Non-equilibrium attractor in high temperature QCD plasma

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DA, Aleksi Kurkela, Michael Strickland, PRL.125, (2020)

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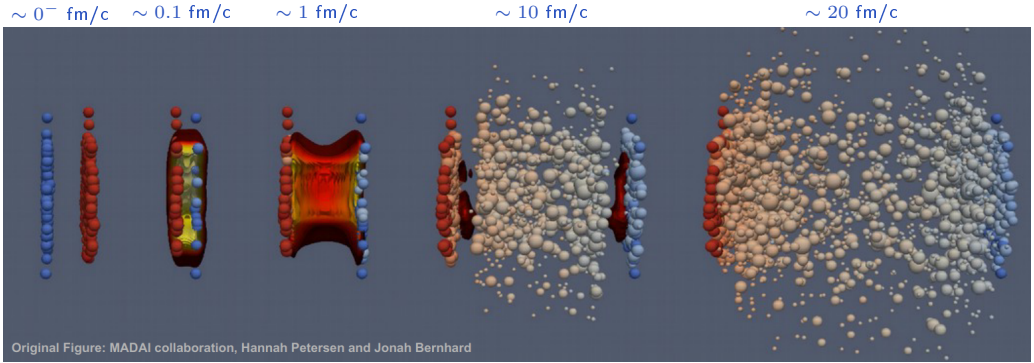
Big picture || Dynamical evolution of ultra-relativistic heavy ion collisions

- Reliable interpertation of data requires matching different theoretical models

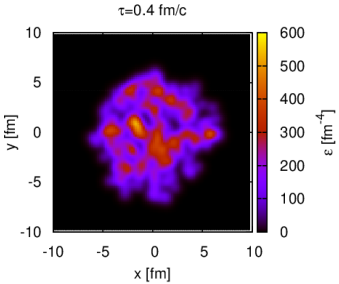


initial state+preequilibrium | Hydrodynamics | Hadronization+Hadronic cascade

QCD Effective kinetic theory

Problem|| Mismatch between theoretical models and degrees of freedom

- Out of equilibrium corrections are the main source of uncertainty



Schenke, Jeon, Gale PRL.106 (2010)

- Large gradients $\partial_\mu \varepsilon/\varepsilon \sim 1/fm$
- Momentum distribution f(x,p)

$$\partial_\mu N^\mu = \partial_\mu \int p^\mu f = 0$$

$$\partial_\mu T^{\mu\nu} = \partial_\mu \int p^\mu p^\nu f = 0$$

$$E\frac{d^3N_s}{d^3p} = \frac{\nu_s}{(2\pi)^3}\int_\sigma f_s(p)$$

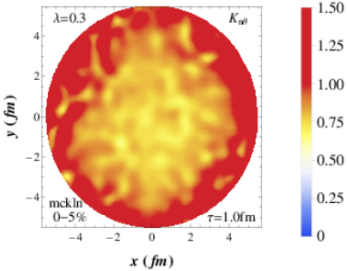
poster by: Christopher Plumberg

Applicability of fluid dynamics far from equilibrium

- Hydrodynamics as an expansion in $(\mathrm{Kn}, \mathrm{Rn}^{-1})$

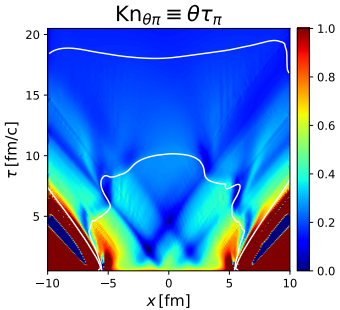
$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}} \leq 0.5$$

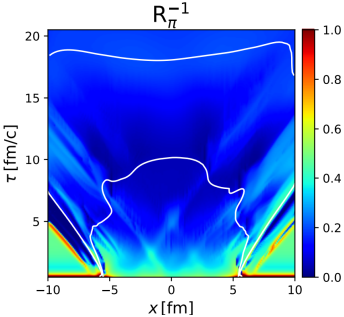
$$\mathrm{R}_\pi^{-1} = \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{P} \leq 0.5$$



$\sqrt{s} = 2.76$ TeV Pb+Pb

Noronha-Hostler, Noronha, Gyulassy PRC 93(2016)





