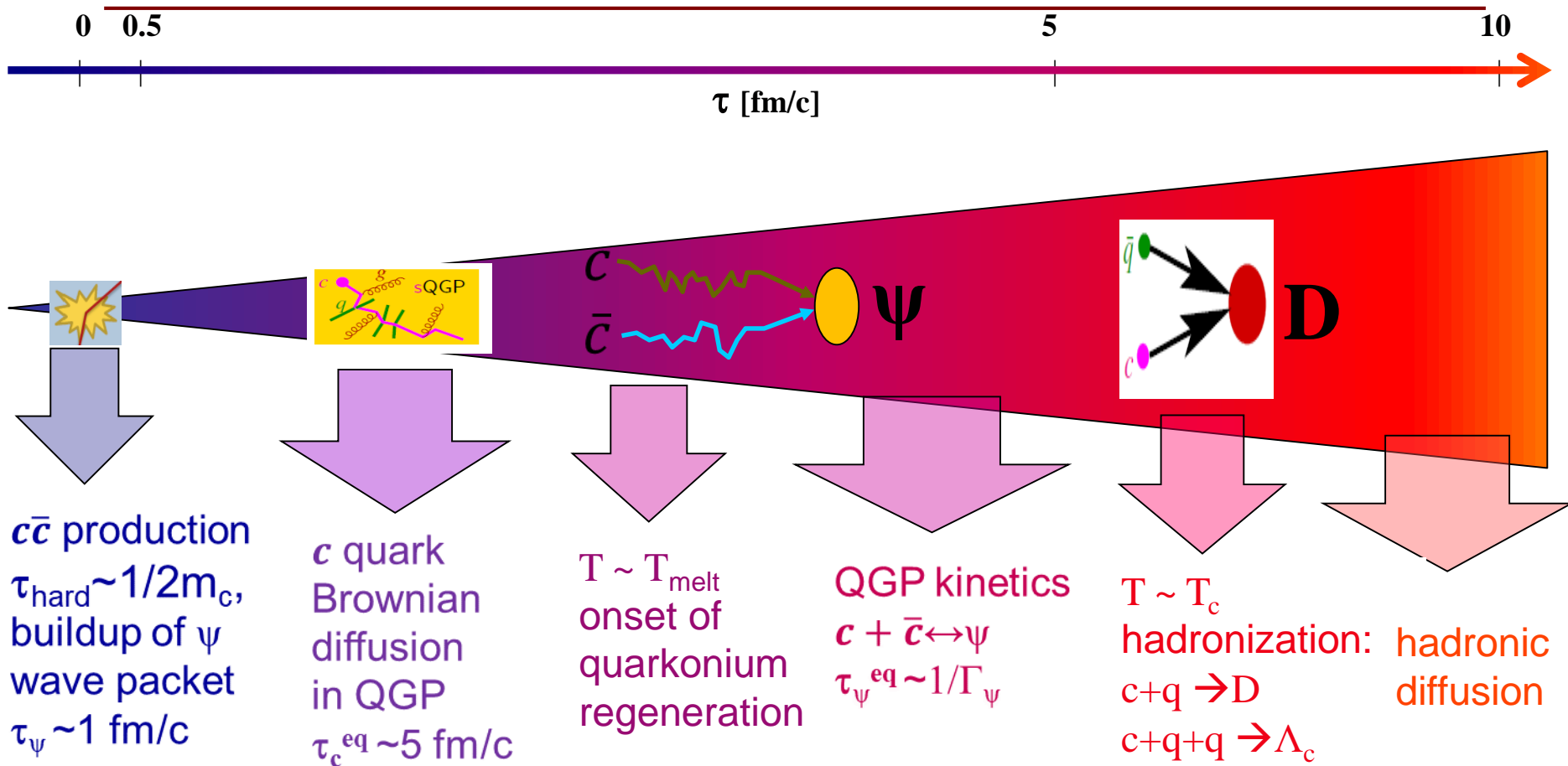


Production & Hadronization of Heavy-Flavor Hadrons

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Heavy flavor transport as probes of QGP



Outline & disclaimer

Part I: charm-hadron production in pp

- Λ_c/D^0 : data vs model
- Σ_c/D^0 & Ξ_c/D^0

Part II: charm-hadron production in AA

- diffusion coefficient & hadronization
- D-meson R_{AA} & v_2 : extracting $\mathcal{D}_s(2\pi T)$
- Λ_c/D^0 : in-medium hadrochemistry

Part III: quarkonium production in AA

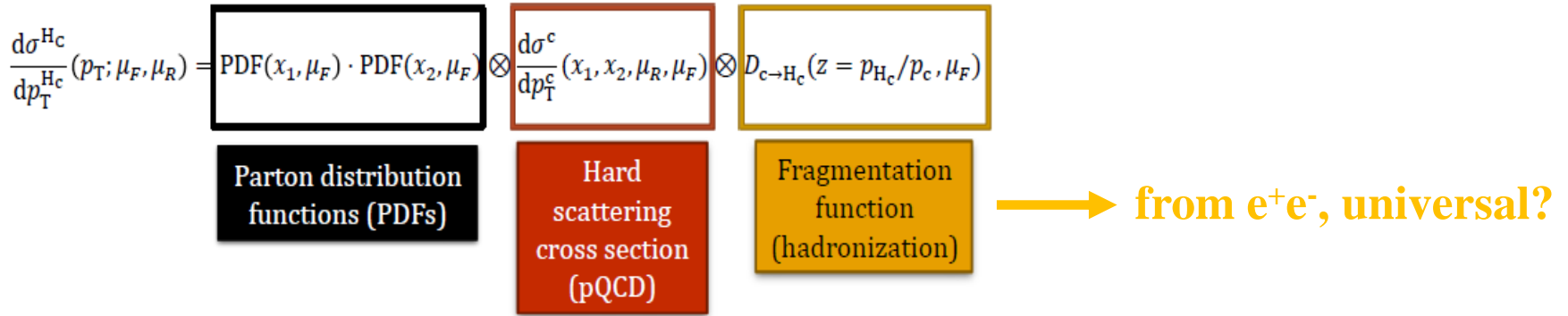
- HQ potential: remnants of confining force
- J/ψ v_2 puzzle
- open quantum approach to Y states

Disclaimer: selection of topics, no high p_T HF jets

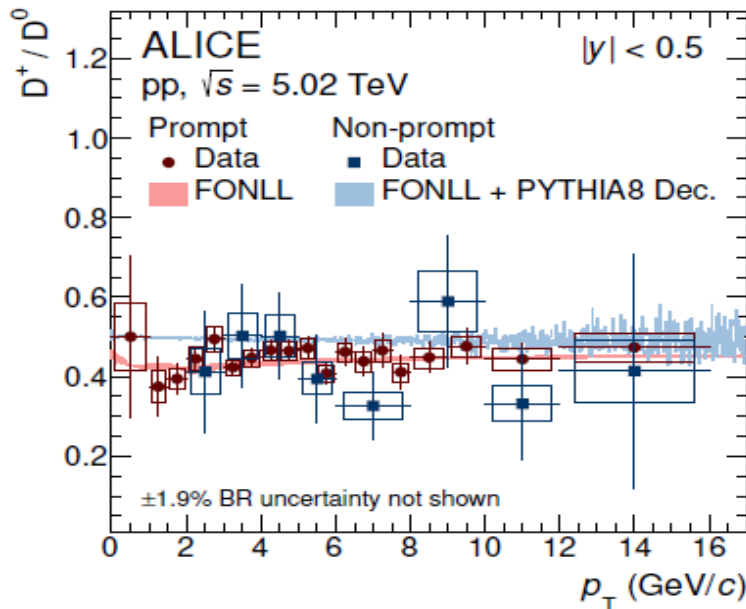
Part I: Charm-hadron production in pp

- pQCD factorization & fragmentation
- Λ_c/D^0 enhancement vs hadronization models
- Σ_c & Ξ_c production

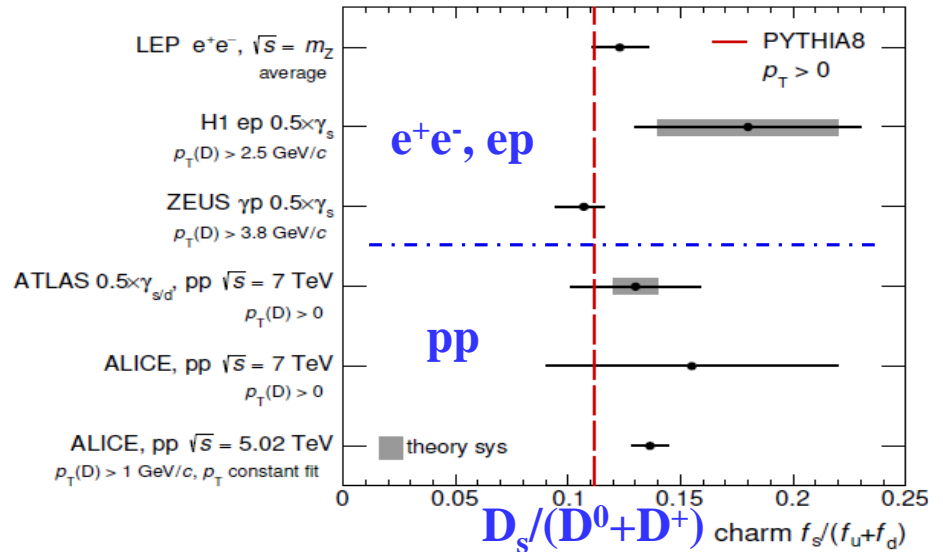
pQCD factorization & fragmentation



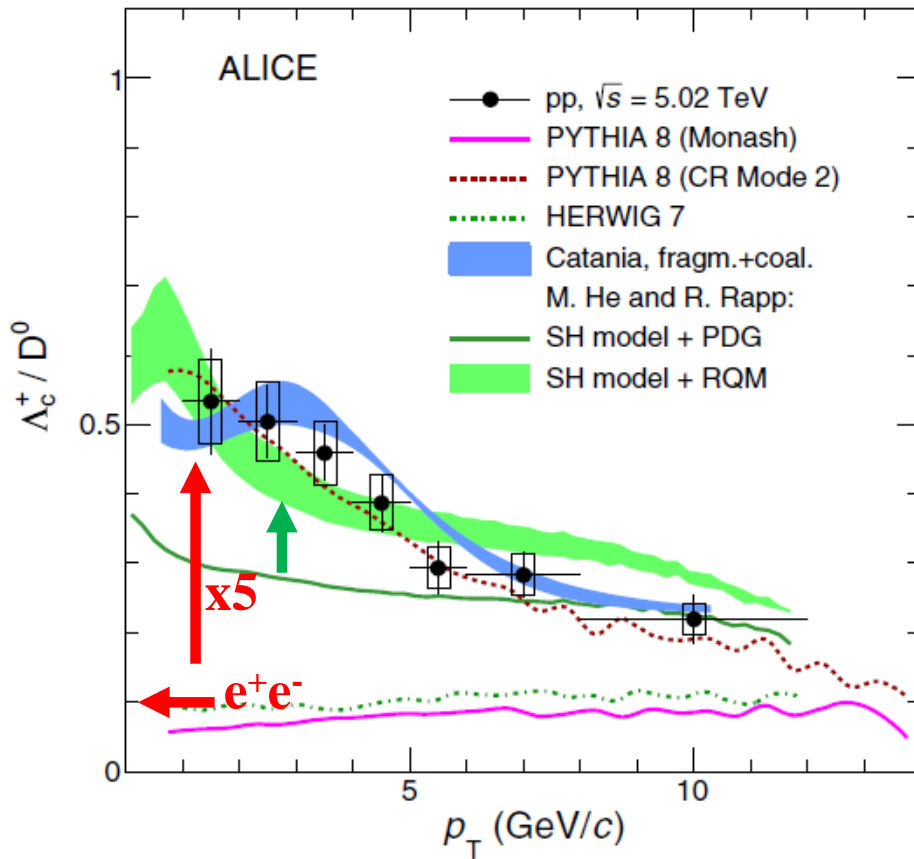
❖ phenomenological FF: assumed universal & constrained by e⁺e⁻



charm-meson ratios in pp reproduced

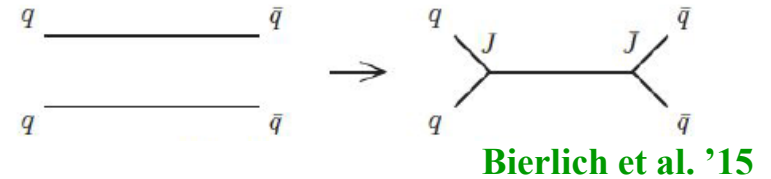


Λ_c^+ / D^0 @ 5 TeV pp collisions



ALICE, PRL 127, 202301(2021)
ALICE, PRC 104, 054905(2021)

- ❖ PYTHIA8: Color-reconnection with junctions frag. into baryons

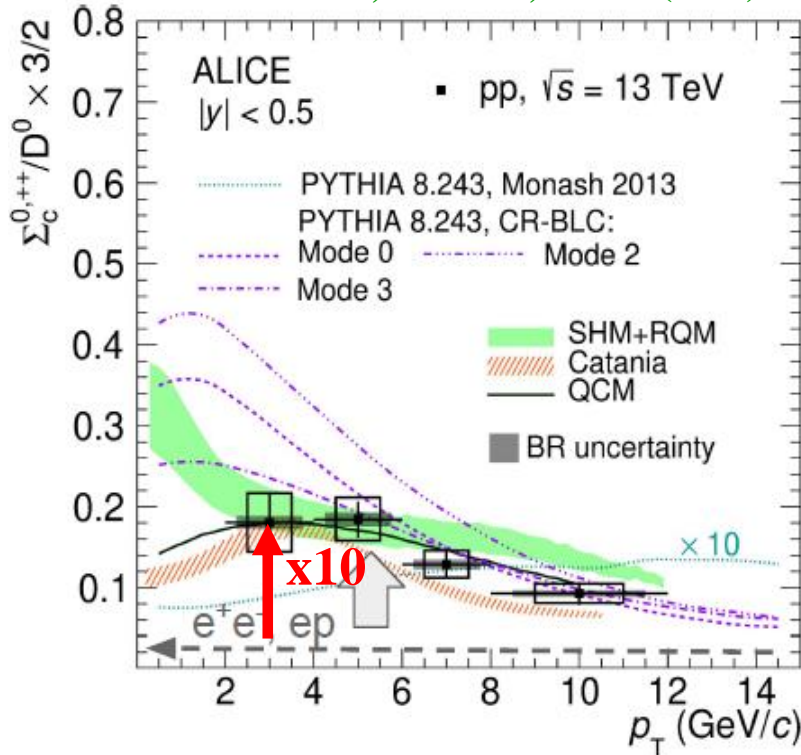


- ❖ Catania: c-q(-q) coalescence in a small QGP fireball [Minissale et al. '21](#)
- ❖ Statistical hadronization in q-rich environment (unlike e^+e^-)
 - augmented by “missing” charm-baryons assuming *relative* chemical equilibrium

