Bayesian inference of the fluctuating proton shape

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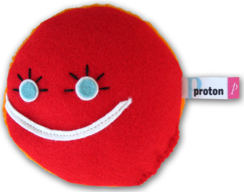
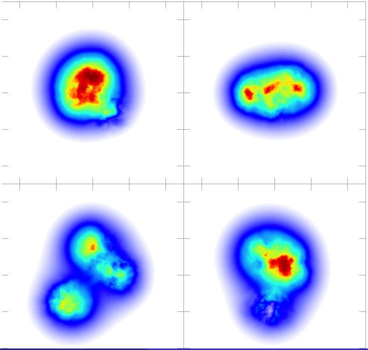
Going beyond a round proton

Motivation

- Initial state geometry fluctuations have a large effect on e.g. flow
- How does the proton geometry fluctuate event-by-event?
- How accurately can we constrain the proton shape fluctuations, and how do these uncertainties propagate from HERA to flow@LHC
  $\Rightarrow$ Bayesian analysis



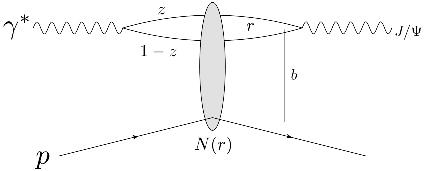
$\Longrightarrow$



Fluctuations and diffractive vector meson production

High energy factorization:

1. $\gamma^* \to q\bar{q}$ splitting, wave function $\Psi^\gamma(r, Q^2, z)$
2. $q\bar{q}$ dipole scatters elastically: $N(r, x, b)$
3. $q\bar{q} \to J/\Psi$, wave function $\Psi^V(r, Q^2, z)$



Diffractive scattering amplitude

$$\mathcal{A} \sim \int \mathrm{d}^2 b \mathrm{d}z \mathrm{d}^2 r \Psi^{\gamma*}\Psi^V(r, Q^2, z) e^{-i\mathbf{b}\cdot\mathbf{\Delta}} N(r, x, b)$$

- Impact parameter is the Fourier conjugate to the momentum transfer $(-t \approx \Delta^2) \to$ **access to the spatial structure**

Diffractive vector meson production

Coherent

$$\sigma_{\text{coherent}} \sim |\langle \mathcal{A} \rangle_\Omega|^2$$

- Proton stays intact

Probes the average interaction
$\Rightarrow$ average shape

Incoherent

$$\sigma_{\text{incoherent}} \sim \langle |\mathcal{A}|^2 \rangle_\Omega - |\langle \mathcal{A} \rangle_\Omega|^2$$

- Proton dissociates

Event-by-event fluctuations in the proton structure

- Experimental signature: rapidity gap
- Theoretically: no net color transfer
- Average over target configurations Ω at amplitude/cross section level

$$\mathcal{A}^{\gamma^* p \to Vp} \sim \int \mathrm{d}^2\mathbf{b}\,\mathrm{d}z\,\mathrm{d}^2\mathbf{r}\,\Psi^{\gamma*}\Psi^{V}(|\mathbf{r}|, z, Q^2)\mathbf{e}^{-\mathbf{i}\mathbf{b}\cdot\mathbf{\Delta}} N(|\mathbf{r}|, x, \mathbf{b}, \Omega)$$

Miettinen, Pumplin, PRD 18, 1978; Caldwell, Kowalski, 0909.1254; H.M, Schenke, 1603.04349; H.M, 2001.10705

Bayesian analysis: parametrization

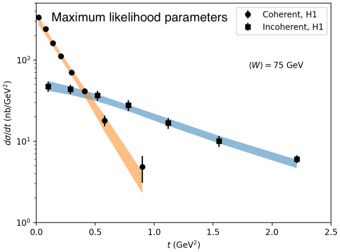
Parametrize geometry

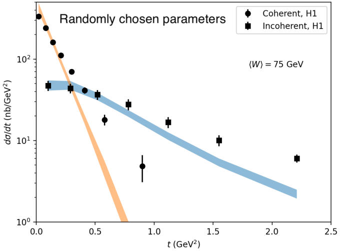
- Number of hot spots N_q
- Proton size (Gaussian width) B_{qc}
- Hot spot size $\sim \exp\left[-b^2/(2B_q)\right]$
- Hot spot density fluctuations: width σ
- Overall color charge density: $Q_s(\mathbf{x})/g^2\mu$
- Infrared regulator m
- Min distance between hot spots $d_{q,\min}$