$\sqrt{s} = 5$ TeV Pb+Pb

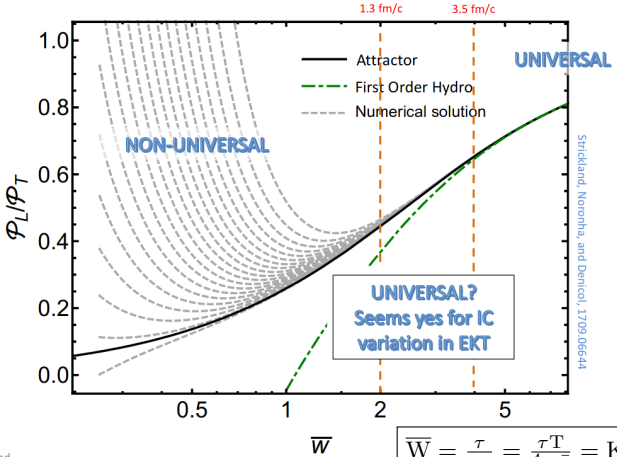
Bazow, Heinz, Strickland Comp. Phys. Comm. 225 (2018)

- How do we construct a controlled hydrodynamical theory?

Tool || attractors: A way to think of out-of-equilibrium hydrodynamics

- Hydrodynamics as a universal attractor Heller and Spalinski, PRL.115 (2015)

- Competition between: expansion and relaxation rates.
- Confirmed for Bjorken flow
- Dictates the dominant physics



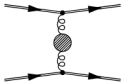
$$\overline{\mathrm{W}} = \frac{\tau}{\tau_R} = \frac{\tau \mathrm{T}}{4\pi\,\overline{\eta}} = \mathrm{K_n^{-1}}$$

QCD medium at high temperatures: Effective kinetic theory AMY JHEP0301 (2003) 030

$$P^\mu \partial_\mu f_{q,g}(\mathbf{p}) = -C[f(\mathbf{p})], \qquad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \qquad g, q(u, d, s, \bar{u}, \bar{d}, \bar{s}))$$

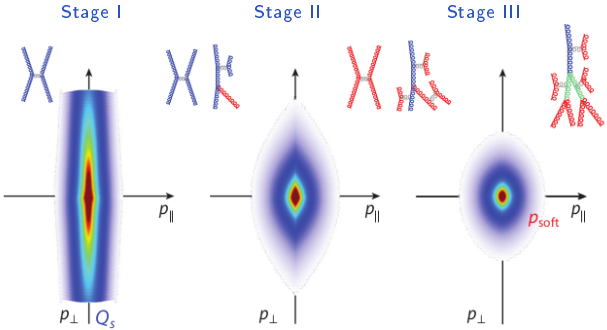
$$\frac{df_{q,g}(\mathbf{p})}{d\tau} - \frac{p_z}{\tau}\partial_{p_z} f_{q,g}(\mathbf{p}) = -\mathcal{C}_{\mathbf{2}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] - \mathcal{C}_{\mathbf{1}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] \qquad \underline{0+1\text{d Bjorken}}$$

At LO, transport at different momentum scales in C[f]



regulated by HTL

LPM included



"Bottom up thermalisation"

Baier, Mueller, Schiff, and Son (2001); J.Berges, M.Heller, A.Mazeliauskas and R.Venugopalan arxiv.2005.12299 (2020); Schlichting, Teaney, Ann. Rev. of Nuc Part. Sci.(2019); Arnold, P. Gorda, T. Iqbal, S. JHEP. 2020, 53

General moments of the Boltzmann equation

Solving for moments of the Boltzmann equation $\Rightarrow$ reconstruction of $f(x,p)$

- A general moment in $0+1\mathrm{d}$ Bjorken flow

$$\boxed{\mathcal{M}^{nm}[f] \equiv \int dP\,(p.u)^n\,(p.z)^{2m}\,f(x,p)}$$

M. Strickland, JHEP2018, 128; 1809.01200.

- Equilibrium values for Bose distribution,

$$\mathcal{M}^{nm}_{\mathrm{eq}} = \frac{T^{n+2m+2}\Gamma(n+2m+2)\zeta(n+2m+2)}{2\pi^2(2m+1)}$$

- Momentum discretization method: 2D grid $\{x_i, p_j\}$ with 250×2000 grid points (Kurkela and Zhu PRL 115, 182301 (2015))