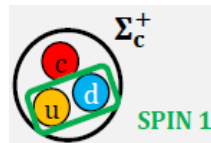
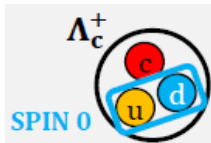
MH & Rapp '19

Σ_c/D^0 & Ξ_c/D^0

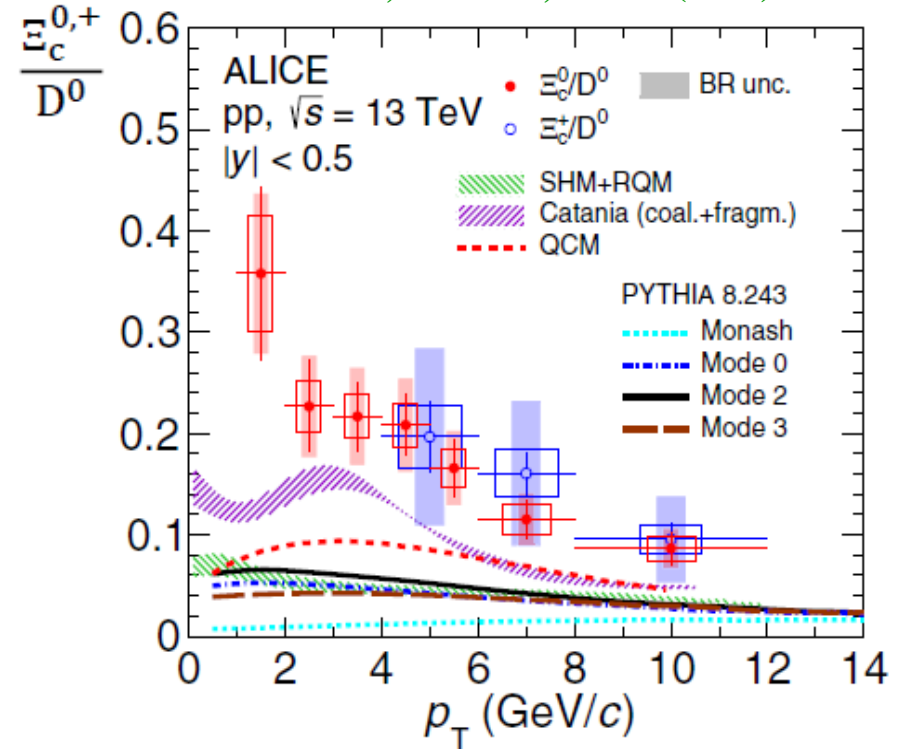
ALICE, PRL 128, 012001(2022)



❖ $\Sigma_c/D^0 \times 10$ enhanced despite more massive spin-1 diquark



ALICE, PRL 127, 272001(2021)



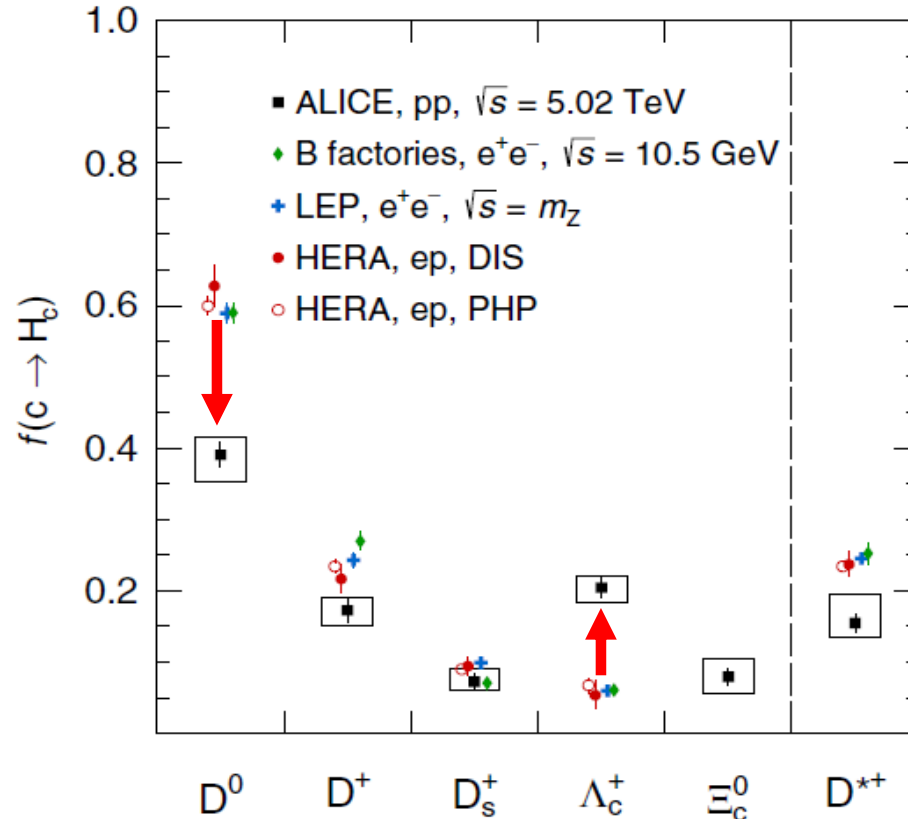
❖ Ξ_c/D^0 ratio underestimated by all models



Take-aways from Part I

- ❖ charm quark fragmentation is **non-universal** from e^+e^- to pp

ALICE, PRD 105, L011103 (2022)



- $c \rightarrow \Lambda_c$ much enhanced vs $c \rightarrow D^0$ reduced
- full charm-hadrons measured $d\sigma^{c\bar{c}}/dy \sim 1.16 \mu\text{b}$ at mid-y
- ➔ significant impact on charmonia production

Part II: Charm-hadron production in AA

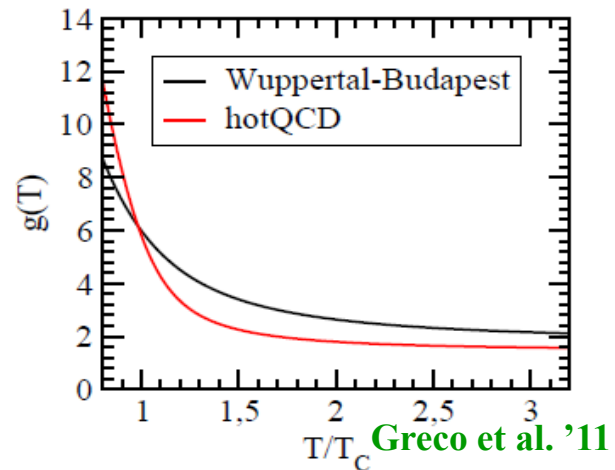
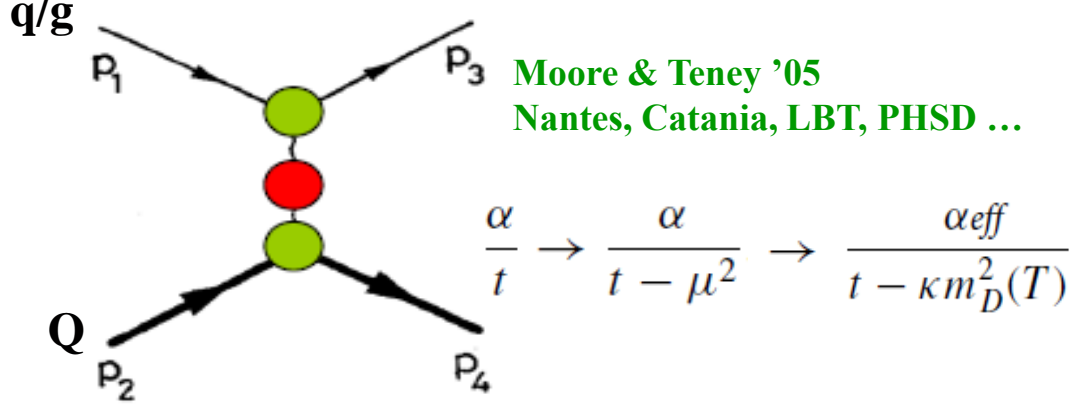
- Microscopic interactions & diffusion coefficient
- Hadronization: Coalescence, Resonance Reco., SHMc
- D-meson R_{AA} & v_2 : Diffusion coefficient
- Charm hadro-chemistry: Λ_c/D^0

Transport coefficient: pQCD vs T-matrix

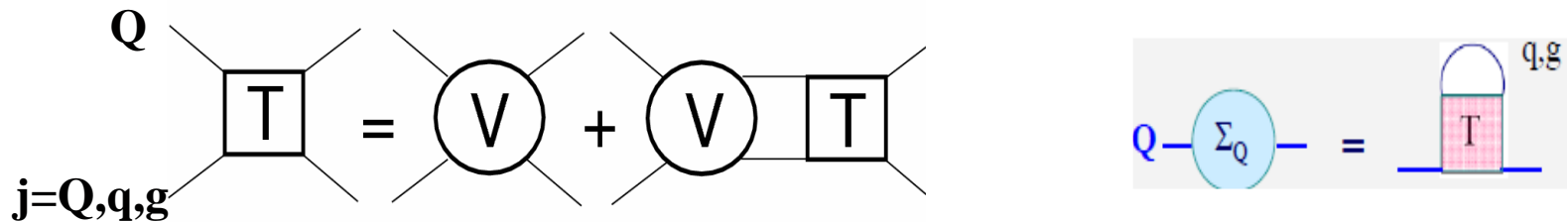
- ❖ HQ Brownian diffusion via Fokker-Planck

$$\frac{\partial}{\partial t} f_Q(t, p) = \gamma \frac{\partial}{\partial p_i} [p_i f_Q(t, p)] + D_p \Delta_{\vec{p}} f_Q(t, p) \quad \gamma = A \sim \int |T_{Qj}|^2 (1 - \cos\theta) f^j$$

- ❖ effective Born diagram w/ large α_s



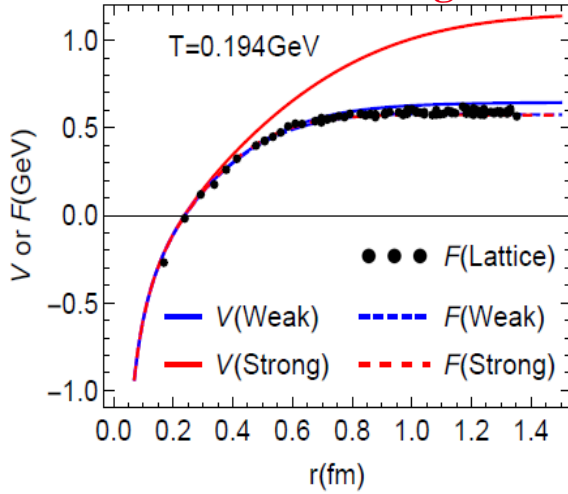
- ❖ T-matrix: coupled two- and one-body integral equations



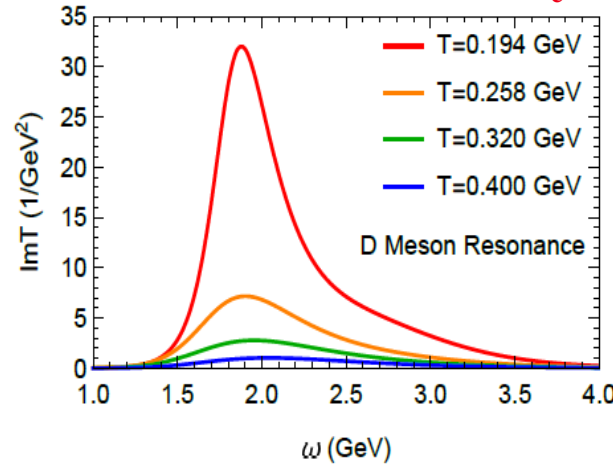
TAMU '05-'19

T-matrix approach: Spectral + Transport Properties

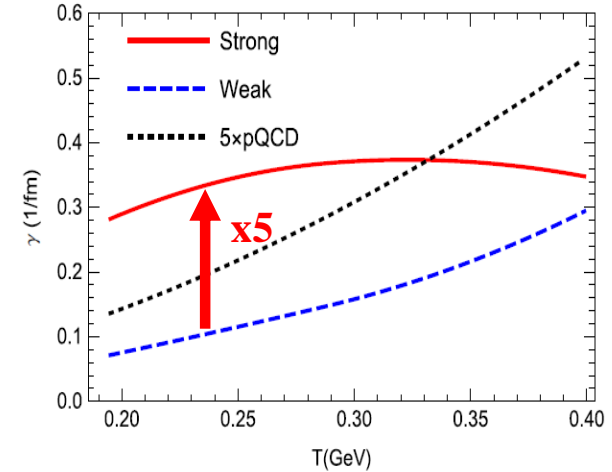
Remnant confining force



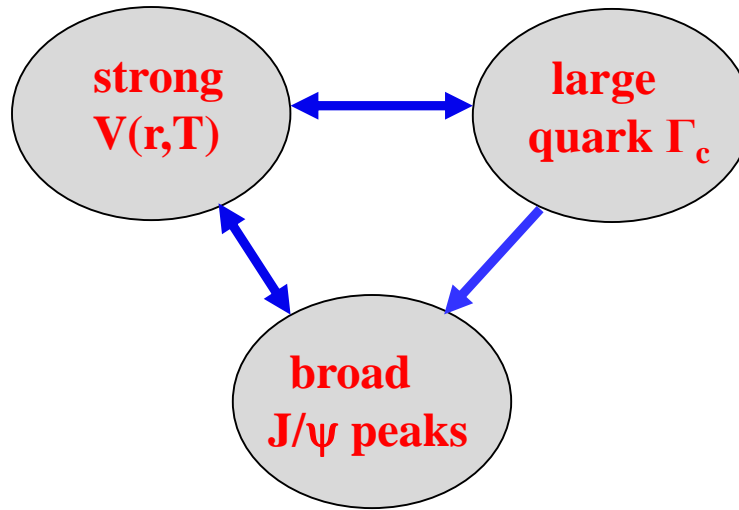
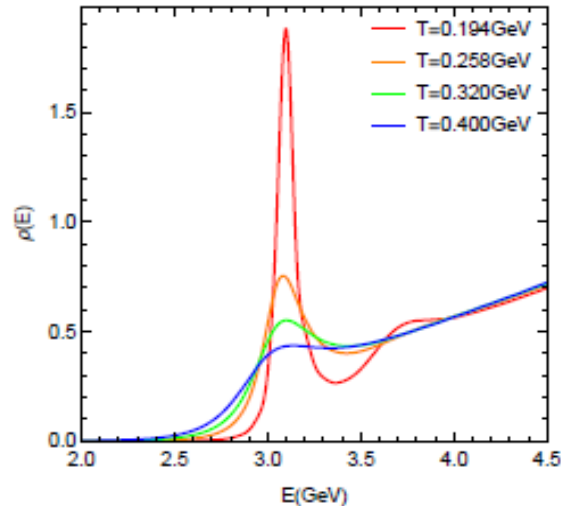
Resonances form near T_c



