Non-equilibrium attractor in high temperature QCD plasma

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DA, Aleksi Kurkela, Michael Strickland, PRL.125, (2020)

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Big picture || Dynamical evolution of ultra-relativistic heavy ion collisions

- Reliable interpretation of data requires matching different theoretical models

\[
\sim 0^{-} \text{ fm/c} \quad \sim 0.1 \text{ fm/c} \quad \sim 1 \text{ fm/c} \quad \sim 10 \text{ fm/c} \quad \sim 20 \text{ fm/c}
\]

Original Figure: MADA1 collaboration, Hannah Petersen and Jonah Bernhard

- Initial state + preequilibrium
- Hydrodynamics
- Hadronization + Hadronic cascade

QCD Effective kinetic theory
Problem || Mismatch between theoretical models and degrees of freedom

▶ Out of equilibrium corrections are the main source of uncertainty

Schenke, Jeon, Gale PRL 106 (2010)

▶ Large gradients $\partial_\mu \varepsilon/\varepsilon \sim 1/fm$
▶ Momentum distribution $f(x,p)$

$E \frac{d^3 N_s}{d^3 p} = \frac{\nu_s}{(2\pi)^3} \int f_s(p)$

Schenke, Jeon, Gale PRL 106 (2010)

$\tau=0.4 \text{ fm}/c$

$\partial_\mu N^\mu = \partial_\mu \int p^\mu f = 0$
$\partial_\mu T^{\mu\nu} = \partial_\mu \int p^\mu p^\nu f = 0$

poster by: Christopher Plumberg
Applicability of fluid dynamics far from equilibrium

- Hydrodynamics as an expansion in $(K_n, R_n^{-1})$

\[ K_n = \frac{\text{microscopic scale}}{\text{macroscopic scale}} \leq 0.5 \]

\[ R_n^{-1} = \frac{\sqrt{\pi} \pi_{\mu\nu} \pi_{\mu\nu}}{P} \leq 0.5 \]

\[ \sqrt{s} = 2.76 \text{ TeV Pb+Pb} \]

Noronha-Hostler, Noronha, Gyulassy PRC 93(2016)

\[ \sqrt{s} = 5 \text{ TeV Pb+Pb} \]


- How do we construct a controlled hydrodynamical theory?
Tool || attractors: A way to think of out-of-equilibrium hydrodynamics

- Hydrodynamics as a universal attractor [Heller and Spalinski. PRL.115 (2015)]

- Competition between: expansion and relaxation rates.
- Confirmed for Bjorken flow
- Dictates the dominant physics

\[
W = \frac{\tau}{\tau_R} = \frac{\tau T}{4\pi \bar{\eta}} = K_n^{-1}
\]
QCD medium at high temperatures: Effective kinetic theory

\[ P^\mu \partial_\mu f_{q,g}(p) = -C[f(p)], \]

\[ f_{q,g} \propto \frac{dN_{g,q}}{d^3 x d^3 p}, \quad g,q(u,d,s,\bar{u},\bar{d},\bar{s}) \]

\[ \frac{d f_{q,g}(p)}{d\tau} - \frac{p_z}{\tau} \partial_{p_z} f_{q,g}(p) = -C_{2\leftrightarrow 2}[f_{q,g}(p)] - C_{1\leftrightarrow 2}[f_{q,g}(p)] \]

At LO, transport at different momentum scales in \( C[f] \)

regulated by HTL

LPM included

"Bottom up thermalisation"

General moments of the Boltzmann equation

Solving for moments of the Boltzmann equation ⇒ reconstruction of $f(x, p)$

- A general moment in $0 + 1d$ Bjorken flow

\[ M^{nm}[f] \equiv \int dP (p.u)^n (p.z)^{2m} f(x, p) \]

M. Strickland, JHEP2018, 128; 1809.01200.

- Equilibrium values for Bose distribution,

\[ M^{nm}_{\text{eq}} = \frac{T^{n+2m+2} \Gamma(n + 2m + 2) \zeta(n + 2m + 2)}{2\pi^2(2m + 1)} \]

- Momentum discretization method: 2D grid \{x_i, p_j\} with 250 × 2000 grid points (Kurkela and Zhu PRL 115, 182301 (2015))

- Low moments ⇒ hydrodynamic fields

\[ M^{10} = \text{number density} \]
\[ M^{20} = \text{energy density} \]
\[ M^{01} = \text{longitudinal pressure} \]

- Measure deviations from equilibrium

\[ \overline{M}^{nm}(\tau) \equiv \frac{M^{nm}(\tau)}{M^{nm}_{\text{eq}}(\tau)} \]
Initial distribution $-\frac{df_p}{d\tau} = C_{1\leftrightarrow 2}[f_p] + C_{2\leftrightarrow 2}[f_p] + C_{\text{exp}}[f_p]$.