- low moments $\Rightarrow$ hydrodynamic fields

$\mathcal{M}^{10} =$ number density

$\mathcal{M}^{20} =$ energy density

$\mathcal{M}^{01} =$ longitudinal pressure

- Measure deviations from equilibrium

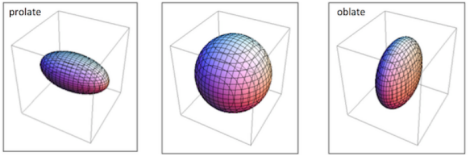
.

$$\boxed{\overline{\mathcal{M}}^{nm}(\tau) \equiv \frac{\mathcal{M}^{nm}(\tau)}{\mathcal{M}^{nm}_{\mathrm{eq}}(\tau)}}$$

Initial distribution $-\frac{d\mathbf{f_p}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{2\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{\rm exp}[\mathbf{f_p}].$

- Thermal Romatschke-Strickland

$$\mathrm{f}_{0,\mathrm{RS}}(\mathbf{p}) = f_{\mathrm{Bose}}\left(\sqrt{\mathbf{p}^2 + \xi_0 p_z^2}/\Lambda_0\right)$$



$-1 < \xi < 0$ $\xi = 0$ $\xi > 0$

anisotropy parameter $(-1 < \xi_0 < \infty)$
Λ_0 is set by Landau matching

Romatschke,Strickland,PRD68, (2003)

- Non-thermal CGC

$$\mathrm{f}_{0,\mathrm{CGC}}(\mathbf{p}) = \frac{2A}{\lambda}\frac{\tilde{\Lambda}_0}{\sqrt{\mathbf{p^2} + \xi_0 p_z^2}}\exp^{-\frac{2}{3}\left(\mathbf{p}^2 + \xi_0 \hat{p}_z^2\right)/\tilde{\Lambda}_0^2}$$

The initial scale $\tilde{\Lambda}_0$ is related to the saturation scale $\tilde{\Lambda}_0 = \langle \mathrm{p_T}\rangle_0 \approx 1.8\,\mathrm{Q_s}$
A is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation

A. Kurkela and Y. Zhu, Phys. Rev. Lett.115, 182301(2015)

T. Lappi, Phys. Lett.B703, 325-330 (2011)

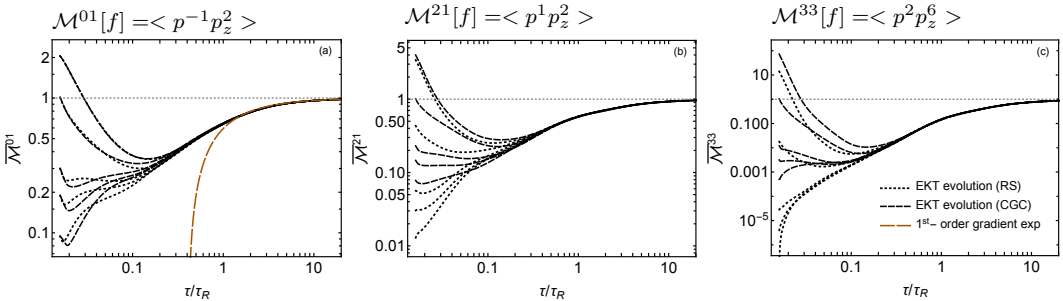
Non-equilibrium QCD attractor at high temperature

$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Forward attractor: late time asymptotics

Pressure anisotropy



Non-equilibrium evolution becomes insensitive to initial conditions at very early times

Non-equilibrium QCD attractor at high temperature

- Pullback attractor: early time asymptotics
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

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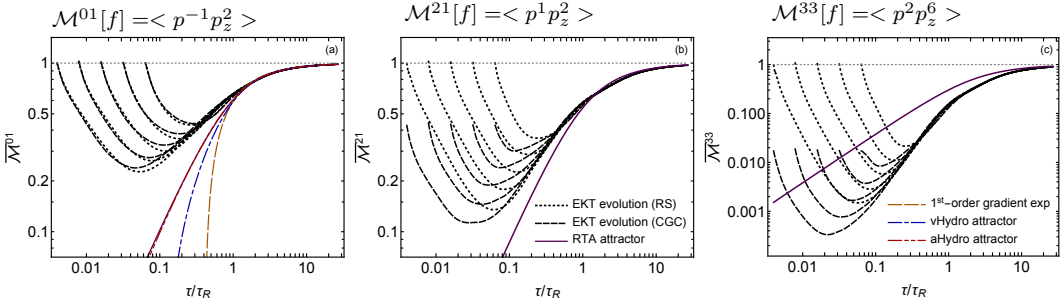
"Pressure anisotropy"