Accelerates thermalization

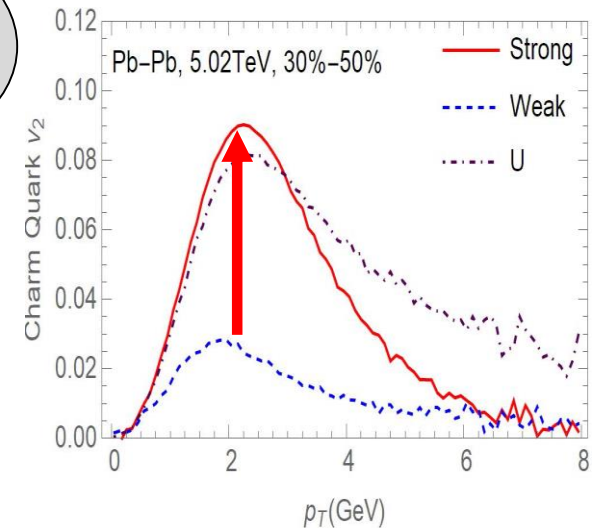


J/ψ survives in QGP



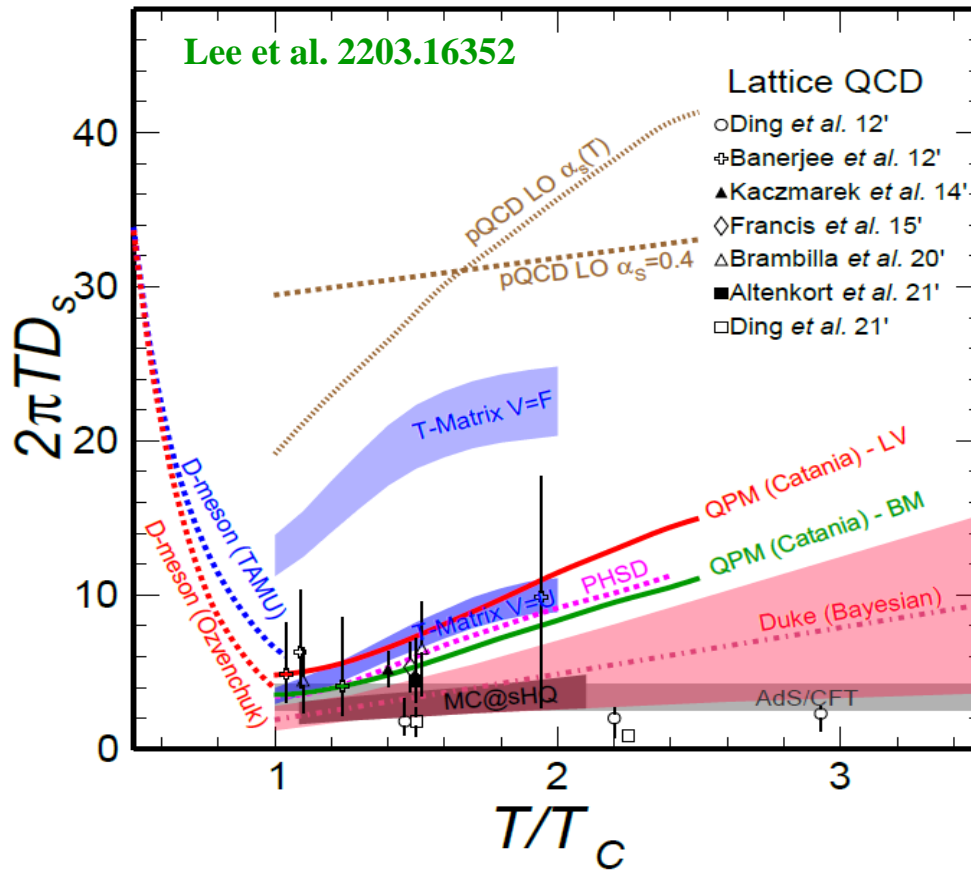
S.Y.F Liu et al '18, '19

Coupling strength \leftrightarrow HQ v_2



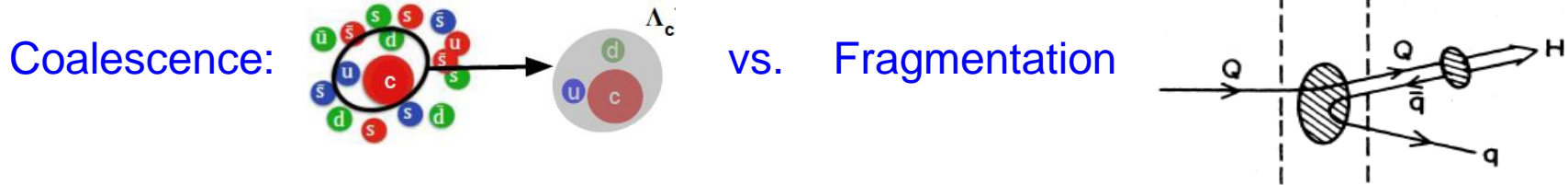
Transport coefficient: $\mathcal{D}_s(2\pi T)$

❖ HQ spatial diffusion coefficient: $\mathcal{D}_s = T/m_Q A(p=0) = T/m_Q \gamma$



- models & lattice $\mathcal{D}_s(2\pi T) \sim 2-4$ near T_c , x10 smaller than pQCD
- maximum coupling strength near T_c , remnant of confining force?

HQ hadronization in QGP



❖ Instantaneous coalescence models (ICM)

$$f_h(\mathbf{p}'_h) = \int \left[\prod_i d\mathbf{p}_i f_i(\mathbf{p}_i) \right] W(\{\mathbf{p}_i\}) \delta(\mathbf{p}'_h - \sum_i \mathbf{p}_i) \quad \text{Fries et al., Greco et al., Voloshin '03}$$

- static Wigner distribution w/ hadron radius
- equilibrium limit challenging at low p_T
- improvement: c-q(-q) form excited cluster + decay

Greco et al. '04, Oh et al.'09, Plumari et al.'18,
Cao et al. '16, '20, Katz '20, Li+Liao '20

Beraudo et al. '15, '22
Cao et al.'20

❖ Statistical hadronization with charm (SHMc) Andronic et al. '21

- *thermalized* c-quarks hadronize at T_c

$$\frac{dN(h_{oc,\alpha}^i)}{dy} = g_c^\alpha V n_i^{\text{th}} \frac{I_\alpha(N_c^{\text{tot}})}{I_0(N_c^{\text{tot}})} \propto g_c^\alpha \leftarrow d\sigma^{c\bar{c}}/dy$$

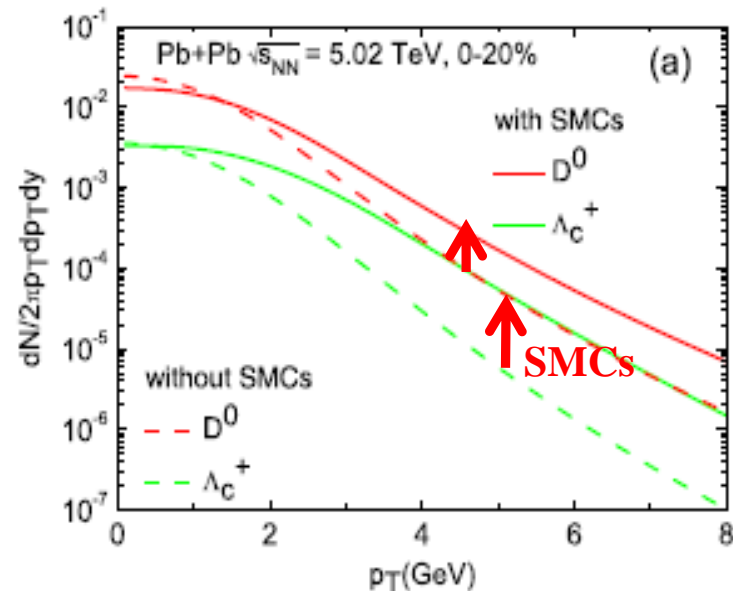
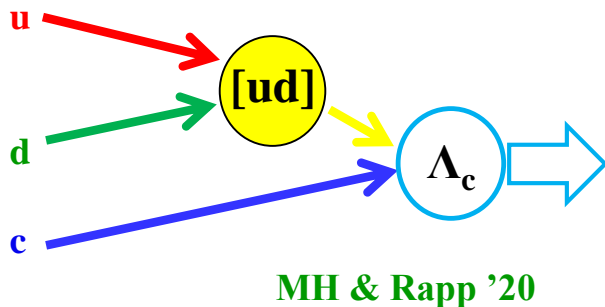
- p_T -spectrum by hydrodynamic blast wave at T_c

HQ hadronization II

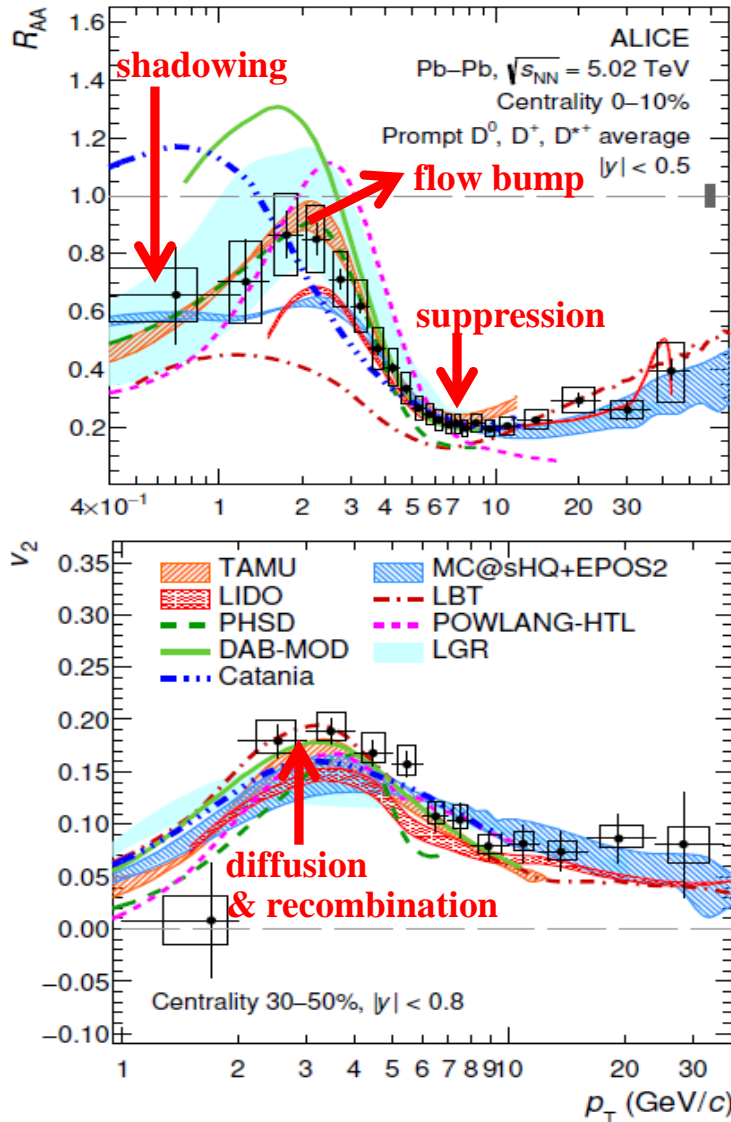
- ❖ Resonance recombination model (RRM) Ravagli et al.'07, MH et al.'12

$$f_M(\vec{x}, \vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2}{(2\pi)^3} f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \underline{\sigma_M(s)} v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

- $\sigma_M(s)$ resonant cross section: energy conservation & equilibrium limit
- 3-body RRM & space-momentum correlations (SMCs)