Dipole amplitude N: MV model on a lattice, similar setup as e.g. in H.M, Schenke, 1603.04349

HERA data: arXiv:1304.5162, web interface to study different parametrizations: click here

$$\gamma^* + p \to \mathrm{J}/\psi + p$$





Bayesian analysis

Bayes' theorem

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{y}_{\rm exp}) \propto \mathcal{P}(\mathbf{y}_{\rm exp}|\boldsymbol{\theta})\mathcal{P}(\boldsymbol{\theta}).$$

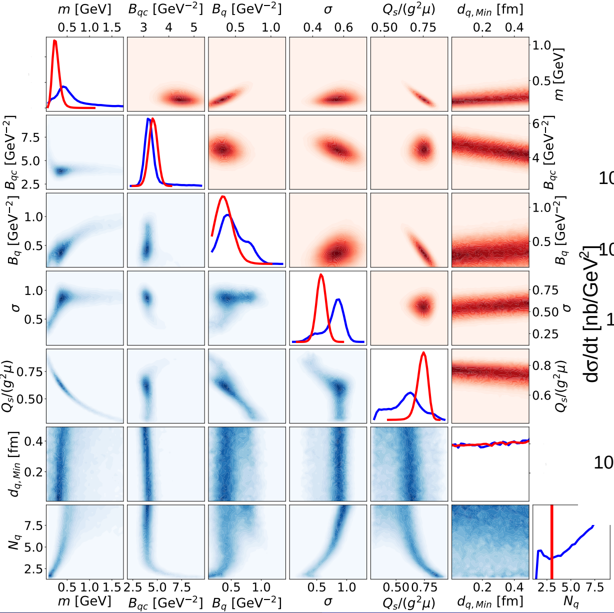
- $\mathcal{P}(\boldsymbol{\theta}|\mathbf{y}_{\rm exp})$: Posterior distribution of model parameters $\boldsymbol{\theta}$, given the data $\mathbf{y}_{\rm exp}$ (HERA) – need this!
- $\mathcal{P}(\mathbf{y}_{\rm exp}|\boldsymbol{\theta})$: likelihood for model with params $\boldsymbol{\theta}$ to describe the data
- Prior likelihood for parametrization $\mathcal{P}(\boldsymbol{\theta})$ (assume uniform)
- Posterior can be used to compute other observables:

$$\langle \mathcal{O} \rangle \sim \int \mathrm{d}\boldsymbol{\theta} \mathcal{P}(\boldsymbol{\theta}|\mathbf{y}_{\rm exp}) \mathcal{O}_{\boldsymbol{\theta}}$$

- Error propagation also straightforward

Technical detail: use Gaussian Process Emulators and Markov Chain Monte Carlo to effectively explore the 7 dimensional parameter space.

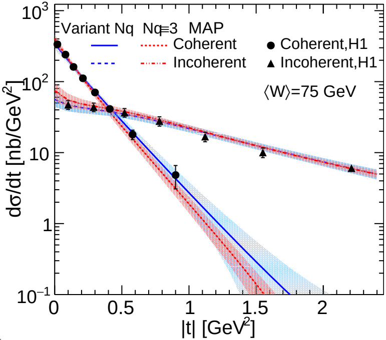
Results ($N_q \equiv 3$ and free N_q)



Two setups:

$N_q \equiv 3$ or free

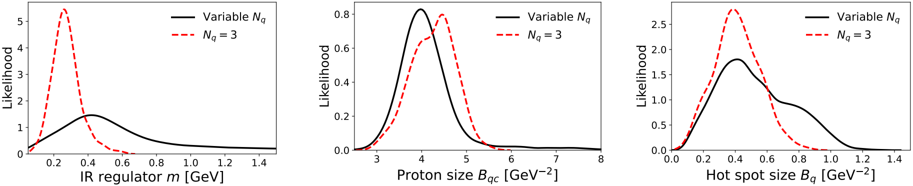
Similar description of the data



H.M, Schenke, Shen, Zhao, arXiv:2203.0584

Selected parameter likelihoods

Most parameters can be constrained accurately

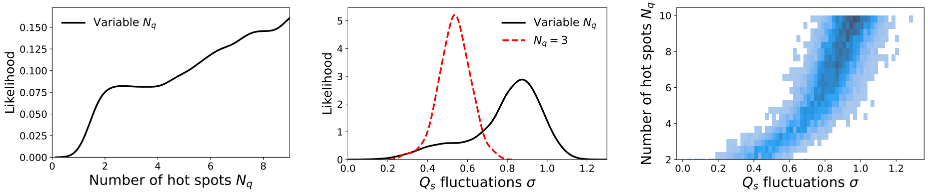


- Somewhat wider distributions if the number of hot spots N_q is free

Constrained by different parts of the J/ψ spectra

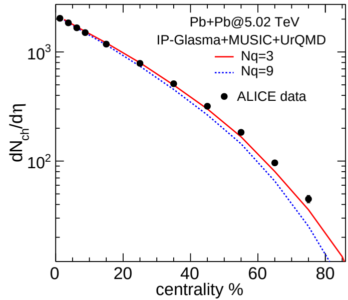
- IR regulator: coherent spectra at small $t \leftrightarrow$ large b H.M, Schenke, 1607.01711
- Proton size: slope of the coherent spectra
- Hot spot size: slope of the incoherent spectra T. Lappi, H.M, 1011.1988
- Weak correlations between these parameters

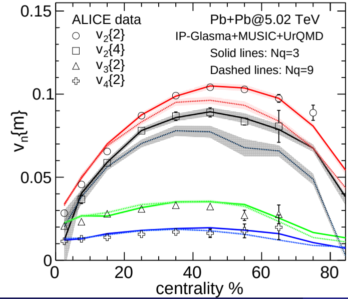
Number of hot spots



- Number of hot spots N_q can not be constrained, only $N_q \geq 2$
- Large N_q partially compensated by large Q_s fluctuations
  - Overlap of the N_q constituents along with Q_s fluctuations
  - Generates an "effective number of hot spots" $< N_q$
- Some constraints from low-t part of incoherent spectra: cross section $\sim$ density fluctuations H.M, B. Schenke, 1607.01711
- Minimum distance between hot spots allowed, not required (backup)

Connection to flow@LHC (Pb+Pb)



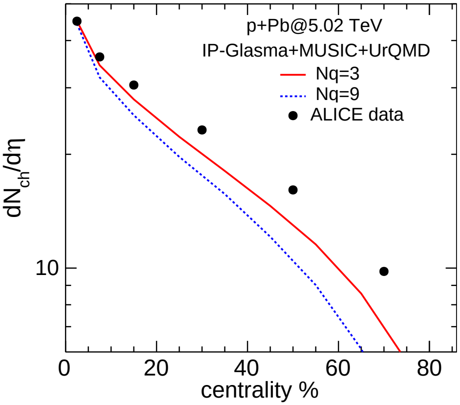


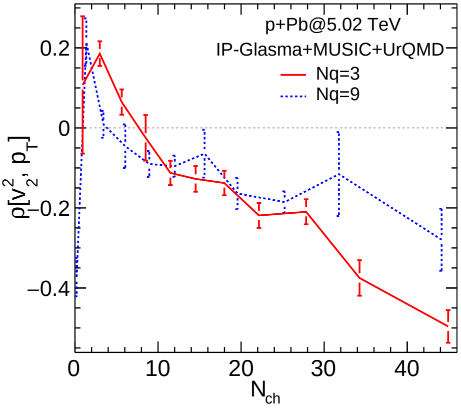
Goal

Include HERA +LHC Pb+Pb & p+Pb data into the Bayesian analysis
$\Rightarrow$ more constraints

- This talk:
  Sensitivity of different observables
- Maximum likelihood parametrizations for $N_q = 3$ and $N_q = 9$
- IP-Glasma + MUSIC + UrQMD simulation
- LHC data prefers $N_q = 3$
  - IR regulators differ by ~ 2
    $\Rightarrow$ different "lumpiness"
    $\Rightarrow$ effect on v_2

p+Pb collisions





- Centrality dependence again prefers $N_q = 3$
- Difficult to get Pb+Pb and p+Pb multiplicities simultaneously
- v_2, p_T correlator in p+Pb identified as a promising observable
- Uncertainty propagation required to quantify the effect of LHC data