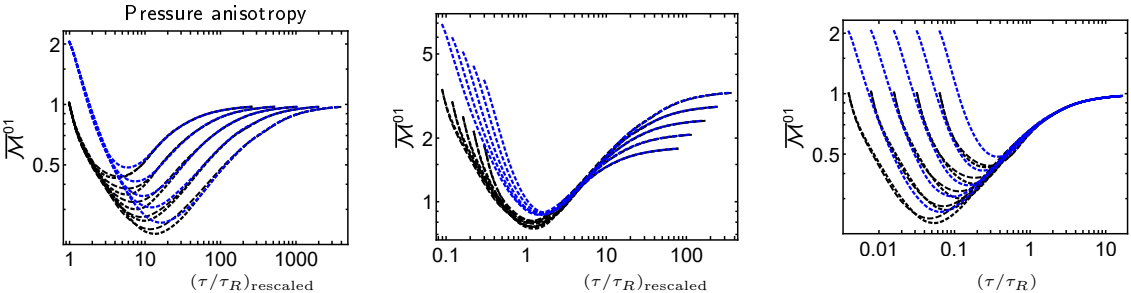


An attractor for the momentum phase space distribution function

Preliminary: Universal Rescaling

DA, Kirill Boguslavski "forthcoming"

- Extended universal scaling prior to the hydrodynamics attractor



- Connecting the early, intermediate, and late dynamics via universal rescaling

Berges, Bogaslavski, Schlichting, and Venugopalan PRD 89 (2014) ; Berges, Bogaslavski, Schlichting, and Venugopalan JHEP (2014) ;

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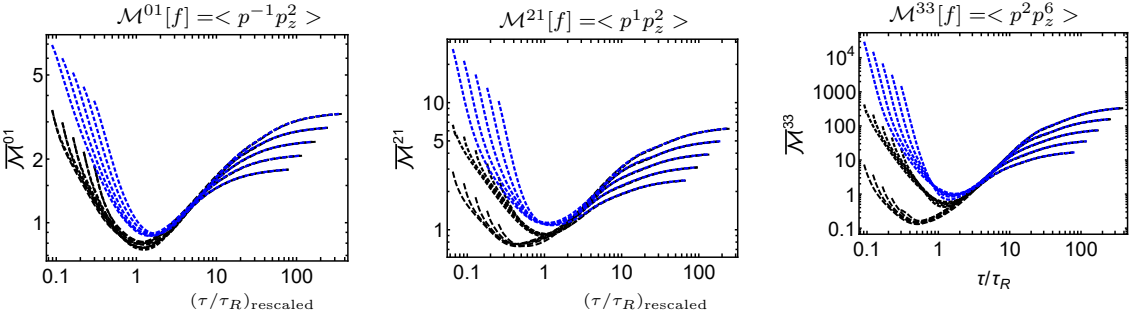

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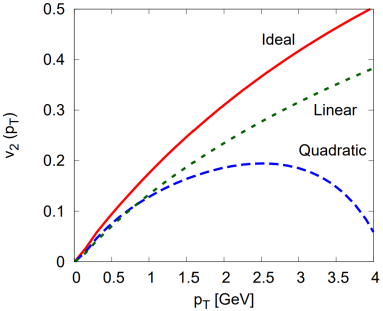
Non-equilibrium effects at Freeze out

Uncertainty in the transition from hydrodynamics: particalization

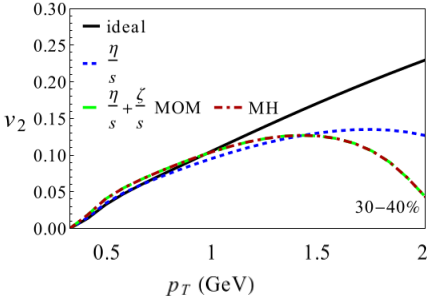
$$E\frac{d^3N_s}{d^3p} = \frac{\nu_s}{(2\pi)^3}\int_\sigma (f_s(\tilde{p}) + \delta f)$$

(Dusling,Moore,Teaney PRC 81, (2008))



- Choice of ansatz improves the high p_T dependence

(Noronha-Hostler, Noronha,Grassi, PRC 90 (2014)



- high p_T flow is affected by δf corrections

Uncertainty in the transition from hydrodynamics: particalization

$N^\mu, T^{\mu\nu}$

The *quadratic ansatz* ($\alpha = 0$)

$$\frac{\delta f_{(i)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{3\overline{\Pi}}{16T^2}(p^2 - 3p_z^2)$$