D-meson R_{AA} & v_2 : extracting $\mathcal{D}_s(2\pi T)$

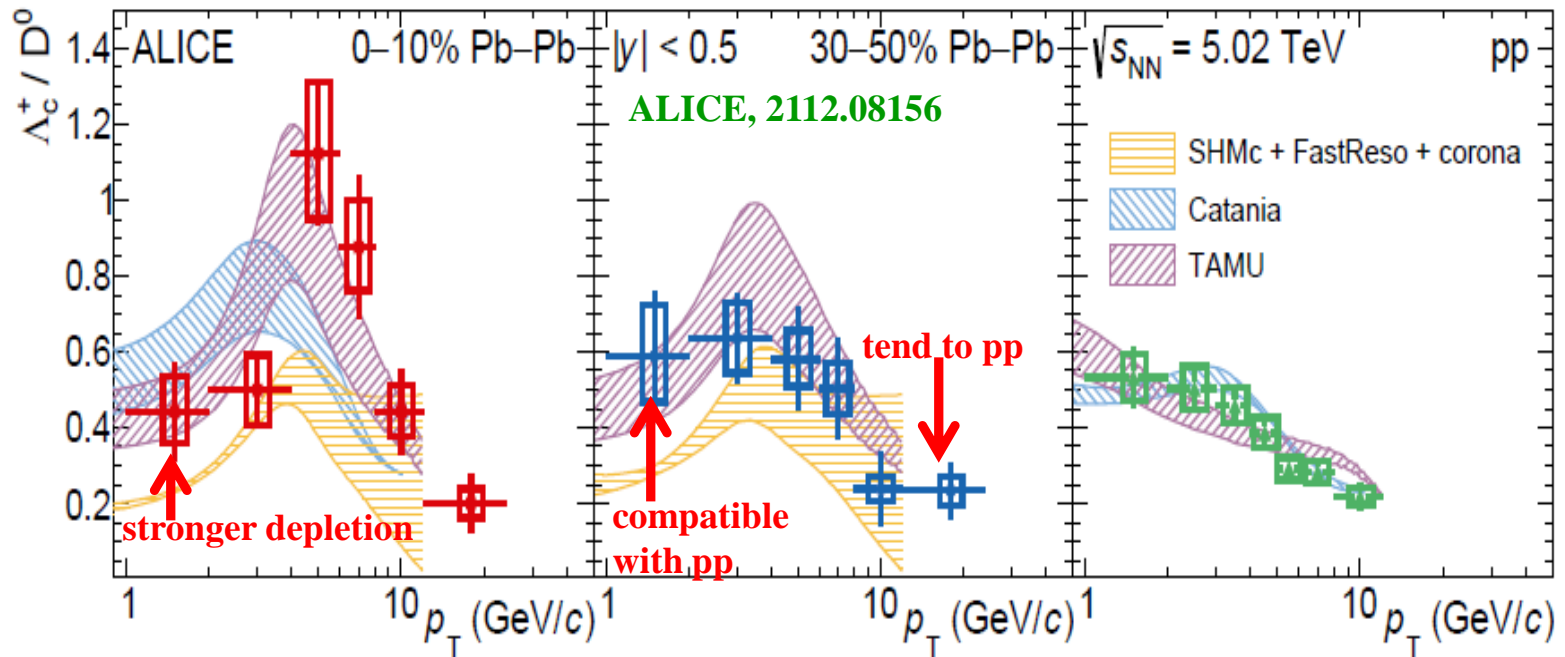


ALICE, JHEP01(2022)174; PLB 813(2021)136054

Model	χ^2/ndf	
	R_{AA}	v_2
Catania [6, 7]	143.8/30	14.0/8
DAB-MOD [9]	234.1/30	9.8/6
LBT [10, 11]	411.8/30	15.8/12
LIDO [13]	46.4/26	62.0/11
LGR [12]	9.2/30	15.5/11
MC@sHQ+EPOS2 [8]	56.6/30	5.7/12
PHSD [5]	294.7/30	19.6/11
POWLANG-HTL [3, 4]	468.6/30	13.5/8
TAMU [2]	30.2/30	8.15/9

- models with $\chi^2/\text{ndf} < 5$ (2) for R_{AA} (v_2)
 $\rightarrow \mathcal{D}_s(2\pi T) = 1.5-4.5$ near T_c
- caveat: also affected by hadronization, hydro, hadronic phase

Charm hadro-chemistry: Λ_c/D^0

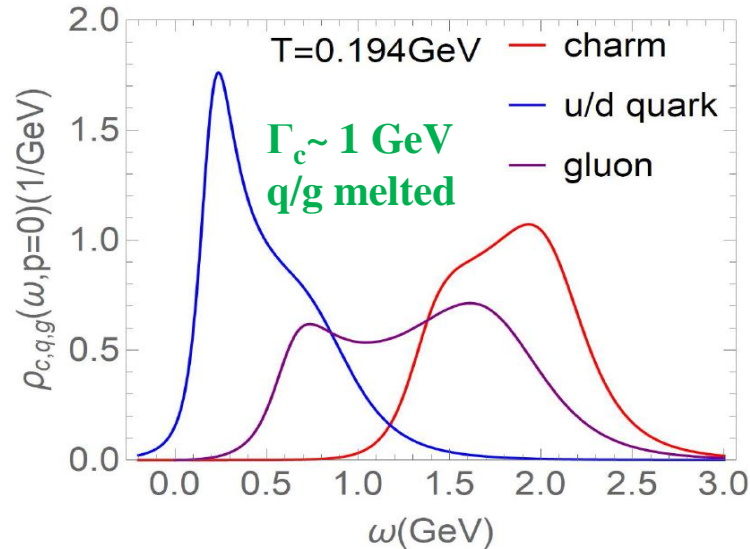


- Catania: instantaneous coalescence + fragmentation Plumari et al. '18
- SHMc: hydrodynamic blast wave spectrum on PDG-only baryons + corona pp Andronic et al. '21
- TAMU: RQM charm-baryons + RRM w/ SMCs
integrated ratio compatible with pp MH & Rapp '20

Take-aways from Part II

❖ Heavy-quark diffusion

- $\mathcal{D}_s(2\pi T)=1.5-4.5$ near $T_c \rightarrow$ scattering rate $\Gamma_{\text{coll}} \sim 3/\mathcal{D}_s \sim 1 \text{ GeV} > M_{q,g}$
 \rightarrow thermal partons melt, Brownian markers survive
- strong coupling via remnants of confining force



❖ Heavy-quark hadronization

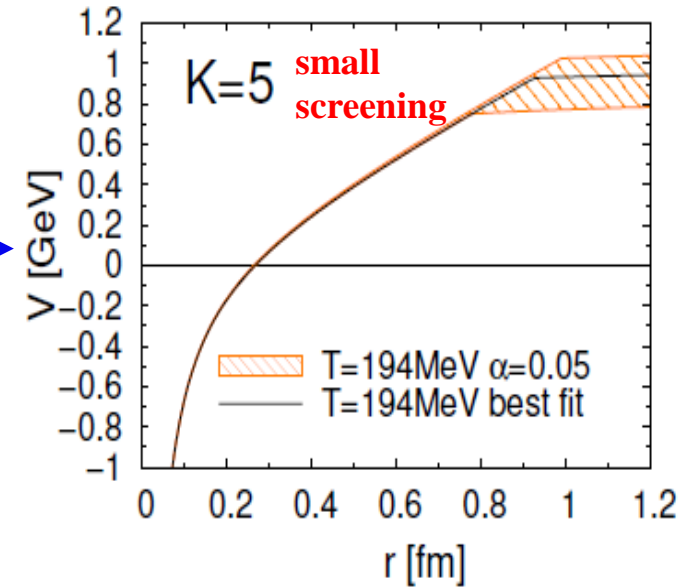
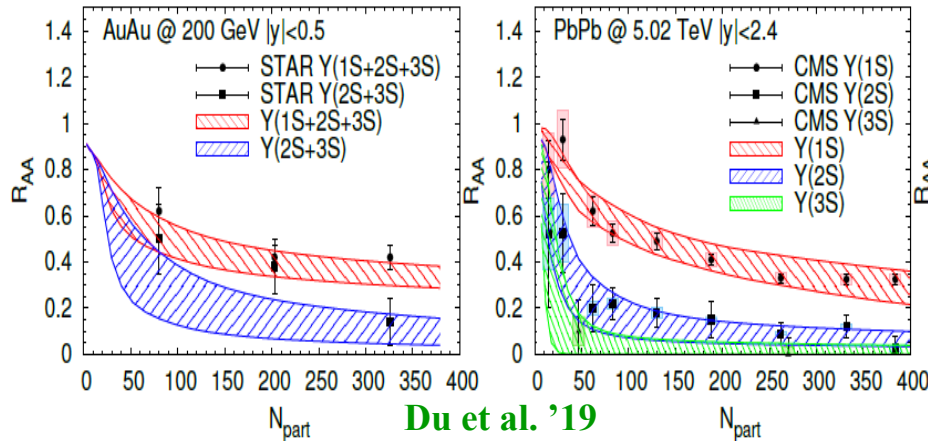
- recombination \rightarrow p_T -dependent charm hadro-chemistry
- p_T -integrated Λ_c/D^0 compatible with pp \rightarrow kinematic redistribution in p_T in AA

Part III: Heavy quarkonium production in AA

- Strong HQ potential
- Semi-classical approach: suppression vs regeneration
- J/ψ v_2 puzzle
- Open quantum system approach to Y states

Extraction of HQ potential

- statistical transport analysis of Y data with trial input potential

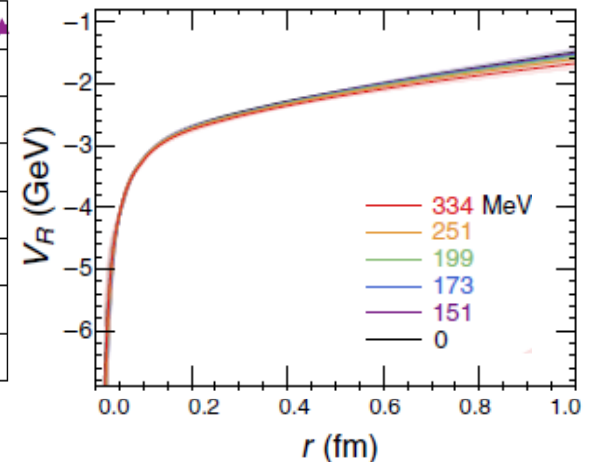
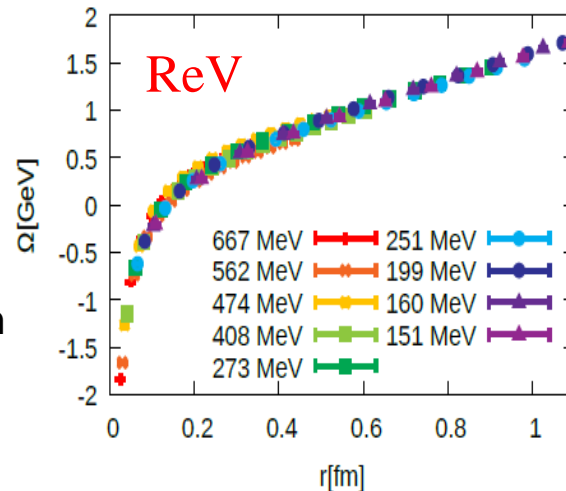


- lattice QCD extraction of static potential

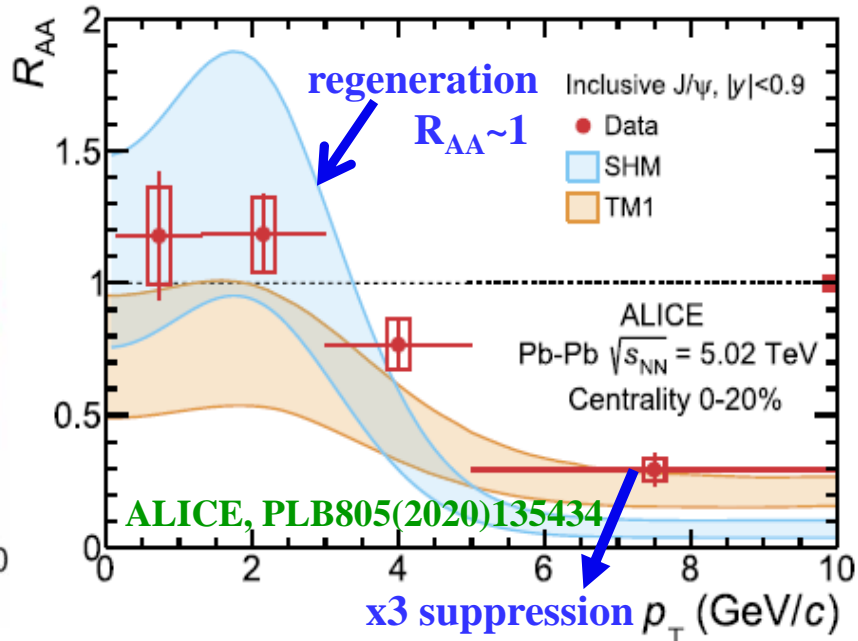
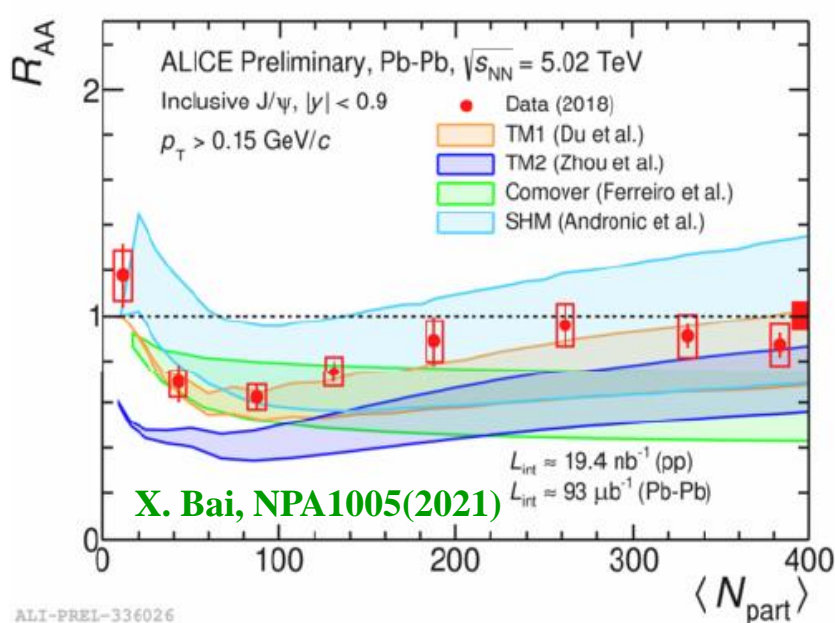
Bala et al., 2110.11659

- neural network approach

Shi et al. '22



J/ψ: suppression vs regeneration



- semi-classical transport **Du et al. '15, Zhou et al. '14, Ferreiro et al. '14**

$$\frac{dN_{\Psi}(\tau(T))}{d\tau} = -\Gamma_{\Psi}(T)[N_{\Psi}(\tau(T)) - N_{\Psi}^{\text{eq}}(T)]$$