**Thermal Romatschke-Strickland**

\[ f_{0,RS}(p) = f_{\text{Bose}}\left(\sqrt{p^2 + \xi_0 p_z^2}/\Lambda_0\right) \]

anisotropy parameter $(-1 < \xi_0 < \infty)$

$\Lambda_0$ is set by Landau matching


**Non-thermal CGC**

\[ f_{0,CGC}(p) = \frac{2A}{\lambda} \frac{\tilde{\Lambda}_0}{\sqrt{p^2 + \xi_0 p_z^2}} \exp^{-\frac{2}{3}(p^2+\xi_0 p_z^2)/\tilde{\Lambda}_0^2} \]

The initial scale $\tilde{\Lambda}_0$ is related to the saturation scale $\tilde{\Lambda}_0 = \langle p_T \rangle_0 \approx 1.8 Q_s$

$A$ is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation


Non-equilibrium QCD attractor at high temperature

- Forward attractor: late time asymptotics

Pressure anisotropy

\[ M^{01}[f] = \langle p^{-1}p_z^2 \rangle \]
\[ M^{21}[f] = \langle p^1p_z^2 \rangle \]
\[ M^{33}[f] = \langle p^2p_z^6 \rangle \]

Non-equilibrium evolution becomes insensitive to initial conditions at very early times
Non-equilibrium QCD attractor at high temperature

- Pullback attractor: early time asymptotics
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

"Pressure anisotropy"

\[ \mathcal{M}^{01}[f] = < p^{-1}p_z^2 > \]
\[ \mathcal{M}^{21}[f] = < p^1p_z^2 > \]
\[ \mathcal{M}^{33}[f] = < p^2p_z^6 > \]

An attractor for the momentum phase space distribution function
Preliminary: Universal Rescaling

DA, Kirill Boguslavski "forthcoming"
▶ Extended universal scaling prior to the hydrodynamics attractor

Pressure anisotropy

▶ Connecting the early, intermediate, and late dynamics via universal rescaling

Berges, Bogaslavski, Schlichting, and Venugopalan PRD 89 (2014) ; Berges, Bogaslavski, Schlichting, and Venugopalan JHEP (2014) ;
Preliminary: Universal Rescaling

DA, Kirill Boguslavski "forthcoming"

- Extended universal scaling prior to the hydrodynamics attractor

\[ \mathcal{M}_{01}^1[f] = \langle p^{-1}p_z^2 \rangle \]
\[ \mathcal{M}_{21}^1[f] = \langle p^1p_z^2 \rangle \]
\[ \mathcal{M}_{33}^3[f] = \langle p^2p_z^6 \rangle \]

- Connecting the early, intermediate, and late dynamics via universal rescaling

Berges, Bogaslavski, Schlichting, and Venugopalan PRD 89 (2014); Berges, Bogaslavski, Schlichting, and Venugopalan JHEP (2014)
Non-equilibrium effects at Freeze out
Uncertainty in the transition from hydrodynamics: particalization

\[ E \frac{d^3 N_s}{d^3 p} = \frac{\nu_s}{(2\pi)^3} \int_\sigma (f_s(\vec{p}) + \delta f) \]

(Dusling, Moore, Teaney PRC 81, (2008))

▶ Choice of ansatz improves the high \( p_T \) dependence

(Noronha-Hostler, Noronha, Grassi, PRC 90 (2014))

▶ high \( p_T \) flow is affected by \( \delta f \) corrections
Uncertainty in the transition from hydrodynamics: partialization

The **quadratic ansatz** \((\alpha = 0)\)

\[
\frac{\delta f(i)}{f_{eq}(1 + f_{eq})} = \frac{3\Pi}{16T^2}(p^2 - 3p_z^2)
\]

The **LPM ansatz** \((\alpha = 0.5)\)

\[
\frac{\delta f(ii)}{f_{eq}(1 + f_{eq})} = \frac{16\Pi}{21\sqrt{\pi} T^{3/2}} \left( \frac{p^{3/2}}{\sqrt{p}} - \frac{3p_z^2}{\sqrt{p}} \right)
\]

The **aHydro freeze-out ansatz**

\[
f(p) = f_{\text{Bose}}(\sqrt{p^2 + \xi p_z^2}/\Lambda)
\]

\[
\mathcal{M}_{\text{aHydro}}^{nm}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{\mathcal{H}^{nm}(\alpha)}{[\mathcal{H}^{20}(\alpha)](n+2m+2)/4}
\]
Insights into the freezeout prescription

- Disagreement increases for higher moments and for earlier times.
- Good agreement between aHydro ansatz and EKT at all times

\[ M_{11}[f] = \langle p_z^2 \rangle \]
\[ M_{21}[f] = \langle p_1 p_z^2 \rangle \]
\[ M_{22}[f] = \langle p_1^4 p_z^4 \rangle \]

<table>
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<th>$\tau/\tau_R$</th>
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<tr>
<td>10</td>
<td>38.5 fm/c</td>
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</tbody>
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For earlier implementation: (Pratt, Torrieri PRC 82(2010)) (Weller, Romatchke PLB 774 (2017))
Adding Quarks into the equation?
Future directions: Presence of quarks in initial state delays hydrodynamisation

Inclusion of quarks increases anisotropy

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced: fusion $gg \rightarrow q\bar{q}$ and splitting $g \rightarrow q\bar{q}$

Understanding equilibration in presence of quarks is essential for hydrodynamics at finite densities

"Attractors at finite densities" by Travis Dore
"Quark production in IS" by Patrick Carzon
Conclusions

- A non-equilibrium attractor for the phase space distribution of QCD at high temperature EKT
- scaling of the distribution function and its implications
- Ahydro distribution for further improvements in th FO
- Inclusion of quarks: chemical equilibration at high moments?
  (DA, A. Mazeliauskas, and M.Strickland, forthcoming)

- $\delta f$ corrections is species dependent, how does this affect final state?
Thank you for your attention!