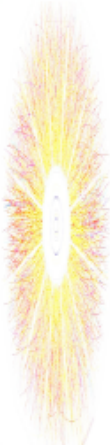
The *LPM ansatz* ($\alpha = 0.5$)

$$\frac{\delta f_{(ii)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{16\overline{\Pi}}{21\sqrt{\pi}\,T^{3/2}}\left(p^{3/2} - \frac{3p_z^2}{\sqrt{p}}\right)$$

The *aHydro freeze-out ansatz*

$$f(p) = f_{\rm Bose}(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda)$$

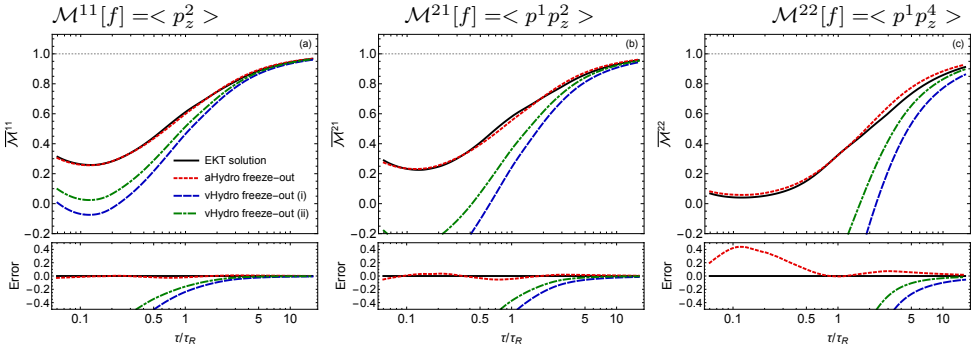
$$\overline{\mathcal{M}}^{nm}_{\rm aHydro}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{\mathcal{H}^{nm}(\alpha)}{[\mathcal{H}^{20}(\alpha)]^{(n+2m+2)/4}}$$



Insights into the freezeout perscription

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Disagreement increases for higher moments and for earlier times.
- Good agreement between aHydro ansatz and EKT at all times



For earlier implementation: (Pratt,Torrieri PRC 82(2010) (Weller, Romatchke PLB 774 (2017)

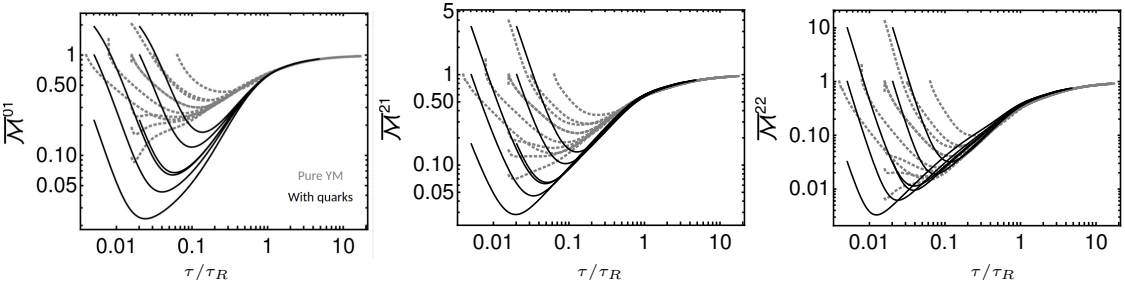
Adding Quarks into the equation?

Future directions: Presence of quarks in initial state delays hydrodynamisation

DA, Mazeliauskas, Strickland. forthcoming

Inclusion of quarks increases anisotropy

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced: fusion $\mathrm{gg} \to \mathrm{q\bar{q}}$ and splitting $\mathrm{g} \to \mathrm{q\bar{q}}$



Understanding equilibration in presence of quarks is essential for hydrodynamics at finite densities

"Attractors at finite densities" by Travis Dore

"Quark production in IS" by Patrick Carzon

Conclusions

- A non-equilibrium attractor for the phase space distribution of QCD at high temperature EKT
- scaling of the distribution function and its implications
- Ahydro distribution for further improvements in th FO
- Inclusion of quarks: chemical equilibration at high moments ?
  (DA, A. Mazeliauskas, and M.Strickland, forthcoming)
- δf corrections is species dependent, how does this affect final state?

Thank you for your attention!