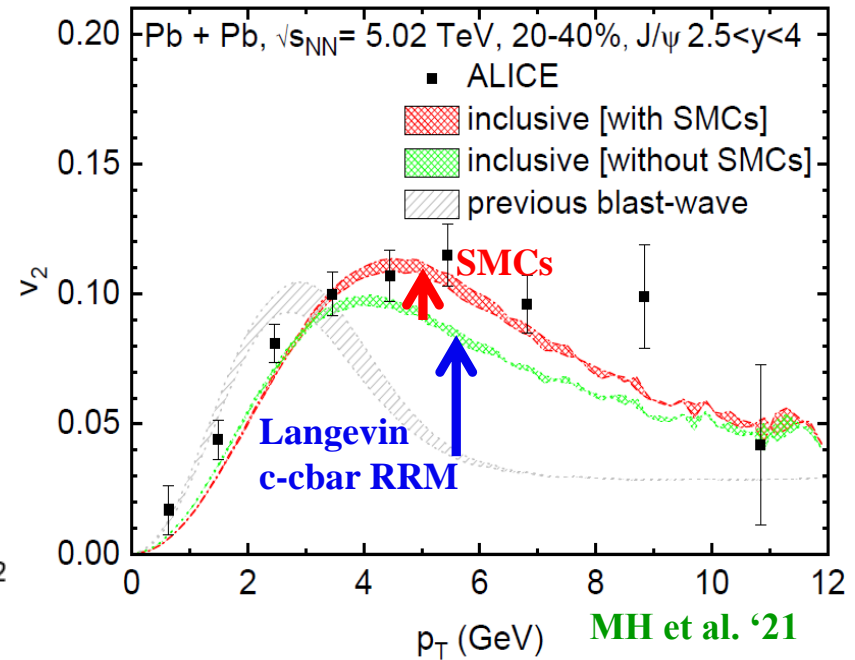
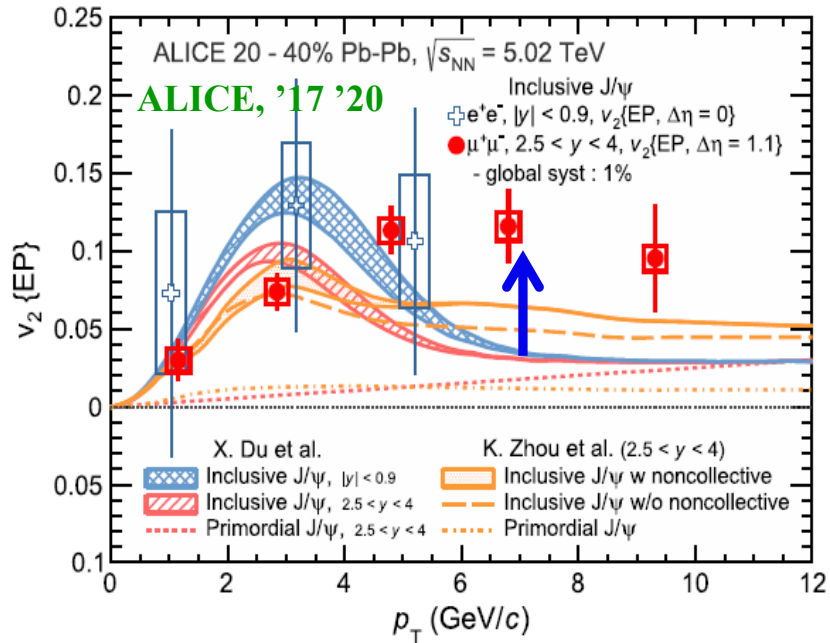
reaction rate Γ_{Ψ}

regeneration toward equilibrium

- SHMc: hydrodynamic blastwave spectrum + pp corona **Andronic et al. '19**

$$\frac{dN(h_{hc}^j)}{dy} = g_c^2 V n_j^{\text{th}} \propto g_c^2 \leftarrow d\sigma^{\text{ccbar}}/dy$$

J/ψ “v₂ puzzle”



- regeneration via RRM

$$f_{\Psi}(\vec{x}, \vec{p}) = C_{\Psi} \frac{E_{\Psi}(\vec{p})}{m_{\Psi} \Gamma_{\Psi}} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2}{(2\pi)^3} \underbrace{f_c(\vec{x}, \vec{p}_1) f_{\bar{c}}(\vec{x}, \vec{p}_2)}_{\text{transported } c \text{ \& } \bar{c} \text{ quark spectra}} \times \sigma_{\Psi}(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

transported c & \bar{c} quark spectra
constrained by D-meson observables

- off-equilibrium c/\bar{c} spectra + space-momentum correlations (SMCs)
→ regeneration up to $p_T \sim 8$ GeV → v_2 enhanced

Open quantum system approach to Y states

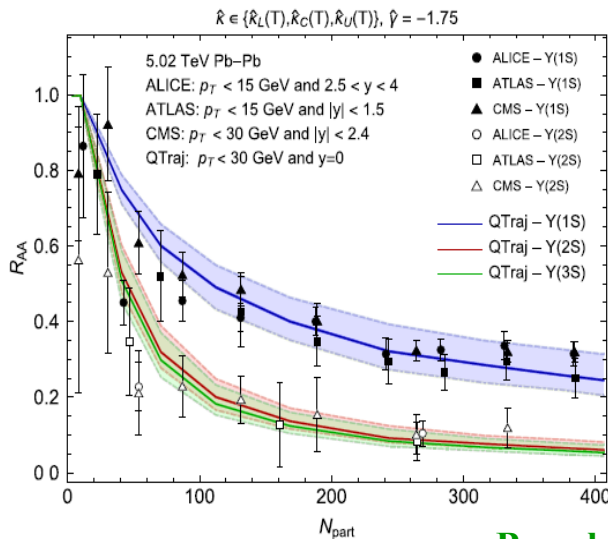
❖ OQS + pNRQCD \rightarrow Lindblad equation

Brambilla et al. '17-21, Yao et al., '21, Blaizot '18
Akamatsu '21, Rothkopf '20, Gossiaux et al. '21

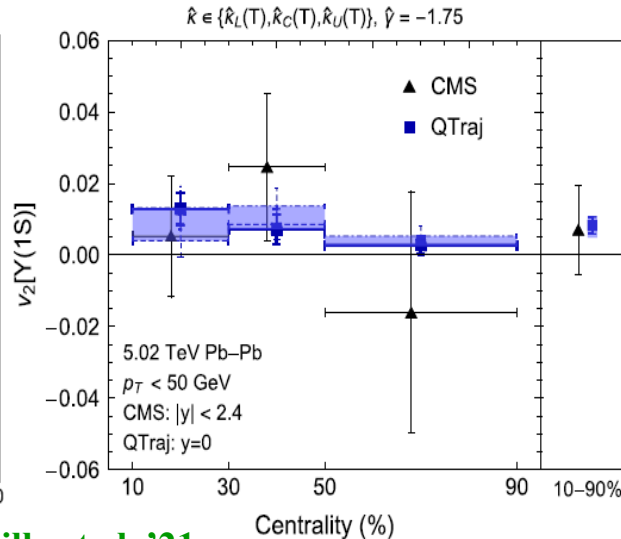
$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_n \left(C_n \rho(t) C_n^\dagger - \frac{1}{2} \{ C_n^\dagger C_n, \rho(t) \} \right)$$

$$M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E$$

- quantum transition between different states included, lacking in semi-classical
- regeneration currently limited to diagonal $b\bar{b}$
- Coulomb potential + **transport coefficient κ** encoded in C_n



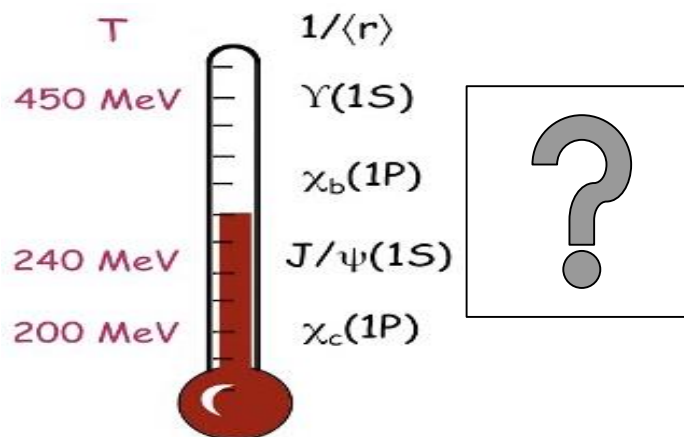
Brambilla et al. '21



- **Y(1S) in-medium width**
 $\Gamma_{Y(1S)} = 3a_0^2 \kappa \sim 50$ MeV at $T \sim 250$ MeV
- values & results comparable to semi-classical approach
Strickland et al. '15

Take-aways from Part III

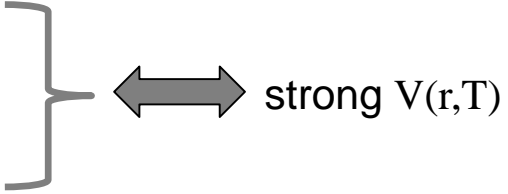
- ❖ strong HQ potential with little screening close to T_c
quarkonia melt through large reaction rates (\leftrightarrow small HQ \mathcal{D}_s)
 - probe of in-medium force via in-medium “spectroscopy”, not “thermometer”



- ❖ Quantitative connections open- \leftrightarrow hidden-charm transport
 - transported c/\bar{c} distributions & $d\sigma^{c\bar{c}}/dy$

Summary & outlook

- ❖ HFs: excellent probes of sQGP structure, transport properties, in-medium force & hadronization

- a small open HF diffusion coefficient \mathcal{D}_s
 - recombination/color neutralization important
 - quarkonia melting by large reaction rates
- 
- strong $V(r,T)$

➔ connection between open- & hidden-HF, e.g. via J/ψ regeneration

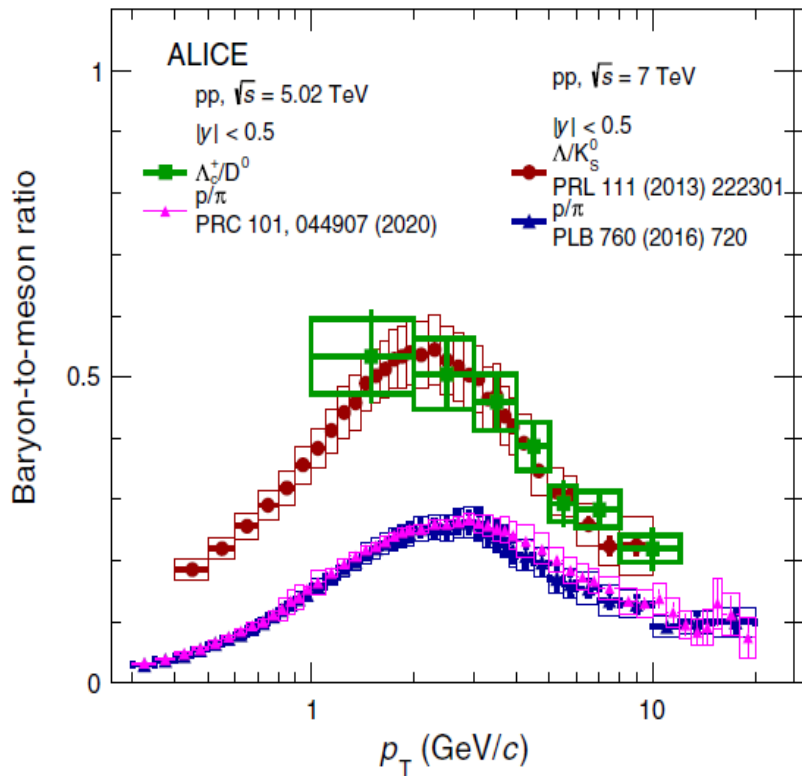
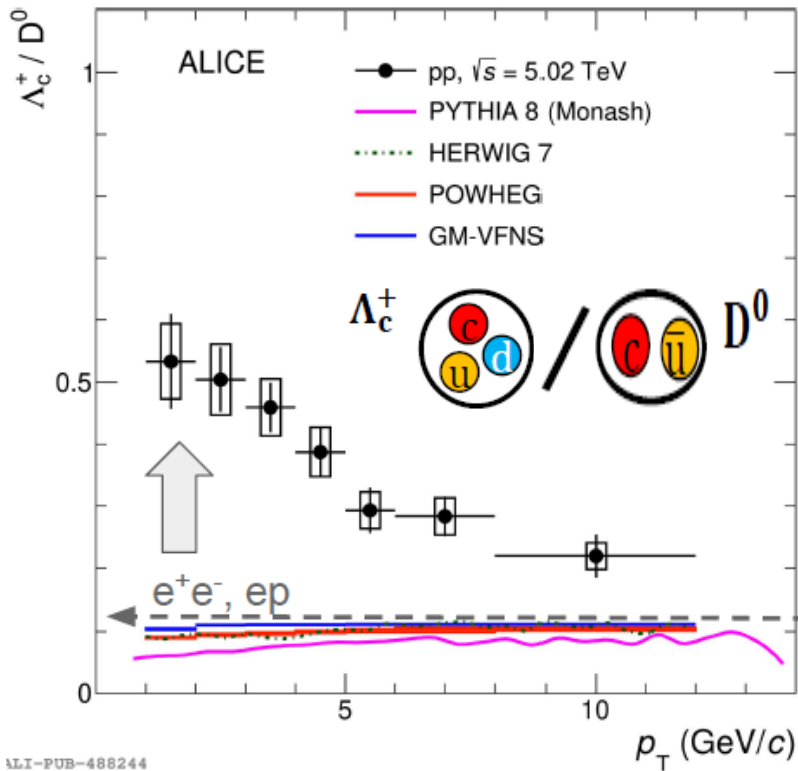
❖ outlook

- Ξ_c production in pp
- p-dependence of \mathcal{D}_s :
nonperturbative diffusion → perturbative radiative e-loss?
- more ...