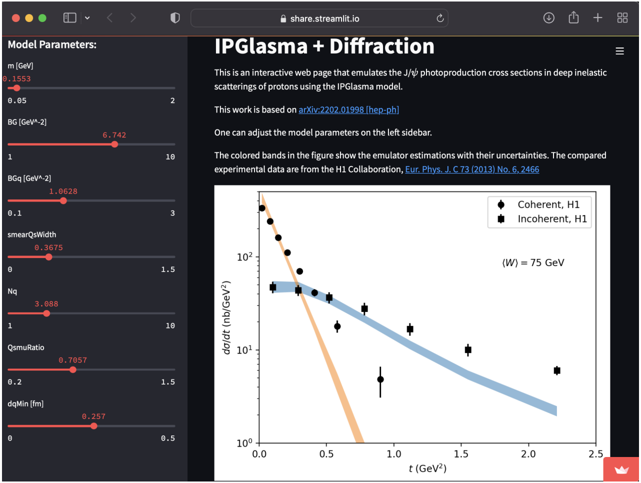
Conclusions

- First Bayesian analysis to extract proton shape fluctuations from diffractive J/ψ data
- Determined posterior distributions publicly available
  - Input to calculations of shape-dependent observables
  - Statistically robust uncertainty analysis possible
- Most model parameters well constrained
  - Except the number of hot spots and minimum distance between them
  - LHC data may provide further constraints
- Outlook
  - Energy dependence using JIMWLK
  - Global analysis of HERA + LHC flow data

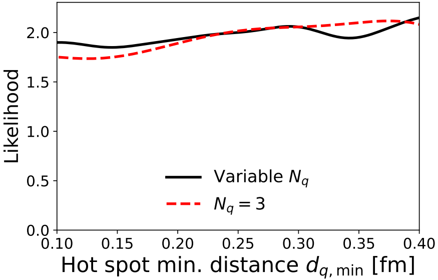
H.M, B. Schenke, C. Shen, W. Zhao, arXiv:2203.05846 [hep-ph]

Try it yourself!

https://share.streamlit.io/chunshen1987/ipglasmadiffractionstreamlit/main/IPGlasmaDiffraction_app.py



Minimum distance



Albacete, Petersen, Soto-Ontoso: repulsive correlations ($d_{q,\mathrm{min}} > 0$) required for v_n correlations in high-multiplicity pp

- HERA vector meson data: repulsive correlations allowed, not required

Best fit parameters

Table: Summary of model parameters, their prior ranges, and constrained maximum likelihood values with uncertainty estimates in 90% credible intervals.

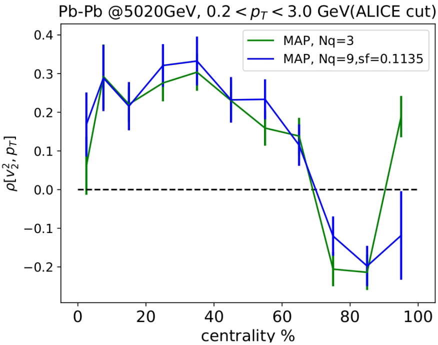
Parameter	Description	MAP	MAP ($N_q \equiv 3$)
m [GeV]	Infrared regulator	$0.506^{+1.12}_{-0.356}$	$0.246^{+0.162}_{-0.103}$
B_{qc} [GeV^{-2}]	Proton size	$4.02^{+1.73}_{-0.728}$	$4.45^{+0.801}_{-0.803}$
B_q [GeV^{-2}]	Hot spot size	$0.474^{+0.434}_{-0.286}$	$0.346^{+0.282}_{-0.202}$
σ	Magnitude of Q_s fluctuations	$0.833^{+0.194}_{-0.441}$	$0.563^{+0.143}_{-0.141}$
$Q_s/(g^2\mu)$	$Q_s \Rightarrow$ color charge density	$0.598^{+0.230}_{-0.264}$	$0.747^{+0.0704}_{-0.0930}$
$d_{q,\text{Min}}$ [fm]	Min hot spot dist	$0.257^{+0.221}_{-0.231}$	$0.254^{+0.222}_{-0.229}$
N_q	Number of hot spots	$6.79^{+2.93}_{-4.83}$	3

H.M, B. Schenke, C. Shen, W. Zhao arXiv:2203.05846 [hep-ph]

Parameters in flow analysis

Parameter	Description	$N_q = 9$	$N_q = 3$
m [GeV]	Infrared regulator	0.780	0.246
B_{qc} [GeV^{-2}]	Proton size	3.98	4.45
B_q [GeV^{-2}]	Hot spot size	0.594	0.346
σ	Magnitude of Q_s fluctuations	0.932	0.563
$Q_s/(g^2\mu)$	$Q_s \Rightarrow$ color charge density	0.492	0.747
$d_{q,\text{Min}}$ [fm]	Min hot spot distance	0.265	0.254
N_q	Number of hot spots	3	9

v_2, p_T correlator in Pb+Pb



- v_2, p_T correlator in Pb+Pb not really sensitive to substructure details