Back-up

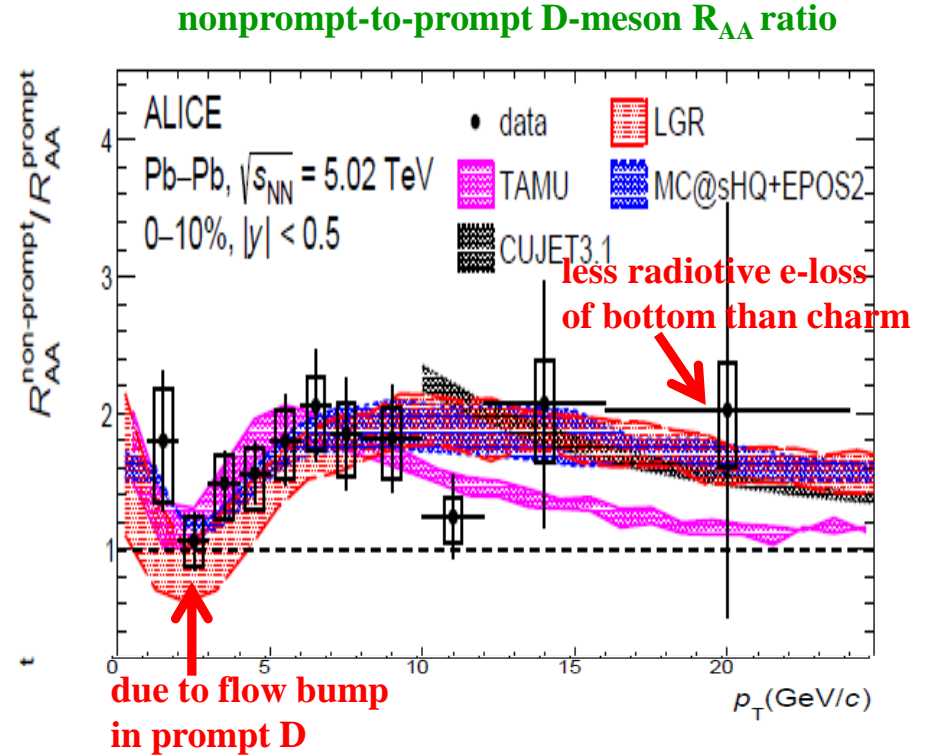
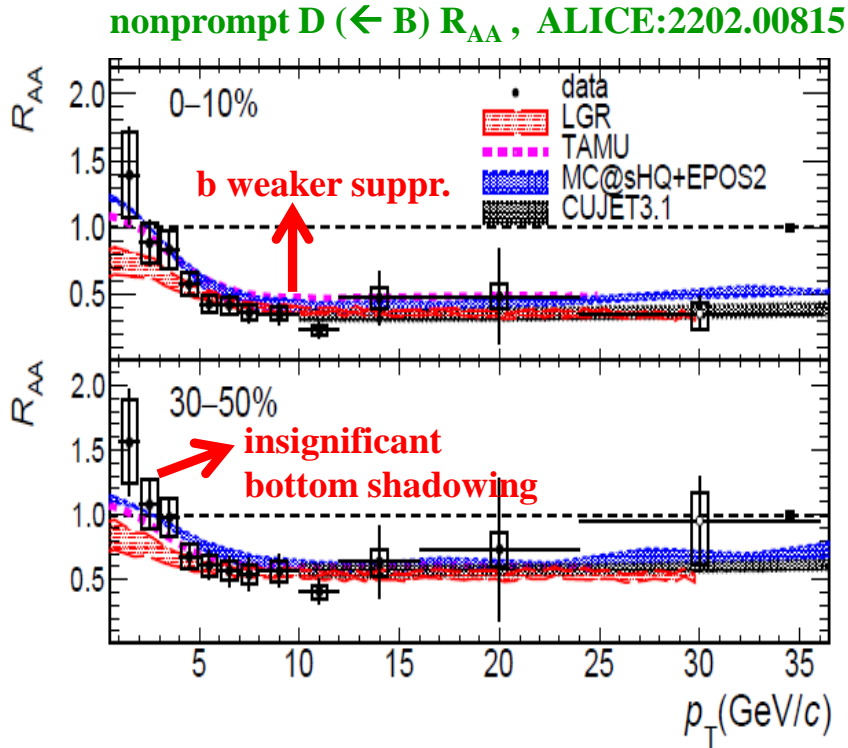
The following are back-up pages

Λ_c^+ / D^0 enhancement surprise



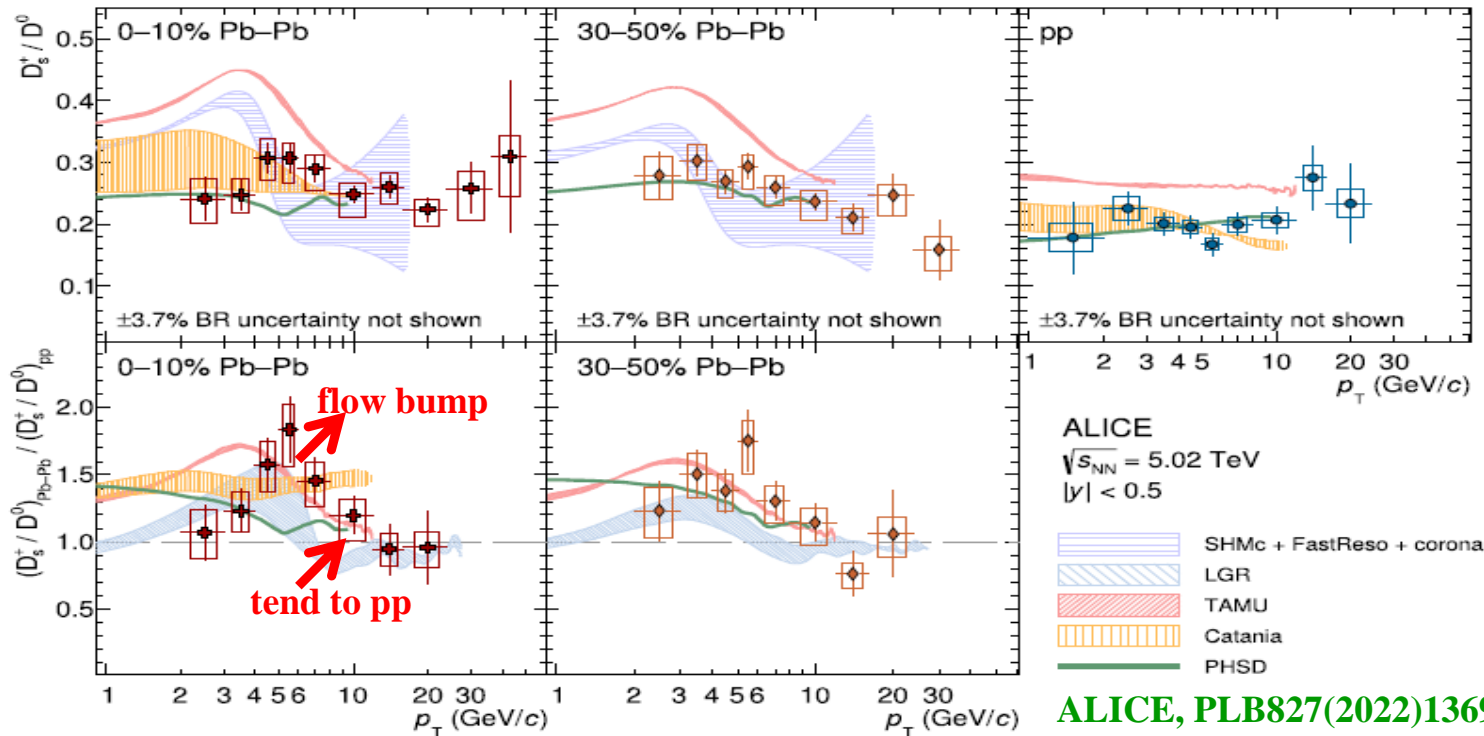
- ❖ a factor ~ 5 enhancement w.r.t. e^+e^- at low p_T , much underestimated by FFs tuned to e^+e^-
- ❖ decreasing toward high p_T , trend similar to Λ/K and p/π

Flavor dependence: charm vs bottom



- ❖ x3 mass: b-quark longer thermalization time at low p_T than charm
less flow added to b from recombination with u/d/s
- ❖ high $p_T > 15$ GeV: b-quark less radiative e-loss \leftarrow stronger “dead cone”

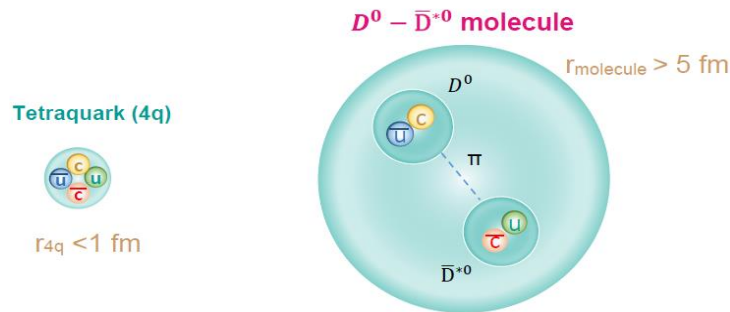
Charm hadro-chemistry: D_s/D^0



- low p_T : enhancement due to charm recombination in a strangeness-equilibrated QGP reproduced by Catania & PHSD; overestimated by TAMU in both pp and PbPb
- high p_T : tending to pp value as fragmentation takes over
- flow bump due to recombination with flowing s-quark heavier than u/d, predicted by TAMU (RRM w/ SMCs) & SHMc (hydro blastwave spectrum)

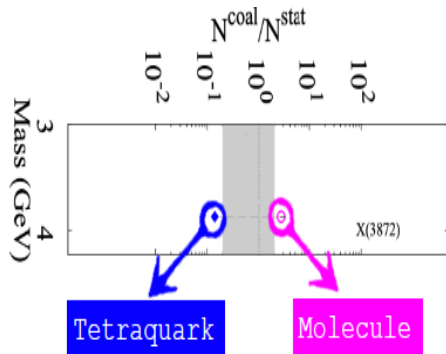
X(3872) production in HIC

❖ inner structure: compact tetraquark vs loosely bound molecule

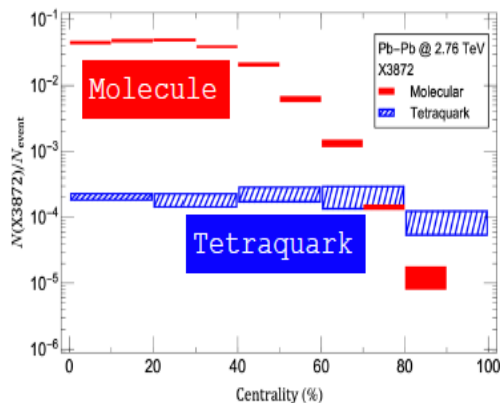


coalescence model

Cho et al. '11

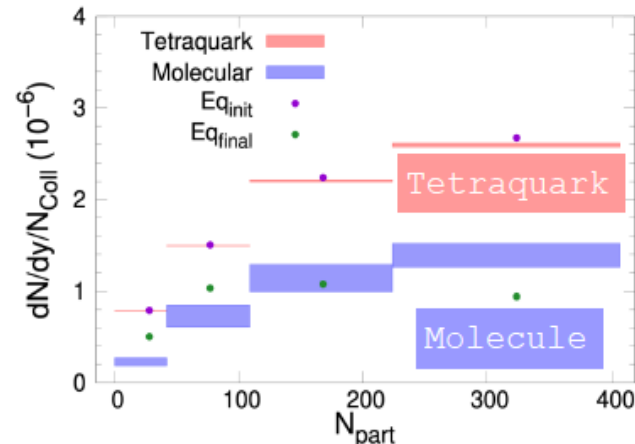


coalescence within AMPT zhang et al. '21



transport model

Wu et al. '21



- $N_{\text{molecule}} > N_{\text{tetraquark}}$ by x10 or 100, yet no account of hadron phase reactions $\pi X \leftrightarrow DD^*$
 → to be better constrained

- $N_{\text{tetraquark}} > N_{\text{molecule}}$ by x2, molecule regenerated in late hadronic phase, tetraquark chem. freezeout at T_c

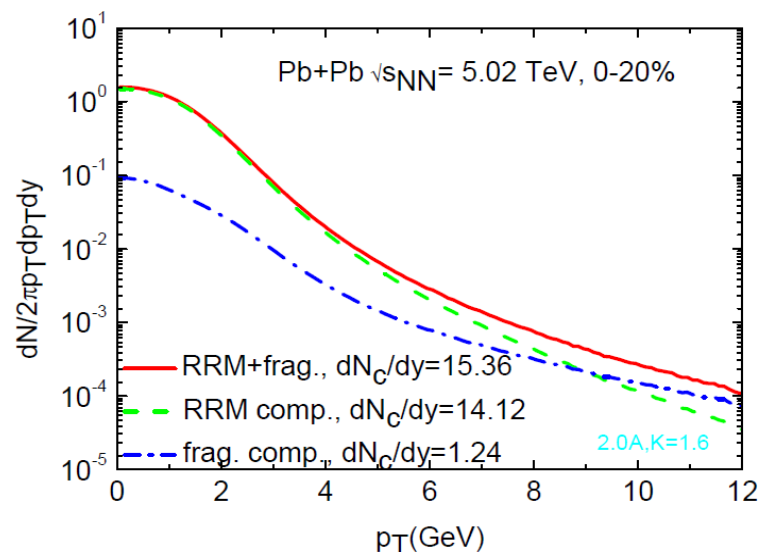
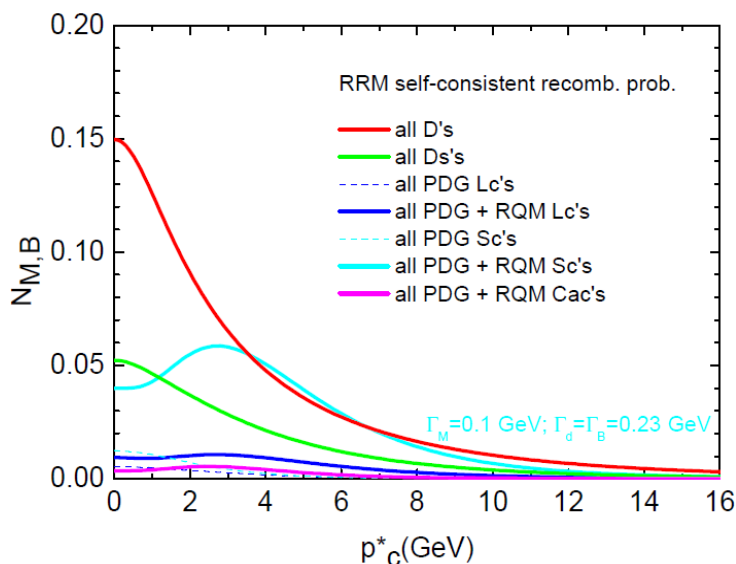
Charm quark recombination probability

- No. of mesons/baryons formed from a single c-quark of rest frame p_c^*

$$N_M(p_c^*) = \int \frac{d^3 \vec{p}_1^*}{(2\pi)^3} g_q e^{-E(\vec{p}_1^*)/T_{pc}} \frac{E_M(\vec{p}^*)}{m_M \Gamma_M} \sigma(s) v_{rel},$$

$$N_B(p_c^*) = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} g_1 e^{-E(\vec{p}_1)/T_c} g_2 e^{-E(\vec{p}_2)/T_c} \frac{E_d(\vec{p}_{12})}{m_d \Gamma_d} \sigma(s_{12}) v_{rel}^{12}(\vec{p}_1, \vec{p}_2) \frac{E_B(\vec{p})}{m_B \Gamma_B} \sigma(s_{d3}) v_{rel}^{d3}(\vec{p}_{12}, \vec{p}_{30}),$$

- Renormalizing $N_M(p_c^*)$ and $N_B(p_c^*)$ by a **common** factor ~ 4 for all charmed mesons/baryons such that $\sum_M P_{coal,M}(p_c^* = 0) + \sum_B P_{coal,B}(p_c^* = 0) = 1$



➔ **charm conservation** consistently built in, in an (e-by-e) way without spoiling the relative chemical equilibrium realized by RRM

Space-momentum correlations: light-q

- hydro: a manifestation of SMCs

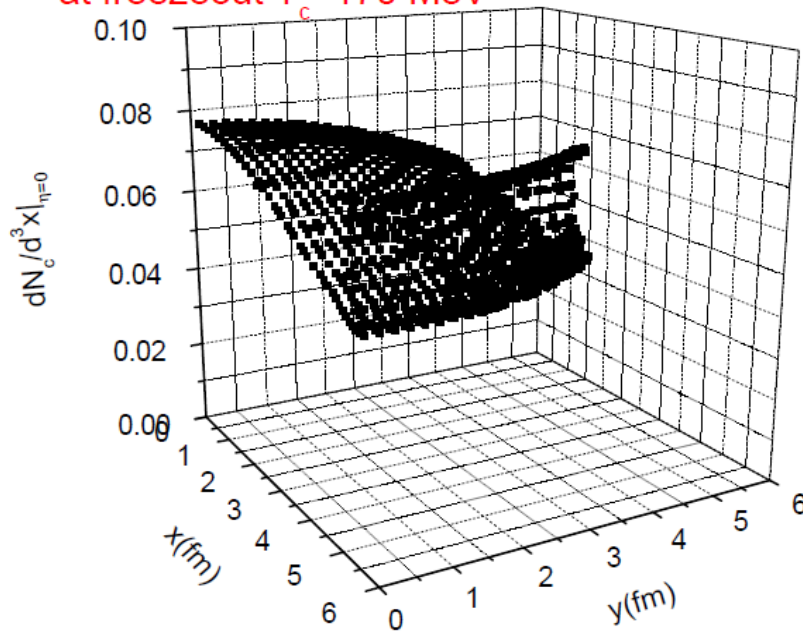
$$f_q^{eq}(\vec{x}, \vec{p}) = g_q e^{-p \cdot u(x)/T(x)} = g_q e^{-\gamma_T(x)[m_T \cosh(y-\eta) - \vec{p}_T \cdot \vec{v}_T(x)]/T(x)}$$

longitudinal boost invariance: $y - \eta$

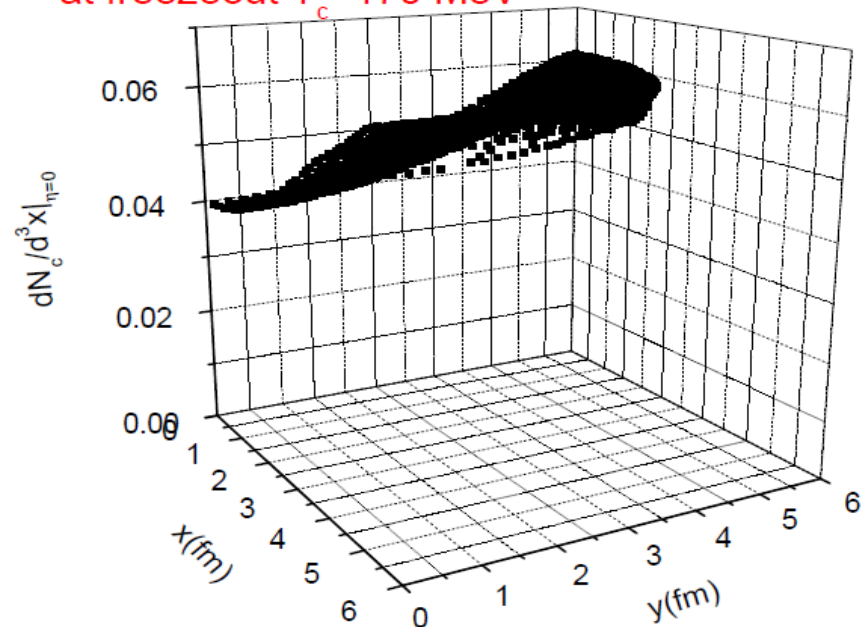
transverse SMCs $p_T \cdot v_T$

- hydro-q: low (high) p_T more concentrated in center (boundary)

hydro light quarks $p_T = 0.0 - 0.3$ GeV,
at freezeout $T_c = 170$ MeV



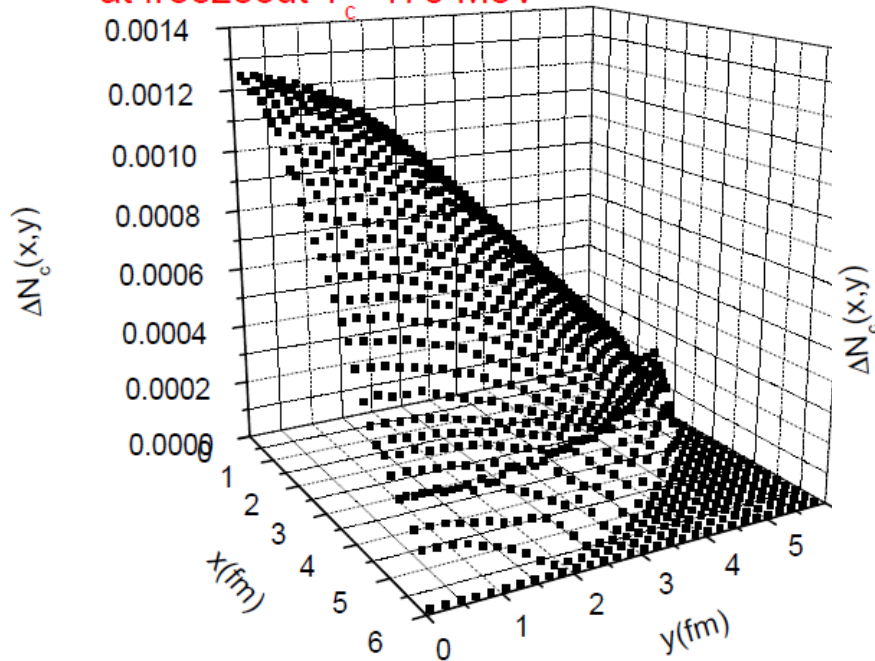
hydro light quarks $p_T = 0.6 - 0.9$ GeV,
at freezeout $T_c = 170$ MeV



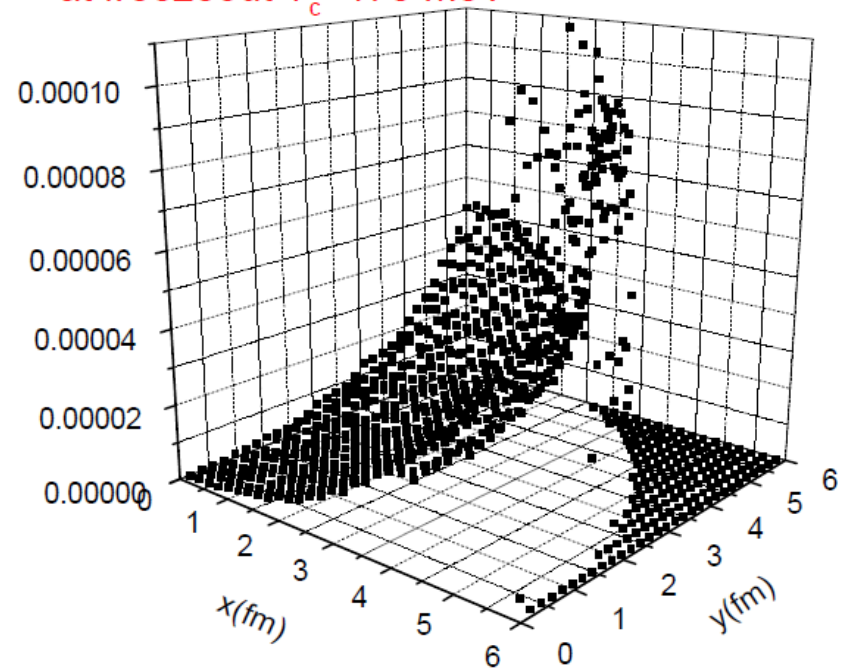
SMCs: Langevin charm quarks

- Langevin-c: low (high) p_T more populated in central (outer)

Langevin charm quarks $p_T=0.0-1.0$ GeV,
at freezeout $T_c=170$ MeV



Langevin charm quarks $p_T=3.0-4.0$ GeV,
at freezeout $T_c=170$ MeV

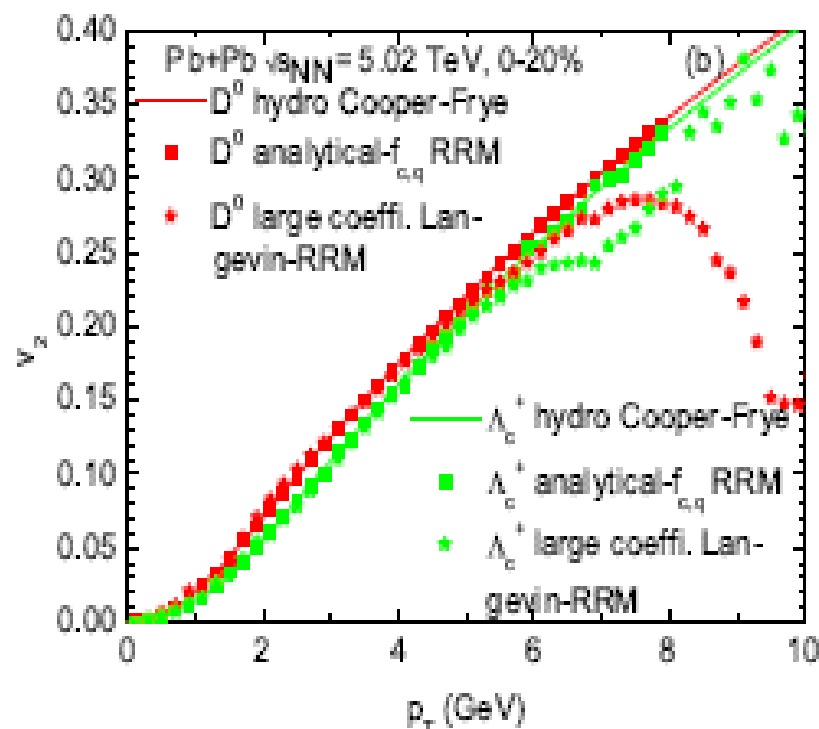
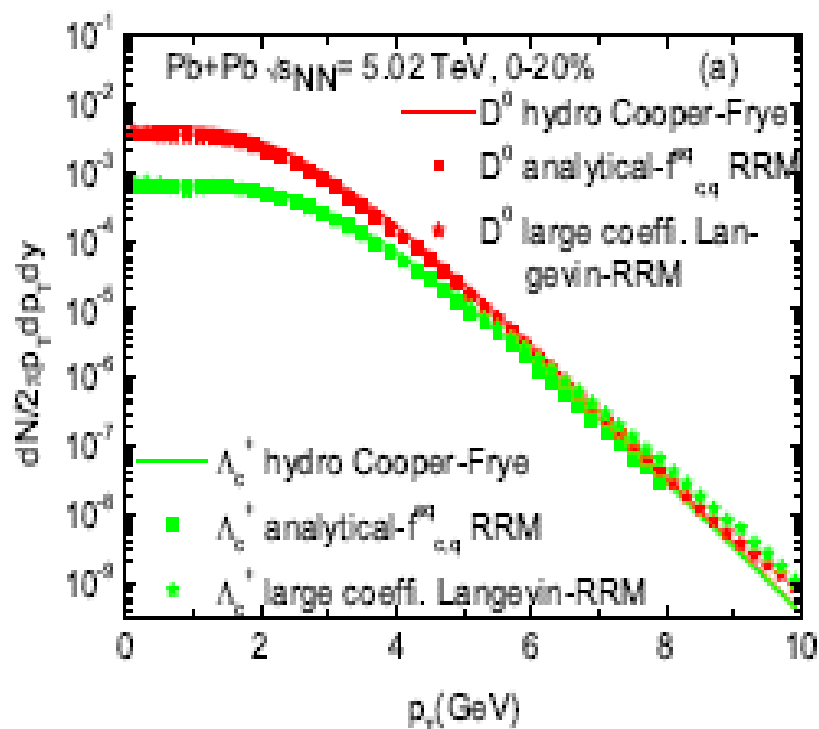


- SMCs usually neglected in ICMs: uniformly distributed independent of p_T

$$f_{c,q}(\vec{x}, \vec{p}) = (2\pi)^3 \frac{dN_{c,q}}{d^3\vec{x}d^3\vec{p}} = \frac{(2\pi)^3}{V E(\vec{p})} \frac{dN_{c,q}}{p_T dp_T d\phi_q dy}$$

RRM equilibrium mapping

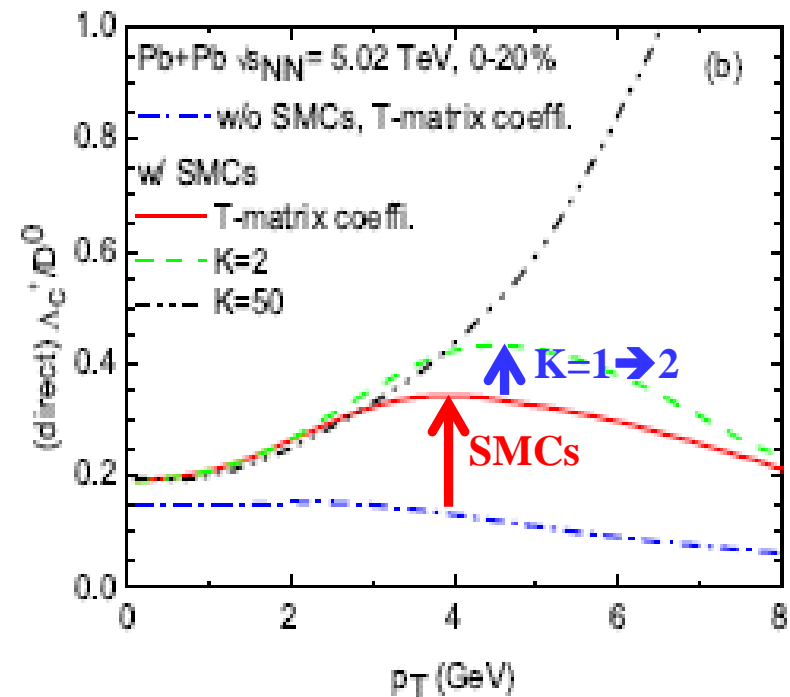
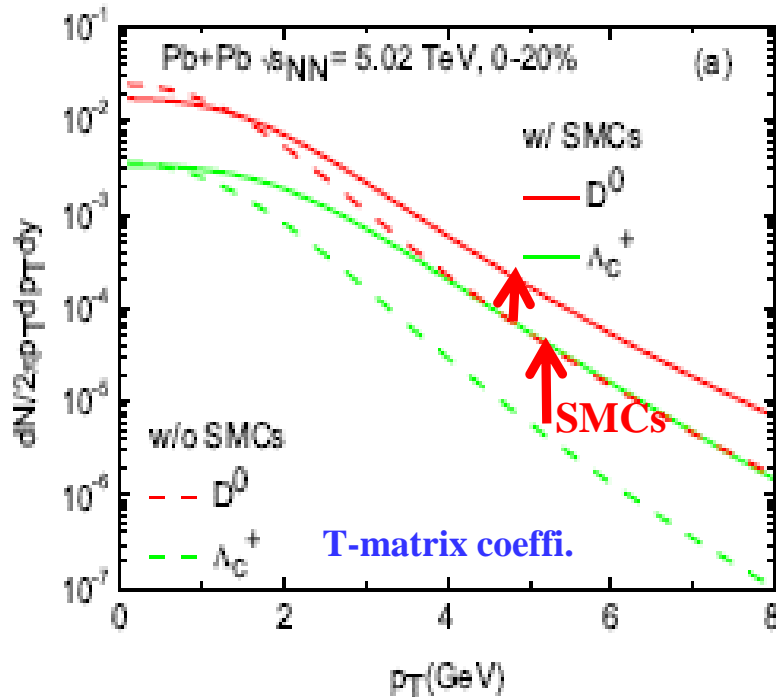
- Event-by-event Langevin-RRM simulation with **very large trans. coeffi.** & with **SMCs** properly incorporated
- ➔ kinetic & chemical equil. mapping



➔ Observables come out as RRM predictions with realistic T-matrix coeffi.

Direct D^0 & Λ_c^+ production via RRM

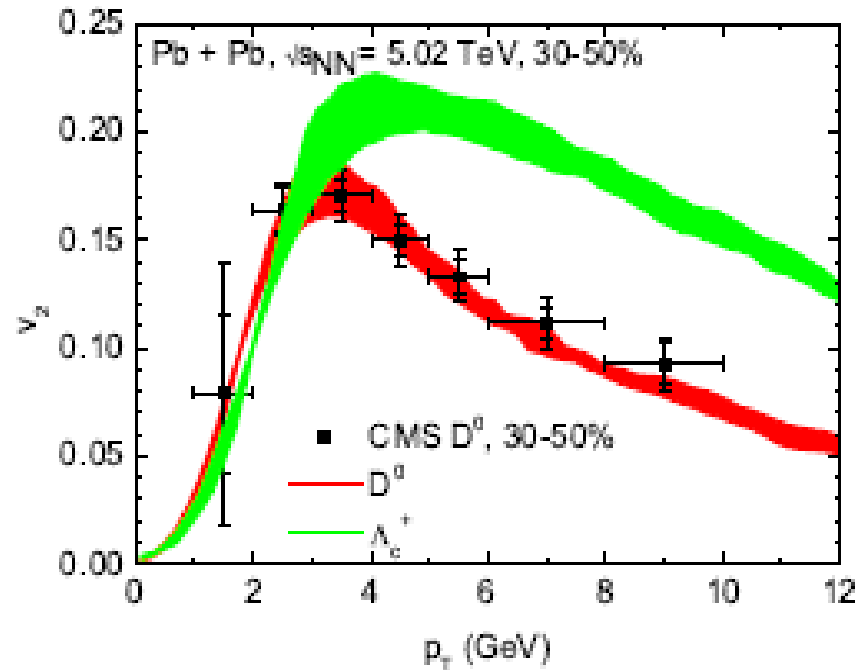
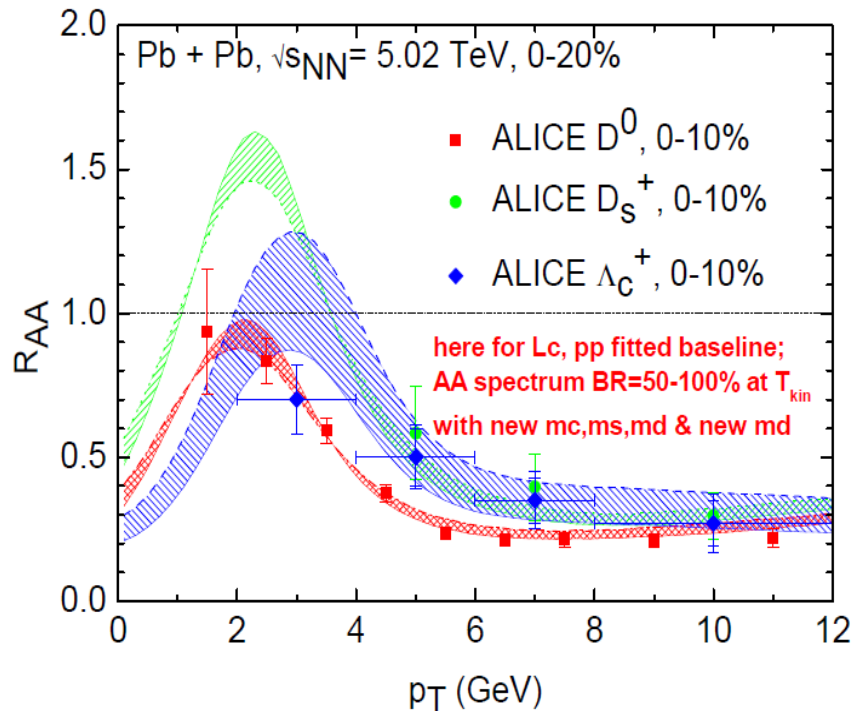
- Including SMCs makes spectra **harder** & **enhances** the Λ_c^+/D^0



- Fast-moving c-quarks [$p_T \sim 3-4$ GeV] moving to outer part of fireball find higher-density of harder [$p_T \sim 0.6-0.9$ GeV] light quarks for recombination
- An effect entering **squared** for the recombination production of Λ_c^+
➔ **larger enhancement for $\Lambda_c^+ \rightarrow \Lambda_c^+/D^0$ ratio enhanced!**

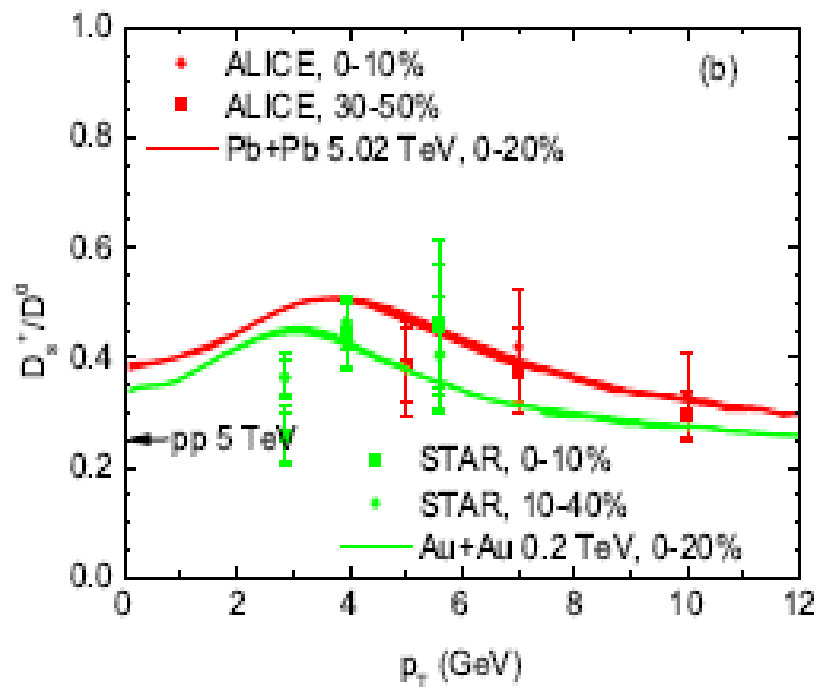
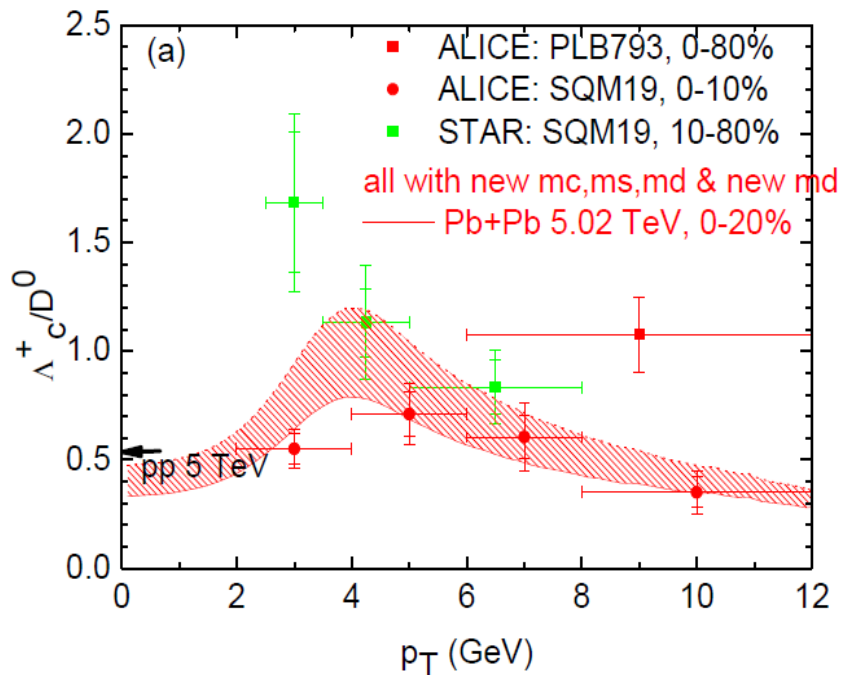
D^0 , D_s^+ & Λ_c^+ suppression & elliptic flow

➤ Final D^0 , D_s^+ & Λ_c^+ , including feeddowns from all RQM baryons



- T-matrix coefficient*K-factor(=1.6), to compensate for radiative e-loss; uncertainty: BR=50-100% to Λ_c^+ for Λ_c^+ 's & Σ_c^+ 's above DN (2805 MeV)
- Hadronic phase diffusion also included: seamlessly connected to hadronization (RRM+frag), increasing D-meson v_2 by $\sim 15\%$

Charm-hadron ratios: Λ_c^+/D^0 & D_s^+/D^0

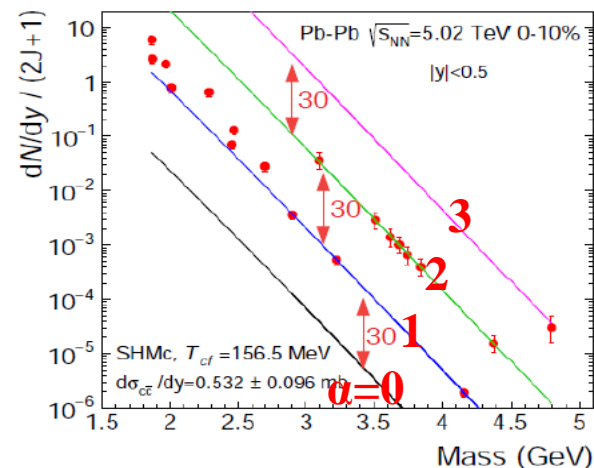


- Λ_c^+/D^0 : low p_T approaching RRM equil. limit = SHM pp; intermediate p_T enhancement from RRM with SMCs; high p_T fragmentation tending to pp value
- D_s^+/D^0 enhancement: recombination of charm in a strangeness-equilibrated QGP

Hadronization: SHMc Andronic, PBM et al. 2104.12754

- SHMc: open-charm statistical hadronization at T_c

$$\frac{dN(h_{oc,\alpha}^i)}{dy} = g_c^\alpha V n_i^{\text{th}} \frac{I_\alpha(N_c^{\text{tot}})}{I_0(N_c^{\text{tot}})}$$
 - ❑ multicharm baryons $\alpha=1,2,3$ emerging pattern
 - ❑ yields enhanced by $g_c^\alpha \sim 30^\alpha$ than pure thermal \rightarrow strong signal of deconfinement



- SHMc yields + blast wave $\rightarrow p_T$ spectra

